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A New Distance Measure of Belief Function in Evidence Theory

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ABSTRACT How to measure the similarity or distance between the basic probability assignment (BPA) in evidence theory is an open issue. The existing evidence distance function has the shortcoming that the cardinality of each subset is not reasonably considered. To address this issue, a new similarity coefficients matrix is presented to model the cardinality of each subset. Based on the proposed similarity coefficients matrix, a novel distance measure of belief function is presented. Some numerical examples are used to compare the proposed distance with existing evidence distance. The results show the new evidence distance has better performance. The application of the proposed measure in target recognition based on sensor data fusion illustrates the promising aspect of real engineering.

INDEX TERMS Evidence theory, belief function, distance measure, sensor fusion, target recognition.

I. INTRODUCTION

Evidence theory, also known as D-S evidence theory [1], [2], has been widely used in many fields such as decision making [3]–[6], target recognition [7], risk and reliability analysis [8]–[11], fault diagnosis [12]–[14], game theory [15], information fusion [16]–[19], uncertainty reasoning and modelling [20]–[23] and other fields [24]–[26]. However, counterintuitive results may obtained when using evidence theory to combine highly conflicting evidence [27], [28]. In order to solve this problem, there are two main kinds of methods: The first one is to modify the rule of combination [29], [30], which considers that the counterintuitive result is caused by the unreasonable normalization of the Dempster combination rule. The second is to preprocess the evidences [31]–[33], mainly to assign the different weight of different evidences before the combination of evidences [34]–[37].

Combining uncertainty information is an open issue [38]–[41]. In order to improve the credibility of the evidence combination results, the effect of conflict evidence on the final fusion result should be reduced. Many methods to measure the similarity between evidences have been proposed to model the conflict degree [42]–[45]. Some of them are based on evidence distance [46]–[48]. As a result, evidence distance has been heavily explored [49], [50]. Bauer [51] introduced two other measures based on pignistic probabilities. Based on bayes a priori distribution matrix, Fixen and Mahler [52] proposed a "classification error distance" between belief functions. Diaz *et al.* [53] proposed a similarity measure between the focal elements used on a distance function of two basic belief assignments in the theory of evidence. The bet commitment distances [54] and the Jousselme *et al.* distance [55] are widely used. The Jousselme *et al.*'s distance is based on the weighted Euclidean distance provided by the geometric interpretation of the evidence theory. Because the calculation of Jousselme *et al.*'s distance is relatively simple and the geometric meaning is clear, it has been widely used to manage conflicts of evidence [56].

The Jousselme *et al.*'s distance [55] measures the difference of evidences by comparing the probability assignment of the evidence and the similarity between the different subsets, basically including all the factors that reflect the information in the evidence. In order to reflect the similarity between subsets, the Jousselme *et al.* [55] distance uses the Jaccard similarity coefficient to modify the probability assignment of each subset, and finally chang the distance between evidences. The more similar the subsets of the two evidences are, and the smaller the difference between the probability assignments of the two evidences for the same subset, the smaller the distance between the two evidences. However, the Jaccard coefficient

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does not consider the number of elements contained in a single subset when calculating the similarity between sets. As a result, the subset has changed, but the distance between the evidences does not change, this will reduce the sensitivity of judging the difference in evidence. This paper will give an example to illustrate this problem.

In this paper, a new similarity coefficient is proposed to replace the Jaccard similarity coefficient in Jousselme *et al.*'s distance [55]. The examples show that the distance proposed in this paper can effectively judge the difference between evidences and more sensitive to the change of evidences. It can obviously improve the convergence speed of information fusion.

The rest of the paper is organized as follows. In Section 2, the related knowledge of D-S theory and Jousselme *et al.*'s distance [55] are briefly introduced. In Section 3, a new distance is proposed. In Section 4 some examples are used to demonstrate the feasibility of the evidence distance proposed in this paper. The conclusion is made in Section 5.

II. PRELIMINARIES

In this section, some preliminaries are briefly introduced.

A. DEMPSTER-SHAFER EVIDENCE THEORY

It's inevitable to handle uncertainty in real world [57]–[60]. Many math tools are developed and widely used, such as fuzzy sets [61]–[65], Z numbers [66], belief structure [67], D numbers [68]–[70], R numbers [71], [72], entropy modelling [73]–[76] and rough sets [77]. Evidence theory is one of the most used math tools due to its efficiency ot deal with uncertainty [78]. Let Θ be a set of N mutually exclusive and exhaustive hypotheses, which means the problem has N possible values, and $H_i (i = 1, 2, 3, \ldots, N)$ are used to represent these hypotheses. The following set is called the frame of discernment [1], [2]

$$\Theta = \{H_1, H_2, \ldots, H_N\}. \tag{1}$$

$P(\theta)$ is the power set composed of 2^N elements A of θ, representing the object is in A

$$P(\Theta)\{\phi, H_1, H_2, \ldots, H_N, (H_1, H_2), (H_1, H_3), \ldots, (H_{N-1}, H_N), \ldots, (H_1, H_2, H_3), \ldots, \Theta\}. \tag{2}$$

A basic probability assignment (BPA) is a function from $P(\theta)$ to [0, 1] defined by:

$$m : P(\Theta) \to [0, 1] \tag{3}$$

and which satisfies the following conditions:

$$\sum_{A \in P(\Theta)} m(A) = 1, \tag{4}$$

$$m(\phi) = 0. \tag{5}$$

where $m(A)$ represents the belief to A [79].

