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Three-Dimensional Coordination Control for Multiple Autonomous Underwater Vehicles

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ABSTRACT In order to deal with coordination control for multiple autonomous underwater vehicles (AUVs), a three-dimensional coordination control scheme is proposed by combining sliding mode control, backstepping technique, and leader-follower strategy. A virtual trajectory is introduced to decrease the information exchange between AUVs. Global leader motion and follower controller constitute the coordination control scheme and are verified by a triangular prism formation simulation of six AUVs. The simulation results demonstrate that the controllers are capable of coordinating AUVs to achieve the desired formation and the coordination control scheme is effective.

INDEX TERMS Autonomous underwater vehicle (AUV), three-dimensional coordination control, sliding mode control, backstepping technique, leader-follower strategy, virtual trajectory.

I. INTRODUCTION

Nowadays, autonomous underwater vehicles (AUVs) have played an important role for underwater exploration and operation with the growing importance of the ocean. As a highly automatic device, AUVs are widely used in scientific, industrial, transport and human interaction area, such as [1]–[3]. Advances in technology make it possible to achieve more complex tasks, which are always difficult to be fulfilled by an individual autonomous vehicle, especially in the uncertain underwater environment. Coordination control for AUVs are becoming of increasing importance to further extend its application areas.

Many scientists and researchers are paying more and more attention to coordination control for multiple AUVs in recent years. In [4], a cooperative control strategy based on virtual bodies and artificial potentials is presented and applied in sea trials at Monterey Bay. In [5], a coordination control scheme based on leader-follower strategy and path following controller is presented to drive a fleet of AUVs forming a triangle to capture 3D images for pipeline inspection. In [6], a novel approach of formation control for AUVs based on the Jacobi shape theory is presented to achieve formation shape and make formation center track a desired trajectory. In [7], a Lyapunov-based backstepping approach for developing cooperative motion is presented to maintain a formation shape for leader and followers while traversing the desired path. Most of them are suitable for horizontal plane and don't consider three-dimensional space formation control. Three-dimensional space coordination control need further study and exploration.

Underwater vehicle is a highly nonlinear, coupled and time-varying dynamic system in three-dimensional space. It is well known that hydrodynamic coefficients and external disturbances are always uncertain, especially for AUVs. In addition, slow information exchange caused by weak underwater communication between AUVs generates another great challenge for formation control of multiple AUVs. Some efforts have been made to handle these challenges. With strong robustness to uncertain parameters and disturbances, sliding mode control has been successfully applied to motion control of autonomous underwater vehicles as shown in [8], [9]. Backstepping technique is a powerful tool to design controllers in accordance with Lyapunov stability [10]. In [11], [12], backstepping technique is combined with sliding mode control to design a path-following controller for autonomous underwater vehicles. In [13], backstepping technique is also applied to fulfill trajectory tracking for underactuated autonomous underwater vehicle. In order to deal with slow information exchange for AUVs, less information need to be transmitted with each other. In [14], a virtual vehicle is constructed to design formation controllers, which

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only need position measurement. In [15], a new approach angle without using velocity information is proposed to achieve formation control. There still need more research on achieve less information exchange.

Formation control methods mainly contain leader-follower strategy [16], behavior [17], virtual structure [6], graph theory [18] and artificial potential field [4] for AUVs according to [19], [20]. For behavior method, it is difficult to make strict theoretical analysis. Controllers based on virtual structure is not flexible for formation motion. Graph theory can make strict theoretical analysis, but it is always complicated for controllers design. For artificial potential field method, artificial potential function is difficult to design and choose. Compared with the above methods, leader-follower strategy is made strict theoretical analysis in [21], [22]. In addition, it is easy to design and realize for engineering application. Considering the merits of leader-follower strategy, this paper adopts leader-follower strategy for formation controllers design. With combing sliding mode control and backstepping technique, formation controllers have better robustness, which is one of requirements for motion control of AUVs.

With the above considerations, this paper proposes a three-dimensional coordination control scheme based on sliding mode control, backstepping technique and leader-follower strategy for multiple AUVs. Leader-follower strategy is adopted, and the coordination control scheme is composed of global leader motion control and follower control. Global leader motion controller is firstly designed with sliding mode control and backstepping technique for three-dimensional trajectory tracking. The controller need acceleration speed and velocity information of the desired three-dimensional trajectory. If it is directly applied in follower controller design, acceleration speed and velocity information of the leader is essential for its follower, which leads to too much information transmission considering slow information exchange between leader and follower. In order to solve the problem, a virtual trajectory only depending on leader position is introduced for the follower controller design. The follower is required to track the virtual trajectory with the virtual trajectory converging to the desired space trajectory of the follower. Global leader motion and follower controller together coordinate the entire group of multiple AUVs to achieve a desired formation. With virtual trajectory, the main features of the proposed three-dimensional coordination control scheme can be summarized as follows:

- Utilizing sliding mode control and backstepping technique, a coordination control scheme in three-dimensional space is presented, which has strong robustness to uncertain parameters and disturbances.
- A virtual trajectory is designed to decrease the amount of information transmission between AUVs, where the leader only transmits its position information to follower.

The rest of this paper is organized as follows. Section II presents mathematical model, formulation of coordination control and preliminaries for AUVs. Section III details controllers design and the virtual trajectory in three-dimensional space. The proposed three-dimensional coordination control scheme is demonstrated by triangular prism formation of six AUVs in section IV. Finally, a conclusion is drawn in section V.

II. PROBLEM FORMULATION AND PRELIMINARIES

This section will present mathematical model and formulation of coordination control for AUVs. Global leader motion controller and follower controller are proposed. At last, some properties and assumptions are listed, which are necessary for controllers design.

