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Guaranteed-Cost Synchronization for Second-Order Wireless Sensor Networks With Nonlinear Dynamics and Given Cost Budgets

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ABSTRACT Guaranteed-cost synchronization problems for the second-order Lipschitz nonlinear wireless sensor network with switching topologies and the given cost budget are investigated. The existing sufficient conditions for the network synchronization are usually proposed on the basis of linear matrix inequality (LMI) tools without taking given cost budgets into account. First, this paper designs a network synchronization protocol with a guaranteed-cost constraint, where the tradeoff between the battery power consumption and the network synchronization performance is established. Second, by the structure characteristic of a piecewise continuous matrix and the Lipschitz condition, the nonlinear term of the dynamics is linearized. Then, sufficient conditions are developed to make the Lipschitz nonlinear network reach guaranteed-cost synchronization, and an upper bound of the cost function is derived for the case not considering the given cost budgets. Third, for the case where the whole energy supply is limited, the relationship between the practical energy consumption and the given limited cost budget is drawn to the criterion as a cost constraint, which can make the nonlinear network reach guaranteed-cost synchronization. In particular, the explicit expressions of control gains in these criteria are derived, which indicates that the existence of control gains in network synchronization criteria can be ensured and most existing references cannot deal with the analytic solutions of control gains. In addition, the proposed criteria depend upon the minimum nonzero and maximum eigenvalues, which means that the scalability of the wireless sensor network can be ensured. Finally, the theoretical results are illustrated by numerical simulations.

INDEX TERMS Wireless sensor network, Lipschitz nonlinear, guaranteed-cost synchronization, switching topology, cost budget.

I. INTRODUCTION

Wireless sensor networks consist of certain wireless devices installed with sensors which can collect the information from the environment. All the components are the active network participant to function as a communication medium. Wireless sensor networks have attracted much attention and been extensively investigated for the widespread use in many fields such as unmanned aerial vehicle control, environmental observation and battlefield surveillance [1]–[9]. In order to forward the information to the destination, many source sensors interact with one another until the packets reach to the sink sensor. Hence, the network can be modeled as the type of the leader-following structure. In fact, due to various network constraints such as the limited memory space, variable transmission rates and the limited energy supply, sensor nodes may suffer from network congestion problems. Moreover, the network congestion may also be worsened caused by connection failures and random changes for environmental disturbances. Thus, both the dynamics and connection topologies are important factors to be considered for the networks to improve the data packet transmission performance decreased by various network constraints [10]–[13]. To suppress the network congestion and improve the transmission performance,

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the information synchronization of the specific performance can be taken as a valid measure, which means that the same states between the sink node and follower nodes are reached for the wireless sensor network.

Due to the environmental disturbances, the nonlinear intrinsic dynamics can be proposed to deal with these random changes for the wireless sensor network. In the literatures [14]–[18], occurring nonlinearities have been modeled for the multi-node network synchronization problem. The paper [14] addressed nonlinear multi-node systems with exogenous disturbances by the assumption that the interaction topology was fixed. Yu *et al.* [15] addressed the second-order multi-node network synchronization problem with the nonlinear dynamics and directed topologies. The paper [16] proposed leader-following synchronization algorithms by introducing the pinning control for second-order multi-node networks with general network interaction topologies. In [17]–[18], control problems for the Lipschitz nonlinear time-delayed systems were addressed and some important and meaningful results were presented. On the other hand, network conditions driven by node failures, mobility and certain intentional misbehaviors can lead to the changing interaction topologies. In addition, the network synchronization with switching topologies is more challenging and meaningful compared with fixed topologies. Wang *et al.* [19] gave the synchronization criteria for the multi-node network with fixed topologies. Cao and Ren [20] focused on the synchronization control problems for linear multi-node networks with undirected and directed graphs in a sampled-data setting. Wen *et al*. [21] explored the global pinning synchronization problem of the multi-node network, where switching directed topologies were considered. In [22], the synchronization problem for homogeneous multi-node networks was invested under switching communication topologies. In above literatures [14]–[22], the synchronization regulation performance was not considered.

When sensor nodes perform the tasks such as the information collection, the data packet transmission and the movement, the limited battery power of the practical multi-node network should be taken into consideration. Meanwhile, the synchronization regulation is required to be considered by some performance indices. Hence, the network synchronization problem with both the limited energy supply and the synchronization performance regulation can be modeled as the optimization problem, which addresses realizing the tradeoff design between two factors for the energy utility optimization. The Laplacian matrix of the interaction topology for the optimal synchronization problem is associated with a complete graph as shown in [23]. Hence, the suboptimal synchronization problem was extensively investigated. In [24], second-order multi-node systems achieved guaranteed performance synchronization by taking advantage of the impulsive control. In [25], distributed guaranteed performance synchronization problems for linear and nonlinear multi-node networks with switching interaction topologies were addressed. It can be seen that the synchronization regulation performance was studied in aforementioned researches without considering the control effort for the multi-node network. Hence, the guaranteed-cost control is an effective approach to solve the tradeoff problem with multi-node networks, which can be regarded as a suboptimal problem to realize the tradeoff design between two factors for the energy utility optimization. Zhou *et al.* [26] addressed guaranteed-cost synchronization problems by an event based control scheme for the distributed multi-node network. Xi *et al.* [27] investigated the nonlinear multi-node network guaranteed-cost synchronization with switching interaction topologies. In [26]–[27], these mentioned approaches cannot make the multi-node network reach guaranteed-cost synchronization in the presence of given cost budgets; that is, they cannot obtain the tradeoff between the system energy consumption and the network synchronization performance by assuming that the cost budget is limited for the multi-node network.

In the literatures [28]–[33], the synchronization criteria were proposed as a numerical algorithm based on the powerful tool of the linear matrix inequality (LMI) technique for the multi-node network. In [28], the distributed guaranteed-cost synchronization problems for the multi-node network were addressed with the general linear models, where synchronization criteria were designed by LMI tools. The paper [29] considered the distributed synchronization problems in terms of LMIs for the multi-node network with the general linear and Lipschitz nonlinear dynamics. In [30], several sufficient conditions were proposed for the network synchronization by LMIs, where the computational complexity greatly increased with the increasing of number of the network nodes. Rezaee and Abdollahi [31] investigated the multi-node network synchronization problems, where the Lipschitz nonlinearities and the jointly connected topologies were both dealt with. In [28]–[33], the criteria for the multi-node network synchronization relied on the LMI tools, which used a feasp solver to get feasible control gain matrices and they may not obtain the feasible solutions. Meanwhile, applying the structure characteristic of the second-order multi-node network, the analytic solutions of control gains are required to be determined for the guaranteed-cost synchronization with the cost budget given previously. It is increasingly recognized that there still remain many open problems to be further studied for the wireless sensor network as the special type of the multi-node network.

In this paper, we focus on the Lipschitz nonlinear guaranteed-cost synchronization problem for the second-order leader-following wireless sensor network with switching topologies and the cost budgets given previously. By utilizing the state errors between the leader node and follower nodes of the network, the dynamics of the network is transformed with the nonlinear dynamics. Then, according to the interaction weight matrix among follower nodes, a piecewise continuous orthonomal matrix and its transpose are proposed. By utilizing the structure characteristic of a piecewise continuous orthonomal matrix and the Lipschitz condition, the existing nonlinear term is eliminated.

For the case not considering the cost budget given previously, analytic solutions of the control gains are determined for the sufficient condition of the synchronization algorithm, which do not subject to the nonlinear constraints. Meanwhile, an upper bound of the cost function composed of the network energy consumption term and the network synchronization performance term is given. For the case considering limited cost budgets, the relationship between given cost budgets and an upper bound of the optimization index is drawn to the synchronization criteria, which means that the whole energy supply of the wireless sensor network is limited. In addition, synchronization criteria are dependent on the initial states, the minimum nonzero eigenvalue and the maximum eigenvalue, which means that the scalability of the wireless sensor network can be ensured.

