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Selective Maintenance Optimization Modelling for Multi-State Deterioration Systems Considering Imperfect Maintenance

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ABSTRACT Motivated by the need to reduce the maintenance cost and improve the system reliability, the issue of selective maintenance has attracted much attention in the industrial and military fields. The existing research mainly focused on binary-state systems or the perfect maintenance strategy. In this paper, we presented a novel selective maintenance model for multi-state deteriorating systems with multi-state components considering imperfect maintenance strategy. The proposed model aims to minimize the total maintenance costs and takes the relationship between the maintenance cost, the system service life and the maintenance quality into account. A case study where the proposed model was applied to an aircraft gas turbine engine system was conducted. The sensitivity analyses for the minimum requirement of the health level, the aging factor, and the number of health states were conducted, respectively. Moreover, a comparative analysis between perfect and imperfect maintenance strategies was conducted, and the results demonstrated that the imperfect maintenance strategy was more cost-effective than perfect maintenance strategy. The findings of this study can guide the maintenance decision-making process for actual systems.

INDEX TERMS Imperfect maintenance, multi-state systems, selective maintenance, system health state.

I. INTRODUCTION

Many industrial and military systems, such as maritime vessels, aircraft and nuclear power plant, are often required to execute a series of missions with a finite break between two adjacent missions [1]. These systems would be unavoidably subject to deterioration with use, which causes partial dysfunction. In most cases, the states of these systems are between ''brand new'' and ''completely failed'', thus they are known as multi-state systems [2]. To ensure the next mission is successfully executed, the maintenance for the multi-state system is indispensable. However, due to the limited duration of the scheduled intermissions break, and budget and maintenance resources constraints, it is often impossible to schedule all the desirable maintenance activities in a break. It is therefore necessary to identify an optimal subset of maintenance activities among all the maintenance options, and this kind of maintenance strategy is defined as selective maintenance [3].

About two decades ago, Rice et al. (1998) [4] firstly developed a mathematic programming model to maximize the reliability of a binary-state series-parallel system consisting of independent and identically distributed ($i.i.d$) components with exponentially distributed lifetimes. Afterwards, selective maintenance problems were extensively studied from various perspectives. For instance, Cassady et al. (2001) [5] considered the case where components' lifetime was subject to the Weibull distribution, and three optional maintenance activities, namely, minimal repair, corrective replacement, and preventive replacements, were available to

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be chosen. To improve efficiency of the optimization process, Rajagopalan and Cassady (2006) [6] proposed four improved enumerative procedures to reduce the computational time. Lust et al. (2009) [7] focused on the selective maintenance problem for systems with a large number of components, and proposed an exact method based on branch-and-bound procedure and a heuristic method based on the Tabu search algorithm. Maillart et al. (2009) [8] studied the selective maintenance problem and considered corrective replacements in a break. Maaroufi et al. (2013) [9] explored the optimal selective maintenance strategy for systems which subject to propagated failures with both global effect and failure isolation phenomena, and proposed a set of rules to reduce the search space. The previous literature mainly focused on the binary-state systems.

Nowadays the multi-state systems have more concern in research field. Liu and Huang (2010) [10] presented a selective maintenance model for multi-state systems composed of binary-state components, and considered the imperfect maintenance that may restore the condition of the system to a state between "brand new" and "completely failed". Pandey et al. (2013) [11] proposed a more generalized model for multi-state systems consisting of multi-state components. More recently, some scholars aimed to address economic dependence [12], stochastic dependence [13], and structural dependence [14] among components in a multi-state system. Some others put emphasis on the dynamic and stochastic conditions in multi-state systems [15], [16]. Furthermore, a few studies have focused on more complex systems, e.g. nonsmooth nonlinear systems [17]–[19], continuous-time fuzzy systems [20], [21], networked nonlinear DC motor systems [22], and uncertain switched stochastic nonlinear systems [23].

As regards the maintenance strategy, a system is usually assumed to be restored to be as good as new one after maintenance. In practice, this assumption is unrealistic. It is more reasonable to assume that a system can be restored to a condition somewhere between two extreme states after maintenance, and this maintenance is viewed as imperfect maintenance [24]. Many scholars made the effort to model the imperfect maintenance in a variety of forms, and the recognized efforts among them include (p, q) model [25], $(p(t), q(t))$ model [26], $(p(n, t), q(n, t), s(n, t))$ model [27], Kijima Type I model [28], Type II model [29], the improvement factor model [30], and the hybrid imperfect model [31]. The above imperfect maintenance models have been applied in various aspects [32], [33]. However, the literature on imperfect maintenance mainly deal with binary-state systems. There is rare research that applied the above models to the multi-state systems.

In this paper, we construct a novel selective maintenance model for multi-state deteriorating systems with multi-state components considering imperfect maintenance strategy. The states of the system and its components include "brand new", "completely failed" and other intermediate states. The available maintenance options include "doing nothing", "minor repair", "major repair" and "replacement". The proposed model aims to minimize the total maintenance costs and takes the relationship between the maintenance cost, the system service life and the improvement of maintenance quality into account. We conducted a case study where the proposed model was applied to an aircraft gas turbine engine system. In contrast to the perfect maintenance strategy, the imperfect maintenance strategy obtained from the proposed model showed significant economic advantages.

