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Finite-Time Stochastic H_∞ Control for Singular Markovian Jump Systems With (x, v)-Dependent Noise and Generally Uncertain Transition Rates

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ABSTRACT This paper is concerned with the problems of finite-time stochastic H_∞ control for singular Itô Markovian jump systems with (x, v)-dependent noise and generally uncertain transition rates (GUTRs). Based on two equivalent sets, a new criterion, ensuring the considered systems with completely known TRs to be finite-time stochastically bounded with H_∞ performance, is first established, which is less conservative than the existing one. Then, the obtained results are extended to the case of GUTRs. To overcome the nonlinear difficulty resulting in the Young inequality for designing a convex controller, an approach called the association of free variables and slack variables are used. The state feedback controller and observer-based controller are respectively designed such that the corresponding closed-loop systems with GUTRs are finite-time stochastically bounded while achieving H_∞ performance. Finally, the numerical examples are addressed to illustrate the effectiveness and efficiency of our obtained results.

INDEX TERMS Singular stochastic Itô systems, Markov jump systems, finite-time H_∞ control, generally uncertain transition rates.

I. INTRODUCTION

During the past two decades, interest on singular stochastic systems driven by the Itô stochastic differential equation has been increasing [1]–[9]. The reason is that such systems possessing double characteristics of singular systems and the Itô stochastic differential equation are much more realistic and advanced than deterministic ones. As is well known, singular systems, also called differential algebraic systems, generalized state-space systems, descriptor or implicit systems, have extensive applications in real areas such as the oil catalytic cracking model, dynamic Leontief model of a multisector economy, biological systems, electric circuits as well as power systems [10]–[12]. At the same time, the environment noise inevitably disturbs any system, and the linear Itô stochastic differential equation has played important roles in many practical fields such as economy, biology, etc, [13]–[18]. To date, abundant results on various control problems of singular systems or linear Itô stochastic systems have been proposed, we refer the reader to [13], [15], [19]–[24] and references therein. It can be found that, however, little works have been made on singular Itô stochastic systems. This is because the essential issue that the condition for existence and uniqueness of a solution to the system equation has not been completely resolved, more details please see, e.g., [2]–[4]. As a result, some suitable conditions have to be assumed to guarantee the existence and uniqueness of solution of this class of stochastic systems.

In some practical applications, we often focus our attention on the transient behavior of a system state response. Followed by this fact, the definition of finite-time stability of systems emerged, which can be specifically described as: the state of system did not exceed some bounds in a prescribed finite-time interval on the condition that the initial state was in

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a fixed bound [25], [26]. Up to now, many important control problems on this subject have been presented. For instance, robust finite-time stabilization for uncertain singular Markovian jump systems were discussed in [27]–[29], and state feedback controller, output feedback controller as well as observed-based controller were respectively designed such that the corresponding closed systems satisfied the finite-time boundedness while achieving H_∞ performance. The finite-time stability and stabilization for singular Itô stochastic systems without Markovian jump parameters was firstly studied in [30], where the concept of finite-time stochastic stability was introduced and a state feedback controller was presented. However, it can be found that all the references mentioned above employed the equality constraint $P_iE^T = EP_i^T$, which can not only lead to some trouble when checking the condition numerically, but also set some obstacles for the analysis of controller. So it is incentive and desirable to substitute that equality constraint with a strict LMI condition.

In another research front line, a great deal of attention has been devoted to a class of Markovian jump systems with generally uncertain transition rates (GUTRs) in recent years. Many encouraging development have been made on this hot topic, such as stability and stabilization [31]–[34], output feedback control [35], H_∞ control [36], [37], passivity and passification [38], delay-dependent H_∞ filtering [39] and so on. The main reason for focusing on such systems can be embodied by the following twofold: one hand, Markovian jump systems can model some dynamics systems whose structure and parameters often suffered random abrupt variations such as component repairs and failures, sudden environmental disturbances [40]–[43]. On the other hand, it is not easy and even impossible to obtain completely exact TRs on the engineering practice. Moreover, as said in [31], uncertain TRs and partially unknown TRs were still too restrictive to describe some actual problems better [44]–[48]. As a result, a kind of more general TRs labeled as generally uncertain transition rates was put forward in [31], in which each TR was allowed to be known, unknown and uncertain. The uncertain TRs meant that estimate values and bounds of TRs were known [44]–[46]. The description on the partially unknown TRs was that each TR whether was completely known or completely unknown [47], [48]. It is easy to see that uncertain TRs as well as partially known TRs can be viewed as the special case of GUTRs. However, it is not trivial to generalize the results of completely known TRs to uncertain TRs or GUTRs. When designing the controller of systems with GUTRs, the nonlinear term resulting from the Young inequality will appear. A natural question is, how to overcome the difficulty of nonlinear term and subsequently design a convex controller? Although [31] explored the stability of linear Markovian jump systems with GUTRs, but the feedback controller was not designed. [32] generalized the results of [31] to singular systems, sufficient conditions of stability for systems under consideration were obtained, and a state feedback controller was presented by using the duality principle. Reference [33] derived less conservative conditions than those in [32] with the aid of free-weighting matrices and slack variable matrices. On the basis of [32], [34] further discussed the stability and stabilization for singular Itô Markovian jump systems with GUTRs, the state feedback controller was presented by introducing a equality constraint to bind the uncertain term. Reference [37] proposed a separated method to decouple the interconnection between Lyapunov variables and controller gains. Reference [38] studied the passivity and passification for normal Itô Markovian jump systems with GUTRs by using free-weighting matrices and the same equality constraint as that in [34]. Reference [39] was concerned with the delay-dependent H_∞ filtering for stochastic singular Itô systems with GUTRs, in which the delay-dependent sufficient conditions were presented to ensure the filtering error system to be stochastically admissible. On the basis of above analysis, to the authors' best of knowledge, the problems of finite-time stochastic H_∞ control for singular Itô Markovian jump systems with GUTRs haven't been fully explored, even for the case of (x, v)-dependent noise, which also motivate us to do this study.

This paper will study finite-time stochastic H_∞ control for singular Itô Markovian jump systems with (x, v)-dependent noise and GUTRs. The main contributions of this paper are summarized as follows: First, taking advantage of two equivalent sets technique, sufficient conditions of finite-time stochastic H_∞ control for the considered systems with completely known TRs are derived. The obtained results not only are less conservative than those in [27]–[30], but also can be viewed as generations from singular Markovian jump systems to singular Itô Markovian jump systems with multiplicative noise. Second, the nonlinear terms resulting of the Young inequality have been eliminated by resorting to the combination of free-weighting matrices and slack variable matrices. A new criterion ensuring singular Itô Markovian jump systems with GUTRs to be finite-time stochastically bounded with H_∞ performance level is established, which makes the controller analysis go smoothly. Third, the state feedback controller and observer-based controller for the corresponding closed-loop systems are presented, respectively. Finally, numerical examples are proposed to verify the effectiveness of our obtained results.

The rest of this paper is organized as follows. In Section II, we give important assumptions and present several useful lemmas. Section III contains main results of this paper on finite-time stochastic H_∞ stabilization via state feedback controller and observer-based controller. Numerical examples are presented in Section IV to illustrate the effectiveness of obtained results. Conclusions are drew in Section V.

Throughout this paper, the following notations will be used. Notations: $\mathbb{R}^n$: the linear space of all $n-$ dimensional real vectors with usual 2-norm $\|\cdot\|$; $\mathbb{R}^{m\times n}$: the linear space of all $m \times n$ real matrices; $A > 0$(resp.$A < 0$): A is a real symmetric positive definite (resp. negative definite) matrix;

A^T: the transpose of A; E^+: Moore-Penrose pseudo inverse of a matrix E; $\mathcal{E}(\cdot)$: the expectation operator; I_n: the $n \times n$ identity matrix; $He(A)$: $A + A^T$; $rank(A)$: the rank of a matrix A.

II. PRELIMINARIES

Consider the following singular Itô-type Markovian jump system with (x, v)-dependent noise and generally uncertain transition rates:

$$\begin{cases} Edx(t) = [A(r_t)x(t) + B(r_t)u(t) + D(r_t)v(t)]dt \\ \qquad\qquad +[A_0(r_t)x(t) + D_0(r_t)v(t)]dw(t), \\ y(t) = F(r_t)x(t), \\ z(t) = C(r_t)x(t) + B_1(r_t)u(t) + D_1(r_t)v(t), \\ Ex(0) = x_0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, v(t) \in \mathbb{R}^q$, $y(t) \in \mathbb{R}^l$ and $z(t) \in \mathbb{R}^p$ are, respectively, the system state, the control input, the disturbance signal, the measure output and the controlled output; $w(t)$ is one-dimensional, standard Wiener process that is defined on the complete filtered probability space $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathcal{P})$ with a filtering $\{\mathcal{F}_t\}_{\{t\geq 0\}}$; E is a constant matrix with $rank(E) = r \leq n$; $A(r_t)$, $B(r_t)$, $D(r_t)$, $A_0(r_t)$, $D_0(r_t)$, $F(r_t)$, $C(r_t)$, $B_1(r_t)$, $D_1(r_t)$ are known matrices of compatible dimensions, $\{r_t, t \geq 0\}$ is a right continuous homogeneous Markovian jump process taking values in a finite state space $\mathcal{S} = \{1, \cdots, N\}$ with transition probability matrix $\prod = \{\hat{\pi}_{ij}\}_{N\times N}$ given by

$$Pr\{r_{t+h} = j|r_t = i\} = \begin{cases} \hat{\pi}_{ij}h + o(h), & i \neq j, \\ 1 + \hat{\pi}_{ii}h + o(h), & i = j, \end{cases}$$

with $h > 0$, $\lim_{h\to 0} o(h)/h = 0$, and $\hat{\pi}_{ij} \geq 0(i \neq j)$ represents the transition rate from i to j, which satisfies $\hat{\pi}_{ii} = -\sum_{j=1,j\neq i}^{N} \hat{\pi}_{ij}$. Suppose that Markovian jump process $\{r_t, t \geq 0\}$ is independent of $w(t)$, and the external disturbance signal $v(t)$ satisfies

$$\mathcal{E}\left\{\int_0^t v^T(s)v(s)ds\right\} \leq h^2, h > 0, t \in [0, T]. \tag{2}$$

In this paper, TRs of Markovian jump process are assumed to be known, unknown as well as uncertain, simultaneously. Concretely speaking, the transition probability matrix of system (1) with N jump modes can be described as

$$\Xi = (\hat{\pi}_{ij})_{N\times N} = \begin{bmatrix} ? & \pi_{12} & ? & \cdots & \pi_{1N} + \Delta\pi_{1N} \\ \pi_{21} + \Delta\pi_{21} & ? & \pi_{33} & \cdots & ? \\ \vdots & \vdots & \vdots & \ddots & \cdots \\ \pi_{N1} + \Delta\pi_{N1} & ? & ? & \cdots & \pi_{NN} + \Delta\pi_{NN} \end{bmatrix}, \tag{3}$$

where π_{ij} and $\Delta\pi_{ij} \in [-\mu_{ij}, \mu_{ij}](\mu_{ij} \geq 0)$ respectively represent the estimate value and the estimate error of uncertain transition rate $\hat{\pi}_{ij}$. Also, π_{ij} and μ_{ij} are known. The notion ? denotes the completely unknown TRs, namely, both the estimate value and the estimate error of $\hat{\pi}_{ij}$ are unknown. For each $r_t = i, i \in \mathcal{S}$, we define $\mathcal{S} = \mathcal{S}_k^i + \mathcal{S}_{uk}^i$, where $\mathcal{S}_k^i \triangleq \{j : \hat{\pi}_{ij}$ is completely known or the estimate value of $\hat{\pi}_{ij}$ is known for $j \in \mathcal{S}\}$, $\mathcal{S}_{uk}^i \triangleq \{j : \hat{\pi}_{ij}$ is unknown for $j \in \mathcal{S}\}$. Note that if $\hat{\pi}_{ij}$ is completely known, its estimate error equals 0. In view of this, if $\mathcal{S}_k^i \neq \emptyset$, we denote $\mathcal{S}_k^i = \{k_1^i, k_2^i, \cdots, k_l^i\}$ with $1 \leq l \leq N$, where $k_m^i \in Z^+(1 \leq k_m^i \leq N, m = 1, 2, \cdots, l)$ represents the index of the mth estimate value-known element in the ith row of matrix Ξ.

