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Distributed Finite-Time Cooperative Control for Quadrotor Formation

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ABSTRACT This paper investigates a finite-time formation control problem for multiple networked quadrotors. A novel distributed control approach is presented under the leader-follower formation framework, and the approach is developed based on the finite-time Lyapunov theory and the homogeneous system theory such that all quadrotors form and maintain a desired geometric pattern within finite time while tracking a reference trajectory. The designed control law is composed of a dynamic observer, a position controller and an attitude controller, in which the observer is adopted to provide estimates of the leader quadrotor information for each follower quadrotor, and the controllers are in a cascade structure. It is shown that the finite-time leader-follower formation of a group of quadrotors can be achieved via the distributed control approach, and the cascade control architecture conforms to quadrotor dynamic characteristics. The constructive procedures on how to synthesize such a control law are also given. The effectiveness of the proposed control approach is verified by the simulation.

INDEX TERMS Finite-time control, formation control, unmanned aerial vehicles.

I. INTRODUCTION

Recent years have seen impressive progress in distributed cooperative control of networked multi-agent systems, see [1]–[6]. Considerable amount of relevant works contribute enormously to the enrichment of the research topic (e.g., [7]–[12] and references therein). In particular, formation control of multiple aircrafts, as one of the most essential cooperative control problems, has attracted extensive interest from control and robotics communities. The objective of formation control of multiple aircrafts is to make a group of networked aircrafts form and maintain some desired positions and orientations so as to achieve a common goal. As of now, much research efforts have been given to flight formation control [13]–[17], and many salient results on the distributed formation control of multiple aircrafts have been achieved, see [18]–[21]. Thanks to the significant progress achieved by aircraft formation control, it has been verified that multiple aircrafts with a desired geometric pattern have the capability to carry out more complicated tasks than a single aircraft, see [22]–[24] for details.

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Although several valuable results have been obtained on formation control design of aircrafts, yet most of them only focus on formation error convergence to prescribed bounds over a fixed time interval. However, for some demanding situations where it is hoped that multiple aircrafts are able to reach and maintain a desired geometric structure thereafter within a finite-time interval, most existing formation control approaches are inapplicable to such scenarios. Moreover, in practice, engineers often concentrate on transient performance of formation control systems besides steady-state performance, and robustness against modeling uncertainties and external disturbances. Fortunately, finite-time control has proved to enjoy advantages in terms of fast response, better robustness and perturbation attenuation [25]–[27]. Considering the aforementioned features owned by finite-time control, more recently, formation control design in conjunction with finite-time control has got increasing attentions in the field of aviation.

Initial work on finite-time control approach to formation of multi-agent systems was considered in [28]. Following this research, in [29], decentralized observers based on the sliding mode control method were employed to facilitate the study on finite-time formation tracking problems for single/double-integrator dynamics. Then in [30], an optimal formation controller was synthesized for networked linear systems to achieve desired formation over finite time. In [31], the finite-time formation-containment control problem for double-integrator-like dynamics subject to uncertain nonlinearities was addressed. Later in [32], finite-time formation maneuvering of single-integrator dynamics was investigated with a distance-based control approach. In [33], a time-varying formation tracking controller was presented for multiple linear systems with mismatched disturbances. It can be observed from the aforementioned works that finite-time formation control has been extensively investigated for certain specific classes of systems, which are insufficient to describe the complex aircraft dynamics caused by strong nonlinear coupling and changeable flight environments. Therefore, it is desirable to develop finite-time formation control approaches for aircrafts.

More recently, several valuable results have been published on finite-time formation control of aircrafts. Zhao *et al.* [34], Zhang *et al.* [35] used the precise feedback linearization method to design distributed finite-time formation controllers for networked aircrafts. However, these results are hardly extended to flight control systems subject to modeling uncertainties and external disturbances due to the dependance on accurate system models. Compared with these works, the proposed control approach can be readily integrated with robust control approaches to deal with uncertainties and disturbances. Wang *et al.* [36] addressed the finite-time formation control problem via the terminal sliding mode control. However, such control approaches usually involve potential singularity problems due to the existence of negative fractional powers in comparison with the proposed control approach. Du *et al.* [37] established a finite-time formation tracking control scheme for multi-UAV systems with input quantization based on the backstepping technique. However, only a few specific classes of nonlinear systems can be handled by the technique because applications of recursive design is complex. By contrast, the proposed formation controller and Lyapunov functions can be constructed without intermediate steps.

In this paper, a novel distributed control approach to finite-time formation of multiple quadrotors is developed, in which a distributed observer is designed to estimate the leader information for each follower, a position tracking controller is proposed for each follower quadrotor to achieve the desired formation pattern within finite time while tracking the reference trajectory generated by the leader quadrotor, and an attitude controller is presented such that each follower quadrotor is able to track the desired attitude. The major advantages of the proposed control approach include: 1) a unified framework for designing distributed control schemes is established to achieve finite-time formation of 6-DOF aircrafts in comparison with most existing works as shown in [34] and [35], where each aircraft is treated as a point-mass system without attitude requirements; 2) all follower quadrotors are not required to receive the leader quadrotor information in comparison with most existing formation control such as [34], [37], where all followers can have access to the leader information; and 3) upper bounds on the time taken for the quadrotors to form a desired geometric structure are calculated while only finite-time formation is achieved without estimating finite settling time in some existing works [38].

The rest of the paper is arranged as follows. Section II outlines a quadrotor model, the graph theory and two technical lemmas. A distributed finite-time formation control approach along with the stability analysis is proposed in Section III. Numerical simulations of an illustrative example are shown in Section IV, and conclusions are given in Section V.

II. PRELIMINARIES

A. QUADROTOR MATHEMATICAL MODEL

Consider a group of $N+1$ three-dimensional quadrotors, of which mathematical models are derived based on the assumption that each quadrotor is regarded as a rigid body and the origin of the body-fixed coordinate system is at its center of mass.

1) COORDINATE SYSTEMS AND TRANSFORMATIONS

To describe the states of quadrotor motion, appropriate coordinate systems are established [39], [40] as follows:

- *Inertial coordinate system $Oxyz$*: coordinate origin O is at a certain point of the earth surface; Ox axis is in the ground plane and points to the East; the negative direction of Oz axis is perpendicular to the ground plane and points to the geocentre, Oy axis is in the ground plane as well and makes up the right-handed coordinate system with Ox axis and Oz axis.
- *Body-fixed coordinate system $o_ix_by_bz_b$*: coordinate origin o_i is at the centroid of the ith quadrotor; o_ix_b axis coincides with the headward direction of the quadrotor; o_iz_b axis is in the quadrotor symmetric plane containing o_ix_b axis and points upward; o_iy_b axis is perpendicular to the symmetric plane and makes up the right-handed coordinate system with o_ix_b axis and o_iz_b axis.

The transformation matrix from the inertial coordinate system $Oxyz$ to the body coordinate system $o_ix_by_bz_b$ for the ith quadrotor is given by

$$R_i^{I\to B} = \begin{bmatrix} c\theta_i c\psi_i & c\theta_i s\psi_i & -s\theta_i \\ s\phi_i s\theta_i c\psi_i - c\phi_i s\psi_i & s\phi_i s\theta_i s\psi_i + c\phi_i c\psi_i & s\phi_i c\theta_i \\ c\phi_i s\theta_i c\psi_i + s\phi_i s\psi_i & c\phi_i s\theta_i s\psi_i - s\phi_i c\psi_i & c\phi_i c\theta_i \end{bmatrix} \tag{1}$$

where $(R_i^{I\to B})^{\mathrm{T}} = (R_i^{I\to B})^{-1}$, s and c denote the trigonometric functions *sin* and *cos* respectively, and θ_i, ψ_i, and ϕ_i are the pitch angle, yaw angle and roll angle of the ith quadrotor, respectively.