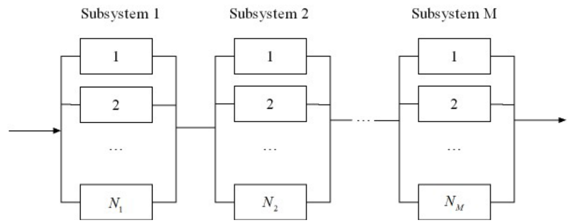

FIGURE 1. Series-parallel system.

Overall, the contributions of this paper can be summarized as follows: First, we constructed the relationship between the maintenance cost, system service life and the improvement of maintenance quality. Second, we proposed a novel selective maintenance model for multi-state systems considering imperfect maintenance strategy. Third, we applied the proposed model to an aircraft gas turbine engine system and obtained some insights to guide the maintenance decision-making process for actual systems.

The remainder of the paper is organized as follows. Section II provides a brief introduction to the multi-state system considered in this paper and states the problem to be addressed. The selective maintenance model is constructed in Section III. Illustrate examples, numerical results and sensitivity analyses are discussed in Section IV. Section V draws the conclusions and recommends the future study.

II. SYSTEM DESCRIPTION

The system in this study consists of M independent subsystems connected in series. There are N_i identical components connected in parallel in subsystem i, $i \in \{1, 2, \cdots, M\}$ (see Fig. 1). For convenience, the components are represented by $1, 2, \cdots, K$.

The system has to execute consecutively a sequence of missions with a break interval between two adjacent missions. As the operating time increases, some functions of the system would be degraded due to wear and tear. Thus, the system may be in different health states, e.g. fully functional, partially functional, completely failed, etc. Therefore, the system belongs to a class of multi-state systems. Corresponding to the health state, a health level in the range of 0 to 100% is used to present the functional status of the system in this study. When the health level was 100%, it indicates that the system is brand new and can fully function; when the health level is 0, it indicates that the system is completely failed. Similarly, a component in the system also has its health state and the corresponding health level.

To ensure that the next mission can be completed successfully, some maintenance activities must be carried out to satisfy the minimum requirement of the health level of the system. The maintenance activities can only be scheduled in the break interval. The maintenance resources (e.g. budget, time, workers, etc.) which can be utilized are limited in the maintenance activities. The system was modeled and analyzed on the base of following assumptions.

Assumption 1: The components in the same subsystem are independent identically distributed.

Assumption 2: There are more than two health states for each component, including but not limited to ''brand new'', ''partially failed'' and ''completely failed''.

Assumption 3: The missions executed by the multi-state system are homogeneous. The missions cannot be interrupted while they are in progress. A minimum requirement of the health level for the multi-state system must be satisfied before the next mission.

Assumption 4: There is only one worker in the maintenance activity. For large projects which need more than one worker, the mission can be divided into small tasks where each one needs only one worker.

Assumption 5: There are multiple maintenance options, including but not limited to ''replacement'', ''major repair'', ''minor repair'' and ''doing nothing''. Note that ''doing nothing'' is deemed to be a kind of ''maintenance'' activity in this study, which doesn't consume any maintenance resource and cannot improve the system's health state.

Based on the above assumptions, the decision-makers have to decide how to allocate the available maintenance resources to each component in the multi-state system so that the whole multi-state system can be restored to ensure that the subsequent mission can be completed successfully.

III. RESEARCH METHODOLOGY

A. NOTATION

The symbols used in this study and their explanations are given as follows.

1) INDICES

- k The indicator of components in the multi-state system, $k \in \{1, 2, \cdots, K\}$;
- s The health state indicator of components, $s \in \{1, 2, \cdots, S\}$;
- i The initial health state indicator of components before maintenance, $i \in \{1, 2, \cdots, S\}$;
- j The health state indicator of components after maintenance, $j \in \{1, 2, \cdots, S\}, j \geq i$.

2) PARAMETERS

- K The total number of components in a multi-state system;
- S The total number of health states for each component;
- h_s^k The health level of component k in the state $s, k \in \{1, 2, \cdots, K\}, s \in \{1, 2, \cdots, S\}$;
- Δh_{ij}^k The increment in the health level of component k from the health state i to j, $k \in \{1, 2, \cdots, K\}, i \in \{1, 2, \cdots, S\}, j \in \{1, 2, \cdots, S\}, j \geq i$;
- h_0^k The initial health level of component k before maintenance, $k \in \{1, 2, \cdots, K\}$;
- c_0^k The fixed maintenance cost of component $k, k \in \{1, 2, \cdots, K\}$;
- c_r^k The replacement cost for component $k, k \in \{1, 2, \cdots, K\}$;
- c_{ij}^k The maintenance cost for component k from the health state i to j, $k \in \{1, 2, \cdots, K\}, i \in \{1, 2, \cdots, S\}, j \in \{1, 2, \cdots, S\}, j \geq i$;
- t_{ij}^k The maintenance time for component k from the health state i to j, $k \in \{1, 2, \cdots, K\}, i \in \{1, 2, \cdots, S\}, j \in \{1, 2, \cdots, S\}, j \geq i$;
- T_0 The available maintenance time in the break between missions;
- ρ^k The aging factor of component $k, k \in \{1, 2, \cdots, K\}$;
- ω^k The weight coefficient of component k in the multi-state system;
- h^* The minimum requirement of health level for the multi-state system to successfully complete the next mission.