Remark 1: Notice that if $\Delta\pi_{ij} = 0$, (3) degenerates to the partially unknown transition probability matrix [47], [48]. Also, if there is no ? appearing in (3), in this case, (3) becomes the uncertain bounded transition probability matrix [44]–[46]. Therefore, the GUTRs matrix considered in this paper is more general and complex than other two cases.

Because of special characteristics of TRs, it is reasonable to give the following assumptions; see. e.g., [31]–[33], [39].

Assumption 2:

(i) If $\mathcal{S}_k^i = \mathcal{S}$, then $\pi_{ij} - \mu_{ij} \geq 0(\forall j \in \mathcal{S}, j \neq i), \pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij} \leq 0$, and $\mu_{ii} = -\sum_{j=1,j\neq i}^{N} \mu_{ij}$.

(ii) If $\mathcal{S}_k^i \neq \mathcal{S}$ and $i \in \mathcal{S}_k^i$, then $\pi_{ij} - \mu_{ij} \geq 0(\forall j \in \mathcal{S}_k^i, j \neq i), \pi_{ii} + \mu_{ii} \leq 0$, and $\sum_{j\in\mathcal{S}_k^i} \hat{\pi}_{ij} \leq 0$;

(iii) If $\mathcal{S}_k^i \neq \mathcal{S}$ and $i \in \mathcal{S}_{uk}^i$ then $\pi_{ij} - \mu_{ij} \geq 0(\forall j \in S_k^i)$ and $\beta = min_{i\in\mathcal{S}_{uk}^i} \hat{\pi}_{ii}$.

For notational simplicity, we denote $A(r_t), B(r_t), D(r_t)$, $A_0(r_t)$, $D_0(r_t)$, $C(r_t)$, $B_1(r_t)$, $D_1(r_t)$, $F(r_t)$ by A_i, B_i, D_i, A_{0i}, D_{0i}, C_i, B_{1i}, D_{1i}, F_i for each $r_t = i, i \in \mathcal{S}$. Furthermore, to make fully use of the known information of TRs, the lower bound of the uncertain element $\hat{\pi}_{ij}$ is expressed as $\underline{\pi}_{ij} = \pi_{ij} - \mu_{ij}$.

Assumption 3: For each $r_t = i, i \in \mathcal{S}$, two equivalent assumptions are given as follows. (i) $rank(E, A_{0i}) = rank(E)$. (ii) There exist a set of matrices Y_i, $i \in \mathcal{S}$ such that $A_{0i} = EY_i$ hold.

Remark 4: It was shown from [1]–[4] that, due to the appearance of diffusion term, the sole regularity condition cannot guarantee the existence of solution to the singular Itô stochastic system equation any more. For this reason, the additional assumptions have to be presented, which implies that the stochastic disturbance term does not really cause the change on system structure. It can be seen that if the input matrix depends on noise in system (1), the condition of Assumption 3 may be destroyed after implementing a state feedback control law on system (1). In addition, the item (i) of Assumption 3 was given in [3], [4], which is less conservative than previous work such as [1], [2]. Recently, the item (ii) of Assumption 3 was presented in [8]. However, it can be proved that two conditions are equivalent. $(i) \Longrightarrow (ii)$: Due to $rank(E, A_{0i}) = rank(E)$, there must exist nonsingular matrices U and V such that

$$UEV = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad UA_{0i}V = \begin{bmatrix} \hat{A}_{01i} & \hat{A}_{02i} \\ 0 & 0 \end{bmatrix}. \tag{4}$$

By (4), $A_{0i}, i \in \mathcal{S}$ are equivalently expressed as

$$A_{0i} = U^{-1}\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} V^{-1}V\begin{bmatrix} \hat{A}_{01i} & \hat{A}_{02i} \\ 0 & 0 \end{bmatrix} V^{-1} = EV\begin{bmatrix} \hat{A}_{01i} & \hat{A}_{02i} \\ 0 & 0 \end{bmatrix} V^{-1}. \quad (5)$$

Let $Y_i = V\begin{bmatrix} \hat{A}_{01i} & \hat{A}_{02i} \\ 0 & 0 \end{bmatrix} V^{-1}$, then we have $A_{0i} = EY_i$.

$(ii) \Longrightarrow (i)$: If $A_{0i} = EY_i$ hold for some matrices $Y_i, i \in \mathcal{S}$, it is easy to see that $rank(E, A_{0i}) = rank(E, EY_i) = rank(E)$.

Next, we present several important Lemmas which will be used in the sequel.

Lemma 5 [1]: For a general stochastic singular system

$$Edx(t) = f_1(x(t), t)dt + f_2(x(t), t)dw(t), \quad (6)$$

where $f_1, f_2 : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ satisfy the local Lipschitz condition and the linear growth condition. Let $V(x(t), t) = x^T(t)E^TPx(t)$ with $E^TP = P^TE \geq 0$. Then $V(x(t), t)$ is again an Itô process with the singular stochastic differential given by

$$\begin{aligned} dV(x(t), t) = {} & [f_1^T(x(t), t)Px(t) + x^TP^Tf_1(x(t), t) \\ & + f_2^T(x(t), t)(E^+)^TE^TPE^+f_2(x(t), t)]dt \\ & + 2x^T(t)P^Tf_2(x(t), t)dw(t). \end{aligned} \quad (7)$$

Remark 6: It should be pointed out that E^+ and the relation $E = EE^+E$ are introduced to compute $dV(x(t), t)$; see, e.g., [1]–[3]. By applying Itô's formula to $V(x(t), t)$, we have

$$dV(x(t), t) = d(x^T(t)E^T)Px(t) + x^T(t)P^Td(Ex(t)) + d(x^T(t)E^T)Pd(x(t)). \quad (8)$$

But we only know the expression of $dEx(t)$ rather than $dx(t)$. In view of this, the third term $d(x^T(t)E^T)Pd(x(t))$ in (8) is rewritten as $d(x^T(t)E^T)(E^+)^TE^TPE^+d(Ex(t))$. Correspondingly, $dV(x(t), t)$ can be easily calculated.

Lemma 7 [49]: For each $r_t = i, i \in \mathcal{S}$, let $P_i \in \mathbb{R}^{n\times n}$ be symmetric such that $E_L^TP_iE_L > 0$, and $R_i \in \mathbb{R}^{(n-r)\times(n-r)}$ are nonsingular. Then, $P_iE + \Phi^TR_i\Lambda^T$ are nonsingular and their inverse matrices are expressed as

$$(P_iE + \Phi^TR_i\Lambda^T)^{-1} = \bar{P}_iE^T + \Lambda\bar{R}_i\Phi, \quad (9)$$

where $\bar{P}_i \in \mathbb{R}^{n\times n}$ are symmetric and $\bar{R}_i \in \mathbb{R}^{(n-r)\times(n-r)}$ are nonsingular such that

$$(E_R^T\bar{P}_iE_R)^{-1} = E_L^TP_iE_L, \quad (10)$$

$$\bar{R}_i = (\Lambda^T\Lambda)^{-1}R_i^{-1}(\Phi\Phi^T)^{-1}, \quad (11)$$

where E_L and E_R have full column ranks with $E = E_LE_R^T$, Φ and Λ are the left and right null matrices of E, respectively.

Lemma 8 [21]: The following sets are equivalent for each $r_t = i, i \in \mathcal{S}$:

$$\begin{aligned} \mathscr{U} &= \{M_i \in \mathbb{R}^{n\times n} : E^TM_i = M_i^TE \geq 0, M_i \text{ are nonsingular}\}, \\ \mathscr{V} &= \{M_i = P_iE + \Phi^TR_i\Lambda^T : P_i = P_i^T \in \mathbb{R}^{n\times n}, E_L^TP_iE_L > 0, \\ & \qquad R_i \in \mathbb{R}^{(n-r)\times(n-r)} \text{ are nonsingular}\}, \end{aligned} \quad (12)$$

where E_L, E_R, Λ and Φ are defined in Lemma 7.

Lemma 9 [50]: Let $\mathcal{Z}$ and $\mathcal{F} > 0$ be real square matrices, then we have the following matrix inequality

$$\varepsilon(\mathcal{Z} + \mathcal{Z}^T) \leq \varepsilon^2\mathcal{F} + \mathcal{Z}\mathcal{F}^{-1}\mathcal{Z}^T \quad (13)$$

for any real number ε.

III. MAIN RESULTS

A. FINITE-TIME STOCHASTIC H_∞ CONTROL VIA STATE FEEDBACK CONTROLLER

This section aims to design a state-feedback controller for system (1) with GUTRs such that the resultant closed-loop system is finite-time stochastically bounded with an H_∞ disturbance attenuation level γ.

Below, we consider the following state feedback controller for system (1)

$$u(t) = K(r_t)x(t), \quad (14)$$

where $K(r_t)$ denoted by K_i for each $i \in \mathcal{S}$, is the state feedback gain matrix to be designed. Substituting (14) into system (1), we obtain the resultant closed-loop system

$$\begin{cases} Edx(t) = [\hat{A}(r_t)x(t) + D(r_t)v(t)]dt \\ \qquad\qquad + [A_0(r_t)x(t) + D_0(r_t)v(t)]dw(t) \\ z(t) = \hat{C}(r_t)x(t) + D_1(r_t)v(t), \\ Ex(0) = x_0, \end{cases} \quad (15)$$

where $\hat{A}(r_t) = A(r_t) + B(r_t)K(r_t), \hat{C}(r_t) = C(r_t) + B_1(r_t)K(r_t)$.

Similar to [27], the following fundamental definitions for the singular Itô stochastic system (15) are introduced, which are crucial to give our main results.

Definition 10: For scalars $0 < c_1 < c_2, T > 0, h > 0, \gamma > 0$ and a positive matrix $Q > 0$,

(i) the closed-loop system (15) with $v(t) = 0$ is said to be finite-time stochastically stable (FTSS) with respect to (wrt) (c_1, c_2, T, Q), if it has an impulse-free solution in the time interval $[0, T]$ and satisfies

$$\mathcal{E}\{x^T(0)E^TQEx(0)\} \leq c_1^2 \Longrightarrow \mathcal{E}\{x^T(t)E^TQEx(t)\} < c_2^2. \quad (16)$$

(ii) The closed-loop system (15) is said to be finite-time stochastically bounded (FTSB) wrt (c_1, c_2, T, h, Q), if it is FTSS and the condition (2) holds for all nonzero $v(t)$.

