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Carrier Frequency Offset Estimation Bound for OFDM-Based Single Relay Networks With Multipath Receptions

CHUN-CHYUAN CHEN[1] AND MENG-LIN KU[2], (Member, IEEE)
[1]Department of Electrical Engineering, National Dong Hwa University, Hualien 97401, Taiwan
[2]Department of Communication Engineering, National Central University, Taoyuan 32001, Taiwan
Corresponding author: Chun-Chyuan Chen (chyuan@gms.ndhu.edu.tw)

ABSTRACT In this paper, the carrier frequency offset (CFO) estimation bound is presented to assess the CFO estimation performance of an orthogonal frequency division multiplexing (OFDM)-based single relay network over multipath channels, for which an amplify-and-forward (AF) protocol is adopted. We theoretically derive the Cramer–Rao bounds (CRBs) of the CFO estimation in multipath channels. Unlike the analysis in flat fading channels where the CFOs of the two hops can be combined together as a single estimation parameter, the CFOs of the source-relay (S-R) and the relay-destination (R-D) links are required to be evaluated separately in multipath channels. The first-order Taylor series expansion is applied to simplify the calculation of the inverse of the covariance matrix which reflects the correlation of multipath channels and relay noise. In addition, we derive the modified CRB (MCRB) of the CFO estimation to get more insight into the influence of the system parameters on the CRB performance. Through computer simulations, we evaluate the impact of multipath channels, the operating signal-to-noise ratio (SNR), and the number of effective multipaths on the CFO estimation error in the dual-hop relay transmission.

INDEX TERMS Orthogonal frequency division multiplexing, relay networks, carrier frequency offset, multipath channels, Cramér-Rao bound

I. INTRODUCTION

With the rapid growth of wireless services, next-generation wireless systems are expected to provide higher data rates and better quality of services. Orthogonal frequency division multiplexing (OFDM) techniques have been widely accepted as the most promising air interface due to its ability to combat the multipath fading [1]. To further improve reliability, multiple antennas are commonly employed to perform spatial diversity [2]. However, multiple-antenna systems often accompany with considerably high implementation cost at the terminals as the number of antennas increases. Cooperative communication, which relies on multiple distributed relays for data transmissions, is an alternative cost-effective solution [3]. By exploiting the distributed nature of relays, a cooperative relay network has a great potential to resist channel fading and to increase data throughput. As such, OFDM-based relay networks have also received a lot of attentions in the applications of multipath environments [4]. In general, wireless relay schemes can be classified into two categories: amplify-and-forward (AF) and decode-and-forward (DF). The AF relay scheme simply retransmits the amplified version of its received signals to the destination without causing too much computation effort. On the other hand, the DF relay scheme firstly decodes and re-encodes the received signals and then forwards the signals to the destination, which usually involves higher complexity than that of the AF scheme. Both of the AF and DF schemes are popular in wireless relay networks. We will focus on OFDM-based AF relay networks in this paper, and the study of the DF scheme is beyond the scope of this paper.

Similar to the OFDM systems, the performance of OFDM-based AF relay networks is sensitive to the carrier frequency offset (CFO) problem caused by the mismatch between the transmitter's and receiver's oscillators [5]. The CFO will destroy the orthogonality of the OFDM signals, leading to the inter-carrier interference (ICI) and thus deteriorating the system performance [6]. Furthermore, the ICI on subcarriers becomes more severe while the system is operated in multipath channels [7], [8]. When it comes to the OFDM-based relay network, multiple CFOs arise due to the distributed nature of the network with discrepant

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oscillators [9]. With multipath receptions, the multiple CFOs estimation is mainly affected by two factors. First, the CFOs at the relay and the destination yield complicated distortion of the received signals at the destination node because of the dual-hop transmission. Second, the Gaussian noise amplified by the relay and filtered by the multipath channels becomes colored noise at the destination. In this regard, the multiple CFOs estimation problem is an essential issue for the successful deployment of OFDM-based relay networks.

The Cramer-Rao bound (CRB) is a lower bound on the variance of any unbiased estimator with nuisance parameters, and it is a useful benchmark for examining the performance of estimation algorithms [10]. The CRB of the CFO estimation usually depends on some nuisance parameters such as channel parameters and the timing delay in wireless channels. In our study, we focus on the CFO estimation under the assumption of perfect timing synchronization. The true CRB of the CFO estimation is obtained by assuming that all the nuisance parameters (channel parameters) are known. In more practical scenarios, however, the CFO estimation is implemented with unknown channel parameters. The modified CRB (MCRB) of the CFO estimation can be computed only based on the statistical distribution of the nuisance parameters (channel parameters) [11]. The MCRB is generally looser than the true CRB. Similar to the derivation of the true CRB, the derivation of the MCRB depends on the definition of the modified Fisher information matrix (MFIM), which is the expectation value of the conventional FIM (Fisher information matrix) with respect to the channel parameters.

For OFDM systems, the CRBs of the CFO estimation were proposed in [12], [13]. Also, the CRBs of the blind CFO estimation for OFDM systems over multipath channels were derived in [12]. By considering virtual, pilot and data subcarriers embedded in one OFDM block, the CRB of the CFO estimation was investigated in [13]. Some works analyzed the CRBs of the CFO estimation in multipath fading channels for the DF cooperative communication systems [14], [15]. In [14], the authors considered two-phase transmission for three-node cooperative communication systems and derived the CRBs of the CFO estimation in multipath fading channels. In [15], the authors derived the CRBs of the joint channel and CFO estimation for DF multi-relay networks. Based on the assumption of the perfect data detection at each relay node, both of these DF cooperative communication systems addressed the CFO estimation problem only for the relay-destination (R-D) link. In [16], the CRBs of the multiple CFOs estimation were proposed for both DF and AF relay networks in flat fading channels. In [17], the CRBs of the multi-parameter estimation were derived for multi-relay networks in the presence of multiple CFOs and multiple timing offsets. Since the channel fading is assumed to be flat in these cases, the two CFOs due to the source-relay (S-R) and R-D links can be merged together as a single estimation parameter, and the overall noise term at the destination is still preserved as white Gaussian noise. With block-rotated preambles design in [18], the authors derived the CRB of the CFO estimation in frequency-selective fading channels for two-way AF relay systems. Although multipath channels were considered in [18], the CRB derivation was degenerated to a single parameter estimation problem, which is only related to the difference between the CFOs of the source and the destination nodes, based on the specific design of block-rotated preambles. However, this requires a particular design of preambles, which may not be applicable to general cases. To the best of our knowledge, there have been no literature addressing the CRBs of the CFOs estimation for OFDM-based AF relay networks over multipath channels.

In this paper, we focus on the CRBs of the CFO estimation for an OFDM-based AF single-relay network in multipath fading channels, which is different from the previous works in flat fading channels where only require single parameter estimation to extract the integrated CFO. On the contrary, the CFOs from the S-R and R-D links are both needed to be estimated in multipath fading channels. A closed form for the true CRB of the CFOs estimation is theoretically presented. The first-order Taylor's series expansion is also applied to ease the difficulty of calculating the inverse of the color noise covariance matrix in the FIM. In addition, we derived the theoretical MCRB with statistical channel state information, but unknown channel parameters, for the CFOs estimation. Based on this, we can get more insight into the impact of the multipath channels and the operating SNRs on the CFO estimation error in the dual-hop relay transmission. By computer simulations, the true CRB performance and the MCRB performance are compared at different operating SNRs and channel scenarios. It is found that the MCRB provides a looser bound than the true CRB does. The CRB performance is dominated by the relay SNR and also affected by the power delay profile of the R-D channel link. The proposed CRBs can serve as an important benchmark for the future design of CFO estimation algorithms in OFDM-based AF relay networks. The main contributions of this paper are listed as follows.

- We derive the true CRB of the CFOs estimation for an OFDM-based AF relay network over multipath fading channels.
- We derive the MCRB of the CFOs estimation with statistical channel state information for an OFDM-based AF relay network over multipath fading channels, and investigate the relationship between the MCRB and the SNRs of the S-R and R-D links.
- We provide the numerical results of the true CRB and the MCRB by computer simulations.

The rest of this paper is organized as follows. In Section II, the system model for OFDM-based AF single relay networks is described. Both the true CRB and the MCRB associated with the CFO estimation for the considered relay networks are derived in Section III and IV, respectively. Numerical results of the CRB performances are presented in Section V. Finally, the conclusions of this paper are drawn in Section VI.

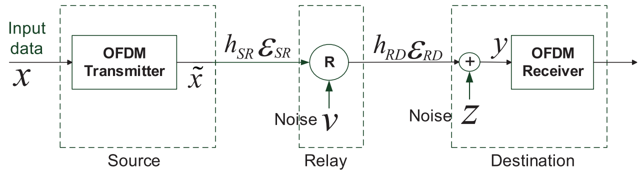

FIGURE 1. **The block diagram of an OFDM-based single relay system.**

II. SYSTEM MODEL

In Fig.1, we consider an OFDM-based single relay network, consisting of a source (S) node, a unit-gain AF relay (R) node, and a destination (D) node. During each OFDM symbol period, the input data vector $\mathbf{x} = [x(0), x(1), \ldots, x(N-1)]^T$, in which each data symbol $x(i)$ has the signal power σ_S^2, is modulated by N orthogonal subcarriers to generate a time-domain OFDM signal vector $\tilde{\mathbf{x}}$. Here we address the CFOs estimation problem, while the timing synchronization is assumed to be perfect in this paper. The two-phase transmission protocol and the AF relay scheme are adopted for the considered relay network. During the first phase, the source broadcasts the OFDM signal to the relay node. For the second phase, the relay amplifies and forwards the OFDM signal received from the source node to the destination node. It is assumed that the wireless links in the network are multipath fading channels. We define an $N \times 1$ channel vector from the source to the relay nodes as

$$\mathbf{h}_{SR} = \Big[h_{SR}(0), h_{SR}(1), \ldots, h_{SR}(L-1), \underbrace{0, 0, \ldots, 0}_{N-L}\Big]^T, \qquad (1)$$

where $h_{SR}(l)$ is the complex Gaussian channel coefficient for the lth path with zero mean and variance $\sigma_{SR,l}^2$, and L is the total number of the effective paths between the source and the relay nodes. The CFO between the source node and the relay node is denoted by ϵ_{SR}, which is normalized by the subcarrier spacing. Similarly, the channel from the relay to the destination nodes is denoted as an $N \times 1$ column vector:

$$\mathbf{h}_{RD} = \Big[h_{RD}(0), h_{RD}(1), \ldots, h_{RD}(L-1), \underbrace{0, 0, \ldots, 0}_{N-L}\Big]^T, \qquad (2)$$

