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Exact and Asymptotic Analysis of Partial Relay Selection for Cognitive RF-FSO Systems With Non-Zero Boresight Pointing Errors

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ABSTRACT In this paper, we investigate the performance of asymmetric radio frequency (RF) and free space optical (FSO) dual-hop cooperative relay network along with a direct RF link between source and destination. The FSO link experiences double generalized Gamma turbulence in the presence of the generalized non-zero boresight pointing errors, and RF links experience non-identically distributed Nakagami-*m* fading. Moreover, considering both cognitive radio and non-cognitive scenarios, a partial relay selection (PRS) strategy has been employed. Furthermore, both relay selection and underlay power restriction are governed with the outdated channel state information (CSI). We also assume both heterodyne and intensity modulation/direct detection methods in the FSO receiver. Under the assumption of amplify-and-forward relaying and PRS, we derive closed-form expressions for outage probability (OP) of both scenarios, while bit-error probability and ergodic capacity of the non-cognitive scenario are also obtained. The asymptotic expressions of OP and diversity order of this network are derived for both perfect CSI and outdated CSI cases. It is demonstrated that the diversity order is a function of the fading severity of the RF links, turbulence parameters of the FSO link, and pointing error, regardless of interference channel parameter of the primary user.

INDEX TERMS Cognitive radio network, double generalized Gamma, free space optical communications, non-zero boresight pointing errors, outdated channel state information, partial relay selection.

I. INTRODUCTION

Free-space optical (FSO) communications has attracted considerable research attention due to their advantages including high bandwidth, full-duplex gigabit Ethernet throughput in certain applications, a huge license-free spectrum, a certain immunity to interference, and a high level of security [1]. These advantages of FSO communications can compensate some shortcomings of the radio frequency (RF) communications.

A. TURBULENCE CHANNEL MODEL

However, the major limitation in the FSO communication systems is the atmospheric turbulence-induced fading that restrict FSO communication systems to short range links. In order to model this random phenomenon, several statistical models in the literature have been proposed. These include the log-normal, log-normal Rician, I-K distribution, Malaga, Gamma-Gamma (G^2) and Double-Weibull [2]. The G^2 is widespread distribution for modeling FSO turbulence channel in the literature, however, G^2 distribution is not completely match with experimental data particularly in tails [2]. Recently, a novel statistical model called Double Generalized Gamma (DGG) distribution has been introduced by [2], where irradiance fluctuations are given by production of small-scale and large-scale fluctuations, both of which are function of generalized Gamma distribution. This model precisely describes the signal propagation under all conditions, (i.e., from weak to strong turbulence conditions) added to the fact that it generalizes other distributions.

B. NON-ZERO BORESIGHT MISALIGNMENT

Additionally, the building sway phenomenon leads to the vibration of transmitter beam that causes a misalignment between transmitter and receiver known as pointing error.

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Pointing error has two components i.e., boresight and jitter. The boresight is static displacement between center of the beam and detector's center which is mostly caused by the thermal expansion of the buildings. On the other hand, the random offset of the beam center at detector plane leads to jitter which is largely caused due to the buildings sway and vibration.

Several statistical models have been proposed to model misalignment such as Rayleigh distribution [3], Hoyt distribution [4], Rician distribution [5] and Beckmann distribution [6]. Among all above models, Beckmann distribution is a general model which includes all other models as special cases. However, due to mathematical intractability, this model has not became popular. Recently, Beckmann distribution is approximated with a modified Rayleigh distribution [7].

C. MOTIVATION

By increasing the data rate demands of wireless users, cooperative communications is introduced in order to improves the capacity of wireless communication and reliability and power-efficient coverage [8]. In this way, an asymmetric dual-hop cooperative relaying systems assuming the RF and FSO links in each hop of information transmission was suggested in [9]. The dual-hop RF-FSO relay network provides an efficient solution to fill the connectivity gap between the RF access network and the fiber optic-based backbone network as well. Due to high capacity of FSO links, a large number of RF users could be multiplexed through a single FSO link.

D. RELATED WORK

The outage performance of dual-hop amplify-and-forward (AF) RF-FSO relaying systems in various turbulence conditions was investigated in [9], [10], while bit error probability (BEP), symbol error rate and ergodic capacity (EC) are achieved in [11]. The study in [12] was extended these results to multiuser selection. The authors in [13] generalized distribution model of the FSO link to DGG turbulence. A cooperative RF-FSO system suffering from co-channel interference was investigated in [14].

Furthermore, due to lack of enough frequency spectrum and high cost of using frequency bands, cognitive radio networks have been proposed to improve the spectrum efficiency [15]. The impact of underlay cognitive radio on the RF-FSO relay network was evaluated in [16]–[21]. In [16], [17] performance analysis were done for channel state information (CSI)-assisted and fixed gain AF relaying techniques assuming Rayleigh and G^2 distributions for the RF and FSO links, respectively. The authors in [21] generalized these results for DGG and Nakagami-m distributions for the RF and FSO links, respectively. Moreover, diversity order of this relay network was presented for single relay system as well. The authors in [18], [19] extended these results for cognitive multiple-input multiple-output (MIMO) RF-FSO relay network while [20] considered impact of imperfect CSI of the primary user (PU) channel. So far, in all of underlay RF-FSO cognitive relay works, only a single relay has been considered.

Considering multiple relays and using relay selection strategy leads to performance improvement [22]. In the partial relay selection (PRS) strategy contrary to best relay selection one, the relay is selected based on CSI of only one hop (source-relay or relay-destination link) [23] which leads to power, bandwidth and time savings. In addition, CSI may be outdated at the time of selection since it takes some time for the destination to acquire the estimates of all source-relay channels. Due to the time varying nature of the RF channels and feedback delay, the instantaneous CSI which is deployed for relay selection may be outdated for the relay selection. The effect of outdated CSI on the performance of multi-relay RF network was investigated in [24], [25] and [26] assuming respectively the Rayleigh and Nakagami-m fading channels. In order to generalize the performance analysis when the best relay is not available due to the practical implementation restriction such as some scheduling or load balancing conditions, selection among all relays except the best one (k-th worst relay selection) was proposed for Rayleigh fading channels in [27].

The idea of the PRS in a dual-hop RF-FSO relay system with outdated CSI was proposed in [28]. In [28], [29], the outage performance of RF-FSO relaying system with consideration of Rayliegh/(G^2) fading was derived and the BEP and EC were achieved in [30] while [31] derived OP assuming Malaga distribution for the FSO links. The authors in [32] generalized it with hardware impairment consideration in transmitters and receivers for fixed gain AF and decode-and-forward (DF) relaying methods, while they considered Malaga distribution for the FSO links. Additionally, in [33] upper-bounded OP, BEP and EC of CSI-assisted AF relaying were derived where RF links were subjected to Nakagami-m fading and co-channel interference and the FSO links was influenced by DGG.

E. CONTRIBUTION

From a realistic point of view, the choice of Nakagami-m fading is to characterize more versatile fading scenarios that are more or less severe than Rayleigh fading via the m fading parameter, which includes the Rayleigh fading ($m = 1$) as a special case. Furthermore, PUs and secondary users (SUs) are often far from each other; as such, independent and non-identically distributed (i.n.i.d.) fading is assumed with distinct fading parameters in the respective links.

While all previous works have improved our knowledge on the performance characterization of RF-FSO systems, the most important differences between our work and [28]–[33] are: 1) We consider underlay cognitive RF-FSO relay network with PRS scheme, which is not considered in [28]–[33]. 2) Unlike [28]–[33], we considered a direct link between source and destination. 3)We derive exact value of OP, BEP and EC of RF-FSO relay network with fixed gain AF relaying technique assuming Nakagami-m and DGG for the RF and the FSO links, respectively while in [33] the upper-bounded of these parameters are obtained for CSI-assisted AF relaying technique. 4) Contrary to [33], we derive diversity order of AF RF-FSO relay network with both imperfect CSI and perfect CSI scenarios for fixed gain AF relaying technique. 5) We consider the generalized pointing error model with non-zero boresight which is not considered in [28]–[33].

Motivated by the above mentioned limitations of [28]–[33], we herein pursue a detailed and generalized performance analysis of dual-hop cognitive mixed RF-FSO AF relaying systems, where k-th worst (or $(N-k)$-th best) relay selection strategy for mixed RF-FSO relay network with outdated CSI are explored. In this light, we derive the OP and BEP at arbitrary signal-to-noise ratio (SNR), along with asymptotic expressions in the low outage regime. The contributions of this paper can be summarized as follows:

- We consider a mixed dual-hop multi-relay RF-FSO configuration along with a direct RF link between source and destination for cognitive and non-cognitive scenarios, where SUs use the resources of a primary network (PN) in an underlay spectrum sharing scenario. This is a practical but complicated setup which has scarcely appeared in the literature. Closed-form expressions of OP of both scenarios and BEP and EC of the non-cognitive scenario are obtained by assuming the i.n.i.d. Nakagami-m fading for the RF links and the DGG atmospheric turbulence for the FSO links with considering generalized pointing errors. Assuming the outdated CSI case and the PRS method, the k-th worst relay selection strategy is performed at the first hop. Additionally, it is worthy to mention that the best relay selection scheme is a special case of our proposed scheme. The destination employs the selection combining scheme where the optical receiver employs both type of detection i.e. heterodyne and IM/DD methods.
- We consider generalized pointing error model in this paper where non-zero boresight pointing errors with non-identical jitter variance are taken into account.
- It is the first time that multi-relay RF-FSO configuration considered in a cognitive radio network (CRN). In other word, the OP of the RF-FSO relay network is obtained with considering a power constraint strategy in an underlay cognitive radio scenario while the CSI of the interference channel between PUs and SUs is considered outdated.
- The asymptotic expression of the OP and the diversity order of this system are given for both outdated and perfect CSI cases. It has been shown that the diversity order of secondary network (SN) strictly depends only on the fading severity of the RF link and turbulence parameters of the FSO link and pointing errors and is independent of the PU parameters. We prove that when CSI of the RF link is outdated, increasing number of relays does not provide more diversity order (i.e. the diversity order reduces to fading severity of a single RF link).