(iii) The closed-loop system (15) is said to be finite-time stochastically bounded (FTSB) with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, Q)$, if, under the zero initial condition, it is FTSB and satisfies

$$\mathcal{E}\left\{\int_0^T z^T(t)z(t)dt\right\} < \gamma^2\mathcal{E}\left\{\int_0^T v^T(t)v(t)dt\right\}. \quad (17)$$

Remark 11: In contrast with the notion of finite-time boundedness for the deterministic system [25], [26], mathematical expectation is introduced to evaluate the boundedness of the state trajectory over a finite time interval, which reveals some differences between deterministic and stochastic systems. In addition, it should be pointed out that finite-time stochastic boundedness of system (15) implies that the state trajectory of the dynamical mode of system (15) is finite-time stochastically bounded, so does the whole mode. The reason is that the static mode of system (15) is impulse free. Therefore, the definition of finite-time stochastic boundedness given in this paper is consistent with those of [25], [26].

In what follows, we will firstly provide sufficient conditions for finite-time stochastic H_∞ control of system (15) with completely known TRs, which pay the way for following discussions.

Lemma 12: The closed-loop system (15) with completely known TRs is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, Q)$, if there exist scalars $\delta \geq 0, \bar{\gamma} > 0$, a set of symmetric matrices $P_i = P_i^T$, nonsingular matrices R_i and positive symmetric matrices $G_i > 0$ such that the following inequalities hold for each $i \in \mathcal{S}$:

$$E_L^T P_i E_L > 0, \tag{18}$$

$$\begin{bmatrix} \sum_{1i} & \sum_{2i} & \hat{C}_i^T \\ * & \sum_{3i} & D_{1i}^T \\ * & * & -I \end{bmatrix} < 0, \tag{19}$$

$$E^T M_i = E^T Q^{1/2} G_i Q^{1/2} E, \tag{20}$$

$$e^{\delta T}(c_1^2 sup_{i\in\mathcal{S}}\{\lambda_{max}(G_i)\} + \bar{\gamma}h^2) < c_2^2 inf_{i\in\mathcal{S}}\{\lambda_{min}(G_i)\}, \tag{21}$$

where

$$\begin{aligned} \sum\nolimits_{1i} &= He(\hat{A}_i^T M_i) + A_{0i}^T (E^+)^T E^T M_i (E^+) A_{0i} \\ &\quad + \sum_{j=1}^{N} \hat{\pi}_{ij} E^T M_j - \delta E^T M_i, \\ \sum\nolimits_{2i} &= A_{0i}^T (E^+)^T E^T M_i (E^+) D_{0i} + M_i^T D_i, \\ \sum\nolimits_{3i} &= D_{0i}^T (E^+)^T E^T M_i (E^+) D_{0i} - \bar{\gamma} I, \\ M_i &= P_i E + \Phi^T R_i \Lambda^T, \quad \gamma = \sqrt{\bar{\gamma} e^{\delta T}}. \end{aligned} \tag{22}$$

Proof: The proof will be divided into three steps.

Step (i): We will prove that system (15) has an impulse-free solution in the time interval $[0, T]$. By (18) and $M_i = P_i E + \Phi^T R_i \Lambda^T$, it is easy to see that a set of matrices $M_i, i \in \mathcal{S}$ satisfy the set $\mathscr{V}$ in Lemma 8, then by the two equivalent sets, $M_i, i \in \mathcal{S}$ satisfy the set $\mathscr{U}$, i.e.

$$E^T M_i = M_i^T E \geq 0. \tag{23}$$

Note that (19) implies $\sum_{1i} < 0$, which together with (23) yields

$$\hat{A}_i^T M_i + M_i^T \hat{A}_i + (\hat{\pi}_{ii} - \delta) E^T M_i < -\sum_{j=1, i\neq j}^{N} \hat{\pi}_{ij} E^T M_j \leq 0. \tag{24}$$

On the other hand, because of $rank(E) = r \leq n$, there must exist a pair of nonsingular matrices U, V such that

$$UEV = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad U\hat{A}_i V = \begin{bmatrix} \tilde{A}_{1i} & \tilde{A}_{2i} \\ \tilde{A}_{3i} & \tilde{A}_{4i} \end{bmatrix}. \tag{25}$$

Setting

$$U^{-T} M_i V = \begin{bmatrix} \tilde{M}_{1i} & \tilde{M}_{2i} \\ \tilde{M}_{3i} & \tilde{M}_{4i} \end{bmatrix}, \tag{26}$$

and substituting (25) and (26) into (23), we obtain

$$\tilde{M}_{1i} = \tilde{M}_{1i}^T \geq 0, \quad \tilde{M}_{2i} = 0. \tag{27}$$

Then applying (25), (26) and (27) to (24), we have $\tilde{A}_{4i}^T \tilde{M}_{4i} + \tilde{M}_{4i}^T \tilde{A}_{4i} < 0$, which implies $\tilde{A}_{4i}, i \in \mathcal{S}$ are invertible. Furthermore, considering Assumption 3, there must exist nonsingular matrices $\hat{U}_i$ and V such that

$$\begin{aligned} &\hat{U}_i E V = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \hat{U}_i \hat{A}_i V = \begin{bmatrix} \tilde{A}_{1i} - \tilde{A}_{2i}\tilde{A}_{4i}^{-1}\tilde{A}_{3i} & 0 \\ \tilde{A}_{3i} & \tilde{A}_{4i} \end{bmatrix}, \\ &\hat{U}_i A_{0i} V = \begin{bmatrix} \tilde{A}_{01i} & \tilde{A}_{02i} \\ 0 & 0 \end{bmatrix}, \end{aligned} \tag{28}$$

with $\hat{U}_i = \begin{bmatrix} I & -\tilde{A}_{2i}\tilde{A}_{4i}^{-1} \\ 0 & I \end{bmatrix} U$. Hence, by Lemma 3 of [4], it follows that system (15) has an impulse-free solution in the time interval $[0, T]$.

Step (ii): The closed-loop system (15) is FTSB wrt (c_1, c_2, T, h, Q). By Schur complement Lemma, (19) leads to

$$\begin{bmatrix} \sum_{1i} & \sum_{2i} \\ * & \sum_{3i} \end{bmatrix} < 0. \tag{29}$$

Let us choose the Lyapunov function candidate as $V(x(t), r_t = i) = x^T(t) E^T M_i x(t)$ and consider $E = EE^+E$. On the basis of Lemma 5 and generalized Itô's formula [see (5.2) of [15]], it follows that

$$\begin{aligned} &LV(x(t), i) \\ &= [\hat{A}_i x(t) + D_i v(t)]^T M_i x(t) + x^T(t) M_i^T [\hat{A}_i x(t) + D_i v(t)] \\ &\quad + [\hat{A}_{0i} x(t) + D_{0i} v(t)]^T (E^+)^T E^T M_i (E^+) [\hat{A}_{0i} x(t) + D_{0i} v(t)] \\ &\quad + \sum_{j=1}^{N} \pi_{ij} E^T M_j \\ &= [x^T(t) \quad v^T(t)] \begin{bmatrix} \Omega_{1i} & \Omega_{2i} \\ * & \Omega_{3i} \end{bmatrix} \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \end{aligned} \tag{30}$$

where

$$\begin{aligned} \Omega_{1i} &= \hat{A}_i^T M_i + M_i^T \hat{A}_i + A_{0i}^T (E^+)^T E^T M_i E^+ A_{0i} \\ &\quad + \sum_{j=1}^{N} \hat{\pi}_{ij} E^T M_j, \\ \Omega_{2i} &= M_i^T D_i + A_{0i}^T (E^+)^T E^T M_i E^+ D_{0i}, \\ \Omega_{3i} &= D_{0i}^T (E^+)^T E^T M_i E^+ D_{0i}. \end{aligned}$$

In view of (29) and (30), we can get

$$\mathcal{LV}(x(t), i) < \delta V(x(t), i) + \bar{\gamma} v^T(t) v(t). \tag{31}$$

Pre-multiplying (31) by $e^{-\delta t}$, which yields

$$\mathcal{L}[e^{-\delta t} V(x(t), i)] < e^{-\delta t} \bar{\gamma} v^T(t) v(t). \tag{32}$$

Integrating (32) from 0 to t and taking expectation on it, (32) becomes

$$e^{-\delta t}\mathcal{E}\{V(x(t),i)<\mathcal{E}\{V(x(0),r_0)\}+\mathcal{E}\{\int_0^t \bar{\gamma}e^{-\delta s}v^T(s)v(s)ds\}. \quad (33)$$

By (33), we have

$$\mathcal{E}\{V(x(t),i)\}<e^{\delta t}\Big\{\mathcal{E}\{V(x(0),r_0\}+\bar{\gamma}\mathcal{E}\{\int_0^t v^T(s)v(s)ds\}\Big\}. \quad (34)$$

By making use of (20), we get

$$\begin{aligned}\mathcal{E}\{V(x(t),i)\} &= \mathcal{E}\{x^T(t)E^TM_ix(t)\}\\ &\geq inf_{i\in\mathcal{S}}\{\lambda_{min}(G_i)\}\mathcal{E}\{x^T(t)E^TQEx(t)\}, \quad (35)\\ \mathcal{E}\{V(x(0),r_0)\} &= \mathcal{E}\{x^T(0)E^TM_ix(0)\}\\ &\leq sup_{i\in\mathcal{S}}\{\lambda_{max}(G_i)\}\mathcal{E}\{x^T(0)E^TQEx(0)\} \quad (36)\end{aligned}$$

As a result, (21), (34), (35) as well as (36) result in

$$\mathcal{E}\{x^T(t)E^TQEx(t)\}<c_2^2, \quad t\in[0,T]. \quad (37)$$

Step (iii): We will show that the closed-loop system (15) is FTSB with an H_∞ disturbance attenuation level γ wrt $(c_1, c_2, T, h, \gamma, Q)$. Based on Definition 10-(iii), it remains to prove that the expression (17) holds. The procedures below are similar to those of [27], [28], [48]. By (19) and (30), one can get

$$\mathcal{L}\mathcal{V}(x(t),i)<\delta V(x(t),i)+\bar{\gamma}v^T(t)v(t)-z^T(t)z(t). \quad (38)$$

Pre-multiplying (38) by $e^{-\delta t}$, it follows that

$$\mathcal{L}[e^{-\delta t}V(x(t),i)]<e^{-\delta t}[\bar{\gamma}v^T(t)v(t)-z^T(t)z(t)]. \quad (39)$$

Integrating (39) from 0 to T and taking expectation on it, we obtain the following inequality under the zero initial condition

$$\mathcal{E}\Big\{\int_0^T e^{-\delta t}[z^T(t)z(t)-\bar{\gamma}v^T(t)v(t)]dt\Big\} < -\mathcal{E}\{e^{-\delta T}V(x(T),i)\}\leq 0. \quad (40)$$

Notice that $e^{-\delta T}\leq e^{-\delta t}, \forall t\in[0,T]$, (40) becomes

$$\mathcal{E}\Big\{\int_0^T e^{-\delta T}z^T(t)z(t)dt\Big\}<\mathcal{E}\Big\{\int_0^T e^{-\delta t}\bar{\gamma}v^T(t)v(t)dt\Big\}. \quad (41)$$

(41) is the same as

$$\mathcal{E}\{\int_0^T z(t)z(t)dt\}<\mathcal{E}\Big\{\int_0^T e^{\delta(T-t)}\bar{\gamma}v^T(t)v(t)dt\Big\}. \quad (42)$$

It is easy to see that the following inequality holds

$$\mathcal{E}\Big\{\int_0^T e^{\delta(T-t)}\bar{\gamma}v^T(t)v(t)dt\}\leq\mathcal{E}\{\int_0^T e^{\delta T}\bar{\gamma}v^T(t)v(t)dt\Big\} \quad (43)$$

Finally, let $\gamma=\sqrt{\bar{\gamma}e^{\delta T}}$, then (42) as well as (43) results in (17). The proof is ended.