A. VEHICLE MODELING

The autonomous underwater vehicles can be completely described by its position and orientation with respect to the earth-fixed coordinate system. It is convenient to define two coordinate frames as shown in Fig. 1 to present the mathematical model of the AUV. The coordinate frame $\{\mathbf{E}\} := \{X, Y, Z\}$ is the earth-fixed coordinate (inertial coordinate) and $\{\mathbf{B}\} := \{x, y, z\}$ is the vehicle-fixed coordinate. The pose vector of the AUV is represented by $\boldsymbol{\eta} = [\boldsymbol{\eta_1}^T, \boldsymbol{\eta_2}^T]^T$, where $\boldsymbol{\eta_1} = [X, Y, Z]^T$ is the position vector and $\boldsymbol{\eta_2} = [\phi, \theta, \psi]^T$ is the orientation vector with respect to the earth-fixed coordinate in the earth-fixed coordinate system. The velocity vector of the AUV is represented by $\mathbf{v} = [\mathbf{v_1}^T, \mathbf{v_2}^T]^T$, which is relative to the origin of the earth-fixed coordinate in the vehicle-fixed coordinate system. Let $\mathbf{v_1} = [u, v, w]^T$ be the linear velocity vector and let $\mathbf{v_2} = [p, q, r]^T$ be the angular velocity vector. The six variables represent six velocity components of motion, namely surge, sway, heave, roll, pitch, and yaw.

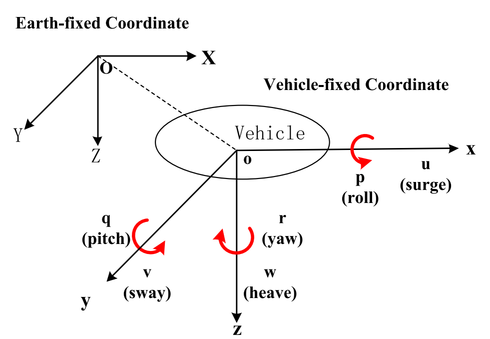

FIGURE 1. The coordinate frames.

The mathematical model of the AUV is composed of kinematic and dynamics equation. The kinematic equation can be given by

$$\dot{\eta_1} = \mathbf{J_1}(\eta_2)\mathbf{v_1}, \tag{1}$$

$$\dot{\eta_2} = \mathbf{J_2}(\eta_2)\mathbf{v_2}. \tag{2}$$

$\mathbf{J_1}(\eta_2)$ and $\mathbf{J_2}(\eta_2)$ are a velocity transformation matrix that transforms velocities of the vehicle-fixed to the earth-fixed coordinate frame as given by,

$$\mathbf{J_1}(\boldsymbol{\eta_2}) = \begin{bmatrix} c_\psi c_\theta & c_\psi s_\theta s_\phi - s_\psi c_\phi & c_\psi s_\theta c_\phi + s_\psi s_\phi \\ s_\psi c_\theta & s_\psi s_\theta s_\phi + c_\psi c_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi \\ -s_\theta & c_\theta s_\phi & c_\theta c_\phi \end{bmatrix}, \tag{3}$$

$$\mathbf{J_2}(\boldsymbol{\eta_2}) = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix}, \tag{4}$$

where $c_{(\cdot)}$, $s_{(\cdot)}$ and $t_{(\cdot)}$ are short notations for cos(·), sin(·) and tan(·) throughout of this paper, respectively.

The dynamics equation can be written as:

$$\mathbf{M}\dot{\mathbf{v}} = -\mathbf{N}(\boldsymbol{\eta}, \mathbf{v}) + \boldsymbol{\tau},$$
$$\mathbf{N}(\boldsymbol{\eta}, \mathbf{v}) = \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}), \tag{5}$$

where $\mathbf{M}$ is a 6×6 inertia matrix as a sum of the rigid body inertia matrix $\mathbf{M_R}$ and the hydrodynamic virtual inertia (added mass) $\mathbf{M_A}$; $\mathbf{C}(\mathbf{v})$ is a 6×6 Coriolis and centripetal matrix including rigid body terms $\mathbf{C_R}(\mathbf{v})$ and terms $\mathbf{C_A}(\mathbf{v})$ due to the added mass; $\mathbf{D}(\mathbf{v})$ is a 6×6 damping matrix including terms due to drag forces; $\mathbf{g}(\boldsymbol{\eta})$ is a 6×1 vector containing the restoring terms formed by the vehicles buoyancy and gravitational terms; $\boldsymbol{\tau}$ is a 6×1 vector including the control forces and moments, namely $[\tau_X, \tau_Y, \tau_Z, \tau_K, \tau_M, \tau_N]^T$.

B. COORDINATION CONTROL FOR MULTIPLE AUVS

The main object of coordination control for multiple AUVs is to coordinate the entire group of multiple AUVs so as to achieve a desired formation. Leader−follower strategy is adopted to achieve the object. One global leader AUV, which does not follow any other AUVs, is selected and traces a three dimension time-varying path. Stable controllers need to be designed for the other AUVs in order to keep a predefined three dimension space distance with respect to its leader. A common leader−follower strategy is shown in Fig. 2. There are n AUVs. $AUV_i (i = 1, 2, \ldots, n)$ represents the i−th AUV. AUV_1 is the global leader. The arrow represents information transmission from leader to follower, namely position information. It can be viewed that there are $n - 1$ pairs of leader and follower within the formation according to leader−follower strategy. In each pair of leader and follower, the follower AUV should maintain a desired three dimension space distance relative to its leader. When all the AUVs keep in the expected positions, the desired formation is established and then coordination control is achieved for multiple AUVs.

In order to better describe the problem, the following notations are introduced in the rest of the paper.

- $\boldsymbol{\eta_{1gl}} = [X_{gl}, Y_{gl}, Z_{gl}]^T$ represents three dimension space position of the global leader; $\boldsymbol{\eta_{1gld}} = [X_{gld}, Y_{gld}, Z_{gld}]^T$ represents the desired three dimension space position of the global leader; $\boldsymbol{\eta_{2gl}} = [\phi_{gl}, \theta_{gl}, \psi_{gl}]^T$ represents orientation vector of the global leader; $\mathbf{v_{gl}} = [\mathbf{v_{1gl}}^T, \mathbf{v_{2gl}}^T]^T$ expresses the velocity vector of the global leader with respect to the origin of the earth-fixed coordinate in the vehicle-fixed coordinate system, where $\mathbf{v_{1gl}} = [u_{gl}, v_{gl}, w_{gl}]^T$ and $\mathbf{v_{2gl}} = [p_{gl}, q_{gl}, r_{gl}]^T$.