where $h_{RD}(l)$ is also the zero-mean complex Gaussian channel coefficient with variance $\sigma_{RD,l}^2$, and without loss of generality, the total number of the effective paths between the relay and the destination nodes is assumed to be L. In addition, the normalized CFO between the relay and the destination nodes is denoted by ϵ_{RD}. Furthermore, we assume that the length of the cyclic prefix (CP) is larger than the maximum channel delay spread $2L$ from the source to the destination via the relay in order to alleviate the inter-symbol interference problem. Therefore, during the first transmission phase, the received OFDM signal at the relay node can be represented as

$$\mathbf{r} = \mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x} + \mathbf{v}, \qquad (3)$$

where $\mathbf{W}$ is an $N \times N$ inverse discrete Fourier transform (IDFT) matrix, in which the $(m, n)th$ element is given by $\mathbf{W}(m, n) = \frac{1}{\sqrt{N}} \exp\left(j\frac{2\pi mn}{N}\right)$, for $m, n = 0, \ldots, N-1$, and $\mathbf{v} = \left[v(0), v(1) \ldots, v(N-1)\right]^T$ is a complex additive white Gaussian noise (AWGN) vector with zero mean and covariance $\sigma_R^2\mathbf{I}$. The CFO matrix $\mathbf{E}_{SR}$ is a diagonal matrix with the linear phase $\exp\left(j\frac{2\pi\epsilon_{SR}\cdot n}{N}\right)$ as its diagonal elements:

$$\mathbf{E}_{SR} = \mathrm{diag}\bigg(\Big[1, \exp\big(j\frac{2\pi\epsilon_{SR}\cdot 1}{N}\big), \exp\big(j\frac{2\pi\epsilon_{SR}\cdot 2}{N}\big), \ldots, \exp\big(j\frac{2\pi\epsilon_{SR}\cdot(N-1)}{N}\big)\Big]^T\bigg). \qquad (4)$$

Moreover, the matrix $\mathbf{H}_{SR}$ represents an $N \times N$ circular convolution channel matrix from the source to the relay nodes, which can be explicitly expressed as

$$\mathbf{H}_{SR} = \begin{bmatrix} h_{SR}(0) & h_{SR}(N-1) & \cdots & h_{SR}(1) \\ h_{SR}(1) & h_{SR}(0) & \cdots & h_{SR}(2) \\ \vdots & \vdots & \ddots & \vdots \\ h_{SR}(N-2) & h_{SR}(N-3) & \cdots & h_{SR}(N-1) \\ h_{SR}(N-1) & h_{SR}(N-2) & \cdots & h_{SR}(0) \end{bmatrix}. \qquad (5)$$

For the sake of simple notations, the desired signal component in $\mathbf{r}$ is defined by

$$\mathbf{s}_R = \mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}. \qquad (6)$$

From (3), the received signal in the second transmission phase at the destination node can be obtained as follows:

$$\begin{aligned}\mathbf{y} &= \mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{r} + \mathbf{z} \\ &= \mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{s}_R + \mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{v} + \mathbf{z}\end{aligned} \qquad (7)$$

where $\mathbf{z} = \left[z(0), z(1), \ldots, z(N-1)\right]^T$ is a zero-mean complex AWGN vector at the destination node with covariance $\sigma_D^2\mathbf{I}$. Similar to the definition in (4) and (5), the CFO matrix $\mathbf{E}_{RD}$ with respect to the R-D link is given as

$$\mathbf{E}_{RD} = \mathrm{diag}\bigg(\Big[1, \exp\big(j\frac{2\pi\epsilon_{RD}\cdot 1}{N}\big), \exp\big(j\frac{2\pi\epsilon_{RD}\cdot 2}{N}\big), \ldots, \exp\big(j\frac{2\pi\epsilon_{RD}\cdot(N-1)}{N}\big)\Big]^T\bigg), \qquad (8)$$

and the $N \times N$ circular convolution channel matrix $\mathbf{H}_{RD}$ from the relay to the destination nodes can be represented by

$$\mathbf{H}_{RD} = \begin{bmatrix} h_{RD}(0) & h_{RD}(N-1) & \cdots & h_{RD}(1) \\ h_{RD}(1) & h_{RD}(0) & \cdots & h_{RD}(2) \\ \vdots & \vdots & \ddots & \vdots \\ h_{RD}(N-2) & h_{RD}(N-3) & \cdots & h_{RD}(N-1) \\ h_{RD}(N-1) & h_{RD}(N-2) & \cdots & h_{RD}(0) \end{bmatrix}. \qquad (9)$$

Definition 1: The true CRB is a CRB requiring the perfect knowledge of the channel coefficients $\mathbf{h}_{SR}$ and $\mathbf{h}_{RD}$.

Definition 2: The MCRB is a CRB without knowing the exact channel coefficients but requiring the statistical information of the channel coefficients $\mathbf{h}_{SR}$ and $\mathbf{h}_{RD}$.

III. DERIVATION OF THE TRUE CRB FOR THE CFOS ESTIMATION

In this section, we derive the true CRB for the CFOs estimation at the destination node by assuming that the instantaneous channel coefficients is known to the destination node. Additionally, the signal power of the data symbol is set by one, i.e., $\sigma_s^2 = 1$. First, the FIM needs to be calculated in order to determine the CRB, and the $(k, m)th$ entry of the FIM can be computed according to the definition in [10]:

$$\mathbf{F}(k,m) = 2\mathrm{Re}\left\{\frac{\partial \boldsymbol{\mu}_y^H}{\partial \varepsilon_k}\mathbf{C}_y^{-1}.\frac{\partial \boldsymbol{\mu}_y}{\partial \varepsilon_m}\right\} + \mathrm{tr}\left\{\mathbf{C}_y^{-1}\frac{\partial \mathbf{C}_y}{\partial \varepsilon_k}\mathbf{C}_y^{-1}\frac{\partial \mathbf{C}_y}{\partial \varepsilon_m}\right\}, \quad k = 1,2 \quad \text{and } m = 1,2, \tag{10}$$

where $\varepsilon_1 = \varepsilon_{SR}$ and $\varepsilon_2 = \varepsilon_{RD}$ for the simplicity of notation, $\mathrm{Re}\{\cdot\}$ denotes the real part of a complex value, $\mathrm{tr}\{\cdot\}$ takes the trace of a matrix, and the notations $\boldsymbol{\mu}_y$ and $\mathbf{C}_y$ represent the mean and the covariance of the received signal $\mathbf{y}$ at the destination node, respectively. Using (7), the mean $\boldsymbol{\mu}_y$ and the covariance $\mathbf{C}_y$ can be calculated as

$$\boldsymbol{\mu}_y = \mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{s}_R; \tag{11}$$

$$\mathbf{C}_y = \mathrm{E}\left[\left(\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{v} + \mathbf{z}\right)\left(\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{v} + \mathbf{z}\right)^H\right] = \sigma_R^2\mathbf{E}_{RD}\mathbf{R}_{RD}\mathbf{E}_{RD}^H + \sigma_D^2\mathbf{I}, \tag{12}$$

where $\mathrm{E}[\cdot]$ takes the expectation, $(\cdot)^H$ is the Hermitian operation, and $\mathbf{R}_{RD}$ represents the channel correlation matrix for the R-D link:

$$\mathbf{R}_{RD} = \mathbf{H}_{RD}\mathbf{H}_{RD}^H = \begin{bmatrix} r_{RD}(0) & r_{RD}(1) & \cdots & r_{RD}(N-2) & r_{RD}(N-1) \\ r_{RD}(N-1) & r_{RD}(0) & \cdots & r_{RD}(N-3) & r_{RD}(N-2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{RD}(2) & r_{RD}(3) & \cdots & r_{RD}(0) & r_{RD}(1) \\ r_{RD}(1) & r_{RD}(2) & \cdots & r_{RD}(N-1) & r_{RD}(0) \end{bmatrix} \tag{13}$$

where $r_{RD}(i) = \sum_{n=0}^{N-1} h_{RD}(n)h_{RD}^*((n+i))_N$ is the circular autocorrelation function of the channel from the relay to the destination nodes, and the brace $((\cdot))_N$ is a modulo-N operator. From (13), it is worth mentioning that the channel power of different channel paths is accumulated in $r_{RD}(0)$, and the off-diagonal entries in $\mathbf{R}_{RD}$ are typically much smaller than the diagonal entries due to the non-coherent combining in $r_{RD}(i)$ as $i \neq 0$. Accordingly, the correlation matrix $\mathbf{R}_{RD}$ possesses a strongly diagonal property under multipath environments. By applying (11) and (12), the derivatives in the FIM of (10) can be explicitly calculated as follows:

$$\frac{\partial \boldsymbol{\mu}_y}{\partial \varepsilon_{SR}} = j\frac{2\pi}{N}\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{D}\mathbf{s}_R; \tag{14}$$

$$\frac{\partial \boldsymbol{\mu}_y}{\partial \varepsilon_{RD}} = j\frac{2\pi}{N}\mathbf{D}\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{s}_R; \tag{15}$$

$$\frac{\partial \mathbf{C}_y}{\partial \varepsilon_{SR}} = 0; \tag{16}$$

$$\frac{\partial \mathbf{C}_y}{\partial \varepsilon_{RD}} = \frac{2\pi}{N}\sigma_R^2\mathbf{E}_{RD}\tilde{\mathbf{Q}}_{RD}\mathbf{E}_{RD}^H, \tag{17}$$

where $\mathbf{D} = \mathrm{diag}([0, 1, \ldots, N-1]^T)$, and $\tilde{\mathbf{Q}}_{RD}$ is given (18), as shown at the top of the next page. To simplify the computation of $\mathrm{tr}\left\{\mathbf{C}_y^{-1}\frac{\partial \mathbf{C}_y}{\partial \varepsilon_k}\mathbf{C}_y^{-1}\frac{\partial \mathbf{C}_y}{\partial \varepsilon_m}\right\}$ in (10), the covariance matrix $\mathbf{C}_y$ in (12) is reformulated in terms of $\mathbf{E}_{RD}$ as follows:

$$\mathbf{C}_y = \sigma_R^2\mathbf{E}_{RD}\mathbf{Q}_{RD}\mathbf{E}_{RD}^H\ , \tag{19}$$

where $\mathbf{Q}_{RD} = \mathbf{R}_{RD} + \sigma_R^{-2}\sigma_D^2\mathbf{I}$. As a result, the inverse of the covariance matrix $\mathbf{C}_y$ can be derived as

$$\mathbf{C}_y^{-1} = \sigma_R^{-2}\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{E}_{RD}^H\,. \tag{20}$$