Remark 13: A new criterion, which guarantees singular stochastic systems (15) with completely known TRs to be FTSB with an H_∞ performance level γ, is established in Lemma 12. It should be pointed that the obtained results are not trivial extensions to singular Itô stochastic systems. By using the two equivalent sets [21], the equality constraint $P_iE^T = EP_i^T$ given in [27]–[29] and [30] has been removed, which makes the numerical computation more reliable and tractable. Moreover, with the help of Lemma 7, the nonlinear term $\sum_{j=1}^N \pi_{ij}P_iP_j^{-1}EP_i^T$ presented in Theorem 1 of [27] can be replaced as follows: $\sum_{j=1}^N \pi_{ij}M_i^{-T}M_j^TEM_i^{-1}=\sum_{j=1}^N \pi_{ij}N_i^TE^TP_jEN_i=\pi_{ii}N_i^TE^T+\sum_{j=1,i\neq j}^N \pi_{ij}N_i^TE_R(E_L^TP_iE_L)E_R^TN_i$ with $N_i=\bar{P}_iE^T+\Lambda\bar{R}_i\Phi$. Note that $\sum_{j=1,i\neq j}^N \pi_{ij}N_i^TE_R(E_L^TP_iE_L)E_R^TN_i$ can be directly converted into the linear term, which lays a solid foundation for designing a controller in terms of strict LMIs.

Theorem 14: The closed-loop system (15) with GUTRs is FTSB with an H_∞ disturbance attenuation level γ wrt $(c_1, c_2, T, h, \gamma, Q)$, if there exist scalars $\delta\geq 0, \bar{\gamma}>0$, a set of symmetric matrices P_i, W_i, S_{ij}, X_{ij}, positive symmetric matrices $G_i>0, H_i>0, T_{ij}>0, Z_{ij}>0$, and nonsingular matrices R_i such that (18), (20), (21) and the following inequalities hold for each $r_t=i, i\in\mathcal{S}$:

$$\begin{bmatrix}\sum_{4i}+\theta_{1i} & \sum_{2i} & \hat{C}_i^T & W_i & \Psi_{1i}\\ * & \sum_{3i} & D_{1i}^T & 0 & 0\\ * & * & -I & 0 & 0\\ * & * & * & -H_i & 0\\ * & * & * & * & -\Psi_{2i}\end{bmatrix}<0, \quad i\in\mathcal{S}_k^i, \quad (44)$$

$$\begin{bmatrix}\sum_{4i}+\theta_{2i} & \sum_{2i} & \hat{C}_i^T & \Psi_{3i}\\ * & \sum_{3i} & D_{1i}^T & 0\\ * & * & -I & 0\\ * & * & * & -\Psi_{4i}\end{bmatrix}<0, \quad i\in\mathcal{S}_{uk}^i, \quad (45)$$

$$E^TM_j-E^TM_i-W_i\leq 0, \quad j\in\mathcal{S}_{uk}^i, i\in\mathcal{S},\ i\neq j, \quad (46)$$

$$E^TM_j-E^TM_i-W_i-S_{ij}\leq 0, \quad j\in\mathcal{S}_k^i, i\in\mathcal{S}_k^i,\ i\neq j, \quad (47)$$

$$E^TM_j-E^TM_i-W_i-X_{ij}\leq 0, \quad j\in\mathcal{S}_k^i, i\in\mathcal{S}_{uk}^i,\ i\neq j, \quad (48)$$

where

$$\sum_{4i}=He(\hat{A}_i^TM_i)+A_{0i}^T(E^+)^TE^TP_iE(E^+)A_{0i}-\delta E^TP_iE,$$

$$\theta_{1i}=\sum_{j\in\mathcal{S}_k^i, i\neq j}[\underline{\pi_{ij}}(E^TM_j-E^TM_i-W_i)+\mu_{ij}S_{ij}+\frac{\mu_{ij}^2}{4}T_{ij}] +\frac{\mu_{ii}^2}{4}H_i-\pi_{ii}W_i,$$

$$\begin{cases}\Psi_{1i} = [S_{ik_1^i}, S_{ik_2^i}, \ldots, S_{ik_l^i}],\\ \Psi_{2i} = diag[T_{ik_1^i}, T_{ik_2^i}, \ldots, T_{ik_l^i}],\\ \Psi_{3i} = [X_{ik_1^i}, X_{ik_2^i}, \ldots, X_{ik_l^i}],\\ \Psi_{4i} = diag[Z_{ik_1^i}, Z_{ik_2^i}, \ldots, Z_{ik_l^i}],\\ k_m^i \in \mathcal{S}_k^i, k_m^i \neq i, m = 1, 2, \ldots, l,\end{cases}$$

$$\theta_{2i} = \sum_{j\in\mathcal{S}_k^i}[\underline{\pi_{ij}}(E^T M_j - E^T M_i - W_i) + \mu_{ij}X_{ij} + \frac{\mu_{ij}^2}{4}Z_{ij}] - \beta W_i,$$

$\sum_{2i}$ and $\sum_{3i}$ are the same as those in Lemma 12.

Proof: Due to $\hat{\pi}_{ii} = -\sum_{j\in\mathcal{S}_k^i, i\neq j}\hat{\pi}_{ij} - \sum_{j\in\mathcal{S}_{uk}^i, i\neq j}\hat{\pi}_{ij}$, it follows that

$$\sum_{j=1}^{N}\hat{\pi}_{ij}E^T M_j = \sum_{j\in\mathcal{S}_k^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i) + \sum_{j\in\mathcal{S}_{uk}^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i). \quad (49)$$

Inspired by [33], [45], [46], [47], we introduce the free weighting matrices $W_i, i \in \mathcal{S}$ and the zero sum equation $\sum_{j=1}^{N}\hat{\pi}_{ij}W_i = 0$, then (49) is converted into

$$\sum_{j=1}^{N}\hat{\pi}_{ij}E^T M_j = \sum_{j\in\mathcal{S}_k^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i - W_i) + \sum_{j\in\mathcal{S}_{uk}^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i - W_i) - \hat{\pi}_{ii}W_i. \quad (50)$$

Case (i) ($i \in \mathcal{S}_k^i$): Taking into account (46) and $\hat{\pi}_{ij} \geq 0$ ($j \in \mathcal{S}_{uk}^i$), it is easy to see that

$$\sum_{j\in\mathcal{S}_{uk}^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i - W_i) \leq 0. \quad (51)$$

Then (50) and (51) can result in

$$\begin{aligned}\sum\nolimits_{1i} &\leq \sum\nolimits_{4i} + \sum_{j\in\mathcal{S}_k^i, i\neq j}\hat{\pi}_{ij}(E^T M_j - E^T M_i - W_i) - \hat{\pi}_{ii}W_i\\ &= \sum\nolimits_{4i} + \sum_{j\in\mathcal{S}_k^i, i\neq j}(\pi_{ij} + \Delta\pi_{ij})(E^T M_j - E^T M_i - W_i)\\ &\quad - (\pi_{ii} + \Delta\pi_{ii})W_i\\ &= \sum\nolimits_{4i} + \sum_{j\in\mathcal{S}_k^i, i\neq j}[\underline{\pi_{ij}}(E^T M_j - E^T M_i - W_i)\\ &\quad + (\Delta\pi_{ij} + \mu_{ij})(E^T M_j - E^T M_i - W_i - S_{ij})\\ &\quad + (\Delta\pi_{ij} + \mu_{ij})S_{ij}] - (\pi_{ii} + \Delta\pi_{ii})W_i, \end{aligned} \quad (52)$$

where $\sum_{1i}$ is given in (19). According to Lemma 9, we obtain

$$\sum_{j\in\mathcal{S}_k^i, i\neq j}\Delta\pi_{ij}S_{ij} \leq \sum_{j\in\mathcal{S}_k^i, i\neq j}(\frac{\mu_{ij}^2}{4}T_{ij} + S_{ij}T_{ij}^{-1}S_{ij}), \quad (53)$$

and

$$-\Delta\pi_{ii}W_i \leq \frac{\mu_{ii}^2}{4}H_i + W_i H_i^{-1}W_i. \quad (54)$$

Via (47) and Assumption 2-(ii), we have

$$(\Delta\pi_{ij} + \mu_{ij})(E^T M_j - E^T M_i - W_i - S_{ij}) \leq 0. \quad (55)$$

Therefore, (52)-(55) can lead to

$$\sum\nolimits_{1i} \leq \sum\nolimits_{4i} + \theta_{1i} + \sum_{j\in\mathcal{S}_k^i, i\neq j}S_{ij}T_{ij}^{-1}S_{ij} + W_i H_i^{-1}W_i. \quad (56)$$

Applying Schur complement Lemma to (44) and considering (55), it can be verified that condition (44) implies (19).

Case (ii)($i \in \mathcal{S}_{uk}^i$) : Along with the similar line as that in Case (i), we get the following inequality

$$\sum\nolimits_{1i} \leq \sum\nolimits_{4i} + \theta_{2i} + \sum_{j\in\mathcal{S}_k^i, i\neq j}X_{ij}Z_{ij}^{-1}X_{ij}. \quad (57)$$

Employing Schur complement Lemma to (45) and using (57), we derive that (45) implies (19). This completes the proof.

Remark 15: As it can be seen, most of existing references investigated the control problems on Markovian jump systems with GUTRs by classifying the TRs as three cases, that is : (i) $i \in \mathcal{S}_k^i, \mathcal{S}_{uk}^i = \emptyset$; (ii) $i \in S_k^i, \mathcal{S}_{uk}^i \neq \emptyset$; (iii) $i \in \mathcal{S}_{uk}^i$. However, the item (i) and the item (ii) have been merged in Theorem 14. In fact, in the case of (i), we only need to set $W_i = 0, H_i = 0$ and delete the fourth row and column in (44), then (44) and (47) can ensure system (15) to be FTSB with an H_∞ disturbance attenuation γ.

Remark 16: As said in [8], [37], [45], [46], it is difficult to design a convex controller for systems with uncertain TRs or GUTRs. This is because the nonlinear association terms caused by the Young inequality often emerge. To obtain a convex controller, the duality principal in [32] was employed to study the stability and stabilization of singular Markovian jump systems, however, which is not applicable to system (1) of this paper. Similarly, the separated approach adopted in [37] is still restricted to the case that the measurement output equation doesn't contain the control input. Therefore, the technique called the association of free-connection weighting matrices with slack matrices is used in Theorem 14. It can be seen that the free-connection weighting matrices W_i are proposed to deal with the completely unknown transition rates, which can provide less conservative results than fix-weighting matrices [38], [47]. Also, the aim introducing the slack matrices S_{ij} and X_{ij} is to conquer the nonlinear difficulty arising in the Young inequality, which provides a convex method for controller synthesis [33].