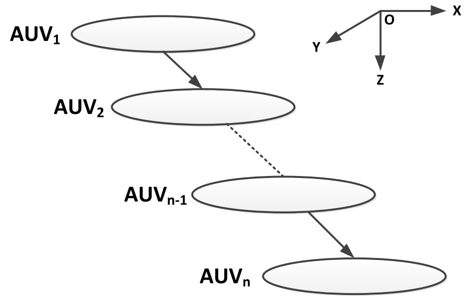

FIGURE 2. A leader−follower strategy for *n* AUVs.

- In each pair of leader and follower, $\boldsymbol{\eta_{1l}} = [X_l, Y_l, Z_l]^T$ represents three dimension space position of the leader; $\boldsymbol{\eta_{1f}} = [X_f, Y_f, Z_f]^T$ represents three dimension space position of the follower; $\boldsymbol{\eta_{1fd}} = [X_{fd}, Y_{fd}, Z_{fd}]^T$ represents the desired three dimension space position of the follower; $\boldsymbol{d_\eta} = [d_x, d_y, d_z]^T$ represents the predefined three dimension space distance of the follower with respect to its leader; $\mathbf{v_f} = [\mathbf{v_{1f}}^T, \mathbf{v_{2f}}^T]^T$ expresses the velocity vector of the follower, where $\mathbf{v_{1f}} = [u_f, v_f, w_f]^T$ and $\mathbf{v_{2f}} = [p_f, q_f, r_f]^T$.
- It is obvious that $\boldsymbol{\eta_{1fd}} = \boldsymbol{\eta_{1l}} + \boldsymbol{d_\eta}$

C. PROBLEM STATEMENT

Without loss of generality, common torpedo-like AUV model is adopted to present coordination control problem in the paper. Torpedo-like AUV is equipped with rudders for steering and diving, and a thruster to provide forward force, that is, the torpedo-like AUV has only three control forces and moments, namely $\boldsymbol{\tau} = [\tau_X, 0, 0, 0, \tau_M, \tau_N]^T$. With the above subsection notation, the coordination control problem under study contains two parts. The first part is called global leader motion controller design and can be formulated as follows:

Consider AUV model with kinematic and dynamic equations given by (1)−(5). *Given the desired* $\boldsymbol{\eta_{1gld}}$ *for the global leader, design feedback control laws for the control forces and moments* $\boldsymbol{\tau}$, *such that position error* $\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}$ *tend to zero asymptotically.*

The second part is called follower controller design and can be formulated as follows:

Consider AUV model with kinematic and dynamic equations given by (1)−(5). *Given the desired* $\boldsymbol{d_\eta}$ *for the follower, design feedback control laws for the control forces and moments* $\boldsymbol{\tau}$, *such that position error of the follower* $\boldsymbol{\eta_{1f}} - \boldsymbol{\eta_{1fd}}$ *tend to zero asymptotically.*

D. PRELIMINARIES

In this subsection, some definitions, properties and assumptions are presented in order to better show controllers design.

Definition 1: A matrix $\mathbf{S}$ is said to be skew-symmetrical if $\mathbf{S} = -\mathbf{S}^T$.

Definition 2: $\mathbf{S}(\mathbf{v_2})$ is defined as:

$$\mathbf{S}(\mathbf{v_2}) = -\mathbf{S}^T(\mathbf{v_2}) = \begin{pmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{pmatrix}$$

Property 1: The linear velocity transformation matrix $\mathbf{J_1}(\boldsymbol{\eta_2})$ has the following properties:

$$\mathbf{J_1}^T = \mathbf{J_1}^{-1}$$
$$\dot{\mathbf{J}}_1 = \mathbf{J_1}\mathbf{S}(\mathbf{v_2})$$

Property 2: The time derivation of the function $f(\epsilon) = \log(\cosh(\epsilon))$ is:

$$\dot{f}(\epsilon) = \dot{\epsilon}\tanh(\epsilon)$$

Assumption 1: The linear velocity of the underwater vehicle, namely $\mathbf{v_1}$, is bounded and its maximum value is denoted as $\mathbf{v_{1lm}}$.

III. CONTROLLERS DESIGN

This section will detail global leader motion controller and follower controller design based on sliding mode control and backstepping technique. Specially, a virtual trajectory depending on leader position measurement is introduced for follower controller design. With the virtual trajectory, the leader only transmits its position information to follower.

A. GLOBAL LEADER MOTION CONTROLLER

The global leader's motion model is described by (1)-(5). To achieve $\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}$ convergence to zero, a new error is proposed as in [13]:

$$\boldsymbol{\sigma} = \boldsymbol{\varepsilon} - \boldsymbol{\rho} \tag{6}$$

where $\boldsymbol{\rho}$ is the radius of the error space, $\boldsymbol{\rho} = [\delta, 0, 0]^T$, δ is designed as relatively small value compared with length of AUV, and $\boldsymbol{\varepsilon}$ is defined as

$$\boldsymbol{\varepsilon} = \mathbf{J}_1^{-1}(\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}). \tag{7}$$