Notation: Throughout this paper, we use $f_h(.)$ and $F_h(.)$ to denote the probability density function (PDF) and cumulative distribution function (CDF) of a random variable (RV) h, respectively. $\Gamma(n) = \int_0^\infty e^{-t} t^{n-1} dt$ is the Gamma function [34, Eq. (8.310.1)], $\Gamma(n, x) = \int_x^\infty e^{-t} t^{n-1} dt$ is the upper incomplete Gamma function [34, Eq. (8.350.2)] and $\Upsilon(n, x) = \Gamma(n) - \Gamma(n, x)$ is the lower incomplete Gamma function. Furthermore, $G_{\cdot,\cdot}^{\cdot,\cdot}[.]$ and $H_{\cdot,\cdot:\cdot,\cdot:\cdot,\cdot}^{\cdot,\cdot:\cdot,\cdot:\cdot,\cdot}[.]$ denote Meijer's-G and extended generalized bivariate Fox-H function (EGBFHF) which are explained in [34, Eq. (9.301)] and [35, Eq. (1)], respectively. $I_\nu(.)$ and *erf (.)* represent the ν-th order modified Bessel function of the first kind [34, Eq. (8.406.1)] and error function [34, Eq. (8.250.1)], respectively. $[x]_p$ shows a vector with identical values equal to x and length p. Assuming that X is a RV, $\overline{X}$ is denoted expected value of X (i.e. $\mathbb{E}[X]$). In the following $\Delta(. : .)$, $\overline{\Delta}(. : .)$ and $\overline{\Delta}(., ., .)$ are respectively defined as

$$\Delta(j : x) \triangleq \frac{x}{j}, \ldots, \frac{x+j-1}{j},$$
$$\overline{\Delta}(j : x) \triangleq \frac{x+j-1}{j}, \ldots, \frac{x}{j},$$
$$\overline{\Delta}(r, j, x) \triangleq \Delta\left(r, \frac{x+j-1}{j}\right), \ldots, \Delta\left(r, \frac{x}{j}\right).$$

The outdated CSI denotes by "ˆ" throughout this paper.

II. SYSTEM MODEL AND FADING STATISTICS

As demonstrated in Fig. 1, a dual-hop RF-FSO relay system in an underlay cognitive network scenario is considered with a single PU P, a single secondary source S, a single secondary destination D and N secondary AF relays R_n, $(n \in \{1, \ldots, N\})$. Indeed for each relays, we consider several secondary sources with different data rate requirements in which their data are amplified and multiplexed through the FSO link. For simplicity's sake and without loss of generality we just show one of the secondary sources and analyze the system for a single random user. Based on partial relay selection strategy, the best available relay with maximum SNR is selected. However, in order to fairly generalized our analysis, sometimes the best relay may not be available for transmission, the k-th worst or $(N-k)$-th best relay selection strategy is considered [27]. The source node transmits the signal via N RF links. The AF relay with k-th worse SNR is being selected. The selected AF relay amplifies the transmitted signal from node S with gain G and by proceeding conversion to the optical signal, retransmits it to the destination. The destination is equipped with both receiver lens aperture and RF antenna for optical and RF signals, respectively. The destination received signal via both the direct RF link and the asymmetric RF-FSO dual-hop link. The destination node deploys selection combining (SC) scheme between these two links.

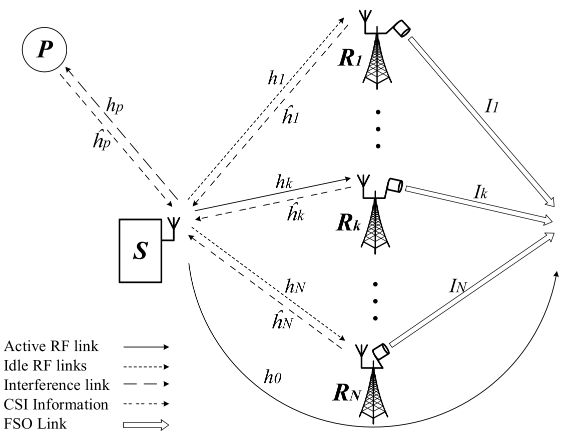

FIGURE 1. System model of multi-relay underlay cognitive RF-FSO transmission system.

A. RF LINK

All $S \rightarrow R_n$ links experience independent and identically distributed (i.i.d.) Nakagami-m fading with the same fading severity and average SNR parameters [23]–[27] which is denoted by h_n. $S \rightarrow P$ and $S \rightarrow D$ links experience Nakagami-m fading, where the channel coefficients are presented by h_p and h_0, respectively. Therefore, $|h_n|^2$, $|h_p|^2$ and $|h_0|^2$ are Gamma distributed with fading severity m_R, m_I and m_D and mean power Ω_R, Ω_I and Ω_D and scale parameter $a_R = m_R/\Omega_R$, $a_I = m_I/\Omega_I$ and $a_D = m_D/\Omega_D$, respectively. The Gamma distribution PDF and CDF are defined as follow [36, Eq. (2.21)].

$$f_{|h|^2}(\gamma) = \frac{a_X^{m_X}\,\gamma^{m_X-1}}{\Gamma\left(m_X\right)}\exp\left(-a_X\gamma\right), \tag{1}$$

$$F_{|h|^2}(\gamma) = \frac{\Upsilon\left(m_X, a_X\gamma\right)}{\Gamma\left(m_X\right)} = 1-\exp\left(-a_X\gamma\right)\sum_{i=0}^{m_X-1}\frac{\left(a_X\gamma\right)^i}{i!}, \tag{2}$$

where $X \in \{R, I, D\}$ and $a_X = m_X/\Omega_X$, while m_X is the severity parameter and Ω_X represents the average SNR. The second expression in (2) can be written when m_X is an integer.

In the underlay scenario of CRN, the transmit power of S has been restricted by maximum tolerable interference power of PN which is defined as Q. Coming from delay and time-varying nature of channel, the available CSI at the selection instant may be differ from the actual one and the CSI of interference link to the PU is outdated due to feedback delay, as well. The correlation coefficients between $|h_n|^2$ and $|\hat{h}_n|^2$ and $|h_p|^2$ and $|\hat{h}_p|^2$ are depicted with ρ_R and ρ_I, respectively. The joint PDF of two Gamma RV with fading severity m_Y, mean power Ω_Y, scale parameter $a_Y = m_Y/\Omega_Y$ and correlation coefficient ρ_Y is as follow [36, Eq. (9.333)].

$$f_{|h|^2,|\hat{h}|^2}(x,y) = \frac{a_Y^{m_Y+1}}{\left(1-\rho_Y\right)\Gamma\left(m_Y\right)}\left(\frac{xy}{\rho_Y}\right)^{\frac{m_Y-1}{2}} \times\exp\left(-\frac{a_Y\left(x+y\right)}{\left(1-\rho_Y\right)}\right)I_{m_Y-1}\left(\frac{2a_Y\sqrt{\rho_Y xy}}{\left(1-\rho_Y\right)}\right), \tag{3}$$

where $Y \in \{R, I\}$. Here we define SNR of the RF link for two circumstances:

1) With neglecting PU and its corresponding link from the system model in Fig. 1 (i.e. non-cognitive scenario) and performing the k-th worst relay selection strategy in the mixed RF-FSO system, the SNR of the RF links $S \rightarrow R_k$ and $S \rightarrow D$ are defined as $\gamma_{SR} = |h_k|^2\overline{\gamma}_S$ and $\gamma_{SD} = |h_0|^2\overline{\gamma}_S$, respectively. Also, the average SNR is defined as $\overline{\gamma}_S = P_s/N_0$ and P_s is power of the transmitted signal from S.
2) Considering the k-th worst relay selection strategy in the mixed RF-FSO system assuming underlay scenario, the SNR of the RF links from S to R_k and D are respectively, defined as

$$\gamma_{SD} = \frac{\alpha_I\overline{\gamma}_{SQ}|h_0|^2}{|\hat{h}_p|^2}, \tag{4}$$

$$\gamma_{SR} = \frac{\alpha_I\overline{\gamma}_{SQ}|h_k|^2}{|\hat{h}_p|^2}, \tag{5}$$

where $\overline{\gamma}_{SQ} = Q/N_0$ and α_I is the power margin factor at S which can be obtained numerically for a given probability of interference to the PU (P_I) [37, Eq. (12)]. The interference probability at the PU can be written as $P_I = \Pr(\alpha_I Q|h_p|^2/|\hat{h}_p|^2 > Q)$.

Without loss of generality, N_0 is noise variance at all nodes.

B. FSO LINK

The $R_n \rightarrow D$ FSO links undergo DGG turbulence channel that is denoted by I_n. The stochastic behavior of the FSO link depends on three parameters, i.e. path loss I_l, atmospheric turbulence I_a and pointing error I_p. The path loss has deterministic value with the exponential Beers-Lambert law $h_l = \exp(-\sigma L)$, where σ and L show the atmospheric attenuation and the distance between relay and destination nodes, respectively. The atmospheric turbulence is RV and follows DGG distribution. With considering non-zero boresight and based on approximated Beckmann distribution proposed in [7], the pointing error is a RV with modified Rayleigh distribution.

The corresponding FSO channel coefficient to the selected k-th worse RF link is defined as $I_k = I_l I_p I_a$. The instantaneous electrical SNR of the received signal at D is $\gamma_{RD} = (\eta I_k)^r/N_0$ (for simplicity we omit the index k for parameters belong to I_k). The PDF of SNR distribution of the FSO link assuming non-zero boresight pointing error is defined as [21]

$$f_{\gamma_{RD}}\left(\gamma_{RD}\right) = \frac{A_1}{\gamma_{RD}}G_{\alpha_2 p,m}^{m,0}\left[B_1\,g^{\alpha_2 p}\left(\frac{\gamma_{RD}}{\mu_r}\right)^{\frac{\alpha_2 p}{r}}\middle|\begin{matrix}\kappa_1\\ \kappa_2\end{matrix}\right], \tag{6}$$

where $A_1 = \xi_{mod}^2 p^{m_2-\frac{1}{2}}q^{m_1-\frac{1}{2}}(2\pi)^{1-\frac{p+q}{2}}\big/(r\Gamma(m_1)\Gamma(m_2))$, $B_1 = m_1^q m_2^p\big/(p^p q^q\Omega_1^q\Omega_2^p)$, $m = p+q+\alpha_2 p$, $\kappa_1 = [\underline{\Delta}(\alpha_2 p : \xi_{mod}^2+1)]$ and $\kappa_2 = [\underline{\Delta}(\alpha_2 p : \xi_{mod}^2), \underline{\Delta}(q : m_1), \underline{\Delta}(p : m_2)]$. The small-scale and large-scale atmospheric fluctuations of the turbulence channel are described by $\{\alpha_1, m_1, \Omega_1\}$ and $\{\alpha_2, m_2, \Omega_2\}$, p and q are positive integer values such that $p/q = \alpha_1/\alpha_2$ and $g =$