Theorem 17: The closed-loop system (15) with GUTRs is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, Q)$, if there exist scalars $\delta \geq 0, \bar{\gamma} > 0, \lambda > 0$, a set of symmetric matrices $\bar{P}_i, \bar{W}_i, \bar{S}_{ij}, \bar{X}_{ij}$, positive symmetric matrices $\bar{H}_i > 0, \bar{T}_{ij} > 0, \bar{Z}_{ij} > 0, J_{2i} > 0$, matrices L_i, and nonsingular matrices $\bar{R}_i$ such that the following inequalities hold for each $r_t = i, i \in \mathcal{S}$:

$$E_R^T \bar{P}_i E_R > 0, \quad (58)$$

$$\begin{bmatrix} \sum_{5i} & D_i & \sum_{6i} & N_i^T A_{0i}^T (E^+)^T E_R & \bar{W}_i & \bar{\Psi}_{1i} & \Upsilon_{1i} \\ * & \bar{\gamma} I & D_{1i}^T & D_{0i}^T (E^+)^T E_R & 0 & 0 & 0 \\ * & * & I & 0 & 0 & 0 & 0 \\ * & * & * & E_R^T \bar{P}_i E_R & 0 & 0 & 0 \\ * & * & * & * & \bar{H}_i & 0 & 0 \\ * & * & * & * & * & \bar{\Psi}_{2i} & 0 \\ * & * & * & * & * & * & \Upsilon_{2i} \end{bmatrix} < 0, \quad i \in \mathcal{S}_k^i, \quad (59)$$

$$\begin{bmatrix} \sum_{7i} & D_i & \sum_{6i} & N_i^T A_{0i}^T (E^+)^T E_R & \bar{\Psi}_{3i} & \Upsilon_{1i} \\ * & \bar{\gamma} I & D_{1i}^T & D_{0i}^T (E^+)^T E_R & 0 & 0 \\ * & * & I & 0 & 0 & 0 \\ * & * & * & E_R^T \bar{P}_i E_R & 0 & 0 \\ * & * & * & * & \bar{\Psi}_{4i} & 0 \\ * & * & * & * & * & \Upsilon_{2i} \end{bmatrix} < 0, \quad i \in \mathcal{S}_{uk}^i, \quad (60)$$

$$\begin{bmatrix} -\bar{W}_i - N_i^T E^T & N_i^T E_R \\ * & -E_R^T \bar{P}_j E_R \end{bmatrix} \le 0, \quad i \in S, \quad j \in \mathcal{S}_{uk}^i, \ i \neq j, \quad (61)$$

$$\begin{bmatrix} -\bar{W}_i - N_i^T E^T - \bar{S}_{ij} & N_i^T E_R \\ * & -E_R^T \bar{P}_j E_R \end{bmatrix} \le 0, \quad i \in S_k^i, \quad j \in \mathcal{S}_k^i, \ i \neq j, \quad (62)$$

$$\begin{bmatrix} -\bar{W}_i - N_i^T E^T - \bar{X}_{ij} & N_i^T E_R \\ * & -E_R^T \bar{P}_j E_R \end{bmatrix} \le 0, \quad i \in S_{uk}^i, \quad j \in \mathcal{S}_k^i, \ i \neq j, \quad (63)$$

$$\lambda I < Q^{\frac{1}{2}} U^{-1} diag\{J_{1i}, J_{2i}\} U^{-T} Q^{\frac{1}{2}} < I, \quad (64)$$

$$-c_2^2 e^{-\delta T} + \bar{\gamma} h^2 + \frac{c_1^2}{\lambda} < 0, \quad (65)$$

where

$$\sum_{5i} = He(A_i N_i + B_i L_i) - \delta N_i^T E^T + \theta_{3i},$$

$$\theta_{3i} = \sum_{j \in \mathcal{S}_k^i, i \neq j} [-\underline{\pi_{ij}}(N_i^T E^T + \bar{W}_i) + \mu_{ij} \bar{S}_{ij} + \frac{\mu_{ij}^2}{4} \bar{T}_{ij}] + \frac{\mu_{ii}^2}{4} \bar{H}_i - \pi_{ii} \bar{W}_i,$$

$$\sum_{6i} = N_i^T C_i^T + L_i^T B_{1i}^T$$

$$\begin{cases} \bar{\Psi}_{1i} = [\bar{S}_{ik_1^i}, \bar{S}_{ik_2^i}, \cdots, \bar{S}_{ik_l^i}], \\ \bar{\Psi}_{2i} = diag[\bar{T}_{ik_1^i}, \bar{T}_{ik_2^i}, \cdots, \bar{T}_{ik_l^i}], \\ \bar{\Psi}_{3i} = [\bar{X}_{ik_1^i}, \bar{X}_{ik_2^i}, \cdots, \bar{X}_{ik_l^i}], \\ \bar{\Psi}_{4i} = diag[\bar{Z}_{ik_1^i}, \bar{Z}_{ik_2^i}, \cdots, \bar{Z}_{ik_l^i}], \\ \Upsilon_{1i} = [\sqrt{\underline{\pi_{ik_1^i}}} N_i^T E_R, \sqrt{\underline{\pi_{ik_2^i}}} N_i^T E_R, \cdots, \\ \qquad \sqrt{\underline{\pi_{ik_l^i}}} N_i^T E_R], \\ \Upsilon_{2i} = diag\{E_R^T \bar{P}_{ik_1^i} E_R, E_R^T \bar{P}_{ik_2^i} E_R, \cdots, \\ \qquad E_R^T \bar{P}_{ik_l^i} E_R\}, \\ (k_m^i \in \mathcal{S}_k^i, k_m^i \neq i, m = 1, 2, \cdots, l), \end{cases}$$

$$\sum_{7i} = He(A_i N_i + B_i L_i) - \delta N_i^T E^T + \theta_{4i},$$

$$\theta_{4i} = \sum_{j \in \mathcal{S}_k^i} [-\underline{\pi_{ij}}(N_i^T E^T + \bar{W}_i) + \mu_{ij} \bar{X}_{ij} + \frac{\mu_{ij}^2}{4} \bar{Z}_{ij}] - \beta \bar{W}_i,$$

$$N_i = \bar{P}_i E^T + \Lambda \bar{R}_i \Phi,$$

$$J_{1i} = [I_r \quad 0] U E N_i U^T [I_r \quad 0]^T.$$

The state feedback gain matrices are presented as $K_i = L_i N_i^{-1}, i \in \mathcal{S}$.

Proof: As observed in Lemma 7, we obtain $(E_L^T P_i E_L)^{-1} = E_R^T \bar{P}_i E_R$ and $N_i = M_i^{-1} = \bar{P}_i E^T + \Lambda \bar{R}_i \Phi$, so inequality (58) implies (18). Next, pre- and post-multiplying (44), (45), (46)-(48) by $diag\{M_i^{-T}, I, I, I, I\}$, $diag\{M_i^{-T}, I, I, I\}$, M_i^{-T} and their transposes, respectively, and introducing the following new matrices $L_i = K_i N_i$, $\bar{W}_i = N_i^T W_i N_i$, $\bar{S}_{ij} = N_i^T S_{ij} N_i$, $\bar{Z}_{ij} = N_i^T Z_{ij} N_i^T$, $\bar{X}_{ij} = N_i^T X_{ij} N_i$, $\bar{H}_i = N_i^T H_i N_i$, $\bar{T}_{ij} = N_i^T T_{ij} N_i$, we derive that inequalities (44) and (45) are respectively equivalent to (59) and (60) by applying Schur complement Lemma and a series of simple computations. Furthermore, inequalities (46)-(48) are equivalently transformed to (61)-(63).

The remainder is to prove that conditions (64) and (65) imply (20) and (21). Note that (20) is equivalent to

$$N_i^T E^T = N_i^T E^T Q^{\frac{1}{2}} G_i Q^{\frac{1}{2}} E N_i. \quad (66)$$

So we will only need to find the suitable matrices $G_i > 0$, $i \in \mathcal{S}$ to satisfy (66). To this end, setting $V^{-1} N_i U^T = \begin{bmatrix} \tilde{N}_{1i} & \tilde{N}_{2i} \\ \tilde{N}_{3i} & \tilde{N}_{4i} \end{bmatrix}$, and taking into account $N_i^T E^T = E N_i \ge 0$, we derive $\tilde{N}_{1i} = \tilde{N}_{1i}^T \ge 0, \tilde{N}_{2i} = 0$. Also, $\tilde{N}_{1i}$ is positive because of the nonsingularity of N_i. Thus, (66) holds when taking $G_i = Q^{-\frac{1}{2}} U^T \begin{bmatrix} \tilde{N}_{1i} & 0 \\ 0 & J_{2i} \end{bmatrix}^{-1} U Q^{-\frac{1}{2}}$ with the arbitrary positive matrices $J_{2i} > 0$. On the other hand, $\tilde{N}_{1i}$ is expressed as the following form of known symmetric matrices

$$\begin{aligned} J_{1i} \triangleq \tilde{N}_{1i} &= [I_r \quad 0] V^{-1} N_i U^T [I_r \quad 0]^T \\ &= [I_r \quad 0] U E N_i U^T [I_r \quad 0]^T. \end{aligned} \quad (67)$$

Hence, condition (66) is satisfied. Finally, it can be seen from (64) that $I < G_i < \lambda^{-1} I$, which together with (65) can result in (21). This completes the proof.

When system (15) includes neither noise nor unknown and uncertain TRs, Theorem 17 degenerates to the conditions of finite-time stochastic H_∞ stabilization by state feedback controller for singular Markovian jump systems with completely known TRs, which is presented as follows.

Corollary 18: There exist the mode-dependent state feedback controller $u(t) = K_i x(t)$ with $K_i = L_i N_i^{-1}, i \in \mathcal{S}$ such that system (15) with completely known TRs and $w(t) = 0$ is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, Q)$, if there exist scalars $\delta \ge 0$, $\bar{\gamma} > 0$, $\lambda > 0$, a set of matrices $\bar{P}_i = \bar{P}_i^T$, $J_{2i} > 0$, L_i and nonsingular matrices $\bar{R}_i$ such that (58),(64),(65) and the following inequalities hold for each $r_t = i, i \in \mathcal{S}$:

$$\begin{bmatrix} \bar{\sum}_{1i} & D_i & \sum_{6i} & \Psi_{5i} \\ * & -\bar{\gamma} I & D_{1i}^T & 0 \\ * & * & -I & 0 \\ * & * & * & -\Psi_{6i} \end{bmatrix} < 0, \quad (68)$$

where

$$\begin{aligned} \bar{\sum}_{1i} &= He(A_iN_i + B_iL_i) + (\pi_{ii} - \delta)E\bar{P}_iE^T, \\ \Psi_{5i} &= [\sqrt{\pi_{i1}}N_i^T E_R, \sqrt{\pi_{i2}}N_i^T E_R, \cdots, \sqrt{\pi_{i,i-1}}N_i^T E_R, \\ &\quad \sqrt{\pi_{i,i+1}}N_i^T E_R, \cdots, \sqrt{\pi_{iN}}N_i^T E_R], \\ \Psi_{6i} &= diag\{E_R^T \bar{P}_{i1} E_R, E_R^T \bar{P}_{i2} E_R, \cdots, E_R^T \bar{P}_{i,i-1} E_R, \\ &\quad E_R^T \bar{P}_{i,i+1} E_R, \cdots, E_R^T \bar{P}_{iN} E_R\}, \end{aligned}$$

and $\sum_{6i}$ is given in Theorem 17.

