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Adaptive Sliding Mode Fault-Tolerant Control for a Class of Uncertain Systems With Probabilistic Random Delays

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ABSTRACT This paper investigates the robust control problem of uncertain nonlinear systems subject to actuator faults via the integral sliding mode control (ISMC) scheme. The actuator failures contain the cases of an outage, loss of effectiveness, and stuck fault, which are frequently encountered in practical applications. Furthermore, in this paper, the time-varying delay considered obeys the probability distribution within certain intervals. By fully considering the information of the probabilistic random delay, new sufficient conditions are given to ensure the asymptotical stability of the controlled system with prescribed H_∞ performance. Moreover, a new control law is proposed to compensate for the effects induced by the actuator faults, and an adaptive mechanism is adapted to estimate the unknown terms. In a word, the proposed ISMC scheme ensures the asymptotical stability with guaranteed H_∞ performance of the closed-loop system and forces that the state trajectories are attracted to the pre-designed sliding surface. Finally, a practical example of the rocket fairing structural acoustic model is used to clarify the validly of the presented control method.

INDEX TERMS Networked system, integral sliding mode control, actuator faults, adaptive mechanism, probabilistic random delays.

I. INTRODUCTION

It is well known that systems would undergo the malfunction when the actuators/sensors suffer from the wear or damage. Then, an effective strategy for fault diagnosis or compensating the effect of faults onto the whole performance is urgently required [1], [2]. Accordingly, considerable contributions have been devoted to the synthesis problem for controlled systems with fault signals [3], [4]. Among them, the fault-tolerant control (FTC) plays a special role in practical systems with crucial safe such as passenger aircraft and modern fighter aircraft, due to its super advantages to compensate the faults by using the configured system redundancy. When the fault-tolerant control scheme is utilized, the systems can still keep stability with an acceptable performance when suffering from actuator faults. Accordingly, a great number of results have been reported to handle the actuator/sensor faults by utilizing robust mechanism [5], [6], observer-based method [7], [8] and adaptive technique [9]. To mention a few, several approaches have been proposed in [10], [11] concerning on the control systems by using the fault-tolerant control design scheme. In [12], a velocity-free controller has been considered for a set of nonlinear systems with external disturbance and multiple actuator faults. In [10], [13], the actuator failures including the cases of outage, loss of effectiveness and stuck fault have been addressed for nonlinear systems based on fuzzy logic scheme. Besides, the approach developed in [14] only addresses the actuator degradation without the consideration of stuck fault.

In the past several decades, the sliding mode control (SMC) has been successfully applied to a variety of engineering domains [15]–[18]. By using this method, if the state

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trajectories are attracted to the sliding surface, the robustness properties against matched disturbance or other features can be obtained [19]–[23]. Due to the superior features of SMC, the SMC methods has been widely used to stabilize the fault-tolerant systems [24], [25]. To mention a few, a novel optimal sliding mode control has been presented in [26] for uncertain linear time-invariant system, where two fault-tolerant control schemes have been described. In [7], an observer-based actuator fault detection and isolation approach has been given for a class of nonlinear systems with faults, where the idea of equivalent output error injection has been utilized for the reconstruction of the fault signal. Moreover, in [10], an integral sliding-mode-based fault-tolerant controller has been designed for uncertain nonlinear system with degradation and stuck. Recently, the analysis of time-varying actuator failures in uncertain systems is appealing [8], [25], thus additional effort is desirable to be made on the developments of efficient control strategies for time-varying actuator failures, which consists of one motivations of the present work.

On the other hand, the time-delays are commonly inevitable during the modeling of realistic problems, which could cause instability, oscillation and chaotic attractors [9], [27]. To clearly discuss the effect of delays, the information of variation range or variation rate of delay has been employed in [5], [28]–[30]. In the real systems, the value of time delay may be large suddenly although this situation does not occur often with small probability. As mentioned in [31], this case might encounter in the wireless network, where the probabilistic distribution delays have been modeled and discussed. Thus, the traditional method for delayed systems [23] relying on the variation rate of delay becomes invalid here since the suddenly changes of delay. Accordingly, new technique for system with probabilistic delay is urgently required. In fact, the main challenging issue is on how to take the probabilistic characteristics of delay into consideration. Owing to these issues, the researchers have paid much attention to study the stability problems of dynamic systems with probabilistic delays. For example, in [32], some sufficient conditions have been established to ensure the exponential stability of impulsive systems subject to random delays via the reciprocal convex technique. Furthermore, in [31], both the information of variant range of time delay and the probabilistic characteristic of the time-varying delay have been fully taken into account, and a new delay-dependent stability criterion has been given. To the best of authors' knowledge, the H_∞ sliding mode control problem has not gained adequate consideration for uncertain systems with probabilistic random delays and actuator fault due probably to the complexities/difficulties with respect to the probabilistic random delays, not to mention the case that the involved bounds might be unknown, which constitutes another motivation of this paper.

Based on the above analysis, we aim to handle the adaptive H_∞ sliding mode control problem for uncertain systems with probabilistic random delays and time-varying actuator faults. New sufficient criteria are given to ensure the asymptotical

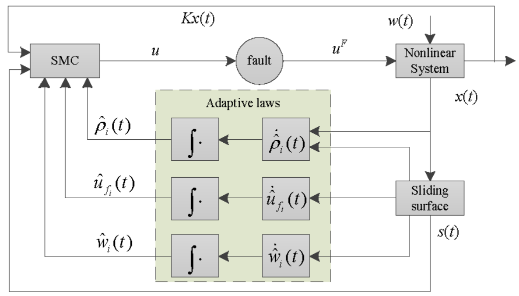

FIGURE 1. The schematic diagram of adaptive sliding mode FTC.

stability with guaranteed H_∞ performance of the sliding mode dynamics. Besides, a new control law is synthesized to ensure the reachability, where an adaptive mechanism is employed to estimate the involved unknown terms. The main advantages of this paper can be highlighted as follows: 1) The considered system has a general form, which covers probabilistic random delays, actuator faults and external disturbance in a same framework. Moreover, the actuator fault contains the cases of outage and stuck fault. 2) New sufficient conditions to guarantee the stability with satisfactory H_∞ performance are presented by fully taking the information of probability of random delays into account, and an algorithm is developed to calculate the gain matrix. 3) An integral sliding mode fault tolerant scheme is presented to cope with the effects of actuator fault and the sliding mode controller ensuring the reachability is constructed.

Notations: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ depict the $n-$dimensional Euclidean space and set of $n \times m$ real matrices, respectively. The symmetric positive definite (semi-definite) matrix P is represented by $P > 0$ ($P \geq 0$). $\|\cdot\|$ is the Euclidean vector norm or its induced norm of a matrix. $\mathbb{E}\{\star\}$ denotes the mathematical expectation of random variable $\star$. The symbol $*$ in symmetric matrix characterizes the term induced by symmetry.

II. PROBLEM FORMULATION AND PRELIMINARIES

The schematic diagram of the presented sliding mode FTC method for nonlinear system is depicted in Fig. 1.

Consider the following class of uncertain systems with time-varying delays and actuator fault:

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A)x(t) + A_\tau x(t-\tau(t)) + Bu^F(t) + D_1 w(t) \\ z(t) &= Cx(t) + A_{\tau 1}x(t-\tau(t)) + D_2 w(t) \\ x(t) &= \phi(t), \quad t \in [-\tau_2, 0] \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state, $z(t)$ denotes the controlled output, $w(t) \in \mathbb{R}^q$ represents the external disturbance in $\mathcal{L}_2[0,\infty)$, $\phi(t)$ are initial conditions. The uncertain term ΔA is of the form $\Delta A = EFH$, where F satisfies $F^T F \leq I$. Moreover, the matrices A, A_τ, $A_{\tau 1}$, B, C, D_1, D_2, E and H are all known matrices and have corresponding suitable dimensions.

The control input vector $u^F(t) \in \mathbb{R}^m$ adopts the following actuator fault model

$$u^F(t) = \rho u(t) + \varsigma u_f(t) \tag{2}$$

where ρ is unknown time-varying diagonal matrix representing the actuator efficiency factor. Specially, ρ and ς satisfy $\rho \in \{\rho^1, \rho^2, \cdots, \rho^L\}$, $\varsigma \in \{\varsigma^1, \varsigma^2, \cdots, \varsigma^L\}$ with L being the number of total faulty modes and $\rho^j \in \text{diag}\{\rho_1^j, \rho_2^j, \cdots, \rho_m^j\}$, $\varsigma^j \in \text{diag}\{\varsigma_1^j, \varsigma_2^j, \cdots, \varsigma_m^j\}$ with $\rho_i^j \in [\underline{\rho}_i, \bar{\rho}_i]$, $\varsigma_i^j = 0$ or 1. Moreover, $u_f(t) = \begin{bmatrix} u_{f,1}(t) & u_{f,2}(t) & \cdots & u_{f,m}(t)\end{bmatrix}^T \in \mathbb{R}^m$ is given to denote the unknown stuck actuator fault with unknown bound. Accordingly, we set $\underline{\rho} = \text{diag}\{\underline{\rho}_1, \underline{\rho}_2, \cdots, \underline{\rho}_m\}$ and $\bar{\rho} = \text{diag}\{\bar{\rho}_1, \bar{\rho}_2, \cdots, \bar{\rho}_m\}$. From a practical viewpoint, this paper considers the following actuator failure cases:

$$\begin{cases} \rho_i^j = 1, \varsigma_i^j = 0, & \text{the } i\text{th actuator is normal;} \\ \rho_i^j = 0, \varsigma_i^j = 0, & \text{the } i\text{th actuator is outage;} \\ \rho_i^j \in (0,1), \varsigma_i^j = 0, & \text{the } i\text{th actuator is loss of effectiveness;} \\ \rho_i^j = 0, \varsigma_i^j = 1, & \text{the } i\text{th actuator is undergoes stuck fault.} \end{cases}$$

The time-varying delay $\tau(t)$ belongs to $[0, \tau_2]$ and there exists a constant real scalar τ_1 $(0 \le \tau_1 < \tau_2)$ such that either $\tau(t) \in [0, \tau_1]$ or $\tau(t) \in (\tau_1, \tau_2]$. Next, introduce the Bernoulli sequence $\alpha(t)$ depicting the situations of $\tau(t)$ taking values in $[0, \tau_1]$ or $(\tau_1, \tau_2]$ with the following probability

$$\text{Prob}\{\alpha(t) = 1\} = \mathbb{E}\{\alpha(t)\} = \bar{\alpha}$$
$$\text{Prob}\{\alpha(t) = 0\} = 1 - \mathbb{E}\{\alpha(t)\} = 1 - \bar{\alpha}$$

where $\bar{\alpha} \in [0, 1]$ denotes the probability of $\tau(t)$ taking values in $[0, \tau_1]$.

