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Design of X-Bar Control Chart for Resampling Under Uncertainty Environment

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ABSTRACT This paper presents the designing of the X-bar control chart using resampling scheme under the uncertainty environment. The necessary measures of the present chart are derived under the neutrosophic system. The neutrosophic average run length (NARL) when the process is in-control and out-of-control are derived under the neutrosophic statistical interval method (NSIM). The neutrosophic control chart coefficients are determined through the algorithm developed under NSIM. The comparative study shows that the proposed chart is better than the existing chart in NARL.

INDEX TERMS Monitoring, Shewhart chart, neutrosophic average run length, statistical interval method, shift.

I. INTRODUCTION

The monitoring of the manufacturing process is very important to improve the quality of the product. Therefore, to meet specifications limits or the target, a quick indication is needed when the process moves away from the given target. The control chart is effective tool for the monitoring of the manufacturing process in the industries and service companies. The Shewhart control charts have many applications in the industries. The Shewhart control charts are easy in the implementation and save time in deciding the state of the process. The attribute and variable Shewhart control charts are used to reduce non-conforming items. In earlier chart is applied when data coming from the process is discrete and later one is used for the monitoring the process when data is measurable. The X-bar control chart is contained more information about the process data than the attribute control chart such as the np-control chart. [1] pointed out that the control charts are applied to monitor the variation in the process. [2] discussed the applications of the control charts in the industry [3] proposed the variable X-bar control chart. [4] proposed the variable control chart using efficient sampling schemes. [5] designed the control chart for monitoring the variability in the process. More details on the applications of the control charts can be read in [6], [7] and [8].

Although, the Shewhart control charts are designed using single sampling is easy to apply but needs a larger sample for the decision-making. Larger the sample size means the high cost for the monitoring of the process. Further, the control charts using the single sample cannot be applied when the experimenters are in-decision at the information of the first sample. The repetitive sampling scheme introduced by [9] provides a decision on less sample than single sampling. The repetitive sampling guides the experiments to resample if in-decision at the first sample information. Because of many applications of the repetitive sampling in the control charts, several authors proposed the control chart using it and showed the efficiency over the control charts using the single sampling. [10] proposed the X-bar control using repetitive sampling. [11] worked on double EWMA chart using this sampling. [12] worked on the process capability index under repetitive sampling. More information can be read in [13]–[18] and [19].

Usually, the control charts are designed under the assumptions that all observations or parameters are known and determined. As mentioned by [1],"observations include human judgments, and evaluations and decisions, a continuous random variable of a production process should include the variability caused by human subjectivity or measurement devices, or environmental conditions. These variability causes create vagueness in the measurement system". To deal with these situations, the control charts using the fuzzy logic are applied for the monitoring of the process. [20] proposed the fuzzy chart for more than two processes. [21] proposed the fuzzy attribute chart. The fuzzy based attribute and variable control

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TABLE 1. The NARL when $n_N \epsilon$ [2], [5].

c	$k_N\epsilon$[(2.512386499,0.014060480), (7.59478947, 0.928857605)]	$k_N\epsilon$[(0.8232839778,0.0270404400), (2.558875933, 2.2815332076),]	$k_N\epsilon$[(2.680640816,0.006220976), (3.164757005, 2.206055728)]
	$n_N\epsilon$ [2,5]	$n_N\epsilon$ [2,5]	$n_N\epsilon$ [2,5]
	ARL_{1N}		
0	[201.334488, 203.327266]	[303.204206, 303.7142]	[370.251830, 372.336796]
0.01	[192.611117, 173.231638]	[293.89580, 283.861913]	[353.544665, 347.002082]
0.02	[184.277293, 147.615477]	[284.91425, 265.426058]	[337.609521, 323.520065]
0.05	[161.438459, 91.454035]	[259.80659, 217.578305]	[294.079835, 262.807467]
0.1	[129.652077, 41.456052]	[223.41358, 157.611521]	[233.890124, 187.315790]
0.2	[84.025461, 9.100441]	[166.94313, 85.465992]	[148.538968, 97.883565]
0.3	[54.823974, 2.623204]	[126.46483, 48.378731]	[94.851564, 53.032203]
0.4	[36.040796,1.325333]	[97.10993, 28.559377]	[60.928024, 29.753056]
0.5	[23.903941, 1.065182]	[75.60171, 17.567593]	[39.404287, 17.284384]
0.6	[16.030053, 1.013048]	[59.70801, 11.253801]	[25.697740, 10.414280]
0.7	[10.904737, 1.002608]	[47.89138, 7.506310]	[16.942232, 6.533377]
0.8	[7.561067, 1.000520]	[39.08280, 5.214100]	[11.337253, 4.293594]
0.9	[5.379364, 1.000103]	[32.53385, 3.773583]	[7.747844, 2.977678]
0.95	[4.593656, 1.000046]	[29.93576, 3.260504]	[6.473350, 2.535726]
1	[3.961825, 1.000021]	[27.71908, 2.846594]	[5.457401, 2.193474]

TABLE 2. The NARL when $n_N \epsilon$ [5], [7].

c	$k_N\epsilon$[(4.734464636,2.256335550), (7.191441746, 4.134635)]	$k_N\epsilon$[(6.600517496,1.4364072075), (5.890285461, 4.235032511)]	$k_N\epsilon$[(3.052004409,2.20987128), (7.680818755, 3.7906169919)]
	$n_N\epsilon$ [5,7]	$n_N\epsilon$ [5,7]	$n_N\epsilon$ [5,7]
	ARL_{1N}		
0	[200.0537, 204.630774]	[300.2889, 305.395668]	[370.8410, 373.865400]
0.01	[184.334153, 183.801065]	[263.679368,279.897180]	[345.754512, 329.814020]
0.02	[169.884640, 165.129943]	[231.557622,256.637477]	[322.497248, 291.001873]
0.05	[133.147624, 119.915836]	[156.935287,198.327410]	[262.337533, 200.091214]
0.1	[89.075932, 70.711132]	[82.321556,130.132749]	[187.456901, 107.592053]
0.2	[40.530285, 25.186878]	[23.167862,57.713462]	[98.554817, 31.758155]
0.3	[18.945157, 9.473911]	[7.056332, 26.628067]	[53.795841, 9.934661]
0.4	[9.217799, 3.987812]	[2.656725, 12.845213]	[30.447129, 3.606120]
0.5	[4.786830, 2.056859]	[1.453363, 6.568200]	[17.860787, 1.761529]
0.6	[2.751807, 1.373896]	[1.123994, 3.647976]	[10.870590, 1.222412]
0.7	[1.811714, 1.131896]	[1.033862, 2.267458]	[6.883843, 1.064775]
0.8	[1.375887, 1.046249]	[1.009225, 1.607626]	[4.556878, 1.018767]
0.9	[1.173570, 1.016066]	[1.002505, 1.290336]	[3.171877, 1.005395]
0.95	[1.117736, 1.009425]	[1.001303, 1.200136]	[2.701457,1.002881]
1	[1.079736, 1.005509]	[1.000677, 1.137586]	[2.334327,1.001534]

charts are studied by [22]. [23] proposed the X-bar and range chart using this approach. More details about such control charts can be seen in [24]–[30] and [31].