3) VARIABLES

$$x_{ij}^k = \begin{cases} 1, & \text{if the health state of component } k \text{ increaces} \\ & \text{from } i \text{ to } j \text{ after maintenance,} \\ & k \in \{1, 2, \cdots, K\}, i, j \in \{1, 2, \cdots, S\}, \\ & j \geq i; \\ 0, & \text{Otherwise.} \end{cases}$$

- f_i^k The health level of component k with the initial health state i after maintenance, $k \in \{1, 2, \cdots, K\}, i \in \{1, 2, \cdots, S\}$.

B. HEALTH STATE IMPROVEMENT FACTORS

As mentioned in the literature [34], the maintenance cost and system life are the two major factors to affect the improvement in the health state of the multi-state system. On one hand, if the maintenance cost spent approaches zero, the health state of the multi-state system will hardly be improved. On the other hand, if the maintenance cost for one component approaches the replacement cost, the health state of the component will be improved to be as good as a new one. In addition, the system life is another important factor which affects the improvement in the health state caused by maintenance activities. When the system is ''young'', a little maintenance resource can make a great improvement in the health state. By contrast, as the system is aged, more maintenance resources have to be consumed to obtain the same performance improvement. The relationship among the maintenance cost, the system aging factor and the maintenance

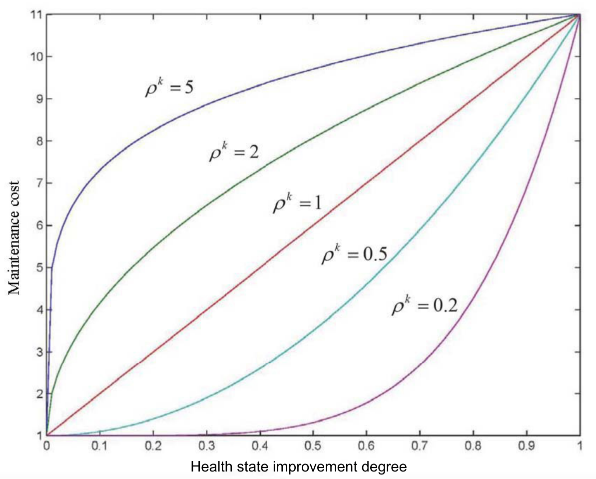

FIGURE 2. The relationships between the maintenance cost and the health state improvement degree for different aging factors.

quality can be expressed by (1).

$$c_{ij}^k = \begin{cases} c_r^k + c_0^k, & i < j = S \\ (\frac{j-i}{S-1})^{\frac{1}{\rho^k}} c_r^k + c_0^k, & i < j < S \\ 0, & i = j \end{cases} \tag{1}$$

where $\rho^k > 0$ is a characteristic constant associated with the service life of component k. It is related to the intrinsic characteristics of the component and its usage life, which can be estimated by historical data, including maintenance cost, usage life, and reliability or failure data of components.

In (1), when $i < j = S$, it indicates that the initial health state of component k is i, and the health state after maintenance is S, which means that the component k with the initial health state i is replaced with a new spare component. Thus, the maintenance cost is the sum of the replacement cost and the fixed maintenance cost. When $i < j < S$, it indicates that the initial health state of component k is i, and the health state after maintenance is $j < S$, which does not reach the maximum state S, so the maintenance cost consists of a variable cost related to the increment in the health state, and a fixed maintenance cost. When $i = j$, the health states of component k before and after maintenance are unchanged, which indicates that no any maintenance activity incurred, so the maintenance cost is zero.

Furthermore, we denote that $\theta = \frac{j-i}{S-1}$ in (1), which means the improvement degree in the health state of component k. We assume that the fixed maintenance cost is 1 unit and the replacement cost is 10 units. For different aging factors ρ^k, Fig. 2 depicts the relationships between the maintenance cost and the maintenance quality.

Fig. 2 illustrates that at the same ρ^k level, the more maintenance cost is spent, the higher maintenance quality can be improved. With the increase of ρ^k, the improvement in the maintenance quality needs to consume more maintenance cost. The smaller value of ρ^k corresponds to the ''younger'' system. For the ''younger'' system, the majority of components are new, so repairing this kind of systems is more economical and effective. For aged systems, the majority of components are very old, and even some components cannot work, so the corresponding ρ^k is larger than ''younger'' system. Thus, both the intrinsic characteristics of the component and its service life are important factors that determine the parameter ρ^k.

C. SELECTIVE MAINTENANCE MODEL

The selective maintenance problem for multi-state systems can be divided into two aspects: the first one is to identify the components to be repaired, and the second one is to determine that how much the maintenance resources should be allocated for each component. Both aspects can be expressed simultaneously using a binary decision variable x_{ij}^k. To ensure that the multi-state system can successfully complete the subsequent mission, the health state of the system must meet the minimum requirement. Therefore, a mixed integer programming model is established with the objective of minimizing the maintenance cost, as shown in Equations eq(2)–eq(8).