Notice that the matrix $\mathbf{Q}_{RD}$ still preserves the strongly diagonal property, since it involves the summation of a strongly diagonal matrix $\mathbf{R}_{RD}$ and an identity matrix. To calculate the inverse of the matrix $\mathbf{Q}_{RD}$, we further define a scalar factor δ_{RD} and a zero-diagonal matrix $\mathbf{B}_{RD}$ which satisfy

$$\mathbf{Q}_{RD} = \delta_{RD}\left(\mathbf{I} + \delta_{RD}^{-1}\mathbf{B}_{RD}\right)\ , \tag{21}$$

where $\delta_{RD} = r_{RD}(0) + \sigma_R^{-2}\cdot\sigma_D^2$. By applying the first-order Taylor's series expansion into (21), the matrix $\mathbf{Q}_{RD}^{-1}$ can be approximated as

$$\mathbf{Q}_{RD}^{-1} \approx \delta_{RD}^{-1}\left(\mathbf{I} - \delta_{RD}^{-1}\mathbf{B}_{RD}\right). \tag{22}$$

By substituting (14)-(17) and (20) into (10) and after some straightforward manipulations, the entries of the FIM $\mathbf{F}$ can be explicitly computed as

$$\mathbf{F}(1,1) = 2\mathrm{Re}\left\{\alpha\mathbf{s}_R^H\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}\mathbf{s}_R\right\}; \tag{23}$$

$$\mathbf{F}(1,2) = 2\mathrm{Re}\left\{\alpha\mathbf{s}_R^H\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{Q}_{RD}^{-1}\mathbf{E}_{RD}^H\mathbf{D}\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{s}_R\right\}; \tag{24}$$

$$\mathbf{F}(2,1) = 2\mathrm{Re}\left\{\alpha\mathbf{s}_R^H\mathbf{H}_{RD}^H\mathbf{E}_{RD}^H\mathbf{D}^H\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}\mathbf{s}_R\right\}; \tag{25}$$

$$\mathbf{F}(2,2) = 2\mathrm{Re}\left\{\alpha\mathbf{s}_R^H\mathbf{H}_{RD}^H\mathbf{E}_{RD}^H\mathbf{D}^H\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{D}\mathbf{H}_{RD}\mathbf{s}_R\right\} + \mathrm{tr}\left[\beta\mathbf{Q}_{RD}^{-1}\tilde{\mathbf{Q}}_{RD}\mathbf{Q}_{RD}^{-1}\tilde{\mathbf{Q}}_{RD}\right], \tag{26}$$

where $\alpha = \sigma_R^{-2}\left(\frac{2\pi}{N}\right)^2$, $\beta = \left(\frac{2\pi}{N}\right)^2$, and $\mathbf{Q}_{RD}^{-1}$ can be obtained from (22) via approximation. Finally, the CRBs of the CFOs estimation for ϵ_{SR} and ϵ_{RD} can be represented as [10]

$$CRB_{\epsilon_{SR}} = \mathbf{F}^{-1}(1,1) = \frac{\mathbf{F}(2,2)}{\mathbf{F}(1,1)\mathbf{F}(2,2) - \mathbf{F}(1,2)\mathbf{F}(2,1)}; \tag{27}$$

$$CRB_{\epsilon_{RD}} = \mathbf{F}^{-1}(2,2) = \frac{\mathbf{F}(1,1)}{\mathbf{F}(1,1)\mathbf{F}(2,2) - \mathbf{F}(1,2)\mathbf{F}(2,1)}, \tag{28}$$

where $\mathbf{F}^{-1}(i,j)$ is the $(i,j)th$ entry of the matrix $\mathbf{F}^{-1}$.

$$\tilde{\mathbf{Q}}_{RD} = j\begin{bmatrix} 0 & (-1)r_{RD}(1) & \cdots & -(N-2)r_{RD}(N-2) & -(N-1)r_{RD}(N-1) \\ (1)r_{RD}(N-1) & 0 & \cdots & -(N-3)r_{RD}(N-3) & -(N-2)r_{RD}(N-2) \\ (2)r_{RD}(N-2) & (1)r_{RD}(N-1) & \ddots & \vdots & \vdots \\ \vdots & \vdots & \cdots & 0 & -(1)r_{RD}(1) \\ (N-1)r_{RD}(1) & (N-2)r_{RD}(2) & \cdots & (1)r_{RD}(N-1) & 0 \end{bmatrix}. \quad (18)$$

IV. DERIVATION OF THE MCRB FOR THE CFOS ESTIMATION

For the derivation of the MCRB, it only relies on the statistical distributions of the channel parameters without the acquisition of the exact channel coefficients. The relay SNR and the destination SNR are denoted by $\Gamma_R(=\frac{1}{\sigma_R^2})$ and $\Gamma_D(=\frac{1}{\sigma_D^2})$ respectively, where the signal power of the data symbol is assumed to be one ($\sigma_s^2 = 1$). Similar to the derivation of the true CRB with the perfect knowledge of the channel coefficients, the derivation of the MCRB requires the calculation of the MFIM, which is the expectation value of the FIM as derived in (23)-(26) with respect to the random channel parameters. Prior to deriving the MFIM of the CFO estimation, some lemmas are provided in the following.

Lemma 1: Assume that γ_l, for $l = 0, 1, \ldots, L-1$, are L independent circular symmetric complex Gaussian random variables with zero mean and distinct variance σ_l^2. Then, the expectation value of the random variable $\left(\sum_{l=0}^{L-1}|\gamma_l|^2 + c\right)^{-1}$, where c is a positive constant, is given by

$$\sum_{l=0}^{L-1}\left(\sigma_l^2\prod_{i=0,i\neq l}^{L-1}\left(1-\frac{\sigma_i^2}{\sigma_l^2}\right)\right)^{-1}\cdot e^{\frac{c}{\sigma l^2}}E_1\left(\frac{c}{\sigma_l^2}\right),$$

where $E_1(\cdot)$ is the exponential integral function and defined by

$$E_1(x) = \int_x^\infty \frac{e^{-t}}{t}dt.$$

Proof: The detailed proof is provided in Appendix A. ∎

Lemma 2: Assume that x and y are two independent random variables with the exponential distribution and generalized chi-square distribution, respectively, and the corresponding probability density functions are

$$f_X(x) = \frac{1}{\sigma_0^2}e^{-\frac{x}{\sigma_0^2}};$$
$$f\left(y;(L-1),\sigma_1^2,\ldots,\sigma_{L-1}^2\right) = \sum_{l=1}^{L-1}\left(\sigma_l^2\prod_{i=1,i\neq l}^{L-1}\left(1-\frac{\sigma_i^2}{\sigma_l^2}\right)\right)^{-1}e^{-\frac{y}{\sigma_l^2}},\quad x,y\geq 0.$$

Let c be a positive constant. Then, the mean of the random variable $\frac{x}{x+y+c}$ is given by

$$\mathsf{E}\left[\frac{x}{x+y+c}\right] = \left(1-e^{\frac{c}{\sigma_0^2}}\cdot\frac{c}{\sigma_0^2}E_1\left(\frac{c}{\sigma_0^2}\right)\right)\cdot\left(1-\sum_{l=1}^{L-1}\omega_l\frac{\sigma_l^2}{\left(1-\frac{\sigma_0^2}{\sigma_l^2}\right)}\right)-\sum_{l=1}^{L-1}\omega_l\left[\frac{\sigma_0^2}{\left(1-\frac{\sigma_0^2}{\sigma_l^2}\right)^2}\cdot\left(e^{\frac{c}{\sigma_0^2}}E_1\left(\frac{c}{\sigma_0^2}\right)-e^{\frac{c}{\sigma_l^2}}E_1\left(\frac{c}{\sigma_l^2}\right)\right)\right],$$

where $\omega_l = \left(\sigma_l^2\prod_{i=1,i\neq l}^{L-1}\left(1-\frac{\sigma_i^2}{\sigma_l^2}\right)\right)^{-1}$.

Proof: The detailed proof is provided in Appendix B. ∎

Lemma 3: Assume that γ_l, for $l = 0, 1, \ldots, (L-1)$, are L independent circular symmetric complex Gaussian random variables with zero mean and distinct variance σ_l^2. Then, the mean of the random variable $\left(\sum_{l=0}^{L-1}|\gamma_l|^2 + c\right)^{-2}$, where c is a positive constant, is given as

$$\mathsf{E}\left[\left(\sum_{l=0}^{L-1}|\gamma_l|^2+c\right)^{-2}\right]=\sum_{l=0}^{L-1}\left(\sigma_l^2\prod_{i=0,i\neq l}^{L-1}\left(1-\frac{\sigma_i^2}{\sigma_l^2}\right)\right)^{-1}\cdot\left(c^{-1}-\sigma_l^{-2}e^{\frac{c}{\sigma_l^2}}\cdot E_1\left(\frac{c}{\sigma_l^2}\right)\right).$$

Proof: The detailed proof is provided in Appendix C. ∎

Before we develop a theorem to establish the MFIM of the CFOs estimation for the relay network with multipath receptions, some notations are defined in the following:

$$\delta_m = \sum_{l=0}^{L-1-m}\sigma_{RD,l}^2\sigma_{RD,l+m}^2,\quad m = 1,2,\ldots,L-1;$$
$$R_c = \frac{\Gamma_R}{\Gamma_D};$$
$$\zeta_l = e^{R_c\sigma_{RD,l}^{-2}}E_1\left(R_c\sigma_{RD,l}^{-2}\right);$$
$$\tau_{i,m} = \frac{\sigma_{RD,i}^2}{\sigma_{RD,m}^2};$$
$$\mathcal{X}_{l,k} = \prod_{i=0,i\neq k,i\neq l}^{L-1}(1-\tau_{i,k})^{-1};$$
$$\rho_l = \left(\sigma_{RD,l}^2\prod_{i=0,i\neq l}^{L-1}(1-\tau_{i,l})\right)^{-1};$$
$$\phi = \sum_{l=0}^{L-1}\rho_l\left(R_c^{-1}-\sigma_{RD,l}^{-2}\zeta_l\right);$$

$$\Delta = 2N\left(\frac{2\pi}{N}\right)^2 \cdot \sum\nolimits_{m=1}^{L-1} m(N-m)\cdot\delta_m\phi;$$

$$\psi_l = \left(1 - R_c\sigma_{RD,l}^{-2}\cdot\zeta_l\right)\left(1-\sum\nolimits_{k=0,k\neq l}^{L-1}\frac{\mathcal{X}_{l,k}}{(1-\tau_{l,k})}\right) - \sum\nolimits_{k=0,k\neq l}^{L-1}\frac{\tau_{l,k}\mathcal{X}_{l,k}}{(1-\tau_{l,k})^2}\cdot(\zeta_l-\zeta_k).$$