Remark 19: It can be seen that conditions (58),(64),(65) and (68) are given in terms of strict LMIs. Thus, what we have obtained are superior to Theorem 1 of [27] and Theorem 22 of [30] whether in theory or numerical computation.

B. FINITE-TIME STOCHASTIC H_∞ CONTROL BY OBSERVER-BASED CONTROLLER

It is well known that state variables of the system are generally difficult to measure in most of practical situations. Therefore, it is desirable to design a controller without resorting to accessing to the state. This subsection is devoted to designing an observer-based output feedback controller that ensures the closed-loop system with GUTRs to be FTSB with an H_∞ disturbance attenuation γ. The observer-based controller is provided as follows:

$$\begin{cases} Ed\hat{x}(t) = A(r_t)\hat{x}(t)dt + B(r_t)u(t)dt + J(r_t)[\hat{y}(t) - y(t)]dt, \\ \hat{y}(t) = F(r_t)\hat{x}(t), \\ u(t) = K(r_t)\hat{x}(t), \end{cases} \quad (69)$$

where $\hat{x}(t) \in \mathbb{R}^n$ is the estimation of $x(t)$; $K(r_t)$ and $J(r_t)$ are the controller gain and observer gain to be designed, respectively.

Let $e(t) = x(t) - \hat{x}(t)$ and $\tilde{x}(t) = [x^T(t) \quad e^T(t)]^T$, then the corresponding closed-loop augment system is given as in the following form:

$$\begin{cases} \tilde{E}d\tilde{x}(t) = [\tilde{A}(r_t)\tilde{x}(t) + \tilde{D}(r_t)v(t)]dt \\ \qquad\qquad + [\tilde{A}_0(r_t)\tilde{x}(t) + \tilde{D}_0(r_t)v(t)]dw(t), \\ z(t) = \tilde{C}(r_t)\tilde{x}(t) + \tilde{D}_1(r_t)v(t), \\ \tilde{E}\tilde{x}(0) = \tilde{x}_0, \end{cases} \quad (70)$$

where

$$\tilde{E} = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}, \quad \tilde{A}_0(r_t) = \begin{bmatrix} A_0(r_t) & 0 \\ A_0(r_t) & 0 \end{bmatrix},$$

$$\tilde{A}(r_t) = \begin{bmatrix} A(r_t) + B(r_t)K(r_t) & -B(r_t)K(r_t) \\ 0 & A(r_t) + J(r_t)F(r_t) \end{bmatrix},$$

$$\tilde{D}(r_t) = \begin{bmatrix} D(r_t) \\ D(r_t) \end{bmatrix}, \quad \tilde{D}_0(r_t) = \begin{bmatrix} D_0(r_t) \\ D_0(r_t) \end{bmatrix}, \ \tilde{D}_1(r_t) = D_1(r_t),$$

$$\tilde{C}(r_t) = [C(r_t) + B_1(r_t)K(r_t) \quad -B_1(r_t)K(r_t)].$$

Definition 20: For scalars $0 < c_1 < c_2, T > 0, \gamma > 0$ and a matrix $\tilde{Q} = diag\{Q, Q\} > 0$, the closed-loop augment system (70) is said to be

(i) FTSS wrt $(c_1, c_2, T, \tilde{Q})$ when $v(t) = 0$, if it has an impulse-free solution in the time interval $[0, T]$ and satisfies the following condition

$$\mathcal{E}\{\tilde{x}^T(0)\tilde{E}^T\tilde{Q}\tilde{E}\tilde{x}(0)\} \le c_1^2 \Longrightarrow \mathcal{E}\{\tilde{x}^T(t)\tilde{E}^T\tilde{Q}\tilde{E}\tilde{x}(t)\} < c_2^2. \quad (71)$$

(ii) FTSB wrt $(c_1, c_2, T, h, \tilde{Q})$, if it is FTSS and condition (2) holds for all nonzero $v(t)$.

(iii) FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, \tilde{Q})$, if it is FTSB while satisfying condition (17) under the zero initial condition.

Theorem 21: The closed-loop augment system (70) with completely known TRs is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, \tilde{Q})$, if there exist scalars $\delta \ge 0, \bar{\gamma} > 0$, a set of matrices $P_i = P_i^T$, nonsingular matrices R_i, and positive symmetric matrices $G_i > 0$ such that (18), (20), (21) and the following inequalities hold for each $r_t = i, i \in \mathcal{S}$

$$\begin{bmatrix} \Pi_{11i} & \Pi_{12i} & \Pi_{13i} & \Pi_{14i} \\ * & \Pi_{22i} & \Pi_{23i} & -K_i^T B_{1i}^T \\ * & * & \Pi_{33i} & D_{1i}^T \\ * & * & * & -I \end{bmatrix} < 0, \quad (72)$$

where

$$\begin{aligned} \Pi_{11i} &= He(A_i^T M_i + K_i^T B_i^T M_i) + 2A_{0i}^T (E^+)^T E^T P_i E E^+ A_{0i} \\ &\quad + \sum_{j=1}^{N} \hat{\pi}_{ij} E^T P_j E - \delta E^T P_i E, \\ \Pi_{12i} &= -M_i^T B_i K_i, \\ \Pi_{13i} &= M_i^T D_i + 2A_{0i}^T (E^+)^T E^T P_i E E^+ D_{0i}, \\ \Pi_{14i} &= C_{1i}^T + K_i^T B_{1i}^T, \\ \Pi_{22i} &= He(A_i^T M_i + F_i^T J_i^T M_i) + \sum_{j=1}^{N} \hat{\pi}_{ij} E^T P_j E - \delta E^T P_i E, \\ \Pi_{23i} &= M_i^T D_i, \quad \Pi_{33i} = -\bar{\gamma} I + 2D_{0i}^T (E^+)^T E^T P_i E E^+ D_{0i}. \end{aligned}$$

Proof: Setting $\tilde{E}_L = diag\{E_L, E_L\}, \tilde{E}_R = diag\{E_R, E_R\}, \tilde{E}^+ = diag\{E^+, E^+\}, \tilde{P}_i = diag\{P_i, P_i\}, \tilde{R}_i = diag\{R_i, R_i\}, \tilde{M}_i = diag\{M_i, M_i\}, \tilde{G}_i = diag\{G_i, G_i\}, G_i > 0$, it is easy to see that (18),(20) together with (21) imply the following conditions hold, respectively

$$\tilde{E}_L \tilde{P}_i \tilde{E}_L > 0, \quad (73)$$

$$\tilde{E}^T \tilde{M}_i = \tilde{E}^T \tilde{Q}^{1/2} \tilde{G}_i \tilde{Q}^{1/2} \tilde{E}, \quad (74)$$

$$e^{\delta T}(c_1^2 \lambda_2 + \bar{\gamma} h^2) < c_2^2 \lambda_1, \quad (75)$$

where $\lambda_2 = sup_{i\in\mathcal{S}}\{\lambda_{max}(G_i)\} = sup_{i\in\mathcal{S}}\{\lambda_{max}(\tilde{G}_i)\}, \lambda_1 = inf_{i\in\mathcal{S}}\{\lambda_{min}(G_i)\} = inf_{i\in\mathcal{S}}\{\lambda_{min}(\tilde{G}_i)\}$. On the other hand,

(72) is just the same as

$$\begin{bmatrix} \tilde{\Pi}_{11i} & \tilde{\Pi}_{12i} & \tilde{C}_i^T \\ * & \tilde{\Pi}_{22i} & D_{1i}^T \\ * & * & -I \end{bmatrix} < 0, \tag{76}$$

with

$$\tilde{\Pi}_{11i} = He(\tilde{A}_i^T \tilde{M}_i) + \tilde{A}_{0i}^T (\tilde{E}^+)^T \tilde{E}^T \tilde{M}_i (\tilde{E}^+) \tilde{A}_{0i} + \sum_{j=1}^{N} \hat{\pi}_{ij} \tilde{E}^T \tilde{M}_j - \delta \tilde{E}^T \tilde{M}_i,$$

$$\tilde{\Pi}_{12i} = \tilde{A}_{0i}^T (\tilde{E}^+)^T \tilde{E}^T \tilde{M}_i (\tilde{E}^+) \tilde{D}_{0i} + \tilde{M}_i^T \tilde{D}_i,$$

$$\tilde{\Pi}_{22i} = \tilde{D}_{0i}^T (\tilde{E}^+)^T \tilde{E}^T \tilde{M}_i (\tilde{E}^+) \tilde{D}_{0i} - \bar{\gamma} I.$$

Hence, based on Lemma 12, conditions (73)-(76) ensure that system (70) with completely known TRs is FTSB with an H_∞ disturbance attenuation γ. This completes the proof.

Theorem 22: The closed-loop augment system (70) with GUTRs is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, \tilde{Q})$, if there exist scalars $\delta \geq 0, \bar{\gamma} > 0, \lambda > 0$, a set of symmetric matrices $\bar{P}_i, \hat{P}_i, \bar{W}_i, \bar{S}_{ij}, \bar{X}_{ij}$, positive symmetric matrices $J_{2i} > 0, \bar{H}_i > 0, \bar{T}_{ij} > 0, \bar{Z}_{ij} > 0$, matrices $L_i, \tilde{J}_i$ and nonsingular matrices $\bar{R}_i, \hat{R}_i$ such that (61)-(65) and the following inequalities hold for each $r_t = i, i \in \mathcal{S}$:

$$E_R^T \bar{P}_i E_R > 0, \tag{77}$$

$$\hat{E}_R^T \hat{P}_i \hat{E}_R > 0, \tag{78}$$

$$F_i N_i = (\hat{P}_i \hat{E}^T + \hat{\Lambda} \hat{R}_i \hat{\Phi}) F_i, \tag{79}$$

$$\begin{bmatrix} \tau_{11i} & \tau_{12i} & \tau_{13i} & \tau_{14i} & \tau_{15i} \\ * & \tau_{22i} & \tau_{23i} & 0 & 0 \\ * & * & \tau_{33i} & 0 & 0 \\ * & * & * & \tau_{44i} & 0 \\ * & * & * & * & \tau_{55i} \end{bmatrix} < 0, \quad i \in \mathcal{S}_k^i, \tag{80}$$

$$\begin{bmatrix} \zeta_{11i} & \zeta_{12i} & \zeta_{13i} & \zeta_{14i} & \zeta_{15i} \\ * & \zeta_{22i} & \zeta_{23i} & 0 & 0 \\ * & * & \zeta_{33i} & 0 & 0 \\ * & * & * & \zeta_{44i} & 0 \\ * & * & * & * & \zeta_{55i} \end{bmatrix} < 0, \quad i \in \mathcal{S}_{uk}^i, \tag{81}$$

where

$$\tau_{11i} = \begin{bmatrix} \sum_{5i} & -B_i L_i \\ * & \sum_{8i} \end{bmatrix}, \quad \tau_{12i} = \begin{bmatrix} D_i & \sum_{6i} \\ D_i & -L_i^T B_{1i}^T \end{bmatrix},$$

$$\tau_{13i} = \begin{bmatrix} N_i^T A_{0i}^T (E^+)^T E_R & N_i^T A_{0i}^T (E^+)^T E_R \\ 0 & 0 \end{bmatrix},$$