Suppose two evidences m_1 and m_2, which can be combined into a new evidential body m through Dempster combination rule

$$m(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - k} \tag{6}$$

with

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C) \tag{7}$$

k represents the degree of conflict between evidences and also called normalization constant. When k=1, the Dempster combination rule cannot be used. The following examples illustrate that the traditional conflict coefficient does not measure the conflict between two evidences well enough. evidence theory cannot used to combination highly conflicting evidence

Example 1: The frame of discernment Θ is {A, B, C}, the BPAs of the two evidence bodies are as follows [28]:

$$m_1(A) = 0.99, \quad m_1(B) = 0.01, \; m_1(C) = 0;$$
$$m_2(A) = 0, \quad m_2(B) = 0.01, \; m_2(C) = 0.99.$$

Although these two evidences have low support for subset B, the result of calculated BPA of B is equal to 1, which is obviously not credible, so counterintuitive results would obtained when using evidence theory to combine highly conflicting evidence. Recently, more and more people use evidence distance to measure the conflict of evidences, and evidence distance is more effective than k when dealing with highly conflicting evidences [80], [81]. The distance is proportional to the degree of conflict between evidences, the greater the conflict between the evidence bodies, the less reliable the results obtained by the Dempster combination rule. Then let's briefly review the existing distance used to deal with conflicting evidence.

B. EXISTING EVIDENCE DISTANCE

Similarity and distance measure play a very important role in real application [82], [83]. How to measure the similarity or distance between two mass functions has been paid great attention [55]. Let m_1 and m_2 be two BPAs on the same frame of discernment Θ, containing N mutually exclusive and exhaustive hypotheses. The distance between m_1 and m_2 is [55]

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\overrightarrow{m_1} - \overrightarrow{m_2})^T \underline{\underline{D}}(\overrightarrow{m_1} - \overrightarrow{m_2})} \tag{8}$$

where $\overrightarrow{m_1}$ and $\overrightarrow{m_2}$ are the associated vectors of BPAs, m_1 and m_2 and $\underline{\underline{D}}$ is a $2^N \times 2^N$ matrix whose elements are

$$D(A, B) = \frac{|A \cap B|}{|A \cup B|},$$
$$A, B \in P(\Theta).$$

Another way to represent d_{BPA} is

$$d_{BPA}(m_1, m_2) = \sqrt{\frac{1}{2}(\|\overrightarrow{m_1}\|^2 + \|\overrightarrow{m_2}\|^2 - 2\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle)} \tag{9}$$

where $\|\overrightarrow{m}\|^2 = \langle \overrightarrow{m}, \overrightarrow{m} \rangle$,and $\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle$ is the scalar product defined by

$$\langle \overrightarrow{m_1}, \overrightarrow{m_2} \rangle = \sum_{i=1}^{2^N} \sum_{j=1}^{2^N} m_1(A_i) m_2(A_j) \frac{|A_i \bigcap A_j|}{|A_i \bigcup A_j|} \tag{10}$$

with $A_i, A_j \epsilon P(\Theta), i, j = 1, 2, \ldots, 2^N$.

It's reasonable to considering the weigh of the factor or evidence in complex systems [84], [85]. In conflicting management, weight also can be determined by the evidence distance function or other machine learning method [86].

III. THE PROPOSED EVIDENCE DISTANCE

In this section, the shortcoming of existing distance measure is analyzed firstly. Then, a new distance of belief function is proposed.

A. THE SHORTCOMING OF EXISTING EVIDENCE DISTANCE

In order to reflect the similarity between subsets, Jousselme *et al.*'s distance introduces matrix $\underline{\underline{D}}$ and modifies the basic probability assignment of two evidences by matrix $\underline{\underline{D}}$. So as to obtain the distance that can reflect the similarity between two evidences. The elements in $\underline{\underline{D}}$ are calculated by the Jaccard similarity coefficient.

For better explanation, the principle of the jaccard similarity coefficient is represented by venn diagram, which is often used to reflect the relationship of sets. The premise is the points in the circle are evenly distributed. The two venn diagrams in Fig. 1 represent two different sets relationship. A circle represent a subset. The number of elements contained in the subset determines the area of the circle. The intersection of two circles represents the same part of the two sets, and areas without intersections represent different parts of the two sets. The combined region of sets A and B is denoted by $|A \bigcup B|$, The region in both A and B, where the two sets overlap is denoted by $|A \bigcap B|$. In Fig. 1, because $|A \bigcap B| = 0$, so set A and B are completely different. Fig. 1 is circles with unequal areas, though $|C \bigcap A| > |B \bigcap A|$, but the area of the circle C is large, the overlap portion is smaller compared to the area of the circle C, so the similarity of the two circles is not large. Though$|C \bigcap B|$ is small, the areas of the circle C and B are relatively small, so the similarity is rather large. It can be concluded that the number of elements contained in a single set of collections has an impact on the size of similarity between sets. The following example is used to illustrate the shortcoming of the existing distance.

Example 2: Given a frame of discernment Θ is {1,2, ...,10}, the BPAs of the two evidence bodies are as follows:

$$m_1(M) = 1.$$
$$m_2(N) = 1.$$

M changed from {1}, {12} to ...{1, 2, ..., 10}. The results are shown in Tab. 1 and Fig. 2.

