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A New Sequential Surrogate Method for Reliability Analysis and Its Applications in Engineering

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ABSTRACT In reliability analysis for the practical engineering problems with the time-consuming model, it has become an important challenge that how to obtain accurate reliability assessment with a minimum number of calls. In order to reduce the computational cost, this paper develops a new sequential surrogate method combining adaptive kriging and Markov chain Monte Carlo simulation with a novel learning strategy for reliability analysis. The proposed method is named AK-MCMC, which takes full advantage of the classification feature of reliability analysis based on the surrogate models, and it can efficiently approximate the classification boundary of the performance function. First, the learning strategy is developed to sequentially pick out the informative samples for updating the experimental design samples. Then, a new stopping criterion is adopted to guarantee the classification accuracy of the constructed kriging model. In this way, the proposed method skillfully makes reliability evaluation independent of an adaptive iterative process, which greatly improves the efficiency of model refinement. Finally, the proposed method is applied to several examples, which contain small failure probability problem, non-linearity problem, and engineering problem with an implicit performance function. In particular, the efficiency of the proposed AK-MCMC method is proved for the problems with small failure probability.

INDEX TERMS Reliability analysis, kriging model, Markov chain, learning strategy, classification accuracy.

I. INTRODUCTION

In practical engineering problems, there are often various uncertainties that ultimately affects the structural safety, such as material properties, structural dimensions, boundary conditions. Reliability analysis has become increasingly important in engineering analysis in recent years, and it can quantify the structural safety by considering these uncertainties. In reliability analysis [1], a given vector $\boldsymbol{x} = [x_1, x_1, \cdots, x_n]$ of random variables with n-dimension affects the performance of a system or structure in variable space Ω. According to the joint probability density function $f(\boldsymbol{x})$ of $\boldsymbol{x}$, probability of failure P_F can be calculated by the following integral:

$$P_F = P\{G(\boldsymbol{x}) \leq 0\} = \int_{G(\mathbf{x})\leq 0} f(\boldsymbol{x})\, d\boldsymbol{x} \tag{1}$$

where $G(\boldsymbol{x})$ is the performance function of a system or structure.

Since the multi-dimensional numerical integration (1) is time-consuming in the failure domain, it is hard to be directly solved for general engineering problems. In the past few decades, several methods have been developed to assess the failure probability, and Monte Carlo simulation (MCS) is the most convincing one. MCS converts the integral problem of (1) into the summation problem of (2).

$$P_F = \int_{R^n} I(\boldsymbol{x}) f(\boldsymbol{x}) d\boldsymbol{x} \approx \frac{1}{N_{mcs}} \sum_{i=1}^{N_{mcs}} I(\boldsymbol{x}) \tag{2}$$

where R^n shows the n-dimensional variable space; $I(\boldsymbol{x})$ is an indicator function as a two-class classifier, $I(\boldsymbol{x}) = 0$ when $G(\boldsymbol{x}) > 0$, and $I(\boldsymbol{x}) = 1$ when $G(\boldsymbol{x}) \leq 0$. The definition of indicator function $I(\boldsymbol{x})$ converts the original problem into a binary classification problem. But for low failure probabilities, the computational burden of using direct MCS becomes intolerable [2]. Many analytical methods have

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been developed to avoid this problem. For the purpose of balancing efficiency and accuracy, the First Order Reliability Method (FORM) and Second Order Reliability Method (SORM) [3], [4] based on most probable point (MPP) are developed to approximately compute the failure probability. However, the results of failure probability computed by FORM or SORM are not accurate for problems with high-dimensional, high non-linear or implicit performance function [3], [4]. Therefore, it is necessary to develop some new methods for accurately evaluating the probability of failure.

In the field of reliability analysis, digital simulation methods and surrogate models have been paid increasing attention in recent years. Many digital simulation methods have been proposed and developed, such as Importance Sampling (IS) method [5], [6], Subset Simulation (SS) method [7], [8] and Line Sampling (LS) method [9], [10]. IS [5] generates the weighted random samples according to an important sampling density distribution function, rather than the original distribution. In IS, the determination of importance sampling density function is a hard and inefficient, especially for problems with high nonlinear performance functions, or multiple design points [6]. SS was proposed by Au and Beck [8], which is a powerful tool for small probabilities, separates the probability space of original variables into a series of subsets, and then computes the probability of failure as a product of conditional probabilities. However, the selection of the proposal distribution of Markov Chain Monte Carlo (MCMC) has vital influence on the result of reliability assessment, and the conditional sample points generated by the MCMC simulation have a certain correlation, which will reduce the computational accuracy of SS [7]. LS [9], [10] is used to evaluate the probability of failure based on the optimal important direction, which is from the origin of coordinate to the MPP in the standard normal space. But the efficiency of LS will gradually decline as the deviation of the important direction from the optimal important direction. Generally, digital simulation methods are not practical in the reliability analysis where the engineering problems are involved.

Surrogate models are also important approach to be used for increasing the computational efficiency in complex engineering analysis. In reliability analysis based on surrogate model, the true performance function is replaced by a surrogate model to evaluate the failure probability. At present, the most common methods are Response Surface Methodology (RSM) [11], [12], Support Vector Machines (SVM) [1], [13], [14], Artificial Neural Networks (ANN) [15] and Kriging model [16], [17]. The RSM uses a polynomial function to replace the original performance function through a sequence of experimental design samples. SVM [1], [14] can explicitly divide the disjoint and non-convex boundaries between failure domains and safety domains. ANN [15] is trained with some actual experimental design samples and then is used to replace the original function. Guarascio *et al.* [18] and Matheron [19] proposed and developed Kriging model, which has been used widely as an exact interpolation method. Due to the stochastic propriety of the Kriging model, it can not only estimate the predicted values in any points, but also calculate estimations of the local variance on the prediction points. This variance characterizes of the prediction is termed as the Kriging variance. However, the key to construct a high-precision surrogate model is how to determine experimental design samples, and the methods to determine samples are often named Design of Experiments (DoE) [20], [21]. However, the prediction accuracy of the surrogate models is usually unsatisfactory in reliability analysis until the adaptive sequential surrogate methods [22], [23] are proposed.

In recent years, Adaptive sequential surrogate methods [14], [24]–[35] have been developed in structural reliability analysis, especially the methods based on Kriging model. To assess the reliability of structures in a more efficient way, Echard et al. proposed an iterative method combining Kriging model with MCS, which is called AK-MCS [24]. Later, Echard et al. developed a method based on IS and Active Kriging model, which is called AK-IS [25]. Zheng *et al.* [26] improved the AK-MCS method to improve its speed of convergence for problems with a connected domain of failure. Zhao *et al.* [27] proposed an efficient reliability method based on adaptive IS and Kriging model with an active learning strategy. Lv *et al.* [28] proposed a new learning function H for Kriging based on information entropy theory. Huang *et al.* [29] proposed a new method based on Kriging model and SS, called AK-SS, which can make full use of all statistical information obtained from the constructed Kriging models to improve the computational efficiency of the AK-MCS. Balesdent *et al.* [30] proposed a method combining Kriging and adaptive IS for problems with a small failure probability. Sun *et al.* [31] proposed the least improvement function (LIF) to enrich DoE, and the accuracy of constructed Kriging model is improved for reliability analysis. Leliévre *et al.* [32] proposed the AK-MCSi method for solving the difficulty to assess small probabilities of failure and inability to parallelize computations with the AK-MCS method.

In addition, available adaptive sequential surrogate methods based on other surrogate model for reliability analysis also have been extensively studied in recent years. Basudhar and Missoum [23] proposed an adaptive explicit decision function method using SVM with a learning strategy and a new stopping criterion, and the method is further expanded as an efficient reliability method combining adaptive SVM and MCS (ASVM-MCS) [33]. Basudhar and Missoumy [14] developed an adaptive reliability assessment method that can quantify the probability of having an error in the approximation of the boundary of failure using probabilistic support vector machines (PSVMs). Xiao *et al.* [34] proposed a new adaptive sequential sampling method base on back-propagation (BP) neural network for efficient reliability analysis, and the method can provide an efficient manner with multiple failure modes. Keshtegara and Kisi [35] proposed the Radial basis M5Tree (RM5Tree) method by improving M5 model tree (M5Tree), and the RM5Tree method

can reduce computational efforts for high dimensional reliability problems. Li *et al.* [36] proposed a sequential surrogate reliability method (SSRM) based on radial basis function, and a special optimization problem in SSRM is solved to iteratively update the surrogate model of the limit state function.

Nevertheless, these adaptive sequential surrogate methods mentioned above have a common disadvantage in iterative process. The disadvantage is that adaptive sampling and reliability assessment are nested with each other, which makes the samples obtained by adaptive sampling dependent with the reliability evaluation method and reduces the iteration efficiency of adaptive sequential sampling algorithm. Since there are fewer available samples in the vicinity of failure boundary, the concentration phenomenon of samples in DoE is more prone to occur with the adaptive sequential surrogate method based MCS. The computational efficiency of those adaptive methods based MCS is low, especially for problems with small failure probabilities. In order to make reliability analysis more efficient, this work develops a new adaptive sequential surrogate method that makes reliability evaluation independent from adaptive iterative process.

This paper includes the following parts. Section 2 presents the basic theory of Kriging model and MCMC simulation. Section 3 describes the detailed principles and process of the proposed AK-MCMC method for reliability analysis. Section 4 illustrates the proposed AK-MCMC method with three classical numerical examples and one practical engineering example, and the reliability analysis results are also compared with some other available adaptive sequential sampling methods in each example. Finally, conclusions and future work is summarized in Section 5.