Compared with the existing researches [34]–[37] on guaranteed-cost synchronization for the wireless sensor network, this paper mainly has following three contributions. Firstly, the current paper addresses influences of the Lipschitz nonlinear dynamics and switching topologies on the guaranteed-cost synchronization and an approach is proposed to linearize the nonlinear term. The methods in aforementioned researches [34] and [35] are no longer applicable to deal with the nonlinear dynamics and changing topologies. Secondly, analytic solutions of control gains are demonstrated for the guaranteed-cost synchronization, which present explicit expressions of control gains in synchronization criteria. The synchronization criteria in [35] and [36] are designed in terms of the gain matrices by LMI tools. In some situations, the synchronization criteria may not have the feasible numerical solutions. Thirdly, the limited cost budget can be introduced, which means that the practical limited energy supply of the wireless sensor network is taken into account. The research approaches in [34]–[37] presented different guaranteed-cost upper bounds not considering the limited energy supply of the wireless sensor network. The current paper concerns with cost budgets given previously as a constraint to design control gains of synchronization protocols.

The outline of this paper is organized as follows. Section II gives certain preliminary knowledge and problem descriptions on the basis of graph theory. In Section III, sufficient conditions of the leader-following guaranteed-cost synchronization are presented for the Lipschitz nonlinear wireless sensor network without the given cost budget and with the given cost budget and an approach to obtain an upper bound of the guaranteed-cost function is given. A simulation example which demonstrates theoretical results is presented in Section IV. Finally, some remarks conclude the paper in Section V.

Notations: Let R^m denote the m-dimensional real column vector space, $R^{m\times m}$ stand for the set of $m\times m$ dimensional real matrices and I and $\mathbf{1}$ stand for the identity matrix and the column vector with components equal to 1, respectively. $A^T = A < 0$ and $A^T = A > 0$ express that the symmetric matrix A is negative definite and positive definite, respectively. Let symbol $\otimes$ denote the Kronecker product and the diagonal matrix with diagonal elements $\lambda_1, \lambda_2, \cdots, \lambda_N$ is denoted by $\mathrm{diag}\{\lambda_1, \lambda_2, \cdots, \lambda_N\}$.

II. PRELIMINARIES AND PROBLEM DESCRIPTION

In this section, some preliminary concepts in terms of the graph theory are introduced and then the problem description is proposed as follows.

A. PRELIMINARIES ON GRAPH THEORY

This paper depicts the network topology by a graph $G = (V(G), E(G))$ for the network with N identical nodes. Graph G is composed of a nonempty vertex set $V(G) = \{v_1, v_2, \cdots, v_N\}$ and the edge set $E = \{e_{ij} = \{(v_i, v_j)\} \in V \times V$. The vertex v_i expresses sensor node i, the edge e_{ij} denotes the connection channel between sensor nodes i and j, and the edge weight w_{ji} of e_{ij} is defined as the interaction strength from sensor nodes i to j, where w_{ji} is zero if sensor nodes i to j is not connected, and is positive otherwise. The index set of all neighbor nodes of vertex v_j is expressed by $N_j = \left\{i : (v_i, v_j) \in E(G)\right\}$. The sequence of edges $\left(v_l, v_{l_1}\right), \left(v_{l_1}, v_{l_2}\right), \cdots, \left(v_{l_{k-1}}, v_k\right)$ describes a path between sensor nodes l and k. Let $L = \left[l_{ji}\right] \in R^{N\times N}$ with $l_{jj} = \sum_{i\in N_j} w_{ji}$ and $l_{ji} = -w_{ji}\ (j \neq i)$ be the Laplacian matrix of graph G. A directed graph has a spanning tree if there exists a directed path from a root node to any other node. For the leader-following wireless sensor network, it is assumed that the network interaction topology has a spanning tree and is connected; that is, the motion of the leader is independent of other follower nodes and partial other nodes have access to the state information of the leader node. The local interaction topology among followers is undirected. If the interaction topology G is connected, then the minimum eigenvalue of the Laplacian matrix L is zero and all the other eigenvalues are larger than zero. One can refer to paper [38] for more basic knowledge and concepts on graph theory.

B. DESCRIBING LIPSCHITZ NONLINEAR GUARANTEED-COST SYNCHRONIZATION

Consider the wireless sensor network with N homogeneous second-order Lipschitz nonlinear network nodes. One can set that sink node 1 as the leader and the others are followers. Consider the dynamics of sensor nodes expressed by

$$\begin{cases} \dot{x}_j(t) = v_j(t), \\ \dot{v}_j(t) = u_j(t) + f\left(x_j(t), v_j(t)\right), \end{cases} \tag{1}$$

where $j = 1, 2, \cdots, N$, $x_j(t) \in R$ and $v_j(t) \in R$ denote the queue length state and the transmission rate state of sensor node j, and $u_j(t) \in R$ is the control input. Since the leader does not obtain local transmission state information from other sensor nodes, we can set control input $u_1(t) \equiv 0$. In (1), $f\left(x_j(t), v_j(t)\right)$ is the intrinsic nonlinear dynamics, which could include unmodeled dynamics, parameter variations and other external disturbances. The Lipschitz nonlinear function $f\colon \mathbb{R}^2 \times [0, +\infty) \to \mathbb{R}$ is continuous and differentiable and satisfies the

Lipschitz condition $\left|f\left(x_i(t), v_i(t)\right) - f\left(x_j(t), v_j(t)\right)\right| \leq \gamma\left(\left|x_i(t)-x_j(t)\right|^2 + \left|v_i(t) - v_j(t)\right|^2\right)^{0.5}$ $(i, j \in \{1, 2, \cdots, N\})$ with the Lipschitz constant $\gamma > 0$.

Let all possible variable connected topologies be expressed by a topology set $\Gamma = \{G_1, G_2, \cdots, G_M\}$ with a finite natural number index set $\mathcal{I}_M = \{1, 2, \cdots, M\}$. The time function $\sigma(t) : [0, \infty) \rightarrow \mathcal{I}_M$ presents a switching signal and the value at time t denotes the index of the switching interaction topology set at time t. Moreover, in the current paper, it is assumed that switching sequences $0 < t_1 < t_2 < \cdots < t_n < \cdots$ satisfy $\inf_n (t_{n+1} - t_n) = T_c (\forall n \geq 0)$ for a positive constant T_c. It should be pointed out that all the $\Gamma = \{G_1, G_2, \cdots, G_M\}$ can be arbitrarily switched.