It can be seen that the two sets measured have changed, but the results do not reflect this change.

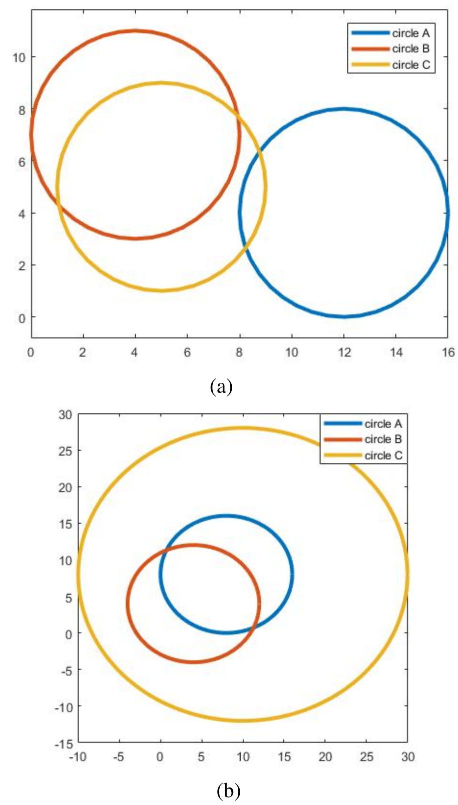

FIGURE 1. Use venn diagram to show the relationship between finite groups of things. (a) Circles with equal area. (b) Circles with unequal area.

TABLE 1. The results of example 2 obtained by Jousselme distance.

M	N	Jousselme *et al.*' [55]
{1}	{1,2,3,...,10}	0.9487
{1,2}	{2,3,4,...,10}	0.9487
{1,2,3}	{3,4,5,...,10}	0.9487
{1,2,3,4}	{4,5,6,...,10}	0.9487
{1,2,3,...,5}	{5,6,7,...,10}	0.9487
{1,2,3,...,6}	{6,7,8,...,10}	0.9487
{1,2,3,...,7}	{7,8,9,10}	0.9487
{1,2,3,...,8}	{8,9,10}	0.9487
{1,2,3,...,9}	{9,10}	0.9487
{1,2,3,...,10}	{10}	0.9487

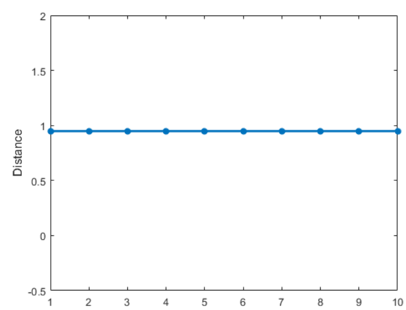

FIGURE 2. The result of example 2.

B. THE NEW DISTANCE MEASURE

Belief function, or mass function, is a generalization of probability distribution. For the two subsets, whether the number of elements contained in the subset changes or the number of identical elements in the two subsets changes, it should be reflected in the similarity. Therefore, this paper proposes a new coefficient of similarity between subsets. The main idea is to measure the similarity by specific to a single subset, so both subsets can determine the degree of similarity.

Definition 1: Given subsets A and B, the similarity coefficients matrix $\underline{\underline{D_\alpha}}$ is defined with the elements are as follows

$$D_\alpha(A,B) = \frac{|A\bigcap B|}{|A|} \times \frac{|A\bigcap B|}{|B|} \tag{11}$$

$\frac{|A\bigcap B|}{|A|}$ represents the proportion of the parts that are shared between two sets in each set, it can be understood as the similarity between $|A\bigcap B|$ and $|A|$. Jaccard similarity coefficient uses the union of two sets as the denominator, and the union of the sets is used as the numerator, so the characteristics of each individual set are easily ignored in the calculation process.

Definition 2: The new evidence distance is defined as

$$d_\alpha(m_1,m_2) = \sqrt{\frac{1}{2}(\vec{m_1}-\vec{m_2})^T \underline{\underline{D_\alpha}}(\vec{m_1}-\vec{m_2})}, \tag{12}$$

and $\underline{\underline{D_\alpha}}$ is used to describe the similarity between the subsets of Θ.

Equation (8) can be transformed as follows

$$d_\alpha(m_1,m_2) = \sqrt{\frac{1}{2}(\|\vec{m_1}\|^2+\|\vec{m_2}\|^2-2\langle\vec{m_1},\vec{m_2}\rangle)}, \tag{13}$$

and

$$\langle\vec{m_1},\vec{m_2}\rangle = \sum_{i=1}^{2^N}\sum_{j=1}^{2^N} m_1(A_i)m_2(A_j)(\frac{|A\bigcap B|}{|A|}\times\frac{|A\bigcap B|}{|B|}) \tag{14}$$

with $A_i, A_j \epsilon P(\Theta)$, $i,j = 1,2,\ldots,2^N$.

Next we prove that the proposed distance satisfies the following requirements for any vectors made of BPAs.

1) Nonnegativity: $d(m_1,m_2) \geq 0$.
2) Symmetry: $d(m_1,m_2) = d(m_2,m_1)$.
3) Triangle inequality: $d(m_1,m_2) \leq d(m_1,m_3) + d(m_2,m_3)$.

First to prove that the new distance satisfies the nonnegative property.