II. BASIC THEORY

A. KRIGING MODEL

Kriging model is a statistical theory-based interpolation technique, and it consists of a parametric linear regression model and a nonparametric stochastic process [24], [25], [28]. Assuming those input variables are defined as $\boldsymbol{x}$ and the corresponding response is defined as $G(\boldsymbol{x})$, Kriging is given as:

$$\hat{G}(\boldsymbol{x}) = \boldsymbol{F}(\boldsymbol{\beta}, \boldsymbol{x}) + z(\boldsymbol{x}) = \boldsymbol{f}^T(\boldsymbol{x})\boldsymbol{\beta} + \delta(\boldsymbol{x}) \tag{3}$$

where an averaged approximation $F(\boldsymbol{\beta}, \boldsymbol{x})$ of the response is the deterministic part, and it can be expressed as an ordinary polynomial regression of $\boldsymbol{x}$. β is the vector of regression coefficients. $F(\boldsymbol{\beta}, \boldsymbol{x})$ is simplified to a constant in the ordinary Kriging, and is taken as $F(\boldsymbol{\beta}, \boldsymbol{x}) = \beta$. All the following formulas are deduced with the simplification of ordinary Kriging. So $\hat{G}(\boldsymbol{x})$ can be simplified as

$$\hat{G}(\boldsymbol{x}) = \beta + \boldsymbol{\delta}(\boldsymbol{x}) \tag{4}$$

Here $\boldsymbol{\delta}(\boldsymbol{x})$ is a zero-mean stationary Gaussian process with auto-covariance at samples $\boldsymbol{x}$ and $\boldsymbol{w}$ defined as

$$\mathrm{cov}(\boldsymbol{\delta}(\boldsymbol{x}), \boldsymbol{\delta}(\boldsymbol{x})) = \sigma^2 R(\boldsymbol{x}, \boldsymbol{w}) \tag{5}$$

where σ^2 is the process variance. Auto-correlation function $R(\boldsymbol{x}, \boldsymbol{w})$ can also be considered several functions, such gauss correlation function, exponential correlation function, and cubic correlation function. In this paper, the Gauss correlation function is employed, and it can be expressed as (6).

$$R(\boldsymbol{x}, \boldsymbol{w}) = \exp\left(-\sum_{i=1}^{n} \theta_i |x_i - w_i|^2\right) \tag{6}$$

where n is the dimension of design variables. x_i and w_i denote the ith component of variable $\boldsymbol{x}$ and variable $\boldsymbol{w}$, respectively. θ_i is the correlation parameter to ensure the Kriging model with an high flexibility.

A sample set DoE with N_d experimental samples $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{N_d}]^T (\boldsymbol{x}_i \in R^n)$ and theirs true responses $\boldsymbol{Z} = [G(\boldsymbol{x}_1), G(\boldsymbol{x}_2), \cdots, G(\boldsymbol{x}_{N_d})]^T G(\boldsymbol{x}_i) \in R$ are given. Defining $\boldsymbol{R} = [R(\boldsymbol{x}_i, \boldsymbol{x}_j)]_{N_d \times N_d}$ and $\boldsymbol{F}$ as a $N_d \times 1$ unit vector, the unknown parameters β and σ^2 can be deduced by least squares method:

$$\hat{\beta} = \left(\boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{F}\right)^{-1} \boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{Z} \tag{7}$$

$$\hat{\sigma}^2 = \frac{1}{N_d} \left(\boldsymbol{Z} - \boldsymbol{F}\hat{\beta}\right)^T \boldsymbol{R}^{-1} \left(\boldsymbol{Z} - \boldsymbol{F}\hat{\beta}\right) \tag{8}$$

The parameter β of regression coefficients and the parameter σ^2 of constant process are dependent on the correlation parameters θ through the correlation matrix. Therefore, the correlation parameters θ must first be determined by maximum likelihood estimation with (9).

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left\{\boldsymbol{R}(\boldsymbol{\theta}) \left[\hat{\sigma}^2\right]^{N_d}\right\} \tag{9}$$

For an unknown predicted sample $\boldsymbol{x}_p$ with θ, β and σ^2 known, the linear unbiased estimator of the response and Kriging variance are computed as

$$\hat{G}(\boldsymbol{x}_p) = \hat{\beta} + \boldsymbol{r}(\boldsymbol{x}_p)^T \boldsymbol{R}^{-1} \left(\boldsymbol{Z} - \boldsymbol{F}\hat{\beta}\right) \tag{10}$$

$$\hat{\sigma}^2_{\hat{G}}(\boldsymbol{x}_p) = \hat{\sigma}^2 \left(1 + \boldsymbol{u}(\boldsymbol{x}_p)^T \left(\boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{F}\right)^{-1} \boldsymbol{u}(\boldsymbol{x}_p) - \boldsymbol{r}(\boldsymbol{x}_p)^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}_p)\right) \tag{11}$$

where $\boldsymbol{r}(\boldsymbol{x}_p)^T$, is a n_s dimensional row vector representing the correlative relations between unknown predicted sample $\boldsymbol{x}_p$ and experimental samples $\boldsymbol{X} = [\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{N_d}]^T$, expressed as $\boldsymbol{r}(\boldsymbol{x}_p)^T = [R(\boldsymbol{x}_p, \boldsymbol{x}_1), R(\boldsymbol{x}_p, \boldsymbol{x}_2), \cdots, R(\boldsymbol{x}_p, \boldsymbol{x}_{N_d})]^T$, and $\boldsymbol{u}(\boldsymbol{x}_p)^T = \boldsymbol{F}^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x}_p) - 1$.

From (10), the gradient $\hat{G}'(\boldsymbol{x}_p) = [\frac{\partial \hat{G}}{\partial x_{p1}}, \frac{\partial \hat{G}}{\partial x_{p2}}, \cdots, \frac{\partial \hat{G}}{\partial x_{pn}}]^T$ can be expressed [37] as

$$\hat{G}'(\boldsymbol{x}_p) = \boldsymbol{J}_r(\boldsymbol{x}_p)^T \boldsymbol{R}^{-1} \left(\boldsymbol{Z} - \boldsymbol{F}\hat{\beta}\right) \tag{12}$$

where $\boldsymbol{J}_r$ is the jacobian of $\boldsymbol{r}(\boldsymbol{x}_p)$,

$$\left(\boldsymbol{J}_r(\boldsymbol{x}_p)\right)_{ij} = \frac{\partial \left(R(\boldsymbol{x}_p, \boldsymbol{x}_i)\right)}{\partial x_{pj}} \tag{13}$$

The process of calculating $\hat{G}(\boldsymbol{x}_p)$, $\hat{\sigma}^2_{\hat{G}}(\boldsymbol{x}_p)$, $\hat{G}'(\boldsymbol{x}_p)$ can be implemented by the MATLAB toolbox DACE [36], which has been applied in several references [28], [31].

B. MARKOV CHAIN MONTE CARLO SIMULATION

Markov Chain Monte Carlo (MCMC) simulation is a powerful approach for sampling according to an arbitrary probability distribution. In MCMC, samples are simulated as the states of a Markov Chain which has the target probability distribution as its limit stationary distribution under the Traversal Hypothesis. Since the MCMC method with the Metropolis-Hastings algorithm becomes inefficient to simulate samples with many independent components, Au and Beck [8] proposed a component-wise (or modified M–H) algorithm to improve the sampling efficiency of MCMC. Instead of using an n-dimensional proposal PDF in the original method, each coordinate ξ_j of the pre-candidate ξ is generated from a one-dimensional proposal PDF $p_j^*(\xi_j \mid x_j)$, which depends on the jth coordinate x_j of the current state. And the proposal PDF $p_j^*(\xi_j \mid x_j)$ has a symmetry property, i.e. $p_j^*(\xi \mid x) = p_j^*(x \mid \xi)$.

In this paper, the MCMC simulation with component-wise algorithm is employed to generate N_m samples that are lying in the required region. The process of generating a sequence of samples $\{\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{N_m}\}$ from a given or randomly generated sample $\boldsymbol{x}_1$ to $\boldsymbol{x}_{N_m}$ is briefly summarized as follows [39]:

1) GENERATE CANDIDATE SAMPLE $\tilde{x}$

(a) Generate ξ_j by sampling from the proposal PDF $p_j^*(\xi_j \mid x_{kj})$.

In the process of Markov Chain, the transfer of a state to another state is controlled by the proposal distribution. In this paper, an n-dimensional uniform distribution within the interval $[x_{kj} - l_j/2, x_{kj} + l_j/2]$ $(j = 1, 2, \cdots, n)$ is selected as the proposal distribution, that is,

$$p_j^*(\xi_j \mid x_{kj}) = \begin{cases} 1/l_j & if\ |\xi_j - x_{kj}| \le l_j/2 \\ 0 & else \end{cases} \tag{14}$$

where ξ_j and x_{kj} are the jth component of n dimensional vector $\boldsymbol{\xi}$ and the kth sample$\boldsymbol{x}$ of MCMC, respectively. l_j is the side length of x_j component of n dimensional hypercube, and it determines the maximum allowable range of the next sample deviate from the current sample. The l_j takes the empirical value $6N_m^{-1/(n+4)}$.

(b) Accept or reject ξ_j

$$\tilde{x}_j = \begin{cases} \xi_j, & \text{with probability } min\left\{1, \dfrac{q_j(\xi_j)}{q_j(x_{kj})}\right\} \\ x_{kj}, & \text{with probability } 1 - min\left\{1, \dfrac{q_j(\xi_j)}{q_j(x_{kj})}\right\} \end{cases} \tag{15}$$

where $q_j(\cdot)$ is the jth coordinate x_j of the limit stationary distribution $\boldsymbol{q}(\boldsymbol{x})$ of Markov Chain.

2) ACCEPT OR REJECT $\tilde{x}$

$$\boldsymbol{x}_{k+1} = \begin{cases} \tilde{x} & \tilde{x}\ in\ \mathbf{D_F} \\ \boldsymbol{x}_k & \tilde{x}\ not\ in\ \mathbf{D_F} \end{cases} \tag{16}$$

where $\mathbf{D_F}$ is the required region. (16) expresses that the next sample $\boldsymbol{x}_{k+1}$ is set to candidate sample $\tilde{x}$ or the last sample $\boldsymbol{x}_k$ according to whether candidate sample $\tilde{x}$ lies in $\mathbf{D_F}$ or not.

Here, a brief introduction to MCMC simulation is only provided, and the details may consult the references [8], [38].

III. THE PROPOSED AK-MCMC METHOD FOR RELIABILITY ANALYSIS

Generally, a given structure or mechanism with a determined failure mode can be expressed as

$$Z = \mathrm{g}(\boldsymbol{y}) \tag{17}$$

where $Z = \mathrm{g}(\boldsymbol{y})$ is the performance function. The safe and failure domains are defined as $Z > 0$ and $Z \le 0$, respectively. The vector $\boldsymbol{y} = (y_1, y_2, \cdots, y_n)$ denotes random variables. The performance function $\mathrm{g}(\boldsymbol{y})$ is often an implicit function in practical engineering problems, and the repeated finite element simulation or dynamic simulation of structures or mechanisms is usually time-consuming. Surrogate models are able to reduce the computational burden significantly. How to efficiently construct an accurate surrogate model with a few samples has been an important challenge in reliability analysis and reliability optimization.

Kriging model is often used to substitute the original performance function to accelerate the process of reliability analysis [24], [26], [28]. In recent studies, adaptive sequential sampling methods based on Kriging model have the disadvantage that adaptive sampling and reliability assessment are nested with each other. Iteration efficiency of adaptive sequential sampling with these algorithms is lower. For the purpose of improving the iteration efficiency of sequential sampling, this work proposes a new adaptive sequential surrogate method combining **A**daptive **K**riging and **M**arkov **C**hain **M**onte **C**arlo simulation, and the method is called AK-MCMC.