$$\min \ C = \sum_{k=1}^{K} \sum_{i=1}^{S} \sum_{j=i}^{S} c_{ij}^k x_{ij}^k \tag{2}$$

$$s.t. \ \sum_{k=1}^{K} \sum_{i=1}^{S} \omega^k f_i^k \geq h^*, \tag{3}$$

$$f_i^k = h_0^k + \sum_{j=i}^{S} \Delta h_{ij}^k x_{ij}^k, \quad \forall i \in \{1, 2, \cdots, S\}, \tag{4}$$

$$\sum_{j=i}^{S} x_{ij}^k = 1, \forall k \in \{1, 2, \cdots, K\}, \quad i \in \{1, 2, \cdots, S\}, \tag{5}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{S} \sum_{j=i}^{S} t_{ij}^k x_{ij}^k \leq T_0, \tag{6}$$

$$x_{ij}^k \in \{0, 1\}, \quad \forall k \in \{1, 2, \cdots, K\}, \quad i, j \in \{1, 2, \cdots, S\}, \ j \geq i, \tag{7}$$

$$f_i^k \geq 0, \forall k \in \{1, 2, \cdots, K\}, \quad i \in \{1, 2, \cdots, S\}. \tag{8}$$

In the above model, the objective (2) minimizes the overall maintenance costs for all components. Here c_{ij}^k is defined by (1). Constraint (3) limits that the health level of the multi-state system should be no less than the minimum requirement of the health level to successfully complete the subsequent mission. Constraints (4) construct the relationship of the health indices before and after maintenance. Constraints (5) indicate that one and only one ''maintenance activity'' should be scheduled for component k with the initial health state i. Note that the ''maintenance activity'' is a generalized term, including ''doing nothing'' when $j = i$, and ''minor repair'', ''overhaul'', ''replacement'' and so on when $j > i$. Constraint (6) limits the total maintenance time for all components to the

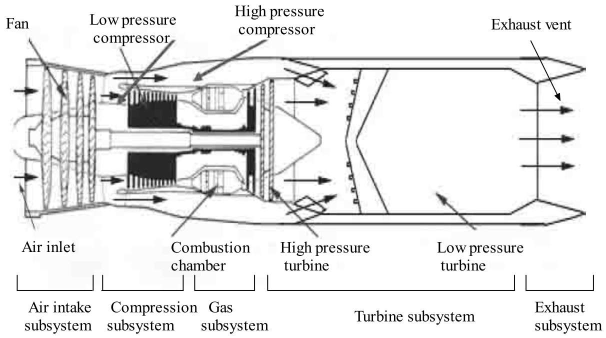

FIGURE 3. System structure of an aircraft gas turbine engine.

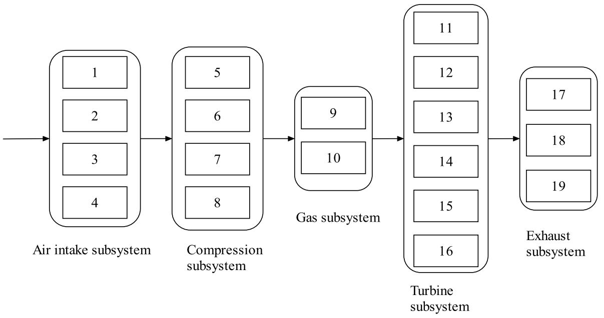

FIGURE 4. The components numbering chart of the aircraft engine.

available time in the break between missions. Constraints (7) and (2) define the decision variables.

IV. RESULTS AND DISCUSSIONS

In this section, the proposed selective maintenance model is applied to an aircraft gas turbine engine system. The basic structure of the engine is shown in Figure 3. Since the aero-engine system and its components have multiple health states, the engine is a typical multi-state system.

A. DATA PREPARATION

The aircraft engine consists of five major functional subsystems, i.e. the air intake subsystem, the compression subsystem, the gas subsystem, the turbine subsystem and the exhaust subsystem, as shown in Fig. 3. Each subsystem in the aircraft engine contains a different number of components. Among them, the intake subsystem consists of four fans; the compression subsystem consists of two low-pressure compressors and two high-pressure compressors; the gas subsystem contains two combustion chambers; and the turbine subsystem consists of two high-pressure rotors. The high pressure turbine and the low pressure turbine consist of four low pressure rotors; the exhaust subsystem consists of a core nozzle and two branch tail nozzles. To facilitate identification, the above components are numbered separately, as shown in Fig. 4.

We assume that each component in the engine has 7 health states, and the corresponding health indices related to each health state are given in Table 1.

TABLE 1. The health state of components and the corresponding health indices.

Health state s	1	2	3	4	5	6	7
Health level h_s^k	0	10%	30%	50%	70%	90%	100%

In Table 1, the health state 1 represents the worst state, and its corresponding health level is 0, which indicate that the component is completely failed and cannot complete the next mission. The health state 7 represents the best health state, and its health level is 100%, which indicate that the component is brand-new and can complete the next mission with a 100% chance. The rest health states are intermediate states between these two extreme states.

We assume that the aging factor ρ^k for all components is 1, and the minimum requirement of the health level h^* is 85%. The available maintenance time T_0 is 500 minutes. The other parameters of the selective maintenance model for the aircraft gas turbine engine system are given in Table 2. Note that the health level values were estimated based on the health condition assessment method proposed in our previous studies [35]–[37].