2) FORCES AND MOMENTS

The forces acting on the ith quadrotor are thrust T_i, gravity G_i and drag D_i. Thrust T_i equals to the resultant thrust from four rotors; gravity G_i is obtained as

$$G_i = m_i g, \tag{2}$$

where g is the gravitational acceleration and m_i the mass of the ith quadrotor; and drag D_i is calculated by

$$D_i = -k_{i,\chi}\,\dot{\chi}_i, \tag{3}$$

where $k_{i,\chi} = diag\left\{k_{i,x}, k_{i,y}, k_{i,z}\right\}$ is the aerodynamic damping coefficient matrix, and $\chi_i = [x_i, y_i, z_i]^{\mathrm{T}}$ is the position vector of the ith quadrotor under the inertial coordinate system.

To facilitate analysis of rotational motion for quadrotors, resultant moment can be decomposed into pitch moment $M_{i,\theta}$, roll moment $M_{i,\phi}$ and yaw moment $M_{i,\psi}$, respectively. Pitch moment $M_{i,\theta}$ and roll moment $M_{i,\phi}$ are generated by thrust difference, and yaw moment $M_{i,\psi}$ is generated by the reaction moment caused by the drag acting on four rotors. The drag makes quadrotors have yaw tendency whose direction is opposite to the corresponding rotor rotation.

3) QUADROTOR DYNAMICS MODEL

It follows from [41] that the 6-DOF dynamics model of the ith quadrotor is described by

$$\begin{bmatrix}\ddot{x}_i\\ \ddot{y}_i\\ \ddot{z}_i\end{bmatrix} = \frac{1}{m_i}\left\{\begin{bmatrix}0\\0\\-m_i g\end{bmatrix} + R_i^{B\to I}\begin{bmatrix}0\\0\\T_i\end{bmatrix} + \begin{bmatrix}-k_{i,x}\dot{x}_i\\-k_{i,y}\dot{y}_i\\-k_{i,z}\dot{z}_i\end{bmatrix}\right\} \tag{4}$$

and

$$\begin{bmatrix}J_{i,\theta}\ddot{\theta}_i\\ J_{i,\psi}\ddot{\psi}_i\\ J_{i,\phi}\ddot{\phi}_i\end{bmatrix} = \begin{bmatrix}-k_{i,\theta}l_{i,\theta}\dot{\theta}_i\\ -k_{i,\psi}\dot{\psi}_i\\ -k_{i,\phi}l_{i,\phi}\dot{\phi}_i\end{bmatrix} + \begin{bmatrix}M_{i,\theta}\\ M_{i,\psi}\\ M_{i,\phi}\end{bmatrix}, \tag{5}$$

where $J_{i,\theta}$, $J_{i,\psi}$, $J_{i,\phi}$ are moments of inertia, $k_{i,\theta}$, $k_{i,\psi}$, $k_{i,\phi}$ are aerodynamic damping coefficients, and $l_{i,\theta}$, $l_{i,\phi}$ represent the distance from the epicenter of rotors to the corresponding longitudinal plane of symmetry of the fuselage as shown in Fig.1.

B. ALGEBRAIC GRAPH THEORY

This paper studies a finite-time formation control problem of networked quadrotors, in which the leader quadrotor labeled 0 decides the reference trajectory and others labeled i, $i = 1, \ldots, N$, are followers. The network among these quadrotors is constituted by onboard sensors and data transmitting/receiving devices, of which topology is described as follows.

The graph is denoted as $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with a set of nodes $\mathcal{V} = \{0, 1, \ldots, N\}$ representing $N+1$ quadrotors and a set of edges $\mathcal{E} = \{(j, i)|i, j \in \mathcal{V}; j \neq i, \}$. A directed edge (j, i), $i, j \in \mathcal{V}$ means that the ith quadrotor can have access to the information of the jth quadrotor, and the node j is also called a neighbor of node i. Let $\mathcal{N}_i$ denotes the set which contains all neighbors of node i. The adjacency matrix of the graph $\mathcal{G}$

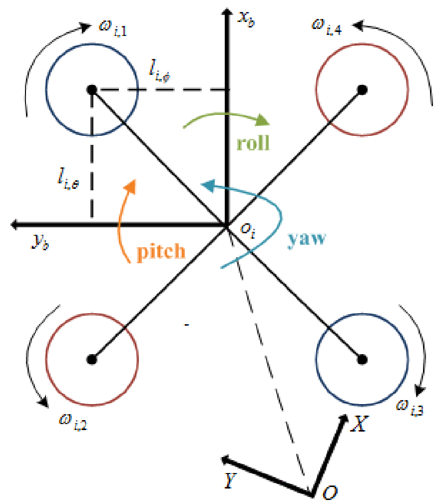

FIGURE 1. Schematic of a quadrotor from top view.

is denoted by $\mathcal{A} = [a_{ij}] \in \mathcal{R}^{(N+1)\times(N+1)}$ with $a_{ii} = 0$ and $a_{ij} = 1$ for $(j, i) \in \mathcal{E}$. The Laplacian matrix of the graph $\mathcal{G}$ is denoted by $\mathcal{L} = [l_{ij}] \in \mathcal{R}^{(N+1)\times(N+1)}$ with $l_{ii} = \sum_{j=0}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $(j, i) \in \mathcal{E}$.

C. RELATED DEFINITIONS AND LEMMAS

To facilitate stability analysis and formation control synthesis for the quadrotors as modeled above, some basic definitions and useful technical lemmas are introduced here.

Definition 1 (Finite-Time Stability): Consider a system of the form

$$\dot{x}(t) = f(x(t)),$$

where $f : \mathcal{D} \to \mathcal{R}^n$ is continuous on a neighborhood $\mathcal{D}$ of the origin and $f(0) = 0$. The origin is said to be a finite-time stable equilibrium if the system satisfies the following conditions:

- finite-time convergent: there exist a neighborhood $\mathcal{D}_0 \subseteq \mathcal{D}$ of the origin and a function $T : \mathcal{D}_0\backslash\{0\} \to (0, \infty)$, called the settling-time function, such that for every $x_0 \in \mathcal{D}_0\backslash\{0\}$, $x(t, x_0) \in \mathcal{D}_0\backslash\{0\}$ for all $t \in [0, T(x_0))$ and $\lim_{t\to T(x_0)} x(t, x_0) = 0$;
- Lyapunov stability.

If $\mathcal{D}_0 = \mathcal{D} = \mathcal{R}^n$, then the origin is globally finite-time stable.

Remark 1: It is known from [27] that the difficulty in finite-time control problems is to find a Lyapunov function V satisfying $\dot{V} \leq -kV^{\alpha}$ with $k > 0$ and $0 < \alpha < 1$ for concerned systems in general. In [26], the authors analyzed finite-time stability of homogeneous systems, which bridges the differences between asymptotical stability and finite-time stability, and inspired from the seminal work, homogeneous system theorems have been increasingly used to deal with finite-time control problems.