$$\tau_{14i} = \begin{bmatrix} \bar{W}_i & \bar{\Psi}_{1i} & \Upsilon_{1i} \\ 0 & 0 & 0 \end{bmatrix}, \quad \tau_{15i} = \begin{bmatrix} 0 & 0 & 0 \\ \bar{W}_i & \bar{\Psi}_{1i} & \Upsilon_{1i} \end{bmatrix},$$

$$\tau_{22i} = \begin{bmatrix} -\bar{\gamma} I & D_{1i}^T \\ * & I \end{bmatrix},$$

$$\tau_{23i} = \begin{bmatrix} D_{0i}^T (E^+)^T E_R & D_{0i}^T (E^+)^T E_R \\ 0 & 0 \end{bmatrix},$$

$$\tau_{33i} = diag\{-E_R^T \bar{P}_i E_R, -E_R^T \bar{P}_i E_R\},$$

$$\tau_{44i} = diag\{-\bar{H}_i, -\bar{\Psi}_{2i}, -\Upsilon_{2i}\},$$

$$\tau_{55i} = \tau_{44i}, \zeta_{11i} = \begin{bmatrix} \sum_{7i} & -B_i L_i \\ * & \sum_{9i} \end{bmatrix},$$

$$\zeta_{12i} = \tau_{12i}, \zeta_{13i} = \tau_{13i},$$

$$\zeta_{14i} = \begin{bmatrix} \bar{\Psi}_{3i} & \Upsilon_{1i} \\ 0 & 0 \end{bmatrix}, \quad \zeta_{15i} = \begin{bmatrix} 0 & 0 \\ \bar{\Psi}_{3i} & \Upsilon_{1i} \end{bmatrix},$$

$$\zeta_{22i} = \tau_{22i}, \quad \zeta_{23i} = \tau_{23i}, \; \zeta_{33i} = \tau_{33i},$$

$$\zeta_{44i} = -diag\{\bar{\Psi}_{4i}, \Upsilon_{2i}\}, \zeta_{55i} = \zeta_{44i},$$

$$\sum_{8i} = He(A_i N_i + \tilde{J}_i F_i) - \delta N_i^T E^T + \theta_{3i},$$

$$\sum_{9i} = He(A_i N_i + \tilde{J}_i F_i) - \delta N_i^T E^T + \theta_{4i},$$

$\hat{E}$ is a suitable singular matrix with $\hat{E} = \hat{E}_L \hat{E}_R^T$, $\hat{\Phi}$ and $\hat{\Lambda}$ are left and right null matrix of $\hat{E}$, respectively, and $\sum_{5i}, \sum_{6i}, \sum_{7i}, \bar{\Psi}_{1i}, \bar{\Psi}_{2i}, \Upsilon_{1i}, \Upsilon_{2i}, \theta_{3i}, \theta_{4i}$ are given in Theorem 17. In this case, the state feedback control gain matrices are presented as $K_i = L_i N_i^{-1}$ with $N_i = \bar{P}_i E^T + \Lambda \bar{R}_i \Phi, i \in \mathcal{S}$, and the observer gain matrices are calculated as $J_i = \tilde{J}_i (\hat{P}_i \hat{E}^T + \hat{\Lambda} \hat{R}_i \hat{\Phi})^{-1}, i \in \mathcal{S}$.

Proof: Performing the congruence transformation on (72) with $diag\{M_i^{-1}, M_i^{-1}, I, I\}$ and setting $L_i = K_i N_i, \tilde{J}_i = J_i (\hat{P}_i \hat{E}^T + \hat{\Lambda} \hat{R}_i \hat{\Phi})$, then using the Schur complement equivalence, (72) becomes

$$\begin{bmatrix} \tilde{\tau}_{11i} & \tau_{12i} & \tau_{13i} \\ * & \tau_{22i} & \tau_{23i} \\ * & * & \tau_{33i} \end{bmatrix} < 0, \tag{82}$$

where

$$\tilde{\tau}_{11i} = \begin{bmatrix} \nabla_{1i} & -B_i L_i \\ * & \nabla_{2i} \end{bmatrix},$$

$$\nabla_{1i} = He(A_i N_i + B_i L_i) + \sum_{j=1}^{N} \hat{\pi}_{ij} N_i^T E^T P_j E N_i - \delta N_i^T E^T,$$

$$\nabla_{2i} = He(A_i N_i + \tilde{J}_i F_i) + \sum_{j=1}^{N} \hat{\pi}_{ij} N_i^T E^T P_j E N_i - \delta N_i^T E^T.$$

Furthermore, $\sum_{j=1}^{N} \hat{\pi}_{ij} N_i^T E^T P_j E N_i$ can be taken the same measures as $\sum_{j=1}^{N} \hat{\pi}_{ij} E^T M_j$ in Theorem 14. Then it can be proved that inequalities (80) and (81) imply (82). The detail proof here is omitted.

As a byproduct of Theorem 22, the following conditions regarding FTSB with an H_∞ performance index γ of the closed-loop augment system (70) with completely known TRs and $w(t) = 0$ are derived.

Corollary 23: There exist the mode-dependent observer-based controller in the form of (69) such that the closed-loop augment system (70) with completely known TRs and $w(t) = 0$ is FTSB with an H_∞ disturbance attenuation γ wrt $(c_1, c_2, T, h, \gamma, \tilde{Q})$, if there exist scalars $\delta \geq 0, \bar{\gamma} > 0, \lambda > 0$, a set of matrices $\bar{P}_i = \bar{P}_i^T, \hat{P}_i = \hat{P}_i^T, J_{2i} > 0, L_i, \tilde{J}_i$ and nonsingular matrices $\bar{R}_i, \hat{R}_i$, such that (64),(65), (77)-(79) and the following inequalities hold for each $i \in \mathcal{S}$:

$$\begin{bmatrix} \bar{\sum}_{1i} & -B_iL_i & D_i & \sum_{6i} & \Psi_{5i} & 0 \\ * & \bar{\sum}_{2i} & D_i & -L_i^TB_{1i}^T & 0 & \Psi_{5i} \\ * & * & -\bar{\gamma}I & D_{1i}^T & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & \Psi_{6i} & 0 \\ * & * & * & * & * & \Psi_{6i} \end{bmatrix} < 0, \tag{83}$$

where $\bar{\sum}_{2i} = He(A_iN_i+\tilde{J}_iF_i)+(\pi_{ii}-\delta)E\bar{P}_iE^T$, $\bar{\sum}_{1i}$, $\Psi_{5i}\Psi_{6i}$ are the same as those in Corollary 18. The state feedback gain and the observer gain are presented as those of Theorem 22.

Remark 24: It is observed that the state feedback gain and observer-based gain are simultaneously resolved at the expense of using the equality constraint (79), how to remove it deserve our further study. Also, to solve (79) by applying Matlab LMI toolbox, we replace it as the following inequality

$$\begin{bmatrix} -\rho I & F_iN_i - (\hat{P}_iE^T + \hat{\Lambda}\hat{R}_i\hat{\Phi})F_i \\ * & -I \end{bmatrix} < 0, \tag{84}$$

where ρ is a sufficiently small positive scalar.

IV. NUMERICAL EXAMPLES

In this section, two examples are proposed to demonstrate the effectiveness of our presented results.

Example 25: Consider a oil catalytic cracking model [4], [12], which is described by

$$\begin{aligned} \dot{x}_1(t) &= a_1x_1(t) + a_2x_2(t) + b_1u(t) + d_1v(t), \\ 0 &= a_3x_1(t) + a_4x_2(t) + b_2u(t) + d_2v(t), \end{aligned} \tag{85}$$

where $x_1(t)$ denotes a vector to be regulated, such as regenerate temperature, valve position, blower capacity, etc; $x_2(t)$ represents the vector reflecting business benefits, administration, policy, etc; $u(t)$ is the regulation value, and $v(t)$ represents a extra disturbance. Obviously, (85) can be equivalently expressed as

$$E\dot{x}(t) = Ax(t) + Bu(t) + Dv(t), \tag{86}$$

with

$$\begin{aligned} x(t) &= [x_1(t)^T \quad x_2(t)^T]^T, \\ E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, \\ B &= \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad D = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}. \end{aligned} \tag{87}$$

To demonstrate the efficiency of our proposed results, without loss of generality, the coefficient matrices of (86) are modeled by a Markovian process, and the control output equation

$$z(t) = C(r_t)x(t) + B_1(r_t)u(t) + D_1(r_t)v(t)$$

is taken into account, then (86) becomes

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) + D(r_t)v(t) \\ z(t) = C(r_t)x(t) + B_1(r_t)u(t) + D_1(r_t)v(t), \\ Ex(0) = x_0. \end{cases} \tag{88}$$

In order to compare Corollary 18 with the existing results of [27], the coefficient matrices of (88) are taken the same data as example 2 of [27]:

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.8 & 1.5 \\ 2 & 3 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -2 & 1.2 \\ 1 & 4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -1 & 0.2 \\ 0.5 & -0.1 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} -1 & 1 \\ 0.5 & -2 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \\ D_2 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \quad C_1 = C_2 = [1\ 1], \\ B_{11} &= B_{12} = [0.1\ 0.2], \quad D_{11} = D_{12} = 0.1. \end{aligned}$$

The Markovian jump transition probability matrix is given by

$$\begin{bmatrix} -1.2 & 1.2 \\ 1 & -1 \end{bmatrix}. \tag{89}$$

Other parameters are given as $h = 1, T = 2, c_1 = 1, Q = I$. According to the method proposed in [27], the expression $min(\gamma^2 + c_2^2)$ was viewed as the optimal objective to get the state feedback gain for FTH_∞SB of system (88), then feasible solutions can be obtained when δ changes the value in [1.85 10.34]. Furthermore, in the case of $\delta = 2.4283$, the optimal values were presented by $c_2 = 20.0345$ and $\gamma = 12.5486$. However, by applying Corollary 18 of this paper, feasible solutions can be found when $\delta \in [0 \quad 10.35]$, and the optimal values are given as $c_2 = 11.3967$ and $\gamma = 1.1348$. Therefore, it can be seen that, even for system (1) without noise, our obtained results are much better than existing one.