In practice, it may not possible that all observations are determined and precise. To deal with the situation, neutrosophic statistics (NS) which is based on the neutrosophic

TABLE 3. The values of NARL when $n_N \epsilon$ [8], [10].

c	$k_N\epsilon$[(6.74426050,5.386273341), (7.621406527,7.4132229808)]	$k_N\epsilon$[(6.402750603,5.268491144), (7.48632502,7.281767770)]	$k_N\epsilon$[(7.171873445,5.145571064), (8.11029551, 7.20390135)]
	$n_N\epsilon$ [8,10]	$n_N\epsilon$ [8,10]	$n_N\epsilon$ [8,10]
	ARL_{1N}		
0	[200.952707, 205.650778]	[300.752316,303.910070]	[370.344288,376.302984]
0.01	[184.061487,187.645865]	[275.258755,276.270372]	[335.564108,340.711581]
0.02	[168.679123,171.370179]	[252.075911,251.371054]	[304.178994,308.735345]
0.05	[130.230859,131.231946]	[194.280884,190.370325]	[227.129484,230.813667]
0.1	[85.485796,85.612217]	[127.322447,121.941421]	[140.720295,144.376096]
0.2	[38.228556,38.881789]	[56.984876,53.435621]	[55.630418, 59.748236]
0.3	[17.975885,19.217141]	[26.894432,25.509876]	[22.925395, 26.558565]
0.4	[8.949448, 10.315067]	[13.400499, 13.236996]	[9.974098, 12.675612]
0.5	[4.794942, 6.005928]	[7.100097, 7.453798]	[4.723584, 6.536769]
0.6	[2.833998, 3.791974]	[4.058898, 4.551683]	[2.557484, 3.694954]
0.7	[1.891246, 2.595254]	[2.552071, 3.013453]	[1.653209, 2.331641]
0.8	[1.432654, 1.921656]	[1.791156, 2.160556]	[1.273250, 1.660859]
0.9	[1.208428, 1.531452]	[1.402296, 1.671328]	[1.113402, 1.325914]
0.95	[1.143945, 1.401628]	[1.285971, 1.509186]	[1.072712, 1.227573]
1	[1.098967, 1.301889]	[1.202613, 1.384660]	[1.046431, 1.158014]

TABLE 4. The NARL for various n_N.

c	$k_N\epsilon$[(8.009442623,5.19141353), (15.855788396,15.41129)]	$k_N\epsilon$[(5.37095783,4.269663496), (11.39003005,10.263297632)]	$k_N\epsilon$[(5.00442865,1.99926409), (17.59261423,16.18589183)]
	$n_N\epsilon$ [8,18]	$n_N\epsilon$ [7,13]	$n_N\epsilon$ [5,19]
	ARL_{1N}		
0	[200.452494,205.282952]	[300.779465,305.970031]	[370.982446, 377.320356]
0.01	[179.592376, 181.426847]	[276.912588,273.306747]	[338.712899, 327.508182]
0.02	[160.949431, 160.591230]	[255.073592,244.366623]	[309.302011, 284.643339]
0.05	[116.052433, 112.402239]	[199.980350,175.669136]	[235.759767, 188.303485]
0.1	[67.699357, 63.916500]	[134.670824,103.231480]	[150.452575, 96.921377]
0.2	[23.696070, 23.057676]	[63.353664,38.085388]	[62.096223, 28.021890]
0.3	[8.819897, 9.589058]	[31.240064,15.325522]	[26.214639, 9.180313]
0.4	[3.716007, 4.610613]	[16.148828,6.796548]	[11.481956, 3.590757]
0.5	[1.946944, 2.590486]	[8.787784,3.417957]	[5.380020, 1.837203]
0.6	[1.330086, 1.712660]	[5.082309,2.024053]	[2.836111, 1.269532]
0.7	[1.114573, 1.315375]	[3.168515,1.433883]	[1.770705, 1.084391]
0.8	[1.039433, 1.133807]	[2.160315,1.181213]	[1.323322, 1.025049]
0.9	[1.013397, 1.052829]	[1.621719,1.073479]	[1.135308, 1.006858]
0.95	[1.007759, 1.032019]	[1.454504,1.046045]	[1.087400, 1.003451]
1	[1.004471, 1.018859]	[1.331648,1.028469]	[1.056379, 1.001684]

logic that is the generalization of fuzzy logic, see [32] and [33]. The NS, which is the extension of classical statistics and can be applied efficiently than classical statistics under the uncertainty environment. [34] and [35] used the neutrosophic numbers in rock measuring problems. [36] introduced the idea of NS in the area of quality control. [37] proposed

TABLE 5. The comparison of both charts when $n_N \epsilon$ [5], [7].

c	Proposed chart	Aslam and Khan (2019) chart
	$k_N \epsilon$ [(3.052004409,2.20987128), (4.264453272,4.215888714)]	$k_N \epsilon$[8.00046837,10.003918430]
0	[370.840980, 372.455359]	[370.968387, 375.196830]
0.01	[345.754512, 343.378510]	[370.056696, 373.904318]
0.02	[322.497248, 316.775177]	[367.345625, 370.074346]
0.05	[262.337533, 249.664424]	[349.323906, 345.119982]
0.1	[187.456901, 170.052216]	[296.188477, 276.341502]
0.2	[98.554817, 82.731112]	[177.974588, 145.528267]
0.3	[53.795841, 42.845466]	[99.670615, 73.427245]
0.4	[30.447129, 23.593892]	[56.658802, 38.647893]
0.5	[17.860787, 13.797759]	[33.436320, 21.558755]
0.6	[10.870590, 8.557377]	[20.583617, 12.767883]
0.7	[6.883843, 5.620211]	[13.224962, 8.020325]
0.8	[4.556878, 3.902287]	[8.862818, 5.334637]
0.9	[3.171877, 2.858728]	[6.189083, 3.749152]
0.95	[2.701457, 2.494607]	[5.250452, 3.206093]
1.0	[2.334327, 2.204049]	[4.498134, 2.776936]

the attribute control chart using the NS. [38] worked on the variance chart using the NS. [39] designed the chart to monitor reliability under the NS. [40] worked on the gamma chart using the NS and [41] proposed the NS based attribute chart using the repetitive sampling. [42] proposed the X-bar control chart using the single sampling. In this existing control chart, a decision about the state of the process is taken on the basis of information obtained from the single sample.