Theorem 1: The 2×2 MFIM of the CFOs estimation with the statistical distribution of the channel parameters can be approximated as

$$\mathsf{E}[F] = \begin{bmatrix}\mathsf{E}[\mathbf{F}(1,1)] & \mathsf{E}[\mathbf{F}(1,2)]\\ \mathsf{E}[\mathbf{F}(2,1)] & \mathsf{E}[\mathbf{F}(2,2)]\end{bmatrix},$$

where

$$\mathsf{E}[\mathbf{F}(1,1)] = \frac{\Gamma_R\left(\frac{2\pi}{N}\right)^2}{3}N(N-1)(2N-1) \cdot\left[1-R_c\cdot\sum\nolimits_{l=0}^{L-1}\rho_l\cdot\zeta_l\right]; \quad (29)$$

$$\mathsf{E}[\mathbf{F}(1,2)] = \mathsf{E}[\mathbf{F}(2,1)] = \frac{\Gamma_R\left(\frac{2\pi}{N}\right)^2}{3} \cdot\sum\nolimits_{l=0}^{L-1}\left[2N^3-3N^2+N+3Nl^2-3N^2l\right]\psi_l; \quad (30)$$

$$\mathsf{E}[\mathbf{F}(2,2)] = \mathsf{E}[\mathbf{F}(1,1)] + \Delta. \quad (31)$$

Proof: To facilitate the derivation, we utilize the strongly diagonal property of the channel correlation matrices $\mathbf{R}_{RD}$ and $\mathbf{H}_{SR}^H\mathbf{H}_{SR}$, which can be approximated as the diagonal matrices $r_{RD}(0)\cdot\mathbf{I}$ and $r_{SR}(0)\cdot\mathbf{I}$, respectively. Accordingly, the inverse matrix $\mathbf{Q}_{RD}^{-1}$ can be obtained by $\lambda_q^{-1}\cdot\mathbf{I}$, where $\lambda_q = r_{RD}(0)+R_c$. In addition, we have $\mathsf{E}\left[\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\right]=\mathbf{I}$ and $\mathsf{E}[r_{SR}(0)] = 1$. In what follows, we in turn derive the expectation values of $\mathbf{F}(1,1)$, $\mathbf{F}(1,2)$, $\mathbf{F}(2,1)$ and $\mathbf{F}(2,2)$, as described in (23)-(26), to obtain the MFIM with the statistical distribution of the channel parameters.

a. Derivation of $\mathsf{E}[\mathbf{F}(1,1)]$
Since $\mathbf{F}(1,1)$ in (23) is a scalar, we can rewrite $\mathbf{F}(1,1)$ and substitute $\mathbf{s}_R$ in (6) as

$$\begin{aligned}\mathbf{F}(1,1) &= 2\mathsf{Re}\{\mathsf{tr}[\alpha\cdot\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H\mathbf{E}_{SR}^H\mathbf{D}^H\mathbf{H}_{RD}^H \cdot\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}]\}\\ &= 2\mathsf{Re}\{\mathsf{tr}[\alpha\cdot\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H \cdot\mathbf{E}_{SR}^H\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}]\},\end{aligned} \quad (32)$$

where $\alpha = \sigma_R^{-2}\left(\frac{2\pi}{N}\right)^2$ and $\mathbf{D} = \mathsf{diag}([0,1,\ldots,N-1]^T)$. Then the expectation value of $\mathbf{F}(1,1)$ is given by

$$\begin{aligned}\mathsf{E}[\mathbf{F}(1,1)] &= 2\sigma_R^{-2}\left(\frac{2\pi}{N}\right)^2\cdot\mathsf{Re}\left\{\mathsf{tr}\left[\mathsf{E}\left[\frac{r_{RD}(0)}{\lambda_q}\right] \cdot\mathsf{E}\left[\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\right]\mathbf{D}^H\mathbf{D}\mathsf{E}[r_{SR}(0)]\cdot\mathbf{I}\right]\right\}\\ &= 2\Gamma_R\left(\frac{2\pi}{N}\right)^2\cdot\mathsf{E}\left[1-\frac{R_c}{\lambda_q}\right]\cdot\mathsf{Re}\left\{\mathsf{tr}\left[\mathbf{D}^H\mathbf{D}\right]\right\}\\ &= 2\Gamma_R\left(\frac{2\pi}{N}\right)^2\left(1-R_c\cdot\mathsf{E}\left[\lambda_q^{-1}\right]\right)\cdot\sum\nolimits_{n=0}^{N-1}n^2\\ &= \frac{\Gamma_R}{3}\left(\frac{2\pi}{N}\right)^2 N(N-1)(2N-1) \cdot\left(1-R_c\cdot\mathsf{E}\left[\lambda_q^{-1}\right]\right).\end{aligned} \quad (33)$$

By using Lemma 1 and the definition of $\lambda_q = r_{RD}(0)+R_c$, we can get $\mathsf{E}\left[\lambda_q^{-1}\right] = \sum_{l=0}^{L-1}\rho_l\cdot\zeta_l$. Thus, it leads to

$$\mathsf{E}[\mathbf{F}(1,1)] = \frac{\Gamma_R}{3}\left(\frac{2\pi}{N}\right)^2 N(N-1)(2N-1) \cdot\left[1-R_c\cdot\sum\nolimits_{l=0}^{L-1}\rho_l\zeta_l\right]. \quad (34)$$

b. Derivation of $\mathsf{E}[\mathbf{F}(1,2)]$
Since $\mathbf{F}(1,2)$ in (24) is a scalar, we can rewrite $\mathbf{F}(1,2)$ and substitute $\mathbf{s}_R$ in (6) as

$$\begin{aligned}\mathbf{F}(1,2) &= 2\mathsf{Re}\Big\{\alpha\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H\mathbf{E}_{SR}^H\mathbf{D}^H\mathbf{H}_{RD}^H \cdot\mathbf{Q}_{RD}^{-1}\mathbf{E}_{RD}^H\mathbf{D}\mathbf{E}_{RD}\mathbf{H}_{RD}\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\Big\}\\ &= 2\alpha\cdot\mathsf{Re}\Big\{\mathsf{tr}\Big[\mathbf{H}_{RD}\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H \cdot\mathbf{E}_{SR}^H\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{Q}_{RD}^{-1}\mathbf{E}_{RD}^H\mathbf{D}\mathbf{E}_{RD}\Big]\Big\}.\end{aligned} \quad (35)$$

The expectation value of $\mathbf{F}(1,2)$ can be obtained by

$$\begin{aligned}&\mathsf{E}[\mathbf{F}(1,2)]\\ &= 2\alpha\cdot\mathsf{Re}\left\{\mathsf{tr}\left[\mathsf{E}\left[\frac{1}{\lambda_q}\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right] \cdot\mathsf{E}\left[\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\right]\mathsf{E}[r_{SR}(0)]\cdot\mathbf{I}\right]\right\}\\ &= 2\Gamma_R\left(\frac{2\pi}{N}\right)^2\cdot\mathsf{Re}\left\{\mathsf{tr}\left[\mathsf{E}\left[\frac{1}{\lambda_q}\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]\right]\right\}\\ &= 2\Gamma_R\left(\frac{2\pi}{N}\right)^2\cdot\left\{\sum\nolimits_{l=0}^{L-1}\left[\frac{1}{6}[2N^3 -3N^2+N+3Nl^2-3N^2l]\right]\cdot\mathsf{E}\left[\frac{|h_{RD}(l)|^2}{\lambda_q}\right]\right\},\end{aligned} \quad (36)$$

where the term $\mathsf{tr}\left[\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]$ is derived in the Appendix D. Moreover, we can get $\mathsf{E}\left[\frac{|h_{RD}(l)|^2}{\lambda_q}\right] = \mathsf{E}\left[\frac{|h_{RD}(l)|^2}{r_{RD}(0)+R_c}\right] = \psi_l$ by applying the Lemma 2. The expectation value $\mathsf{E}[\mathbf{F}(1,2)]$ can be finally represented by

$$\mathsf{E}[\mathbf{F}(1,2)] = \frac{1}{3}\Gamma_R\left(\frac{2\pi}{N}\right)^2\cdot\sum\nolimits_{l=0}^{L-1}[2N^3-3N^2 +N+3Nl^2-3N^2l]\psi_l. \quad (37)$$

c. Derivation of $\mathsf{E}[\mathbf{F}(2,1)]$

Similar to the derivation of the expectation value $\mathsf{E}[\mathbf{F}(1,2)]$, we can first represent $\mathbf{F}(2,1)$ by

$$\begin{aligned}\mathbf{F}(2,1) &= 2\mathrm{Re}\Big\{\mathrm{tr}\Big[\alpha\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H\mathbf{E}_{SR}^H\mathbf{H}_{RD}^H\mathbf{E}_{RD}^H \\ &\quad\cdot\mathbf{D}^H\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\Big]\Big\} \\ &= 2\mathrm{Re}\Big\{\mathrm{tr}\Big[\alpha\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H \\ &\quad\cdot\mathbf{E}_{SR}^H\mathbf{H}_{RD}^H\mathbf{E}_{RD}^H\mathbf{D}^H\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{H}_{RD}\mathbf{D}\Big]\Big\}.\end{aligned}$$

The expectation value of $\mathbf{F}(2,1)$ can be derived by

$$\begin{aligned}&\mathsf{E}[\mathbf{F}(2,1)] \\ &= 2\alpha\mathrm{Re}\Big\{\mathrm{tr}\Big[\mathsf{E}[r_{SR}(0)] \\ &\quad\cdot\mathsf{E}\Big[\frac{1}{\lambda_q}\mathbf{H}_{RD}\mathbf{D}\mathbf{H}_{RD}^H\mathbf{D}^H\Big]\Big]\Big\} \\ &= 2\Gamma_R\left(\frac{2\pi}{N}\right)^2\mathrm{Re}\left\{\mathsf{E}\left[\mathrm{tr}\left[\frac{1}{\lambda_q}\mathbf{H}_{RD}\mathbf{D}\mathbf{H}_{RD}^H\mathbf{D}^H\right]\right]\right\} \\ &= \mathsf{E}[\mathbf{F}(1,2)]. \qquad (38)\end{aligned}$$

d. Derivation of $\mathsf{E}[\mathbf{F}(2,2)]$

We can derive $\mathbf{F}(2,2)$ as

$$\begin{aligned}\mathbf{F}(2,2) &= 2\mathrm{Re}\Big\{\mathrm{tr}\Big[\alpha\mathbf{x}^H\mathbf{W}^H\mathbf{H}_{SR}^H\mathbf{E}_{SR}^H\mathbf{H}_{RD}^H\mathbf{E}_{RD}^H \\ &\quad\cdot\mathbf{D}^H\mathbf{E}_{RD}\mathbf{Q}_{RD}^{-1}\mathbf{D}\mathbf{H}_{RD}\mathbf{E}_{SR}\mathbf{H}_{SR}\mathbf{W}\mathbf{x}\Big]\Big\} \\ &\quad+\mathrm{tr}\left[\beta\mathbf{Q}_{RD}^{-1}\widetilde{\mathbf{Q}}_{RD}\mathbf{Q}_{RD}^{-1}\widetilde{\mathbf{Q}}_{RD}\right], \qquad (39)\end{aligned}$$