In AK-MCMC, the adaptive iterative process is independent with reliability evaluation, and Kriging model is used to avoid a great amount of calls to the true performance function. The algorithm of AK-MCMC can be divided into three phases including initial preparation phase, model refinement phase and reliability evaluation phase. The initial preparation phase consists of determining the sampling space, constructing the initial Kriging model, and generating the test samples. The model refinement phase consists of generating MCMC samples, selecting the optimal sample and updating DoE, updating the Kriging model and calculating the test index. After obtained the final Kriging model, the failure probability is calculated in reliability evaluation phase. The process of the proposed AK-MCMC method for reliability analysis can be seen in Fig.1.

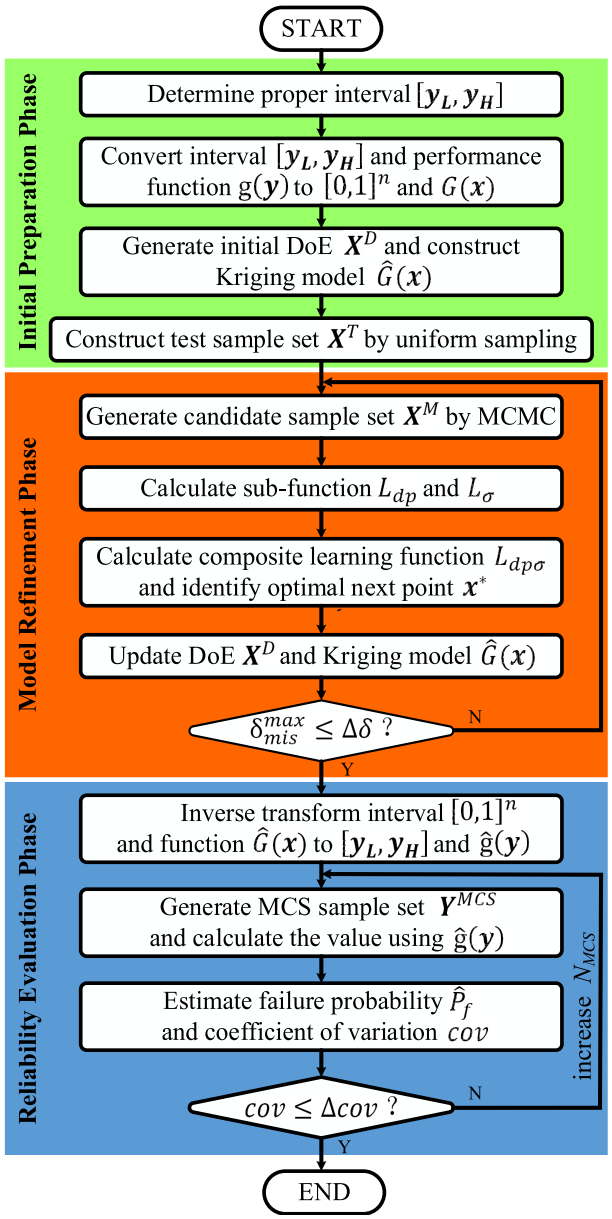

FIGURE 1. The algorithm flowchart of the proposed AK-MCMC method.

A. INITIAL PREPARATION

1) DETERMINE PROPER INTERVALS

In the initial preparation phase of the proposed AK-MCMC method, the first step is to define a proper interval $y_i \epsilon [y_{iL}, y_{iH}]\,(i = 1, 2, \cdots, n)$ for each random variable. Two approaches for determining the proper interval have been developed by Rashki *et al.* [39] and Hamzehkolaei *et al.* [40]. The first one is to generate MCS samples according to probability distribution of each random variable. Another one is based on a conservative assumption of the reliability index. According to the approaches proposed by Rashki *et al.* [39], two other similar methods are developed. One method to determine interval is that the user directly gives it according to the actual situation. Another is a practical and intuitive approach according to the inverse cumulative distribution function (CDF). With the practical approach, the lower and upper bounds of the intervals $[y_{iL}, y_{iH}]$ can be obtained from (18) and (19), respectively.

$$y_{iL} = F_i^{-1}(\Phi(-\tau_i)) \tag{18}$$

$$y_{iH} = F_i^{-1}(1 - \Phi(-\tau_i)) \tag{19}$$

where $\Phi(\cdot)$ denotes the CDF of standard normal distribution. F_i^{-1} is defined as the inverse CDF of the ith dimension y_i of random variable $\boldsymbol{y}$. τ_i denotes a scale parameter of the ith dimension y_i of random variable $\boldsymbol{y}$, and it can represent the length of the variable interval $[y_{iL}, y_{iH}]$. The value of the scale parameter τ is recommended between 4 and 8.

2) SPACE TRANSFORMATION

For the purpose of eliminating the influence of the variable magnitudes, the second step is the transformation of sampling space $[\boldsymbol{y_L}, \boldsymbol{y_H}]$ and performance function (g). The sampling space $[\boldsymbol{y_L}, \boldsymbol{y_H}]$ is converted to $[0, 1]^n$, and the transformation between $\boldsymbol{x} \in [0, 1]^n$ and $\boldsymbol{y} \in [\boldsymbol{y_L}, \boldsymbol{y_H}]$ can be expressed as

$$\boldsymbol{x} = \frac{\boldsymbol{y} - \boldsymbol{y_L}}{\boldsymbol{y_H} - \boldsymbol{y_L}} \tag{20}$$

Then, the performance function (g) is also converted to the corresponding function (G). The relationship between function (G) and performance function (g) can be expressed as

$$\mathrm{g}(\boldsymbol{y}) = \mathrm{g}\left(\boldsymbol{x}\left(\boldsymbol{y_H} - \boldsymbol{y_L}\right) + \boldsymbol{y_L}\right) = G(\boldsymbol{x}) \tag{21}$$

3) GENERATE INITIAL DOE AND CONSTRUCT KRIGING MODEL

The next step is to generate N_d Latin Hypercube Design (LHD) [24], [25] samples for initial DoE $\boldsymbol{X}^D$ in the converted sampling space $[0, 1]^n$, and the ith random sample of $\boldsymbol{X}^D$ can be shown by $\boldsymbol{x}_i^D = \left[x_{i1}^D, x_{i2}^D, \cdots, x_{in}^D\right]$. Then the according performance values $[G(\boldsymbol{x}_1^D), G(\boldsymbol{x}_2^D), \cdots, G(\boldsymbol{x}_{N_d}^D)]^T$ are evaluated by calling to the performance function (G) for all samples in $\boldsymbol{X}^D$. The sample point set $\left(\boldsymbol{X}^D, G\left(\boldsymbol{X}^D\right)\right) = \left\{(\boldsymbol{x}_i, G(\boldsymbol{x}_i)) | \boldsymbol{x}_i \in \boldsymbol{X}^D, i = 1, 2, \cdots, N_d\right\}$ is used to train the Kriging model $\hat{G}(\boldsymbol{x})$. The number N_d of initial DoE is taken as $\lceil a\sqrt{n} \rceil$ (the symbol '$\lceil\cdot\rceil$' is an upwardly integer operator symbol), and it means that the average number of samples projected on the unit diagonal is equal to a. a can generally be selected as 6-12, and it is taken as 6 in this paper. For more accurate initial Kriging model, a larger value should be selected. $\sqrt{n}$ equals to the length of the diagonal of $[0, 1]^n$, and it can be used to represent the volume of the hypercube.

4) CONSTRUCT TEST SAMPLES

A constructed surrogate model should have a higher classification accuracy in reliability analysis. A test sample set is adopted to catch the global capacity of the model classification and update training samples, and it is termed by $\boldsymbol{X}^T$. There are many available uniform sampling approaches, such as Uniform Design, Halton Sampling and Latin Hypercube

Sampling (LHS). For making the samples in $\boldsymbol{X}^T$ with maximum possible uniformity, the LHS is used to generate N_T uniform samples in $[0, 1]^n$ for constructing the test sample set $\boldsymbol{X}^T = \left[\boldsymbol{x}_1^T, \boldsymbol{x}_2^T, \cdots, \boldsymbol{x}_{N_T}^T\right]^T$. In this work, the reference value of N_T is recommended to be set as $\left[b\sqrt{n}\right]$, and it means that the average number of samples projected on the unit diagonal is equal to b. As a result of the inexpensive calculation of Kriging model, the parameter b can be set to a larger value, and $b = 10^5$ is sufficient for a more accurate result in the proposed AK-MCMC method.

B. MODEL REFINEMENT

For adaptive sequential sampling methods, one of the main concerns is to determine the locations of the selected new training samples in iterations. A learning function is necessary to the proposed AK-MCMC method. The learning function is used to select the optimal new training samples to evaluate performance by calling the true performance function. For the purpose of selecting the optimal new training samples at each iteration, a novel composite learning function $L_{dp\sigma}$ is developed, and it consists of sub-functions L_{dp} and L_σ. The first sub-function L_{dp} focuses on the nearest distance $d_{nearest}$ from the next optimal sample to the existing training samples and the misclassification probability P_m of the next optimal sample, simultaneously. The sub-function L_{dp} contributes to select the optimal new training samples that locate not only far away from the existing training samples, but also having a higher chance of being misclassified. The second sub-function function L_σ focuses on the points having a higher uncertainty with the current Kriging model $\hat{G}(\boldsymbol{x})$. The composite learning function $L_{dp\sigma}$ takes into account the advantages and characteristics of the two sub-functions mentioned above. In the following subsections, these two sub-functions and composite learning function are introduced in details.

1) GENERATION OF CANDIDATE SAMPLES

The candidate sample set $\boldsymbol{X}^M$ should be constructed before selecting the new optimal sample. In this work, the shortcomings of candidate samples near the limit-state of $\hat{G}(\boldsymbol{x})$ in the adaptive methods based on MCS are overcome. The MCMC simulation as described in section 2.2 is employed to construct candidate samples.

The limit stationary distribution $\boldsymbol{q}(\boldsymbol{x})$ and the proposal distribution $p_j^*\left(\xi_j \mid x_j\right)$ of the MCMC are set to be evenly distributed to make the candidate samples as evenly distributed as possible near the limit state. Simultaneously, these candidate samples should be located in the area, where the distance any sample to the limit-state of $\hat{G}(\boldsymbol{x})$ is not greater than the allowable value Δd. The distance between the sample $\boldsymbol{x}_i^M (i = 1, 2, \cdots, N_M)$ in $\boldsymbol{X}^M$ and the hyper-surface $\hat{G}(\boldsymbol{x}) = 0$ can be approximately expressed as

$$d\left(\boldsymbol{x}_i^M\right) = \frac{\left|\hat{G}\left(\boldsymbol{x}_i^M\right)\right|}{\left\|\hat{G}'\left(\boldsymbol{x}_i^M\right)\right\|} \tag{22}$$

In order to balance the efficiency and range of MCMC sampling, the allowable value Δd is set to $\frac{\sqrt{n}}{20}$, where n is the dimension of sample space in the paper. $d\left(\boldsymbol{x}_i^M\right) \leq \Delta d$ can be used to characterize that the occupy space of the generated candidate samples by MCMC is no more than 10% of the total $[0, 1]^n$ space. The number N_m of candidate samples is taken as $N_M = [c \times \Delta d \times 2]$, and $c = 10^4$ can be used to obtain a more accurate classification result in this paper.