According to (1) and property 1, the derivative of error $\boldsymbol{\sigma}$ with respect to time is:

$$\begin{aligned}
\dot{\boldsymbol{\sigma}} &= \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\rho}} = \dot{\boldsymbol{\varepsilon}} \\
&= \dot{\mathbf{J}}_1^{-1}(\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}) + \mathbf{J}_1^{-1}(\dot{\boldsymbol{\eta}}_{1gl} - \dot{\boldsymbol{\eta}}_{1gld}) \\
&= \dot{\mathbf{J}}_1^{T}(\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}) + \mathbf{J}_1^{-1}(\dot{\boldsymbol{\eta}}_{1gl} - \dot{\boldsymbol{\eta}}_{1gld}) \\
&= S^T(\mathbf{v_{2gl}})\mathbf{J}_1^{\mathbf{T}}(\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}) + \mathbf{J}_1^{-1}(\dot{\boldsymbol{\eta}}_{1gl} - \dot{\boldsymbol{\eta}}_{1gld}) \\
&= S^T(\mathbf{v_{2gl}})\mathbf{J}_1^{\mathbf{T}}(\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}) - S^T(\mathbf{v_{2gl}})\boldsymbol{\rho} \\
&\quad + S^T(\mathbf{v_{2gl}})\boldsymbol{\rho} + \mathbf{J}_1^{-1}(\dot{\boldsymbol{\eta}}_{1gl} - \dot{\boldsymbol{\eta}}_{1gld}) \\
&= -S(\mathbf{v_{2gl}})\boldsymbol{\sigma} - S(\mathbf{v_{2gl}})\boldsymbol{\rho} + \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gl} - \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gld} \\
&= -S(\mathbf{v_{2gl}})\boldsymbol{\sigma} - S(\mathbf{v_{2gl}})\boldsymbol{\rho} + \mathbf{v_{1gl}} - \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gld} \\
&= -S(\mathbf{v_{2gl}})\boldsymbol{\sigma} - \begin{bmatrix} 0 & -r_{gl} & q_{gl} \\ r_{gl} & 0 & -p_{gl} \\ -q_{gl} & p_{gl} & 0 \end{bmatrix}\begin{bmatrix} \delta \\ 0 \\ 0 \end{bmatrix} \\
&\quad + \begin{bmatrix} u_{gl} \\ v_{gl} \\ w_{gl} \end{bmatrix} - \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gld} \\
&= -S(\mathbf{v_{2gl}})\boldsymbol{\sigma} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\delta \\ 0 & \delta & 0 \end{bmatrix}\begin{bmatrix} u_{gl} \\ q_{gl} \\ r_{gl} \end{bmatrix} \\
&\quad + \begin{bmatrix} 0 \\ v_{gl} \\ w_{gl} \end{bmatrix} - \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gld}
\end{aligned} \tag{8}$$

If it is chose that:

$$\begin{bmatrix} u_{gl} \\ q_{gl} \\ r_{gl} \end{bmatrix} = \begin{bmatrix} u_{gld} \\ q_{gld} \\ r_{gld} \end{bmatrix} = \mathbf{P}^{-1}\left(-\mathbf{K}\boldsymbol{\sigma} - \begin{bmatrix} 0 \\ v_{gl} \\ w_{gl} \end{bmatrix} + \mathbf{J}_1^{-1}\dot{\boldsymbol{\eta}}_{1gld}\right) \tag{9}$$

where

$$\mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -\delta \\ 0 & \delta & 0 \end{bmatrix},$$
$$\mathbf{K} = k\mathbf{I},\ k > 0 \text{ and } \mathbf{I} \in R^{3\times 3} \text{ is unite matrix.} \tag{10}$$

then

$$\dot{\boldsymbol{\sigma}} = -(S(\mathbf{v_{2gl}}) + \mathbf{K})\boldsymbol{\sigma} \tag{11}$$

According to [23], (11) is uniformly exponential stable, that is, $\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}$ converges to zero with a relatively small error space. In order to make $[u_{gl}, q_{gl}, r_{gl}]$ converge to the desired ones, namely (9), the sliding mode method is implemented as shown in [24]. The desired velocity is:

$$\mathbf{v_{gld}} = [u_{gld}, 0, 0, 0, q_{gld}, r_{gld}]^T \tag{12}$$

The sliding surface is proposed that:

$$\mathbf{s} = [s_1, s_2, s_3]^T = \mathbf{h}\mathbf{v_{gle}} \tag{13}$$

where $\mathbf{h} \in R^{3\times 6}$ is a design matrix, and $\mathbf{v_{gle}} = \mathbf{v_{gl}} - \mathbf{v_{gld}}$.

Let $\boldsymbol{\tau} = \mathbf{Bu}$ where

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14}$$

and

$$\mathbf{u} = [\tau_X, \tau_M, \tau_N]^T \tag{15}$$

Let $\mathbf{A} = \mathbf{hM}^{-1}$. According to dynamics equation, namely (5), taking the time derivative of $\mathbf{s}$, it is obtained that:

$$\begin{aligned}
\dot{\mathbf{s}} &= \mathbf{h}\dot{\mathbf{v}}_{gle} = \dot{\mathbf{v}}_{gl} - \dot{\mathbf{v}}_{gld} \\
&= \mathbf{hM}^{-1}(-\mathbf{N} + \boldsymbol{\tau}) - \dot{\mathbf{v}}_{gld} \\
&= \mathbf{A}(-\mathbf{N} + \boldsymbol{\tau}) - \dot{\mathbf{v}}_{gld} \\
&= \mathbf{A}(-\mathbf{N} + \mathbf{Bu}) - \dot{\mathbf{v}}_{gld}
\end{aligned} \tag{16}$$

Reaching law is adopted to design sliding mode controller, that is, the following equation is adopted:

$$\mathbf{F} = [\xi\tanh(s_1/\kappa), \xi\tanh(s_2/\kappa), \xi\tanh(s_3/\kappa)]^T,$$
$$\xi > 0 \text{ and } \kappa > 0 \text{ is design parameters.} \tag{17}$$

Based on sliding mode method, let $\dot{\mathbf{s}} = -\mathbf{F}$. Then, according to (16) and (17), the sliding mode controller can be obtained that:

$$\mathbf{u} = (\mathbf{AB})^{-1}(-\mathbf{F} + \mathbf{AN} + \mathbf{h}\dot{\mathbf{v}}_{gld}) \tag{18}$$

Finally, based on backstepping technique, a feedback control law, namely (18), is proposed for the global leader, where $\mathbf{v_{gld}}$ can be obtained by (9) and (12). The control law ensures position error $\boldsymbol{\eta_{1gl}} - \boldsymbol{\eta_{1gld}}$ tending to zero asymptotically with a designed error space.