TABLE 2. The parameter values for components.

k	i	h_s^k	c_0^k	c_r^k	ω^k
1	2	0.1	3.04	68.99	0.051
2	4	0.5	3.04	68.99	0.051
3	2	0.1	3.04	68.99	0.051
4	5	0.7	3.04	68.99	0.051
5	4	0.5	1.96	44.42	0.033
6	5	0.7	1.96	44.42	0.033
7	5	0.7	4.92	111.51	0.082
8	5	0.7	4.92	111.51	0.082
9	1	0	1.83	41.58	0.031
10	1	0	1.83	41.58	0.031
11	3	0.3	5.83	132.30	0.097
12	4	0.5	5.83	132.30	0.097
13	5	0.7	3.48	78.91	0.058
14	3	0.3	3.48	78.91	0.058
15	6	0.9	3.48	78.91	0.058
16	5	0.7	3.48	78.91	0.058
17	7	1	2.50	56.70	0.042
18	4	0.5	1.17	26.46	0.019
19	3	0.3	1.17	26.46	0.019

* k represents the serial number of components; i represents the initial health state; h_s^k represents the initial health level; c_0^k represents the fixed maintenance cost; c_r^k represents the replacement cost; ω^k means the weight coefficient. The values in the fourth and fifth columns are in $10, 000 units.

B. NUMERICAL RESULTS

Since the established selective maintenance model for multi-state systems is a mixed integer programming model, it can be programmed using AMPL, an algebraic modeling language for mathematical programming, and solved using the commercial solver CPLEX 12.4 [38]. After calculation, the optimal objective is $4,269,700. The health level of the aircraft engine system is 85%, which is equal to the minimum requirement of the health level to successfully complete the next mission. The optimal solution, the optimal selective maintenance strategy and the corresponding maintenance cost for each component in the aircraft engine system

TABLE 3. The results of selective maintenance for the aircraft engine system.

k	i	h_i^k	j	f_j^k	Δh	Maintenance activity	Cost*
1	2	0.1	6	0.9	0.8	Major repair	49.03
2	4	0.5	6	0.9	0.4	Intermediate repair	26.04
3	2	0.1	6	0.9	0.8	Major repair	49.03
4	5	0.7	6	0.9	0.2	Minor repair	14.54
5	4	0.5	6	0.9	0.4	Intermediate repair	16.77
6	5	0.7	6	0.9	0.2	Minor repair	9.36
7	5	0.7	5	0.7	0	Doing nothing	0
8	5	0.7	5	0.7	0	Doing nothing	0
9	1	0	6	0.9	0.9	Major repair	36.48
10	1	0	6	0.9	0.9	Major repair	36.48
11	3	0.3	6	0.9	0.6	Intermediate repair	71.98
12	4	0.5	6	0.9	0.4	Intermediate repair	49.93
13	5	0.7	5	0.7	0	Doing nothing	0
14	3	0.3	6	0.9	0.6	Intermediate repair	42.94
15	6	0.9	6	0.9	0	Doing nothing	0
16	5	0.7	5	0.7	0	Doing nothing	0
17	7	1	7	1	0	Doing nothing	0
18	4	0.5	6	0.9	0.4	Intermediate repair	9.99
19	3	0.3	6	0.9	0.6	Intermediate repair	14.4

* The values in the last column are in \$10, 000 units.

are listed in Table 3. According to the definition, x_{ij}^k is a 0-1 variable, and it takes a value of 0 in most cases. In fact, we only care about the cases where it takes a value of 1. The first, second and fourth columns in Table 3 correspond to all cases where x_{ij}^k takes the value 1. For example, when $k = 1$, $i = 2$ and $j = 6$, which indicates that $x_{26}^1 = 1$. Similarly, the values of f_i^k are listed in the fifth column in Table 3, which constitute the optimal solution together with the values of x_{ij}^k.

The maintenance activities for components listed in the seventh column of Table 3 are corresponding to the increment in the health level Δh listed in the sixth column of Table 3. Due to the different maintenance costs for different components even though they have the same improvement in the health level, the maintenance activities were classified into five kinds based on the maintenance quality: doing nothing, minor repair, intermediate repair, major repair and replacement. When $\Delta h = 0$, namely no any improvement in the maintenance quality, it was resulted from that no any maintenance activity was scheduled; when $0 < \Delta h \leq 30\%$, namely a minor improvement in the maintenance quality, it was resulted from that minor repair activities were scheduled; similarly, when $30\% < \Delta h \leq 60\%$, it was resulted from that intermediate repair activities were executed; when $60\% < \Delta h \leq 100\%$, it was resulted from that major repair activities were executed; when $\Delta h = 100\%$, it means that the component was replaced with a new one. According to this classification standard, it can be seen that components ♯ 7, 8, 13, 15, 16 and 17 do not require any maintenance activity; components ♯ 4 and 6 only require minor repair activities; components ♯ 1, 3, 9 and 10 require major repair activities; components ♯ 2, 5, 11, 12, 14, 18, and 19 require intermediate repair activities. The maintenance cost consumed for each component are listed in the last column of Table 3, which were computed according to (1).

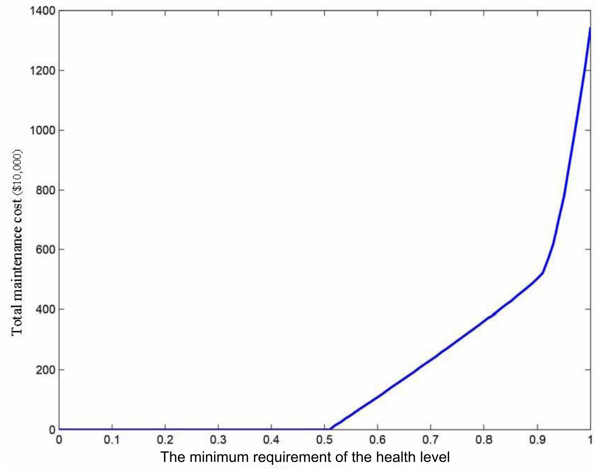

FIGURE 5. Maintenance cost of aircraft engine system for different minimum requirements of the health level.