To make transmission nodes follow the sink node with the same desired states for the wireless sensor network with the given $\gamma_1, \gamma_2, \eta \in R > 0$, consider a guaranteed-cost synchronization control protocol with a linear quadratic cost function and switching topologies designed as follows:

$$\begin{cases} u_j(t) = \sum\limits_{i \in N_{\sigma(t),j}} w_{\sigma(t),ji} \left(k_1 \left(x_i(t) - x_j(t)\right)\right. \\ \qquad\left. + k_2 \left(v_i(t) - v_j(t)\right)\right), \\ J_s = \int_0^{\infty} \left(J_u(t) + J_x(t)\right) dt, \end{cases} \tag{2}$$

where $j = 2, 3, \cdots, N$, $k_1 > 0$ and $k_2 > 0$ are control gains, $w_{\sigma(t),ji}$ denotes the weight of the transmission channel (v_i, v_j) with $w_{\sigma(t),ii} = 0$, $w_{\sigma(t),ji} = w_{\sigma(t),ij} \geq 0$ and $w_{\sigma(t),ij} > 0$ if $(v_j, v_i) \in E\left(G_{\sigma(t)}\right)$, $N_{\sigma(t),j}$ denotes the neighbor set of node j at time t, and

$$J_u(t) = \sum_{j=1}^{N} \eta u_j^2(t),$$

$$J_x(t) = \sum_{j=1}^{N} \sum_{i \in N_{\sigma(t),j}} w_{\sigma(t),ji} \left(\gamma_1 \left(x_i(t) - x_j(t)\right)^2 + \gamma_2 \left(v_i(t) - v_j(t)\right)^2\right).$$

One can take $J_s^* > 0$ as a given cost budget; that is, the whole energy consumption budget is limited. Then, the definition of the leader-following wireless sensor network guaranteed-cost synchronization with switching interaction topologies and given cost budgets is defined as shown below.

Definition 1: For any given $J_s^* > 0$, wireless sensor network (1) is said to be leader-following guaranteed-cost synchronizable with the given cost budget by protocol (2) if there exist control gains k_1 and k_2 such that $\lim_{t\rightarrow\infty} \left(x_j(t) - x_1(t)\right) = 0$, $\lim_{t\rightarrow\infty} \left(v_j(t) - v_1(t)\right) = 0$ $(j = 2, 3, \cdots, N)$ and $J_s \leq J_s^*$ for any bounded disagreement initial states $x_j(0)$ and $v_j(0)$ $(j = 2, 3, \cdots, N)$.

The following section will present an approach to solve explicit expressions of control gains k_1 and k_2 of protocol (2) such that leader-following wireless sensor network (1) achieves Lipschitz nonlinear guaranteed-cost synchronization with the switching interaction topologies and the given cost budget.

Remark 1: J_s is a distributed energy optimization index related to $w_{\sigma(t),ji}$, which means that the information exchanges between the node and its neighboring nodes. J_s is composed of both $J_u(t)$ and $J_x(t)$. For the isolated systems, the energy consumption term is related to the control input and the synchronization performance term is designed by the state information. In protocol (2), $J_u(t)$ is constructed by the control input state $u_j(t)$ and $J_x(t)$ is constructed by the relative state information $x_i(t) - x_j(t)$ and $v_i(t) - v_j(t)$. Moreover, from an engineering perspective, $J_u(t)$ represents the battery power consumption term of the sensor network while $J_x(t)$ represents the network synchronization performance term on the queue length state and the transmission rate state. It should be pointed out that the relative states between the leader node and follower nodes are convergent not the states of the network nodes, which means that the follower nodes follow the leader node with the same desired states for the network synchronization. The targets of control protocol (2) with different parameters γ_1, γ_2 and η are to realize the guaranteed-cost synchronization and the different tradeoff design between $J_u(t)$ and $J_x(t)$ for the Lipschitz nonlinear wireless sensor network. In addition, in [26]–[28], an upper bound of the linear quadratic index is determined when the guaranteed-cost synchronization is achieved. In this paper, the design of control gains of synchronization protocols is more complex and meaningful for the wireless sensor network under the condition of the given bound of the linear quadratic index, which means the limited given cost budget.

Remark 2: Compared with control protocols in [35]–[37] about guaranteed-cost synchronization, two major characteristics of synchronization criteria are mentioned. The first one is that aforementioned results achieve guaranteed-cost synchronization on the basis of LMI tools, which rely on a numerical software to get solvable gain matrices. It is known that as a numerical algorithm, a feasp solver cannot demonstrate the analytic solutions. To tackle this problem, we propose an approach to present explicit feasible values of k_1 and k_2 for the guaranteed-cost synchronization. The second one is that the cost budget of the wireless sensor network is limited and previously given, which means that the limited practical energy supply of the wireless sensor network is given previously and should be considered as a constraint for the synchronization criterion design. The major challenge is that the relation between given cost budgets and an upper bound of cost function should be established when feasible values of k_1 and k_2 are designed. It is noted that the cost budget drawn to the synchronization criteria can ensure that the upper bound of the optimization index is less than the given cost budgets.

III. GUARANTEED-COST SYNCHRONIZATION CRITERIA FOR THE LIPSCHITZ NONLINEAR WIRELESS SENSOR NETWORK

For the second-order leader-following wireless sensor network with switching topologies, this section mainly presents the guaranteed-cost synchronization criterion not considering the given cost budget by linearizing nonlinear terms of the dynamics and gives an upper bound of the guaranteed-cost function composed of the energy consumption term and the synchronization regulation term. Furthermore, the guaranteed-cost synchronization criterion is determined for the case where the cost budget of the wireless sensor network is given previously.

Let $L^{ff}_{\sigma(t)}$ be the Laplacian matrix of the interaction topology among follower nodes and $\Lambda^{fl}_{\sigma(t)} = \mathrm{diag}\left\{w^{21}_{\sigma(t)}, w^{31}_{\sigma(t)}, \cdots, w^{N1}_{\sigma(t)}\right\}$ represent the interconnection weight matrix from the leader node to followers, which are piecewise continuous. Thus, one can obtain that the interaction weight matrix among follower nodes is $L^{ff}_{\sigma(t)} + \Lambda^{fl}_{\sigma(t)}$ associated with the whole Laplacian matrix.

Firstly, the dynamics of wireless sensor networks is converted by the structure decomposition as follows. It is assumed that all switching topologies have a spanning tree and the interaction topologies among follower sensor nodes are undirected, then there exists a piecewise continuous orthonomal matrix $\tilde{U}_{\sigma(t)}$ such that $\tilde{U}^T_{\sigma(t)}\left(L^{ff}_{\sigma(t)} + \Lambda^{fl}_{\sigma(t)}\right)\tilde{U}_{\sigma(t)} = \mathrm{diag}\left\{\lambda^2_{\sigma(t)}, \lambda^3_{\sigma(t)}, \cdots, \lambda^N_{\sigma(t)}\right\}$ with the general assumption that $0 < \lambda^2_{\sigma(t)} \leq \lambda^3_{\sigma(t)} \leq \cdots \leq \lambda^N_{\sigma(t)}$ being eigenvalues of $L^{ff}_{\sigma(t)} + \Lambda^{fl}_{\sigma(t)}$. Let $\Delta x_j(t) = x_j(t) - x_1(t)$, $\Delta v_j(t) = v_j(t) - v_1(t)$, $\Delta f_j(t) = f(x_j(t), v_j(t)) - f(x_1(t), v_1(t))$ $(j = 2, 3, \cdots, N)$, $\Delta X(t) = [\Delta x_2(t), \Delta v_2(t), \Delta x_3(t), \Delta v_3(t), \cdots, \Delta x_N(t), \Delta v_N(t)]^T$, and

$$\begin{cases}\tilde{U}^T_{\sigma(t)}[\Delta x_2(t), \Delta x_3(t), \cdots, \Delta x_N(t)]^T \\ \quad = \left[\tilde{x}_2(t), \tilde{x}_3(t), \cdots, \tilde{x}_N(t)\right]^T, \\ \tilde{U}^T_{\sigma(t)}[\Delta v_2(t), \Delta v_3(t), \cdots, \Delta v_N(t)]^T \\ \quad = \left[\tilde{v}_2(t), \tilde{v}_3(t), \cdots, \tilde{v}_N(t)\right]^T, \\ \tilde{U}^T_{\sigma(t)}[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)]^T \\ \quad = \left[\tilde{f}_2(t), \tilde{f}_3(t), \cdots, \tilde{f}_N(t)\right]^T. \end{cases} \tag{3}$$