Remark 1: Backstepping technique is a convenient tool to deal with controller design for a highly nonlinear, coupled and time-varying dynamic system in three-dimensional space. However, it is hard to handle unknown parameters and uncertain disturbances only by itself. Sliding mode control is powerful to design controllers when facing unknown parameters and uncertain disturbances. It is very effective to combine backstepping technique and sliding mode for three-dimensional motion control and coordination of AUVs.

B. FOLLOWER CONTROLLER

Similarly to the global leader motion controller, feedback control laws for the follower can be obtained, where velocity and acceleration information of the desired space trajectory of the follower ($\boldsymbol{\eta_{1fd}}$) is necessary by observing (18). Considering $\boldsymbol{\eta_{1fd}} = \boldsymbol{\eta_{1l}} + \boldsymbol{d_\eta}$, it means that velocity and acceleration information of the leader need to be provided to its follower if the follower uses the similar controller as global leader. As said in section I, the leader only transmits positional information to its follower. In order to solve the problem, a virtual trajectory is introduced to design feedback control laws for the follower. The follower controller design contains two parts, namely the follower converges to the virtual trajectory and the virtual trajectory converges to the desired space trajectory of the follower.

The following notations are introduced to present the virtual trajectory and controllers design. $\boldsymbol{\eta_{1v}} = [X_v, Y_v, Z_v]^T$ represents the virtual trajectory. $\mathbf{e_v} = \boldsymbol{\eta_{1v}} - \boldsymbol{\eta_{1fd}}$ and represents the error of the virtual trajectory and the desired position of the follower. $\mathbf{v_v} = [u_v, v_v, w_v]^T$ represents the velocity of the virtual trajectory. $\mathbf{v_v}$ is considered as:

$$\dot{\boldsymbol{\eta}}_{1v} = \mathbf{v_v} \tag{19}$$

Then, follower controller design contain two steps. Firstly, $\mathbf{v_v}$ need to be designed to fulfill $\mathbf{e_v} = \boldsymbol{\eta_{1v}} - \boldsymbol{\eta_{1fd}}$ tending to zero with (19). Secondly, $\boldsymbol{\tau}$ need to be designed to fulfill $\boldsymbol{\eta_{1f}} - \boldsymbol{\eta_{1v}}$ tending to zero.

Step 1: A new virtual trajectory error $\mathbf{r_e}$ is defined as:

$$\mathbf{r_e} = \mathbf{e_v} + \boldsymbol{\epsilon} \tag{20}$$

where $\boldsymbol{\epsilon} = [\epsilon_1, \epsilon_2, \epsilon_3]^T$. $\boldsymbol{\epsilon}$ is defined as:

$$\dot{\boldsymbol{\epsilon}} = -\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}\mathbf{r_e} \tag{21}$$

where $\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) = [\lambda_1 tanh(\epsilon_1/\lambda_1), \lambda_2 tanh(\epsilon_2/\lambda_2), \lambda_3 tanh(\epsilon_3/\lambda_3)]^T$, $\lambda_i \in R^+$ is design parameters, $\mathbf{K} = diag[k_1, k_2, k_3]$ is diagonal matrix and $\epsilon_i(0) = 0$, $i = 1, 2, 3$.

$\mathbf{v_v}$ is proposed as:

$$\mathbf{v_v} = \boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) + \boldsymbol{\alpha}_2(\boldsymbol{\epsilon}) \tag{22}$$

where $\boldsymbol{\alpha}_2(\boldsymbol{\epsilon}) = [\mu_1 tanh(\epsilon_1/\mu_1), \mu_2 tanh(\epsilon_2/\mu_2), \mu_3 tanh(\epsilon_3/\mu_3)]^T$ and $\mu_i \in R^+$ is design parameters, $i = 1, 2, 3$.

Taking the time derivative of $\mathbf{r_e}$, it is obtained that:

$$\begin{aligned}\dot{\mathbf{r}}_e &= \dot{\mathbf{e}}_v + \dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\eta}}_{1v} - \dot{\boldsymbol{\eta}}_{1fd} + \dot{\boldsymbol{\epsilon}} \\ &= \mathbf{v_v} - \dot{\boldsymbol{\eta}}_{1fd} - \boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}\mathbf{r_e}\end{aligned} \tag{23}$$

Substitute (22) into (23):

$$\dot{\mathbf{r}}_e = -\mathbf{K}\mathbf{r_e} - \dot{\boldsymbol{\eta}}_{1fd} + \boldsymbol{\alpha}_2(\boldsymbol{\epsilon}) \tag{24}$$

Consider the following Lyapunov function candidate:

$$V_1 = \frac{1}{2}\mathbf{r_e}^T\mathbf{r_e} + \frac{1}{2}\boldsymbol{\epsilon}_{std}^T\boldsymbol{\epsilon}_{std} \tag{25}$$

where

$$\boldsymbol{\epsilon}_{std} = \begin{bmatrix}\sqrt{\mu_1 log(cosh(\epsilon_1/\mu_1))} \\ \sqrt{\mu_2 log(cosh(\epsilon_2/\mu_2))} \\ \sqrt{\mu_3 log(cosh(\epsilon_3/\mu_3))}\end{bmatrix} \tag{26}$$