Proof 1: When all subsets in the body of evidence are different, we can obtained tha

$$d_\alpha(m_1,m_2) = \sqrt{\frac{1}{2}(\vec{m_1}-\vec{m_2})^T(\vec{m_1}-\vec{m_2})}$$

Obviously, in this case $d_\alpha(m_1,m_2) \geq 0$. Otherwise, For $\underline{\underline{D_\alpha}}$ is positive definite.

$$\underline{\underline{D_\alpha}} = C^T C \tag{15}$$

where $C \in R^{2^N\times 2^N}$ is an invertible matrix, So we can obtain the following equation

$$\begin{aligned} d_\alpha(m_1,m_2) &= \sqrt{\frac{1}{2}(\vec{m_1}-\vec{m_2})^T C^T C(\vec{m_1}-\vec{m_2})} \\ &= \sqrt{\frac{1}{2}(C(\vec{m_1}-\vec{m_2}))^T(C(\vec{m_1}-\vec{m_2}))} \\ &= \frac{\sqrt{2}}{2}\|C(\vec{m_1}-\vec{m_2})\| \end{aligned} \tag{16}$$

and $\|C(\vec{m_1}-\vec{m_2})\|$ represents the modulus of the vector, The role of matrix C is to change the modulus of the vector $(\vec{m_1}-\vec{m_2})$, the greater the similarity between subsets, the more obvious the scaling effect of matrix C will be. As $\|C(\vec{m_1}-\vec{m_2})\| \geq 0$, so it is proved that $d(m_1,m_2) \geq 0$.

Then to prove that the new distance satisfies the symmetry property.

Proof 2:

$$\begin{aligned} d_\alpha(m_2,m_1) &= \sqrt{\frac{1}{2}(\vec{m_2}-\vec{m_1})^T \underline{\underline{D_\alpha}}(\vec{m_2}-\vec{m_1})} \\ &= \sqrt{\frac{1}{2}[-(\vec{m_1}-\vec{m_2})^T]\underline{\underline{D_\alpha}}[-(\vec{m_1}-\vec{m_2})]} \\ &= \sqrt{\frac{1}{2}(\vec{m_1}-\vec{m_2})^T \underline{\underline{D_\alpha}}(\vec{m_1}-\vec{m_2})} \\ &= d_\alpha(m_1,m_2) \end{aligned} \tag{17}$$

Thus, new distance satisfies symmetry.

Next to prove that the new distance satisfies the Triangle inequality.

Proof 3: Take 2-norm vector as an example, the BPAs of the two evidence bodies are as follows:

$$\begin{aligned} m_1(A) &= 0.4, m_1(AB) = 0.6; \\ m_2(A) &= 0.9, m_2(AB) = 0.1; \\ m_3(A) &= 0.7, m_3(AB) = 0.3 \end{aligned}$$

then we have

$$\underline{\underline{D_\alpha}} = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$$

The Euclidean distance between $\vec{m_1}$ and $\vec{m_2}$ is

$$\|(\vec{m_1}-\vec{m_2})\| = \sqrt{(-0.5,0.5)^T(-0.5,0.5)} = \sqrt{0.5}$$

The Euclidean distance between $\vec{m_2}$ and $\vec{m_3}$ is

$$\|(\vec{m_2}-\vec{m_3})\| = \sqrt{(0.2,-0.2)^T(-0.2,0.2)} = \sqrt{0.8}$$

According to the new distance

$$\begin{aligned} \|C(\vec{m_1}-\vec{m_2})\| &= \sqrt{(-0.5,0.5)^T \underline{\underline{D_\alpha}}(-0.5,0.5)} \\ &= \sqrt{(-0.25,0.25)^T(-0.5,0.5)} \\ &= \sqrt{0.5\times 0.5} \\ &= \sqrt{0.5}\times\|(\vec{m_1}-\vec{m_2})\| \end{aligned} \tag{18}$$

$$\begin{aligned}\|C(\overrightarrow{m_1}-\overrightarrow{m_2})\| &= \sqrt{(-0.2, 0.2)^T \underline{\underline{D_\alpha}}(-0.2, 0.2)} \\ &= \sqrt{(-0.1, 0.1)^T (-0.2, 0.2)} \\ &= \sqrt{0.5 \times 0.8} \\ &= \sqrt{0.5} \times \|(\overrightarrow{m_2}-\overrightarrow{m_3})\| \end{aligned} \tag{19}$$

It is obvious that

$$\|C(\overrightarrow{m_1}-\overrightarrow{m_2})\| \leq \|(\overrightarrow{m_1}-\overrightarrow{m_2})\|$$

This example demonstrates that matrix C changes the modulus of the vector, the similarity between the two sets is reflected by the matrix $\underline{\underline{D_\alpha}}$ to the distance value.

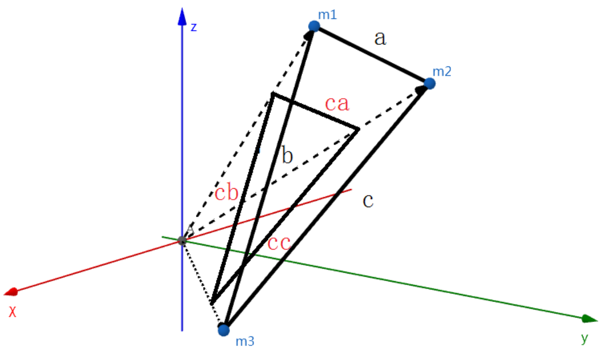

FIGURE 3. Proof using three dimensional space vector.