2) SUB-FUNCTION L_{dp}

Selected new training samples should be kept away from the existing training samples to refrain information redundancy. Since two samples are very close to each other, the problem may be ill-conditioned for the constructing Kriging model. From the discussion mentioned above, the optimal new training samples should be located not only far away from the existing training samples, but also have a higher chance of being misclassified. In order to achieve this purpose, the sub-function L_{dp} is developed as

$$L_{dp}(\boldsymbol{x}) = d_{nearest}(\boldsymbol{x}) \times P_m(\boldsymbol{x}) \tag{23}$$

where $d_{nearest}(\boldsymbol{x})$ represents the distance between the sample $\boldsymbol{x}$ in $\boldsymbol{X}^M$ and its closest sample in $\boldsymbol{X}^D$. $P_m(\boldsymbol{x})$ is the misclassification probability of the sample $\boldsymbol{x}$ in $\boldsymbol{X}^M$. The indexes $d_{nearest}$ and P_m of the ith sample $\boldsymbol{x}_i^M$ can be calculated by (24) [32] and (25) [24], respectively.

$$d_{nearest}\left(\boldsymbol{x}_i^M\right) = \left|\boldsymbol{x}_i^M - \boldsymbol{x}_{nearest}\right| \tag{24}$$

$$P_m(\boldsymbol{x}_i^M) = \Phi\left(-\frac{\left|\hat{G}\left(\boldsymbol{x}_i^M\right)\right|}{\sigma_{\hat{G}}\left(\boldsymbol{x}_i^M\right)}\right) \tag{25}$$

3) SUB-FUNCTION L_σ

The variance of prediction is used to choose an optimal training sample. The candidate sample with the highest variance of prediction means that it has a higher uncertainty than the other candidate samples. For this reason, it should be selected as the optimal training sample to improve the classification accuracy of the constructed Kriging model. The sub-function L_σ is defined to determine the sample which has more significant effect for updating the current Kriging model $\hat{G}(\boldsymbol{x})$. The score for the ith candidate sample $\boldsymbol{x}_i^M$ is calculated by using the proposed sub-function L_σ.

$$L_\sigma\left(\boldsymbol{x}_i^M\right) = \sigma^2(\boldsymbol{x}_i^M) \tag{26}$$

where $\sigma^2\left(\boldsymbol{x}_i^M\right)$ is the variance of prediction of the ith candidate sample $\boldsymbol{x}_i^M$.

In this paper, Jackknifing and cross-validation techniques [41] are employed to estimate $\sigma^2(\boldsymbol{x}_i^M)$. Jackknifing is a classic resampling approach especially for variance estimation. The following steps [41], [42] are to estimate variance $\sigma^2(\boldsymbol{x}_i^M)$ of the prediction.

Step 1: Rewrite the existing N_d training samples to construct the combination as

$$\begin{aligned} \mathrm{S}_P &= (\boldsymbol{X}_D, G(\boldsymbol{X}_D)) \\ &= \left(\left(\boldsymbol{x}_1^D, G\left(\boldsymbol{x}_1^D\right)\right), \left(\boldsymbol{x}_2^D, G\left(\boldsymbol{x}_2^D\right)\right), \cdots, \left(\boldsymbol{x}_{N_d}^D, G\left(\boldsymbol{x}_{N_d}^D\right)\right)\right) \end{aligned} \tag{27}$$

In the light of the principle of k_s fold cross-validation, the existing training samples are partitioned into k_s sub-sets. The combinations of cross-validation samples are obtained by leaving one of the sub-sets out. For instance, the combinations of cross-validation samples with $k_s = N_d$ can be defined as

$$\begin{aligned} \mathrm{S}_P^{(-j)} &= \left(\boldsymbol{X}_D^{(-j)}, G\left(\boldsymbol{X}_D^{(-j)}\right)\right) \\ &= (\left(\boldsymbol{x}_1^D, G\left(\boldsymbol{x}_1^D\right)\right), \cdots, \left(\boldsymbol{x}_{j-1}^D, G\left(\boldsymbol{x}_{j-1}^D\right)\right), \\ &\quad \left(\boldsymbol{x}_{j+1}^D, G\left(\boldsymbol{x}_{j+1}^D\right)\right), \cdots, \left(\boldsymbol{x}_{N_d}^D, G\left(\boldsymbol{x}_{N_d}^D\right)\right)) \end{aligned} \tag{28}$$

where $j = 1, 2, \cdots, k_s$.

Step 2: Construct Kriging model based on the combination $\mathrm{S}_P^{(-j)}$ of k_s cross-validation samples and the complete combination S_P of all samples. Then the number of $k_s + 1$ Kriging models are constructed that can be expressed as

$$\tilde{G}^{(-j)}(\boldsymbol{x}) = \hat{G}^{(-j)}(\boldsymbol{x}) \quad \text{and} \quad \tilde{G}(\boldsymbol{x}) = \hat{G}(\boldsymbol{x}) \tag{29}$$

where $\hat{G}^{(-j)}(\boldsymbol{x})$ is the jth constructed Kriging model utilizing the jth combination $\mathrm{S}_P^{(-j)}$ of cross-validation samples, and $\hat{G}(\boldsymbol{x})$ is the constructed Kriging model adopting the complete combination S_P of all samples.

Step 3: Compute the pseudo-value of each candidate sample by employing Jackknifing approach. The pseudo-value of the ith candidate sample $\boldsymbol{x}_i^M$ can be obtained by

$$\tilde{G}_p^{ij} = k_s \times \tilde{G}\left(\boldsymbol{x}_i^M\right) - (k_s - 1) \times \tilde{G}^{(-j)}\left(\boldsymbol{x}_i^M\right) \tag{30}$$

where $i = 1, 2, \cdots, N_M$.

Step 4: Estimate the jackknife variance $\sigma^2(\boldsymbol{x}_i^M)$ of each candidate sample, which can be expressed as

$$\sigma^2\left(\boldsymbol{x}_i^M\right) = \frac{1}{k_s\left(k_s - 1\right)} \sum_{j=1}^{k_s} \left(\tilde{G}_p^{ij} - \overline{G}_p^i\right)^2 \tag{31}$$

where $\overline{G}_p^i = \frac{1}{k_s} \sum_{j=1}^{k_s} \tilde{G}_p^{ij}$.

4) COMPOSITE LEARNING FUNCTION $L_{dp\sigma}$

A composite learning function called $L_{dp\sigma}$ is proposed for the purpose of comprehensively considering the advantages and characteristics of the two sub-functions. To achieve this purpose, these two measures $\mathrm{L_{dp}}$ and L_σ must be normalized. The comprehensive score for the ith candidate sample $\boldsymbol{x}_i^M$ utilizing $L_{dp\sigma}$ can be expressed as

$$L_{dp\sigma}\left(\boldsymbol{x}_i^M\right) = \frac{\alpha L_{dp}\left(\boldsymbol{x}_i^M\right)}{\max\left(L_{dp}\left(\boldsymbol{X}^M\right)\right)} + \frac{(1-\alpha) L_\sigma\left(\boldsymbol{x}_i^M\right)}{\max\left(L_\sigma\left(\boldsymbol{X}^M\right)\right)} \tag{32}$$

where $\alpha \in [0, 1]$ denotes a weight coefficient, and its value can be given by a user. In this work, $\alpha = 0.5$ is adopted to demonstrate the proposed AK-MCMC method. It should be noted that the value of α can be adjusted according to the importance of the two measures L_{dp} and L_σ. For example, a small value can be given to α if the measure L_σ is more important than the measure L_{dp}. On the contrary, a larger value of α can be selected, if the measure L_{dp} is more important.

According to the proposed composite learning function $L_{dp\sigma}$, the property of the next optimal sample in both of input parameter space and output parameter space can be considered comprehensively. One of the main features of the proposed composite learning function $L_{dp\sigma}$ is that it can furnish a trade-off between these two measures. $\alpha = 0$ and $\alpha = 1$ represent that the composite learning function $L_{dp\sigma}$ is retrograded to the L_{dp} and L_σ, separately.

At each iteration, the candidate sample that maximizes the criterion in (32) is selected as the optimal training sample, and it response value is evaluated by calling to performance function $G(\boldsymbol{x})$.

5) UPDATING OF TRAINING SAMPLES

According to the proposed composite learning function $L_{dp\sigma}$, one training sample $\boldsymbol{x}^* \in \boldsymbol{X}^M$ is picked out from the candidate samples $\boldsymbol{X}^M$ at each iteration. Then the sample point set $\left(\boldsymbol{X}^D, G\left(\boldsymbol{X}^D\right)\right)$ can be enriched by adding the sample point $(\boldsymbol{x}^*, G(\boldsymbol{x}^*))$. With the enriched sample point set, the constructed Kriging models $\hat{G}^{(-j)}$ and $\hat{G}(\boldsymbol{x})$ will be reconstructed correspondingly at each iteration.

6) STOPPING CRITERION

The training sample point set and the constructed Kriging models are successively updated. A new stopping criterion is developed to fold up the model refinement phase of the proposed AK- MCMC method. The cross validation approach [34], [41] has been diffusely applied for evaluating the accuracy of the constructed surrogate models. The main advantage of this approach is that no extra performance function is needed to evaluate. In many existing cross validation methods, the k-fold cross validation is one of the most widely used [34].

To fully utilize the classification feature in reliability analysis, a misclassification index based on the principle of k fold cross validation is proposed to evaluate the impact of the last k samples of sample set $\boldsymbol{X}^D$ on the Kriging model. The misclassification index can be calculated by

$$\delta_{mis}^{(-j)} = \frac{1}{N_T} \sum_{i=1}^{N_T} \frac{\left|C_{label}(i) - C_{label}^{(-j))}(i)\right|}{2}, \quad j = 1, 2, \cdots, k \tag{33}$$

where $C_{label}(i) = \begin{cases} 1 & \hat{G}\left(\boldsymbol{x}_i^T\right) > 0 \\ -1 & \hat{G}\left(\boldsymbol{x}_i^T\right) \le 0 \end{cases}, i = 1, 2, \cdots, N_T,$ and C_{label} indicates the classification of samples in $\boldsymbol{X}^T$ under the Kriging model $\hat{G}(\boldsymbol{x})$ that is constructed by employing the complete combination S_P; $C_{label}^{(-j)}$ indicates the classification

of samples in $\boldsymbol{X}^T$ under the Kriging model $\hat{G}^{(-(N_d-j+1))}(\boldsymbol{x})$ that is constructed by employing the (N_d-j+1)th combination $S_P^{(-(N_d-j+1))}$, and it can be expressed as $C_{label}^{(-j)}(i) = \begin{cases} 1 & \hat{G}^{(-(N_d-j+1))}\left(\boldsymbol{x}_i^T\right) > 0 \\ -1 & \hat{G}^{(-(N_d-j+1))}\left(\boldsymbol{x}_i^T\right) \le 0 \end{cases}, i = 1, 2, \cdots, N_T$; $\delta_{mis}^{(-j)}$ is defined as the misclassification rate that can be used to describe the degree of difference between $\hat{G}^{(-(N_d-j+1))}(\boldsymbol{x})$ and $\hat{G}(\boldsymbol{x})$ on their respective classification boundaries; $\boldsymbol{x}_i^T$ is the ith sample in $\boldsymbol{X}^T$.