Taking the time derivative of V_1 combing with property 2 and (21), it is obtained that:

$$\begin{aligned}\dot{V}_1 &= \mathbf{r_e}^T\dot{\mathbf{r}}_e + \mathbf{K}^{-1}\dot{\boldsymbol{\epsilon}}^T\boldsymbol{\alpha}_2(\boldsymbol{\epsilon}) \\ &= \mathbf{r_e}^T(-\mathbf{K}\mathbf{r_e} - \dot{\boldsymbol{\eta}}_{1fd} + \boldsymbol{\alpha}_2(\boldsymbol{\epsilon})) + \mathbf{K}^{-1}\dot{\boldsymbol{\epsilon}}^T\boldsymbol{\alpha}_2(\boldsymbol{\epsilon}) \\ &= -\mathbf{r_e}^T\mathbf{K}\mathbf{r_e} - \mathbf{r_e}^T\dot{\boldsymbol{\eta}}_{1fd} - \mathbf{K}^{-1}\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2 \\ &= -\mathbf{r_e}^T\mathbf{K}\mathbf{r_e} - \mathbf{r_e}^T\dot{\boldsymbol{\eta}}_{1l} - \mathbf{K}^{-1}\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2 \\ &= -\mathbf{r_e}^T\mathbf{K}\mathbf{r_e} - \mathbf{r_e}^T\mathbf{J_1}\mathbf{v_{1l}} - \mathbf{K}^{-1}\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2\end{aligned} \tag{27}$$

According to assumption 1 and [25], it follows that

$$\dot{V}_1 \le -(min\{k_i\} - \frac{1}{2} - \frac{3\mathbf{v_{1lm}}}{\|\mathbf{r_e}\|})\|\mathbf{r_e}\|^2 - \mathbf{K}^{-1}\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2 \tag{28}$$

With β given positive constant, $\mathbf{K}$ is chosen as:

$$min\{k_i\} - \frac{1}{2} - \frac{3\mathbf{v_{1lm}}}{\beta} > 0 \tag{29}$$

Then the following bound on $\dot{V}_1$ can be obtained that:

$$\|\mathbf{r_e}\|^2 \ge \beta^2 \Rightarrow \dot{V}_1 \le -\|\mathbf{r_e}\|^2 - \mathbf{K}^{-1}\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2 \tag{30}$$

Since $\boldsymbol{\alpha}_1^T\boldsymbol{\alpha}_2 \ge 0$, $\dot{V}_1 \le 0$. It is noted that V_1 is positive definite and radially unbounded. Considering the linear dependency of $\frac{1}{\beta}$ and the results in [26], the closed-loop state is uniformly practically asymptotically stable, namely $\mathbf{e_v} = \boldsymbol{\eta_{1v}} - \boldsymbol{\eta_{1fd}}$ tending to zero.

Step 2: Similarly to the global leader motion controller, in order to fulfill $\boldsymbol{\eta_{1f}} - \boldsymbol{\eta_{1v}}$ tending to zero, a feedback control law can be obtained for the follower:

$$\mathbf{u} = (\mathbf{AB})^{-1}(-\mathbf{F} + \mathbf{AN} + \mathbf{h}\dot{\mathbf{v}}_v) \tag{31}$$

By introducing a virtual trajectory, namely (19) and (22), the follower controller is proposed as (31) to ensures position error $\boldsymbol{\eta_{1f}} - \boldsymbol{\eta_{1fd}}$ tending to zero asymptotically.

Remark 2: It is noted that $\dot{\mathbf{v}}_v$ can be calculated by (22), namely

$$\dot{\mathbf{v}}_v = \dot{\boldsymbol{\alpha}}_1(\boldsymbol{\epsilon}) + \dot{\boldsymbol{\alpha}}_2(\boldsymbol{\epsilon}) \tag{32}$$

where $\dot{\boldsymbol{\alpha}}_1(\boldsymbol{\epsilon}) = [\dot{\epsilon}_1 - \dot{\epsilon}_1 tanh^2(\epsilon_1/\lambda_1), \dot{\epsilon}_2 - \dot{\epsilon}_2 tanh^2(\epsilon_2/\lambda_2), \dot{\epsilon}_3 - \dot{\epsilon}_3 tanh^2(\epsilon_3/\lambda_3)]^T$ and $\dot{\boldsymbol{\alpha}}_1(\boldsymbol{\epsilon}) = [\dot{\epsilon}_1 - \dot{\epsilon}_1 tanh^2(\epsilon_1/\mu_1), \dot{\epsilon}_2 - \dot{\epsilon}_2 tanh^2(\epsilon_2/\mu_2), \dot{\epsilon}_3 - \dot{\epsilon}_3 tanh^2(\epsilon_3/\mu_3)]^T$. It is apparent that $\dot{\mathbf{v}}_v$

is depending on $\boldsymbol{\epsilon}$ and $\dot{\boldsymbol{\epsilon}}$. Substitute (20) into (21), according to $\boldsymbol{\eta}_{1fd} = \boldsymbol{\eta}_{1l} + \boldsymbol{d}_\eta$ and $\mathbf{e}_\mathbf{v} = \boldsymbol{\eta}_{1v} - \boldsymbol{\eta}_{1fd}$, it is obtained that:

$$\begin{aligned}\dot{\boldsymbol{\epsilon}} &= -\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}(\mathbf{e}_\mathbf{v} + \boldsymbol{\epsilon}) \\ &= -\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}(\boldsymbol{\eta}_{1v} - \boldsymbol{\eta}_{1fd} + \boldsymbol{\epsilon}) \\ &= -\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}(\boldsymbol{\eta}_{1v} - \boldsymbol{\eta}_{1l} - \boldsymbol{d}_\eta + \boldsymbol{\epsilon}) \end{aligned} \tag{33}$$

By observing (19) and (22), it is obvious that

$$\boldsymbol{\eta}_{1v} = \int \mathbf{v}_\mathbf{v} dt = \int (\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) + \boldsymbol{\alpha}_2(\boldsymbol{\epsilon}))dt \tag{34}$$

Substitute (34) into (33), considering $\epsilon_i(0) = 0$, $\boldsymbol{\epsilon}$ and $\dot{\boldsymbol{\epsilon}}$ can be calculate by the following equation:

$$\dot{\boldsymbol{\epsilon}} = -\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) - \mathbf{K}(\int (\boldsymbol{\alpha}_1(\boldsymbol{\epsilon}) + \boldsymbol{\alpha}_2(\boldsymbol{\epsilon}))dt - \boldsymbol{\eta}_{1l} - \boldsymbol{d}_\eta + \boldsymbol{\epsilon}) \tag{35}$$

It is apparent that $\boldsymbol{\epsilon}$ and $\dot{\boldsymbol{\epsilon}}$ is only depending on $\boldsymbol{\eta}_{1l}$ and $\boldsymbol{d}_\eta$ by observing (35). That is to say, $\dot{\mathbf{v}}_v$ is only depending on $\boldsymbol{\eta}_{1l}$ and $\boldsymbol{d}_\eta$. Finally, the feedback control law (31) for follower only needs position information of its leader ($\boldsymbol{\eta}_{1l}$) when considering $\boldsymbol{d}_\eta$ is predefined.