It follows from (1) to (3) that

$$\begin{cases}\Delta\dot{x}_j(t) = \dot{x}_j(t) - \dot{x}_1(t) \\ \quad = v_j(t) - v_1(t) = \Delta v_j(t), \\ \Delta\dot{v}_j(t) = \dot{v}_j(t) - \dot{v}_1(t) \\ \quad = \sum\limits_{i\in N_{\sigma(t),j}} w_{\sigma(t),ji}\left(k_1\left(\Delta x_i(t) - \Delta x_j(t)\right)\right. \\ \quad \left. + k_2\left(\Delta v_i(t) - \Delta v_j(t)\right)\right) + \Delta f_j(t), \end{cases} \tag{4}$$

$$\begin{cases}\tilde{U}^T_{\sigma(t)}[\Delta\dot{x}_2(t), \Delta\dot{x}_3(t), \cdots, \Delta\dot{x}_N(t)]^T \\ \quad = \left[\tilde{v}_2(t), \tilde{v}_3(t), \cdots, \tilde{v}_N(t)\right]^T, \\ \tilde{U}^T_{\sigma(t)}[\Delta\dot{v}_2(t), \Delta\dot{v}_3(t), \cdots, \Delta\dot{v}_N(t)]^T \\ \quad = \left[-k_1\lambda^2_{\sigma(t)}\tilde{x}_2(t) - k_2\lambda^2_{\sigma(t)}\tilde{v}_2(t) + \tilde{f}_2(t),\right. \\ \quad -k_1\lambda^3_{\sigma(t)}\tilde{x}_3(t) - k_2\lambda^3_{\sigma(t)}\tilde{v}_3(t) + \tilde{f}_3(t), \cdots, \\ \quad \left.-k_1\lambda^N_{\sigma(t)}\tilde{x}_N(t) - k_2\lambda^N_{\sigma(t)}\tilde{v}_N(t) + \tilde{f}_N(t)\right]^T. \end{cases} \tag{5}$$

Hence, by (3), (4) and (5), one can convert the network system into

$$\begin{cases}\dot{\tilde{x}}_j(t) = \tilde{v}_j(t), \\ \dot{\tilde{v}}_j(t) = -k_1\lambda^j_{\sigma(t)}\tilde{x}_j(t) - k_2\lambda^j_{\sigma(t)}\tilde{v}_j(t) + \tilde{f}_j(t), \end{cases} \tag{6}$$

where $j = 2, 3, \cdots, N$. Since $\tilde{U}_{\sigma(t)}$ is a time-varing orthonomal matrix, we can obtain that $\Delta x_j(t) = 0$ and $\Delta v_j(t) = 0$ when $\tilde{x}_j(t) = 0$ and $\tilde{v}_j(t) = 0$ $(j = 2, 3, \cdots, N)$, which means that the leader-following wireless sensor network achieves synchronization. Let $\tilde{\lambda}_2 = \min\left\{\lambda^{(2)}_m, \forall m \in \mathcal{I}_M\right\}$ and $\tilde{\lambda}_N = \max\left\{\lambda^{(N)}_m, \forall m \in \mathcal{I}_M\right\}$. In the following, a sufficient condition of the guaranteed-cost synchronization is proposed for Lipschitz nonlinear wireless sensor network (1) with synchronization protocol (2).

Theorem 1: Wireless sensor network (1) is leader-following guaranteed-cost synchronizable with switching interaction topologies by protocol (2) if $1 < 2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N \leq 3$. In this case,

$$k_1 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N - 1}},$$
$$k_2 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_2 + 2k_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N - 1}}.$$

Proof: Firstly, sufficient conditions are designed such that $\lim_{t\to\infty}\tilde{x}_j(t) = 0$ and $\lim_{t\to\infty}\tilde{v}_j(t) = 0$ $(j = 2, 3, \cdots, N)$. One can construct a Lyapunov function candidate as follows:

$$V_j(t) = k\tilde{x}^2_j(t) + k_2\tilde{v}^2_j(t) + 2k_1\tilde{x}_j(t)\tilde{v}_j(t), \tag{7}$$

where $j = 2, 3, \cdots, N$ and $k = k_1k_2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N\right)$. Since $k_1 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_1+2\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}^2_N-1}}$, $k_2 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_2+2k_1+2\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}^2_N-1}}$ and $k = k_1k_2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N\right)$, one can show that

$$kk_2 - k^2_1 = k_1\left(k^2_2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N\right) - k_1\right). \tag{8}$$

Duo to $1 < 2\tilde{\lambda}_2 - \eta\tilde{\lambda}^2_N \leq 3$, one has

$$kk_2 - k^2_1 > k_1\left(k^2_2 - k_1\right) > 0. \tag{9}$$

Hence, it can obtain that $k \geq k^2_1/k_2$ and $\det\begin{bmatrix} k & k_1 \\ k_1 & k_2\end{bmatrix} > 0$. Then, one obtains that $V_j(t) > 0$. Let $V(t) = \sum_{j=2}^N V_j(t)$. The time derivative of $V(t)$ by (6) is

$$\begin{aligned}\dot{V}(t) = \sum_{j=2}^{N}&\left(2k\tilde{x}_j(t)\tilde{v}_j(t) + 2k_2\tilde{v}_j(t)\right. \\ &\times\left(-k_1\lambda^j_{\sigma(t)}\tilde{x}_j(t) - k_2\lambda^j_{\sigma(t)}\tilde{v}_j(t) + \tilde{f}_j(t)\right) \\ &+ 2k_1\left(\tilde{v}^2_j(t) + \tilde{x}_j(t)\left(-k_1\lambda^j_{\sigma(t)}\tilde{x}_j(t)\right.\right. \\ &\left.\left.\left.- k_2\lambda^j_{\sigma(t)}\tilde{v}_j(t) + \tilde{f}_j(t)\right)\right)\right).\end{aligned} \tag{10}$$

It can be shown that

$$\begin{aligned}
&\sum_{j=2}^{N} 2k_2\tilde{v}_j(t)\tilde{f}_j(t) + 2k_1\tilde{x}_j(t)\tilde{f}_j(t) \\
&= 2k_1\left[\tilde{x}_2(t), \tilde{x}_3(t), \cdots, \tilde{x}_N(t)\right]\tilde{U}_{\sigma(t)}^T \\
&\quad \times[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)]^T \\
&\quad +2k_2\left[\tilde{v}_2(t), \tilde{v}_3(t), \cdots, \tilde{v}_N(t)\right] \\
&\quad \times \tilde{U}_{\sigma(t)}^T[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)]^T \\
&\le \sum_{j=2}^{N}\left(k_1^2\tilde{x}_j^2(t) + k_2^2\tilde{v}_j^2(t)\right) \\
&\quad +2\left[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)\right]\left(\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\right) \\
&\quad \times[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)]^T \\
&\le \sum_{j=2}^{N}\left(k_1^2\tilde{x}_j^2(t) + k_2^2\tilde{v}_j^2(t)\right) \\
&\quad +2\left[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)\right] \\
&\quad \times[\Delta f_2(t), \Delta f_3(t), \cdots, \Delta f_N(t)]^T \\
&\le \sum_{j=2}^{N}\left(k_1^2\tilde{x}_j^2(t) + k_2^2\tilde{v}_j^2(t)\right) + 2\sum_{j=2}^{N}\Delta f_j^2(t) \\
&\le \sum_{j=2}^{N}\left(k_1^2\tilde{x}_j^2(t) + k_2^2\tilde{v}_j^2(t)\right) \\
&\quad +2\sum_{j=2}^{N}\left(f(x_j(t), v_j(t)) - f(x_1(t), v_1(t))\right)^2.
\end{aligned}$$