Subsequently, a new stopping criterion is defined as

$$\delta_{mis}^{max} \le \Delta\delta \tag{34}$$

where $\delta_{mis}^{max} = \max(\delta_{mis}^{(-j)})$ is the maximum misclassification rate, $j = 1, 2, \cdots, k$; $\Delta\delta$ is a small positive number that can be set by user. For the purpose of considering both accuracy and efficiency, $\Delta\delta$ can usually be taken as a number between $1 \times 10^{-4} \sim 2 \times 10^{-4}$. A smaller value should be taken for a more accurate result. In this paper, $\Delta\delta$ is set to 1×10^{-4}, and $k = 10$ is sufficient for an accurate and efficient classification result in the AK-MCMC method

C. RELIABILITY EVALUATION

1) SPACE INVERSE TRANSFORMATION

After the model refinement phase, the final constructed Kriging model can be defined as $\tilde{G}(\boldsymbol{x}) = \hat{G}(\boldsymbol{x})$. Then the inverse transformation of sampling space $[0, 1]^n$ and the final Kriging model function $\hat{G}(\boldsymbol{x})$ are executed by (35).

$$\hat{G}(\boldsymbol{x}) = \hat{G}\left(\frac{\boldsymbol{y}-\boldsymbol{y_L}}{\boldsymbol{y_H}-\boldsymbol{y_L}}\right) = \hat{g}(\boldsymbol{y}) \tag{35}$$

where $\hat{g}(\boldsymbol{y})$ can be regarded as the Kriging model function of the original performance function (g) in the original variable space.

2) ESTIMATE FAILURE PROBABILITY

In this stage, N_{MCS} samples are generated to create a sample set $\boldsymbol{Y}^{MCS} = [\boldsymbol{y}_1^T, \boldsymbol{y}_2^T, \cdots, \boldsymbol{y}_{N_T}^T]^T$ by random Monte Carlo sampling, and then the probability of failure $\hat{P}_F$ and the coefficient of variation cov with function $\hat{g}(\boldsymbol{y})$ can be estimated by (36) and (37).

$$\hat{P}_f = \frac{1}{N_{MCS}} \sum_{i=1}^{N_{MCS}} I\left(\boldsymbol{y}_i^{MCS}\right) \tag{36}$$

$$cov = \sqrt{\frac{1-\hat{P}_F}{(N_{MCS}-1)\,\hat{P}_F}} \tag{37}$$

If cov is higher than the allowable value Δcov, the number N_{MCS} of samples in sample set $\boldsymbol{Y}^{MCS}$ should be enlarged. The stage of estimating the failure probability is repeated until cov is less than Δcov. Generally, the initial value N_{MCS0} is taken to 10^5, and the value of Δcov is set to 0.01.

IV. VALIDATION EXAMPLES

In this section, four examples are studied to illustrate the applicability of the proposed AK-MCMC method, and both of its efficiency and accuracy are demonstrated. Cases 1-4 are classic reliability analysis problems with nonlinear and explicit performance functions, and they are often used in literature. Case 4 is an aircraft door lock mechanism reliability analysis problem with an implicit performance function. In addition, comparisons of efficiency and accuracy between the proposed AK-MCMC method, the AK-MCS + U method [24], the AK-MCS + EFF method [24], the AK-MCS + H method [28] and MCS are also given for all cases. The relative percentage error ε_{Pf} compared to MCS is expressed as

$$\varepsilon_{Pf} = \frac{\left|P_f - P_{Fmcs}\right|}{P_{Fmcs}} \times 100\% \tag{38}$$

where P_{Fmcs} is average probability of failure estimated with MCS with 50 runs, and it is considered as a reference value; P_f is estimated by the proposed AK-MCMC method, and other three methods mentioned above. In order to consider the impact of randomness, each method is executed 20 runs in each example. When these algorithms are executed, the total number N_{call} of calls to true performance function, probability of failure P_f, relative percentage error ε_{Pf}, and computational time T_c are obtained with 20 runs, respectively. Then, the interval $\left[N_{call}^{min}, N_{call}^{max}\right]$ of N_{call}, the average total number $\bar{N}_{call}$, the interval $\left[P_f^{min}, P_f^{max}\right]$ of P_f, maximum relative percentage error $\varepsilon_{P_f}^{max}$ and average computational time $\bar{T}_c$ are counted. All algorithms are executed in MATLAB2016b with Intel Core i7-7700K CPU@4.20GHz 8 cores.

Before executing the algorithm, the termination condition should be set. The learning criterions, stopping conditions and the other condition of the proposed AK-MCMC method and other three AK-MCS are summarized in Table 1.

A. EXAMPLE 1: A 2D DESCRIPTION CASE WITH A SMALL FAILURE PROBABILITY

A two-dimensional problem with a small failure probability, adopted from references [25], [29], is considered to illustrate

TABLE 1. Definition of different parameters for each method.

Method	AK-MCMC	AK-MCS+U [24]	AK-MCS+EFF [24]	AK-MCS+H [28]
Learning criterion	$\max(L_{dp\sigma})$	$\min(U(\boldsymbol{x}))$	$\max(\mathrm{EFF}(\boldsymbol{x}))$	$\max(H(\boldsymbol{x}))$
Stopping condition	$\delta_{mis}^{max} \le 0.0001$	$\min(U(\boldsymbol{x})) \ge 2$	$\max(\mathrm{EFF}(\boldsymbol{x})) \le 0.001$	$\max(H(\boldsymbol{x})) \le 0.1$
Other condition	$\Delta cov \le 0.01$	$\Delta cov \le 0.01$	$\Delta cov \le 0.01$	$\Delta cov \le 0.01$

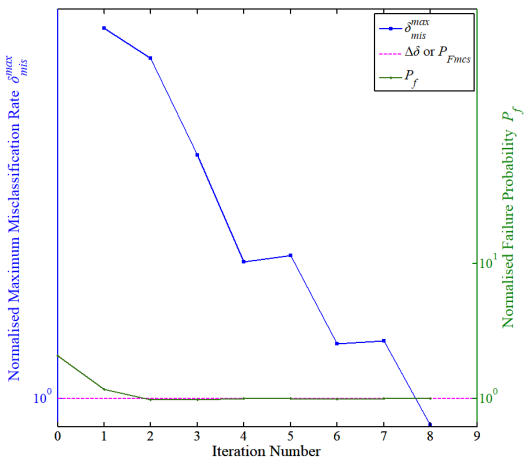

FIGURE 2. Variations of misclassification rate and failure probability with iteration number for example 1.

the proposed AK-MCMC method. The two variables x_1 and x_2 obey the standard normal distribution, and its performance function is defined as

$$G(x_1, x_2) = 0.5(x_1-2)^2-1.5(x_2-5)^3-3 \qquad (39)$$

MCS, the proposed AK-MCMC method, the AK-MCS + U method, the AK-MCS + EFF method and the AK-MCS + H method are executed to evaluate the probability of failure. The reference value P_{Fmcs} is defined as the average probability of failure obtained out of 50 different MCS runs with $N_{MCS} = 4\times10^8$ samples each. The number of MCS samples is determined to have a coefficient of variation *cov* on the probability of failure less than 1%. It is obviously seen that the size of MCS samples is relatively large to accurately predict the failure probability of the problem with a small failure probability, and the predicted time cost of Kriging model will increase significantly. For the purpose of examining the robustness of AK-MCMC, AK-MCS + U, AK-MCS + EFF and AK-MCS + H, each method is performed with 20 different runs.

The reliability analysis results of the each method are summarized in Table 2. They are compared by the average number of calls $\bar{N}_{call}$ to the original performance function, their maximum relative percentage error $\varepsilon_{P_f}^{max}$ and average computational time $\bar{T}_c$ with 20 different runs. As presented in Table 2 , the probabilities of failure acquired by the four methods are very similar to the reference value acquired by MCS with 50 runs. However, the proposed AK-MCMC method obtains a good accuracy with a smallest average number $\bar{N}_{call} = 17.45$ than the other methods. Meanwhile, the terms of $\varepsilon_{P_f}^{max}$ and $\bar{T}_c$ with the proposed AK-MCMC method are smallest in the four methods.

In order to better demonstrate the proposed AK-MCMC method, the iterative process for updating samples with one run is presented in Fig. 2 and Fig. 3. Fig. 2 shows the variations of the normalized maximum misclassification rate δ_{mis}^{max} and normalized failure probability P_f with iteration number. The term of maximum misclassification rate δ_{mis}^{max} is computed in iterative process of the proposed AK-MCMC method. The term of failure probability P_f is obtained after the algorithm is executed, and it is only calculated to show the relationship between it and maximum misclassification rate δ_{mis}^{max} with the number of iterations. As depicted in Fig. 2, the failure probability is basically convergent after the 4th iteration, and this corresponds to Fig. 3(e). However, maximum misclassification rate δ_{mis}^{max} is not met the allowable value $\Delta\delta = 1\times10^{-4}$ currently, and it can be found that the limit state $G_z(\boldsymbol{x}) = 0$ of obtained Kriging model and the limit state $G(\boldsymbol{x}) = 0$ of original performance function are not perfectly coincident in Fig. 3(e). Then, the failure probability