where $\beta = \left(\frac{2\pi}{N}\right)^2$. Accordingly, the expectation value of $\mathbf{F}(2,2)$ is given by

$$\begin{aligned}&\mathsf{E}[\mathbf{F}(2,2)] \\ &= 2\mathrm{Re}\Big\{\mathrm{tr}\Big[\Gamma_R\left(\frac{2\pi}{N}\right)^2\cdot\mathsf{E}\left[\frac{r_{RD}(0)}{\lambda_q}\right]\mathbf{D}^H\mathbf{D} \\ &\quad\cdot\mathsf{E}[r_{SR}(0)]\cdot\mathbf{I}\Big]\Big\}+\beta\cdot\mathsf{E}\left[\mathrm{tr}\left[\frac{1}{\lambda_q^2}\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]\right] \\ &= \mathsf{E}[\mathbf{F}(1,1)]+\left(\frac{2\pi}{N}\right)^2\cdot\mathsf{E}\left[\mathrm{tr}\left[\frac{1}{\lambda_q^2}\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]\right] \\ &= \mathsf{E}[\mathbf{F}(1,1)]+\left(\frac{2\pi}{N}\right)^2\cdot\sum\nolimits_{m=1}^{L-1}[2Nm(N-m)] \\ &\quad\cdot\mathsf{E}\left[\frac{|r_{RD}(m)|^2}{\lambda_q^2}\right], \qquad (40)\end{aligned}$$

where the value of $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$ can be explicitly derived in the Appendix E. Since $|r_{RD}(m)|^2$ and λ_q^2 are uncorrelated with each other, for $m = 1, 2, \ldots, (L-1)$, we can further obtain $\mathsf{E}[\mathbf{F}(2,2)]$ by

$$\begin{aligned}&\mathsf{E}[\mathbf{F}(2,2)] \\ &= \mathsf{E}[\mathbf{F}(1,1)]+\beta\cdot\sum\nolimits_{m=1}^{L-1}[2Nm(N-m)] \\ &\quad\cdot\mathsf{E}\left[|r_{RD}(m)|^2\right]\mathsf{E}\left[\lambda_q^{-2}\right] \\ &= \mathsf{E}[\mathbf{F}(1,1)]+2N\beta\cdot\sum\nolimits_{m=1}^{L-1}m(N-m)\cdot\delta_m\phi \\ &, \qquad (41)\end{aligned}$$

where one can obtain $\mathsf{E}\left[\lambda_q^{-2}\right] = \phi$ by using Lemma 3. ■

From (34), (37), (38) and (41), the MFIM can be determined, and the MCRBs of the CFOs estimation for ϵ_{SR} and ϵ_{RD} can be calculated by [10]

$$MCRB_{\epsilon_{SR}} = \frac{\mathsf{E}[\mathbf{F}(2,2)]}{\mathsf{E}[\mathbf{F}(1,1)]\cdot\mathsf{E}[\mathbf{F}(2,2)]-\mathsf{E}[\mathbf{F}(1,2)]\cdot\mathsf{E}[\mathbf{F}(2,1)]}; \qquad (42)$$

$$MCRB_{\epsilon_{RD}} = \frac{\mathsf{E}[\mathbf{F}(1,1)]}{\mathsf{E}[\mathbf{F}(1,1)]\cdot\mathsf{E}[\mathbf{F}(2,2)]-\mathsf{E}[\mathbf{F}(1,2)]\cdot\mathsf{E}[\mathbf{F}(2,1)]}. \qquad (43)$$

To facilitate the analysis of the MCRBs, we have the following two assumptions:

Assumption 1: The total channel power is set to one.

Assumption 2: The channel power delay profile of the L-paths fading channels keeps unchanged.

According to (42) and (43) and the above assumptions, we can get two corollaries for the MCRBs.

Corollary 1: The MCRB of ϵ_{SR} is always larger than that of ϵ_{RD}. Moreover, the MCRB of ϵ_{SR} gets close to that of ϵ_{RD} when the ratio of the relay SNR to the destination SNR approaches infinity, i.e., $R_c \to \infty$.

Proof: From (31), (42) and (43), since the term Δ is positive, it implies that the MCRB of ϵ_{SR} is always larger than that of ϵ_{RD}. In addition, the term Δ varies with the parameter R_c in ϕ. Actually, the term ϕ can be approximated as [19]

$$\phi \cong \sum\nolimits_{m=0}^{L-1}\rho_m\left(\frac{1}{R_c}-\frac{\ln\left(1+\frac{\sigma_{RD,m}^2}{R_c}\right)}{\sigma_{RD,m}^2}\right). \qquad (44)$$

When the parameter R_c approaches infinity, it can be observed that the term ϕ degrades to zero:

$$\begin{aligned}\lim_{R_c\to\infty}\phi &= \sum_{m=0}^{L-1}\rho_m\cdot\lim_{R_c\to\infty}\left(\frac{1}{R_c}-\frac{\ln\left(1+\frac{\sigma_{RD,m}^2}{R_c}\right)}{\sigma_{RD,m}^2}\right) \\ &= \sum_{m=0}^{L-1}\rho_m(0-0)=0. \qquad (45)\end{aligned}$$

Therefore, the term Δ approaches zero as the ratio of the relay SNR to the destination SNR goes to infinity, and the corresponding MCRB of ϵ_{SR} is comparable to that of ϵ_{RD}. ■

Corollary 2: If the relay SNR Γ_R is equal to the destination SNR Γ_D, i.e., $R_c = 1$, the MCRB is approximately inversely proportional to the SNR.

Proof: Under the setting of $R_c = 1$ and Assumption 2, we can represent (29) by $\mathsf{E}[\mathbf{F}(1,1)] = \Gamma_R \Lambda$, where $\Lambda = \frac{\left(\frac{2\pi}{N}\right)^2}{3} N(N-1)(2N-1)\left[1 - R_c \cdot \sum_{l=0}^{L-1} \rho_l \cdot \zeta_l\right]$ is a constant. Similarly, the expectation values $\mathsf{E}[\mathbf{F}(1,2)]$ and $\mathsf{E}[\mathbf{F}(2,1)]$ can be expressed by $\mathsf{E}[\mathbf{F}(1,2)] = \mathsf{E}[\mathbf{F}(2,1)] = \Gamma_R \Theta$, where $\Theta = \frac{\left(\frac{2\pi}{N}\right)^2}{3} \sum_{l=0}^{L-1} \left[2N^3 - 3N^2 + N + 3Nl^2 - 3N^2\ l\right] \psi_l$ is a constant. In addition, the expectation value $\mathsf{E}[\mathbf{F}(2,2)]$ is nearly the same as $\mathsf{E}[\mathbf{F}(1,1)]$, since the term Δ in (31) is much smaller than $\mathsf{E}[\mathbf{F}(1,1)]$. Accordingly, we can approximate (42) and (43) by

$$MCRB_{\epsilon_{RD}} = MCRB_{\epsilon_{SR}} \cong \frac{\Gamma_R \cdot \Lambda}{\Gamma_R^2 \cdot \Lambda^2 - \Gamma_R^2 \cdot \Theta^2} = \frac{1}{\Gamma_R} \cdot \frac{\Lambda}{\Lambda^2 - \Theta^2}. \quad (46)$$

Therefore, it is concluded that the MCRBs of ϵ_{SR} or ϵ_{RD} is almost inversely proportional to the SNR Γ_R. ■

V. NUMERICAL RESULTS AND DISCUSSIONS

We evaluate the true CRBs with the exact channel coefficients and the MCRBs with the statistical information of the channel parameters for the OFDM-based single-relay networks by computer simulations. In the simulation, the training signals are randomly generated from the binary phase-shift keying (BPSK) modulation scheme and assumed to be perfectly known to the destination. The system bandwidth is 5 MHz, and the number of subcarriers, N, is set to 64. The normalized CFOs are uniformly distributed between -0.5 and 0.5. An International Telecommunication Union (ITU) Veh. A channel model with six effective paths is adopted to investigate the impact of multipath channels on the true CRBs and the MCRBs, where the relative path power profiles are set as 0, -1, -9, 10, -15, -20 (dB) [20]. The channel gains are generated by independent identically distributed (i.i.d.) zero-mean complex Gaussian random variables. In the evaluation of the true CRBs, it is assumed that the exact channel coefficients are perfectly available at the destination. The channel path delays are random and uniformly distributed between zero and eight sampling periods. Finally, the average true CRBs performances are obtained by averaging the results over ten thousands of randomly generated channel realizations through the Monte Carlo simulations.

In the evaluation of the MCRBs, the channel path delays for the six-path ITU Veh. A channel are assumed to be ranged between zero and five sampling periods, in order to examine the impact of the unknown channel gains on the MCRBs performances. The numerical results of the MCRBs, which are obtained via computer simulations, are provided to validate the theoretical results of the MCRBs derived in Section IV. We also investigate the impact of the number of effective channel paths on the true CRBs and the MCRBs performances by taking an exponentially-decay fading channel model in [21] into account. For this model, the average channel power of the *lth* effective path, $\mathsf{E}\left[|h_l|^2\right]$, is proportional to $e^{-l\frac{T_s}{\sigma_T}}$, where T_s is the sampling period and σ_T is the maximum channel delay spread. Moreover, the channel paths are assumed to be uncorrelated with each other, and the total channel power is normalized to one. The ratio of the sampling period to the maximum channel delay spread is set to 0.2. Finally, the timing synchronization is assumed to be perfect throughout the simulation.

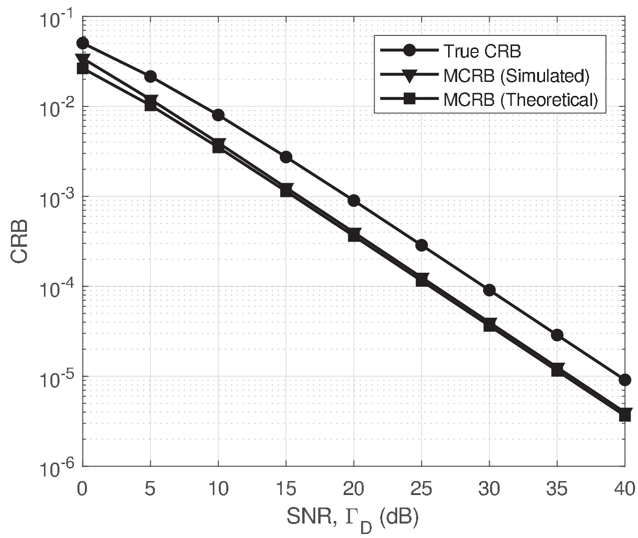

FIGURE 2. The true CRBs and the MCRBs performances for ϵ_{SR} ($\Gamma_R = \Gamma_D$).