Definition 2 (Homogeneous Function and Homogeneous System): A continuous function $V : \mathcal{R}^n \to \mathcal{R}$ is said to be homogeneous of degree $k_1 > 0$ with respect to $(r_1, \ldots, r_n)$ where $r_i > 0, i = 1, \ldots, n$, if for any given $\varepsilon > 0$,

$$V(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{k_1} V(x), \quad x \in \mathcal{R}^n.$$

A continuous vector-field function $f(x) = [f_1(x), \ldots, f_n(x)]^{\mathrm{T}}$ is said to be homogeneous of degree k_2 with respect to $(r_1, \ldots, r_n)$ where $k_2 > -\min_{i=1,\ldots,n} r_i$ and $r_i > 0$, $i = 1, \ldots, n$, if for any given $\varepsilon > 0$,

$$f_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^{k_2 + r_i} f_i(x), \quad i = 1, \ldots, n, \ x \in \mathcal{R}^n.$$

A system $\dot{x} = f(x)$ is said to be homogeneous if $f(x)$ is homogeneous.

The following lemma provides an essential tool to analyze finite-time stability for homogeneous systems.

Lemma 1 ([26, Th. 2]): Suppose that a system $\dot{x} = f(x)$ is homogeneous of degree k. Then the system is finite-time stable if and only if the system is asymptotically stable with $k < 0$.

Before introduce the next lemma, let the Laplacian matrix $\mathcal{L}$ of the graph $\mathcal{G}$ be partitioned as follows.

$$\mathcal{L} = \left(\begin{array}{c|c} \sum_{i=1}^{N} a_{0i} & [a_{01}, \ldots, a_{0N}] \\ -\mathcal{A}_0 1_N & \mathcal{H} \end{array} \right) \tag{6}$$

where $\mathcal{A}_0 = diag\{a_{10}, \ldots, a_{N0}\}^{\mathrm{T}}$ and each entry of 1_N is 1.

Lemma 2 ([42, Lemma 1]): All nonzero eigenvalues of $\mathcal{H}$ are with positive real parts. Furthermore, $\mathcal{H}$ is nonsingular if and only if the graph $\mathcal{G}$ contains a directed spanning tree with the node 0 as the root.

D. PROBLEM FORMULATION

Throughout this paper, the communication topology among quadrotors satisfies the following assumptions.

Assumption 1: There is no directed path from followers to the leader; conversely, there exists at least one path from the leader to any one of the followers. The data communication between two followers is mutual.

Assumption 2: The graph $\mathcal{G}$ contains a directed spanning tree with the node 0 being the root.

The objective of this paper is to synthesize a suitable control law such that the finite-time formation control problem of multiple quadrotors defined as follows is addressed. It should be noted that the formation trajectory is decided by the leader quadrotor and the formation structure is determined by the desired relative position $d_{ij} := [d_{ij,x}, d_{ij,y}, d_{ij,z}]^{\mathrm{T}}$, $i, j \in \mathcal{V}$, from the ith quadrotor to its neighbor, the jth quadrotor.

Definition 3 (Finite-Time Formation Control Problem): Consider N follower quadrotors and a leader quadrotor as described in (4)-(5), and define the formation tracking errors as

$$\begin{aligned} e_{i,x} &= x_0 - x_i + d_{i0,x}, \\ e_{i,y} &= y_0 - y_i + d_{i0,y}, \\ e_{i,z} &= z_0 - z_i + d_{i0,z}. \end{aligned} \tag{7}$$

Given the graph $\mathcal{G}$ and the formation trajectory (x_0, y_0, z_0) decided by the uncontrolled leader quadrotor, find a dynamic control law such that

$$\begin{aligned} \lim_{t \to T^*} e_{i,x}(t) &= 0, \\ \lim_{t \to T^*} e_{i,y}(t) &= 0, \\ \lim_{t \to T^*} e_{i,z}(t) &= 0, \end{aligned} \tag{8}$$

and

$$\begin{aligned} e_{i,x}(t) &= 0, \\ e_{i,y}(t) &= 0, \\ e_{i,z}(t) &= 0, \quad t \geq T^*, \end{aligned} \tag{9}$$

where T^* is finite time.

Remark 2: We only consider the formation structure on the x axis as an example. It follows from (7) and (8) that

$$\begin{aligned} \lim_{t \to T^*} e_{ij,x}(t) :&= \lim_{t \to T^*} \left[e_{i,x}(t) - e_{j,x}(t) \right] \\ &= \lim_{t \to T^*} \left[d_{ij,x} - (x_i - x_j) \right] \\ &= 0, \end{aligned}$$

where $d_{ij} = d_{i0} - d_{j0}$, and one has $\lim_{t \to T^*} (x_i - x_j) = d_{ij,x}$.

Similarly, $(x_i - x_j) = d_{ij,x}$ for $t \geq T^*$ followed from (7) and (9).

Therefore, one obtains that the finite-time formation control problem of networked quadrotors can be formulated in terms of the formation tracking errors (7).

Remark 3: Quadrotor orientations (θ, ψ, ϕ) are physically restricted, i.e., $\theta \in (-\pi/2, \pi/2)$, $\phi \in (-\pi/2, \pi/2)$ and $\psi \in [0, 2\pi]$. In fact, the inertial measurement unit (IMU) of quadrotors made up by onboard accelerometers and gyroscopes can provide orientation measurements in $[0, 2\pi]$ or $(-\pi, \pi)$.

III. MAIN RESULTS

In what follows, the main results of this paper will be presented in two steps. First, a distributed observer is designed to provide the estimated information on the leader for each follower. Second, a finite-time controller under a cascade structure is designed to achieve leader-follower formation.

A. DISTRIBUTED FINITE-TIME OBSERVER DESIGN

It is known from Assumption 1 that not all follower quadrotors have knowledge of information on the leader quadrotor. Moreover, it is also known from the communication network setup among quadrotors that each quadrotor only has access to local information. Therefore, the formation tracking errors in (7) cannot be directly used in finite-time formation controller design. To circumvent the problem, a distributed observer is designed to estimate the leader information for each follower, which takes the form of

$$\dot{\hat{x}}_i = \frac{1}{\sum_{j \in \mathcal{N}_i} a_{ij}} \sum_{j \in \mathcal{N}_i} a_{ij} \dot{\hat{x}}_j - \frac{\eta_1}{\sum_{j \in \mathcal{N}_i} a_{ij}} \mathrm{sig}^{\alpha_1} \left(\sum_{j \in \mathcal{N}_i} a_{ij} (\hat{x}_i - \hat{x}_j) \right), \tag{10}$$

$$\dot{\hat{y}}_i = \frac{1}{\sum\limits_{j\in\mathcal{N}_i} a_{ij}} \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\hat{y}}_j - \frac{\eta_2}{\sum\limits_{j\in\mathcal{N}_i} a_{ij}} \mathrm{sig}^{\alpha_2}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{y}_i - \hat{y}_j)\right), \tag{11}$$

$$\dot{\hat{z}}_i = \frac{1}{\sum\limits_{j\in\mathcal{N}_i} a_{ij}} \sum_{j\in\mathcal{N}_i} a_{ij}\dot{\hat{z}}_j - \frac{\eta_3}{\sum\limits_{j\in\mathcal{N}_i} a_{ij}} \mathrm{sig}^{\alpha_3}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{z}_i - \hat{z}_j)\right), \quad i = 1, \ldots, N, \tag{12}$$

where $[\hat{x}_i, \hat{y}_i, \hat{z}_i, \dot{\hat{x}}_i, \dot{\hat{y}}_i, \dot{\hat{z}}_i]^{\mathrm{T}}$ are the states of the ith observer, which are the estimates of positions and velocities of the leader quadrotor for the ith follower quadrotor, $0 < \alpha_1, \alpha_2, \alpha_3 < 1$ and $\eta_1, \eta_2, \eta_3 > 0$.

The distributed observer (10)–(12) has the following property.