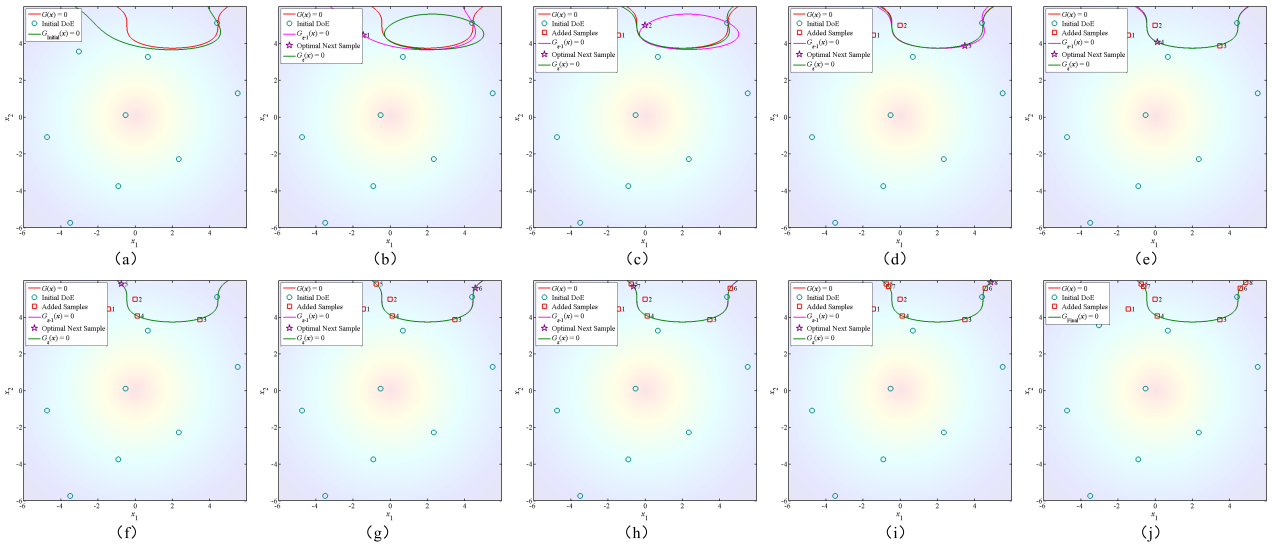

FIGURE 3. Iterative process of AK-MCMC for example 1. (a) The initial surrogate model. (b) The 1st iteration. (c) The 2nd iteration. (d) The 3rd iteration. (e) The 4th iteration. (f) The 5th iteration. (g) The 6th iteration. (h) The 7th iteration. (i) The 8th iteration. (j) The final surrogate model.

TABLE 2. Reliability analysis results for example 1.

Methods	$[N_{call}^{min}, N_{call}^{max}]$	$\bar{N}_{call}$	$[P_f^{min}, P_f^{max}]$	$\varepsilon_{P_f}^{max}$	$\bar{T}_c$(/S)
MCS	-	4×10^8	$P_{Fmcs} = 2.875\times10^{-5}$	-	-
AK-MCMC	[15, 20]	17.00	$[2.861\times10^{-5},2.891\times10^{-5}]$	0.53%	1997
AK-MCS+U	[32, 41]	36.35	$[2.860\times10^{-5},2.893\times10^{-5}]$	0.59%	20202
AK-MCS+EFF	[19, 28]	21.30	$[2.861\times10^{-5},2.893\times10^{-5}]$	0.61%	9660
AK-MCS+H	[35, 42]	37.65	$[2.860\times10^{-5},2.893\times10^{-5}]$	0.60%	41716

TABLE 3. Reliability analysis results for example 2.

Methods	$[N_{call}^{min}, N_{call}^{max}]$	$\bar{N}_{call}$	$[P_f^{min}, P_f^{max}]$	$\varepsilon_{P_f}^{max}$	$\bar{T}_c$(/S)
MCS	-	7×10^6	$P_{Fmcs} = 1.513\times10^{-3}$	-	-
AK-MCMC	[34, 46]	40.65	$[1.499\times10^{-3},1.523\times10^{-3}]$	0.82%	355
AK-MCS+U	[85, 103]	93.35	$[1.492\times10^{-3},1.521\times10^{-3}]$	1.28%	3297
AK-MCS+EFF	[39, 56]	48.95	$[1.492\times10^{-3},1.521\times10^{-3}]$	1.28%	1718
AK-MCS+H	[90, 108]	98.15	$[1.492\times10^{-3},1.521\times10^{-3}]$	1.28%	6856

deviates from the standard value at the 6th iteration. Finally, maximum misclassification rate δ_{mis}^{max} is met and a higher accuracy estimated value of failure probability is obtained at the 8th iteration.

As shown in Fig. 3(a), the N_d =9 initial DoE samples are generated by LHD, and the initial Kriging model is constructed with the DoE. Subsequently, 8 extra samples are added iteratively to the DoE as shown in Fig. 3(b)- Fig. 3(i). These added samples are deposited in the vicinity of the failure boundary and maintain a certain distance from the existing samples in DoE. As shown in Fig. 3(j), the final Kriging model is obtained after 8 iterations, and it is employed to estimate the probability of failure.

B. EXAMPLE 2: A HIGH-ORDER CASE WITH NONLINEAR PERFORMANCE FUNCTION

In this example, a structure with a high-order nonlinear [43] is considered, and its performance function given by

$$G(x_1, x_2, x_3) = 0.025x_1^4 + 2x_2^2 + x_3 + 2.5 \qquad (40)$$

where x_1, x_2 and x_3 are random variables, and they obey the standard normal distribution.

Like example 1, all methods mentioned are employed to evaluate the probability of failure. The reference value P_{Fmcs} is calculated out of 50 different MCS runs with $N_{MCS} = 7\times10^6$ samples each. The reliability analysis results of the proposed AK-MCMC, AK-MCS + U, AK-MCS + EFF, AK-MCS + H and MCS are summarized in Table 3.

It is found that the probability of failure estimated by each method is very similar to the reference value of P_{Fmcs}, and those four methods have a good accuracy as shown in Table 3. Compared with other methods, the proposed AK-MCMC method has more advantages in terms of average number of calls $\bar{N}_{call}$ to the original performance function, maximum relative percentage error $\varepsilon_{P_f}^{max}$ and the average computational time $\bar{T}_c$ with 20 runs. There is a shortest interval $[N_{call}^{min}, N_{call}^{max}]$ with the proposed AK-MCMC method, which means that it is more stable than other methods.

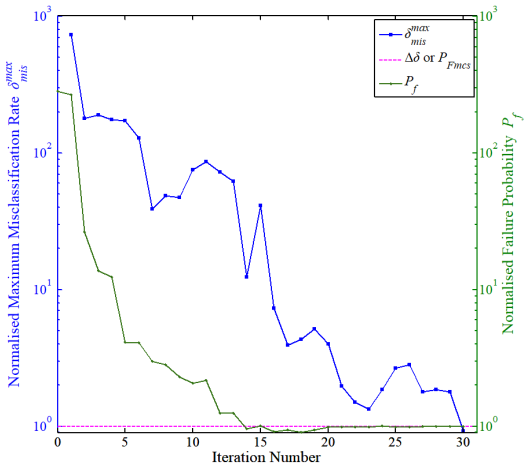

FIGURE 4. Variations of misclassification rate and failure probability with iteration number for example 2.

Fig. 4 is shown to describe the variations of the normalized maximum misclassification rate δ_{mis}^{max} and normalized failure probability P_f with iteration number. Fig. 5 shows the iterative process for updating samples with the proposed

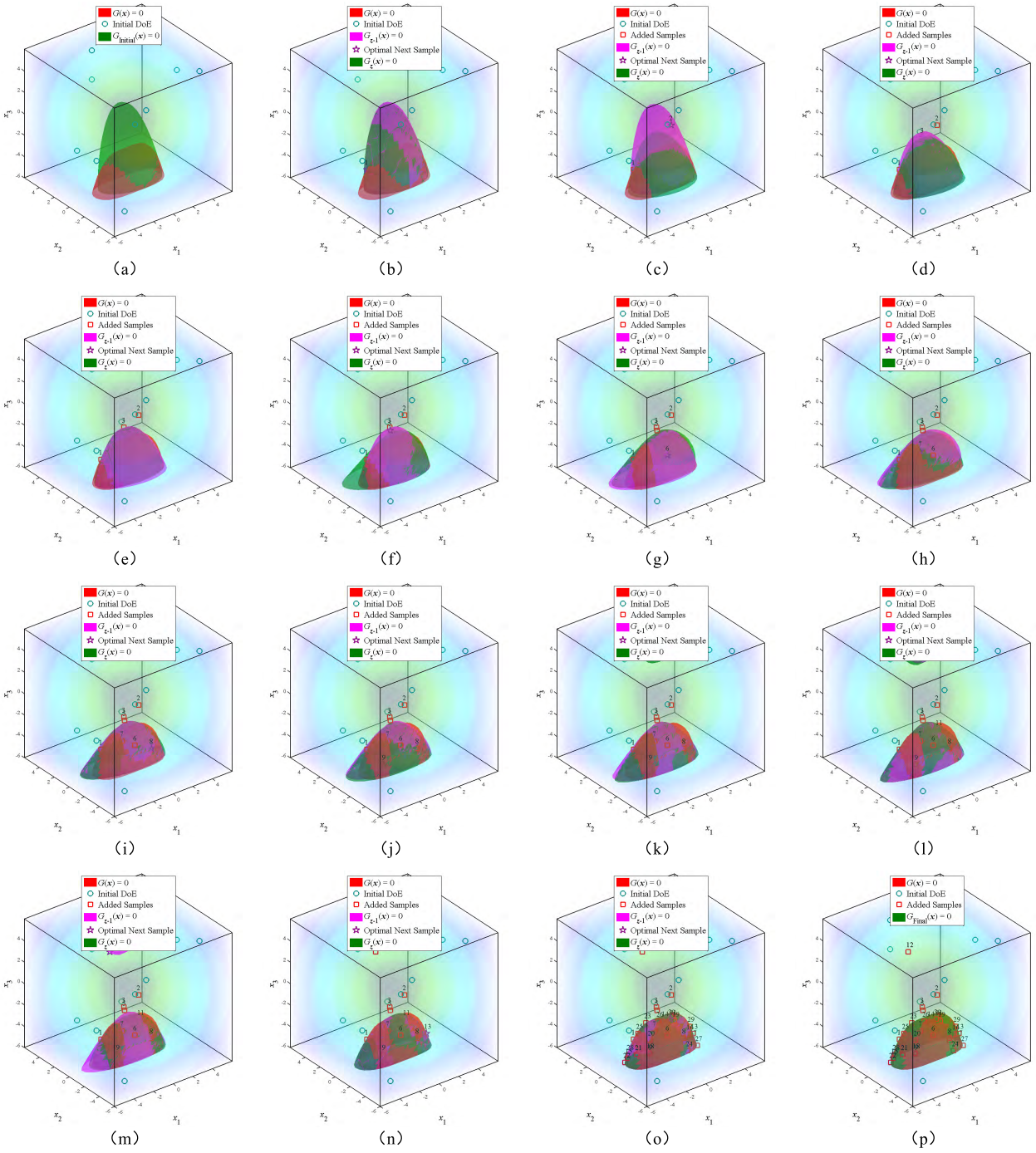

FIGURE 5. Iterative process of AK-MCMC for example 2. (a) The initial surrogate model. (b) The 1st iteration. (c) The 2nd iteration. (d) The 3rd iteration. (e) The 4th iteration. (f) The 5th iteration. (g) The 6th iteration. (h) The 7th iteration. (i) The 8th iteration. (j) The 9th iteration. (k) The 10th iteration. (l) The 11th iteration. (m) The 12th iteration. (n) The 13th iteration. (o) The 14th 30th iterations. (p) The final surrogate model.