IV. SIMULATION RESULTS

To verify the performance of the coordination control scheme for multiple AUVs proposed in this paper, six AUVs are together controlled to form triangular prism in the simulation. The main parameters of the six AUVs are adapted from [27]. The initial position of the global leader is $\boldsymbol{\eta}_{1gl}(0) = [0, 0, 0]^T$. It is adopted that $\boldsymbol{\eta}_{1fi}$ and $\boldsymbol{d}_{\eta i}$ represent the position of the i-th AUV and the predefined space distance of the i-th AUV with respect to its leader for the other AUVs, respectively. Initial positions of the other AUVs, namely followers, is $\boldsymbol{\eta}_{1f1}(0) = [0, 1, 0]^T$, $\boldsymbol{\eta}_{1f2}(0) = [0, -1, 0]^T$, $\boldsymbol{\eta}_{1f3}(0) = [0, -0.5, 0]^T$, $\boldsymbol{\eta}_{1f4}(0) = [0, 1.5, 0]^T$, $\boldsymbol{\eta}_{1f5}(0) = [0, 0.5, 0]^T$. Initial parameters for the other motion state are zero. The desired position of the global leader is:

$$\begin{aligned} X_{gld} &= t, \\ Y_{gld} &= 0.5 * t, \\ Z_{gld} &= 5 \end{aligned} \tag{36}$$

where t represents time. The desired space distance of the i-th AUV with respect to its leader are $\boldsymbol{d}_{\eta 1} = [8, 8, 0]^T$, $\boldsymbol{d}_{\eta 2} = [0, -16, 0]^T$, $\boldsymbol{d}_{\eta 3} = [0, 0, -3]^T$, $\boldsymbol{d}_{\eta 4} = [0, 16, 0]^T$ and $\boldsymbol{d}_{\eta 5} = [-8, -8, 0]^T$.

Controller parameters for global leader are chosen as $\delta = 0.1$, $k = 0.5$, $\mathbf{h} = [1\ 0\ 0\ 0\ 0\ 0; 0\ 0\ 0\ 0\ 1\ 0; 0\ 0\ 0\ 0\ 0\ 1\ ]$, $\xi = 50$ and $\kappa = 0.5$. Controller parameters for followers are chosen as $\lambda_1 = \mu_1 = 2$, $\lambda_2 = \mu_2 = 1$, $\lambda_3 = \mu_3 = 0.1$, and $\mathbf{K} = diag[50, 45, 40]$. Sliding mode controller parameters for followers are the same as global leader.

Simulation results are shown in Fig. 3–Fig. 11. The actual and desired trajectory of the global leader are represented by $\boldsymbol{\eta}_{1gl}$ and $\boldsymbol{\eta}_{1gld}$. The trajectory error of three directions in space for the global leader is represented by ex, ey and ez, namely space trajectory error of the global leader. The actual space distance of the i-th follower with respect to its leader is adopted to describe space tracking performance of followers, represented by ex_i, ey_i and $ez_i (i = 1, 2, 3, 4, 5)$. The actual trajectories of six AUVs are represented by $\boldsymbol{\eta}_{1gl}$ and $\boldsymbol{\eta}_{1fi} (i = 1, 2, 3, 4, 5)$, where $\boldsymbol{\eta}_{1gl}$ is indicated to the global leader and $\boldsymbol{\eta}_{1fi}$ is indicated to the other AUVs.

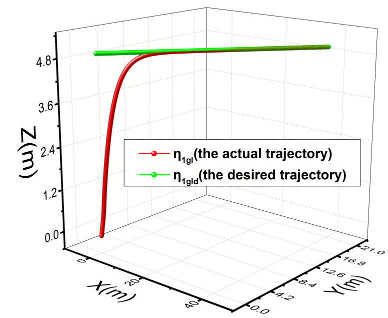

FIGURE 3. The actual and desired trajectory of the global leader.

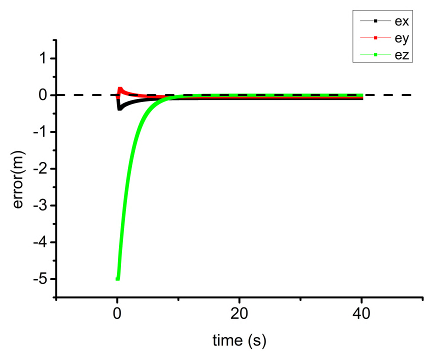

FIGURE 4. Space trajectory error of the global leader.

Fig. 3 shows the actual and desired trajectory of the global leader. The actual trajectory of the global leader converges to desired trajectory in the end. Fig. 4–Fig. 9 show space trajectory of six AUVs with respect to its leader. Space trajectory error of the global leader converges to zero at around 10 second as depicted in Fig. 4. Space trajectory of followers are nearly identical to desired space distance with respect to its leader in the end as shown in Fig. 5–Fig. 9. It is observed that there is a little deviation for space trajectory. As presented in global leader motion controller, $\boldsymbol{\rho}$ is the radius of the error space and result in the deviation. It is also applied in follower controller, which leads to deviation for space trajectory error of followers. With a designed $\boldsymbol{\rho}$ as relatively small value compared with length of AUV, namely $\boldsymbol{\rho} = [0.1, 0, 0]^T$, space trajectory deviation of global leader and followers is tolerant. Fig. 10 shows the trajectories of six AUVs. Fig. 11 shows position of six AUVs at some specific time. The dot represent position of an AUV at a specific time. The dashed line connects two AUV at the same specific time. Different color represent different time. It is observed that six AUVs form triangular prism in the end from Fig. 10 and Fig. 11.