The presence of uncertain observations in the production data may mislead the industrial engineers for the monitoring of the manufacturing process. For example, the control chart under classical statistics when the data have uncertain values or neutrosophic numbers can show that the process is in control when actually some plotting statistic are within the indeterminacy limits. Therefore, in this case, the process monitoring should be done using the control chart under NS. By exploring the literature and best of our knowledge, there is no work on the design of X-bar control using repetitive sampling under the neutrosophic statistical interval method (NSIM). This manuscript presents the designing of the X-bar control chart using repetitive sampling under the uncertainty environment. We will discuss the advantages of the present chart over available in the literature. We expect that the proposed control chart will perform better in detecting the shift in the process than the existing charts under the uncertainty environment.

II. DESIGN OF THE PROPOSED CHART

Let X_L and X_U are the lower and upper values of an indeterminacy interval. Suppose that $X_{Ni} \epsilon \{X_L, X_U\} = i = 1, 2, 3, \ldots, n_N$ be a neutrosophic random variable expressed in indeterminacy interval follows the neutrosophic normal distribution with neutrosophic mean $\mu_N \epsilon \{\mu_L, \mu_U\}$ and neutrosophic standard deviation $\sigma_N \epsilon \{\sigma_L, \sigma_U\}$. Let $\bar{X}_N \epsilon \left\{\bar{X}_L, \bar{X}_U\right\}$ and $s_N = \{s_L, s_U\}$ be the neutrosophic sample mean and standard, respectively and considered as the best linear unbiased estimator of $\mu_N \epsilon \{\mu_L, \mu_U\}$ and$\sigma_N \epsilon \{\sigma_L, \sigma_U\}$. We define $m_N \epsilon \{m_L, m_U\}$ be the target mean for the in-control process. The neutrosophic upper and lower control limits are given as follows

$$LCL_{1N} = m_N - k_{1N}\frac{\sigma_N}{\sqrt{n_N}};\ \mu_N \epsilon \{\mu_L, \mu_U\},\quad \sigma_N \epsilon \{\sigma_L, \sigma_U\},\quad m_N \epsilon \{m_L, m_U\} \tag{1}$$

$$UCL_{1N} = m_N + k_{1N}\frac{\sigma_N}{\sqrt{n_N}};\ \mu_N \epsilon \{\mu_L, \mu_U\},\quad \sigma_N \epsilon \{\sigma_L, \sigma_U\},\quad m_N \epsilon \{m_L, m_U\} \tag{2}$$

$$LCL_{2N} = m_N - k_{2N}\frac{\sigma_N}{\sqrt{n_N}};\ \mu_N \epsilon \{\mu_L, \mu_U\},$$

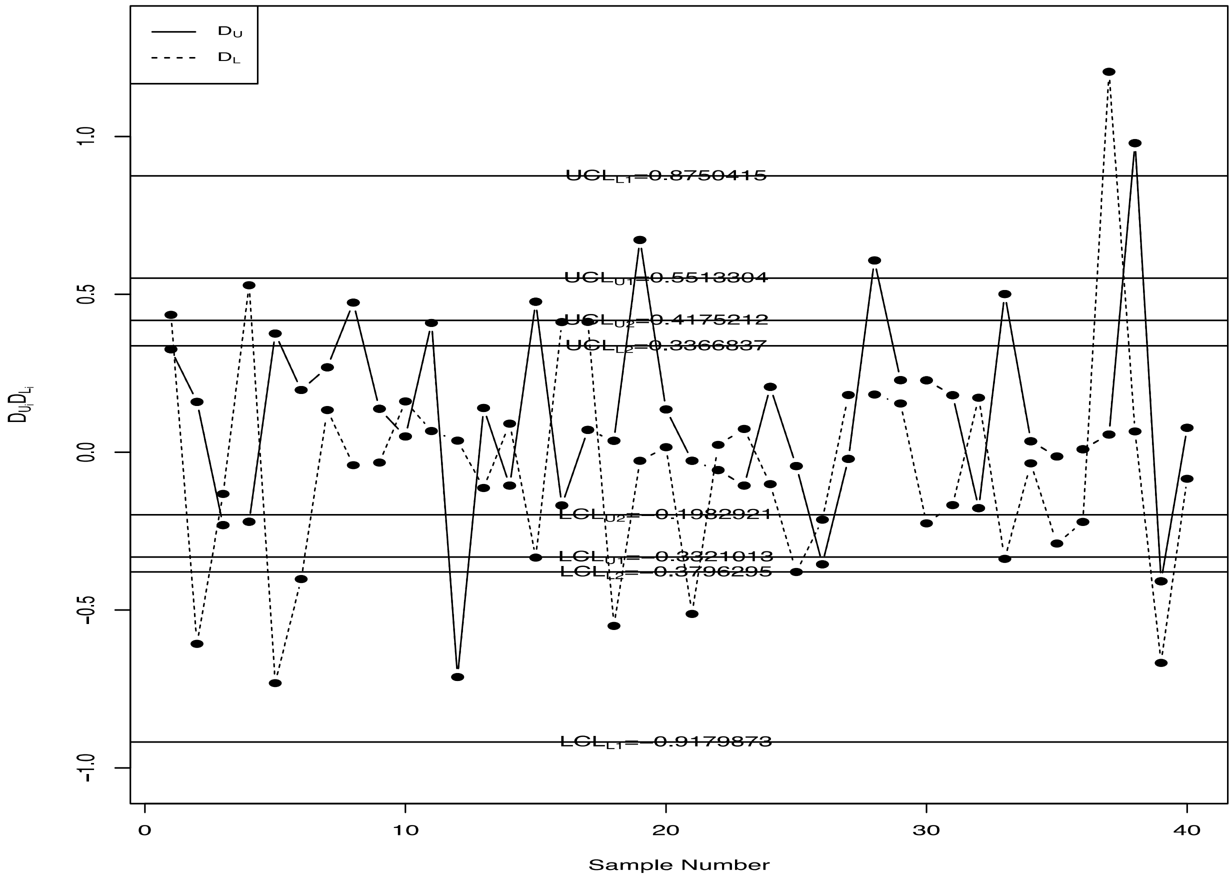

FIGURE 1. The proposed chart for the simulated data.