Lemma 3: Consider the graph $\mathcal{G}$ and the distributed observer (10)–(12) Under Assumptions 1 and 2, the estimates $\hat{x}_i(t), \hat{y}_i(t), \hat{z}_i(t), i = 1, \ldots, N$, converge to $x_0(t), y_0(t), z_0(t)$ within finite time T_{ob}.

Proof: Take the distributed observer design on the x axis as an example. It follows from (10) that

$$\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{\hat{x}}_i - \dot{\hat{x}}_j) = -\eta_1 \mathrm{sig}^{\alpha_1}\left(\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_i - \hat{x}_j)\right), \quad i = 1, \ldots, N. \tag{13}$$

Define $\Delta_i := \sum\limits_{j\in\mathcal{N}_i} a_{ij}(\hat{x}_i - \hat{x}_j)$ and rewrite (13) as

$$\dot{\Delta}_i = -\eta_1 \mathrm{sig}^{\alpha_1}(\Delta_i), \tag{14}$$

where $\mathrm{sig}^{\alpha_1}(\Delta_i) = |\Delta_i|^{\alpha_1}\mathrm{sgn}(\Delta_i)$ with $\mathrm{sgn}(\cdot)$ being the signum function.

Consider Lyapunov functions $V_i = \frac{1}{2}\Delta_i^2, i = 1, \ldots, N$.

The time derivative of V_i along the trajectory of (14) satisfies

$$\begin{aligned}\dot{V}_i &= -\eta_1|\Delta_i|^{1+\alpha_1} \\ &= -\eta_1 2^{\frac{1+\alpha_1}{2}} V_i^{\frac{1+\alpha_1}{2}} \\ &= -\bar{\eta}_1 V_i^{\bar{\alpha}_1},\end{aligned} \tag{15}$$

where $\bar{\eta}_1 = \eta_1 2^{\frac{1+\alpha_1}{2}} > 0$ and $0 < \bar{\alpha}_1 = \frac{1+\alpha_1}{2} < 1$. Upon using finite-time stability theorem in [27], one has that Δ_i reaches zero within finite-time $t_i^* = \frac{V_i(\Delta_i(t_0))^{(1-\bar{\alpha}_1)}}{\bar{\eta}_1(1-\bar{\alpha}_1)}$.

Recall the Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ as shown in (6). Since there exist no edges from followers to the leader, one has that $\sum_{i=1}^{N} a_{0i} = 0$ and $[a_{01}, \ldots, a_{0N}] = 0$.

Denote $\hat{x} = [\hat{x}_1, \ldots, \hat{x}_N]^{\mathrm{T}}$. Since $\hat{x}_0 = x_0$, one has

$$[\Delta_1(t), \ldots, \Delta_N(t)]^{\mathrm{T}} = \mathcal{H}(\hat{x}(t) - 1_N \otimes x_0(t)) = 0, \quad \forall t \geq T_{ob,x} := \max\{t_1^*, \ldots, t_N^*\}, \tag{16}$$

where $\Delta_i(t_i^*) = 0$.

Applying Assumption 2 and Lemma 2, one knows that $\mathcal{H}$ is nonsingular. It thus follows from (16) that $\hat{x}_i = x_0$ for all $t \geq T_{ob,x}$.

Similarly, it is obtained that $\hat{y}_i = y_0$ and $\hat{z}_i = z_0$ for all $t \geq T_{ob} := \max\{T_{ob,x}, T_{ob,y}, T_{ob,z}\}$.

The proof is thus completed. ■

B. DISTRIBUTED FINITE-TIME FORMATION CONTROLLER DESIGN

The proposed controller is composed of an inner-loop attitude controller and an outer-loop position controller in terms of cascade structures since it is known from the analysis of dynamic characteristics for aircrafts that quadrotors have slower position dynamics and faster attitude dynamics. The distributed position controller receives quadrotor states to make a group of quadrotors achieve a desired formation pattern, while the attitude controller receives desired angles generated by the position controller to regulate quadrotor attitudes.

1) POSITION CONTROL

For the convenience of description, the translational dynamics of the ith quadrotor in (4) is expressed as

$$\begin{cases} \ddot{x}_i = -\dfrac{k_{i,x}}{m_i}\dot{x}_i + u_{i,x}, \\ \ddot{y}_i = -\dfrac{k_{i,y}}{m_i}\dot{y}_i + u_{i,y}, \\ \ddot{z}_i = -\dfrac{k_{i,z}}{m_i}\dot{z}_i - g + u_{i,z}, \end{cases} \tag{17}$$

where

$$\begin{cases} u_{i,x} = \dfrac{T_i}{m_i}(\cos\psi_i \sin\theta_i \cos\phi_i + \sin\psi_i \sin\phi_i), \\ u_{i,y} = \dfrac{T_i}{m_i}(\sin\psi_i \sin\theta_i \cos\phi_i - \cos\psi_i \sin\phi_i), \\ u_{i,z} = \dfrac{T_i}{m_i}(\cos\theta_i \cos\phi_i). \end{cases} \tag{18}$$

To address the formation control problem as formulated in Definition 3, the distributed position controller for each quadrotor is designed as follows:

$$\begin{aligned} u_{i,x} = \sum_{j\in\mathcal{N}_i} a_{ij}\Big[&k_{i,1}sig^{\alpha_{i,1}}(\hat{x}_i - \hat{x}_j - x_i + x_j + d_{ij,x}) \\ &+ k_{i,2}sig^{\alpha_{i,2}}(\hat{v}_{i,x} - \hat{v}_{j,x} - v_{i,x} + v_{j,x})\Big] \\ &+ \dot{\hat{v}}_{i,x} + \frac{k_{i,x}}{m_i}\dot{x}_i, \end{aligned} \tag{19}$$

$$\begin{aligned} u_{i,y} = \sum_{j\in\mathcal{N}_i} a_{ij}\Big[&k_{i,1}sig^{\alpha_{i,1}}(\hat{y}_i - \hat{y}_j - y_i + y_j + d_{ij,y}) \\ &+ k_{i,2}sig^{\alpha_{i,2}}(\hat{v}_{i,y} - \hat{v}_{j,y} - v_{i,y} + v_{j,y})\Big] \\ &+ \dot{\hat{v}}_{i,y} + \frac{k_{i,y}}{m_i}\dot{y}_i, \end{aligned} \tag{20}$$

$$\begin{aligned} u_{i,z} = \sum_{j\in\mathcal{N}_i} a_{ij}\Big[&k_{i,1}sig^{\alpha_{i,1}}(\hat{z}_i - \hat{z}_j - z_i + z_j + d_{ij,z}) \\ &+ k_{i,2}sig^{\alpha_{i,2}}(\hat{v}_{i,z} - \hat{v}_{j,z} - v_{i,z} + v_{j,z})\Big] \\ &+ \dot{\hat{v}}_{i,z} - g + \frac{k_{i,z}}{m_i}\dot{z}_i, \end{aligned} \tag{21}$$

where $k_{i,1}, k_{i,2} > 0$, $0 < \alpha_{i,1} < 1$ and $\alpha_{i,2} = \frac{2\alpha_{i,1}}{1+\alpha_{i,1}}$, $i = 1, \ldots, N$.

Theorem 1: Consider the graph $\mathcal{G}$ and the closed-loop system consisting of the system (17) and the distributed dynamic control law (10)–(12) and (19)–(21). Under Assumptions 1-2, then the leader-follower finite-time formation control problem is solved via the proposed control law.

Proof: When designing the position controller, we only take the proof on the x axis as an example.