To describe the probability distribution of the time-varying delay, introduce two sets

$$\mathscr{F}_1 = \tau(t) \in [0, \tau_1],$$
$$\mathscr{F}_2 = \tau(t) \in (\tau_1, \tau_2],$$

and two functions $\tau_1(t) : \mathbb{R} \to [0, \tau_1]$ and $\tau_2(t) : \mathbb{R} \to [\tau_1, \tau_2]$ such that

$$\tau_1(t) = \begin{cases} \tau(t), & \text{if } \alpha(t) = 1 \\ \tau_1, & \text{if } \alpha(t) = 0 \end{cases}$$
$$\tau_2(t) = \begin{cases} \tau_1, & \text{if } \alpha(t) = 1 \\ \tau(t), & \text{if } \alpha(t) = 0. \end{cases}$$

It is obvious that $\alpha(t) = 1$ means that the event $\mathscr{F}_1$ occurs and $\alpha(t) = 0$ means that the event $\mathscr{F}_2$ occurs. Hence, the above system (1) can be equivalently written as

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A)x(t) + \alpha(t)A_\tau x(t - \tau_1(t)) \\ &\quad + (1 - \alpha(t))A_\tau x(t - \tau_2(t)) + Bu^F(t) + D_1 w(t) \\ &= (A + \Delta A)x(t) + \bar{\alpha}A_\tau x(t - \tau_1(t)) + Bu^F(t) \\ &\quad + (1 - \bar{\alpha})A_\tau x(t - \tau_2(t)) + D_1 w(t) \\ &\quad + (\alpha(t) - \bar{\alpha})\left(A_\tau x(t - \tau_1(t)) - A_\tau x(t - \tau_2(t))\right), \end{aligned}$$

$$\begin{aligned} z(t) &= Cx(t) + \bar{\alpha}A_{\tau 1}x(t - \tau_1(t)) \\ &\quad + (1 - \bar{\alpha})A_{\tau 1}x(t - \tau_2(t)) + D_2 w(t) \\ &\quad + (\alpha(t) - \bar{\alpha})A_{\tau 1}(x(t - \tau_1(t)) - x(t - \tau_2(t))) \end{aligned} \tag{3}$$

The purpose of this paper is to design a robust sliding mode controller such that, in the simultaneous presence of the probabilistic random delays and actuator fault, the state trajectories of the addressed systems are driven onto the specified sliding surface in mean square sense and the stability of sliding motion with specified H_∞ performance is guaranteed.

To end this section, the following lemmas are presented which will be used in the subsequent analysis.

Lemma 1 [31]: For a given scalar $h \in [0, 1]$, constant matrices X_1 and X_2 with same dimensions, the inequality $qX_1 + (1 - q)X_2 < 0$ holds for any $q \in [h, 1]$ if and only if $X_1 < 0$ and $hX_1 + (1 - h)X_2 < 0$.

Lemma 2 [31]: For the probabilistic random delays $\tau_i(t)$ $(i = 1, 2)$, constant matrices Φ_i, Π_i $(i = 1, 2)$, and $\Psi = \Psi^T$ with appropriate dimensions, if the following inequalities hold:

$$\tau_1\Phi_1 + (\tau_2 - \tau_1)\Pi_1 + \Psi < 0,$$
$$\tau_1\Phi_1 + (\tau_2 - \tau_1)\Pi_2 + \Psi < 0,$$
$$\tau_1\Phi_2 + (\tau_2 - \tau_1)\Pi_1 + \Psi < 0,$$
$$\tau_1\Phi_2 + (\tau_2 - \tau_1)\Pi_2 + \Psi < 0,$$

then one has

$$\tau_1(t)\Phi_1 + (\tau_1 - \tau_1(t))\Phi_2 + (\tau_2(t) - \tau_1)\Pi_1 + (\tau_2 - \tau_2(t))\Pi_2 + \Psi < 0.$$

Lemma 3 [15]: Let E, F and H be real matrices of compatible dimensions with F satisfying $F^T F \le I$. Then, for matrix $Q = Q^T$, we have $Q + EFH + H^T F^T E^T < 0$ if and only if there exists a scalar $\varepsilon > 0$ such that $Q + \varepsilon EE^T + \varepsilon^{-1}H^T H < 0$.

Lemma 4 [15]: For given matrices X, Y, vectors x, y with suitable dimensions and positive scalar ε, the following inequality

$$2xX^T Yy \le \varepsilon x^T X^T Xx + \varepsilon^{-1}y^T Y^T Yy$$

is satisfied.

III. SLIDING MODE CONTROLLER DESIGN

In this section, an integral sliding surface is firstly constructed and the stochastically stability with specified H_∞ performance of the sliding mode dynamic is guaranteed by proposing a new sufficient condition. What's more, the feedback gain matrix is obtained by solving the sub-optimization problem. Finally, the integral SMC (ISMC) law is synthesized to guarantee that the state trajectories are attracted to the sliding surface and remain there thereafter.

Firstly, factorize the matrix B as $B = B_1 B_2$ where $B_1 \in \mathbb{R}^{n\times l}$ and $B_2 \in \mathbb{R}^{l\times m}$ both have rank $l \le m$. To proceed, the following assumptions are provided.

Assumption 1: For any efficiency vector $\rho \in \{\rho^1, \rho^2, \cdots, \rho^L\}$, $\text{rank}(B_2\rho) = \text{rank}(B_2)$ and (A, B) is completely controllable.

Assumption 2: The actuator stuck fault $u_f(t)$ and disturbance $w(t)$ are both bounded, i.e. there exist unknown positive constants $\bar{u}_{fi}$ and $\bar{w}_i$ such that $\|u_{fi}\| \le \bar{u}_{fi}$ and $\|w_i\| \le \bar{w}_i$, respectively.

It is worthwhile to note that Assumption 1 is quite natural and common in FTC systems and has been used in [10], [13]. In what follows, we consider the variable controller of the form $u(t) = Kx(t) + u_1(t)$ with the explicit form of feed-back gain matrix K being provided later.

A. SLIDING SURFACE DESIGN

In this paper, we consider the integral switching surface defined by

$$s(x(t)) = Gx(t) - Gx(0) - G\int_0^t [Ax(\xi) + BKx(\xi)]\,d\xi$$

where G is a design matrix satisfying $GB_1 = I$. Introducing the free weighting matrix L, a general solution of G is given by $G = B_1^+ + L(I - B_1B_1^+)$ with $B_1^+ = (B_1^TB_1)^{-1}B_1^T$.

Once the state trajectories enter into the sliding surface, the condition $\dot{s}(x(t)) = 0$ is satisfied, i.e.

$$\begin{aligned}\dot{s}(x(t)) &= G\Delta Ax(t) + GB(\rho - I)Kx(t) + GB(\rho u_1(t) \\ &\quad + \varsigma u_f(t)) + GD_1w(t) + \bar{\alpha}GA_\tau x(t - \tau_1(t)) \\ &\quad + (\alpha(t) - \bar{\alpha})(GA_\tau x(t - \tau_1(t)) - GA_\tau x(t - \tau_2(t))) \\ &\quad + (1 - \bar{\alpha})GA_\tau x(t - \tau_2(t)) = 0. \qquad (4)\end{aligned}$$

Then, we can derive the equivalent control law $u_{1,eq}(t)$ as

$$\begin{aligned}u_{1,eq}(t) &= -(B_2\rho)^+G\Delta Ax(t) - (B_2\rho)^+GB(\rho - I)Kx(t) \\ &\quad - (B_2\rho)^+GB\varsigma u_f(t) - (B_2\rho)^+GD_1w(t) \\ &\quad - \bar{\alpha}(B_2\rho)^+GA_\tau x(t - \tau_1(t)) \\ &\quad - (1 - \bar{\alpha})(B_2\rho)^+GA_\tau x(t - \tau_2(t)) \qquad (5)\end{aligned}$$

where $(B_2\rho)^+$ is the Moore-Penorse inverse of $B_2\rho$.

Substituting (5) into (3), one gets the sliding mode dynamics as follows:

$$\begin{aligned}\dot{x}(t) &= [A + BK + (I - B_1G)\Delta A]\,x(t) + (I - B_1G) \\ &\quad \times D_1w(t) + \bar{\alpha}(I - B_1G)A_\tau x(t - \tau_1(t)) \\ &\quad + (1 - \bar{\alpha})(I - B_1G)A_\tau x(t - \tau_2(t)) \\ &\quad + (\alpha(t) - \bar{\alpha})\left(A_\tau x(t - \tau_1(t)) - A_\tau x(t - \tau_2(t))\right).\end{aligned}$$

Set $\mathscr{A} = A + BK$, $\bar{A}_\tau = (I - B_1G)A_\tau$, $E_1 = (I - B_1G)E$ and $\bar{D}_1 = (I - B_1G)D_1$. The sliding mode dynamics can be rewritten as:

$$\begin{aligned}\dot{x}(t) &= [\mathscr{A} + E_1FH]\,x(t) + \bar{\alpha}\bar{A}_\tau x(t - \tau_1(t)) \\ &\quad + (1 - \bar{\alpha})\bar{A}_\tau x(t - \tau_2(t)) + \bar{D}_1w(t) \\ &\quad + (\alpha(t) - \bar{\alpha})\,[A_\tau x(t - \tau_1(t)) - A_\tau x(t - \tau_2(t))]. \qquad (6)\end{aligned}$$

By defining

$$\begin{aligned}y(t) &= [\mathscr{A} + E_1FH]\,x(t) + \bar{\alpha}\bar{A}_\tau x(t - \tau_1(t)) \\ &\quad + (1 - \bar{\alpha})\bar{A}_\tau x(t - \tau_2(t)) + \bar{D}_1w(t),\end{aligned}$$

$$\begin{aligned}M &= [0 \quad A_\tau \quad 0 \quad -A_\tau \quad 0 \quad 0 \quad 0], \\ \xi^T(t) &= \big[x^T(t) \quad x^T(t - \tau_1(t)) \quad x^T(t - \tau_1) \\ &\qquad x^T(t - \tau_2(t)) \quad x^T(t - \tau_2) \quad w^T(t) \quad y^T(t)\big],\end{aligned}$$

then (6) can be expressed as

$$\dot{x}(t) = y(t) + (\alpha(t) - \bar{\alpha})M\xi(t). \qquad (7)$$

It is noted that time delays in (7) are probability dependent, hence the stability analysis should incorporate the information of the occurrence probability. Moreover, when $w(t) = 0$, the closed-loop system (7) is described by

$$\dot{x}(t) = y(t) + (\alpha(t) - \bar{\alpha})\bar{M}\zeta(t) \qquad (8)$$

with

$$\begin{aligned}\bar{M} &= [0 \quad A_\tau \quad 0 \quad -A_\tau \quad 0 \quad 0], \\ \zeta^T(t) &= \big[x^T(t) \quad x^T(t - \tau_1(t)) \quad x^T(t - \tau_1) \\ &\qquad x^T(t - \tau_2(t)) \quad x^T(t - \tau_2)qy^T(t)\big].\end{aligned}$$