Example 26: In many practical cases, the environment noise often exist and cannot be neglected. If we take white noise $\dot{w}(t)$ into account system (88), then the singular Itô stochastic Markovian jump system in the form of (1) is presented. To verify the efficiency of Theorem 17 and Theorem 22, the three-mode singular Itô Markovian jump system (1) with GUTRs was taken the following data:

For mode 1,

$$\begin{aligned} A_1 &= \begin{bmatrix} -0.5 & -0.8 & -1 \\ 1.5 & -1 & 1 \\ 0 & -1 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.2 & 0.4 \\ 1 & 1 \\ -1 & 1 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0.2 \\ 0.4 \\ 0 \end{bmatrix}, \quad A_{01} = \begin{bmatrix} 0.4 & 0.2 & -0.1 \\ 0.5 & 0 & 0.3 \\ 0 & 0 & 0 \end{bmatrix}, \\ D_{01} &= \begin{bmatrix} 0 \\ 0 \\ -0.6 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 1 & 1 \\ 4 & 1 & 1 \end{bmatrix}, \end{aligned}$$

$$B_{11} = \begin{bmatrix} 2 & -0.4 \\ 0.7 & 0.8 \\ 0 & 1 \end{bmatrix}, \quad D_{11} = \begin{bmatrix} 0.5 \\ -0.4 \\ 0.8 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} 1 & -0.7 & -2 \\ 2 & 1 & 0 \end{bmatrix}.$$

For mode 2,

$$A_2 = \begin{bmatrix} -5 & 0.3 & 0 \\ 0.2 & -8.6 & 0 \\ 0 & -1 & -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0.5 \\ 0 & 1 \\ -1 & 1 \end{bmatrix},$$
$$D_2 = \begin{bmatrix} 0 \\ 0.6 \\ -0.8 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} 0.3 & -0.1 & 0.2 \\ -0.4 & 0.2 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$D_{02} = \begin{bmatrix} -0.2 \\ 0 \\ 0.3 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -3 & 2 & 1 \\ 3 & 4.2 & 1.3 \\ 4 & 1 & 1 \end{bmatrix},$$
$$B_{12} = \begin{bmatrix} 2.6 & 0.6 \\ 0.2 & 0.9 \\ 1.2 & 1.4 \end{bmatrix}, \quad D_{12} = \begin{bmatrix} 0 \\ -0.6 \\ 0.8 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -0.7 & 0.9 & -1 \\ 0.9 & 0.8 & 1 \end{bmatrix}.$$

For mode 3,

$$A_3 = \begin{bmatrix} -10 & 0 & 0 \\ 0 & -9 & 0 \\ 1 & -1 & -6 \end{bmatrix}, \quad B_3 = \begin{bmatrix} 0 & 0.5 \\ 0.9 & 1 \\ 1 & 0.8 \end{bmatrix},$$
$$D_3 = \begin{bmatrix} 0.2 \\ -0.4 \\ 0.2 \end{bmatrix}, \quad A_{03} = \begin{bmatrix} -0.5 & 0.4 & -0.3 \\ -0.2 & 0.3 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$
$$D_{03} = \begin{bmatrix} 0.1 \\ 0 \\ 0.4 \end{bmatrix}, \quad C_3 = \begin{bmatrix} 1.1 & 0 & -1 \\ -0.1 & -0.5 & 0 \\ -0.2 & 0 & 0 \end{bmatrix},$$
$$B_{13} = \begin{bmatrix} -0.5 & 0.7 \\ 0.9 & 0.4 \\ 0.1 & 0.2 \end{bmatrix}, \quad D_{13} = \begin{bmatrix} 0.6 \\ -0.4 \\ -0.9 \end{bmatrix},$$
$$F_3 = \begin{bmatrix} -3 & 0 & -1 \\ 0.9 & -4.5 & -3 \end{bmatrix}.$$

The Markovian jump matrix with GUTRs is given as

$$\begin{bmatrix} -1+\Delta\pi_{11} & ? & ? \\ ? & ? & 2+\Delta\pi_{23} \\ 0.2+\Delta\pi_{31} & 0.3+\Delta\pi_{32} & -0.5+\Delta\pi_{33} \end{bmatrix}. \quad (90)$$

In addition, we choose

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad E_L = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \; E_R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$
$$\hat{E} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Phi = [0 \quad 0 \quad 0.7], \; \Lambda = [0 \quad 0 \quad 0.9]^T,$$
$$\hat{\Phi} = [0 \quad 0.5], \quad \hat{\Lambda} = [0 \quad 0.6]^T, \; \mu_{ij} = 0.1 * |\Delta\pi_{ij}|.$$

In this paper, we mainly concern with the H_∞ performance of system, so $\bar{\gamma}$ is considered as optimized variable to obtain an optimized finite-time stabilized controller. Given $c_1 = 0.3$, $c_2 = 5, h = 1, T = 2, \delta = 0.2, \beta = 5, Q = I, \rho = 1.0 \times 10^{-10}$, then by solving inequalities (58)-(65) in Theorem 17, the optimal value of $\bar{\gamma}$ is 1.3400, and the state feedback gain matrices are presented by

$$K_1 = \begin{bmatrix} -1.1328 & -1.4788 & -0.7780 \\ -5.0131 & -1.6555 & -0.7776 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 2.0603 & 1.0426 & 0.1517 \\ -4.4259 & -2.7252 & -0.8340 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 1.1864 & -0.3998 & -0.7173 \\ -1.9402 & -1.3976 & 1.0412 \end{bmatrix}.$$

Also, by solving inequalities (61)-(65) and (77)-(81) in Theorem 22, the optimal value of $\bar{\gamma}$ is 1.3541. The state feedback controller gains are obtained as follows:

$$K_1 = \begin{bmatrix} 1.1977 & -1.6600 & -0.8449 \\ -4.6190 & -1.0782 & -0.5892 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 2.1343 & 1.0335 & 0.1827 \\ -4.2151 & -2.2946 & -0.7128 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} 0.9321 & -0.2976 & -0.6477 \\ -0.0982 & -0.7014 & 0.0132 \end{bmatrix}.$$

The observer gains are given as

$$J_1 = \begin{bmatrix} 0.2874 & -1.5799 \\ 0.2579 & -0.0475 \\ 0.2191 & 1.2021 \end{bmatrix},$$
$$J_2 = \begin{bmatrix} 0.2674 & -0.3510 \\ -0.2029 & -1.4795 \\ 0.3902 & 0.0352 \end{bmatrix},$$
$$J_3 = \begin{bmatrix} 13.2093 & -0.2293 \\ -28.9715 & 22.4344 \\ -25.3593 & 23.5054 \end{bmatrix}.$$

Remark 27: To perform the simulation, we assume $v(t) = 0.6^t(\int_0^2 0.6^{2t}dt = 0.8519 < 1)$ and the initial value $x(0) = [0.3\ 0.3\ 0.2]^T$. By using the Euler-Maruyama approach to simulate Wiener process, state response curves $x(t)$ in Figure 1, $x^T(t)E^TQEx(t)$ and $\mathcal{E}\{x^T(t)E^TQEx(t)\}$

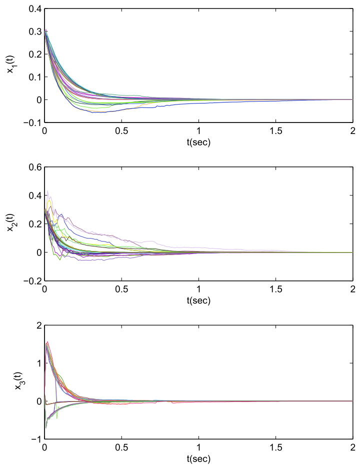

FIGURE 1. State response curves of the closed-loop system (15).

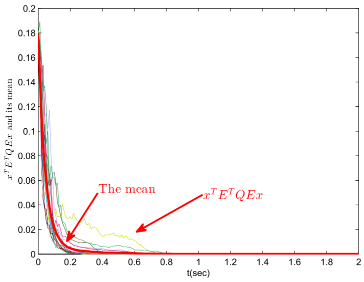

FIGURE 2. $\mathcal{E}\{x^T(t)E^TQEx(t)\}$ for the closed-loop system (15).

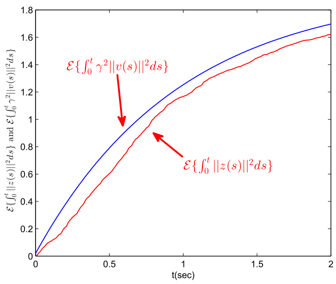

FIGURE 3. H_∞ performance analysis for the closed-loop system (15).

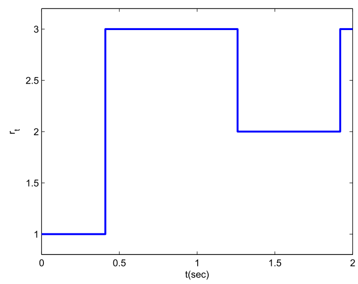

FIGURE 4. Curves of r_t.

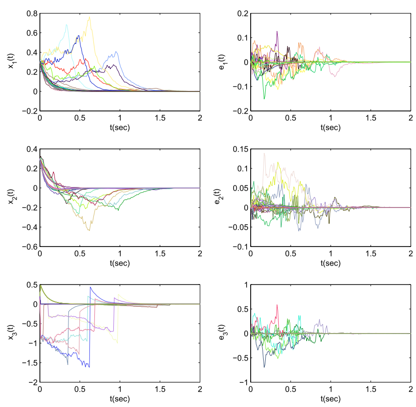

FIGURE 5. State response curves of the closed-loop system (70).

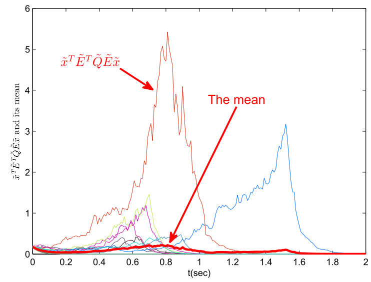

FIGURE 6. $\mathcal{E}\{\tilde{x}^T(t)\tilde{E}^T\tilde{Q}\tilde{E}\tilde{x}(t)\}$ for the closed-loop system (70).

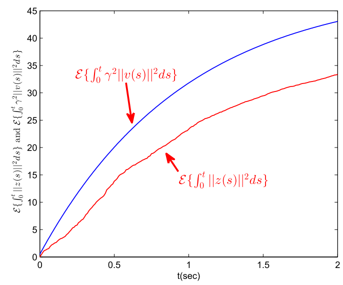

FIGURE 7. H_∞ performance analysis for the closed-loop system (70).

in Figure 2 for the closed-loop system (15) are obtained. Figure 1 shows that the curves asymptotically converge 0, which denotes that the closed-loop system (15) with GUTRs is mean square admissible at infinite time. This is because LMIs (58)-(65) still have feasible solutions in the case of $\delta = 0$. Furthermore, it can be seen from Figure 2 that system (15) with GUTRs is FTSB wrt $(0.3, 5, 2, 1, I)$. Under the zero initial condition, Figure 3 exhibits that the desired H_∞ performance is achieved in the finite-time interval $[0, 2]$. So what we have obtained in Theorem 17 are effective. Similarly, to illustrate the efficiency of Theorem 22, the initial value for system (70) is taken as $\tilde{x}(0) = [0.3\ 0.3\ 0.2\ 0\ 0\ 0]^T$. Then state response curves $x(t)$ and error response curves $e(t)$ are proposed in Figure 5, and the response for $\mathcal{E}\{\tilde{x}^T(0)\tilde{E}^T\tilde{Q}\tilde{E}\tilde{x}(0)\}$ is presented in Figure 6. Therefore, we can also draw a conclusion that the closed-loop augment system (70) with GUTRs not only is FTSB wrt $(0.3, 5, 2, 1, I)$ but also mean square admissible in the infinite time. Besides, from Figure 7, H_∞ performance for system (70) is satisfied as well.

V. CONCLUSION

In this paper, we have discussed the problems of finite-time stochastic H_∞ control for singular Itô-type Markovian jump systems with (x, v)-dependent noise, including both cases of completely known TRs and generally uncertain TRs. First, sufficient conditions ensuring the considered systems with completely known TRs to be FTSB with H_∞ performance have been presented, which, even for the system without noise, are still less conservative than the results of [27]–[29]. Then by utilizing the association of free-weighting matrices and slack matrices, we have obtained the new criteria of finite-time stochastic H_∞ control for the underlying systems with GUTRs. The state feedback controller and observer-based controller have successfully been designed.

It is interesting for us to extend the obtained results of this paper to discrete-time singular systems or semi-Markovian jump systems in the near future.

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