Next we use 3-norm vectors to prove that the new distance satisfy the triangle inequality. In Fig. 3, let $a = \|(\overrightarrow{m_1}-\overrightarrow{m_2})\|$, $b = \|(\overrightarrow{m_1}-\overrightarrow{m_3})\|$, $c = \|(\overrightarrow{m_2}-\overrightarrow{m_3})\|$, $ca = \|C(\overrightarrow{m_1}-\overrightarrow{m_2})\|$, $cb = \|C(\overrightarrow{m_1}-\overrightarrow{m_3})\|$, $cc = \|C(\overrightarrow{m_2}-\overrightarrow{m_3})\|$.

According to the side length relationship of the triangle, we have $a < b + c$. Only when $m_1 = m_3$ or $m_2 = m_3$, $d_\alpha(m_1, m_2) = d_\alpha(m_1, m_3) + d_\alpha(m_2, m_3)$. otherwise, $d_\alpha(m_1, m_2) < d_\alpha(m_1, m_3) + d_\alpha(m_2, m_3)$. Thus, it is proved that the 3-norm satisfies the triangle inequality, in the same way, we can conclude that the proposed distance satisfies the triangle inequality under other conditions.

IV. EXAMPLES AND APPLICATION

In this section, some numerical examples and the real application in target recognition are used to illustrate the efficiency of the proposed distance function.

A. NUMBERICAL EXAMPLES

Example 3: The data is the same as Example 2. Given a frame of discernment Θ is {1,2,...,10}, the BPAs of the two evidence bodies are as follows:

$$m_1(M) = 1.$$
$$m_2(N) = 1.$$

M changed from {1}, {1, 2} to ... {1, 2, ..., 10}, In contrast, N changed from {1, 2, ..., 10} to {1}. The results of our proposed measure are shown in Tab. 3 and Fig. 4.

TABLE 2. The results of example 2 obtained by new distance.

M	N	New distance
{1}	{1,2,3,...,10}	0.9487
{1,2}	{1,2,3,...,9}	0.9718
{1,2,3}	{1,2,3,...,8}	0.9789
{1,2,3,4}	{1,2,3,...,7}	0.9820
{1,2,3,...,5}	{1,2,3,...,6}	0.9832
{1,2,3,...,6}	{1,2,3,...,5}	0.9832
{1,2,3,...,7}	{1,2,3,4}	0.9820
{1,2,3,...,8}	{1,2,3}	0.9789
{1,2,3,...,9}	{1,2}	0.9718
{1,2,3,...,10}	{1}	0.9487

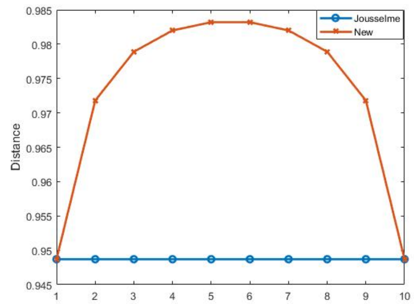

FIGURE 4. The comparison of the two distances.

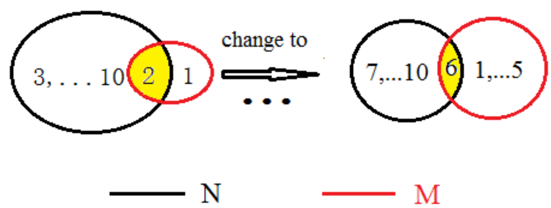

FIGURE 5. Situation that both evidences change.

Although both subset M and subset N are changing, the number of elements shared between the two sets is unchanged. Venn diagram is used to show the relationship between the two subsets, as shown in Fig. 5. Under the premise of ensuring correctness, Drawing a venn diagram to show the relationship of four evidence groups in the example. On the premise of correctness, we only give the venn diagram corresponding to the subset $M = \{1, 2\}$ and $M = \{1, 2, 3, 4, 5, 6\}$. Two circles are used to represent the different subsets, the intersection of the two circles represents the part shared by the two sets, and the numbers in the circle represent the elements in the subset.

It can be seen from the Fig. 3 that the results obtained by the two distances are not much different, but there is a significant difference in the trend between the two curve. The result of calculating with the Jousselme *et al.*'s distance is equal to a constant and does not change with the change of subsets. This is because the Jousselme *et al.*'s distance does not consider the change of a single subset when calculating the similarity, so it does not reflect the specific value of the difference between the two subsets. The value calculated by the new distance is correspondingly changed with the subset. The symmetry of the curve is exactly the same as the trend of evidence. By measuring the difference by specific to a single subset, so the new evidence distance is more reasonable in measuring similarity, and it also significantly improves the sensitivity of calculating the similarity between the two evidences.

Example 4: Given a frame of discernment Θ is $\{1, 2, \ldots, 10\}$, the BPAs of the two evidence bodies are as follows:

$$m_1\{A\} = 1.$$
$$m_2\{2, 3, 4, 5, 6, 7, 8\} = 1.$$

With A changed from $\{1\}$ to $\{1, \ldots, 10\}$. The results are shown in Tab. 3 and Fig. 6.

TABLE 3. The results of example 4.