$A_1 B_1^{-1/(\alpha_2 p)} \prod_{i=1}^{p+q} \Gamma\left(1/(\alpha_2 p)+\kappa_{0,i}\right)/(\xi_{mod}^2+1)$ where $\kappa_0 = [\Delta(q,m_1), \Delta(p,m_2)]$ while $\kappa_{l,i}$ represents the ith term of κ_l. Moreover, μ_r denotes the average electrical SNR of the FSO link for both type of detectors which is defined as $\mu_r = (\eta \mathbb{E}[I_k])^r/N_0$ and the value of this is obtained as $\mu_r = (h_l \eta A_{mod} g)^r/N_0$. ξ_{mod} shows the ratio between the equivalent beam radius and the pointing error standard deviation at the receiver [3] which is given as $\xi_{mod} = w_{z_{eq}}/2\sigma_{mod}$, while $w_{z_{eq}}$ is the equivalent beam radius at the receiver given through $w_{z_{eq}}^2 = \sqrt{\pi} erf(\nu) w_z^2/2\nu \exp(-\nu^2)$, $\nu = \sqrt{\pi} a_r/\sqrt{2} w_z$ where w_z and a_r are Gaussian beam radius on the receiver plane and radius of receiver aperture, respectively. σ_{mod} is defined in terms of Beckmann distribution parameters as [7].

$$\sigma_{mod}^2 = \left(\frac{3\mu_x^2\sigma_x^4 + 3\mu_y^2\sigma_y^4 + \sigma_x^6 + \sigma_y^6}{2}\right)^{1/3}, \tag{7}$$

where μ_x, μ_y, σ_x and σ_y are Beckmann parameters. The (μ_x, σ_x) and (μ_y, σ_y) represent static boresight displacement and the standard deviations (jitter) for horizontal and vertical directions, respectively. Moreover, with matching the first moments of I_p when pointing error displacement follows Beckmann distribution and modified Rayleigh in the log-domain, the A_{mod} is obtained as [7]

$$A_{mod} = A_0 \exp\left(\frac{1}{\xi_{mod}^2} - \frac{1}{2\xi_x^2} - \frac{1}{2\xi_y^2} - \frac{\mu_x^2}{2\xi_x^2\sigma_x^2} - \frac{\mu_y^2}{2\xi_y^2\sigma_y^2}\right), \tag{8}$$

where $\xi_x = w_{z_{eq}}/2\sigma_x$ and $\xi_y = w_{z_{eq}}/2\sigma_y$. The A_0 is amount of collected power at the center of receiver ($A_0 = erf^2(\nu)$). As a special case, with considering zero-mean Gaussian ($\mu_x = \mu_y = 0$) and identical standard deviation ($\sigma_s = \sigma_x = \sigma_y$) for each directions, the non-zero boresight pointing error reduces to zero boresight pointing error in [3] (i.e. $\xi_{mod} = w_{z_{eq}}/2\sigma_s$ and $A_{mod} = A_0$). Accordingly, the relationship between μ_r and $\overline{\gamma}_D$ is readily derived as

$$\overline{\gamma}_{RD} = \begin{cases} \mu_1 & \text{if } r=1 \\ \left(\sigma_{si}^2+1\right)\mu_2 & \text{if } r=2, \end{cases} \tag{9}$$

where σ_{si}^2 is the scintillation index [3].

In the semi-blind fixed gain AF relaying technique, $G = 1/\sqrt{N_0(\mathbb{E}(\gamma_{SR})+1)}$. Therefore, the SNR's distribution of fixed gain AF dual-hop mixed RF-FSO system at the destination can be written as

$$\gamma_{SRD} = \frac{\gamma_{SR}\,\gamma_{RD}}{\gamma_{RD}+C}. \tag{10}$$

Using a semi-blind relaying technique, $C = \mathbb{E}\left(\gamma_{SR}\right)+1$ [28]. By using the SC method at the destination, where the higher SNR path is selected, the end-to-end SNR of the system under consideration can be written as

$$\gamma_{e2e} = \max\left(\gamma_{SRD}, \gamma_{SD}\right). \tag{11}$$

III. PERFORMANCE ANALYSIS OF NON-COGNITIVE RELAY NETWORK SCENARIO

Here, we consider PRS strategy with neglecting the effect of PU and its corresponding link in Fig. 1. Note that, although this scheme is suboptimal compared with the traditional relay selection scheme, it incurs lower implementation complexity. In fact, in this scheme, the relay chooses the best channel based on the source-relay channels, whereas in the traditional relay selection, the destination chooses the link with maximum end-to-end SNR, which requires full instantaneous CSI of all links. This requirement renders the implementation of such schemes laborious. To the best of our knowledge, the OP, BEP and EC of dual-hop fixed gain AF relaying in mixed RF-FSO network with a direct link assuming Nakagami-m and DGG fading channels respectively for the RF and FSO links and employing the PRS with outdated CSI have not been derived, yet.

A. OUTAGE PROBABILITY

The OP is defined as the probability that the end-to-end SNR falls below a specified threshold, γ_{th}. It is defined as

$$P_{out}(\gamma_{th}) = \Pr\left[\gamma_{e2e} \le \gamma_{th}\right] = F_{\gamma_{e2e}}(\gamma_{th}), \tag{12}$$

where $F_{\gamma_{e2e}}(\gamma_{th})$ denotes the CDF of γ_{e2e} evaluated at $\gamma = \gamma_{th}$. In the following, we derive the CDF of γ_{e2e} required to calculate (12).

As mentioned above, due to the time variations of channel and the feedback delay, k-th worst relay selection strategy is done based on the outdated CSI of the RF links. With inspiring from deviation method in [38] for calculating PDF of best relay and generalizing to k-th worst relay selection, the CDF of $|h_k|^2$ is given by

$$F_{|h_k|^2}(x) = 1 - \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{m_R+j-1} \binom{m_R+\beta-1}{\beta-j} \times \binom{m_R+j-1}{j} \frac{\beta! k\, \Xi \left(a_R \chi x\right)^s \exp\left(-a_R \chi x\right)}{s! \chi^{m_R+j}}, \tag{13}$$

where $\chi = 1/(1-\rho_R) - \lambda^2/\varpi$, $\varpi = \rho_R/(1-\rho_R) + N + l - k + 1$ and $\lambda = \sqrt{\rho_R}/(1-\rho_R)$. $\Psi\left(k, N, m_R\right)$ and Ξ are defined respectively as follow

$$\Psi\left(k, N, m_R\right) = \sum_{l=0}^{k-1} \sum_{p_1=0}^{N+l-k} \sum_{p_2=0}^{p_1} \cdots \sum_{p_{m_R-1}=0}^{p_{m_R-2}} \binom{N}{k} \binom{k-1}{l} \times \frac{(-1)^l (N+l-k)!}{p_{m_R-1}!} \prod_{i=1}^{m_R-1} \left(\frac{1}{(p_{i-1}-p_i)!(i!)^{p_i - p_{i+1}}}\right), \tag{14}$$

$$\Xi = \frac{\lambda^{m_R+2j-1}}{\varpi^{m_R+\beta+j} \left(1-\rho_R\right) \rho_R^{(m_R-1)/2}}, \tag{15}$$

TABLE 1. Parameters of binary modulation.

Modulation	δ	τ
Coherent Binary Frequency Shift Keying (CBFSK)	0.5	0.5
Non-Coherent Binary Frequency Shift Keying (NBFSK)	0.5	1
Coherent Binary Phase Shift Keying (CBPSK)	1	0.5
Differential Binary Phase Shift Keying (DBPSK)	1	1

while $p_0 = N + l - k$, $p_{m_R} = 0$ and $\beta = \sum_{q=1}^{m_R-1} p_q$. Here, we assume that m_R and m_D have integer values.

Based on the definition of γ_{SD} and γ_{SR} for non-cognitive scenario in II-A, the CDFs of SNR distribution for the RF links are derived by substituting (13) and (2) (for $X = D$) into $F_{\gamma_{SR}}(\gamma) = F_{|h_k|^2}\left(\gamma/\overline{\gamma}_S\right)$ and $F_{\gamma_{SD}}(\gamma) = F_{|h_0|^2}\left(\gamma/\overline{\gamma}_S\right)$, respectively.

Based on (10), the CDF of γ_{SRD} is derived with following integral

$$F_{\gamma_{SRD}}(\gamma) = \int_0^\infty F_{\gamma_{SR}}\left(\frac{\gamma_{RD}+C}{\gamma_{RD}}\gamma\right) f_{\gamma_{RD}}(\gamma_{RD})\, d\gamma_{RD}. \tag{16}$$

Due to the fact that RF-FSO dual-hop link and direct RF link are independent, the CDF of γ_{e2e} is given by [39]

$$F^{nc}_{\gamma_{e2e}}(\gamma) = F_{\gamma_{SRD}}(\gamma)\, F_{\gamma_{SD}}(\gamma), \tag{17}$$

where index "nc" indicates the non-cognitive scenario. By substituting the CDF of γ_{SR} and (6) into (16), we obtain an integral involving a Meijer's-G function. The exponential function could be altered to a Meijer's-G function using [40, Eq. (8.4.3.2)]. Subsequently, by employing binomial expansion [34, Eq. (1.111)] and integral identity [40, Eq. (2.24.1.1)], the CDF of γ_{SRD} is obtained.