C. SENSITIVITY ANALYSES

In this section, the sensitivity analyses for the minimum requirement of the health level, the aging factor, and the number of health states were conducted, respectively.

1) THE MINIMUM REQUIREMENT OF THE HEALTH LEVEL

To investigate the impact of the minimum requirement of the health level on maintenance decisions, several experiments were conducted where h^* is varied within the range of [0,1] and other parameters remain unchanged. Rerun the selective maintenance decision model and the curve of maintenance cost under different minimum requirements of the health level can be obtained, as shown in Fig. 5.

From Fig. 5, it can be seen that under different minimum requirements of the health level, the curve of maintenance cost can be divided into three phases. When $0 \leq h^* \leq 51\%$, the maintenance cost is zero and all components do not require any maintenance activity. When $51\% \leq h^* \leq 91\%$, the maintenance cost increases steadily from 0 to 5.21 million dollars with the increase in the minimum requirements of the health level. Namely, when the minimum requirement of the health level increases by 1%, the maintenance cost will increase \$130,275 on average. When $91\% \leq h^* \leq 1$, the maintenance cost increases sharply with the increase in the minimum requirement of the health level. At the same time, if the minimum requirement of the health level increases by 1%, the maintenance cost spent will increase by an average of \$914,333. Compared with the previous phase, the growth rate has increased by more than six times. When $h^* = 100\%$, the maintenance cost is as high as 13.44 million dollars. The health states of all components after maintenance are 7, which means that all components are replaced with new ones. From the trend of the maintenance cost curve in Fig. 5, it can be seen that the maintenance cost increases with the increase in the minimum requirement of the health level. When the minimum requirement of the health level is relatively low, the maintenance cost increases relatively slowly; when the

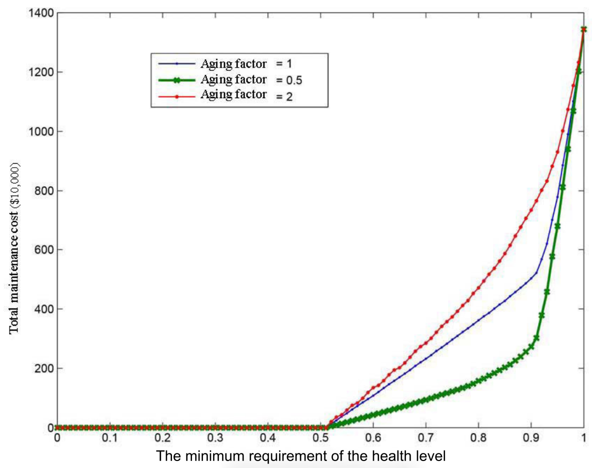

FIGURE 6. Maintenance cost curves at different levels of the aging factor.

minimum requirement is more than a certain level, the unit maintenance cost consumed will be greatly increased. Therefore, it is necessary to find an optimal trade-off between the maintenance cost and the minimum requirement of the health level, where the aircraft engine system can successfully complete the required mission, meanwhile, the maintenance cost is as low as possible.

2) THE AGING FACTOR OF THE AIRCRAFT ENGINE SYSTEM

To analyze the impact of the aging factor on the maintenance strategy, several experiments were conducted where the aging factor ρ^k was set as 0.5, 1, and 2, respectively. The maintenance cost curves at different levels of the aging factor were plotted in the Fig. 6.

Fig. 6 depicts that no matter how much the level of the aging factor is, the total maintenance cost is zero when the minimum requirement of the health state $h^* \in [0, 51\%]$. This is because the aircraft engine system's health level before maintenance has reached 51%, it is therefore no maintenance activity is required to meet the requirement for the next mission. When $h^* \in [52\%, 100\%)$, the maintenance cost will increase at different rates along with the increase in the minimum requirement of the health state. This indicates that the improvement in the system health level requires to consume a certain maintenance cost. When $h^* = 100\%$, the total maintenance cost for the three aging factor levels keeps unchanged. This is because that all components have been replaced, and the aircraft engine system is equivalent to a brand new one.

Furthermore, when $h^* \in [52\%, 100\%)$, the maintenance cost increases with the increase in the value of the aging factor, which indicates that the "older" system consumes more maintenance resource than the "younger" one. As the minimum requirement of the health level increases, the gap of the total maintenance cost between the aging factor $\rho^k = 0.5$ and $\rho^k = 1$ gradually increases at first. When $h^* = 88\%$, the gap reaches its maximum value, then gradually decreases until it reaches zero. Meanwhile, the gap of the maintenance

TABLE 4. Maintenance decision results for the aero-engine system with three-state components.