$$\sigma_N \epsilon \{\sigma_L, \sigma_U\}, \quad m_N \epsilon \{m_L, m_U\} \tag{3}$$

$$UCL_{2N} = m_N + k_{2N}\frac{\sigma_N}{\sqrt{n_N}}; \ \mu_N \epsilon \{\mu_L, \mu_U\}, \quad \sigma_N \epsilon \{\sigma_L, \sigma_U\}, \quad m_N \epsilon \{m_L, m_U\} \tag{4}$$

Note here that $k_{1N} \epsilon \{k_{1L}, k_{1U}\}$ and $k_{2N} \epsilon \{k_{2L}, k_{2U}\}$ are the neutrosophic control chart coefficients associated with neutrosophic lower control limit (NLCL) and neutrosophic upper control limit (NUCL), respectively.

Based on this information, we propose the following X-bar control chart for using the repetitive sampling under the NS.

Step-1: Compute $\bar{X}_N \epsilon \left\{\bar{X}_L = \sum_{i=1}^{n_L X_L} / n_L, \bar{X}_U = \sum_{i=1}^{n_U X_U} / n_U\right\}$ based on $X_{Ni} \epsilon \{X_L, X_U\} = i = 1, 2, 3, \ldots, n_N$.

Step-2: Declare the in-control state if $LCL_{N2} \leq \bar{X}_N \leq UCL_{N2}$.

Step-3: Declare the out-of-control state if $\bar{X}_N \geq UCL_{N1}$ or $\bar{X}_N \leq UCL_{N1}$.

The proposed control chart is the generalized form of several control charts. For example, it reduces to [42] chart when $k_{1N} = k_{2N}$; $k_{1N} \epsilon \{k_{1L}, k_{1U}\}, k_{2N} \epsilon \{k_{2L}, k_{2U}\}$. The proposed chart becomes the traditional repetitive chart proposed by [43] under classical statistics when $k_{1L} = k_{1U}$ and $k_{2L} = k_{2U}$. The proposed chart reduces to the traditional Shewhart control chart under classical statistics when $k_{1N} = k_{2N}$ and $k_{1L} = k_{1U}$ and $k_{2L} = k_{2U}$.

Now, we derive the necessary measures of the proposed control to evaluate its performance. The neutrosophic probability for the in-control process for the single sampling is given by

$$P_{inN1}^{0} = P\left(LCL_{2N} \leq \bar{X}_N \leq UCL_{2N}\right) = P\left(\frac{LCL_{2N} - m_N}{\sigma_N/\sqrt{n_N}} \leq \frac{\bar{X}_N - m_N}{\sigma_N/\sqrt{n_N}} \leq \frac{UCL_{2N} - m_N}{\sigma_N/\sqrt{n_N}}\right); \quad \bar{X}_N \epsilon \left\{\bar{X}_L, \bar{X}_U\right\}, m_N \epsilon \{m_L, m_U\} \tag{5}$$

Let $\frac{LCL_{2N} - m_N}{\sigma_N/\sqrt{n_N}} = Z_N$ be the neutrosophic standard normal variable. After, some simplification, we have,

$$P_{inN1}^{0} = 1 - \Phi_N(k_{2N}) + \Phi_N(k_{1N}); \quad k_{1N} \epsilon \{k_{1L}, k_{1U}\}, k_{2N} \epsilon \{k_{2L}, k_{2U}\} \tag{6}$$

where $\Phi_N(q)$ shows the neutrosophic cumulative distribution function. The neutrosophic repetitive probability for the in-control process is given by

$$P_{PrepN}^{0} = P\left(LCL_{1N} \leq \bar{X}_N \leq LCL_{2N}\right) + P\left(UCL_{2N} \leq \bar{X}_N \leq UCL_{1N}\right) \tag{7}$$