A	Jousselme *et al.*' [55]	New distance
{1}	1	1
{1,2}	0.9354	0.9636
{1,2,3}	0.866	0.8997
{1,2,3,...,4}	0.7906	0.8237
{1,2,3,...,5}	0.707	0.7368
{1,2,3,...,6}	0.6124	0.6362
{1,2,3,...,7}	0.5	0.515
{1,2,3,...,8}	0.3536	0.3536
{1,2,3,...,9}	0.4714	0.4714
{1,2,3,...,10}	0.5477	0.5477

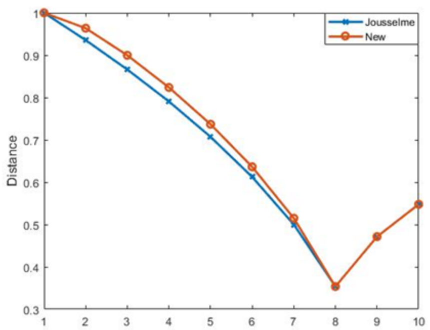

FIGURE 6. Situation that only one set changes.

Dividing the curve into two intervals for discussion according to the number of elements contained in the subset A, Before the curve reaches the lowest point, as the number of elements in the subset A gradually increases, the same part between the two evidences is also reduced. and because the same part between the two evidence does not change, the distance between the two evidences gradually decreases,indicated the similarity between the two evidences gradually increases. When the subset $A = \{1, 2, \ldots, 8\}$, the distance curve reaches lowest point, because the number of the same elements between two evidences is the maximum, while the number of different elements is minimum. Then the number of elements in subset A continues to increase, but the number of identical elements between the two evidences does not change, the value of the evidence distance begins to increase. Since the same part between evidences is equal to m_2, the distance between the two pieces of evidence is determined by m_1, in which case the evidence distance proposed degenerates into Jousselme *et al.*'s distance, so the two curves coincide together.

The above two examples illustrate that the distance proposed in this paper can effectively deal with the various cases of the single subset evidence. Then to prove that the new distance can measure the similarity of the multi-subset evidences equally effectively.

Example 5: Given a frame of discernment Θ is $\{1, 2, \ldots, 15\}$, the BPAs of the two evidence bodies are as follows:

$$m_1(A) = 0.5, m_1(7) = 0.1, m_1(2, 8, 15) = 0.4;$$
$$m_2(1, 2, 3, 4) = 1$$

With A changed from $\{1\}$ to$\{1, 2, \ldots, 15\}$. The results are shown in Tab. 4 and Fig. 7.

TABLE 4. The results of example 5.

A	Jousselme	New
{1}	0.7200	0.7427
{1,2}	0.6658	0.6780
{1,2,3}	0.5553	0.5690
{1,2,3,...,4}	0.4200	0.4401
{1,2,3,...,5}	0.5214	0.5385
{1,2,3,...,6}	0.5788	0.5953
{1,2,3,...,7}	0.6221	0.6366
{1,2,3,...,8}	0.6664	0.6828
{1,2,3,...,9}	0.6830	0.6997
{1,2,3,...,10}	0.6962	0.7130
{1,2,3,...,11}	0.7067	0.7236
{1,2,3,...,12}	0.7153	0.7324
{1,2,3,...,13}	0.7180	0.7397
{1,2,3,...,14}	0.7285	0.7462
{1,2,3,...,15}	0.7440	0.7660

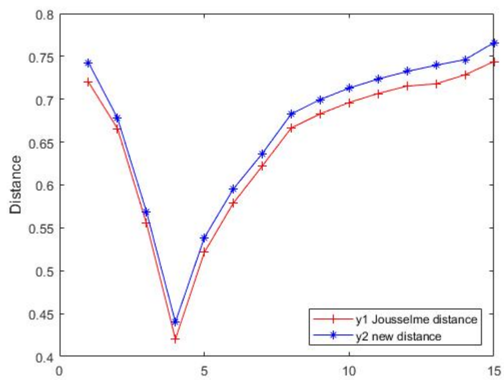

FIGURE 7. Comparison of the distances in measuring the similarity between multiple subsets of evidence.

It can be seen from Fig. 7 that the changing trend of the two distance curves is consistent, and the distance is first decreased and then increased, but when the number of elements in the subset A is relatively small. When the number of elements in subset A is relatively small, the change of distance is relatively large. When the number of elements in subset A increases gradually, the distance between evidences keeps increasing, but the rate of increase gradually slows down. When $A = \{1, 2, 3, 4\}$, the distance reaches the minimum point, because A has a large probability assignment in both evidences, and the difference between the subsets contained in the two evidences is also the smallest. This example shows that the distance function proposed in this paper can effectively measure the similarity between multiple subsets of evidence.

B. TARGET RECOGNITION

Target recognition is a typical sensor data fusion application under uncertainty [87]–[89], and this section will demonstrate the usefulness of the new distance by applying the new distance to the target recognition.

This application is an automatic target recognition method based on sensor data fusion, where the sensors are radars and the targets are flights. The target recognition system is based on Radar/IR seeker, there are two different types of sensor, one is active radar and the other is infrared imaging seeker. After the data processing, the sensor reports are transformed into BPA for futher data fusion. Assume that there are six objects A, B, C, D, E, F in a target recognition system, there are five difference kinds of sensors to observe objects which are sensor(S_1), sensor(S_2), sensor(S_3), sensor(S_4), sensor(S_5). The evidences obtained from these kinds of sensors are shown in Tab. 5.