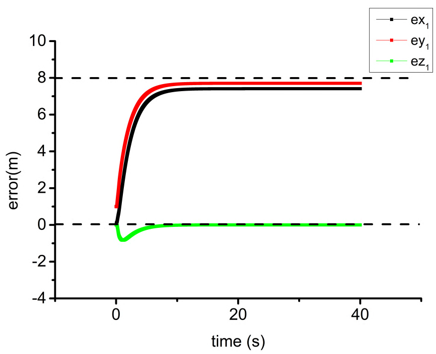

FIGURE 5. Space trajectory of the first follower AUV with respect to its leader.

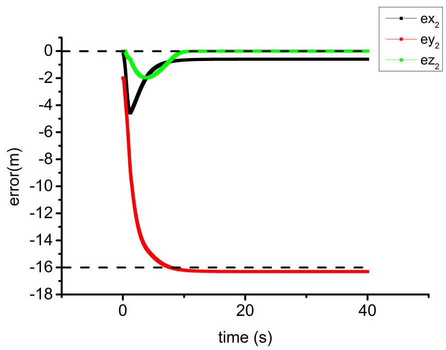

FIGURE 6. Space trajectory of the second follower AUV with respect to its leader.

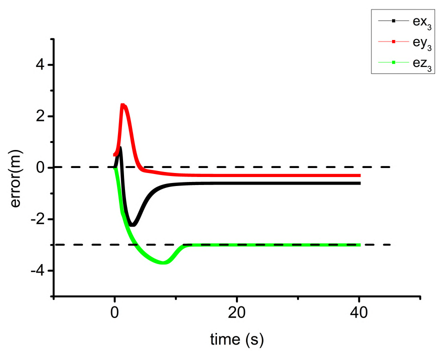

FIGURE 7. Space trajectory of the third follower AUV with respect to its leader.

Simulation results show six AUVs successfully achieve triangular prism formation in three-dimensional space with proposed controllers for global leader and followers, where there are only position information transmission between leader and follower. It is obvious that controllers proposed in the paper can guarantee the entire group coordination for multiple AUVs to achieve a desired formation. It is noted that the proposed controllers need continuously monitor motion state of AUVs. For practical applications, it is better to make further research on controllers based on sampled-data control, such as [28], [29], which will save limited resources, especially limited information transmission speed for AUVs.

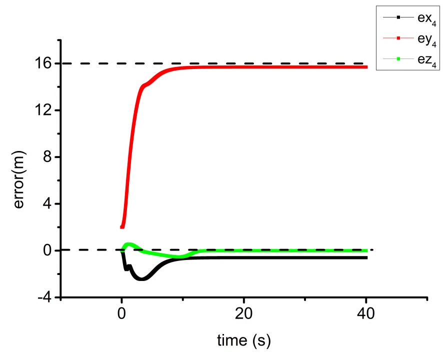

FIGURE 8. Space trajectory of the fourth follower AUV with respect to its leader.

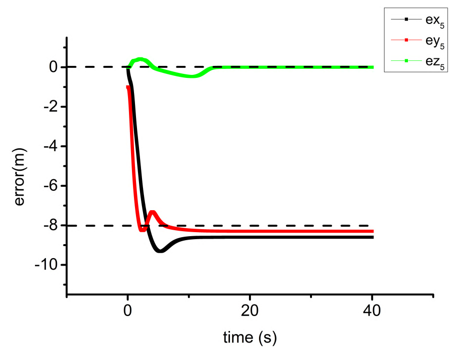

FIGURE 9. Space trajectory of the fifth follower AUV with respect to its leader.

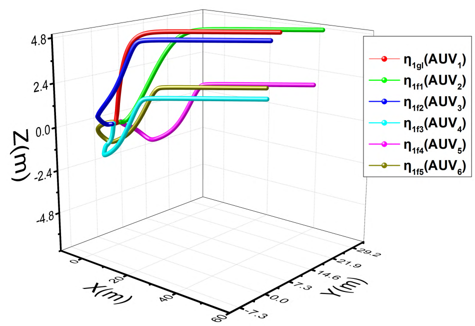

FIGURE 10. The trajectories of six AUVs.

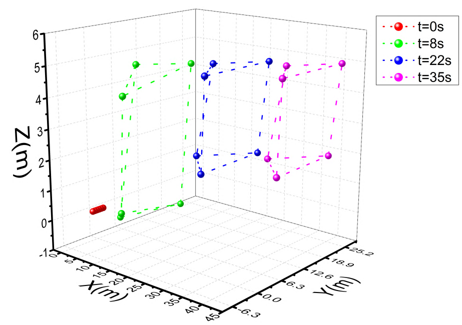

FIGURE 11. Triangular prism formation of six AUVs.

V. CONCLUSION

In this paper, a three-dimensional coordination control scheme based leader-follower strategy is presented with a introduced virtual trajectory for Multiple AUVs. Global leader motion and follower controller are designed to achieve three-dimensional coordination control. Based on sliding mode control and backstepping technique, the proposed control law can ensure trajectory tracking in three-dimensional space for the global leader. A virtual trajectory is proposed for follower controller to achieve only positional information transmission between leader and follower. Sliding mode control and backstepping technique are also applied to obtain final follower controller. A triangular prism formation composed of six AUVs tests the performance of this coordination control scheme. Simulation results shows that six AUVs reach the expected positions and accomplish triangular prism formation in the end. The proposed coordination control scheme is validated and effective in three-dimensional space.

In future research, the proposed controllers will be tested by pool experiment. Limited resources of AUVs should be further considered for controllers design, and it is worth of extending results in this paper to sampled-data control based on some consensus criteria [29], [30].

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