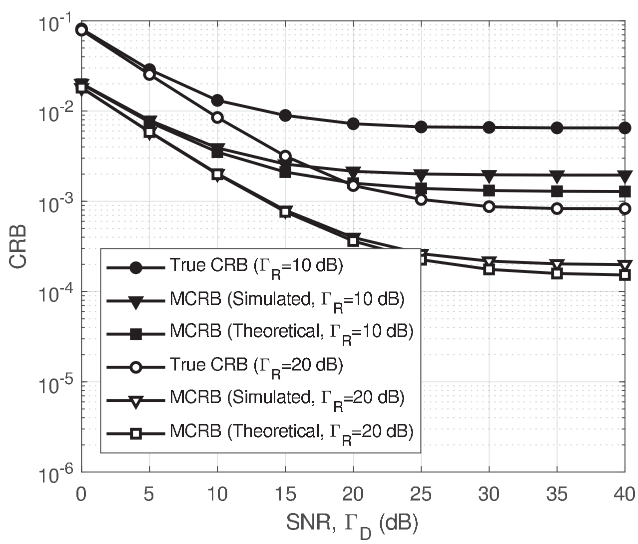

FIGURE 3. The true CRBs and the MCRBs performances for ϵ_{SR} at different operating SNR values of Γ_R.

In Fig.2, the true CRBs and the MCRBs performances of ϵ_{SR} are demonstrated in the ITU Veh. A channel, and the relay SNR is set equal to the destination SNR. It is found that both the true CRBs and the MCRBs of ϵ_{SR} monotonically decrease as the operating SNR increases. It can be observed that for the MCRBs, the theoretical result is slightly lower than the simulation results due to the approximation of the channel correlation matrix by a diagonal matrix in the derivation of the theoretical MCRB. Moreover, the MCRB is a little bit looser than the true CRB. For example, the true CRB is 3×10^{-4}

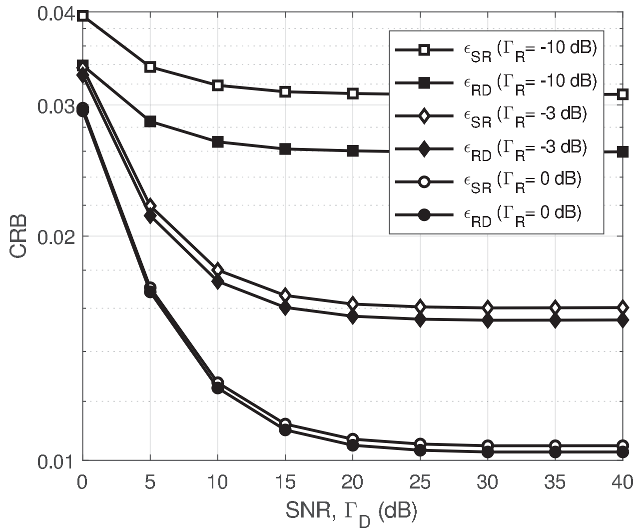

FIGURE 4. The simulated MCRBs performances for ϵ_{SR} and ϵ_{RD} at different operating SNR values of Γ_R.

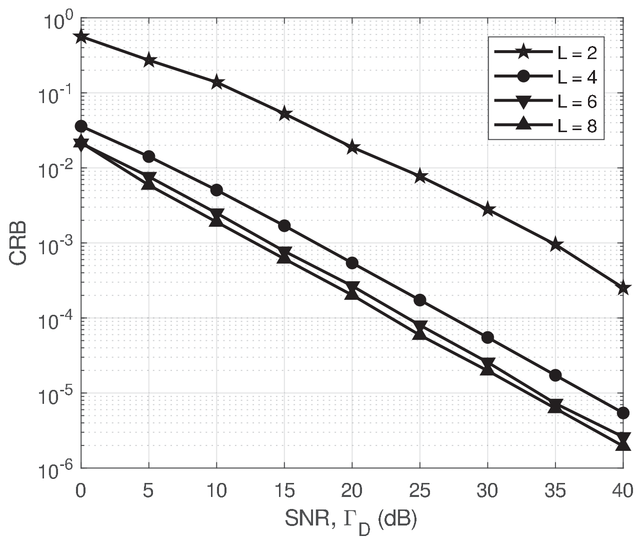

FIGURE 5. The true CRBs performance of ϵ_{SR} in the exponential fading channel as $\Gamma_R = \Gamma_D$.

at $\Gamma_R = \Gamma_D = 25$ dB, while the MCRB is given by 10^{-4}. In Fig.3, we compare the true CRBs and the MCRBs of ϵ_{SR} at different operating values of the relay SNR Γ_R. The values of Γ_R are set as 10 dB or 20 dB. The theoretical MCRB is slightly lower than the simulated MCRB as the destination SNR is large. It is also observed that the relay network with a higher relay SNR setting can achieve better true CRB and MCRB performances. Overall, the performance can be improved as the destination SNR Γ_D increases, whereas the improvement becomes gradually saturated, since the noise at the relay becomes a dominant factor for the CFO estimation. A closer look at this figure reveals that the performance saturation occurs when the destination SNR Γ_D is ten times larger than the relay SNR Γ_R.

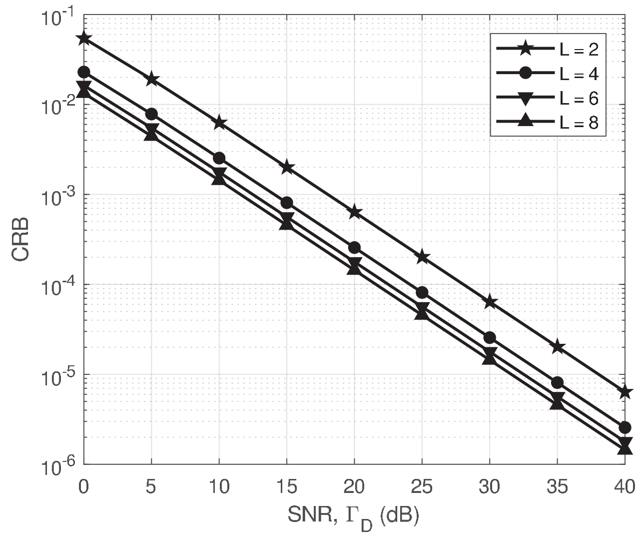

FIGURE 6. The simulated MCRBs performance of ϵ_{SR} in the exponential fading channel as $\Gamma_R = \Gamma_D$.

To investigate the performance difference between the CFOs ϵ_{SR} and ϵ_{RD}, we evaluate the simulated MCRBs at different values of the operating relay SNR Γ_R in Fig. 4. From this figure, we can find that the performance of ϵ_{SR} is worse than that of ϵ_{RD}, since the term $\mathsf{E}[\mathbf{F}(2,2)]$ is larger than the term $\mathsf{E}[\mathbf{F}(1,1)]$ in (42). Moreover, the performance difference between ϵ_{SR} and ϵ_{RD} becomes more obvious when the relay SNR Γ_R is poor. As the relay SNR Γ_R increases and is larger than 0 dB, the performance gap between them is almost zero. Fig. 5 and Fig. 6 show the true CRBs and the simulated MCRBs performances of ϵ_{SR} in the exponentially-decay fading channels with the different numbers of the effective paths, respectively, in order to capture the effect of the multipaths on the CFO estimation bound. We can find that the true CRBs and the MCRBs both decrease as the number of the effective paths increases. The performance improvement of the true CRBs is larger than that of the MCRBs as the number of the effective paths increases, since the channel parameters are known for the true CRB calculation. In addition, the performance improvement is gradually saturated as the number of the effective paths is larger than six. This is because for the case of the eight effective paths, the accumulated power of the first six effective paths occupies up to 88% of the total channel power.

VI. CONCLUSIONS

The CFO estimation bounds for an OFDM-based AF relay network in multipath fading channels have been theoretically analyzed. Due to the multipath effect, the CFOs of the S-R link and the R-D link for the AF relay network are needed to be estimated separately. Both the true CRBs and the MCRBs for the CFO estimation have been derived in closed forms. Computer simulations are used to investigate the impact of the multipath fading channels, the operating SNR and the number of the effective paths on the CFO estimation errors.

The MCRB of the CFO estimation is about 5 dB looser than the true CRB in terms of the SNR. The CFO estimation bound is mainly dominated by the relay SNR, and the true CRBs performance gets saturated when the destination SNR is larger than the relay SNR. The CFO estimation bound for the S-R link is higher than that for the R-D link, and the performance gap between the two links is enlarged as the relay SNR is much smaller than the destination SNR. The CRBs performance decreases when the number of the effective paths in the multipath fading channels increases, and the performance degradation gradually becomes saturated when the number of the effective paths is larger than six. The proposed true CRBs and MCRBs can provide an important insight into assessing any unbiased CFO estimation algorithms for an OFDM-based relay network in multipath fading channels.

APPENDIX A
PROOF OF LEMMA 1

Let us first define $x = \sum_{l=0}^{L-1} |h_l|^2$, which is a generalized chi-square distributed random variable and has the probability density function

$$f(x; L, \sigma_0^2, \ldots, \sigma_{L-1}^2) = \sum_{l=0}^{L-1} \rho_l e^{-\frac{x}{\sigma_l^2}} \quad \mathit{for}\ x \geq 0, \quad \text{(A.1)}$$

where $\rho_l = (\sigma_l^2 \prod_{i=0,i\neq l}^{L-1} (1 - \frac{\sigma_i^2}{\sigma_l^2}))^{-1}$. Thus, the probability density function of $u = \frac{1}{x+c}$ is given by

$$g(u) = \frac{1}{u^2} f\left(\frac{1}{u} - c\right) = \frac{1}{u^2} \sum_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} e^{-\frac{1}{u\sigma_l^2}}, \quad \text{(A.2)}$$

where u is varied between 0 and $\frac{1}{c}$. The mean of u can be derived as

$$\begin{aligned} \mathsf{E}[u] &= \int_0^{\frac{1}{c}} u g(u) du = \sum_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} \int_0^{\frac{1}{c}} \frac{1}{u} e^{-\frac{1}{u\sigma_l^2}} du \\ &= \sum_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} \int_{\frac{c}{\sigma_l^2}}^{\infty} e^{-\lambda} \left(\frac{1}{\lambda}\right) d\lambda \\ &= \sum_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} E_1\left(\frac{c}{\sigma_l^2}\right), \end{aligned} \quad \text{(A.3)}$$

where $E_1(a) = \int_a^\infty e^{-\lambda}\left(\frac{1}{\lambda}\right) d\lambda$ is the exponential integral function.