Theorem 1: Consider the closed-loop control system (8) without external disturbance, i.e. $w(t) = 0$. For given constants τ_1, τ_2 and matrix K, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $T_1 > 0$, $T_2 > 0$ and $\bar{N}_j$ $(j = 1, 2, \cdots, 5)$ of suitable dimensions satisfying

$$\Omega = \begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13}^i \\ * & \Omega_{22} & 0 \\ * & * & \Omega_{33}\end{bmatrix} < 0, \quad (i = 1, 2, 3, 4) \qquad (9)$$

where

$$\begin{aligned}
\Omega_{11} &= \left[\Omega_{11}^{ij}\right]_{6\times 6}, \\
\bar{N}_i &= \left[\bar{N}_{i1}^T \quad \bar{N}_{i2}^T \quad \cdots \quad \bar{N}_{i6}^T\right]^T, \\
\bar{N}_5 &= \left[\bar{N}_{51}^T \quad \bar{N}_{52}^T \quad \bar{N}_{53}^T \quad \bar{N}_{54}^T\right]^T, \\
\Omega_{12} &= \left[\sqrt{\sigma\tau_1}\bar{M}^T \quad \sqrt{\sigma(\tau_2 - \tau_1)}\bar{M}^T\right], \\
\Omega_{22} &= \text{diag}\{-T_1^{-1}, -T_2^{-1}\}, \\
\Omega_{13}^1 &= \left[\tau_1\bar{N}_2 \quad \tau_1\bar{N}_2 \quad \tau_{2-1}\bar{N}_4 \quad \tau_{2-1}\bar{N}_4\right], \\
\Omega_{13}^2 &= \left[\tau_1\bar{N}_1 \quad \tau_1\bar{N}_1 \quad \tau_{2-1}\bar{N}_4 \quad \tau_{2-1}\bar{N}_4\right], \\
\Omega_{13}^3 &= \left[\tau_1\bar{N}_2 \quad \tau_1\bar{N}_2 \quad \tau_{2-1}\bar{N}_3 \quad \tau_{2-1}\bar{N}_3\right], \\
\Omega_{13}^4 &= \left[\tau_1\bar{N}_1 \quad \tau_1\bar{N}_1 \quad \tau_{2-1}\bar{N}_3 \quad \tau_{2-1}\bar{N}_3\right], \\
\Omega_{33} &= \text{diag}\{-\tau_1R_1, -\tau_1T_1, -\tau_{2-1}R_2, -\tau_{2-1}T_2\}, \\
\Omega_{11}^{11} &= \bar{N}_{11} + \bar{N}_{11}^T + \bar{N}_{51}\mathscr{A} + \mathscr{A}^TN_{51}^T + Q_1, \\
\Omega_{11}^{12} &= \bar{N}_{12}^T - \bar{N}_{11} + \bar{N}_{21} + \bar{\alpha}\bar{N}_{51}\bar{A}_\tau + \mathscr{A}^TN_{52}^T, \\
\Omega_{11}^{13} &= \bar{N}_{13}^T - \bar{N}_{21} + \bar{N}_{31}, \\
\Omega_{11}^{14} &= \bar{N}_{14}^T - \bar{N}_{31} + \bar{N}_{41} + \mathscr{A}^TN_{53}^T + \beta\bar{N}_{51}\bar{A}_\tau, \\
\Omega_{11}^{15} &= \bar{N}_{15}^T - \bar{N}_{41}, \\
\Omega_{11}^{16} &= P + \bar{N}_{16}^T - \bar{N}_{51} + \mathscr{A}^TN_{54}^T, \\
\Omega_{11}^{22} &= -\bar{N}_{12}^T - \bar{N}_{12} + \bar{N}_{22} + \bar{N}_{22}^T \\
&\quad + \bar{\alpha}(\bar{N}_{52}\bar{A}_\tau + \bar{A}_\tau^TN_{52}^T), \\
\Omega_{11}^{23} &= -\bar{N}_{13}^T + \bar{N}_{23}^T - \bar{N}_{22} + \bar{N}_{32}, \\
\Omega_{11}^{24} &= -\bar{N}_{14}^T + \bar{N}_{24}^T - \bar{N}_{32} + \bar{N}_{42} + \bar{\alpha}\bar{A}_\tau^TN_{53}^T \\
&\quad + \beta\bar{N}_{52}\bar{A}_\tau,
\end{aligned}$$

$$\begin{aligned}
\Omega_{11}^{25} &= -\bar{N}_{15}^T + \bar{N}_{25}^T - \bar{N}_{42},\\
\Omega_{11}^{26} &= -\bar{N}_{16}^T + \bar{N}_{26}^T + \bar{\alpha}\bar{A}_\tau^T N_{54}^T - \bar{N}_{52},\\
\Omega_{11}^{33} &= -Q_1 + Q_2 - \bar{N}_{23} - \bar{N}_{23}^T + \bar{N}_{33} + \bar{N}_{33}^T,\\
\Omega_{11}^{34} &= -\bar{N}_{24}^T + \bar{N}_{34}^T - \bar{N}_{33} + \bar{N}_{43},\\
\Omega_{11}^{35} &= -\bar{N}_{25}^T + \bar{N}_{35}^T - \bar{N}_{43}, \quad \Omega_{11}^{36} = -\bar{N}_{26}^T + \bar{N}_{36}^T,\\
\Omega_{11}^{44} &= -\bar{N}_{34} - \bar{N}_{34}^T + \bar{N}_{44} + \bar{N}_{44}^T\\
&\quad + \beta(\bar{N}_{53}\bar{A}_\tau + \bar{A}_\tau^T N_{53}^T),\\
\Omega_{11}^{45} &= -\bar{N}_{35}^T + \bar{N}_{45}^T - \bar{N}_{44},\\
\Omega_{11}^{46} &= -\bar{N}_{36}^T + \bar{N}_{46}^T + \beta\bar{A}_\tau^T N_{54}^T - \bar{N}_{53},\\
\Omega_{11}^{55} &= -Q_2 - \bar{N}_{45} - \bar{N}_{45}^T, \; \Omega_{11}^{56} = -\bar{N}_{46}^T,\\
\Omega_{11}^{66} &= \tau_1 R_1 + (\tau_2 - \tau_1)R_2 - \bar{N}_{54} - \bar{N}_{54}^T,\\
\tau_{2-1} &= \tau_2 - \tau_1,
\end{aligned}$$

with $\bar{\mathscr{A}} = \mathscr{A} + E_1 FH$, $\beta = 1 - \bar{\alpha}$, $\sigma = \bar{\alpha}(1 - \bar{\alpha})$, then the closed-loop control system (7) is asymptotically stable in mean-square sense.

Proof: Firstly, construct the following Lyapunov functional

$$V(\zeta(t)) = \sum_{i=1}^{7} V_i(\zeta(t))$$

where

$$\begin{aligned}
V_1(\zeta(t)) &= x^T(t)Px(t),\\
V_2(\zeta(t)) &= \int_{t-\tau_1}^{t} x^T(s)Q_1 x(s)ds,\\
V_3(\zeta(t)) &= \int_{t-\tau_2}^{t-\tau_1} x^T(s)Q_2 x(s)ds,\\
V_4(\zeta(t)) &= \int_{t-\tau_1}^{t}\int_{s}^{t} y^T(v)R_1 y(v)dvds,\\
V_5(\zeta(t)) &= \int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t} y^T(v)R_2 y(v)dvds,\\
V_6(\zeta(t)) &= \sigma\int_{t-\tau_1}^{t}\int_{s}^{t} \zeta^T(v)\bar{M}^T T_1\bar{M}\zeta(v)dvds,\\
V_7(\zeta(t)) &= \sigma\int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t} \zeta^T(v)\bar{M}^T T_2\bar{M}\zeta(v)dvds, \qquad (10)
\end{aligned}$$

with $P > 0$, $Q_i > 0$, $R_i > 0$ and $T_i > 0$ $(i = 1, 2)$ to be determined. Taking the infinitesimal operator $\mathcal{L}$ of $V(\zeta(t))$, one gets

$$\begin{aligned}
&\mathcal{L}V(\zeta(t))\\
&= 2x^T(t)Py(t) + 2(\alpha(t) - \bar{\alpha})x^T(t)P\bar{M}\zeta(t)\\
&\quad + x^T(t)Q_1x(t) + x^T(t-\tau_1)(Q_2 - Q_1)x(t-\tau_1)\\
&\quad - x^T(t-\tau_2)Q_2x(t-\tau_2) + \tau_1 y^T(t)R_1y(t)\\
&\quad + (\tau_2 - \tau_1)y^T(t)R_2y(t) + \sigma\tau_1\zeta^T(t)\bar{M}^T T_1\bar{M}\zeta(t)\\
&\quad + \sigma(\tau_2 - \tau_1)\zeta^T(t)\bar{M}^T T_2\bar{M}\zeta(t)\\
&\quad - \int_{t-\tau_1}^{t} y^T(s)R_1y(s)ds - \int_{t-\tau_2}^{t-\tau_1} y^T(s)R_2y(s)ds\\
&\quad - \sigma\int_{t-\tau_1}^{t} \zeta^T(s)\bar{M}^T T_1\bar{M}\zeta(s)ds\\
&\quad - \sigma\int_{t-\tau_2}^{t-\tau_1} \zeta^T(s)\bar{M}^T T_2\bar{M}\zeta(s)ds. \qquad (11)
\end{aligned}$$

Furthermore, on the basic of Newton-Leibnitz formula, we have the following equations:

$$\begin{aligned}
0 &= 2\zeta^T(t)\bar{N}_1\psi(x, 0, \tau_1(t)), &(12)\\
0 &= 2\zeta^T(t)\bar{N}_2\psi(x, \tau_1(t), \tau_1), &(13)\\
0 &= 2\zeta^T(t)\bar{N}_3\psi(x, \tau_1, \tau_2(t)), &(14)\\
0 &= 2\zeta^T(t)\bar{N}_4\psi(x, \tau_2(t), \tau_2), &(15)\\
0 &= 2r^T(t)\bar{N}_5\big[(\mathscr{A} + E_1FH)x(t) + \bar{\alpha}\bar{A}_\tau x(t-\tau_1(t))\\
&\quad + (1-\bar{\alpha})\bar{A}_\tau x(t-\tau_2(t)) - y(t)\big], &(16)
\end{aligned}$$

where

$$r^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau_1(t)) & x^T(t-\tau_2(t)) & y^T(t)\end{bmatrix},$$

$$\psi(x, y, z) \triangleq x(t-y) - x(t-z) - \int_{t-z}^{t-y} \dot{x}(s)ds,$$

and $\bar{N}_i$ $(i = 1, 2, \cdots, 5)$ are free-weighting matrices.