Define

$$\begin{aligned} \hat{e}_{i,x} &= \hat{x}_i - x_i + d_{i0,x}, \\ \hat{e}_{i,vx} &= \hat{v}_{i,x} - v_{i,x}, \end{aligned} \tag{22}$$

where $\hat{x}_i$ and $\hat{v}_{i,x}$ are the estimates of the leader information on position and velocity for the ith follower.

From (17)–(19) and (22), one has that

$$\begin{aligned} \dot{\hat{e}}_{i,x} &= \hat{e}_{i,xv}, \\ \dot{\hat{e}}_{i,vx} &= \dot{\hat{v}}_{i,x} + \frac{k_{i,x}}{m_i}\dot{x}_i - u_{i,x} \\ &= -\sum_{j\in\mathcal{N}_i} a_{ij}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x} - \hat{e}_{j,x}) \\ &\quad - \sum_{j\in\mathcal{N}_i} a_{ij}k_{i,2}sig^{\alpha_{i,2}}(\hat{e}_{i,vx} - \hat{e}_{j,vx}). \end{aligned} \tag{23}$$

Next, we will prove the finite-time stability of system (23) in three steps.

Step 1: To show system (23) is globally asymptotically stable.

Consider a Lyapunov function as

$$V = \sum_{i=1}^{N}\left[\sum_{j\in\mathcal{N}_i} a'_{ij}k_{i,1}\int_0^{\hat{e}_{i,x}-\hat{e}_{j,x}} sig^{\alpha_{i,1}}(\tau)d\tau + \hat{e}_{i,vx}^2\right], \tag{24}$$

where $a'_{ij} = a_{ij}$ for $i, j \in \mathcal{V}$ except that $a'_{i0} = 2a_{i0}$ for $i \in \mathcal{V}$.

Taking the time derivative of V along the trajectory of system (23) with $a_{ij} = a_{ji}$ for $i, j \in \mathcal{V}\backslash\{0\}$ yields that

$$\begin{aligned} \dot{V} &= \sum_{i=1}^{N}\left[\sum_{j\in\mathcal{N}_i} a'_{ij}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x} - \hat{e}_{j,x})(\hat{e}_{i,vx} - \hat{e}_{j,vx})\right] \\ &\quad - 2\sum_{i=1}^{N}\hat{e}_{i,vx}\left[\sum_{j\in\mathcal{N}_i} a_{ij}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x} - \hat{e}_{j,x})\right. \\ &\quad \left. + \sum_{j\in\mathcal{N}_i} a_{ij}k_{i,2}sig^{\alpha_{i,2}}(\hat{e}_{i,vx} - \hat{e}_{j,vx})\right] \\ &= \sum_{i=1}^{N}\sum_{j=1}^{N} a'_{ij}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x} - \hat{e}_{j,x})(\hat{e}_{i,vx} - \hat{e}_{j,vx}) \\ &\quad + \sum_{i=1}^{N} a'_{i0}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x})\hat{e}_{i,vx} \\ &\quad - \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x} - \hat{e}_{j,x})(\hat{e}_{i,vx} - \hat{e}_{j,vx}) \\ &\quad - 2\sum_{i=1}^{N} a_{i0}k_{i,1}sig^{\alpha_{i,1}}(\hat{e}_{i,x})\hat{e}_{i,vx} \\ &\quad - 2\sum_{i=1}^{N}\hat{e}_{i,vx}\sum_{j\in\mathcal{N}_i} a_{ij}k_{i,2}sig^{\alpha_{i,2}}(\hat{e}_{i,vx} - \hat{e}_{j,vx}) \\ &= -\sum_{i=1}^{N}\sum_{j\in\mathcal{N}_i} a'_{ij}k_{i,2}sig^{\alpha_{i,2}}(\hat{e}_{i,vx} - \hat{e}_{j,vx})(\hat{e}_{i,vx} - \hat{e}_{j,vx}) \\ &\le 0. \end{aligned} \tag{25}$$

Let $\Delta_{e,i} := \sum_{j\in\mathcal{N}_i} a'_{ij}(\hat{e}_{i,vx} - \hat{e}_{j,vx})$, then

$$\left[\Delta_{e,1}, \ldots, \Delta_{e,N}\right]^{\mathrm{T}} = \mathcal{H}'\left[\hat{e}_{1,vx} - e_{0,vx}, \ldots, \hat{e}_{N,vx} - e_{0,vx}\right]^{\mathrm{T}}, \tag{26}$$

where $\mathcal{H}' = \mathcal{H} + \mathcal{A}_0$ with $\mathcal{H}$ and $\mathcal{A}_0$ as defined in (6).

Upon using Lemma 2, one has that $\left[\Delta_{e,1}, \ldots, \Delta_{e,N}\right]^{\mathrm{T}} = [0, \ldots, 0]^{\mathrm{T}}$ is equivalent to $\hat{e}_{i,vx} = 0$ for $i = 1, \ldots, N$, since $\mathcal{H}'$ is nonsingular and $e_{0,vx} = 0$.

From (25) and (26), it is known that $\dot{V} = 0$ implies $\hat{e}_{i,vx} = 0$ and thus $\dot{\hat{e}}_{i,vx} = 0$. It is also known from (23) that $(\hat{e}_{i,vx}, \dot{\hat{e}}_{i,vx}) = 0$ implies $\hat{e}_{i,x} = 0$. Applying LaSalle's theorem, one has that system (23) is globally asymptotically stable.

Step 2: To show system (23) is homogeneous.

It is obtained from Definition 2 that system (23) is homogeneous of degree $k_r = \frac{\alpha_{i,1}-1}{2}$ with respect to the dilation

$$(r_{1,1}, r_{1,2}, \ldots, r_{i,1}, r_{i,2}, \ldots, r_{N,1}, r_{N,2}),$$

where $r_{i,1} = 1$, $r_{i,2} = \frac{\alpha_{i,1}+1}{2}$, $0 < \alpha_{i,1} = \kappa_1 < 1$, and $\alpha_{i,2} = \frac{2\alpha_{i,1}}{1+\alpha_{i,1}}$.

Thus, by Lemma 1, system (23) is globally finite-time stable. Then, with Lemma 3 and Definition 3, one concludes that the quadrotors can form a desired geometric structure within finite time.

Step 3: To estimate the finite upper bound on the settling time of system (23).

Let $\hat{e}_x := \left[\hat{e}_{1,x}, \ldots, \hat{e}_{N,x}\right]^{\mathrm{T}}$ and $\hat{e}_{vx} := \left[\hat{e}_{1,vx}, \ldots, \hat{e}_{N,vx}\right]^{\mathrm{T}}$.