AK-MCMC method. In order to reduce the space, Fig. 5 shows only parts of the iterative process. As shown in Fig. 5, 11 initial samples (Fig. 5(a)) are generated and 30 additional samples are selected sequentially. Since the approximation of the limit state can be progressively improved in the process of subjoining samples, the estimation accuracy of failure probability with the proposed AK-MCMC method is significantly improved as shown in Fig. 4.

C. EXAMPLE 3: DYNAMIC RESPONSE OF A NONLINEAR OSCILLATOR

As shown in Fig. 6, a non-linear undamped single degree of freedom system is considered. The reason this example is selected is that it is a classical illustration in literatures [24]–[26], [43] and involves a medium number of stochastic variables. The statistical parameters of basic variables are given in Table 4. The performance function is

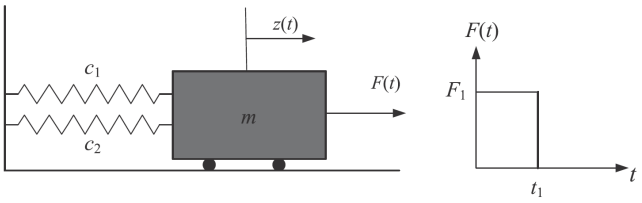

FIGURE 6. A nonlinear oscillator for example 3.

TABLE 4. The statistical parameters of basic variables for example 3.

Variables	Mean	Standard Deviation	Distribution
r	0.5	0.05	Normal
F_1	1	0.2	Normal
m	1	0.05	Normal
c_1	1	0.1	Normal
c_2	0.1	0.01	Normal

expressed by

$$G = 3r - \left| \frac{2F_1}{m\omega_0^2} \sin\left(\frac{\omega_0}{2}t\right) \right| \tag{41}$$

where $\omega_0 = \sqrt{(c_1 + c_2)/m}$.

The reliability analysis results obtained by MCS and the four adaptive sequential sampling methods are summarized in Table 5. It is found that probabilities of failure acquired by AK-MCMC, AK-MCS + U, AK-MCS + EFF, and AK-MCS + H are quite close to the reference value P_{Fmcs} obtained out of 50 different MCS runs with $N_{MCS} = 1\times10^6$ samples each. As listed in the Table 5, the proposed AK-MCMC method only needs average 105.40 calls to the original performance function, whereas the AK-MCS + H requires the most on the term of average calls $\bar{N}_{call}$ (139.95 calls) with 20 runs. Although the AK-MCMC method isn't the most efficient one, it is more accurate with the term of maximum relative percentage error $\varepsilon_{P_f}^{max} = 1.00\%$ than the most efficient one (AK-MCS + EFF) with $\varepsilon_{P_f}^{max} = 1.01\%$, and the average computational time of the proposed AK-MCMC method is fewer than the one with AK-MCS + EFF. Therefore, the proposed AK-MCMC method is still superior to the other three methods.

The variations of the normalized maximum misclassification rate δ_{mis}^{max} and normalized failure probability P_f with iteration number are shown in Fig. 7. It can be found that the fluctuations of failure probability are more severe before the 45th iteration, and it gradually converges between the 45th iteration and the 90th iteration. The maximum misclassification rate δ_{mis}^{max} is met and a higher accuracy estimated value of failure probability is obtained at the 90th iteration.

D. EXAMPLE 4: MODIFIED RASTRIGIN FUNCTION

The next example is based on the modified Rastrigin function, and it is often used in the literature as an application example for testing reliability methods [24], [32]. This function

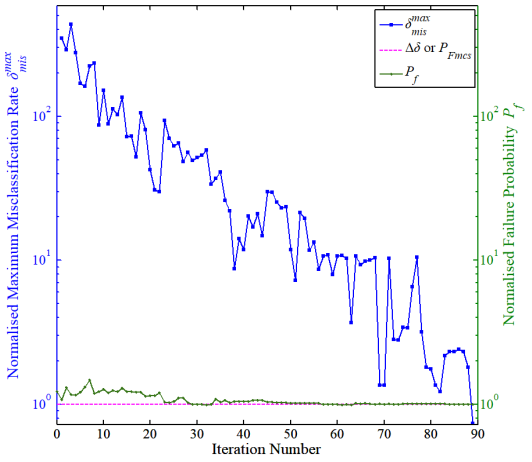

FIGURE 7. Variations of misclassification rate and failure probability with iteration number for example 3.

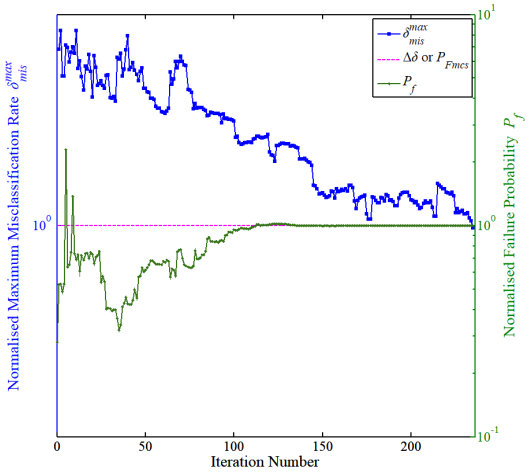

FIGURE 8. Variations of misclassification rate and failure probability with iteration number for example 4.

involving non-convex and non-connex domains of failure characterizes highly nonlinear performance behavior, and it is defined by (42). The two random variables x_1 and x_2 are standard normally distributed.

$$G(x_1, x_2) = 10 - \sum_{i=1}^{2} \left(x_i^2 - 5\cos(2\pi x_i)\right) \tag{42}$$

The proposed AK-MCMC method is compared with Monte Carlo Simulation and other three AK-MCS methods, and the reliability analysis results are listed in Table 6.

The probability of failure estimated by AK-MCMC with average 243.54 calls is found to be the same than the one obtained by MCS for the same population (Table 6). Furthermore, in this case, AK-MCS + U, AK-MCS + EFF and AK-MCS + EFF perform the same population. However, the average number $\bar{N}_{call}$ of required calls to the performance function with the three AK-MCMC methods is much more than the one with AK-MCMC for obtaining a similar reliability evaluation result.

TABLE 5. Reliability analysis results for example 3.

Methods	$[N_{call}^{min}, N_{call}^{max}]$	$\overline{N}_{call}$	$[P_f^{min}, P_f^{max}]$	$\varepsilon_{P_f}^{max}$	$\overline{T}_c$(/S)
MCS	-	1×10^6	$P_{Fmcs} = 2.859\times10^{-2}$	-	-
AK-MCMC	[91, 115]	105.40	$[2.839\times10^{-2},2.886\times10^{-2}]$	1.00%	1676
AK-MCS+U	[126, 142]	133.50	$[2.830\times10^{-2},2.886\times10^{-2}]$	1.00%	355
AK-MCS+EFF	[83, 109]	97.90	$[2.832\times10^{-2},2.887\times10^{-2}]$	1.01%	1830
AK-MCS+H	[132, 146]	136.95	$[2.829\times10^{-2},2.886\times10^{-2}]$	1.00%	728

TABLE 6. Reliability analysis results for example 4.

Methods	$[N_{call}^{min}, N_{call}^{max}]$	$\overline{N}_{call}$	$[P_f^{min}, P_f^{max}]$	$\varepsilon_{P_f}^{max}$	$\overline{T}_c$(/S)
MCS	-	1.3×10^5	$P_{Fmcs} = 7.30\times10^{-2}$	-	-
AK-MCMC	[187, 301]	243.54	$[7.178\times10^{-2},7.406\times10^{-2}]$	1.67%	2427
AK-MCS+U	[451, 494]	469.63	$[7.154\times10^{-2},7.450\times10^{-2}]$	2.05%	2590
AK-MCS+EFF	[502, 544]	524.96	$[7.161\times10^{-2},7.458\times10^{-2}]$	2.16%	2879
AK-MCS+H	[557, 600]	576.22	$[7.152\times10^{-2},7.451\times10^{-2}]$	2.07%	3505

Fig. 8 is also shown to describe the variations of the normalized maximum misclassification rate δ_{mis}^{max} and normalized failure probability P_f with iteration number. It can be found that the fluctuations of failure probability are more severe before about 120th iteration, and it gradually converges after the 120th iteration. The maximum misclassification rate δ_{mis}^{max} is met, and a surrogate model with a higher accuracy for estimating failure probability is obtained at the 235th iteration. The failure boundary of final surrogate model with 244 experimental samples by AK-MCMC to satisfy the stopping condition is shown in Fig. 9. It is seen that the samples are scattered in the design space, and they are located in the vicinity of the limit state. Furthermore, Fig. 9 shows that the limit state is well approximated by AK-MCMC. This example shows that the proposed AK-MCMC can be used on highly nonlinear limit states and on problems involving non-convex and non-connex domains of failure.

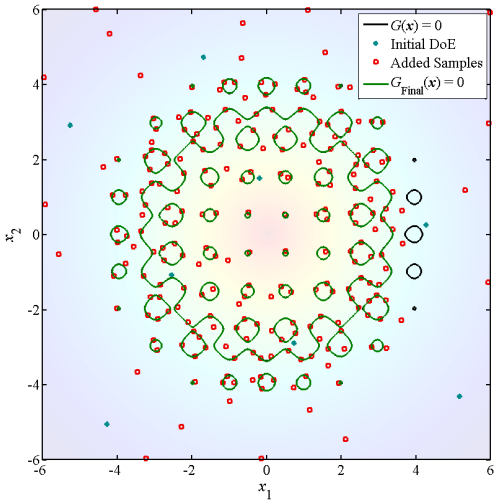

FIGURE 9. The final surrogate model with 244 experimental samples by AK-MCMC for example 4.

E. EXAMPLE 5: AN AIRCRAFT DOOR LOCK MECHANISM

In this example, the reliability of an aircraft door lock mechanism on the locking position accuracy is analyzed. The aircraft door lock mechanism [44] consists of 8 parts: lock-body, lock-hook and lock-ring, push-rod, connecting-rod 1, rocker, connecting-rod 2, tension-spring, which is described in detail in Fig. 10 (a). Due to the randomness of components in the lock mechanism, the locking position of the lock mechanism is a random variable. This causes the position of the aircraft door to overlap with the aircraft fuselage to be misaligned, and the flight performance of the aircraft is affected. The functional feature is defined as the angular deviation α between the actual position of the lock-hook and the ideal position at the lock position, as showed in Fig. 10(b).