By substituting the CDF of γ_{SRD} and (2) with $X = D$ into (17), the CDF of γ_{e2e} can be written as

$$\begin{aligned} F^{nc}_{\gamma_{e2e}}(\gamma) = &\Bigg(1 - \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{j+m_R-1} \sum_{i=0}^{s} \\ &\times \binom{\beta + m_R - 1}{\beta - j} \binom{j + m_R - 1}{j} \binom{s}{i} A (2\pi)^{(1-\alpha_2 p)/2} \\ &\times \frac{\beta!\, k\, \Xi (\alpha_2 p)^{i+\frac{1}{2}}}{s!\, \chi^{m_R + j}} \left(\frac{a_R \chi \gamma}{\overline{\gamma}_S}\right)^{s-i} \exp\left(-\frac{a_R \chi \gamma}{\overline{\gamma}_S}\right) \\ &\times G^{rm+\alpha_2 p, 0}_{r\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R \chi C_{nc} \gamma}{\alpha_2 p \overline{\gamma}_S\, \mu_r}\right)^{\alpha_2 p} \middle| \begin{matrix} \kappa_A \\ \kappa_3 \end{matrix}\right]\Bigg) \\ &\times \frac{\Upsilon\left(m_D, \frac{a_D \gamma}{\overline{\gamma}_S}\right)}{\Gamma\left(m_D\right)}, \end{aligned} \tag{18}$$

where $\kappa_A = \left[\underline{\Delta}\left(r, \alpha_2 p, \xi^2_{mod} + 1\right)\right]$, $\kappa_3 = [\kappa_B, \Delta(\alpha_2 p : i)]$ and $\kappa_B = \left[\Delta\left(r, \alpha_2 p, \xi^2_{mod}\right), \Delta(r, q, m_1), \Delta(r, p, m_2)\right]$; while A and B are respectively defined as

$$A = \frac{\xi^2_{mod}\, p^{m_2 - \frac{1}{2}}\, q^{m_1 - \frac{1}{2}}\, (2\pi)^{1 - \left(\frac{p+q}{2}\right)} r^{\mu - 1}}{\Gamma(m_1)\, \Gamma(m_2)\, \alpha_2 p},$$

$$B = \left(\frac{g^{\alpha_2 p} m_1^q\, m_2^p}{p^p\, q^q\, \Omega_1^q\, \Omega_2^p\, r^{(p+q)}}\right)^r,$$

where $\mu = \sum_{j=1}^{\alpha_2 p} \kappa_{2,j} - \sum_{i=1}^{m} \kappa_{1,i} - \frac{p+q}{2} + 1$. Based on probability laws, the expected value of γ_{SR} is derived in term of CDF of γ_{SR} as follow

$$\mathbb{E}\left(\gamma_{SR}\right) = \int_0^\infty \left(1 - F_{\gamma_{SR}}(\gamma)\right) d\gamma. \tag{19}$$

By substituting the CDF of γ_{SR} into (19) and utilizing [34, Eqs. (3.326.2)], we obtain

$$\begin{aligned} C_{nc} = 1 + \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \binom{\beta + m_R - 1}{\beta - j} \binom{j + m_R - 1}{j} \\ \times \frac{\left(m_R + j\right) \beta!\, k\, \Xi}{\chi^{m_R + j + 1}} \left(\frac{\overline{\gamma}_{SR}}{a_R}\right). \end{aligned} \tag{20}$$

Special Case: The Rayleigh and G^2 distributions are respectively special cases of Nakagami-m and DGG distributions. As a special case, it can be shown that for $m_R = m_I = 1$, $\alpha_1 = \alpha_2 = \Omega_1 = \Omega_2 = 1$, $m_1 = \alpha$ and $m_2 = \beta$ (i.e. Rayleigh/G^2 fading) with zero boresight pointing error ($\mu_x = \mu_y = 0$ and $\sigma_x = \sigma_y$) and without consideration of direct link, eq. (18) reduces to [29, Eq. (17.28)].

B. BIT ERROR PROBABILITY

We now turn our attention to the BEP. We mention that the BEP for most binary modulations can be expressed as [14, Eq. (21)]

$$\overline{P_e} = \frac{\delta^\tau}{2\Gamma(\tau)} \int_0^\infty e^{-\delta\gamma} \gamma^{\tau - 1} F_{\gamma_{e2e}}(\gamma) d\gamma, \tag{21}$$

where τ and δ are the modulation's parameters which can be selected from set $\{0.5, 1\}$ in Table 1.

By substituting (18) into (21) and employing integral identities [40, Eq. (2.24.1.1)] and [34, Eq. (3.351.3)], we arrive to (22), as shown at the top of the next page.

C. ERGODIC CAPACITY

Another important figure of metric is EC. The EC is defined as $\overline{C} = \mathbb{E}[\log_2(1 + c\gamma)]$, where $c = 1$ for heterodyne detection (i.e. $r = 1$) [41, Eq. (14)] and $c = e/(2\pi)$ for IM/DD (i.e. $r = 2$) [42, Eq. (26)]. Additionally, this expression is exact for the case of $r = 1$ while it is a lower-bound for $r = 2$. The EC can be written in terms of the complementary CDF of γ_{e2e} as follow

$$\overline{C} = \frac{1}{\ln(2)} \int_0^\infty \frac{1 - F_{\gamma_{e2e}}(\gamma)}{1 + c\gamma} d\gamma. \tag{23}$$

$$\overline{P_e}^{nc} = \frac{1}{2} + \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{j+m_R-1} \sum_{i=0}^{s} \binom{\beta + m_R - 1}{\beta - j} \binom{j + m_R - 1}{j} \binom{s}{i} A (2\pi)^{1-\alpha_2 p}$$
$$\times \frac{\beta! \delta^\tau k \,\Xi(\alpha_2 p)^{\tau+s}}{2\Gamma(\tau) s! \chi^{m_R+j}} \left(\frac{a_R \chi}{\overline{\gamma}_S}\right)^{s-i} \left(\sum_{t=0}^{m_D-1} \frac{1}{t!} \left(\frac{a_D \alpha_2 p}{\overline{\gamma}_S}\right)^t \left(\frac{a_R \chi + a_D}{\overline{\gamma}_S} + \delta\right)^{i-\tau-s-t}\right.$$
$$\times G^{rm+\alpha_2 p, \alpha_2 p}_{(r+1)\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R \chi C_{nc}}{\left(a_R \chi + a_D + \delta \overline{\gamma}_S\right)\mu_r}\right)^{\alpha_2 p} \middle| \begin{matrix} \Delta\left(\alpha_2 p : 1 + i - \tau - s - t\right), \kappa_A \\ \kappa_3 \end{matrix} \right]$$
$$- \left(\frac{a_R \chi}{\overline{\gamma}_S} + \delta\right)^{i-\tau-s} G^{rm+\alpha_2 p, \alpha_2 p}_{(r+1)\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R \chi C_{nc}}{\left(a_R \chi + \delta \overline{\gamma}_S\right)\mu_r}\right)^{\alpha_2 p} \middle| \begin{matrix} \Delta\left(\alpha_2 p : 1 + i - \tau - s\right), \kappa_A \\ \kappa_3 \end{matrix} \right] \Bigg)$$
$$- \sum_{t=0}^{m_D-1} \frac{\Gamma(\tau + t)\delta^\tau}{2\Gamma(\tau) t!} \left(\frac{a_D}{\overline{\gamma}_S}\right)^t \left(\frac{a_D}{\overline{\gamma}_S} + \delta\right)^{-\tau-t}. \tag{22}$$

$$\overline{C}^{nc} = \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{j+m_R-1} \sum_{i=0}^{s} \binom{\beta + m_R - 1}{\beta - j} \binom{j + m_R - 1}{j} \binom{s}{i} \frac{\beta! k \,\Xi(\alpha_2 p)^{i-\frac{1}{2}} A (2\pi)^{(1-\alpha_2 p)/2}}{\ln(2) s! \chi^{m_R+j}}$$
$$\times \left(B^{\frac{1}{\alpha_2 p}} \left(\frac{a_R \chi C_{nc} \gamma}{\alpha_2 p \overline{\gamma}_S \mu_r}\right)\right)^{i-s} \left(H^{rm+\alpha_2 p, \alpha_2 p ;1,1;1,0}_{(r+1)\alpha_2 p, rm+\alpha_2 p;1,1;0,1}\left[\begin{matrix} \Lambda\left(\kappa_3, s-i, rm+\alpha_2 p\right) \\ \Lambda\left(\kappa_A, s-i, r\alpha_2 p\right) \end{matrix} \middle| \begin{matrix} (0,1) \\ (0,1) \end{matrix} \middle| \begin{matrix} - \\ (0,1) \end{matrix} \middle| D_1, D_2 \right] - \sum_{t=0}^{m_D-1} \frac{1}{t!}\right.$$
$$\times \left(\frac{a_D}{\overline{\gamma}_S}\right)^t \left(B^{\frac{1}{\alpha_2 p}} \left(\frac{a_R \chi C_{nc} \gamma}{\alpha_2 p \overline{\gamma}_S \mu_r}\right)\right)^{-t} H^{rm+\alpha_2 p, \alpha_2 p ;1,1;1,0}_{(r+1)\alpha_2 p, rm+\alpha_2 p;1,1;0,1}\left[\begin{matrix} \Lambda\left(\kappa_3, s-i+t, rm+\alpha_2 p\right) \\ \Lambda\left(\kappa_A, s-i+t, r\alpha_2 p\right) \end{matrix} \middle| \begin{matrix} (0,1) \\ (0,1) \end{matrix} \middle| \begin{matrix} - \\ (0,1) \end{matrix} \middle| D_1, D_3 \right] \Bigg)$$
$$+ \sum_{t=0}^{m_D-1} \frac{1}{\ln(2) t} \left(\frac{a_D}{c \overline{\gamma}_S}\right)^t \exp\left(\frac{a_D}{c \overline{\gamma}_S}\right) \Gamma\left(1 - t, \frac{a_D}{c \overline{\gamma}_S}\right). \tag{24}$$

We transform the fractional and exponential functions to Meijer's-G functions by employing [40, Eq. (8.4.2.5)] and [40, Eq. (8.4.3.2)], respectively. Then, we have an integration on product of three Meijer's-G functions. By employing [40, Eq. (8.3.1.21)] and [43, Eq. (6.2.3)], we can transform these Meijer's-G functions into Fox-H functions [44, Eq. (1.2)]. This integral can be solved in terms of EGBFHFs by using [35, Eq. (2.3)]; then by using [34, Eq. (3.383.10)], we end up with (24), as shown at the top of this page, where $D_1 \triangleq \left(B^{-1/\alpha_2 p} c \alpha_2 p \overline{\gamma} \mu_r\right) / \left(a_R \chi C_{nc} \gamma\right)$, $D_2 \triangleq \left(B^{-1/\alpha_2 p} \alpha_2 p \chi \mu_r\right) / \left(\chi C_{nc} \gamma\right)$ and $D_3 \triangleq \left(B^{-1/\alpha_2 p} \alpha_2 p (a_R \chi + a_D) \mu_r\right) / \left(a_R \chi C_{nc} \gamma\right)$, while $\Lambda\left(.,.,.\right)$ is defined as

$$\Lambda\left(\kappa_G, \varphi, \theta\right) \triangleq \left(1 - \kappa_G - \varphi\left[\frac{1}{\alpha_2 p}\right]_\theta ; \left[\frac{1}{\alpha_2 p}\right]_\theta, \left[\frac{1}{\alpha_2 p}\right]_\theta\right). \tag{25}$$

Note that, the EGBFHF is implemented by [45] and [46] in MATHEMATICA® and MATLAB®, respectively.