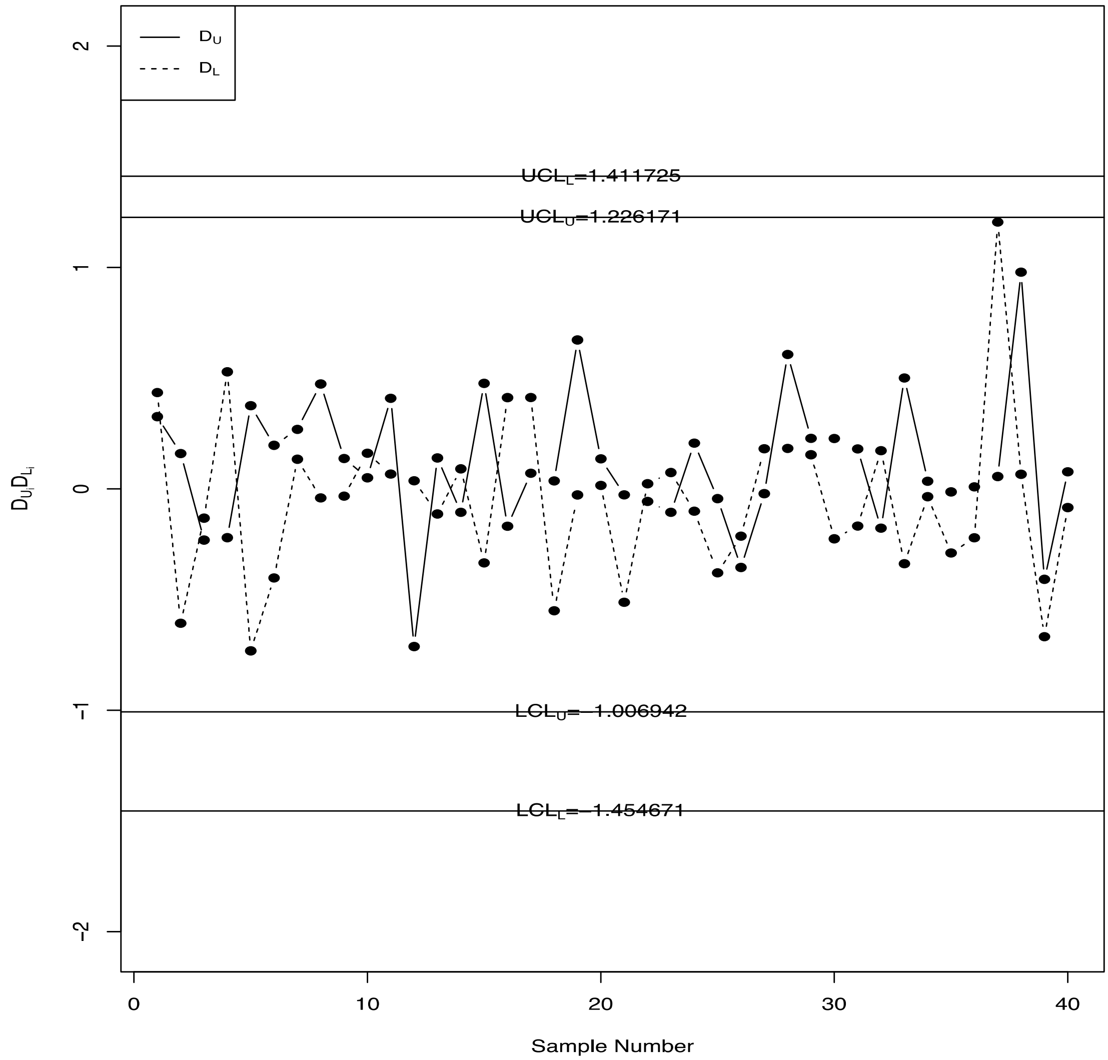

FIGURE 2. The existing chart for the simulated data.

The Eq. (7) can be written as

$$P^0_{PrepN} = P\left(\frac{LCL_{1N}-m_N}{\sigma_N/\sqrt{n_N}} \le \frac{\bar{X}_N-m_N}{\sigma_N/\sqrt{n_N}} \le \frac{LCL_{2N}-m_N}{\sigma_N/\sqrt{n_N}}\right) + P\left(\frac{UCL_{2N}-m_N}{\sigma_N/\sqrt{n_N}} \le \frac{\bar{X}_N-m_N}{\sigma_N/\sqrt{n_N}} \le \frac{UCL_{1N}-m_N}{\sigma_N/\sqrt{n_N}}\right); \bar{X}_N\epsilon\left\{\bar{X}_L,\bar{X}_U\right\}, m_N\epsilon\{m_L,m_U\} \quad (8)$$

The simplified form of Eq. (8) is given by

$$P^0_{PrepN} = (\Phi_N(-k_{2N}) - \Phi_N(-k_{1N})) + (\Phi_N(k_{1N}) - \Phi_N(k_{2N})); \quad k_{1N}\epsilon\{k_{1L},k_{1U}\}, \ k_{2N}\epsilon\{k_{2L},k_{2U}\} \quad (9)$$

Finally, the resampling probability for an in-control process under NSIM is given by (10), as shown at the bottom of this page. The neutrosophic average run length (NARL) indicates when on the average the process will be out-of-control under the indeterminacy. The NARL for the in-control process is given as

$$\text{NARL}_{0N} = \frac{1}{1-P^0_{inN}}; \ \text{ARL}_{0N}\epsilon\{\text{ARL}_{0L}, \text{ARL}_{0U}\} \quad (11)$$

Now, we assume that the process has shifted with a shift constant c, to a new neutrosophic mean $m_{1N} = m_N + c\sigma_N$; $m_{1N}\epsilon\{m_{1L},m_{1U}\}$, $\sigma_N\epsilon\{\sigma_L,\sigma_U\}$. The neutrosophic probability for the in-control process when the process has shifted at $m_{1N} = m_N + c\sigma_N$; $m_{1N}\epsilon\{m_{1L},m_{1U}\}$, $\sigma_N\epsilon\{\sigma_L,\sigma_U\}$ for the

$$P^0_{inN} = \frac{1-\Phi_N(k_{2N})+\Phi_N(k_{1N})}{1-[(\Phi_N(-k_{2N})-\Phi_N(-k_{1N}))+(\Phi_N(k_{1N})-\Phi_N(k_{2N}))]}; \quad P^0_{inN}\epsilon\left\{P^0_{inL},P^0_{inU}\right\} \quad (10)$$

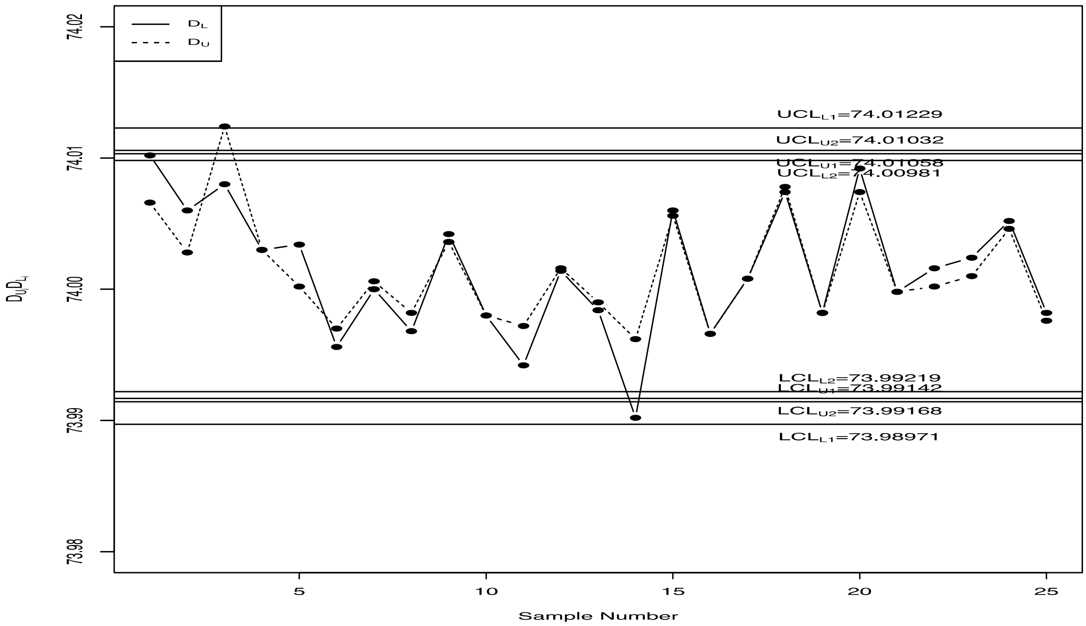

FIGURE 3. The proposed chart for the real data.