Then, for any given $\varepsilon > 0$, the Lyapunov function (24) and its derivative satisfy

$$V(\varepsilon^{\frac{2}{1+\kappa_1}}\hat{e}_x, \varepsilon\hat{e}_{vx}) = \varepsilon^2 V(\hat{e}_x, \hat{e}_{vx}), \tag{27}$$

$$\dot{V}(\varepsilon^{\frac{2}{1+\kappa_1}}\hat{e}_x, \varepsilon\hat{e}_{vx}) = \varepsilon^{1+\kappa_1}\dot{V}(\hat{e}_x, \hat{e}_{vx}). \tag{28}$$

Denote $\varepsilon = V(\hat{e}_x, \hat{e}_{vx})^{-\frac{1}{2}}$. Thus, one has

$$\frac{\dot{V}(\hat{e}_x, \hat{e}_{vx})}{V(\hat{e}_x, \hat{e}_{vx})^{\frac{1+\kappa_1}{2}}} = \dot{V}(V^{-\frac{1}{1+\kappa_1}}\hat{e}_x, V^{-\frac{1}{2}}\hat{e}_{vx}). \tag{29}$$

Moreover, it follows from (27) that

$$V(V^{-\frac{1}{1+\kappa_1}}\hat{e}_x, V^{-\frac{1}{2}}\hat{e}_{vx}) = 1. \tag{30}$$

Based on the above mentioned analysis on the asymptotical stability of system (23), there exists a positive constant c_1

such that

$$\frac{\dot{V}(\hat{e}_x, \hat{e}_{vx})}{V(\hat{e}_x, \hat{e}_{vx})^{\frac{1+\kappa_1}{2}}} = \dot{V}(V^{-\frac{1}{1+\kappa_1}}\hat{e}_x, V^{-\frac{1}{2}}\hat{e}_{vx}) \leq -c_1, \quad (31)$$

that is

$$\dot{V}(\hat{e}_x, \hat{e}_{vx}) \leq -c_1 V(\hat{e}_x, \hat{e}_{vx})^{\frac{1+\kappa_1}{2}}. \quad (32)$$

Applying the finite-time stability theorem in [27], one has that

$$T_{out,x} \leq \frac{2V(\hat{e}_x(t_0), \hat{e}_{vx}(t_0))^{\frac{1-\kappa_1}{2}}}{c_1(1-\kappa_1)}. \quad (33)$$

Thus, the proof is completed. ■

Remark 4: It can be observed from (33) and the finite-time stability theory that the settling time $T_{out,x}$ is related with two parameters c_1 and κ_1. A smaller c_1 leads to a larger $T_{out,x}$, which is easy to be obtained, while the influence of κ_1 on $T_{out,x}$ is more complex because of the dependence on $V(\hat{e}_x(t_0), \hat{e}_{vx}(t_0))$. If $V(\hat{e}_x(t_0), \hat{e}_{vx}(t_0)) \leq 1$, a larger κ_1 leads to a larger $T_{out,x}$. However, if $V(\hat{e}_x(t_0), \hat{e}_{vx}(t_0)) > 1$, the influence of κ_1 on $T_{out,x}$ is not monotonic any longer.

2) ATTITUDE CONTROL

The desired attitude of the quadrotors calculated by intermediate control inputs $u_i = [u_{i,x}, u_{i,y}, u_{i,z}]^{\mathrm{T}}$, $i = 1, \ldots, N$, is given by

$$\begin{cases} T_i^d = m_i\sqrt{u_{i,x}^2 + u_{i,y}^2 + u_{i,z}^2}, \\ \phi_i^d = \arcsin\left(\frac{m_i(u_{i,x}\sin\psi_i^d - u_{i,y}\cos\psi_i^d)}{T_i^d}\right), \\ \theta_i^d = \arctan\left(\frac{u_{i,x}\cos\psi_i^d + u_{i,y}\sin\psi_i^d}{u_{i,z}}\right), \end{cases} \quad (34)$$

where the yaw angle ψ_i^d is a degree of freedom, which may be set to 0 for the sake of convenience.

To facilitate the attitude controller design, rewriting the rotational dynamics of the ith quadrotor in (5) gives that

$$\begin{cases} \ddot{\theta}_i = -\frac{k_{i,\theta}l_{i,\theta}}{J_{i,\theta}}\dot{\theta}_i + \tau_{i,\theta}, \\ \ddot{\psi}_i = -\frac{k_{i,\psi}}{J_{i,\psi}}\dot{\psi}_i + \tau_{i,\psi}, \\ \ddot{\phi}_i = -\frac{k_{i,\phi}l_{i,\phi}}{J_{i,\phi}}\dot{\phi}_i + \tau_{i,\phi}, \end{cases} \quad (35)$$

where $\tau_{i,\theta} = \frac{M_{i,\theta}}{J_{i,\theta}}$, $\tau_{i,\psi} = \frac{M_{i,\psi}}{J_{i,\psi}}$ and $\tau_{i,\phi} = \frac{M_{i,\phi}}{J_{i,\phi}}$.

Thus, the attitude controller for each quadrotor is designed as follows:

$$\begin{aligned} \tau_{i,\theta} &= k_{i,3}sig^{\alpha_{i,3}}(\theta_i^d - \theta_i) \\ &\quad + k_{i,4}sig^{\alpha_{i,4}}(\dot{\theta}_i^d - \dot{\theta}_i) + \ddot{\theta}_i^d + \frac{k_{i,\theta}l_i}{J_{i,\theta}}\dot{\theta}_i, \end{aligned} \quad (36)$$

$$\begin{aligned} \tau_{i,\psi} &= k_{i,3}sig^{\alpha_{i,3}}(\psi_i^d - \psi_i) \\ &\quad + k_{i,4}sig^{\alpha_{i,4}}(\dot{\psi}_i^d - \dot{\psi}_i) + \ddot{\psi}_i^d + \frac{k_{i,\psi}}{J_{i,\psi}}\dot{\psi}_i, \end{aligned} \quad (37)$$

$$\begin{aligned} \tau_{i,\phi} &= k_{i,3}sig^{\alpha_{i,3}}(\phi_i^d - \phi_i) \\ &\quad + k_{i,4}sig^{\alpha_{i,4}}(\dot{\phi}_i^d - \dot{\phi}_i) + \ddot{\phi}_i^d + \frac{k_{i,\phi}l_i}{J_{i,\phi}}\dot{\phi}_i, \end{aligned} \quad (38)$$

where $k_{i,3}, k_{i,4} > 0$ and $0 < \alpha_{i,3}, \alpha_{i,4} < 1$ for $i = 1, \ldots, N$.

Theorem 2: Consider the graph $\mathcal{G}$ and the closed-loop system consisting of the system (35) and the control law (36)–(38). Then with the proposed control law, the desired attitudes in (34) can be tracked within finite time.

Proof: We take pitch angle θ as an example.

Define

$$e_{i,\theta} = \theta_i^d - \theta_i. \quad (39)$$

From (35), (36) and (39), one has that

$$\begin{aligned} \dot{e}_{i,\theta} &= \dot{\theta}_i^d - \dot{\theta}_i, \\ \ddot{e}_{i,\theta} &= -k_{i,3}sig^{\alpha_{i,3}}(e_{i,\theta}) - k_{i,4}sig^{\alpha_{i,4}}(\dot{e}_{i,\theta}). \end{aligned} \quad (40)$$

Choose the following Lyapunov function

$$W_i = k_{i,3}\int_0^{e_{i,\theta}} sig^{\alpha_{i,3}}(\tau)d\tau + \frac{1}{2}\dot{e}_{i,\theta}^2, \quad (41)$$

and its time derivative is

$$\begin{aligned} \dot{W}_i &= k_{i,3}sig^{\alpha_{i,3}}(e_{i,\theta})\dot{e}_{i,\theta} + \dot{e}_{i,\theta}\ddot{e}_{i,\theta} \\ &= -k_{i,4}|\dot{e}_{i,\theta}|^{1+\alpha_{i,4}} \leq 0. \end{aligned} \quad (42)$$

It follows from (42) that $\dot{W}_i = 0$ implies $\dot{e}_{i,\theta} = 0$ and then $\ddot{e}_{i,\theta} = 0$. Moreover, system (40) indicates that $(\dot{e}_{i,\theta}, \ddot{e}_{i,\theta}) = 0$ implies $e_{i,\theta} = 0$. Thus, it is known from LaSalle's theorem that system (40) is globally asymptotically stable.