Since

$$\begin{aligned}
&\left|f\left(x_1(t), v_1(t)\right) - f\left(x_j(t), v_j(t)\right)\right| \\
&\quad \le \gamma\left(\left|x_1(t) - x_j(t)\right|^2 + \left|v_1(t) - v_j(t)\right|^2\right)^{0.5} \\
&\quad \times (j \in \{1, 2, \cdots, N\}),
\end{aligned}$$

one can show that

$$\begin{aligned}
&2\sum_{j=2}^{N}\left(f(x_j(t), v_j(t)) - f(x_1(t), v_1(t))\right)^2 \\
&\le 2\sum_{j=2}^{N}\left(\gamma^2\left(x_1(t) - x_j(t)\right)^2 + \gamma^2\left(v_1(t) - v_j(t)\right)^2\right) \\
&\le 2\sum_{j=2}^{N}\left(\gamma^2\Delta x_j^2(t) + \gamma^2\Delta v_j^2(t)\right) \\
&= 2\gamma^2\left[\Delta x_2(t), \Delta x_3(t), \cdots, \Delta x_N(t)\right] \\
&\quad \times\left(\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\right)\left[\Delta x_2(t), \Delta x_3(t), \cdots, \Delta x_N(t)\right]^T \\
&\quad +2\gamma^2\left[\Delta v_2(t), \Delta v_3(t), \cdots, \Delta v_N(t)\right] \\
&\quad \times\left(\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\right)\left[\Delta v_2(t), \Delta v_3(t), \cdots, \Delta v_N(t)\right]^T \\
&= 2\gamma^2\sum_{j=2}^{N}\tilde{x}_j^2(t) + 2\gamma^2\sum_{j=2}^{N}\tilde{v}_j^2(t).
\end{aligned} \tag{11}$$

Then, from (7) to (12), one has

$$\begin{aligned}
\dot{V}(t) \le& \sum_{j=2}^{N}\left(2k - 4k_2k_1\lambda_{\sigma(t)}^j\right)\tilde{x}_j(t)\tilde{v}_j(t) \\
&+\sum_{j=2}^{N}\left(2\gamma^2 + 2k_1 + k_2^2 - 2k_2^2\lambda_{\sigma(t)}^j\right)\tilde{v}_j^2(t) \\
&+\sum_{j=2}^{N}\left(2\gamma^2 + k_1^2 - 2k_1^2\lambda_{\sigma(t)}^j\right)\tilde{x}_j^2(t).
\end{aligned} \tag{12}$$

Due to

$$\begin{cases}
3\tilde{\lambda}_N\gamma_1 + k_1^2 + 2\gamma^2 + k_1^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right) = 0, \\
k_1k_2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right) + k = 0, \\
3\tilde{\lambda}_N\gamma_2 + k_2^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right) + 2k_1 + k_2^2 + 2\gamma^2 = 0,
\end{cases} \tag{13}$$

it can be obtained by (13) and (14) that

$$\begin{aligned}
&\dot{V}(t) \\
&\le \sum_{j=2}^{N} 2\left(-2k_2k_1\lambda_{\sigma(t)}^j - \left(k_1k_2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right)\right)\tilde{x}_j(t)\tilde{v}_j(t) \\
&\quad +\sum_{j=2}^{N}\left(-2k_2^2\lambda_{\sigma(t)}^j - \left(3\tilde{\lambda}_N\gamma_2 + k_2^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right)\right)\tilde{v}_j^2(t) \\
&\quad +\sum_{j=2}^{N}\left(-2k_1^2\lambda_{\sigma(t)}^j - \left(3\tilde{\lambda}_N\gamma_1 + k_1^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right)\right)\tilde{x}_j^2(t).
\end{aligned} \tag{14}$$

Due to

$$\begin{bmatrix}
-2k_1^2\lambda_{\sigma(t)}^j - \left(3\tilde{\lambda}_N\gamma_1 + k_1^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right) & -2k_2k_1\lambda_{\sigma(t)}^j - \left(k_1k_2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right) \\
-2k_2k_1\lambda_{\sigma(t)}^j - \left(k_1k_2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right) & -2k_2^2\lambda_{\sigma(t)}^j - \left(3\tilde{\lambda}_N\gamma_2 + k_2^2\left(\eta\tilde{\lambda}_N^2 - 2\tilde{\lambda}_2\right)\right)
\end{bmatrix} < 0,$$

one has $\dot{V}(t) \le 0$ and $\dot{V}(t) \equiv 0$ if and only if $\tilde{x}_j(t) \equiv 0$ and $\tilde{v}_j(t) \equiv 0$. By the above proof, wireless sensor network (1) can achieve leader-following synchronization with switching interaction topologies and the Lipschitz nonlinearity.

Next, one can analyze and determine an upper bound of the guaranteed-cost function. Let $J_u = \int_0^{\infty} J_u(t)dt$ and $J_x = \int_0^{\infty} J_x(t)dt$, then it can be obtained by (2) that

$$\begin{aligned}
J_u =& \eta\int_0^{\infty}\Delta X^T(t)\left(\left(L_{\sigma(t)}^{ff} + \Lambda_{\sigma(t)}^{fl}\right)^2\right. \\
&\left.\otimes\begin{bmatrix} k_1^2 & k_1k_2 \\ k_1k_2 & k_2^2 \end{bmatrix}\right)\Delta X(t)dt,
\end{aligned} \tag{15}$$

$$\begin{aligned}
J_x =& 2\int_0^{\infty}\Delta X^T(t)\left(\left(L_{\sigma(t)}^{ff} + \Lambda_{\sigma(t)}^{fl}\right)\right. \\
&\left.\otimes\begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}\right)\Delta X(t)dt.
\end{aligned} \tag{16}$$

Due to

$$\Delta X^T(t)\left(\left(L^{ff}_{\sigma(t)}+\Lambda^{fl}_{\sigma(t)}\right)^2\otimes\begin{bmatrix}k_1^2 & k_1k_2\\ k_1k_2 & k_2^2\end{bmatrix}\right)\Delta X(t)$$
$$=\sum_{j=2}^{N}(\lambda^j_{\sigma(t)})^2\big(k_1\tilde{x}_j(t)+k_2\tilde{v}_j(t)\big)^2, \tag{17}$$
$$\Delta X^T(t)\left(\left(L^{ff}_{\sigma(t)}+\Lambda^{fl}_{\sigma(t)}\right)\otimes\begin{bmatrix}\gamma_1 & 0\\ 0 & \gamma_2\end{bmatrix}\right)\Delta X(t)$$
$$=\sum_{j=2}^{N}\lambda^j_{\sigma(t)}\left(\gamma_1\tilde{x}_j^2(t)+\gamma_2\tilde{v}_j^2(t)\right), \tag{18}$$

one can obtain that

$$\begin{aligned}J_s&=\int_0^\infty (J_u(t)+J_x(t))\,dt\\
&=\int_0^\infty\Bigg((\eta\sum_{j=2}^{N}\left(\lambda^j_{\sigma(t)}\right)^2\big(k_1\tilde{x}_j(t)+k_2\tilde{v}_j(t)\big)^2\\
&\quad+2\sum_{j=2}^{N}\lambda^j_{\sigma(t)}\left(\gamma_1\tilde{x}_j^2(t)+\gamma_2\tilde{v}_j^2(t)\right)\Bigg)\,dt\\
&=\sum_{j=2}^{N}\int_0^\infty\left(\eta(\lambda^j_{\sigma(t)})^2k_1^2+2\lambda^j_{\sigma(t)}\gamma_1\right)\tilde{x}_j^2(t)dt\\
&\quad+\sum_{j=2}^{N}\int_0^\infty\left(\eta(\lambda^j_{\sigma(t)})^2k_2^2+2\lambda^j_{\sigma(t)}\gamma_2\right)\tilde{v}_j^2(t)dt\\
&\quad+\sum_{j=2}^{N}\int_0^\infty 2\eta(\lambda^j_{\sigma(t)})^2k_1k_2\tilde{x}_j(t)\tilde{v}_j(t)dt.\end{aligned} \tag{19}$$