APPENDIX B
PROOF OF LEMMA 2

The cumulative density function (CDF) of the random variable $\frac{x}{x+y+c}$ can be computed as

$$\begin{aligned} \mathsf{Pr}\left[\frac{x}{x+y+c} < t\right] &= \mathsf{Pr}\left[y > x\left(\frac{1}{t} - 1\right) - c\right] \\ &= \mathsf{Pr}\left[x\left(\frac{1}{t} - 1\right) - c < 0\right] \\ &\quad + \mathsf{Pr}\left[y \geq x\left(\frac{1}{t}-1\right) - c \,\middle|\, x\left(\frac{1}{t}-1\right) - c \geq 0\right]. \end{aligned} \quad \text{(B.1)}$$

Note that the first term in (B.1) can be explicitly calculated as

$$\begin{aligned} \mathsf{Pr}\left[x\left(\frac{1}{t} - 1\right) - c < 0\right] &= \mathsf{Pr}\left[x < \frac{c}{\left(\frac{1}{t} - 1\right)}\right] \\ &= \int_0^{\frac{c}{\left(\frac{1}{t}-1\right)}} \frac{1}{\sigma_0^2} e^{-\frac{x}{\sigma_0^2}} dx \\ &= 1 - e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}}, \end{aligned} \quad \text{(B.2)}$$

and the second term in (B.1) can be derived as

$$\begin{aligned} &\mathsf{Pr}\left[y \geq x\left(\frac{1}{t} - 1\right) - c \,\middle|\, x\left(\frac{1}{t} - 1\right) - c \geq 0\right] \\ &= \int_{\frac{c}{\left(\frac{1}{t}-1\right)}}^{\infty} \frac{1}{\sigma_0^2} e^{-\frac{x}{\sigma_0^2}} \int_{x\left(\frac{1}{t}-1\right)-c}^{\infty} \sum_{l=1}^{L-1} \omega_l e^{-\frac{y}{\sigma_l^2}} dy dx \\ &= \sum_{l=1}^{L-1} \omega_l \frac{1}{\sigma_0^2} \int_{\frac{c}{(\frac{1}{t}-1)}}^{\infty} e^{-\frac{x}{\sigma_0^2}} \int_{x(\frac{1}{t}-1)-c}^{\infty} e^{-\frac{y}{\sigma_l^2}} dy dx \\ &= \sum_{l=1}^{L-1} \omega_l \frac{1}{\sigma_0^2} \int_{\frac{c}{(\frac{1}{t}-1)}}^{\infty} e^{-\frac{x}{\sigma_0^2}} \sigma_l^2 e^{-\frac{(x(\frac{1}{t}-1)-c)}{\sigma_l^2}} dx \\ &= \sum_{l=1}^{L-1} \omega_l \frac{1}{\sigma_0^2} \sigma_l^2 \frac{1}{\frac{1}{\sigma_0^2} + \frac{\frac{1}{t}-1}{\sigma_l^2}} e^{-\left(\frac{c}{\sigma_0^2(\frac{1}{t}-1)}\right)}. \end{aligned} \quad \text{(B.3)}$$

From (B.1)-(B.3), we can obtain the CDF of the random variable $\frac{x}{x+y+c}$:

$$\begin{aligned} &\mathsf{Pr}\left[\frac{x}{x+y+c} < t\right] \\ &= 1 - e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}} \\ &\quad + \sum_{l=1}^{L-1} \omega_l \frac{\sigma_l^2}{\sigma_0^2} \cdot \frac{1}{\frac{1}{\sigma_0^2} + \frac{\left(\frac{1}{t}-1\right)}{\sigma_l^2}} e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}}. \end{aligned} \quad \text{(B.4)}$$

By using [22] and (B.4), we can derive the mean of $\frac{x}{x+y+c}$ by

$$\begin{aligned} &\mathsf{E}\left[\frac{x}{x+y+c}\right] \\ &= \int_0^{\infty} \left[1 - \mathsf{Pr}\left[\frac{x}{x+y+c} < t\right]\right] dt \\ &= \int_0^1 \left(e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}} \cdot \left(1 - \sum_{l=1}^{L-1} \omega_l \frac{\sigma_l^2}{\sigma_0^2} \cdot \frac{1}{\frac{1}{\sigma_0^2} + \frac{\left(\frac{1}{t}-1\right)}{\sigma_l^2}}\right)\right) dt \\ &= \int_0^1 \left(e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}}\right) dt - \sum_{l=1}^{L-1} \omega_l \frac{\sigma_l^2}{\sigma_0^2} \end{aligned}$$

$$\cdot \int_0^1 \left(e^{-\frac{c}{\sigma_0^2\left(\frac{1}{t}-1\right)}} \cdot \frac{1}{\frac{1}{\sigma_0^2} + \frac{\left(\frac{1}{t}-1\right)}{\sigma_l^2}} \right) dt. \tag{B.5}$$

By letting $u = \frac{1}{\sigma_0^2(\frac{1}{t}-1)}$, the mean of $\frac{x}{x+y+c}$ in (B.5) can be rewritten by

$$\begin{aligned}
\mathsf{E}\left[\frac{x}{x+y+c}\right]
&= \int_0^\infty \left(e^{-cu}\right) \frac{\sigma_0^2}{\left(1+\sigma_0^2 u\right)^2} du \\
&\quad + \sum_{l=1}^{L-1} \omega_l \frac{\sigma_l^2}{\sigma_0^2} \cdot \int_0^\infty \left(e^{-cu} \cdot \frac{\sigma_l^2\sigma_0^2}{\sigma_l^2 + \frac{1}{u}} \right) \frac{\sigma_0^2}{\left(1+\sigma_0^2 u\right)^2} du \\
&= \int_0^\infty e^{-cu} \cdot \frac{\sigma_0^2}{(1+\sigma_0^2 u)^2} du - \sum\nolimits_{l=1}^{L-1} \omega_l \\
&\quad \cdot \int_0^\infty (e^{-cu} \cdot \frac{\sigma_l^4 \sigma_0^2 u}{\sigma_l^2 u + 1}) \frac{1}{(1+\sigma_0^2 u)^2} du.
\end{aligned} \tag{B.6}$$

Since $\left(\frac{u}{\sigma_l^2 u+1}\right) \frac{1}{\left(1+\sigma_0^2 u\right)^2} = \frac{k_1}{\sigma_l^2 u+1} + \frac{k_2}{\sigma_0^2 u+1} + \frac{k_3}{\left(1+\sigma_0^2 u\right)^2}$, where $k_1 = \frac{-1}{\sigma_l^2\left(1-\frac{\sigma_0^2}{\sigma_l^2}\right)^2}$, $k_2 = \frac{\sigma_0^2}{\sigma_l^4\left(1-\frac{\sigma_0^2}{\sigma_l^2}\right)^2}$, $k_3 = \frac{1}{\sigma_l^2\left(1-\frac{\sigma_0^2}{\sigma_l^2}\right)}$, it then gives

$$\begin{aligned}
\mathsf{E}\left[\frac{x}{x+y+c}\right] &= \int_0^\infty e^{-cu} \cdot \frac{\sigma_0^2}{\left(1+\sigma_0^2 u\right)^2} du \\
&\quad - \sum\nolimits_{l=1}^{L-1} \omega_l \Bigg[\int_0^\infty e^{-cu} \sigma_l^4 \sigma_0^2 \cdot \frac{k_1}{\sigma_l^2 u+1} du \\
&\quad + \int_0^\infty e^{-cu} \sigma_l^4 \sigma_0^2 \cdot \frac{k_2}{\sigma_0^2 u + 1} du \\
&\quad + \int_0^\infty e^{-cu} \sigma_l^4 \sigma_0^2 \cdot \frac{k_3}{(1+\sigma_0^2 u)^2} du \Bigg].
\end{aligned} \tag{B.7}$$

By mathematical integration, we can obtain the integration results:

$$\int_0^\infty e^{-cu} \frac{\sigma_0^2}{\left(1+\sigma_0^2 u\right)^2} du = e^{\frac{c}{\sigma_0^2}} \left[e^{-\frac{c}{\sigma_0^2}} - \frac{c}{\sigma_0^2} E_1\left(\frac{c}{\sigma_0^2}\right) \right]. \tag{B.8}$$

$$\int_0^\infty e^{-cu} \frac{k_1}{\sigma_l^2 u + 1} du = \frac{k_1}{\sigma_l^2} e^{\frac{c}{\sigma_l^2}} E_1\left(\frac{c}{\sigma_l^2}\right). \tag{B.9}$$

$$\int_0^\infty e^{-cu} \frac{k_2}{\sigma_0^2 u + 1} du = \frac{k_2}{\sigma_0^2} e^{\frac{c}{\sigma_0^2}} E_1\left(\frac{c}{\sigma_0^2}\right). \tag{B.10}$$

$$\int_0^\infty e^{-cu} \frac{k_3}{\left(1+\sigma_0^2 u\right)^2} du = \frac{k_3}{\sigma_0^2} e^{\frac{c}{\sigma_0^2}} \left[e^{-\frac{c}{\sigma_0^2}} - \frac{c}{\sigma_0^2} E_1\left(\frac{c}{\sigma_0^2}\right) \right]. \tag{B.11}$$

From (B.7)-(B.11), we can finally derive the mean by

$$\begin{aligned}
\mathsf{E}\left[\frac{x}{x+y+c}\right]
&= e^{\frac{c}{\sigma_0^2}} \cdot \left[e^{-\frac{c}{\sigma_0^2}} - \frac{c}{\sigma_0^2} E_1\left(\frac{c}{\sigma_0^2}\right) \right] \\
&\quad - \sum\nolimits_{l=1}^{L-1} \omega_l \sigma_l^4 \sigma_0^2 \cdot \Bigg[k_1 \cdot \frac{e^{\frac{c}{\sigma_l^2}}}{\sigma_l^2} E_1\left(\frac{c}{\sigma_l^2}\right) \\
&\quad + k_2 \cdot \frac{e^{\frac{c}{\sigma_0^2}}}{\sigma_0^2} E_1\left(\frac{c}{\sigma_0^2}\right) + k_3 \cdot \frac{e^{\frac{c}{\sigma_0^2}}}{\sigma_0^2} \cdot \left[e^{-\frac{c}{\sigma_0^2}} - \frac{c}{\sigma_0^2} E_1\left(\frac{c}{\sigma_0^2}\right) \right] \Bigg] \\
&= \left(1 - \frac{c}{\sigma_0^2} \cdot e^{\frac{c}{\sigma_0^2}} E_1\left(\frac{c}{\sigma_0^2}\right)\right) \left(1 - \sum\nolimits_{l=1}^{L-1} \omega_l \frac{\sigma_l^2}{\left(1 - \frac{\sigma_0^2}{\sigma_l^2}\right)} \right) \\
&\quad - \sum\nolimits_{l=1}^{L-1} \omega_l \Bigg[\frac{\sigma_0^2}{\left(1 - \frac{\sigma_0^2}{\sigma_l^2}\right)^2} \\
&\quad \cdot \left(e^{\frac{c}{\sigma_0^2}} E_1\left(\frac{c}{\sigma_0^2}\right) - e^{\frac{c}{\sigma_l^2}} E_1\left(\frac{c}{\sigma_l^2}\right) \right) \Bigg].
\end{aligned} \tag{B.12}$$