Furthermore, system (40) is homogeneous of degree $k_s = \frac{\alpha_{i,3}-1}{2}$ with respect to the dilation

$$(r_{1,3}, r_{1,4}, \ldots, r_{i,3}, r_{i,4}, \ldots, r_{N,3}, r_{N,4}),$$

where $r_{i,3} = 1$, $r_{i,4} = \frac{\alpha_{i,3}+1}{2}$, $0 < \alpha_{i,3} = \kappa_2 < 1$, and $\alpha_{i,4} = \frac{2\alpha_{i,3}}{1+\alpha_{i,3}}$.

Therefore, one concludes from Lemma 1 that system (40) is globally finite-time stable, that is, with the proposed controller (36)–(38), the desired attitudes are tracked by the quadrotors within finite time.

Following the similar proof to obtain the upper bound as shown in (33), the settling time of system (40) is upper bounded by

$$T_{in,\theta}^i \leq \frac{2W_i(e_{i,\theta}(t_0), \dot{e}_{i,\theta}(t_0))^{\frac{1-\kappa_2}{2}}}{c_2(1-\kappa_2)}, \quad (43)$$

where c_2 is a positive constant satisfying

$$\begin{aligned} \frac{\dot{W}_i(e_{i,\theta}, \dot{e}_{i,\theta})}{W_i(e_{i,\theta}, \dot{e}_{i,\theta})^{\frac{1+\kappa_2}{2}}} &= \dot{W}_i(W_i^{-\frac{1}{1+\kappa_2}}e_{i,\theta}, W_i^{-\frac{1}{2}}\dot{e}_{i,\theta}) \\ &\leq -c_2. \end{aligned} \quad (44)$$

The proof is thus completed. ■

Remark 5: It should be noted that both the derivatives of observer information and the derivatives of desired attitudes are used in the design of formation controllers (19)-(21) and (36)–(38). However, their explicit expressions are too

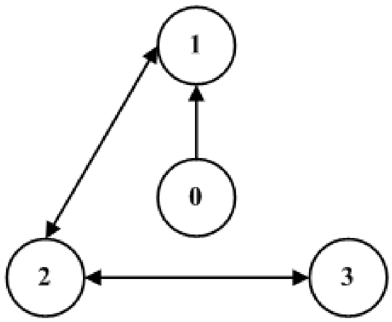

FIGURE 2. Topology graph and the desired formation pattern.

TABLE 1. Quadrotor simulation parameters.

Parameter	Description	Value
m_i, kg	quadrotor mass	1
$g, m/s^2$	gravity acceleration	9.8
$J_{i,\theta}, J_{i,\phi}, kg \cdot m^2$	moments of inertia	1.466×10^{-2}
$J_{i,\psi}, kg \cdot m^2$	moment of inertia	2.848×10^{-2}
$k_{i,x}, k_{i,y}, kg/s$	aerodynamic damping coefficients	6.579×10^{-2}
$k_{i,z}, kg/s$	aerodynamic damping coefficient	0.054
$k_{i,\theta}, k_{i,\phi}, kg \cdot m/s$	aerodynamic damping coefficients	0.01
$k_{i,\psi}, kg \cdot m^2/s$	aerodynamic damping coefficient	0.02
$l_{i,\theta}, l_{i,\phi}, m$	length	0.025
i	index of quadrotors	0,1,2,3

intricate to be described. To solve this problem, the technique of finite-time differentiator as shown in [43] and [44] is adopted. The details on how to use this technique are given in the following simulation section.

Remark 6: The settling time $T^i_{in,\theta}$ is related with two parameters c_2 and κ_2. Following the arguments presented in Remark 4, one has the similar result regarding the influence of c_2 and κ_2 on $T^i_{in,\theta}$. Furthermore, it is worth mentioning that in practice, the settling time $T_{out} := \max\left\{T_{out,x}, T_{out,y}, T_{out,z}\right\}$ should be more than twice as long as the settling time $T_{in} := \max\limits_{i=1,\ldots,N}\left\{T^i_{in,\theta}, T^i_{in,\phi}, T^i_{in,\phi}\right\}$ to ensure that the inner-loop control systems can accurately receive the signals from the outer-loop control systems [45].

IV. NUMERICAL SIMULATIONS

Consider four quadrotors with dynamics (4)-(5) to demonstrate the performance of the proposed finite-time control algorithm. Simulations with three quadrotors following a leader quadrotor while maintaining a triangle formation have been conducted.

The topology graph describing the information exchange among quadrotors is shown in Fig. 2, which satisfies Assumptions 1-2, and the quadrotor simulation parameters are given in Table 1 .

The desired formation trajectory is $[x_0(t), y_0(t), z_0(t)]^{\mathrm{T}} = [5\sin(0.2t), 5\cos(0.2t), t]^{\mathrm{T}}$, the desired formation structure is a triangle, and the desired relative positions from each follower to the leader are given by $d_{10} = [0, 0, 10]^{\mathrm{T}}$, $d_{20} = [10\cos(\frac{7\pi}{6}), 0, 10\sin(\frac{7\pi}{6})]^{\mathrm{T}}$, $d_{30} = [10\cos(-\frac{\pi}{6}), 0, 10\sin(-\frac{\pi}{6})]^{\mathrm{T}}$.

TABLE 2. Initial position and linear velocity of each quadrotor.

Label	$[x_i(0), y_i(0), z_i(0)]^{\mathrm{T}}, m$	$[\dot{x}_i(0), \dot{y}_i(0), \dot{z}_i(0)]^{\mathrm{T}}, m/s$
1	$[0.8, 0, -5]^{\mathrm{T}}$	$[0, 0.1, 0]^{\mathrm{T}}$
2	$[0, 0.8, 5]^{\mathrm{T}}$	$[0, 0.5, 0]^{\mathrm{T}}$
3	$[-0.5, 0.8, 0]^{\mathrm{T}}$	$[0, 0.8, 0]^{\mathrm{T}}$

TABLE 3. Initial angle and angular velocity of each quadrotor.

Label	$[\theta_i(0), \psi_i(0), \phi_i(0)]^{\mathrm{T}}, rad$	$[\dot{\theta}_i(0), \dot{\psi}_i(0), \dot{\phi}_i(0)]^{\mathrm{T}}, rad/s$
1	$[0, \pi/6, 0]^{\mathrm{T}}$	$[0, 0, 0]^{\mathrm{T}}$
2	$[0, 5\pi/6, 0]^{\mathrm{T}}$	$[0, 0, 0]^{\mathrm{T}}$
3	$[0, -\pi/6, 0]^{\mathrm{T}}$	$[0, 0, 0]^{\mathrm{T}}$

TABLE 4. Initial states of each finite-time observer.

Label	$\hat{x}_i(0), m$	$\hat{y}_i(0), m$	$\hat{z}_i(0), m$
1	$x_0(0)$	$y_0(0)$	$z_0(0)$
2	0.15	-0.21	-0.18
3	0.10	0.20	-0.13

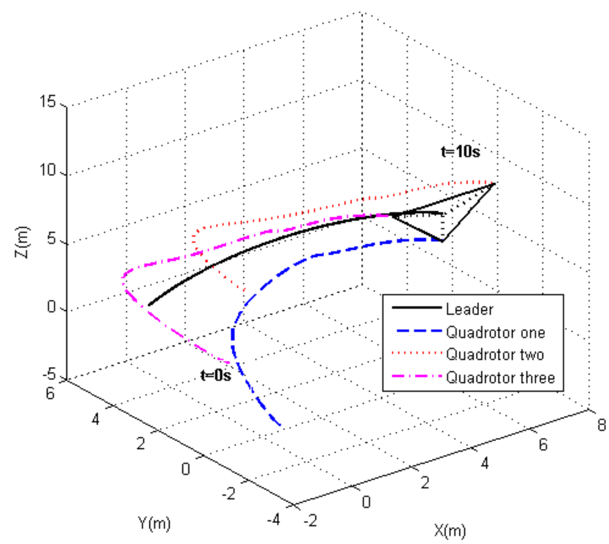

FIGURE 3. Trajectories of all quadrotors.