k	i	h_i^k	j	f_j^k	Δh	Activity	Cost*
1	2	0.5	3	1	0.5	Replacement	72.03
2	2	0.5	3	1	0.5	Replacement	72.03
3	2	0.5	3	1	0.5	Replacement	72.03
4	2	0.5	3	1	0.5	Replacement	72.03
5	2	0.5	3	1	0.5	Replacement	46.38
6	2	0.5	2	0.5	0	Doing nothing	0
7	2	0.5	3	1	0.5	Replacement	116.43
8	2	0.5	3	1	0.5	Replacement	116.43
9	1	0	3	1	1	Replacement	43.41
10	1	0	3	1	1	Replacement	43.41
11	2	0.5	2	0.5	0	Doing nothing	0
12	2	0.5	2	0.5	0	Doing nothing	0
13	2	0.5	3	1	0.5	Replacement	82.39
14	2	0.5	3	1	0.5	Replacement	82.39
15	2	0.5	3	1	0.5	Replacement	82.39
16	2	0.5	2	0.5	0	Doing nothing	0
17	3	1	3	1	0	Doing nothing	0
18	2	0.5	2	0.5	0	Doing nothing	0
19	2	0.5	3	1	0.5	Replacement	27.63

* The values in the last column are in $10, 000 units.

cost between the aging factor $\rho^k = 1$ and $\rho^k = 0.5$ reaches 2.3 million dollars. When $\rho^k = 2$, which means that the aero-engine system is "oldest", the maintenance cost is highest. As the minimum requirement for the next mission increases, the gap between the total maintenance cost between the aging factor $\rho^k = 1$ and $\rho^k = 2$ levels also gradually increases at first and then gradually decreases until it reaches zero. When $h^* = 91\%$, this gap reaches at a maximum value 245.337 million dollars.

From the above analysis, it is obvious that when the system is "young", a little maintenance resource can improve the maintenance quality significantly. With the aging of the system, the functions of various components gradually degrade, which results in the increase in the maintenance cost.

3) THE NUMBER OF HEALTH STATES

In this section, the experiment for the aircraft engine system with 3 health states (i.e., completely failure, malfunction, and perfect functioning) was conducted, and the results were compared with the maintenance strategies for the aircraft engine system with 7 health states in Section IV-B.

We assumed that each component in the aero-engine system has only three health states, denoted by 1, 2, and 3 respectively, and the corresponding health indices are 0, 50%, and 100%, respectively. Here state 1 indicates that the component is completely failed; state 2 indicates that the component is partially failed; state 3 indicates that the component is functioning as a brand new one. After calculation, the optimal objective is 9,298,800 dollars. Table 4 lists the results of the maintenance decision of the aero-engine system with three-state components.

As can be seen from Table 4, the components ♯6, 11, 12, 16, 17 and 18 do not require maintenance, and the remaining components need to be replaced with new ones. Compared to Table 3, the total maintenance cost increased by 118%. When there were only three health states for each

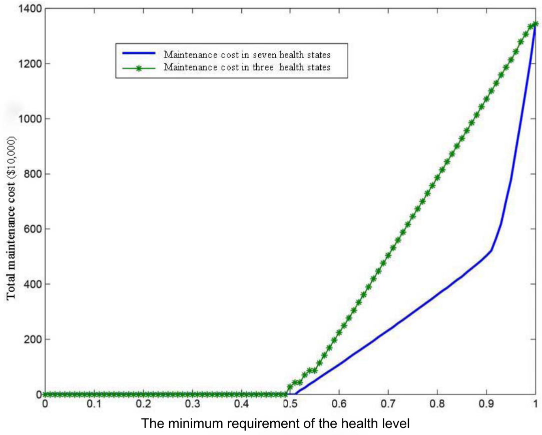

FIGURE 7. Maintenance cost under different number of health states.

component, most health states except for the best state would be considered to be faulty. In this situation, the only way to improve the performance level of a faulty component is to replace it with a new one. Some components that only require minor repair activities may also be replaced with new ones, which increases the total maintenance cost. Fig. 7 plots the maintenance cost curve under different number of health states. Fig. 7 shows that when the minimum requirement for the next mission is lower than a certain level (here is 50%), the maintenance cost under different health states is always zero, indicating that no any maintenance activity was required in this situation. As the minimum requirement increases, the maintenance cost under different health states also increases. However, to meet the minimum requirements of the health state, the maintenance cost for the system with three-state components is higher than the system with seven-state components. The gap between these two kinds of maintenance cost gradually increases at first, then gradually decreases when it reaches the maximum value. When $h^* = 100\%$, the gap between both situations reaches zero. The reason behind is that all the faulty or failed components need to be replaced with new ones when $h^* = 100\%$. When $h^* = 91\%$, the gap reaches a maximum value of 5,855,000 dollars, which further shows that more health states for components can significantly reduce the total maintenance cost.

D. COMPARATIVE ANALYSIS

In this section, we analyzed the perfect maintenance strategy and compared its advantages and disadvantages with imperfect maintenance strategies. We assumed that there are only two kinds of maintenance activities for each component in the aero-engine system: either doing nothing or replacement. Thus, it is necessary to add constraints (9) in the imperfect maintenance decision model in Section III-C.

$$x_{ij}^k = 0,\ \forall k \in \{1, 2, \cdots, K\},\ i, j \in \{1, 2, \cdots, S\},\ i < j < S \quad (9)$$

Constraints (9) indicate that there is no maintenance activity in the case of $i < j < S$. Together with (5), it can be concluded that maintenance activities can only be scheduled

TABLE 5. The perfect maintenance decision results for the aero-engine system.