Considering the results in (12)-(16), we have

$$\begin{aligned}
&2\zeta^T(t)\bar{N}_1\psi(x, 0, \tau_1(t))\\
&\quad \le 2\zeta^T(t)\bar{N}_1x(t) - 2\zeta^T(t)\bar{N}_1x(t-\tau_1(t))\\
&\qquad + \tau_1(t)\zeta^T(t)\bar{N}_1(R_1^{-1} + T_1^{-1})\bar{N}_1^T\zeta(t)\\
&\qquad + \int_{t-\tau_1(t)}^{t} y^T(s)R_1y(s)ds\\
&\qquad + \int_{t-\tau_1(t)}^{t} (\alpha(s) - \bar{\alpha})^2\zeta^T(s)\bar{M}^T T_1\bar{M}\zeta(s)ds, \qquad (17)\\
&2\zeta^T(t)\bar{N}_2\psi(x, \tau_1(t), \tau_1)\\
&\quad \le 2\zeta^T(t)\bar{N}_2x(t-\tau_1(t)) - 2\zeta^T(t)\bar{N}_2x(t-\tau_1)\\
&\qquad + (\tau_1 - \tau_1(t))\zeta^T(t)\bar{N}_2(R_1^{-1} + T_1^{-1})\bar{N}_2^T\zeta(t)\\
&\qquad + \int_{t-\tau_1}^{t-\tau_1(t)} y^T(s)R_1y(s)ds\\
&\qquad + \int_{t-\tau_1}^{t-\tau_1(t)} (\alpha(s) - \bar{\alpha})^2\zeta^T(s)\bar{M}^T T_1\bar{M}\zeta(s)ds, \qquad (18)\\
&2\zeta^T(t)\bar{N}_3\psi(x, \tau_1, \tau_2(t))\\
&\quad \le 2\zeta^T(t)\bar{N}_3x(t-\tau_1) - 2\zeta^T(t)\bar{N}_3x(t-\tau_2(t))\\
&\qquad + (\tau_2(t) - \tau_1)\zeta^T(t)\bar{N}_3(R_2^{-1} + T_2^{-1})\bar{N}_3^T\zeta(t)\\
&\qquad + \int_{t-\tau_2(t)}^{t-\tau_1} y^T(s)R_2y(s)ds\\
&\qquad + \int_{t-\tau_2(t)}^{t-\tau_1} (\alpha(s) - \bar{\alpha})^2\zeta^T(s)\bar{M}^T T_2\bar{M}\zeta(s)ds, \qquad (19)\\
&2\zeta^T(t)\bar{N}_4\psi(x, \tau_2(t), \tau_2)\\
&\quad \le 2\zeta^T(t)\bar{N}_4x(t-\tau_2(t)) - 2\zeta^T(t)\bar{N}_4x(t-\tau_2)\\
&\qquad + (\tau_2 - \tau_2(t))\zeta^T(t)\bar{N}_4(R_2^{-1} + T_2^{-1})\bar{N}_4^T\zeta(t)\\
&\qquad + \int_{t-\tau_2}^{t-\tau_2(t)} y^T(s)R_2y(s)ds\\
&\qquad + \int_{t-\tau_2}^{t-\tau_2(t)} (\alpha(s) - \bar{\alpha})^2\zeta^T(s)\bar{M}^T T_2\bar{M}\zeta(s)ds. \qquad (20)
\end{aligned}$$

Combining the results in (11)-(20) and taking the expectation yield

$$\begin{aligned}
&\mathbb{E}\{\mathcal{L}V(\zeta(t))\}\\
&\quad\leq \mathbb{E}\{\zeta^T(t)[\Omega_{11}+\bar{\alpha}(1-\bar{\alpha})\tau_1\bar{M}^T T_1\bar{M}+\bar{\alpha}(1-\bar{\alpha})\\
&\qquad\times(\tau_2-\tau_1)\bar{M}^T T_2\bar{M}+\tau_1(t)\bar{N}_1(R_1^{-1}+T_1^{-1})\bar{N}_1^T\\
&\qquad+(\tau_1-\tau_1(t))\bar{N}_2(R_1^{-1}+T_1^{-1})\bar{N}_2^T+(\tau_2(t)-\tau_1)\\
&\qquad\times\bar{N}_3(R_2^{-1}+T_2^{-1})\bar{N}_3^T+(\tau_2-\tau_2(t))\bar{N}_4\\
&\qquad\times(R_2^{-1}+T_2^{-1})\bar{N}_4^T]\zeta(t)\}, \qquad (21)
\end{aligned}$$

where Ω_{11} is defined below (8).

Considering the results in (21) and utilizing Lemma 2, it can be seen that (9) are sufficient conditions for guaranteeing that $\mathbb{E}\{\mathcal{L}V(\zeta(t))\}$ is less than zero. Thus, the system (8) is asymptotically stable in mean-square sense when $w(t) = 0$. ■

In order to examine the effect from external disturbance onto whole control performance, the following definition is adopted.

Definition 1 [33]: Letting $\gamma > 0$ be a given scalar, under the zero initial condition, the system (7) is said to have specified H_∞ performance, if the following inequality holds

$$\mathbb{E}\left\{\int_0^\infty z^T(t)z(t)dt\right\} < \int_0^\infty \gamma^2 w^T(t)w(t)dt.$$

Next, sufficient conditions will be derived for guaranteeing the stability under satisfactory H_∞ performance.

Theorem 2: The closed-loop control system (7) is asymptotically stable in mean-square sense with disturbance attenuation level γ, if for the given constants τ_1, τ_2 and matrix K, there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $T_1 > 0$, $T_2 > 0$ and N_j $(j = 1, 2, \cdots, 5)$ of suitable dimensions such that the following inequalities hold:

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13}^i & \Omega_{14} \\ * & \Omega_{22} & 0 & 0 \\ * & * & \Omega_{33} & 0 \\ * & * & * & -I \end{bmatrix} < 0, \quad (i = 1, 2, 3, 4) \qquad (22)$$

where

$$\begin{aligned}
\Omega_{11} &= \left[\Omega_{11}^{ij}\right]_{7\times7},\\
N_i^T &= [N_{i1}^T \quad N_{i2}^T \quad \cdots \quad N_{i7}^T], \quad (i = 1, 2, 3, 4),\\
N_5^T &= [N_{51}^T \quad N_{52}^T \quad \cdots \quad N_{55}^T],\\
\Omega_{12} &= \left[\sqrt{\sigma\tau_1}M^T \quad \sqrt{\sigma(\tau_2-\tau_1)}M^T\right],\\
\Omega_{13}^1 &= \left[\tau_1 N_2 \quad \tau_1 N_2 \quad \tau_{2-1}N_4 \quad \tau_{2-1}N_4\right],\\
\Omega_{13}^2 &= \left[\tau_1 N_1 \quad \tau_1 N_1 \quad \tau_{2-1}N_4 \quad \tau_{2-1}N_4\right],\\
\Omega_{13}^3 &= \left[\tau_1 N_2 \quad \tau_1 N_2 \quad \tau_{2-1}N_3 \quad \tau_{2-1}N_3\right],\\
\Omega_{13}^4 &= \left[\tau_1 N_1 \quad \tau_1 N_1 \quad \tau_{2-1}N_3 \quad \tau_{2-1}N_3\right],\\
\Omega_{22} &= \mathrm{diag}\{-T_1^{-1}, -T_2^{-1}\},\\
\Omega_{33} &= \mathrm{diag}\{-\tau_1 R_1, -\tau_1 T_1, -\tau_{2-1}R_2, -\tau_{2-1}T_2\},\\
\Omega_{14} &= \begin{bmatrix} C & \bar{\alpha}A_{\tau1} & 0 & (1-\bar{\alpha})A_{\tau1} & 0 & D_2 & 0 \\ 0 & \sqrt{\sigma}A_{\tau1} & 0 & -\sqrt{\sigma}A_{\tau1} & 0 & 0 & 0 \end{bmatrix}^T,
\end{aligned}$$

$$\begin{aligned}
\Omega_{11}^{11} &= N_{11}+N_{11}^T+N_{51}\bar{\mathscr{A}}+\bar{\mathscr{A}}^T N_{51}^T+Q_1,\\
\Omega_{11}^{12} &= N_{12}^T-N_{11}+N_{21}+\bar{\alpha}N_{51}\bar{A}_\tau+\bar{\mathscr{A}}^T N_{52}^T,\\
\Omega_{11}^{13} &= N_{13}^T-N_{21}+N_{31},\\
\Omega_{11}^{14} &= N_{14}^T-N_{31}+N_{41}+\bar{\mathscr{A}}^T N_{53}^T+\beta N_{51}\bar{A}_\tau,\\
\Omega_{11}^{15} &= N_{15}^T-N_{41},\\
\Omega_{11}^{16} &= N_{16}^T+N_{51}\bar{D}_1+\bar{\mathscr{A}}^T N_{54}^T,\\
\Omega_{11}^{17} &= P+N_{17}^T-N_{51}+\bar{\mathscr{A}}^T N_{55}^T,\\
\Omega_{11}^{22} &= -N_{12}^T-N_{12}+N_{22}+N_{22}^T\\
&\quad+\bar{\alpha}(N_{52}\bar{A}_\tau+\bar{A}_\tau^T N_{52}^T),\\
\Omega_{11}^{23} &= -N_{13}^T+N_{23}^T-N_{22}+N_{32},\\
\Omega_{11}^{24} &= -N_{14}^T+N_{24}^T-N_{32}+N_{42}+\bar{\alpha}\bar{A}_\tau^T N_{53}^T\\
&\quad+\beta N_{52}\bar{A}_\tau,\\
\Omega_{11}^{25} &= -N_{15}^T+N_{25}^T-N_{42},\\
\Omega_{11}^{26} &= -N_{16}^T+N_{26}^T+N_{52}\bar{D}_1+\bar{\alpha}\bar{A}_\tau^T N_{54}^T,\\
\Omega_{11}^{27} &= -N_{17}^T+N_{27}^T+\bar{\alpha}\bar{A}_\tau^T N_{55}^T-N_{52},\\
\Omega_{11}^{33} &= -Q_1+Q_2-N_{23}-N_{23}^T+N_{33}+N_{33}^T,\\
\Omega_{11}^{34} &= -N_{24}^T+N_{34}^T-N_{33}+N_{43},\\
\Omega_{11}^{35} &= -N_{25}^T+N_{35}^T-N_{43},\\
\Omega_{11}^{36} &= -N_{26}^T+N_{36}^T,\ \Omega_{11}^{37} = -N_{27}^T+N_{37}^T,\\
\Omega_{11}^{44} &= -N_{34}-N_{34}^T+N_{44}+N_{44}^T\\
&\quad+\beta(N_{53}\bar{A}_\tau+\bar{A}_\tau^T N_{53}^T),\\
\Omega_{11}^{45} &= -N_{35}^T+N_{45}^T-N_{44},\\
\Omega_{11}^{46} &= -N_{36}^T+N_{46}^T+N_{53}\bar{D}_1+\beta\bar{A}_\tau^T N_{54}^T,\\
\Omega_{11}^{47} &= -N_{37}^T+N_{47}^T+\beta\bar{A}_\tau^T N_{55}^T-N_{53},\\
\Omega_{11}^{55} &= -Q_2-N_{45}-N_{45}^T,\ \Omega_{11}^{56} = -N_{46}^T,\\
\Omega_{11}^{57} &= -N_{47}^T,\ \Omega_{11}^{66} = N_{54}\bar{D}_1+\bar{D}_1^T N_{54}^T-\gamma^2 I,\\
\Omega_{11}^{67} &= \bar{D}_1^T N_{55}^T-N_{54},\ \tau_{2-1} = \tau_2-\tau_1\\
\Omega_{11}^{77} &= \tau_1 R_1+(\tau_2-\tau_1)R_2-N_{55}-N_{55}^T,
\end{aligned}$$

with $\bar{\mathscr{A}} = \mathscr{A}+E_1FH$, $\beta = 1-\bar{\alpha}$ and $\sigma = \bar{\alpha}(1-\bar{\alpha})$.

Proof: The proof is same with the analysis as in Theorem 1. Construct the following Lyapunov functional

$$V(\xi(t)) = \sum_{i=1}^{7} V_i(\xi(t))$$

where

$$\begin{aligned}
V_1(\xi(t)) &= x^T(t)Px(t),\\
V_2(\xi(t)) &= \int_{t-\tau_1}^{t} x^T(s)Q_1x(s)ds,\\
V_3(\xi(t)) &= \int_{t-\tau_2}^{t-\tau_1} x^T(s)Q_2x(s)ds,\\
V_4(\xi(t)) &= \int_{t-\tau_1}^{t}\int_s^t y^T(v)R_1y(v)dvds,\\
V_5(\xi(t)) &= \int_{t-\tau_2}^{t-\tau_1}\int_s^t y^T(v)R_2y(v)dvds,
\end{aligned}$$

$$V_6(\xi(t)) = \sigma \int_{t-\tau_1}^{t} \int_{s}^{t} \xi^T(v) M^T T_1 M \xi(v) dv ds,$$
$$V_7(\xi(t)) = \sigma \int_{t-\tau_2}^{t-\tau_1} \int_{s}^{t} \xi^T(v) M^T T_2 M \xi(v) dv ds, \quad (23)$$

with $P > 0$, $Q_i > 0$, $R_i > 0$ and $T_i > 0$ $(i = 1, 2)$ to be determined.