There are 13 variables with the truncated normal distribution, which can be seen in Table 7. On the basis of the kinematics principle of lock mechanism, the angular deviation α can be established based on massless rod clearance simulation analysis model in LMS Virtual.Lab Motion, and it is easy to find that the angular deviation α is a function of the 13 variables. When the angular deviation α exceeds a given threshold α_0, the lock mechanism is considered to be invalid at the lock position. The performance function can be defined as

$$G(\boldsymbol{x}) = \alpha_0 - \alpha(\boldsymbol{x}) \tag{43}$$

TABLE 7. The statistical parameters of basic variables for example 5.

Variables	Symbol description	Mean	Standard deviation	Lower limit	Upper limit	Distribution
x_1	Length of AB on lock-hook	34.80mm	0.005mm	34.770mm	34.830mm	Truncated Normal
x_2	Length of BC on connecting-rod 2	42.65mm	0.005mm	42.620mm	42.680mm	Truncated Normal
x_3	Length of CD on rocker	22.90mm	0.005mm	22.870mm	22.930mm	Truncated Normal
x_4	Length of CE on rocker	46.50mm	0.005mm	46.470mm	46.530mm	Truncated Normal
x_5	Length of DE on rocker	24.50mm	0.005mm	24.470mm	24.530mm	Truncated Normal
x_6	Length of DF on connecting-rod 2	18.50mm	0.005mm	18.470mm	18.530mm	Truncated Normal
x_7	Outer diameter of the lock ring O	12.00mm	0.003mm	11.982mm	12.018mm	Truncated Normal
x_8	Clearance size at hinge A	37.00μm	2.92μm	25.00μm	49.00μm	Truncated Normal
x_9	Clearance size at hinge B	17.00μm	2.92μm	5.00μm	29.00μm	Truncated Normal
x_{10}	Clearance size at hinge C	20.50μm	3.52μm	6.00μm	35.00μm	Truncated Normal
x_{11}	Clearance size at hinge D	20.50μm	3.52μm	6.00μm	35.00μm	Truncated Normal
x_{12}	Clearance size at hinge E	20.50μm	3.52μm	6.00μm	35.00μm	Truncated Normal
x_{13}	Clearance size at hinge F	17.00μm	2.92μm	5.00μm	29.00μm	Truncated Normal

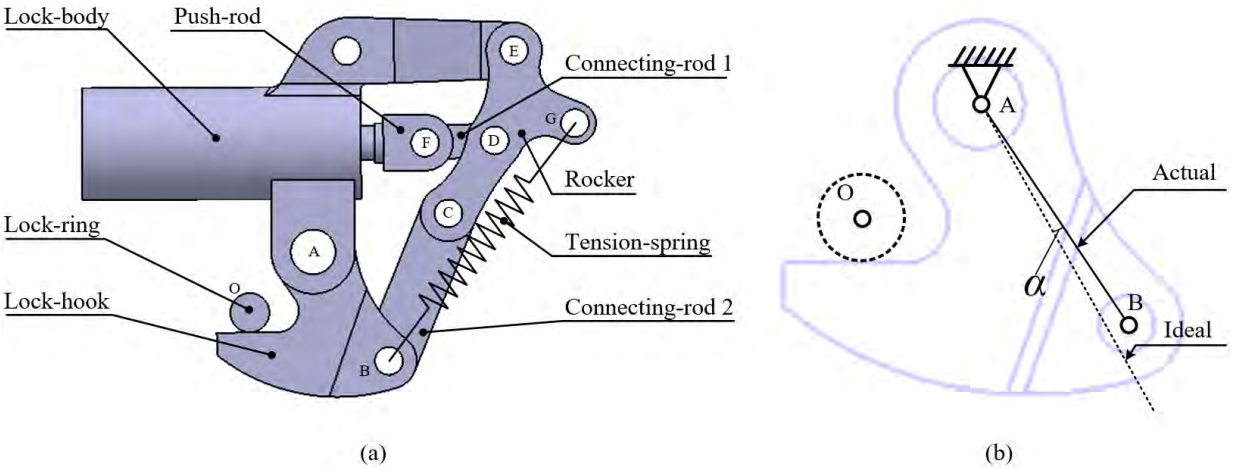

FIGURE 10. The schematics of an aircraft door lock mechanism for example 5: (a) the assembly of the lock mechanism, (b) the characterization of the locking position accuracy.

where $\boldsymbol{x} = [x_1, x_2, \cdots, x_{13}]$ and $\alpha(\boldsymbol{x})$ is the angular deviation between the actual position of the lock-hook and its ideal position at the locking position. The threshold α_0 is set to be 1.3°. Since $\alpha(\boldsymbol{x})$ is computed by calling to the LMS analysis model, this case is a problem with an implicit and multivariable performance function and the calculation of the analysis model is time-consuming.

Due to that the calculation process of MCS is time consuming, the reference value P_{Fmcs} estimated out of just one MCS run with $N_{MCS} = 7 \times 10^6$ samples is 1.566×10^{-3} in this example. First, 7×10^6 samples are randomly generated by MCS under the statistical parameters of variables, and these samples are divided into 70 sample sub-sets containing 100,000 samples. Then, the data for sub-sets are stored in 70 numbered excel files, respectively. The samples in these files are sequentially calculated for the corresponding simulation results on 8 computers with Intel Core i7-7700K CPU@4.20GHz 8 cores in LMS Virtual.Lab 13.6. The average cost time of each computer is about 18 days in the case of 24-hour work. Obviously, the calculation of MCS is time consuming for reliability analysis of the lock mechanism on the locking position accuracy.

Reliability analysis results are summarized in Table 8. It can be seen that probabilities of failure acquired by AK-MCMC, AK-MCS + U, AK-MCS + EFF, and AK-MCS + H are considerably close to the reference value of P_{Fmcs}. The proposed AK-MCMC method only needs average 188.40 calls to the time-consuming simulation model, which is the most efficient one, whereas the AK-MCS + H requires the most average calls $\bar{N}_{call}$ (317.60 calls) with

TABLE 8. Reliability analysis results for example 5.

Methods	$[N_{call}^{min}, N_{call}^{max}]$	$\bar{N}_{call}$	$[P_f^{min}, P_f^{max}]$	$\varepsilon_{P_f}^{max}$	$\bar{T}_c$(/S)
MCS	-	7×10^6	$P_{Fmcs} = 1.566\times10^{-3}$	-	1.225×10^7
AK-MCMC	[176, 202]	188.40	$[1.551\times10^{-3},1.581\times10^{-3}]$	0.94%	14586
AK-MCS+U	[291, 318]	307.70	$[1.524\times10^{-3},1.573\times10^{-3}]$	2.66%	7799
AK-MCS+EFF	[192, 213]	201.45	$[1.525\times10^{-3},1.571\times10^{-3}]$	2.63%	21759
AK-MCS+H	[304, 321]	317.60	$[1.524\times10^{-3},1.573\times10^{-3}]$	2.70%	16159

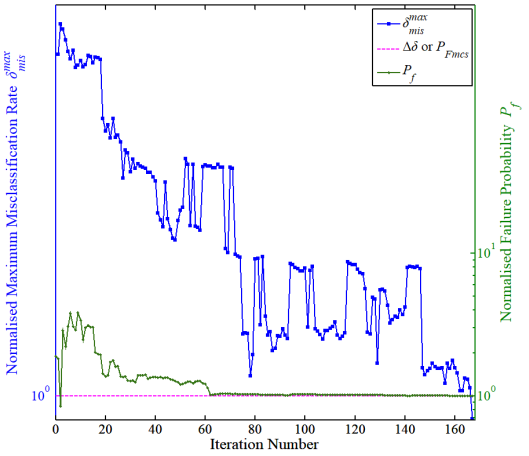

FIGURE 11. Variations of misclassification rate and failure probability with iteration number for example 5.

20 runs. Meanwhile, the proposed AK-MCMC method is the most efficient one with a maximum relative percentage error $\varepsilon_{P_f}^{max} = 0.94\%$ than the most inefficient one (AK-MCS + EFF) with $\varepsilon_{P_f}^{max} = 2.70\%$. Although the AK-MCMC method has a higher average computational time with $\bar{T}_c = 14586s$ than the minimal one (AK-MCS + U) with $\bar{T}_c = 7799s$, there are a smaller $\varepsilon_{P_f}^{max}$ and a less number $\bar{N}_{call}$ of average calls with the proposed AK-MCMC method. So, the proposed AK-MCMC method is quite advantageous compared to the other three methods.

For this example, the failure probability starts to converge around 65th iteration as shown in Fig. 11. However, there are more obvious fluctuations of failure probability during the 65th iteration to the 160th iteration, and the term of maximum misclassification rate δ_{mis}^{max} is substandard and very unstable during this period. The maximum misclassification rate δ_{mis}^{max} is met and a higher accuracy estimated value of failure probability is obtained at the 166th iteration.

V. CONCLUSIONS

A new sequential surrogate method for reliability analysis, AK-MCMC, is proposed by combining adaptive Kriging and Markov Chain Monte Carlo simulation. The proposed AK-MCMC method is subtly divided into three stages: (1) initial preparation phase; (2) model refinement phase; (3) reliability evaluation phase, which makes reliability evaluation and adaptive iterative process independent of each other for improving the efficiency of model refinement. In model refinement phase of the proposed AK-MCMC method, a new learning function and a novel stopping criterion based on the principle of leave one out cross validation are developed. To investigate the efficiency and accuracy of the proposed AK-MCMC method, it is employed to evaluate the failure probabilities of four classic cases and an implicit and 13-dimensional engineering problem.

The applications of the first two examples and the 4th example indicate that probabilities of failure assessed by all methods mentioned above are quite similar to the reference value estimated by MCS, and the relative percentage errors are very small. However, in terms of the average number of calls to the original performance function and calculation time, the proposed AK-MCMC method is more advantageous than the other three AK-MCS methods, especially in the first case with a small failure probability and the 4th example involving non-convex and non-connex domains of failure. This fully demonstrates the advantage of the proposed method to make the adaptive process and the reliability analysis independent of each other. In example 3 and example 5, the proposed AK-MCMC method has no longer an advantage in the term of average computational cost. The primary reason is that the generation of candidate samples by MCMC becomes inefficient as the dimension of problem increases. However, the proposed AK-MCMC method is still more advantageous than the other three methods in terms of the number of relative percentage error calls to performance function.

In reliability analysis based on surrogate models, there are three main directions in our future efforts for practical engineering. The first is that how to improve the efficiency of determining next optimal sample in a more efficient way. The second is to employ a more efficient simulation method in the reliability evaluation stage of the proposed AK-MCMC method. The third is to find a more efficient alternative surrogate model to replace the Kriging model.

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