Let $\Theta_{j11}=-2k_1^2\lambda^j_{\sigma(t)}-\left(3\tilde{\lambda}_N\gamma_1+k_1^2\left(\eta\tilde{\lambda}_N^2-2\tilde{\lambda}_2\right)\right)+\eta(\lambda^j_{\sigma(t)})^2k_1^2+2\lambda^j_{\sigma(t)}\gamma_1$, $\Theta_{j12}=-2k_2k_1\lambda^j_{\sigma(t)}-\left(k_1k_2\left(\eta\tilde{\lambda}_N^2-2\tilde{\lambda}_2\right)\right)+\eta(\lambda^j_{\sigma(t)})^2k_1k_2$ and $\Theta_{j22}=-2k_2^2\lambda^j_{\sigma(t)}-\left(3\tilde{\lambda}_N\gamma_2+k_2^2\left(\eta\tilde{\lambda}_N^2-2\tilde{\lambda}_2\right)\right)+\eta(\lambda^j_{\sigma(t)})^2k_2^2+2\lambda^j_{\sigma(t)}\gamma_2$ $(j=2,3,\cdots,N)$, then one can obtain that

$$\begin{aligned}&\begin{bmatrix}\Theta_{j11} & \Theta_{j12}\\ \Theta_{j12} & \Theta_{j22}\end{bmatrix}\\
&=\begin{bmatrix}2\tilde{\lambda}_2k_1^2-2\lambda^j_{\sigma(t)}k_1^2 & 2\tilde{\lambda}_2k_2k_1-2\lambda^j_{\sigma(t)}k_2k_1\\ 2\tilde{\lambda}_2k_2k_1-2\lambda^j_{\sigma(t)}k_2k_1 & 2\tilde{\lambda}_2k_2^2-2\lambda^j_{\sigma(t)}k_2^2\end{bmatrix}\\
&\quad+\begin{bmatrix}\eta(\lambda^j_{\sigma(t)})^2k_1^2-\eta\tilde{\lambda}_N^2k_1^2 & \eta(\lambda^j_{\sigma(t)})^2k_1k_2-\eta\tilde{\lambda}_N^2k_1k_2\\ \eta(\lambda^j_{\sigma(t)})^2k_1k_2-\eta\tilde{\lambda}_N^2k_1k_2 & \eta(\lambda^j_{\sigma(t)})^2k_2^2-\eta\tilde{\lambda}_N^2k_2^2\end{bmatrix}\\
&\quad+\begin{bmatrix}2\lambda^j_{\sigma(t)}\gamma_1-3\tilde{\lambda}_N\gamma_1 & 0\\ 0 & 2\lambda^j_{\sigma(t)}\gamma_2-3\tilde{\lambda}_N\gamma_2\end{bmatrix}\\
&\le 0.\end{aligned}$$

Due to

$$\begin{aligned}J_s&=\sum_{j=2}^{N}\int_0^\infty\left(\eta(\lambda^j_{\sigma(t)})^2k_1^2+2\lambda^j_{\sigma(t)}\gamma_1\right)\tilde{x}_j^2(t)\,dt\\
&\quad+\sum_{j=2}^{N}\int_0^\infty\left(\eta(\lambda^j_{\sigma(t)})^2k_2^2+2\lambda^j_{\sigma(t)}\gamma_2\right)\tilde{v}_j^2(t)\,dt\\
&\quad+\sum_{j=2}^{N}\int_0^\infty 2\eta(\lambda^j_{\sigma(t)})^2k_1k_2\tilde{x}_j(t)\tilde{v}_j(t)\,dt\\
&\quad+\sum_{j=2}^{N}\int_0^\infty \dot{V}_j(t)dt-V(t)|_{t\to\infty}+V(0)\\
&=\sum_{j=2}^{N}\int_0^\infty\big(\tilde{x}_j(t),\tilde{v}_j(t)\big)\begin{bmatrix}\Theta_{j11} & \Theta_{j12}\\ \Theta_{j12} & \Theta_{j22}\end{bmatrix}\\
&\quad\times\big(\tilde{x}_j(t),\tilde{v}_j(t)\big)^T dt-V(t)|_{t\to\infty}+V(0),\end{aligned} \tag{20}$$

we can get that

$$J_s\le V(0). \tag{21}$$

On the basis of the above process, results of Theorem 1 can be concluded, which can guarantee that Lipschitz nonlinear wireless sensor network (1) achieves guaranteed-cost synchronization by protocol (2) with switching interaction topologies.

Remark 3: For the general linear leader-following wireless sensor network, by the state error information and eigenvalues of interaction weight matrices among follower sensor nodes, the system dynamics can be decomposed, which can be applied to determine the synchronization criteria. However, for the nonlinear network, the aforementioned methods are no longer valid since the dynamics of the sensor nodes contains the nonlinear terms and the interaction topologies are switching. To linearize the Lipschitz nonlinear term, we use the structure characteristic of the piecewise continuous orthonomal matrix $\tilde{U}_{\sigma(t)}$; that is $\tilde{U}_{\sigma(t)}\tilde{U}^T_{\sigma(t)}=I_{N-1}$ in Theorem 1. Then, the term $2\gamma^2$ represents the existence of the nonlinear dynamics, which is eliminated by k, k_1 and k_2 in Theorem 1. Hence, based on the above methods and the Lipschitz condition, we propose a sufficient condition for the network synchronization in terms of analytic solutions, which is not influenced by the Lipschitz nonlinear term. In addition, one can see that when the linear optimization index is taken into consideration, the criteria proposed for the network synchronization can also be applicable for the guaranteed-cost synchronization by the proof. Moreover, not all the eigenvalues of switching interaction topologies are required to be obtained necessarily. Control gains k_1 and k_2 are determined dependent upon the minimum nonzero eigenvalue $\tilde{\lambda}_2$ and the maximum eigenvalue $\tilde{\lambda}_N$ of switching topologies and $\tilde{\lambda}_2$ and $\tilde{\lambda}_N$ can be estimated by the approach used in [39] and [40], which can decrease the computational complexity. It can also be concluded that k_1 can be calculated independently and k_2 is dependent on k_1.

Remark 4: In the associated works [26]–[28] about achieving guaranteed-cost synchronization, LMI tools on the basis of the feasp solver are used to determine the feasible control gain matrices. However, in some situations, a feasp solver may not achieve feasible control gain matrices for the multi-node network to realize guaranteed-cost synchronization. Hence, control gains k_1 and k_2 are required to be designed as the analytic algorithm and the explicit expressions can ensure that the wireless sensor network achieves leader-following guaranteed-cost synchronization. It should be pointed out that we can choose control gains k_1 and k_2 without the verification of feasibility. Furthermore, expressions of control gains k_1 and k_2 can eliminate the influence of the nonlinear term while the aforementioned researches [33], [34] are no longer valid to deal with the Lipschitz nonlinearity.

When the limited energy supply is taken into account, an approach to achieve the feasible values of control gains k_1 and k_2 is proposed such that the leader-following wireless sensor network achieves guaranteed-cost synchronization with both switching interaction topologies and the cost budget given previously.