The following equations along the same way in (12)-(16) are true

$$0 = 2\xi^T(t) N_1 \psi(x, 0, \tau_1(t)), \quad (24)$$
$$0 = 2\xi^T(t) N_2 \psi(x, \tau_1(t), \tau_1), \quad (25)$$
$$0 = 2\xi^T(t) N_3 \psi(x, \tau_1, \tau_2(t)), \quad (26)$$
$$0 = 2\xi^T(t) N_4 \psi(x, \tau_2(t), \tau_2), \quad (27)$$
$$0 = 2r^T(t) N_5 \Big[(\mathscr{A} + E_1 F H) x(t) + \bar{\alpha} \bar{A}_\tau x(t - \tau_1(t)) + (1 - \bar{\alpha}) \bar{A}_\tau x(t - \tau_2(t)) + \bar{D}_1 w(t) - y(t) \Big], \quad (28)$$

where $r^T(t) = [x^T(t)\, x^T(t-\tau_1(t))\, x^T(t-\tau_2(t))\, w^T(t)\, y^T(t)]$. Here, it should be noted that N_i $(i = 1, 2, \cdots, 5)$ are different from the definitions of $\bar{N}_i$ in Theorem 1 due to the existence of external disturbance.

Along the analysis process of (16)-(20) and using Lemma 2, it is easy to verify that (22) can guarantee $\mathbb{E}\{\mathcal{L}V(\xi(t))\} + \mathbb{E}\left\{z(t)^T z(t)\right\} - \gamma^2\, w^T(t) w(t)$ less than zero. Therefore, the closed-loop system (7) is asymptotically stable in mean-square sense with disturbance attenuation level γ. ■

Theorem 2 shows the stability of (7) with specified H_∞ performance by utilizing Definition 1, where the gain matrix K is assumed to be given. The following theorem further gives the design way for the gain matrix K according to the results in Theorem 2.

Theorem 3: For given constants τ_1, τ_2 and γ, if there exist matrices $\hat{P} > 0$, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{R}_1 > 0$, $\hat{R}_2 > 0$, $\hat{T}_1 > 0$, $\hat{T}_2 > 0$, X and $\hat{N}_j$ $(j = 1, 2, \cdots, 5)$ with appropriate dimensions such that the following inequalities

$$\hat{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13}^i & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} \\ * & \hat{\Omega}_{22} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 \\ * & * & * & \bar{\Omega}_{44} & 0 & 0 \\ * & * & * & * & \bar{\Omega}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \quad i = 1, 2, 3, 4 \quad (29)$$

are satisfied, where

$$\bar{\Omega}_{12} = \begin{bmatrix} \sqrt{\sigma\tau_1}\hat{M} & \sqrt{\sigma(\tau_2 - \tau_1)}\hat{M} \end{bmatrix},$$
$$\bar{\Omega}_{13}^1 = \begin{bmatrix} \tau_1\hat{N}_2 & \tau_1\hat{N}_2 & \tau_{2-1}\hat{N}_4 & \tau_{2-1}\hat{N}_4 \end{bmatrix},$$
$$\bar{\Omega}_{13}^2 = \begin{bmatrix} \tau_1\hat{N}_1 & \tau_1\hat{N}_1 & \tau_{2-1}\hat{N}_4 & \tau_{2-1}\hat{N}_4 \end{bmatrix},$$
$$\bar{\Omega}_{13}^3 = \begin{bmatrix} \tau_1\hat{N}_2 & \tau_1\hat{N}_2 & \tau_{2-1}\hat{N}_3 & \tau_{2-1}\hat{N}_3 \end{bmatrix},$$
$$\bar{\Omega}_{13}^4 = \begin{bmatrix} \tau_1\hat{N}_1 & \tau_1\hat{N}_1 & \tau_{2-1}\hat{N}_3 & \tau_{2-1}\hat{N}_3 \end{bmatrix},$$
$$\bar{\Omega}_{14}^T = \begin{bmatrix} \bar{\Omega}_{14}^{(1)} & \bar{\Omega}_{14}^{(2)} \end{bmatrix}, \quad \bar{\Omega}_{15}^T = \begin{bmatrix} \bar{\Omega}_{15}^{(1)} & \bar{\Omega}_{15}^{(2)} \end{bmatrix}^T,$$
$$\bar{\Omega}_{16}^T = \begin{bmatrix} \bar{\Omega}_{16}^{(1)} & \bar{\Omega}_{16}^{(2)} \end{bmatrix}^T,$$
$$\bar{\Omega}_{14}^{(1)} = \begin{bmatrix} E_1^T & \rho_2 E_1^T & 0 & \rho_3 E_1^T \\ HS^T & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\Omega}_{14}^{(2)} = \begin{bmatrix} 0 & E_1^T N_{54}^T & \rho_5 E_1^T \\ 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\Omega}_{15}^{(1)} = \begin{bmatrix} AS + BK & \bar{\alpha}\bar{A}_\tau S & 0 & \beta\bar{A}_\tau S \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\bar{\Omega}_{15}^{(2)} = \begin{bmatrix} 0 & 0 & -S \\ 0 & N_{54}^T & 0 \end{bmatrix},$$
$$\bar{\Omega}_{16}^{(1)} = \begin{bmatrix} CS & \bar{\alpha}A_{\tau 1}S & 0 & (1-\bar{\alpha})A_{\tau 1}S \\ 0 & \sqrt{\sigma}A_{\tau 1}S & 0 & -\sqrt{\sigma}A_{\tau 1}S \end{bmatrix},$$
$$\bar{\Omega}_{16}^{(2)} = \begin{bmatrix} 0 & D_2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$\hat{\Omega}_{22} = \mathrm{diag}\{-S - S^T + \hat{T}_1, -S - S^T + \hat{T}_2\},$$
$$\bar{\Omega}_{33} = \mathrm{diag}\{-\tau_1\hat{R}_1, -\tau_1\hat{T}_1, -\tau_{2-1}\hat{R}_2, -\tau_{2-1}\hat{T}_2\},$$
$$\bar{\Omega}_{44} = \mathrm{diag}\{-\varepsilon_1 I, -\varepsilon_1^{-1} I\},$$
$$\hat{M}^T = [0\, A_\tau S^T\ 0\ -A_\tau S^T\ 0\, 0\, 0],$$
$$\bar{\Omega}_{55} = \mathrm{diag}\{-\varepsilon_2 I, -\varepsilon_2^{-1} I\},$$
$$\hat{\Omega}_{11}^{11} = \hat{N}_{11} + \hat{N}_{11}^T + \Phi + \Phi^T + \hat{Q}_1,$$
$$\hat{\Omega}_{11}^{12} = \hat{N}_{12}^T - \hat{N}_{11} + \rho_2\Phi + \hat{N}_{21} + \bar{\alpha}\bar{A}_\tau S^T,$$
$$\hat{\Omega}_{11}^{13} = \hat{N}_{13}^T - \hat{N}_{21} + \hat{N}_{31},$$
$$\hat{\Omega}_{11}^{14} = \hat{N}_{14}^T - \hat{N}_{31} + \hat{N}_{41} + \rho_3\Phi + \beta\bar{A}_\tau S^T,$$
$$\hat{\Omega}_{11}^{15} = \hat{N}_{15}^T - \hat{N}_{41}, \quad \hat{\Omega}_{11}^{16} = \bar{D}_1 + \hat{N}_{16}^T,$$
$$\hat{\Omega}_{11}^{17} = \hat{P} + \hat{N}_{17}^T - S^T + \rho_5\Phi,$$
$$\hat{\Omega}_{11}^{22} = -\hat{N}_{12}^T - \hat{N}_{12} + \hat{N}_{22} + \hat{N}_{22}^T + \bar{\alpha}\rho_2(\bar{A}_\tau S^T + S\bar{A}_\tau^T),$$
$$\hat{\Omega}_{11}^{23} = -\hat{N}_{13}^T + \hat{N}_{23}^T - \hat{N}_{22} + \hat{N}_{32},$$
$$\hat{\Omega}_{11}^{24} = -\hat{N}_{14}^T + \hat{N}_{24}^T - \hat{N}_{32} + \hat{N}_{42} + \bar{\alpha}\rho_3 S\bar{A}_\tau^T + \beta\rho_2\bar{A}_\tau S^T,$$
$$\hat{\Omega}_{11}^{25} = -\hat{N}_{15}^T + \hat{N}_{25}^T - \hat{N}_{42},$$
$$\hat{\Omega}_{11}^{26} = \rho_2\bar{D}_1 - \hat{N}_{16}^T + \hat{N}_{26}^T,$$
$$\hat{\Omega}_{11}^{27} = -\hat{N}_{17}^T + \hat{N}_{27}^T + \bar{\alpha}\rho_5 S\bar{A}_\tau^T - \rho_2 S^T,$$
$$\hat{\Omega}_{11}^{33} = -\hat{Q}_1 + \hat{Q}_2 - \hat{N}_{23} - \hat{N}_{23}^T + \hat{N}_{33} + \hat{N}_{33}^T,$$
$$\hat{\Omega}_{11}^{34} = -\hat{N}_{24}^T + \hat{N}_{34}^T - \hat{N}_{33} + \hat{N}_{43},$$
$$\hat{\Omega}_{11}^{35} = -\hat{N}_{25}^T + \hat{N}_{35}^T - \hat{N}_{43}, \quad \hat{\Omega}_{11}^{36} = -\hat{N}_{26}^T + \hat{N}_{36}^T,$$
$$\hat{\Omega}_{11}^{37} = -\hat{N}_{27}^T + \hat{N}_{37}^T, \quad \hat{\Omega}_{11}^{46} = -\hat{N}_{36}^T + \hat{N}_{46}^T + \rho_3\bar{D}_1,$$
$$\hat{\Omega}_{11}^{44} = -\hat{N}_{34} - \hat{N}_{34}^T + \hat{N}_{44} + \hat{N}_{44}^T + \beta\rho_3(\bar{A}_\tau S^T + S\bar{A}_\tau^T),$$
$$\hat{\Omega}_{11}^{45} = -\hat{N}_{35}^T + \hat{N}_{45}^T - \hat{N}_{44},$$
$$\hat{\Omega}_{11}^{47} = -\hat{N}_{37}^T + \hat{N}_{47}^T + \beta\rho_5 S\bar{A}_\tau^T - \rho_3 S^T,$$
$$\hat{\Omega}_{11}^{55} = = -\hat{Q}_2 - \hat{N}_{45} - \hat{N}_{45}^T,$$
$$\hat{\Omega}_{11}^{56} = -\hat{N}_{46}^T, \quad \hat{\Omega}_{11}^{57} = -\hat{N}_{47}^T,$$
$$\hat{\Omega}_{11}^{66} = N_{54}\bar{D}_1 + \bar{D}_1^T N_{54}^T - \gamma^2 I, \ \hat{\Omega}_{11}^{67} = \rho_5\bar{D}_1^T,$$
$$\hat{\Omega}_{11}^{77} = \tau_1\hat{R}_1 + (\tau_2 - \tau_1)\hat{R}_2 - \rho_5(S + S^T) \quad (30)$$

with $\tau_{2-1} = \tau_2 - \tau_1$, $\Phi = S^TA^T + X^TB^T$, then system (7) is asymptotically stable with disturbance attenuation level γ and the gain matrix K can be obtained by $K = XS^{-1}$.