APPENDIX C
PROOF OF LEMMA 3

From (A.1), we can represent the probability density function of $u = \left(\left(\sum_{l=0}^{L-1} |h_l|^2 + c\right)^2\right)^{-1}$ by

$$\begin{aligned}
g(u) &= \frac{\sqrt{u}}{2u^2} f\left(\frac{1}{\sqrt{u}} - c\right) \\
&= \frac{\sqrt{u}}{2u^2} \sum\nolimits_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} e^{-\frac{1}{\sigma_l^2\sqrt{u}}}.
\end{aligned} \tag{C.1}$$

Then the mean of u can be derived by

$$\begin{aligned}
\mathsf{E}[u] &= \int_0^{\frac{1}{c^2}} u g(u)\, du \\
&= \sum\nolimits_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} \left(\int_0^{\frac{1}{c^2}} \frac{\sqrt{u}}{2u} e^{\frac{-1}{\sigma_l^2\sqrt{u}}} du \right).
\end{aligned} \tag{C.2}$$

By the change of variables $\frac{1}{\sqrt{u}} = t$, the integral in (C.2) can be evaluated by

$$\begin{aligned}
&\int_0^{\frac{1}{c^2}} \frac{\sqrt{u}}{2u} e^{\frac{-1}{\sigma_l^2\sqrt{u}}} du \\
&= \int_\infty^c (-2)\, t^{-3} \frac{1}{2t} t^2 e^{\frac{-t}{\sigma_l^2}} dt \\
&= \int_c^\infty \frac{1}{t^2} e^{\frac{-t}{\sigma_l^2}} dt = \frac{e^{\frac{-c}{\sigma_l^2}}}{c} - \frac{1}{\sigma_l^2} \cdot E_1\left(\frac{c}{\sigma_l^2}\right).
\end{aligned} \tag{C.3}$$

As a result, we can represent the mean of u by

$$\mathsf{E}[u] = \sum\nolimits_{l=0}^{L-1} \rho_l e^{\frac{c}{\sigma_l^2}} \left(\frac{e^{\frac{-c}{\sigma_l^2}}}{c} - \frac{1}{\sigma_l^2} \cdot E_1\left(\frac{c}{\sigma_l^2}\right) \right). \tag{C.4}$$

APPENDIX D
DERIVATION OF $\mathrm{tr}\left[\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]$

From (9) and $\mathbf{D} = \mathrm{diag}([0, 1, \ldots, N-1]^T)$, we can directly compute $\mathrm{tr}\left[\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]$ as

$$\begin{aligned}\mathrm{tr}&\left[\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]\\ &= \sum_{m=1}^{N-1} m^2\cdot|h_{RD}(0)|^2\\ &\quad+\sum_{m=1}^{N-2} m\,(m+1)\cdot|h_{RD}(1)|^2\\ &\quad+\sum_{l=2}^{L-1}\sum_{m=1}^{N-l-1} m\,(m+1)\cdot|h_{RD}\,(l)|^2\\ &\quad+\sum_{l=2}^{L-1}\sum_{m=1}^{l-1} m\,(N-l+m)\cdot|h_{RD}\,(l)|^2. \end{aligned}\tag{D.1}$$

Since the first three terms in (D.1) can be combined by $\sum_{l=0}^{L-1}\sum_{m=1}^{N-l-1} m\,(m+1)\cdot|h_{RD}\,(l)|^2$, and through some mathematical manipulations, we have

$$\begin{aligned}\mathrm{tr}&\left[\mathbf{D}^H\mathbf{H}_{RD}^H\mathbf{D}\mathbf{H}_{RD}\right]\\ &=\sum_{l=0}^{L-1}\frac{1}{6}(N-l)(N-l-1)(2N-2l-1)|h_{RD}(l)|^2\\ &\quad+\sum_{l=0}^{L-1}\frac{1}{6}l(l-1)(3N-l-1)|h_{RD}(l)|^2\\ &=\sum_{l=0}^{L-1}\left[\frac{1}{6}(2N^3-3N^2+N+3Nl^2-3N^2l)\right]|h_{RD}(l)|^2. \end{aligned}\tag{D.2}$$

APPENDIX E
DERIVATION OF $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$

From (18), we can directly calculate $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$ as

$$\begin{aligned}\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right] &= \sum_{k=1}^{N-2}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2\right.\\ &\quad\left.+\sum_{m=1}^{N-k-1} m^2|r_{RD}(m)|^2\right]\\ &\quad+\sum_{m=1}^{N-1} 2m^2|r_{RD}(m)|^2. \end{aligned}\tag{E.1}$$

Then, we divide the first term of $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$ into three terms θ_1, θ_2 and θ_3, which are

$$\theta_1 = \sum_{k=1}^{L-2}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2 +\sum_{m=1}^{N-k-1} m^2|r_{RD}(m)|^2\right];\tag{E.2}$$

$$\theta_2 = \sum_{k=L-1}^{N-L}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2 +\sum_{m=1}^{N-k-1} m^2|r_{RD}(m)|^2\right];\tag{E.3}$$

$$\theta_3 = \sum_{k=N-L+1}^{N-2}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2 +\sum_{m=1}^{N-k-1} m^2|r_{RD}(m)|^2\right].\tag{E.4}$$

Since $r_{RD}(m) = 0$ for $m = L, L+1, \ldots, (N-L)$ due to the L-path channel and $|r_{RD}(m)| = |r_{RD}\,(N-m)\,|$, we can rewrite θ_1 by

$$\begin{aligned}\theta_1 &= \sum_{k=1}^{L-2}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2\right.\\ &\quad+\sum_{m=k+1}^{L-1}(N-m)^2\,|r_{RD}(N-m)|^2\\ &\quad\left.+\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2\right]\\ &=\sum_{k=1}^{L-2}\left[\sum_{m=1}^{k} m^2|r_{RD}(m)|^2\right]\\ &\quad+\sum_{k=1}^{L-2}\left[\sum_{m=k+1}^{L-1}(N-m)^2\,|r_{RD}(m)|^2\right]\\ &\quad+(L-2)\cdot\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2\\ &=\sum_{m=1}^{L-1}(L-m-1)m^2|r_{RD}(m)|^2\\ &\quad+\sum_{m=1}^{L-1}(m-1)\,(N-m)^2\,|r_{RD}(m)|^2\\ &\quad+(L-2)\cdot\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2\\ &=\sum_{m=1}^{L-1}\Big[(2L-m-3)\,m^2\\ &\quad+(m-1)\,(N-m)^2\Big]|r_{RD}(m)|^2. \end{aligned}\tag{E.5}$$

From (E.3), similarly, θ_2 can be further calculated as

$$\begin{aligned}\theta_2 &= \sum_{k=L-1}^{N-L}\left[\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2 +\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2\right]\\ &=2\,(N-2L+2)\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2. \end{aligned}\tag{E.6}$$

For the derivation of θ_3, the first term in (E.4) can be rewritten as

$$\begin{aligned}&\sum_{k=N-L+1}^{N-2}\sum_{m=1}^{k} m^2|r_{RD}(m)|^2\\ &\quad=\sum_{k=N-L+1}^{N-2}\left[\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2 +\sum_{m=N-L+1}^{k} m^2|r_{RD}(m)|^2\right]\\ &\quad=(L-2)\cdot\sum_{m=1}^{L-1} m^2|r_{RD}(m)|^2\\ &\qquad+\sum_{m=1}^{L-1}(m-1)(N-m)^2|r_{RD}(m)|^2. \end{aligned}\tag{E.7}$$

On the other hand, the second term in (E.4) can be rewritten by

$$\begin{aligned}&\sum_{k=N-L+1}^{N-2}\sum_{m=1}^{N-k-1} m^2|r_{RD}(m)|^2\\ &\quad=\sum_{m=1}^{L-1}(L-m-1)\,m^2|r_{RD}(N-m)|^2. \end{aligned}\tag{E.8}$$

From (E.4), (E.7) and (E.8), we can obtain θ_3 as

$$\begin{aligned}\theta_3 &= \sum_{m=1}^{L-1}\Big[(2L-m-3)\,m^2\\ &\quad+(m-1)\,(N-m)^2\Big]|r_{RD}(m)|^2. \end{aligned}\tag{E.9}$$

Furthermore, the second term of $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$ can be rewritten by

$$\sum_{m=1}^{N-1} 2m^2|r_{RD}(m)|^2$$

$$= \sum_{m=1}^{L-1} \left(2m^2 + 2(N-m)^2\right) |r_{RD}(m)|^2. \quad \text{(E.10)}$$

From (E.5), (E.6), (E.9) and (E.10), we can represent $\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right]$ by

$$\begin{aligned}\mathrm{tr}\left[\widetilde{\mathbf{Q}}_{RD}\widetilde{\mathbf{Q}}_{RD}\right] &= \theta_1 + \theta_2 + \theta_3 \\ &\quad + \sum_{m=1}^{L-1} \left(2m^2 + 2(N-m)^2\right) |r_{RD}(m)|^2 \\ &= \sum_{m=1}^{L-1} 2Nm(N-m) |r_{RD}(m)|^2. \quad \text{(E.11)}\end{aligned}$$

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CHUN-CHYUAN CHEN received the Ph.D. degree in communication engineering from National Chiao Tung University (NCTU), in 2001. From 2001 to 2002, he was an Engineer with Computer and Communications Research Laboratories, Industrial Technology Research Institute, Chu-Tung, Taiwan, where he was involved in mobile communications. Since 2002, he has been with the Department of Electrical Engineering, National Dong Hwa University, Hualien, Taiwan, as an Assistant Professor. His research interests include the design of baseband receivers in communication systems and orthogonal frequency division multiplexing (OFDM) systems, especially for mobile communications.

MENG-LIN KU (M'11) received the B.S., M.S., and Ph.D. degrees in communication engineering from National Chiao Tung University, Hsinchu, Taiwan, in 2002, 2003, and 2009, respectively. From 2009 to 2010, he was a Postdoctoral Research Fellow with Prof. L.-C. Wang in the Department of Electrical and Computer Engineering, National Chiao Tung University, and Prof. V. Tarokh in the School of Engineering and Applied Sciences, Harvard University. In 2010, he became a Faculty Member of the Department of Communication Engineering, National Central University, Jungli, Taiwan, where he is currently an Associate Professor. In 2013, he was a Visiting Scholar with Prof. K. J. R. Liu in the Signals and Information Group, University of Maryland. His current research interests include the areas of green communications, cognitive radios, and optimization and learning for radio access. He was a recipient of the Exploration Research Award from the Pan Wen Yuan Foundation, Taiwan, in 2013. He was a recipient of the Best Counseling Award, in 2012, the university-level Best Teaching Award, in 2014, 2015, and 2016, and the Research Excellence Award, in 2018 and 2019, all from National Central University.