IV. PERFORMANCE ANALYSIS OF RF-FSO RELAY NETWORK IN UNDERLAY COGNITIVE SCENARIO

Cognitive radio is a promising technique for improving spectrum utilization efficiency. In a CRN, there are two networks (i.e. PN and SN). In an underlay system, a SU can have full spectrum access if the imposed interference on the PU's spectrum is less than a threshold limit. To this end, the SU should confine their transmission powers to a predefined threshold. In this section, we analysis underlay cognitive mixed RF-FSO relay network in two scenario 1) with direct link 2) without direct link.

A. OUTAGE PROBABILITY ANALYSIS WITHOUT DIRECT LINK

Here, we analysis RF-FSO relay network when the direct link between source and destination is not available. In order to obtain the CDF of γ_{e2e} without direct link, we obtain the CDF of γ_{SR}. Based on the definition of γ_{SR} in (5), we have

$$F_{\gamma_{SR}}(\gamma) = \int_0^\infty F_{|h_k|^2}\left(\frac{\gamma}{\alpha_I \overline{\gamma}_{SQ}} x\right) f_{|\hat{h}_p|^2}(x)\, dx. \tag{26}$$

By substituting (13) and (1) (for $X = I$) into (26) and employing [34, Eq. (3.351.3)], the CDF of γ_{SR} is derived as

$$F^{co}_{\gamma_{SR}}(\gamma) = 1 - \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{m_R+j-1} \binom{m_R + \beta - 1}{\beta - j}$$
$$\times \binom{m_R + j - 1}{j} \binom{m_I + s - 1}{s} \frac{\beta! k \,\Xi a_I^{m_I}}{\chi^{m_R+j}}$$
$$\times \frac{\left(\frac{a_R \chi \gamma}{\alpha_I \overline{\gamma}_{SQ}}\right)^s}{\left(\frac{a_R \chi \gamma}{\alpha_I \overline{\gamma}_{SQ}} + a_I\right)^{m_I+s}}, \tag{27}$$

where "co" indicates the underlay cognitive scenario. We substitute (27) and (6) into (16) and employ the definition of binomial coefficients in [34, Eq. (1.111)]. Then by deploying [40, Eq. (2.24.2.4)] and after some algebraic manipulations, the CDF of γ_{e2e} is obtained as

$$F^{cnd}_{\gamma_{e2e}}(\gamma) = 1 - \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{j+m_R-1} \sum_{i=0}^{s} \times \binom{\beta + m_R - 1}{\beta - j} \binom{j + m_R - 1}{j} \binom{s}{i} \frac{\beta! k\, \Xi a_I^{m_I} (\alpha_2 p)^{m_I + s}}{\Gamma(m_I)\, s!\, \chi^{m_R + j}} \times \left(\frac{a_R \chi \gamma}{\alpha_I \overline{\gamma}_{SQ}}\right)^{s-i} \left(\frac{a_R \chi \gamma}{\alpha_I \overline{\gamma}_{SQ}} + a_I\right)^{i - m_I - s} A(2\pi)^{1-\alpha_2 p} \times G^{rm+\alpha_2 p, \alpha_2 p}_{(r+1)\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R \chi C_{co} \gamma}{\mu_r \left(a_I \alpha_I \overline{\gamma}_{SQ} + a_R \chi \gamma\right)}\right)^{\alpha_2 p} \middle| \begin{matrix} \kappa_4 \\ \kappa_5 \end{matrix}\right], \tag{28}$$

where index "cnd" indicates cognitive and non-direct link scenario, while $\kappa_4 = \left[\Delta\left(\alpha_2 p : 1 + i - m_I - s\right), \kappa_A\right]$ and $\kappa_5 = \left[\Delta\left(\alpha_2 p : i\right), \kappa_B\right]$.

By substituting (27) into (19) and using [34, Eqs. (3.194.3, 8.384.1)], we obtain the constant value as follow

$$C_{co} = 1 + \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \binom{m_R + \beta - 1}{\beta - j} \binom{m_R + j - 1}{j} \times \frac{\left(m_R + j\right) \beta! k\, \Xi a_I}{\left(m_I - 1\right) \chi^{m_R + j + 1}} \left(\frac{\alpha_I \overline{\gamma}_{SQ}}{a_R}\right), \tag{29}$$

which is valid for $m_I > 1$.

B. OUTAGE PROBABILITY ANALYSIS WITH DIRECT LINK

In the underlay scenario of CRN with direct link, due to power restriction at S with respect to RV $|\hat{h}_p|^2$, the SNR of the $S \to R_k$ and $S \to D$ links are not independent. Therefore, the RVs γ_{SRD} and γ_{SD} are not independent, as well. Hence, we cannot utilize (17) in order to calculate the CDF of γ_{e2e}. Since both γ_{SRD} and γ_{SD} depends on $|\hat{h}_p|^2$, we employ the conditional CDF of γ_{SD} and γ_{SRD} on RV $Z = |\hat{h}_p|^2$ as bellow

$$F_{\gamma_{e2e}}\left(\gamma \middle| Z\right) = F_{\gamma_{SRD}}\left(\gamma \middle| Z\right) F_{\gamma_{SD}}\left(\gamma \middle| Z\right). \tag{30}$$

Then the overall CDF is obtained by using the following integral

$$F^{co}_{\gamma_{e2e}}(\gamma) = \int_0^{\infty} F_{\gamma_{e2e}}\left(\gamma \middle| Z\right) f_Z(z)\, dz. \tag{31}$$

Based on (4) and (5), the conditional CDF of γ_{SR} and γ_{SD} can be written respectively as

$$F_{\gamma_{SD}}\left(\gamma \middle| Z\right) = F_{|h_0|^2}\left(\frac{\gamma Z}{\alpha_I \overline{\gamma}_{SQ}}\right), \tag{32}$$

$$F_{\gamma_{SR}}\left(\gamma \middle| Z\right) = F_{|h_k|^2}\left(\frac{\gamma Z}{\alpha_I \overline{\gamma}_{SQ}}\right). \tag{33}$$

Based on (10), the conditional CDF of γ_{SRD} by employing the following integral is derived as

$$F_{\gamma_{SRD}}\left(\gamma \middle| Z\right) = \int_0^{\infty} F_{\gamma_{SR}}\left(\frac{\gamma_{RD} + C}{\gamma_{RD}} \gamma \middle| Z\right) f_{\gamma_{RD}}\left(\gamma_{RD}\right) d\gamma_{RD}. \tag{34}$$

In order to obtain the conditional CDF of γ_{SRD}, we put (33) and (6) in (34). The exponential function is transformed to Meijer's-G function with [40, Eq. (8.4.3.2)]. Employing binomial expansion [34, Eq. (1.111)] and integral identity [40, Eq. (2.24.1.1)], the conditional CDF of γ_{SRD} is derived as

$$F^{co}_{\gamma_{SRD}}\left(\gamma \middle| Z\right) = 1 - \Psi\left(k, N, m_R\right) \sum_{j=0}^{\beta} \sum_{s=0}^{m_R + j - 1} \sum_{i=0}^{s} \times \binom{m_R + \beta - 1}{\beta - j} \binom{m_R + j - 1}{j} \binom{s}{i} \times \frac{\beta!\, k \lambda^{m_R + 2j - 1} (\alpha_2 p)^{i + \frac{1}{2}} A (2\pi)^{(1-\alpha_2 p)/2}}{s! \varpi^{m_R + \beta + j} \chi^{m_{R+j}} \left(1 - \rho_R\right) \rho_R^{(m_R - 1)/2}} \times \left(\frac{a_R \chi Z \gamma}{\alpha_I \overline{\gamma}_{SQ}}\right)^{s-i} \exp\left(-\frac{a_R \chi Z \gamma}{\alpha_I \overline{\gamma}_{SQ}}\right) \times G^{rm+\alpha_2 p, 0}_{r\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R \chi C_{co} Z \gamma}{\alpha_2 p \alpha_I \overline{\gamma}_{SQ}\, \mu_r}\right)^{\alpha_2 p} \middle| \begin{matrix} \kappa_A \\ \kappa_3 \end{matrix}\right]. \tag{35}$$

Based on (30) and by substituting (35), (32) and (1) (for $X = I$) into (31) and applying integral identity [40, Eq. (2.24.3.1)] we get (36), as shown at the top of the next page. The C parameter for with direct link scenario is same as (29).

C. SPECIAL CASES

In order to fulfill our analysis, we consider two extreme cases: 1) without CSI ($\rho_R = 0$) and 2) with perfect CSI ($\rho_R = 1$). We consider some special cases for all scenarios which are mentioned in this paper (i.e. non-cognitive and cognitive scenarios). Therefore, the resulted derivations are valid for the OP in (18), (28) and (36), the BEP in (22) and the EC in (24).