Proof: Firstly, define $N_{51} = S^{-1}$, $N_{52} = \rho_2 S^{-1}$, $N_{53} = \rho_3 S^{-1}$, $N_{55} = \rho_5 S^{-1}$, where $\rho_5 \neq 0$ and $S > 0$.

Set $\mathscr{L}_1 = \text{diag}\{S, S, S, S, S, I, S\}$. Then pre- and post-multiplying Ω in (22) with $\mathscr{L}$=diag$\{\mathscr{L}_1, I, I, S, S, S, S, I, I\}$ and its transpose, respectively, one gets $\bar{\Omega} = \mathscr{L}\Omega\mathscr{L} = [\bar{\Omega}_{ij}]_{3\times 3}$ with $\bar{\Omega}_{11} = \mathscr{L}_1\Omega_{11}\mathscr{L}_1^T$. Next, set $\hat{P} = SPS^T$, $\hat{Q}_i = SQ_iS^T$, $\hat{R}_i = SR_iS^T$, $\hat{T}_i = ST_iS^T$ $(i = 1, 2)$, $\hat{N}_{ij} = SN_{ij}S^T$, $\hat{N}_{i6} = N_{i6}S^T$ $(i = 1, 2, 3, 4,\ j = 1, 2, 3, 4, 5, 7)$, $\hat{N}_i^T = [\hat{N}_{i1}^T\ \hat{N}_{i2}^T\ \cdots\ \hat{N}_{i7}^T]$.

Subsequently, notice

$$\bar{\Omega}_{11} = \hat{\Omega}_{11} + \Phi_1F^T\Phi_2 + \Phi_2^TF\Phi_1^T + \Phi_3\Phi_4 + \Phi_4^T\Phi_3^T$$

where

$$\Phi_1 = \begin{bmatrix} SH^T \\ 0 \end{bmatrix}, \quad \Phi_4 = \begin{bmatrix} 0_{n\times 5n} & N_{54}^T & 0 \end{bmatrix},$$
$$\Phi_2 = \begin{bmatrix} E_1^T & \rho_2E_1^T & 0 & \rho_3E_1^T & 0 & E_1^TN_{54}^T & \rho_5E_1^T \end{bmatrix},$$
$$\Phi_3^T = \begin{bmatrix} AS + BX & \bar{\alpha}\bar{A}_\tau S & 0 & \beta\bar{A}_\tau S & 0 & 0 & -S \end{bmatrix}.$$

By applying Lemmas 3-4 and using Schur complement, the inequality (22) can be transformed into

$$\bar{\Omega}_i = \begin{bmatrix} \hat{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13}^i & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} \\ * & \bar{\Omega}_{22} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 \\ * & * & * & \bar{\Omega}_{44} & 0 & 0 \\ * & * & * & * & \bar{\Omega}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0, \quad i = 1, 2, 3, 4 \tag{31}$$

with $\hat{\Omega}_{11} = [\hat{\Omega}_{11}^{pq}]_{6\times 6}$, $\bar{\Omega}_{1l}$ $(l = 2, 4, 5, 6)$, $\bar{\Omega}_{13}^i$ $(i = 1, 2, 3, 4)$, $\bar{\Omega}_{jj}$ $(j = 2, 3, 4, 5)$ being defined as in (30).

It can be seen that inequality (31) is a nonconvex one due to the existence of $-T_i^{-1}$ in (31) and ε_1^{-1}, ε_2^{-1}. Noticing $(ST_i - I)T_i^{-1}(ST_i - I)^T \geq 0$, we can easily obtain $-T_i^{-1} \leq -S - S^T + \hat{T}_i$ $(i = 1, 2)$. Considering the above analysis, it's not difficult to verify that the inequality (29) guarantees $\bar{\Omega}_i < 0$. ■

Remark 1: Notice the existence of ε_1^{-1} and ε_2^{-1} in (29), by setting $\bar{\varepsilon}_1 = \varepsilon_1^{-1}$, $\bar{\varepsilon}_2 = \varepsilon_2^{-1}$ and applying cone-complementarity, then (29) can be approximated by the following sub-optimization problem:

$$\min(\varepsilon_1\bar{\varepsilon}_1 + \varepsilon_2\bar{\varepsilon}_2)$$
$$\vec{\Omega} = \begin{bmatrix} \hat{\Omega}_{11} & \bar{\Omega}_{12} & \bar{\Omega}_{13}^i & \bar{\Omega}_{14} & \bar{\Omega}_{15} & \bar{\Omega}_{16} \\ * & \hat{\Omega}_{22} & 0 & 0 & 0 & 0 \\ * & * & \bar{\Omega}_{33} & 0 & 0 & 0 \\ * & * & * & \hat{\Omega}_{44} & 0 & 0 \\ * & * & * & * & \hat{\Omega}_{55} & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0,$$
$$i = 1, 2, 3, 4 \tag{32}$$
$$\begin{bmatrix} \varepsilon_j & I \\ I & \bar{\varepsilon}_j \end{bmatrix} > 0, \quad j = 1, 2 \tag{33}$$

with $\hat{\Omega}_{44} = \text{diag}\{-\varepsilon_1 I, -\bar{\varepsilon}_1 I\}$, $\hat{\Omega}_{55} = \text{diag}\{-\varepsilon_2 I, -\bar{\varepsilon}_2 I\}$. According to [34], if the solution of the above minimization problem is 2, then (29) is solvable.

Based on the above analysis, we provide the following algorithm for the solvability of (29).

Algorithm for Theorem 3:

Step1: Given constants ρ_m $(m = 2, 3, 5)$ and maximum number N of iterations, set $k = 0$.

Step2: Find feasible solutions

$$\{\hat{P}, \hat{Q}_i, \hat{R}_i, \hat{T}_i, S, \hat{N}_j, \varepsilon_l, \bar{\varepsilon}_l, X\}$$

$(i = 1, 2,\ j = 1, 2, \cdots, 5,\ l = 1, 2)$ satisfying the following minimization problem:

$$\min \sum_{i=1}^{2}(\varepsilon_i\bar{\varepsilon}_{i,k} + \varepsilon_{i,k}\bar{\varepsilon}_i)$$
subject to (32) and (33)
$$\varepsilon_{i,k+1} = \varepsilon_i, \quad \bar{\varepsilon}_{i,k+1} = \bar{\varepsilon}_i, \quad i = 1, 2$$

if there is no feasible solution, then go to *Step* 3. Otherwise, exit (found the feasible solutions).

Step3: Set $k = k + 1$ and try another constants ρ_m $(m = 2, 3, 5)$. If $k < N$, then go to *Step* 2. Otherwise, exit (no feasible solution is found).

B. REACHABILITY ANALYSIS

Consider the variable controller as follows:

$$u(t) = Kx(t) + u_1(t)$$

where

$$u_1(t) = -\eta(x, t)B_2^T \text{sign}(s(x(t))), \tag{34}$$

with K being designed in Theorem 3, $s(x(t))$ being sliding function and

$$\begin{aligned}\eta(x, t) = \frac{1}{\mu}\Big[&\left(\|GE\|\|H\| + \|B_2(\hat{\rho}(t) - I)K\|\right)\|x(t)\| \\ &+ \bar{\alpha}\|GA_\tau\|\|x(t - \tau_1(t))\| + \sum_{i=1}^{m}\|B_{2i}\|\hat{u}_{fi}(t) \\ &+ (1 - \bar{\alpha})\|GA_\tau\|\|x(t - \tau_2(t))\| \\ &+ \sum_{i=1}^{q}\|GD_{1,i}\|\hat{w}_i(t) + \epsilon\Big].\end{aligned} \tag{35}$$

Here, μ is a positive scalar to be given, $\hat{\rho}$, $\hat{u}_{fi}$ and $\hat{w}_i$ are the estimates of ρ, u_{fi} and w_i, $\epsilon > 0$ is a real number.

We now present the following adaptive laws to estimate the unknown parameters:

$$\dot{\hat{\rho}}_i(t) = \text{Proj}_{[\underline{\rho}_i, \bar{\rho}_i]}\{\mathcal{F}\} = \begin{cases} 0 \text{ if } & \hat{\rho}_i = \underline{\rho}_i \text{ and } \mathcal{F} \leq 0 \\ & \text{or } \hat{\rho}_i = \bar{\rho}_i \text{ and } \mathcal{F} \geq 0, \\ \mathcal{F} \text{ otherwise} \end{cases}$$
$$\dot{\hat{u}}_{fi}(t) = \gamma_{1i}\|s(x(t))\|\|B_{2i}\|, \quad \hat{u}_{fi}(0) = u_{fi0},$$
$$\dot{\hat{w}}_i(t) = \gamma_{2i}\|s(x(t))\|\|GD_{1,i}\|, \quad \hat{w}_{i0} = w_{i0}, \tag{36}$$

where $\mathcal{F} = \gamma_{0i}s^T B_{2i}K_i x(t)$, and γ_{0i}, γ_{1i}, γ_{2i} are positive design parameters. u_{fi0} and w_{i0} are given initial values. B_{2i} and $D_{1,i}$ are, respectively, the ith column of B_2 and D_1, K_i is ith row of gain matrix K.

Let

$$\begin{aligned}\tilde{\rho}(t) &= \hat{\rho}(t) - \rho,\\ \tilde{u}_f(t) &= \hat{u}_f(t) - \bar{u}_f,\ \tilde{w}(t) = \hat{w}(t) - \bar{w},\end{aligned} \tag{37}$$

then the time-derivative of the error systems can be rewritten as

$$\dot{\tilde{\rho}}(t) = \dot{\hat{\rho}}(t),\ \dot{\tilde{u}}_f(t) = \dot{\hat{u}}_f(t),\ \dot{\tilde{w}}(t) = \dot{\hat{w}}(t) \tag{38}$$

use the fact that ρ, $\bar{u}_s$ and $\bar{w}$ are all unknown parameters.

Remark 2: It is worth mentioning that the projection operator in (36) forces $\hat{\rho}_i$ limit to convex set $[\underline{\rho}_i, \bar{\rho}_i]$, which can be solved by the LMI approach. The initial condition of $\hat{\rho}_i$ can be selected as $\hat{\rho}_{i0} = (\underline{\rho}_i + \bar{\rho}_i)/2$ to guarantee the projection constrained.

Remark 3: The free parameters γ_{0i}, γ_{1i} and γ_{2i} in (36) are introduced to adjust the convergence rates of $\hat{\rho}_i$, $\hat{u}_{fi}$ and $\hat{w}_i$, respectively. Note that the higher parameters may lead to a big overshoot and the lower parameters could cause slow response of adaptive parameters.

Lemma 5 [10]: The matrix $B_2\rho B_2^T$ is invertible for every ρ and there exist two positive real numbers μ and κ such that the inequality

$$\mu I \leq B_2\rho B_2^T \leq \kappa I$$

holds.