Let $\Delta x(t) = [\Delta x_2(t), \Delta x_3(t), \cdots, \Delta x_N(t)]$, $\Delta v(t) = [\Delta v_2(t), \Delta v_3(t), \cdots, \Delta v_N(t)]$, $w_1(t) = \sum_{j=2}^{N} \Delta x_j^2(t)$, $w_2(t) = \sum_{j=2}^{N} \Delta v_j^2(t)$ and $w_3(t) = \sum_{j=2}^{N} \left|\Delta v_j(t)\Delta x_j(t)\right|$, then Theorem 2 is demonstrated as follows.

Theorem 2: For any given $J_s^* > 0$, wireless sensor network (1) achieves leader-following guaranteed-cost synchronization with the given cost budget and switching interaction topologies by protocol (2) if $1 < 2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 \leq 3$, $\gamma_1 > \gamma_2$ and $\frac{9\tilde{\lambda}_N\gamma_1+6\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}_N^2-1} \leq \min\left(\frac{J_s^*}{w_1(0)+w_2(0)+2w_3(0)}, \frac{27\tilde{\lambda}_N^2(\gamma_1-\gamma_2)^2}{4}\right)$. In this case,

$$k_1 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}},$$
$$k_2 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_2 + 2k_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}}.$$

Proof: Firstly, according to (5) and (7), one has

$$\begin{aligned} V(t) &= \sum_{j=2}^{N}\left(k\tilde{x}_j^2(t) + k_2\tilde{v}_j^2(t) + 2k_1\tilde{x}_j(t)\tilde{v}_j(t)\right) \\ &= k\Delta x(t)\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\Delta x^T(t) + k_2\Delta v(t)\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\Delta v^T(t) \\ &\quad + 2k_1\Delta v(t)\tilde{U}_{\sigma(t)}\tilde{U}_{\sigma(t)}^T\Delta x^T(t) \\ &= k\sum_{j=2}^{N}\Delta x_j^2(t) + k_2\sum_{j=2}^{N}\Delta v_j^2(t) + 2k_1\sum_{j=2}^{N}\Delta v_j(t)\Delta x_j(t) \\ &\leq kw_1(t) + k_2w_2(t) + 2k_1w_3(t). \end{aligned} \tag{22}$$

Thus,

$$V(0) \leq kw_1(0) + k_2w_2(0) + 2k_1w_3(0). \tag{23}$$

By (22) and (24), one can see that $V(0) \leq kw_1(0) + k_2w_2(0) + 2k_1w_3(0) \leq J_s^*$ can guarantee that $J_s \leq J_s^*$. Due to $k = k_1k_2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2\right)$, $k_1 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_1+2\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}_N^2-1}}$, $k_2 = \sqrt{\frac{3\tilde{\lambda}_N\gamma_2+2k_1+2\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}_N^2-1}}$, $\gamma_1 > \gamma_2$ and $\frac{9\tilde{\lambda}_N\gamma_1+6\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}_N^2-1} \leq \frac{27\tilde{\lambda}_N^2(\gamma_1-\gamma_2)^2}{4}$, one can obtain that $k_1 \geq k_2$ and $3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2 \geq 2k_1$. Hence, one can obtain that

$$\begin{aligned} V(0) &\leq kw_1(0) + k_2w_2(0) + 2k_1w_3(0) \\ &= k_1k_2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2\right)w_1(0) + k_2w_2(0) + 2k_1w_3(0) \\ &\leq k_1^2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2\right)w_1(0) + k_1\left(w_2(0) + 2w_3(0)\right) \\ &\leq k_1^2\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2\right)w_1(0) \\ &\quad + \frac{3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2}{2}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{3\tilde{\lambda}_N\gamma_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}\left(2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2\right)w_1(0) \\ &\quad + \frac{3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2}{2}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{3\tilde{\lambda}_N\gamma_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}\left(3w_1(0)\right) \\ &\quad + \frac{3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2}{2}\left(w_2(0) + 2w_3(0)\right). \end{aligned} \tag{24}$$

Due to $\frac{9\tilde{\lambda}_N\gamma_1+6\gamma^2}{2\tilde{\lambda}_2-\eta\tilde{\lambda}_N^2-1} \leq \frac{J_s^*}{w_1(0)+w_2(0)+2w_3(0)}$, one has

$$\begin{aligned} V(0) &\leq \frac{3\tilde{\lambda}_N\gamma_1 + 2\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}\left(3w_1(0)\right) \\ &\quad + \frac{3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2}{2}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{J_s^*}{w_1(0) + w_2(0) + 2w_3(0)}w_1(0) \\ &\quad + \frac{3\tilde{\lambda}_N\gamma_1 - 3\tilde{\lambda}_N\gamma_2}{2}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{J_s^*}{w_1(0) + w_2(0) + 2w_3(0)}w_1(0) \\ &\quad + \frac{9\tilde{\lambda}_N\gamma_1 + 6\gamma^2}{2}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{J_s^*}{w_1(0) + w_2(0) + 2w_3(0)}w_1(0) \\ &\quad + \frac{9\tilde{\lambda}_N\gamma_1 + 6\gamma^2}{2\tilde{\lambda}_2 - \eta\tilde{\lambda}_N^2 - 1}\left(w_2(0) + 2w_3(0)\right) \\ &\leq \frac{J_s^*}{w_1(0) + w_2(0) + 2w_3(0)}w_1(0) \\ &\quad + \frac{J_s^*}{w_1(0) + w_2(0) + 2w_3(0)}\left(w_2(0) + 2w_3(0)\right) \\ &\leq J_s^*. \end{aligned} \tag{25}$$

Hence, Theorem 2 can guarantee that the Lipschitz nonlinear guaranteed-cost synchronization can be reachable for leader-following wireless sensor network (1) with switching interaction topologies and the given cost budget J_s^*. Hence, the above process completes the proof.

Remark 5: As an optimization index, the guaranteed-cost function is constructed by the relative state between the sink node and follower nodes and an upper bound of the guaranteed-cost function is obtained by the value of the Lyapunov function candidate at time zero. From the practical point of view, the cost budgets given previously are introduced. Following the lines of the aforementioned proof, in Theorem 2, the limited energy budgets as a constraint are dealt with when designing explicit expressions of the control gains k_1 and k_2, which can guarantee that the upper bound of the optimization index is less than the given cost budget. There are two difficulties in obtaining Theorem 2. The first one is to determine an upper bound of the cost function based on Theorem 1. The second key difficulty is to draw the relation between the cost budget given previously and an upper bound of the cost function to synchronization criteria. On the basis of an upper bound of the guaranteed-cost function, it can be implied that $V(0) \leq kw_1(0) + k_2w_2(0) + 2k_1w_3(0) \leq J_s^*$ can ensure that the cost budgets given previously meet the needs of the energy supply. Hence, based on Theorem 1, one adds $kw_1(0) + k_2w_2(0) + 2k_1w_3(0) \leq J_s^*$ as a constraint when designing explicit values of control gains k_1 and k_2 for the guaranteed-cost network synchronization with leader-following structure and the cost budgets given previously. It can be concluded that the added conditions are related to $\tilde{\lambda}_2$ and $\tilde{\lambda}_N$ of the interconnection weight matrix. Hence, expressions of control gains k_1 and k_2 are still only dependent upon $\tilde{\lambda}_2$ and $\tilde{\lambda}_N$ without further computational complexity and the scalability of the wireless sensor networks can also be ensured. Moreover, J_s^* should be given properly, for example, the too small cost budget cannot guarantee whether the wireless sensor network starts to work or not.

IV. NUMERICAL SIMULATION

In this section, the illustration of theoretical results proved in above sections is demonstrated in the following example.