1) OUTAGE PROBABILITY ANALYSIS WITHOUT CSI (I.E. $\rho_R = 0$)

Here, we suppose $\rho_R = 0$, which means the outdated version of CSI are not related to actual ones at all. Therefore the relay selection is performed randomly. When $\rho_R \to 0$, we have $\varpi = N + l - k + 1$, $\lambda = \sqrt{\rho_R}$ and $\chi = 1$. Therefore, Ξ in (15) when ρ_R goes to zero can be rewritten as

$$\lim_{\rho_R \to 0} \Xi = \frac{\rho_R^j}{(N + l - k + 1)^{m_R + \beta + j}}. \tag{37}$$

This equation is zero when $\rho_R = 0$ and is non-zero when $j = 0$. With i.i.d. assumption for all RF links, it is similar to existence of only one RF link with Nakagami-m fading with m_R and a_R parameters. Therefore, the OP is obtained

$$F^{co}_{\gamma_{e2e}}(\gamma) = 1 + \Psi\left(k, N, m_R\right)\sum_{j=0}^{\beta}\sum_{s=0}^{m_R+j-1}\sum_{i=0}^{s}\binom{m_R+\beta-1}{\beta-j}\binom{m_R+j-1}{j}\binom{s}{i}A(2\pi)^{1-\alpha_2 p}$$
$$\times \frac{\beta!k\,\Xi(\alpha_2 p)^{m_I+s}a_I^{m_I}}{\Gamma(m_I)s!\chi^{m_R+j}}\left(\frac{a_R\chi\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)^{s-i}\left(\sum_{t=0}^{m_D-1}\frac{(\alpha_2 p)^t}{t!}\left(\frac{\left(a_R\chi+a_D\right)\gamma}{\alpha_I\overline{\gamma}_{SQ}}+a_I\right)^{i-m_I-s-t}\right.$$
$$\times\left(\frac{a_D\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)^{t} G^{rm+\alpha_2 p,\alpha_2 p}_{(r+1)\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R\chi C_{co}\gamma}{\left(\left(a_R\chi+a_D\right)\gamma+\alpha_I\overline{\gamma}_{SQ}a_I\right)\mu_r}\right)^{\alpha_2 p}\middle|\begin{matrix}\Delta\left(\alpha_2 p: 1+i-m_I-s-t\right), \kappa_A\\ \kappa_3\end{matrix}\right]$$
$$\left.-\left(\frac{a_R\chi\gamma}{\alpha_I\overline{\gamma}_{SQ}}+a_I\right)^{i-m_I-s} G^{rm+\alpha_2 p,\alpha_2 p}_{(r+1)\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R\chi C_{co}\gamma}{\left(a_R\chi\gamma+\alpha_I\overline{\gamma}_{SQ}a_I\right)\mu_r}\right)^{\alpha_2 p}\middle|\begin{matrix}\Delta\left(\alpha_2 p: 1+i-m_I-s\right), \kappa_A\\ \kappa_3\end{matrix}\right]\right)$$
$$-\sum_{t=0}^{m_D-1}\frac{\Gamma\left(m_I+t\right)a_I^{m_I}}{\Gamma\left(m_I\right)t!}\left(\frac{a_D\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)^{t}\left(\frac{a_D\gamma}{\alpha_I\overline{\gamma}_{SQ}}+a_I\right)^{-m_I-t}. \quad (36)$$

as in (36) when $N = 1$ and $k = 1$. Moreover, when $N = 1$ and $k = 1$ (35) reduces to [21, Eq. (14)].

2) OUTAGE PROBABILITY ANALYSIS WITH PERFECT CSI (I.E. $\rho_R = 1$)

Here, we suppose $\rho_R = 1$ which means we have perfect CSI of the RF links (i.e. $|h_n|^2$). In this case, we have $\varpi = \lambda = 1/(1-\rho_R)$ and $\chi = N + l - k + 1$, thus, Ξ in (15) when ρ_R goes to 1 is equal to

$$\lim_{\rho_R\to 1}\Xi = \left(1-\rho_R\right)^{\beta-j}. \quad (38)$$

We should note that, (38) is equal to zero except for $j = \beta$.

V. ASYMPTOTIC ANALYSIS OF OUTAGE PROBABILITY

Since the derived exact closed-form expressions provide limited physical insights, we now focus on the high SNR analysis and obtain diversity order. Here, we analysis the OP of cognitive scenario which is presented in Section IV-B. We consider two circumstances: 1) $0 \le \rho_R < 1$ and 2) $\rho_R = 1$ and derive the diversity order for these two different cases.

A. ASYMPTOTIC EXPRESSION FOR $0 \le \rho_R < 1$

With employing integration in [34, Eq. (3.381.2)], we can reexpress the CDF of $|h_k|^2$ in (13) as follow

$$F_{|h_k|^2}(x) = \Psi\left(k, N, m_R\right)\sum_{j=0}^{\beta}\sum_{s=0}^{\infty}\binom{m_R+\beta-1}{\beta-j}$$
$$\times\binom{m_R+j-1}{j}\frac{1}{\Gamma\left(m_R+j+s+1\right)}$$
$$\times\frac{\beta!k\;\Xi(a_R\chi x)^{m_R+j+s}\exp\left(-a_R\chi x\right)}{\chi^{m_R+j}}. \quad (39)$$

By substituting (39) into (33), the conditional CDF of γ_{SR} can be obtained.

In order to derive the conditional CDF of γ_{SRD}, we put CDF of γ_{SR} and (6) in (34). Using [40, Eq. (8.4.3.2)], the exponential function is altered to the Meijer's-G function. By employing the binomial coefficients in [34, Eq. (1.111)] and integral identity [40, Eq. (2.24.1.1)], we have

$$F^{co}_{\gamma_{SRD}}(\gamma|Z) = \Psi\left(k, N, m_R\right)\sum_{j=0}^{\beta}\sum_{s=0}^{\infty}\sum_{i=0}^{m_R+j+s}\binom{m_R+\beta-1}{\beta-j}$$
$$\times\binom{m_R+j-1}{j}\binom{m_R+j+s}{i}\frac{\beta!\,k\,\Xi(\alpha_2 p)^{i+\frac{1}{2}}}{\Gamma\left(m_R+j+s+1\right)\chi^{m_R+j}}$$
$$\times A(2\pi)^{(1-\alpha_2 p)/2}\left(\frac{a_R\chi Z\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)^{m_R+j+s-i}\exp\left(-\frac{a_R\chi Z\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)$$
$$\times G^{rm+\alpha_2 p,0}_{r\alpha_2 p, rm+\alpha_2 p}\left[B\left(\frac{a_R\chi C_{co}Z\gamma}{\alpha_2 p\alpha_I\overline{\gamma}_{SQ}\mu_r}\right)^{\alpha_2 p}\middle|\begin{matrix}\kappa_A\\ \kappa_3\end{matrix}\right]. \quad (40)$$

Using [34, Eq. (3.381.2)], we can reexpress the conditional CDF of γ_{SD} as follow

$$F^{co}_{\gamma_{SD}}(\gamma|Z) = \exp\left(-\frac{a_D\gamma Z}{\alpha_I\overline{\gamma}_{SQ}}\right)\sum_{t=0}^{\infty}\frac{\left(\frac{a_D\gamma Z}{\alpha_I\overline{\gamma}_{SQ}}\right)^{m_D+t}}{\Gamma\left(m_D+t+1\right)}. \quad (41)$$

Without loss of generality, we suppose $\overline{\gamma}_{SQ}$ is equal to $d_1\mu_r$ such that $\overline{\gamma} = \overline{\gamma}_{SQ} = d_1\mu_r$, where d_1 is an arbitrary positive value. By substituting (41), (40) and (1) (when $X = I$) into (31) and employing [40, Eq. (2.24.3.1)], we arrive to

$$F^{co}_{\gamma_{e2e}}(\gamma) = \Psi\left(k, N, m_R\right)\sum_{j=0}^{\beta}\sum_{s=0}^{\infty}\sum_{i=0}^{m_R+j+s}\sum_{t=0}^{\infty}\binom{m_R+\beta-1}{\beta-j}$$
$$\times\binom{m_R+j-1}{j}\binom{m_R+j+s}{i}\frac{A(2\pi)^{1-\alpha_2 p}}{\Gamma\left(m_D+t+1\right)\Gamma\left(m_I\right)}$$
$$\times\frac{\beta!\,k\,\Xi(\alpha_2 p)^{m_R+m_I+m_D+j+s+t}a_I^{m_I}}{\Gamma\left(m_R+j+s+1\right)s!\chi^{m_R+j}}$$

$$\times\left(\frac{a_R\chi}{\alpha_I}\right)^{m_R+j+s-i}\left(\frac{a_D}{\alpha_I}\right)^{m_D+t}\left(\frac{\gamma}{\overline{\gamma}}\right)^{m_R+m_D+j+s+t-i}$$
$$\times\left(\frac{(a_R\chi+a_D)\gamma}{\alpha_I\overline{\gamma}}+a_I\right)^{i-m_R-m_I-m_D-j-s-t}\Theta(6,3,\chi), \tag{42}$$

where

$$\Theta(\vartheta,\nu,\omega) = G^{rm+\alpha_2 p,\alpha_2 p}_{(r+1)\alpha_2 p,rm+\alpha_2 p}\left[B\left(\frac{d_1\frac{a_R\omega C_{co}\gamma}{a_I\alpha_I\overline{\gamma}}}{\left(\frac{(a_R\omega+a_D)\gamma}{a_I\alpha_I\overline{\gamma}}+1\right)\overline{\gamma}}\right)^{\alpha_2 p}\middle|\begin{matrix}\kappa_\vartheta\\ \kappa_\nu\end{matrix}\right], \tag{43}$$

where $\kappa_6=\left[\Delta\left(\alpha_2 p:1+i-m_R-m_I-m_D-j-s-t\right),\kappa_A\right]$.

At high SNRs, when $\overline{\gamma}\to\infty$, (29) is approximately equal to $C_{co}\simeq d_2\overline{\gamma}$ for $0\le\rho_R<1$, where d_2 is defined as

$$d_2=\Psi\left(k,N,m_R\right)\sum_{j=0}^{\beta}\binom{m_R+\beta-1}{\beta-j}\binom{m_R+j-1}{j}\times\frac{\left(m_R+j-1\right)\Gamma\left(m_I-1\right)\beta!k\,\Xi a_I}{\Gamma\left(m_I\right)\chi^{m_R+j+1}}\left(\frac{\alpha_I}{a_R}\right). \tag{44}$$

By using the Taylor series expansion of Meijer's-G function in [34, Eq. (9.303)], we can reexpress (43) approximately as follow

$$\Theta(\vartheta,\nu,\omega)\simeq\sum_{f=1}^{rm+\alpha_2 p}\frac{\prod\limits_{l=1,f\neq l}^{rm+\alpha_2 p}\Gamma\left(\kappa_{\nu,l}-\kappa_{\nu,f}\right)\prod\limits_{l=1}^{\alpha_2 p}\Gamma\left(1-\kappa_{\vartheta,l}+\kappa_{\nu,f}\right)}{\prod\limits_{l=\alpha_2 p+1}^{(r+1)\alpha_2 p}\Gamma\left(\kappa_{\vartheta,l}-\kappa_{\nu,f}\right)}\times\left(B\left(\frac{d_1\frac{a_R\omega d_2\gamma}{a_I\alpha_I}}{\left(\frac{(a_R\omega+a_D)\gamma}{a_I\alpha_I\overline{\gamma}}+1\right)\overline{\gamma}}\right)^{\alpha_2 p}\right)^{\kappa_{\nu,i}}. \tag{45}$$