TABLE 5. Settling time of observation errors for each follower quadrotor.

Label	$x_i - \hat{x}_i$	$y_i - \hat{y}_i$	$z_i - \hat{z}_i$
1	0.185s	0.539s	0.194s
2	0.189s	0.543s	0.198s
3	0.189s	0.543s	0.198s

In the simulation, the initial states of each follower quadrotor are listed in Tables 2-3 and the initial states of each finite-time observer are given in Table 4. It should be noted that the initial states in Table 2 are the same as that in [38], and the initial states in Tables 3-4 are chosen randomly.

Then, apply Lemma 3, Theorem 1 and Theorem 2 with the following control parameters to each follower quadrotor. The

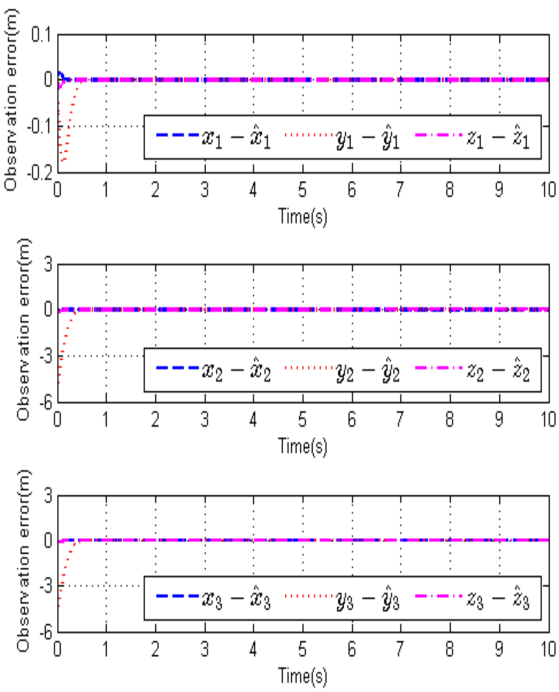

FIGURE 4. Observation errors of all follower quadrotors.

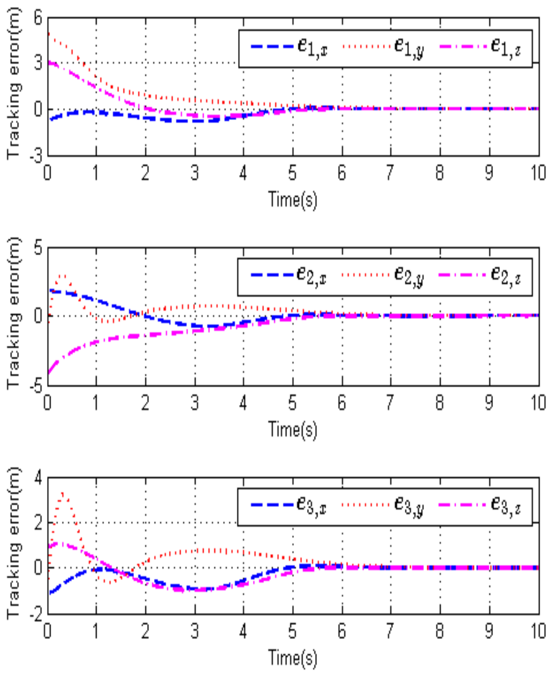

FIGURE 5. Formation position errors of all follower quadrotors.

finite-time differentiator used in the simulation is the same as that in [43].

$$\eta_1 = \eta_2 = \eta_3 = 10, \quad \alpha_1 = \alpha_2 = \alpha_3 = 0.7,$$
$$k_{i,1} = 10, \quad k_{i,2} = 10, \ \alpha_{i,1} = 0.7, \ \alpha_{i,2} = 2\alpha_{i,1}/(1+\alpha_{i,1}),$$
$$k_{i,3} = 10, \quad k_{i,4} = 10, \ \alpha_{i,3} = 0.7, \ \alpha_{i,4} = 2\alpha_{i,3}/(1+\alpha_{i,3}),$$
$$i = 1, 2, 3.$$

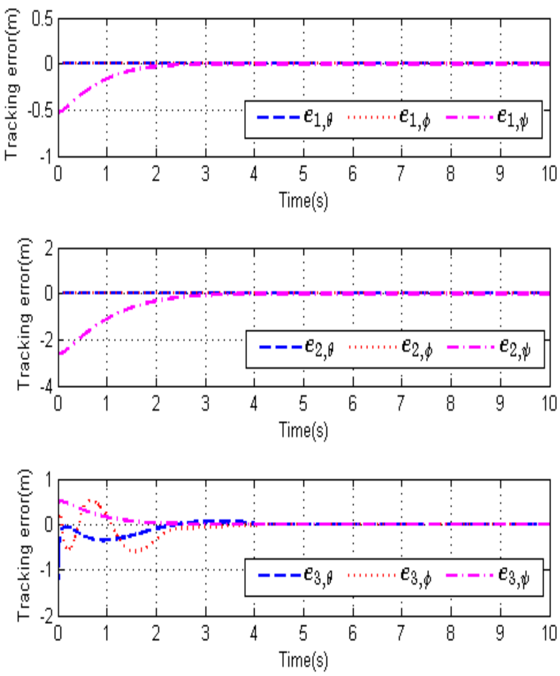

FIGURE 6. Formation attitude errors of all follower quadrotors.

TABLE 6. Settling time of formation errors for each follower quadrotor.

Label	$e_{i,x}$	$e_{i,y}$	$e_{i,y}$	$e_{i,\theta}$	$e_{i,\psi}$	$e_{i,\phi}$
1	8.456s	9.005s	8.352s	0s	3.981s	0s
2	8.651s	8.902s	8.507s	0s	3.474s	0s
3	8.618s	9.013s	8.960s	4.395s	3.981s	4.212s

Fig. 3 shows the formation trajectory of all quadrotors during 10s. It is observed that quadrotors converge to the desired formation.

The observation errors $x_i - \hat{x}_i$, $y_i - \hat{y}_i$ and $z_i - \hat{z}_i$ are shown in Fig. 4, and the formation tracking errors $e_{i,x}$, $e_{i,y}$, $e_{i,z}$, $e_{i,\theta}$, $e_{i,\psi}$ and $e_{i,\phi}$ are shown in Figs. 5-6. The settling time taken for the observation errors and formation errors to converge to the origins is shown in Tables 5-6 , respectively. It is observed that the observer of each follower quadrotor can estimate the leader quadrotor information within 0.55s at most, and the desired formation can be achieved at about 9s. The effectiveness of the proposed control approach is verified by the simulation results.

V. CONCLUSIONS

In this paper, a distributed control approach to the finite-time formation of networked quadrotors has been proposed. It is shown that leader-follower formation of the quadrotors can be achieved within finite time via the proposed control approach. Moreover, the control law is designed under a cascade structure with an inner-loop attitude controller and an outer-loop position controller, and the control architecture accords with quadrotor dynamic characteristics. Based on the obtained results, it is of interest to extend the static network to the time-varying one, and to take into account the presence of disturbances and uncertainties.

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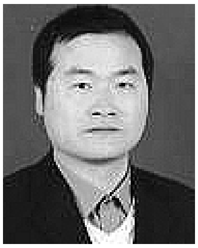

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