TABLE 5. The evidence obtained by sensors.

	{A, B}	{B, C}	{A, B, C, D, E, F}
$S_1 : m_1$	0.5	0.5	0.0
$S_2 : m_2$	0.0	0.5	0.5
$S_3 : m_3$	0.5	0.0	0.5
$S_4 : m_4$	0.5	0.0	0.5
$S_5 : m_5$	0.5	0.0	0.5

TABLE 6. The result obtained by proposed distance.

Distance	m_1	m_2	m_3	m_4	m_5
m_1	0	0.408	0.408	0.408	0.408
m_2	0.408	0	0.433	0.433	0.433
m_3	0.408	0.433	0	0	0
m_4	0.408	0.433	0	0	0
m_5	0.408	0.433	0	0	0

First, we measure the similarity between the three evidences and then obtain the weight of each evidence [90]. The results obtained by Jousselme *et al*. distance are all equal to 0.408, so the weight of each evidence is equal to 0.2. Since the evidence obtained by the sensors S_3, S_4, S_5 is the same with each other, the reliability of those three evidence is higher than that of the other two evidence, and S_3, S_4, S_5 should be given a higher weight. It is obvious that Jousselme *et al*. distance cannot effectively classify these evidences. The result obtained by the proposed distance are shown as Tab.6,

Let's analyze the inequality that $d_\alpha(m_1, m_2) < d_\alpha(m_3, m_2)$, because the more elements a subset contains, the less useful information it can provide when making a decision, so even though m_3(A, B)=m_3(A, B, C,D, E, F), the property of the evidence described by m_1(A, B) is more important. The difference between m_1 and m_2 is caused by (A, B) and (A, B, C, D, E, F), while the difference between m_3 and m_2 is caused by (A, B) and (B, C), so the result that $d_\alpha(m_1, m_2) < d_\alpha(m_3, m_2)$ is reasonable.

The weights assigned to different evidences according to the new distance results are as follows:

$$w_1 = 0.1675$$
$$w_2 = 0.1622$$
$$w_3 = 0.2234$$
$$w_4 = 0.2234$$
$$w_5 = 0.2234$$

Finally, modify the BPAs by weights and combine the weighted averaging evidence four times. The final results are listed in Tab. 7, according to the final results, the real target is A, B. Because the evidence distance proposed in this paper is more sensitive when measure the similarity between evidence, it can find conflict evidence more quickly and accurately, thus minimizing the impact of unreliable evidence when combining evidence. This example proves that the new distance has high sensitivity and can be used in target recognition.

TABLE 7. The evidence combination result of target recognition.

	{A, B}	{B, C}	{A, B, C, D, E, F}
Deng *et al.*' [90]	0.8266	0.1708	0.0026
New distance	0.8558	0.1169	0.0273

TABLE 8. The BPAs of evidences.

m	{1,2}	{2,3}	{3,4,5}	{1,2,3,4,5}
m_1	0.5	0.3	0.2	0.0
m_2	0.0	0.8	0.1	0.1
m_3	0.6	0.2	0.0	0.2
m_4	0.6	0.1	0.0	0.3
m_5	0.65	0.2	0.0	0.15

Then compare the new distance with multiple classic distances. The frame of discernment Θ is $\{1, 2, 3, 4, 5\}$, the BPAs of the two evidence bodies is shown as Tab. 8, the rusults are shown as Tab. 9.

It can be seen from the Tab. 9 that there are significant differences in the results obtained by combining the evidence in different ways. The D-S evidence combination rule produces counterintuitive behavior when dealing with highly conflicting evidence. Only in the evidence m_1, the BPA of the subset $\{1, 2, 3, 4, 5\}$ is equal to 0. In other evidences, the BPA of the subset $\{1, 2, 3, 4, 5\}$ is greater than 0, but the BPA of the subset $\{1, 2, 3, 4, 5\}$ obtained after the fusion is all equal to 0, and the result is not credible. Murphy's simple average fusion method converges faster and robustness is better than the D-S evidence combination rule, but simply averaging the BPA of the evidences and assigning the same weight to all the evidence makes conflict evidences have a greater impact on the combined results. Taking the subset $\{1, 2\}$ as an example, the BPA of the subset $\{1, 2\}$ is still smaller after completing the evidence combination twice, it indicate that the conflict evidence body m_2 still has a great influence on the fusion result. therefore the result of this combination is not good enough.

Compared with the above two methods, the results is more ideal to combine evidences after changing the weight of the

TABLE 9. The results of the evidence combination. For the layout of the table, let A = 1, 2. B = 2, 3. C = 3, 4, 5. D = 1, 2, 3, 4, 5.