In fact, let $\mu = \lambda_{\min}(B_2\underline{\rho}B_2^T)$ and $\kappa = \lambda_{\max}(B_2\bar{\rho}B_2^T)$, then Lemma 5 can be easily obtained. Based on the presented ISMC laws in (34)-(36) and using Lemma 5, the following theorem further reveals the reachability.

Theorem 4: Let ISMC law be given in (34) and (36), then the state trajectory of the closed-loop FTC system (1) with actuator fault (2) can be driven onto the integral sliding manifold $\mathscr{S} =: \{x \in \mathbb{R}^n | s(x(t)) = 0\}$.

Proof: Construct the following candidate Lyapunov function for the reachability analysis

$$V(s(x), \tilde{\rho}, \tilde{u}_f, \tilde{w}) = V_1(t) + V_2(t) \tag{39}$$

with

$$\begin{aligned}V_1(t) &= \frac{1}{2}s^T(x(t))s(x(t)),\\ V_2(t) &= \sum_{i=1}^{m}\left(\frac{\tilde{\rho}_i^2}{2\gamma_{0i}} + \frac{\tilde{u}_{fi}^2}{2\gamma_{1i}}\right) + \sum_{i=1}^{q}\frac{\tilde{w}_i^2}{2\gamma_{2i}}.\end{aligned}$$

For simplicity, here and below, define s or $s(x)$ as $s(x(t))$. Taking the infinitesimal operator $\mathcal{L}$ of $V_1(t)$ along the system (3) leads to

$$\begin{aligned}\mathcal{L}V_1(t) &= s^T(x)\big[G\Delta Ax(t) + \bar{\alpha}GA_\tau x(t-\tau_1(t))\\ &\quad + (1-\bar{\alpha})GA_\tau x(t-\tau_2(t)) + GB(\rho - I)u_0(t)\\ &\quad + (\alpha(t)-\bar{\alpha})(GA_\tau x(t-\tau_1(t)) - GA_\tau x(t-\tau_2(t)))\\ &\quad + GBu_1(t) + B_2\varsigma u_f(t) + GD_1 w(t)\big]\\ &= s^T(x)\Big\{\Big[GEF^TH + B_2(\hat{\rho}(t) - I)K\Big]x(t)\\ &\quad + B_2(\rho - \hat{\rho}(t))Kx(t) + B_2\rho u_1(t) + B_2\varsigma u_f(t)\\ &\quad + \bar{\alpha}GA_\tau x(t-\tau_1(t)) + (1-\bar{\alpha})GA_\tau x(t-\tau_2(t))\\ &\quad + GD_1w(t) + (\alpha(t)-\bar{\alpha})(GA_\tau x(t-\tau_1(t))\\ &\quad - GA_\tau x(t-\tau_2(t)))\Big\}\\ &\leq \|s\|\big[\|GE\|\|H\| + \|B_2(\hat{\rho} - I)K\|\big]\|x(t)\|\\ &\quad + \bar{\alpha}\|s\|\|GA_\tau x(t-\tau_1(t))\| + (1-\bar{\alpha})\|s\|\\ &\quad \times\|GA_\tau x(t-\tau_2(t))\| - s^TB_2\tilde{\rho}Kx(t)\\ &\quad + s^TB_2\rho u_1(t) + s^TB_2\varsigma u_f(t) + s^TGD_1w(t)\\ &\quad + (\alpha(t)-\bar{\alpha})s^T(x)(GA_\tau x(t-\tau_1(t))\\ &\quad - GA_\tau x(t-\tau_2(t))).\end{aligned} \tag{40}$$

Hence, we see that

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V_1(t)\}\\ &\quad\leq \|s\|\Big[\|GE\|\|H\| + \|B_2(\hat{\rho} - I)K\|\|x(t)\| + \bar{\alpha}\|GA_\tau\|\\ &\qquad\times\|x(t-\tau_1(t))\| + (1-\bar{\alpha})\|GA_\tau\|\|x(t-\tau_2(t))\|\Big]\\ &\qquad - s^TB_2\tilde{\rho}Kx(t) + s^TB_2\rho u_1(t) + s^TB_2\varsigma u_f(t)\\ &\qquad + s^TGD_1w(t).\end{aligned} \tag{41}$$

Now, considering the several terms in the above equation, the following inequalities are true according to Assumption 2,

$$s^TB_2\tilde{\rho}Kx(t) = \sum_{i=1}^{m}s^TB_{2i}K_ix(t)\tilde{\rho}_i, \tag{42}$$

$$s^TB_2\varsigma u_f(t) = \sum_{i=1}^{m}s^TB_{2i}\varsigma_i u_{fi} \leq \sum_{i=1}^{m}\|s\|\|B_{2i}\|\bar{u}_{fi}, \tag{43}$$

$$s^TGD_1w(t) = \sum_{i=1}^{q}s^TGD_{1,i}w_i \leq \sum_{i=1}^{q}\|s\|\|GD_{1,i}\|\bar{w}_i. \tag{44}$$

Substituting (42)-(44) into (41) yields

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V_1(t)\}\\ &\quad\leq \|s\|\Big[\|GE\|\|H\| + \|B_2(\hat{\rho} - I)K\|\|x(t)\| + \bar{\alpha}\|GA_\tau\|\\ &\qquad\times\|x(t-\tau_1(t))\| + (1-\bar{\alpha})\|GA_\tau\|\|x(t-\tau_2(t))\|\Big]\\ &\qquad - \sum_{i=1}^{m}s^TB_{2i}K_ix(t)\tilde{\rho}_i + s^TB_2\rho u_1(t)\\ &\qquad + \sum_{i=1}^{m}\|s\|\|B_{2i}\|\bar{u}_{fi} + \sum_{i=1}^{q}\|s\|\|GD_{1,i}\|\bar{w}_i.\end{aligned} \tag{45}$$

Considering the several terms in (45) and taking (36) into account, we have

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V_1(t)\}\\ &\quad\leq \|s\|\Big[\|GE\|\|H\| + \|B_2(\hat{\rho} - I)K\|\|x(t)\| + \bar{\alpha}\|GA_\tau\|\\ &\qquad\|x(t-\tau_1(t))\| + (1-\bar{\alpha})\|GA_\tau\|\|x(t-\tau_2(t))\|\Big]\end{aligned}$$

$$
\begin{aligned}
&-\sum_{i=1}^{m} s^T B_{2i} K_i x(t)\tilde{\rho}_i + s^T B_2 \rho u_1(t)\\
&+\sum_{i=1}^{m} \|s\|\|B_{2i}\|(\hat{u}_{fi} - \tilde{u}_{fi})\\
&+\sum_{i=1}^{q} \|s\|\|GD_{1,i}\|(\hat{w}_i - \tilde{w}_i). \qquad (46)
\end{aligned}
$$

Furthermore, noticing the formula of $u_1(t)$, we get

$$
s^T B_2 \rho u_1(t) = -\eta s^T B_2 \rho B_2^T \operatorname{sign}(s) \leq -\eta\mu\|s\|_1. \qquad (47)
$$

Noticing that $\|s\|_1 \geq \|s\|$ and combining (46) with (47) result in

$$
\begin{aligned}
\mathbb{E}\{\mathcal{L}V_1(t)\} \leq& -\epsilon\|s\| - \sum_{i=1}^{m} s^T B_{2i} K_i x(t)\tilde{\rho}_i\\
&-\sum_{i=1}^{q} \|s\|\|GD_{1,i}\|\tilde{w}_i - \sum_{i=1}^{m} \|s\|\|B_{2i}\tilde{u}_{fi}. \qquad (48)
\end{aligned}
$$

Moreover, in view of the adaptive laws (36) and noticing the fact (38), one can further get

$$
\begin{aligned}
&\mathbb{E}\{\mathcal{L}V(t)\}\\
&\leq -\epsilon\|s\| - \sum_{i=1}^{m} s^T B_{2i} K_i x(t)\tilde{\rho}_i - \sum_{i=1}^{q} \|s\|\|GD_{1,i}\|\tilde{w}_i\\
&\quad -\sum_{i=1}^{m} \|s\|\|B_{2i}\tilde{u}_{fi} + \sum_{i=1}^{m} \frac{\tilde{\rho}_i\dot{\tilde{\rho}}_i}{\gamma_{0i}} + \sum_{i=1}^{m} \frac{\tilde{u}_{fi}\dot{\tilde{u}}_{fi}}{\gamma_{1i}}\\
&\quad +\sum_{i=1}^{q} \frac{\tilde{w}_i\dot{\tilde{w}}_i}{\gamma_{2i}} = -\epsilon\|s\|. \qquad (49)
\end{aligned}
$$

It means that $V(t)$ is decrease when $s(x) \neq 0$. Thus the state trajectory will arrive at sliding manifold $\mathscr{S}$ in finite time and force to remain there subsequently. ■

Remark 4: It should be noticed that the large initial values of $\hat{\rho}_{i0}$, $\hat{u}_{fi0}$ and $\hat{w}_{i0}$ can compensate the effect of faults and external disturbance in the systems. Also, we can confine the sliding function to the convergence set in finite time by adaptive laws, even though the bound of the actuator fault is unknown.

Remark 5: So far, a new sliding-mode-based FTC scheme for delayed system has been proposed. Owing to the existence of probabilistic delay, actuator fault and bounded-unknown disturbance, the existing techniques using variant rate of delay, exact actuator information and upper bound of disturbance become invalid here. Accordingly, some challenges/difficulties are faced in the process of deriving the new results. To be specific, 1) how to utilize the information of variation range of delay rather than the variation rate of delay; 2) how to reduce the effect caused by actuator fault; and 3) how to cope with bounded-unknown disturbance and stuck fault effectively. To overcome these obstacles, the information of variation range of delay has been used with the help of Lemmas 1-2 to guarantee the stability. Moreover, a new sliding mode controller with adaptive mechanism has been proposed to attenuate the effects from actuator and bounded-unknown disturbance. Specifically, the integral switching surface is firstly given by introducing the gain matrix and design matrix. With the help of Lemmas 1-2, the stability for the sliding motion is then guaranteed and an algorithm for calculating gain matrix is presented. Finally, the adaptive laws for estimating the unknown parameters are presented to compensate the effects from actuator fault and external disturbance. It is worthwhile to note that the proposed method is analyzed from the viewpoint of the theoretical analysis and verified by the rocket fairing structural acoustic model given later.

IV. A PRACTICAL EXAMPLE

Example 1: In this section, we consider a practical example on rocket fairing structural acoustic [6], [35].