B. ASYMPTOTIC EXPRESSION FOR $\rho_R=1$

Here, we assume that the PRS is performed based on the perfect CSI case (i.e. $\rho_R=1$). Therefore, the actual values of CSI are sorted for relay selection. The CDF of k-th order statistic of perfect CSI case can be attained as follow

$$F_{|h_k|^2}(x)=\sum_{l=k}^{N}\binom{N}{l}\left[F_{|h_n|^2}(x)\right]^l\left[1-F_{|h_n|^2}(x)\right]^{N-l}. \tag{46}$$

With substituting (2) (for $X=R$) in (46) and applying power series expansion of lower incomplete Gamma, we have

$$F_{|h_k|^2}(x)=\sum_{l=k}^{N}\sum_{j=0}^{N-l}\binom{N}{l}\binom{N-l}{j}(-1)^j\times\left[\sum_{i=0}^{\infty}\frac{(a_Rx)^{m_R+i}}{\Gamma\left(m_R+i+1\right)}\right]^{l+j}\exp\left(-a_R(l+j)x\right). \tag{47}$$

The conditional CDF of γ_{SR} is achieved by substituting (47) into (33). The conditional CDF of γ_{SRD}, when $\rho=1$, is derived by substituting the conditional CDF of γ_{SR} and (6) into (34) and keeping only the dominant terms. The exponential function is altered to the Meijer's-G function with using [40, Eq. (8.4.3.2)]. Employing the binomial coefficients [34, Eq. (1.111)] and integral identity [40, Eq. (2.24.1.1)], we end up with

$$F^{co}_{\gamma_{SRD}}(\gamma|Z)\overset{\gamma_{SQ}\gg1}{\simeq}\sum_{l=k}^{N}\sum_{j=0}^{N-l}\sum_{i=0}^{m_R(l+j)}\binom{N}{l}\binom{N-l}{j}\times\binom{m_R(l+j)}{i}\frac{(-1)^j(\alpha_2 p)^{i+\frac{1}{2}}A(2\pi)^{(1-\alpha_2 p)/2}}{\left(\Gamma\left(m_R+1\right)\right)^{(l+j)}}\times\left(\frac{a_RZ\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)^{m_R(l+j)-i}\exp\left(-\frac{a_R(l+j)Z\gamma}{\alpha_I\overline{\gamma}_{SQ}}\right)\times G^{rm+\alpha_2 p,0}_{r\alpha_2 p,rm+\alpha_2 p}\left[B\left(\frac{a_R(l+j)C_{co}Z\gamma}{\alpha_2 p\alpha_I\overline{\gamma}_{SQ}\mu_r}\right)^{\alpha_2 p}\middle|\begin{matrix}\kappa_A\\ \kappa_3\end{matrix}\right]. \tag{48}$$

As mentioned in subsection V-A, we consider $\overline{\gamma}=\overline{\gamma}_{SQ}=d_1\mu_r$ for high SNR regime. By substituting (41), (48) and (1) (for $X=I$) into (31) and employing [40, Eq. (2.24.3.1)], the CDF of γ_{e2e} when $\rho=1$ is obtained as

$$F^{co}_{\gamma_{e2e}}(\gamma)\overset{\gamma\gg1}{\simeq}\sum_{l=k}^{N}\sum_{j=0}^{N-l}\sum_{i=0}^{m_R(l+j)}\sum_{t=0}^{\infty}\binom{N}{l}\binom{N-l}{j}\binom{m_R(l+j)}{i}\times\frac{(-1)^j(\alpha_2 p)^{m_R(l+j)+m_I+m_D+t}A(2\pi)^{1-\alpha_2 p}a_I^{m_I}}{\left(\Gamma\left(m_R+1\right)\right)^{(l+j)}\Gamma(m_I)\Gamma(m_D+t+1)}\times\left(\frac{a_R}{\alpha_I}\right)^{m_R(l+j)-i}\left(\frac{a_D}{\alpha_I}\right)^{m_D+t}\left(\frac{\gamma}{\overline{\gamma}}\right)^{m_R(l+j)+m_D+t-i}\times\left(\frac{\left(a_R(l+j)+a_D\right)\gamma}{\alpha_I\overline{\gamma}}+a_I\right)^{i-m_R(l+j)-m_I-m_D-t}\times\Theta(7,3,l+j), \tag{49}$$

where $\kappa_7=\left[\Delta\left(\alpha_2 p:1+i-m_R(l+j)-m_I-m_D-t\right),\kappa_A\right]$.

Therefore, in the high SNR regime, (42) and (49) are given respectively when $0\le\rho_R<1$ and $\rho_R=1$.

The diversity order determines the slope of the OP and BEP versus average SNR curve at asymptotically high SNR in a log-log scale. Therefore, the diversity order is the power of SNR in the dominant term (which means the lowest power of SNR). Based on high SNR definition of Meijer's-G function in (45), the lowest power of SNR for (42) and (49) are respectively given in (50) and (51), as shown at the bottom of the next page.

Hence, the lowest feasible values of power of SNR can be found when $t=0$, $j=0$, $s=0$ and $i=m_R$ for (50) and when $l=k$, $j=0$ and $i=m_R(l+j)$ for (51). Consequently, the diversity order of this relay network is obtained as

$$G_d = \begin{cases} \min\left\{m_R, \frac{\xi_{mod}^2}{r}, \frac{\alpha_1 m_1}{r}, \frac{\alpha_2 m_2}{r}\right\} + m_D & 0 \le \rho_R < 1 \\ \min\left\{k m_R, \frac{\xi_{mod}^2}{r}, \frac{\alpha_1 m_1}{r}, \frac{\alpha_2 m_2}{r}\right\} + m_D & \rho_R = 1 \end{cases}. \tag{52}$$

The value of k in (52) depends on k-th worst relay selection strategy which can be selected from set $k \in \{1, 2, \ldots, N\}$.

For the non-cognitive scenario in Section III, we can obtain the same diversity order expression as in (52). This indicates that the diversity order is a function of RF link severity parameters (i.e. m_R and m_D) and FSO turbulence parameters (i.e. α_1, m_1, α_2 and m_2), pointing error (i.e. ξ_{mod}) and detection method in the destination (i.e. r). Assuming that the effect of the relay-to-destination link becomes dominant in (52), then enhancing the source-to-relay link does not lead to improving the overall link performance and vice versa, however enhancing the quality of direct source-to-destination link leads to enhancing the overall system performance.

VI. NUMERICAL RESULTS

In this section, we compare the analytical expressions against Monte-Carlo simulations. We have assumed that the channel mean powers of $S \to R_n$, $S \to P$ and $S \to D$ are $\Omega_R = 16$, $\Omega_I = 4$ and $\Omega_D = 1$, respectively. The attenuation coefficient of the FSO link is equal to $\sigma = 0.43$ dB/Km which is considered for clear air conditions [47]. We considered the Euclidean distance between R_k and D is 0.5 Km. For the FSO link, the following parameters respectively for strong, moderate and weak turbulence conditions are assumed:

- $m_1 = 0.5$, $m_2 = 1.8$, $\Omega_1 = 1.5074$, $\Omega_2 = 0.928$, $\alpha_1 = 1.8621$, $\alpha_2 = 1$,
- $m_1 = 0.55$, $m_2 = 2.35$, $\Omega_1 = 1.5793$, $\Omega_2 = 0.9671$, $\alpha_1 = 2.169$, $\alpha_2 = 1$,
- $m_1 = 4.02$, $m_2 = 4.52$, $\Omega_1 = 1.0676$, $\Omega_2 = 1.06$, $\alpha_1 = 2.1$, $\alpha_2 = 2$.

Furthermore, the effect of pointing error parameters (the jitter and boresight error) on the performance are investigated. We consider normalized beam width value $w_z/a_r = \{10\}$. Otherwise stated, we assume $(\mu_x/a_r, \mu_y/a_r) = \{(1, 1)\}$ and $(\sigma_x/a_r, \sigma_y/a_r) = \{(3, 3)\}$ for non-zero boresight ($\xi_{mod} = 1.6$) while $(\mu_x/a_r, \mu_y/a_r) = \{(0, 0)\}$ and $(\sigma_x/a_r, \sigma_y/a_r) = \{(1, 1)\}$ are considered for zero boresight ($\xi_{mod} = 5.03$). Unless otherwise stated, we have $m_I = 3$, $P_I = 0.1$ and $\rho_I = 0.707$ and hence $\alpha_I = 0.53$.

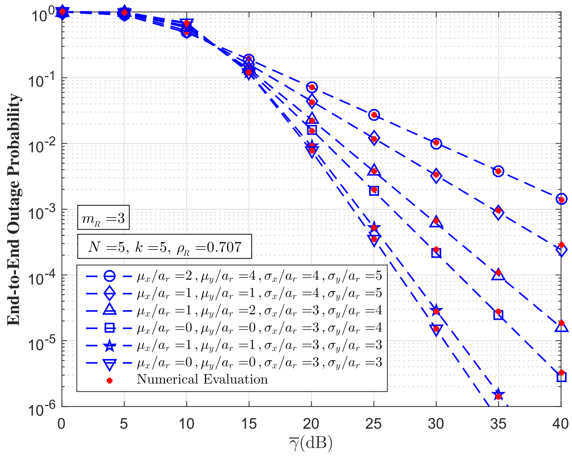

FIGURE 2. Outage probability versus ($\overline{\gamma}$) under weak turbulence conditions under zero and non-zero boresight displacement with IM/DD detection technique.

Figure 2 illustrate the analytical expression for the OP of fixed gain relaying in 18 for non-cognitive scenario without direct link, versus the global average SNR ($\overline{\gamma} = \overline{\gamma}_S = d_1 \mu_r$). The fading severity of the RF links are assumed $m_R = 3$. Weak turbulence conditions and variety conditions of pointing error are supposed. In this Figure, we examine the impact of boresight and jitter parameters of pointing error on the OP performance. We can notice that increasing jitter and boresight parameters leads to performance degradation. The good agreement between the derived expression in 18 and the simulation results in different pointing error conditions, shows the accuracy of modified Rayleigh approximation and our analytical expression.