rule	$\{m_1, m_2\}$	$\{m_1, m_2, m_3\}$	$\{m_1, \ldots, m_4\}$	$\{m_1, \ldots, m_5\}$
Dempster	m(A)= 0.1388 m(B)=0.7501 m(C)=0.1111 m(D)=0	m(A)=0.2565 m(B)=0.6920 m(C)=0.0515 m(D)=0	m(A)= 0.4412 m(B)=0.5292 m(C)=0.0296 m(D)=0	m(A)= 0.6506 m(B)= 0.3411 m(C)=0.0083 m(D)=0
Murphy [91]	m(A)= 0.1804 m(B)= 0.7372 m(C)= 0.0772 m(D)=0.0052	m(A)= 0.3883 m(B)= 0.5809 m(C)= 0.0269 m(D)=0.0039	m(A)= 0.6279 m(B)= 0.3578 m(C)= 0.0126 m(D)=0.0027	m(A)= 0.7897 m(B)=0.2052 m(C)= 0.0039 m(D)= 0.0012
Deng *et al.*' [90]	m(A)= 0.2690 m(B)= 0.6372 m(C)= 0.0903 m(D)=0.0035	m(A)= 0.5346 m(B)= 0.4345 m(C)= 0.0268 m(D)= 0.0041	m(A)= 0.7581 m(B)= 0.2284 m(C)= 0.0107 m(D)= 0.0028	m(A)= 0.8847 m(B)= 0.1121 m(C)= 0.0025 m(D)= 0.0007
New	m(A)= 0.2681 m(B)= 0.6408 m(C)=0.0876 m(D)=0.0035	m(A)= 0.5483 m(B)= 0.4210 m(C)=0.0266 m(D)=0.0041	m(A)= 0.7686 m(B)= 0.2181 m(C)= 0.0105 m(D)=0.0028	m(A)= 0.8911 m(B)= 0.1063 m(C)= 0.0019 m(D)= 0.0007

evidence, as the BPA of the subset {1, 2} is greater than the result of fusion with the Dempster's method twice after the first fusion.

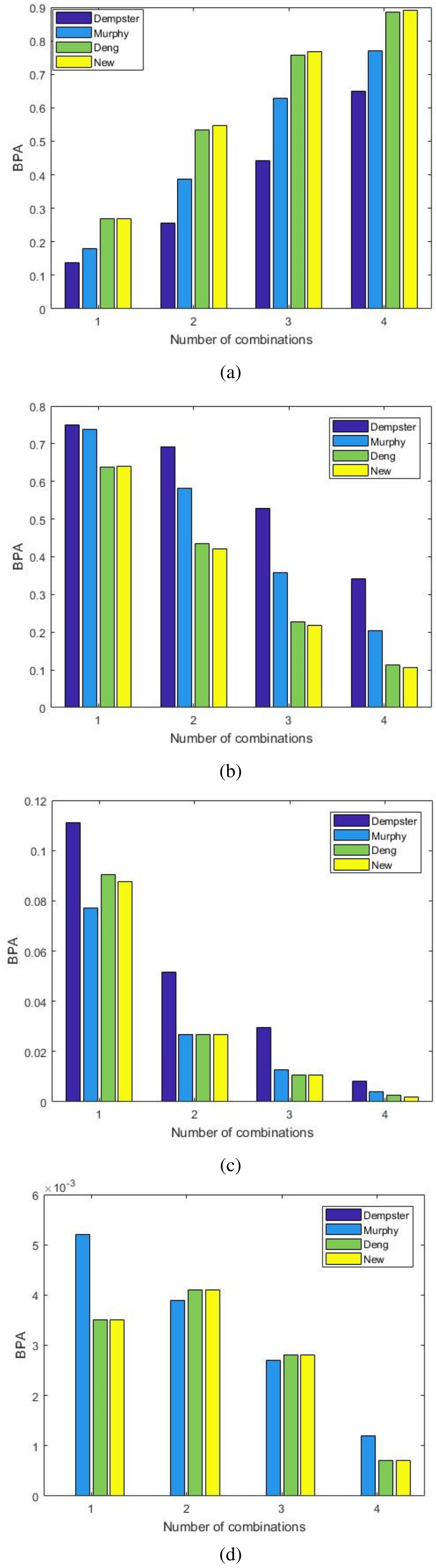

FIGURE 8. Comparison of different combination methods . (a) Combination result of m(1, 2). (b) Combination result of m(2, 3). (c) Combination result of m(3, 4, 5). (d) Combination result of m(1, 2, 3, 4, 5).

Through the Fig. 8, we can see the convergence speed of different methods in the combination of evidence. The BPA of target subset {1, 2} calculated by the proposed method has the fastest rate of increase, after four fusions, m(1, 2) = 0.8911, while the results obtained by other methods are smaller than it. Except for the target subset, the BPAs of other subsets are gradually decreasing as the number of fusions increases, and the BPAs of all non-target subsets obtained by the proposed method is minimal. Due to the existence of conflict evidence m_2, after the combination of evidence m_1, m_2 and m_3, the BPA of {2, 3} is still large. Average the BPA of {2, 3} of evidence m_1, m_2 and m_3, we have m(2, 3) = 4.333, only the distance value obtained by the new method is less than this average value. Because the new distance has high sensitivity in measuring the similarity between evidences, it has a fast convergence speed in the application of target recognition. The above application shows that the proposed method can recognize the target with the minimum number of fusion times, and less affected by conflict evidences, so it is excellent in the sensor data fusion.

V. CONCLUSION

This paper proposes a new distance measure of belief function by the presented similarity coefficients matrix. The new similarity coefficient takes the cardinality of the subsets of the power set. Therefore, the distance of evidence presented in this paper can more sensitively measure the change of evidence. The ability to measure differences between two BPAs is also significantly improved. The application of the proposed measure in target recognition based on sensor data fusion illustrates the promising aspect in real engineering [17], [40], [80], [81], [92].

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