The following parameters are considered:

$$
A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415\\ -0.1980 & -0.115 & -0.0318 & 0.3\\ -3.0500 & 1.1880 & -0.4650 & 0.9\\ 0 & 0.0805 & 1 & 0 \end{bmatrix},
$$

$$
A_\tau = \begin{bmatrix} -0.04 & 0.0311 & 0.012 & -0.53\\ 0.03 & -0.71 & 0 & -1.02\\ 0.10 & 0.01 & -1.503 & 0.40\\ 0.02 & 0 & 0.1 & -0.87 \end{bmatrix},
$$

$$
B = \begin{bmatrix} 1 & 1.55 & 0.75\\ 0.975 & 0.8 & 0.85\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix},
$$

$$
D_1 = \begin{bmatrix} 0.1563 & -0.0876\\ 0.3067 & 0.2562\\ 0 & -0.2155\\ -0.3144 & -0.2362 \end{bmatrix}, \quad E = \begin{bmatrix} 1\\ 0\\ 0\\ 0 \end{bmatrix},
$$

$$
H = \begin{bmatrix} 0 & 1 & 1 & 0 \end{bmatrix}, \quad F = 0.1,
$$

$$
C = \begin{bmatrix} 0.1324 & -0.1121 & 0.0132 & 0.4015\\ -0.1534 & -0.5427 & 0.0237 & -0.0064 \end{bmatrix},
$$

$$
D_2 = \begin{bmatrix} 0.1437 & 0.0983\\ 0.2812 & 0.2631 \end{bmatrix},
$$

$$
A_{\tau 1} = \begin{bmatrix} -0.1532 & 0.321 & 0.0054 & -0.1561\\ -0.1049 & -0.2927 & 0.1739 & 0.0602 \end{bmatrix}.
$$

In addition, consider the variation range of time delay as $\tau_1 = 0.1$, $\tau_2 = 0.3$ and the corresponding probability is $\text{Prob}\{\alpha(t) = 1\} = 0.95$. Furthermore, set the bounds of the actuator efficiency as $\underline{\rho} = \text{diag}\{0.65, 0.7, 0.7\}$ and $\bar{\rho} = \text{diag}\{0.8, 0.8, 0.8\}$. It is assumed that the third actuator accurse stuck fault between $15s$-$15.3s$ with stuck fault $u_{f2}(t) = 20 + 10\cos(20(t-5))$ and the disturbance $w(t)$ is set as

$$
w(t) = \begin{bmatrix} \frac{1}{(1+t)^2} & \frac{1}{1+\exp(0.5t)} \end{bmatrix}^T.
$$

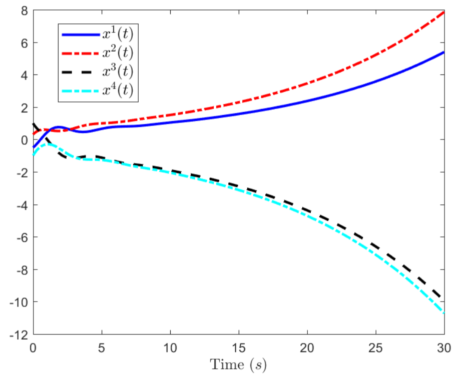

FIGURE 2. State response of the open-loop system.

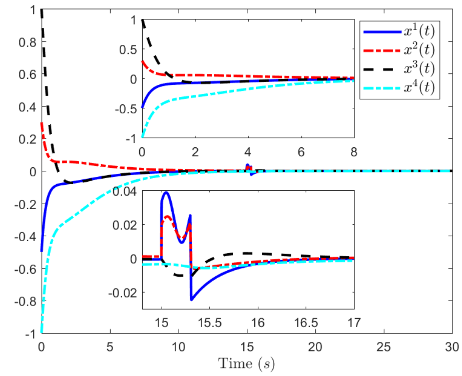

FIGURE 3. The state trajectory of the closed-loop system.

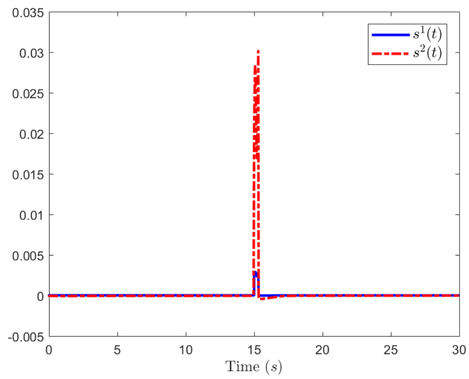

FIGURE 4. The trajectories of the sliding function.

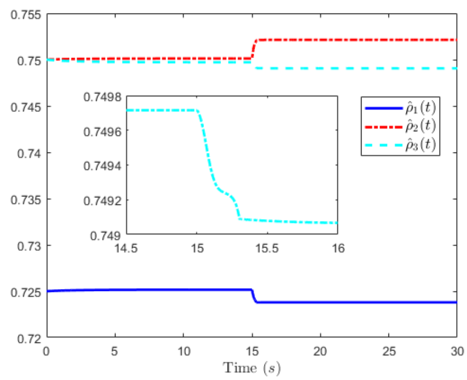

FIGURE 5. The estimations of actuator efficiency factors.

Let the adaptive gains in (30) be $\gamma_{0i} = 0.01$, $\gamma_{1i} = 0.01$, $\gamma_{2i} = 0.01$ $(i = 1, 2)$. Then, select

$$B_1 = \begin{bmatrix} 1 & 1.55 \\ 0.975 & 0.8 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 & 0 & 1.0088 \\ 0 & 1 & -0.1670 \end{bmatrix},$$

which satisfy $B = B_1 B_2$. Accordingly, choose

$$G = \begin{bmatrix} -1.1248 & 2.1793 & 0 & 0 \\ 1.3708 & -1.4060 & 0 & 0 \end{bmatrix}.$$

Based on the results in Theorems 2-3, solving the Algorithm yields

$$X = \begin{bmatrix} -3.0806 & -4.9684 & 0.1177 & -0.7103 \\ 0.5072 & 0.8231 & -0.0196 & 0.1172 \\ 3.0561 & 4.9222 & -0.1165 & 0.7039 \end{bmatrix} \times 10^3,$$

$$S = \begin{bmatrix} 0.5998 & 0.1796 & 0.0721 & 0.1612 \\ 0.1796 & 0.8974 & -0.0901 & -0.0854 \\ 0.0721 & -0.0901 & 0.8894 & -0.0604 \\ 0.1612 & -0.0854 & -0.0604 & 0.9691 \end{bmatrix}.$$

Then, by calculating $K = XS^{-1}$ leads to

$$K = \begin{bmatrix} -3.4931 & -4.9060 & -0.1219 & -0.5921 \\ 0.5729 & 0.8140 & 0.0208 & 0.0987 \\ 3.4686 & 4.8586 & 0.1199 & 0.5853 \end{bmatrix} \times 10^3.$$

In the simulation, select the initial conditions as $x_0 = \begin{bmatrix} -0.5 & 0.3 & 1 & -1 \end{bmatrix}^T$, $\hat{\rho}_i(0) = (\underline{\rho}_i + \bar{\rho}_i)/2$, $\hat{u}_{f1}(0) = 0.1$, $\hat{u}_{f2}(0) = 0.2$, $\hat{u}_{f3}(0) = 0.15$, $\hat{w}_1(0) = 0.1$, $\hat{w}_2(0) = 0.2$. To eliminate the phenomenon of chattering, we take the strategy by replacing $\mathrm{sign}(s(x))$ with $\mathrm{sat}(s(x))$ satisfying

$$\mathrm{sat}(s(x)) = \begin{cases} 1 & \text{if } s(x) > d, \\ s(x)/d & \text{if } -d \le s(x) \le d, \\ -1 & \text{if } s(x) < d. \end{cases}$$

Select $d = 0.003$ and index $\gamma = 3$, then we know all parameters and the simulation results can be depicted in Figs. 2-8. The state responses without control are shown in Fig. 2, it can be seen that the original open-loop system is unstable. However, based on the proposed control method, the state trajectory of the closed-loop system presented in Fig. 3 is stable ultimately. The estimations of the actuator efficiency factors are presented in Fig. 5. Figs. 6-7 depict the estimations of the bounds of $u_f(t)$ and $w(t)$.

On the other hand, a sever fault scenario occurred at $t = 15s$, which leads to suddenly tremor, but doesn't affect the ultimate stability. At the same time, the adaptive laws changed suddenly to adapt the sever fault. It should be noticed that the estimations $\hat{\rho}$, $\hat{u}_f$ and $\hat{w}$ are not necessary to converge to their exact values. Noting the depicting in Fig. 4-7, the adaptive laws do not converge to the ideal values within

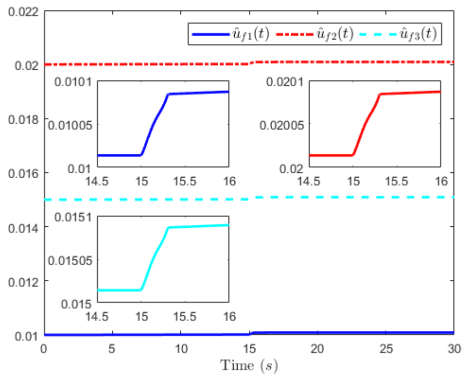

FIGURE 6. The estimation $\hat{u}_f(t)$.

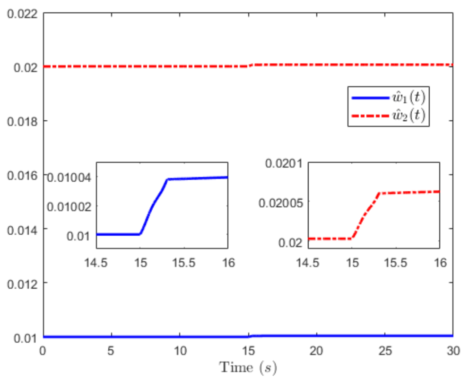

FIGURE 7. The estimation $\hat{w}(t)$.

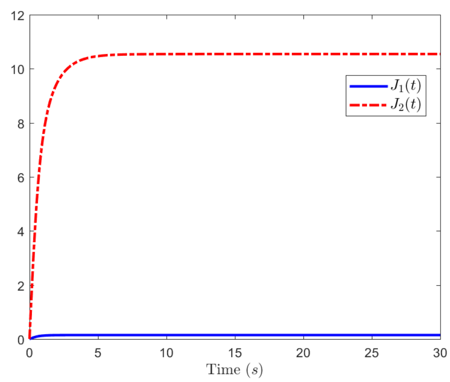

FIGURE 8. The responses of $J_1(t)$ and $J_2(t)$.

the first few seconds which causes a small deviation of the sliding function in Fig. 4 and the sliding mode takes place after the convergence of the adaptive laws. It further shows the efficiency of the proposed ISM control scheme. Moreover, noting Fig. 3, we can see that the sudden actuator fault leads to a transient instability which further causes a small deviation of sliding surface in Fig. 4. Then, in view of the adaptive laws in (36), it reveals that the parameters $\hat{u}_{fi}(t)$, $\hat{w}_i(t)$ increase and $\hat{\rho}_i(t)$ changes in a moment which is consistent with Figs. 5-7. Introducing $J_1(t) = \int_0^t z^T(s)z(s)ds$ and $J_2(t) = \gamma^2 \int_0^t w^T(s)w(s)ds$, the responses of $J_1(t)$ and $J_2(t)$ are depicted in Fig. 8, which further explains that the desired H_∞ performance can be achieved. Overall, it follows from the simulation results that the validity of the achieved FTC scheme is revealed.

V. CONCLUSION

In this paper, the sliding mode control problem has been discussed for uncertain system with the probabilistic random delays, actuator fault and external disturbance. By fully utilizing the information of the probability distribution delay, the asymptotically stable under satisfactory H_∞ performance has been guaranteed when the state trajectory enters into the sliding surface. An algorithm has been firstly given to transform the traditional nonconvex problem into a convex one, then new sufficient results have been obtained. In the meanwhile, a novel adaptive ISM controller has been designed, which not only attenuate the effect of actuator fault but guarantee the stability with probabilistic delay and unknown external noises in a unified framework as well. Further topics include the discussions on the designs of FTC with time-varying actuator efficiency factor and communication protocols.

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