single sampling is given by

$$P^1_{inN1} = 1 - \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) + \Phi_N\left(-k_{2N} + c\sqrt{n_N}\right); \quad k_{1N}\epsilon\{k_{1L}, k_{1U}\}, k_{2N}\epsilon\{k_{2L}, k_{2U}\} \tag{12}$$

The neutrosophic repetitive probability for the in-control at the process at $m_{1N} = m_N + c\sigma_N; m_{1N}\epsilon\{m_{1L}, m_{1U}\}, \sigma_N\epsilon\{\sigma_L, \sigma_U\}$ is given by

$$P^1_{PrepN}|m_{1N} = P\left(LCL_{1N} \le \bar{X}_{1N} \le LCL_{2N}\right) + P\left(UCL_{2N} \le \bar{X}_{2N} \le UCL_{1N}\right); \quad \bar{X}_{1N}\epsilon\left\{\bar{X}_{1L}, \bar{X}_{1U}\right\}, \bar{X}_{2N}\epsilon\left\{\bar{X}_{2L}, \bar{X}_{2U}\right\} \tag{13}$$

or

$$P^1_{PrepN} = \Phi_N\left(-k_{1N} + c\sqrt{n_N}\right) - \Phi_N\left(-k_{2N} + c\sqrt{n_N}\right) + \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) - \Phi_N\left(k_{1N} + c\sqrt{n_N}\right); \quad k_{1N}\epsilon\{k_{1L}, k_{1U}\}, k_{2N}\epsilon\{k_{2L}, k_{2U}\} \tag{14}$$

Finally, the neutrosophic probability that the process is in-control at $m_{1N} = m_N + c\sigma_N$; $m_{1N}\epsilon\{m_{1L}, m_{1U}\}, \sigma_N\epsilon\{\sigma_L, \sigma_U\}$ using the repetitive sampling is given by (15), as shown at the bottom of this page. The NARL at$m_{1N} = m_N + c\sigma_N$; $m_{1N}\epsilon\{m_{1L}, m_{1U}\}, \sigma_N\epsilon\{\sigma_L, \sigma_U\}$ is given by

$$\mathrm{NARL}_{1N} = \frac{1}{1 - P^1_{inN}}; \mathrm{ARL}_{1N}\epsilon\{\mathrm{ARL}_{1L}, \mathrm{ARL}_{1U}\} \tag{16}$$

Tables 1-3 show the values of NARL for $n_N\epsilon$ [2], [5], $n_N\epsilon$ [5], [7], $n_N\epsilon$ [8], [10], various values of c and various values of specifiedNARL$_{0N}$, say r_{0N}. Table 4 is presented the values of NARL for various values of $n_N\epsilon[n_L, n_U]$. From Tables 1-4, we note the following trends in the control chart parameters under the NS.

1) For a fixed value of $n_N\epsilon[n_L, n_U]$, the indeterminacy interval of NARL decreases as c increases.
2) For other fixed neutrosophic parameters, the indeterminacy interval of NARL decreases as $n_N\epsilon[n_L, n_U]$ incerses.
3) From Table 4, we note that as $n_N\epsilon[n_L, n_U]$ increases, indeterminacy interval of NARL increases.

A. ALGORITHM

The following algorithm under the NSIM is used to evaluate the values of $k_{1N}\epsilon\{k_{1L}, k_{1U}\}, k_{2N}\epsilon\{k_{2L}, k_{2U}\}$ and ARL$_{1N}\epsilon$ {ARL$_{1L}$, ARL$_{1U}$}.

1) Specify the suitable values of ARL$_{0N}\epsilon$ {ARL$_{0L}$, ARL$_{0U}$}, c and $n_N\epsilon[n_L, n_U]$.
2) Determine $P^0_{inN}\epsilon\left\{P^0_{inL}, P^0_{inU}\right\}$ and NARL$_{0N}$ using Eq. (10) and Eq. (11), respectively.
3) Determine the values of $k_{1N}\epsilon\{k_{1L}, k_{1U}\}, k_{2N}\epsilon\{k_{2L}, k_{2U}\}$ such that NARL$_{0N} \ge r_{0N}$.
4) Choose those combinations of $k_{1N}\epsilon\{k_{1L}, k_{1U}\}, k_{2N}\epsilon\{k_{2L}, k_{2U}\}$ where NARL$_{0N}$ is very close to r_{0N}.

$$P^1_{inN} = \frac{1 - \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) + \Phi_N\left(-k_{2N} + c\sqrt{n_N}\right)}{1 - \left[\Phi_N\left(-k_{1N} + c\sqrt{n_N}\right) - \Phi_N\left(-k_{2N} + c\sqrt{n_N}\right) + \Phi_N\left(k_{2N} + c\sqrt{n_N}\right) - \Phi_N\left(k_{1N} + c\sqrt{n_N}\right)\right]}; \ P^1_{inN}\epsilon\left\{P^1_{inL}, P^1_{inU}\right\} \tag{15}$$