Consider a two-dimensional wireless sensor network constructed by six nodes labeled as $1, 2, \cdots, 6$. Node 1 is set as the leader and the others are follower nodes. The dynamics of each sensor is modeled by (1) and the Lipschitz nonlinearity is described as $f(x_i(t)) = -0.2\sin(x_i(t))$. FIGURE 1 depicts the changing topology set Γ with four different interaction topologies, where the interaction topologies among follower nodes are undirected. In order to simplify the calculation process, the associated adjacency matrices are assumed to be 0-1. The communication topologies are arbitrarily switched with $T_d = 0.5s$, which are shown in FIGURE 2.

All initial state values of sensor nodes are given as follow:

$$[x_1(0), v_1(0)]^T = [3.2, 0.9]^T, \quad [x_2(0), v_2(0)]^T = [1.6, 1.6]^T,$$
$$[x_3(0), v_3(0)]^T = [2.5, 0.9]^T, \quad [x_4(0), v_4(0)]^T = [4.9, 1.7]^T,$$
$$[x_5(0), v_5(0)]^T = [0.7, 1.9]^T, \quad [x_6(0), v_6(0)]^T = [3.9, 0.8]^T.$$

In control protocol (2), positive scalars $\eta = 0.004$, $\gamma_1 = 1.64$ and $\gamma_2 = 0.2$ are given previously and the limited cost budget is given as $J_s^* = 26000$. According to Theorem 2, one can obtain that $344.46 \leq \min(344.46, 374.32)$. Then, $k_1 = 10.71$ and $k_2 = 10.52$.

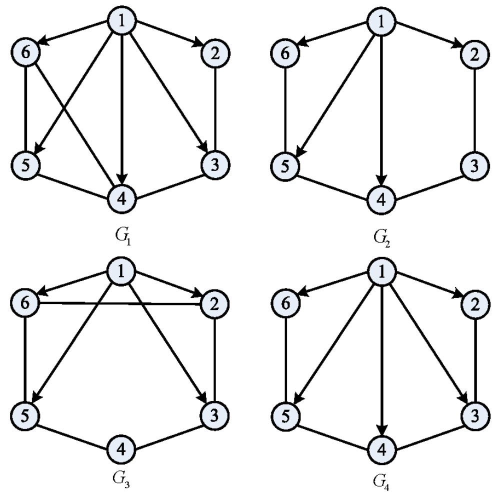

FIGURE 1. Switching topology set.

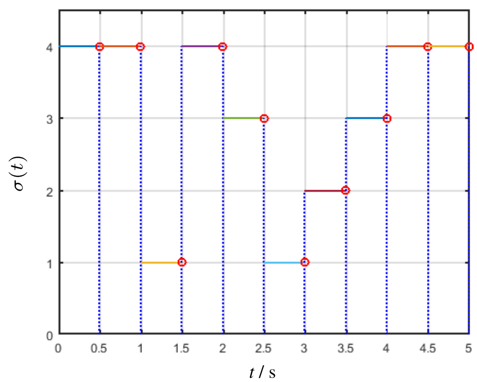

FIGURE 2. Switching signal.

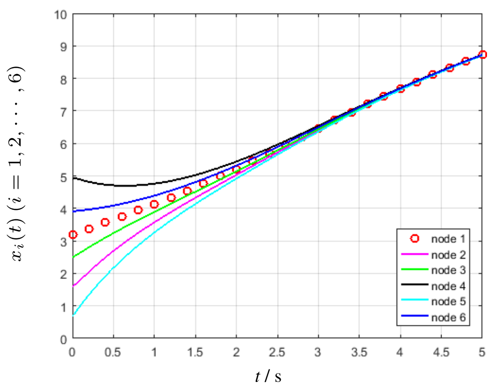

FIGURE 3. State trajectory of $x_i(t)$ $(i = 1, 2, \cdots, 6)$.

FIGURE 3 and FIGURE 4 depict the queue length states and the transmission rate states of six nodes, where the states of follower nodes converge to the states of the leader node. FIGURE 5 presents the cost function J_s and the given cost budget J_s^*. It can also been obtained that the guaranteed-cost

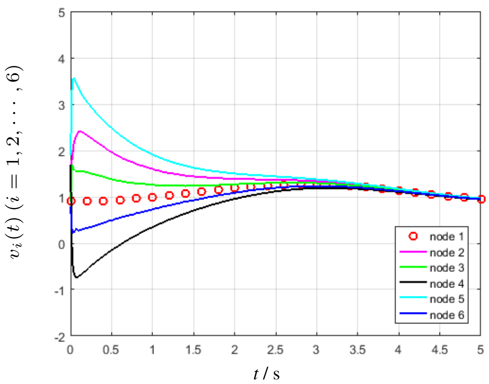

FIGURE 4. State trajectory of $v_i(t)$ $(i = 1, 2, \cdots, 6)$.

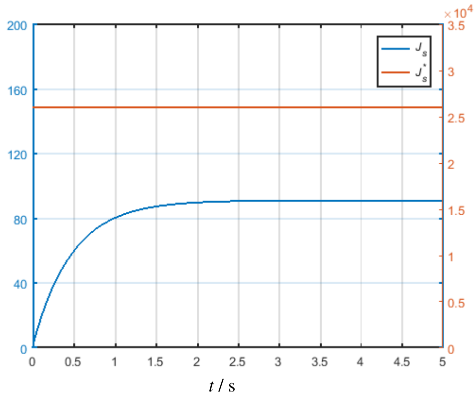

FIGURE 5. Trajectories of J_s and J_s^*.

value converges to a finite value with $J_s < J_s^*$ from FIGURE 5. One can conclude that wireless sensor network (1) achieves Lipschitz nonlinear guaranteed-cost synchronization with given cost budgets and switching topologies by synchronization protocol (2) from these figures.

In above simulations, the guaranteed-cost value J_s and the cost budget J_s are demonstrated, where J_s is less than J_s^*. In numerical simulations in [19], the feasible control gains can be obtained on the basis of the LMIs. Notice that in the current paper, the value of control gains can be solved by the analytic solution in the synchronization criteria when other parameters are given properly. In addition, the given cost budget and the nonlinear disturbance are not taken into consideration for the multi-node network in [19].

V. CONCLUSIONS

For the second-order leader-following wireless sensor network with switching topologies and the Lipschitz nonlinearities, the guaranteed-cost synchronization protocol was proposed, where the tradeoff between the battery power consumption and the network synchronization performance was established. Based on the structure characteristic of the piecewise continuous orthonomal matrix, the Lipschitz nonlinearity term was eliminated. The guaranteed-cost criteria of the Lipschitz nonlinear synchronization were determined for the wireless sensor network without the given cost budget and an upper bound of the guaranteed-cost function was derived. For the case where the cost budget was given, by establishing the relation between an upper bound of the guaranteed-cost value and the given cost budget, the criteria of the guaranteed-cost synchronization considering limited cost budgets were determined, which were independent of the number of nodes and related to the minimum nonzero eigenvalue and the maximum eigenvalue. Moreover, the scaling of the inequality of the quadratic form was utilized to draw the given cost budget to synchronization criteria. Especially, the explicit expressions of control gains were derived rather than control gain matrices solved by LMIs in synchronization criteria.

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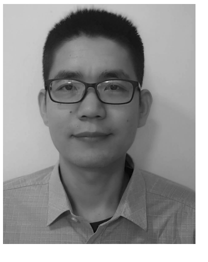

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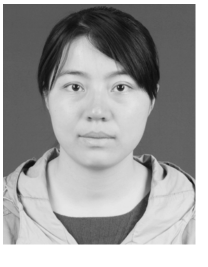

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