k	i	h_i^k	j	f_j^k	Δh	Activity	Cost*
1	2	0.1	7	1	0.9	Replacement	72.03
2	4	0.5	4	0.5	0	Doing nothing	0
3	2	0.1	7	1	0.9	Replacement	72.03
4	5	0.7	5	0.7	0	Doing nothing	0
5	4	0.5	7	1	0.5	Replacement	46.38
6	5	0.7	5	0.7	0	Doing nothing	0
7	5	0.7	5	0.7	0	Doing nothing	0
8	5	0.7	5	0.7	0	Doing nothing	0
9	1	0	7	1	1	Replacement	43.41
10	1	0	7	1	1	Replacement	43.41
11	3	0.3	7	1	0.7	Replacement	138.13
12	4	0.5	7	1	0.5	Replacement	138.13
13	5	0.7	5	0.7	0	Doing nothing	0
14	3	0.3	7	1	0.7	Replacement	82.39
15	6	0.9	6	0.9	0	Doing nothing	0
16	5	0.7	5	0.7	0	Doing nothing	0
17	7	1	7	1	0	Doing nothing	0
18	4	0.5	4	0.5	0	Doing nothing	0
19	3	0.3	7	1	0.7	Replacement	27.63

* The values in the last column are in \$10,000 units.

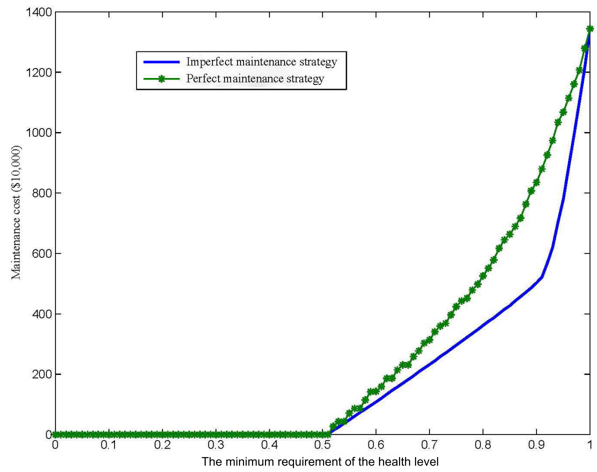

FIGURE 8. Maintenance cost for the aeroengine system under different maintenance strategies.

in the case of $i = j$ or $j = S$. After calculation, the optimal objective is 6.63 million dollars, an increase by 55.4% over the total maintenance cost under the imperfect maintenance strategy. In the meanwhile, the health level of the aero-engine system is 85.2%. Table 5 lists the perfect maintenance decision results for the aero-engine system.

From the seventh column of Table 5, it can be seen that there are only two types of maintenance activities: replacement or doing nothing. If "replacement" is scheduled, the health state of the components after maintenance will be as good as new ones. The maintenance activities for each component under the perfect maintenance strategy are as follows: except components ♯ 6, 11, 12, 16, 17 and 18 which do not require maintenance, the remaining components need to be replaced with new ones. Furthermore, the maintenance cost curves under perfect and imperfect maintenance strategies are plotted in Fig. 8.

Fig. 8 illustrates that when h^* is below a certain critical value (here is 51%), the maintenance cost is zero. Because the health level of the aero-engine system before maintenance has satisfied the minimum requirement for the next mission, so it doesn't require any maintenance activity. When h^* is between this critical value and a maximum value of 100%, the total maintenance cost under the perfect maintenance strategy is always higher than the maintenance cost under the imperfect maintenance strategy. When h^* is equal to 92%, the gap of the maintenance costs between these two strategies reaches a maximum value of 3.59 million dollars. At this situation, the maintenance cost under the perfect maintenance strategy increased by about 63.3% compared to the imperfect maintenance strategy. This further verifies that the imperfect maintenance strategy can significantly reduce the total maintenance cost while meeting the minimum requirement for the next mission. This is because, under the perfect maintenance strategy, many components which only require minor repairs have to be replaced with new ones, which would greatly increase maintenance cost and waste maintenance resources. Therefore, compared with the perfect maintenance strategy, the imperfect maintenance strategy has obvious economic advantages.

In addition, when $h^* = 100\%$, under both perfect maintenance and imperfect maintenance strategy, the total maintenance costs incurred are equal. This is because in this case all faulty or failed components were replaced with new ones, which indicates that the imperfect maintenance strategy becomes the perfect maintenance strategy.

V. CONCLUSIONS

In this paper, a novel selective maintenance model for a series-parallel multi-state system consisting of multi-state components is proposed. Imperfect maintenance for each component is considered in the maintenance options, along with the replacement and the do-nothing options. The relationships among maintenance quality, maintenance cost and the system life are formulated. A mixed integer programming model is constructed to minimize the total maintenance cost. A case study was presented to illustrate the proposed selective maintenance model, which was applied to an aircraft gas turbine engine system. The results showed that the constructed model can provide effective guidance in the maintenance decision process. The sensitivity analyses for the minimum requirement of the health level for the next mission, the system aging factor, the number of health states were conducted. Compared to the perfect maintenance strategy, it highlights the cost-effectiveness of the imperfect maintenance strategy. The results and sensitivity analyses can provide guidance in the maintenance decision-making process for actual multi-state systems.

There are several extensions to this study. Firstly, a more complex dependence relationship between components should be considered; Secondly, the assumption that all components in the same subsystem are independent identically distributed will be relaxed in our future work; Thirdly, the uncertain factors in the maintenance process will be explored and considered in our future work.

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