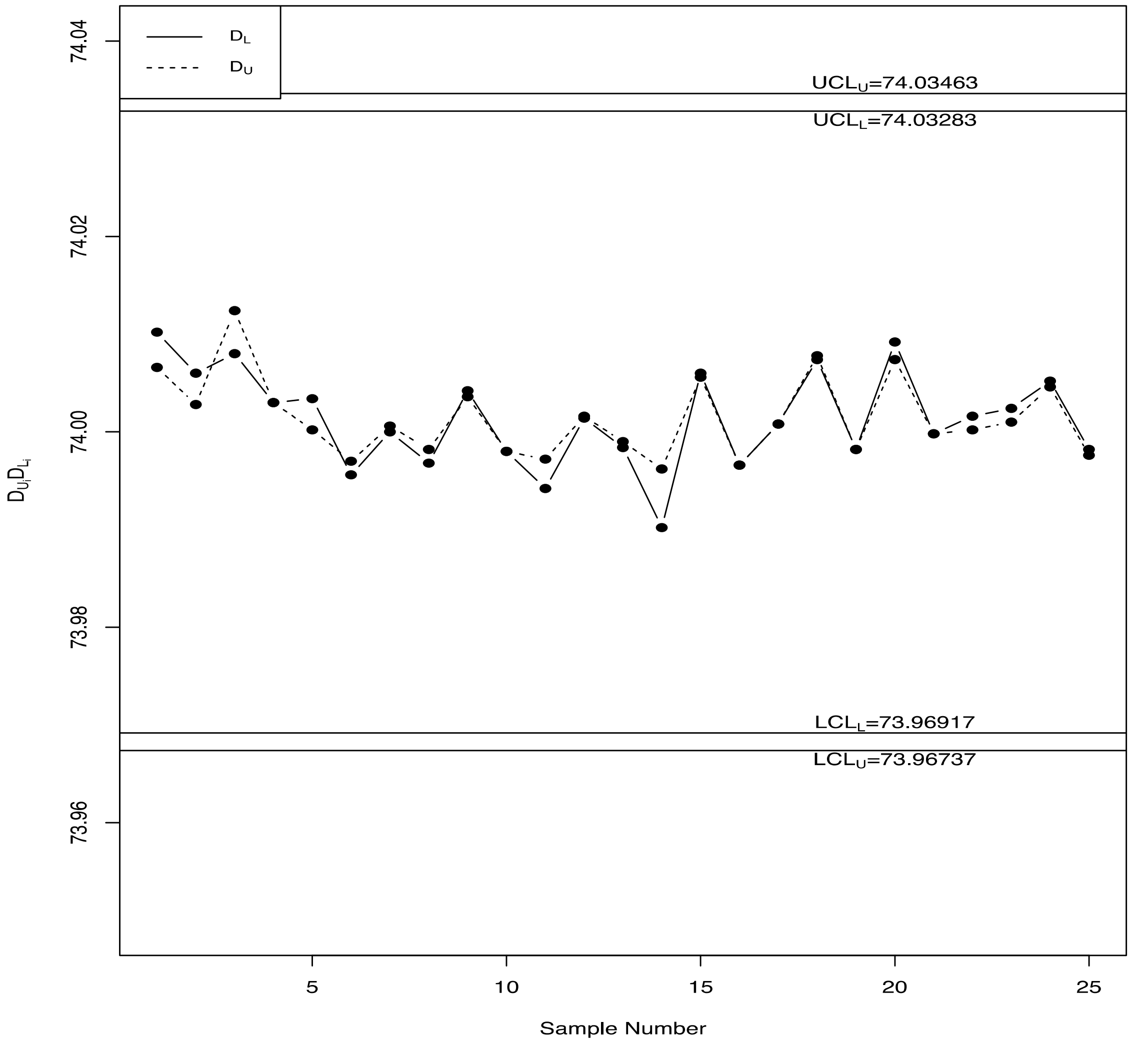

FIGURE 4. **The existing chart for the real data.**

5) Use $k_{1N}\epsilon\{k_{1L}, k_{1U}\}$, $k_{2N}\epsilon\{k_{2L}, k_{2U}\}$ to find $\text{ARL}_{1\text{N}}\epsilon\{\text{ARL}_{1\text{L}}, \text{ARL}_{1\text{U}}\}$ for various c.

III. COMPARATIVE STUDIES

The efficiency of the proposed control chart with the available competitor control chart under the NSIM proposed by [42] is discussed in this section. [42] showed that X-bar under NS performs better than the X-bar chart under classical statistics in ARL. We will discuss the advantages of the proposed chart with [42] in terms of NARL and using the simulated data at the same values of specified parameters of both control charts. For this comparison, let $\text{ARL}_{0\text{N}}\epsilon\{370, 370\}$ and $n_N\epsilon[5, 7]$. The values of the proposed chart and Aslam and Khan (2019) are shown in Table 5.

From Table 5, it can be noted that the proposed control chart has the smaller values of NARL than the existing control chart at all values of c. Note that when $c = 0.05$, the indeterminacy interval of NARL values for the proposed control chart is $\text{ARL}_{1\text{N}}\epsilon$ [262.337533, 249.664424] while it is $\text{ARL}_{1\text{N}}\epsilon$ [349.323906, 345.119982] from the Aslam and Khan (2019) control chart. From this comparison, it is concluded that the proposed control will detect the shift in the process between the 249$^{\text{th}}$ sample and the 262$^{\text{nd}}$ sample while the existing chart will indicate the shift in the process between 345$^{\text{th}}$ sample and the 349$^{\text{th}}$ sample. Therefore, the proposed control chart is more sensitive to detect a shift in the process as compared to the competitor chart under the uncertainty.

Now, the efficiency of the proposed chart with the existing chart is discussed using the simulated data drawn from the neutrosophic normal distribution with neutrosophic mean $\mu_N\epsilon\{0, 0\}$ and $\sigma_N\epsilon\{1, 1\}$. The first 20 observations are generated assuming the process is in-control and next same observations are generated for the shifted process when $c = 0.07$, $n_N\epsilon[5, 5]$ and $\text{ARL}_{0\text{N}}\epsilon\{370, 370\}$. The statistic $\bar{X}_N\epsilon\left\{\bar{X}_L, \bar{X}_U\right\}$ is computed and plotted on the control chart given in Figure 1 for the proposed chart and Figure 2 for the existing control chart. From Figures 1-2, we note that the proposed control chart detects the shift at around the 36$^{\text{th}}$ sample while the competitor's chart shows that all values

TABLE 6. The data of diameter.

	Sample Observation					$\bar{X}_N$
1	[74.03, 74.03]	[74.002, 73.991]	[74.019,74.0.19]	[73.992,73.992]	[74.008,74.001]	[74.0102,74.0066]
2	[73.995, 73.995]	[73.992, 74.003]	[74.001,74.001]	[74.011,74.011]	[74.004,74.004]	[74.0006,74.0028]
3	[73.988, 74.017]	[74.024, 74.024]	74.021,74.021]	[74.005,74.005]	[74.002,73.995]	[74.008,74.0124]
4	[74.002, 74.002]	[73.996, 73.996]	[73.993,73.993]	[74.015,74.015]	[74.009,74.009]	[74.003,74.003]
5	[73.992, 73.992]	[74.007, 74.007]	[74.015,74.015]	[73.989,73.989]	[74.014,73.998]	[74.0034,74.0002]
6	[74.009, 74.009]	[73.994, 74.001]	[73.997,73.997]	[73.985,73.985]	[73.993,73.993]	[73.9956,73.997]
7	[73.995, 73.998]	[74.006, 74.006]	[73.994,73.994]	[74,74]	[74.005,74.005]	[74,74.0006]
8	[73.985, 73.985]	[74.003,74.01]	[73.993,73.993]	[74.015,74.015]	[73.988,73.988]	[73.9968,73.9982]
9	[74.008, 74.005]	[73.995, 73.995]	[74.009,74.009]	[74.005,74.005]	[74.004,74.004]	[74.0042,74.0036]
10	[73.998, 73.998]	[73.998,73.998]	[73.99,73.99]	[74.007,74.007]	[73.995,73.995]	[73.998,73.998]
11	[73.994, 73.998]	[74,74.002]	[73.994,73.994]	[73.995,73.995]	[73.99,74.001]	[73.9942,73.9972]
12	[74.004, 74.004]	[74,74.002]	[74.007,74.005]	[74,74.001]	[73.996,73.996]	[74.0014,74.0016]
13	[73.983, 73.993]	[74.002,74.002]	[73.998,73.998]	[73.997,73.997]	[74.012,74.005]	[73.9984,73.999]
14	[74.006, 74.006]	[73.967,73.985]	[73.994,73.994]	[74,74]	[73.984,73.996]	[73.9902,73.9962]
15	[74.012, 74.012]	[74.014,74.012]	[73.998,73.998]	[73.999,73.999]	[74.007,74.007]	[74.006,74.0056]
16	[74, 74]	[73.984,73.984]	[74.005,74.005]	[73.998,73.998]	[73.996,73.996]	[73.9966,73.9966]
17	[73.994, 73.994]	[74.012,74.012]	[73.986,73.986]	[74.005,74.005]	[74.007,74.007]	[74.0008,74.0008]
18	[74.006, 74.006]	[74.01,74.011]	[74.018,74.018]	[74.003,74.003]	[74,74.001]	[74.0074,74.0078]
19	[73.984, 73.984]	[74.002,74.002]	[74.003,74.003]	[74.005,74.005]	[73.997,73.997]	[73.9982,73.9982]
20	[74,74]	[74.01,74.01]	[74.013,74.009]	[74.02,74.015]	[74.003,74.003]	[74.0092,74.0074]
21	[73.982, 73.982]	[74.001,74.001]	[74.015,74.015]	[74.005,74.005]	[73.996,73.996]	[73.9998,73.9998]
22	[74.004, 74.004]	[73.999,73.999]	[73.99,73.99]	[74.006,74.006]	[74.009,74.002]	[74.0016,74.0002]
23	[74.01, 74.01]	[73.989,73.989]	[73.99,73.99]	[74.009,74.005]	[74.014,74.011]	[74.0024,74.001]
24	[74.015, 74.011]	[74.008,74.008]	[73.993,73.993]	[74,74]	[74.01,74.011]	[74.0052,74.0046]
25	[73.982, 73.982]	[73.984,73.989]	[73.995,73.995]	[74.017,74.012]	[74.013,74.01]	[73.9982,73.9976]

of statistic within the control limits. Therefore, we conclude that the proposed chart has the ability to detect the shift in the process earlier than the existing control chart.

IV. INDUSTRIAL APPLICATION

The application of the proposed control chart is given for the automobile data taken from the automobile industry. Similar data is used by Aslam and Khan (2019). According to Aslam and Khan (2019) ''In this industry, the inside diameter measurement of automobile engine piston rings is a continuous random variable is obtained by the measurement process''. In addition, as mentioned by [44] ''all observations and measurements of continuous variables are not precise numbers but more or less non-precise. This imprecision is different from variability and errors. Therefore, also lifetime data are not precise numbers but more or less fuzzy. The best up-to-date mathematical model for this imprecision is so-called non-precise numbers''. Therefore, it may possible that some observations of the diameter obtained from the measurement have imprecise observations. The data is shown in Table 6 is taken from Aslam and Khan (2019) for easy reference. For this data, let $n_N \epsilon$ [5, 5] and $ARL_{0N} \epsilon$ {370, 370}. The neutrosophic control limits and statistics for this data given as

$$\sigma_N \epsilon [0.008896, 0.009399], \quad m_N \epsilon [74.001, 74.001]$$
$$LCL_{L1} = (73.98971), \quad LCL_{L2} = (73.99219)$$
$$LCL_{U1} = (73.99142), \quad LCL_{U2} = (73.99168)$$
$$UCL_{L1} = (74.01229), \quad UCL_{L2} = (74.00981)$$
$$UCL_{U1} = (74.01058), \quad UCL_{U2} = (74.01032)$$

The statistic $\bar{X}_N \epsilon \left\{\bar{X}_L, \bar{X}_U\right\}$ is computed and reported in the last column of Table 6. The values of $\bar{X}_N \epsilon \left\{\bar{X}_L, \bar{X}_U\right\}$ is plotted on the control chart in Figure 3 for the proposed control chart and in Figure 4 for the existing control chart. We note that some values of the statistic fall in indeterminacy interval and one point is at UCL_{L1}. From Figure 4, we note that the process is an in-control state. By comparing Figure 3 and Figure 4, it is concluded that the proposed chart indicates that there is some issue to be fixed in the process as one point is at the control limit while the existing control chart indicates that no action is needed as the process is an in-control state.

V. CONCLUDING REMARKS

We proposed the X-bar control chart using the repetitive sampling under the NS. The necessary measureless were derived to evaluate the performance of the proposed control. From the comparison study, we conclude that the proposed control chart has edge over the existing control chart. The proposed chart has the ability to perform well under the uncertainty environment. Therefore, the use of the proposed control chart when observations are imprecise, incomplete and uncertain is more effective to be applied than the existing control chart under classical statistics and the NS. The use of the proposed control chart in the industry will be helpful in reducing the non-conforming items. The proposed control chart using other sampling schemes can be considered as future research. The proposed control chart for monitoring median or monitoring location and scale for some non-normal distributions can be considered as future research.

AUTHOR CONTRIBUTIONS

Conceived and designed the experiments: MA. Performed the experiments: MA. Analyzed the data: MA. Contributed reagents/materials/analysis tools: MA. Wrote the paper: MA

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DATA AVAILABILITY

The data is given in the paper.

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