In Fig. 3 the analytic expression of OP in 18 for non-cognitive scenario with direct link, versus the average electrical SNR of the FSO link are illustrated for various turbulence conditions with considering non-zero boresight pointing error. The average SNR at S is $\overline{\gamma}_S = 20$dB and the

$$\begin{aligned} \alpha_2 p \kappa_6 + m_R + m_D + t + j + s - i = \big[& i + m_R + m_D + t + j + s - i, \ldots, \\ & \xi_{mod}^2/r + m_R + m_D + t + j + s - i, \ldots, \\ & \alpha_1 m_1/r + m_R + m_D + t + j + s - i, \ldots, \\ & \alpha_2 m_2/r + m_R + m_D + t + j + s - i, \ldots\big]. \end{aligned} \tag{50}$$

$$\begin{aligned} \alpha_2 p \kappa_7 + m_R(l + j) - i = \big[& i + m_R(l + j) - i, \ldots, \\ & \xi_{mod}^2/r + m_R(l + j) - i, \ldots, \\ & \alpha_1 m_1/r + m_R l + j - i, \ldots, \\ & \alpha_2 m_2/r + m_R(l + j) - i, \ldots\big]. \end{aligned} \tag{51}$$

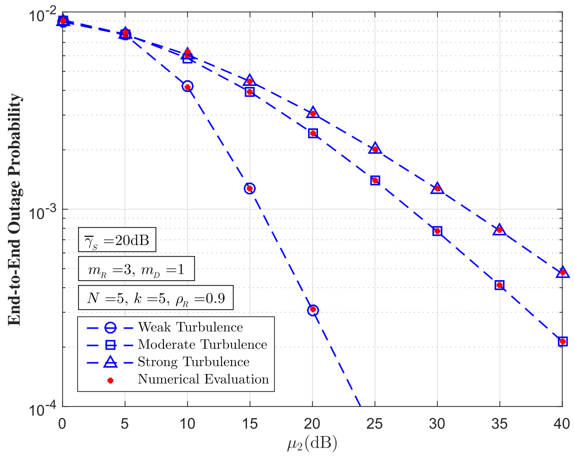

FIGURE 3. Outage probability versus (μ_2) under various turbulence conditions with IM/DD detection technique and non-zero boresight displacement.

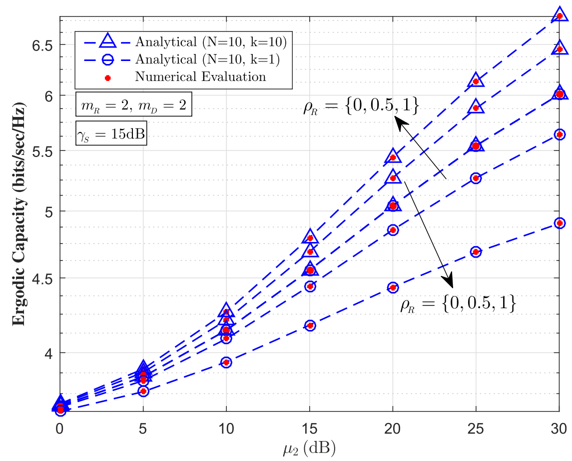

FIGURE 5. Ergodic Capacity versus (μ_2) under strong turbulence conditions for fixed gain relaying with IM/DD technique and non-zero boresight displacement.

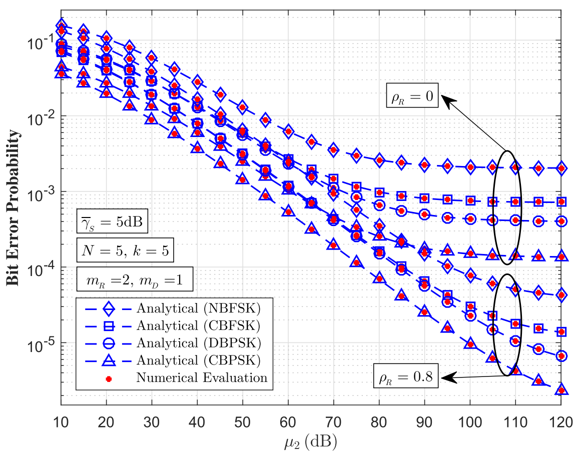

FIGURE 4. Bit error probability of direct detection technique under strong turbulence conditions for fixed gain relaying with non-zero boresight displacement.

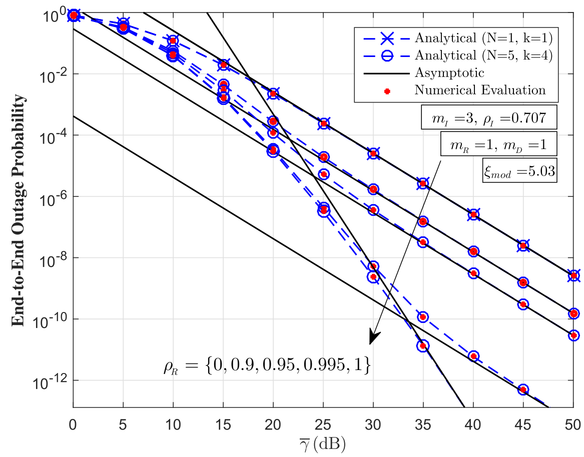

FIGURE 6. Outage probability versus ($\overline{\gamma}$) under weak turbulence conditions for fixed gain relaying with IM/DD technique and zero boresight displacement.

fading severity of the RF links are considered $m_R = 2$ and $m_D = 1$. We can notice that high turbulence effect degrades the OP of the system.

In Fig. 4, we present the analytical expression of BEP of binary modulations in (22) versus the average electrical SNR of the FSO link. The fading severity of the RF links are assumed $m_R = 2$ and $m_D = 1$ and the SNR at S is $\gamma_S = 5$dB. We assume that $N = 5$ relays are available and the best relay selection strategy is applied (i.e. $k = 5$). Furthermore, the two correlation factors $\rho_R = \{0, 0.8\}$ are assumed. Also, strong turbulence conditions and non-zero boresight are supposed for the FSO link and $\xi_{mod} = 1.6$. It is observed that by increasing the average electrical SNR of the FSO link, the BEP decreases; however, as the SNR increases an error floor takes place. It comes from the fact that if SNR of one link grows with no bound, the other link's SNR becomes dominant. For the reason of superiority of PSK versus FSK from spectral efficiency point of view, the BEP of PSK outperforms the BEP of FSK. As expected, increasing the value of correlation factor leads to significant enhancement of the BEP.

In Fig. 5, we demonstrate analytical expression of EC in (24) versus the average electrical SNR of the FSO link over strong turbulence conditions assuming IM/DD technique and non-zero boresight pointing errors. A fixed average SNR $\overline{\gamma}_S = 15$dB is assumed at S with $m_R = 2$ and $m_D = 2$. In order to make visual distinction between curves, we increase the number of available relays to 10. We consider two scenarios of best relay selection (i.e. $k = 10$) and worst relay selection (i.e. $k = 1$) for various values of ρ_R. As can be seen, the best relay selection strategy always outperforms the worst relay selection strategy. As observed, when the best relay is selected, increasing ρ_R leads to increasing EC and vice versa. However, when the worst relay is selected, increasing ρ_R leads to decreasing EC.

Figure 6 illustrates the analytical expression for the OP of fixed gain relaying in (36) for outdated CSI case with direct link, against the global average SNR (i.e. $\overline{\gamma}$). We assume

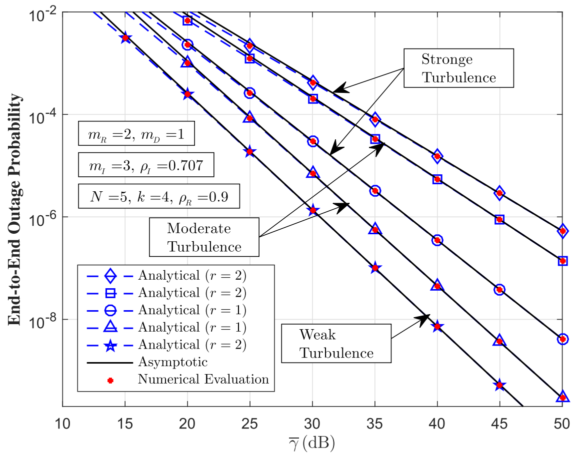

FIGURE 7. Outage probability versus ($\bar{\gamma}$) for fixed gain relaying under various turbulence conditions and non-zero boresight pointing errors for IM/DD and heterodyne detection techniques.

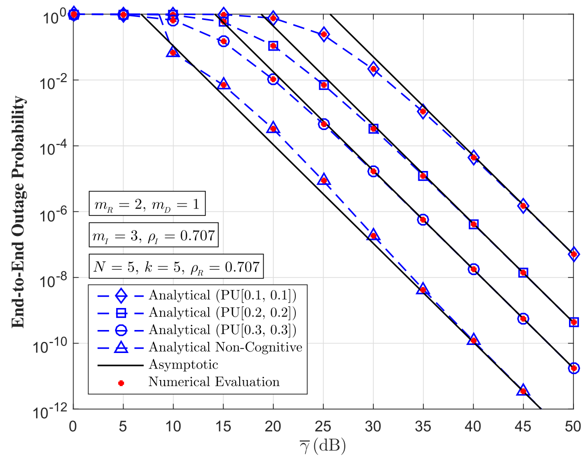

FIGURE 8. Outage probability versus ($\bar{\gamma}$) under weak turbulence conditions for fixed gain relaying with IM/DD technique and zero boresight displacement.

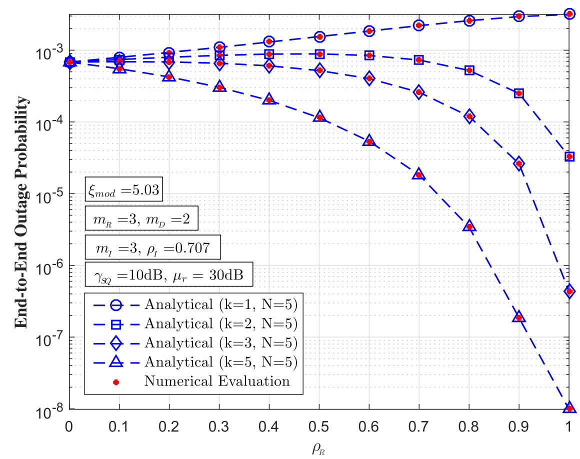

FIGURE 9. Outage probability versus (ρ_R) under weak turbulence conditions for fixed gain relaying with IM/DD technique and zero boresight displacement.

weak turbulence conditions and IM/DD technique with zero boresight ($\xi_{mod} = 5.03$), $m_R = 1$ and $m_D = 1$. As observed, by increasing ρ_R, the OP curves move toward the OP curve of perfect CSI case, however the diversity of multi-relay case is never obtained and increasing ρ_R just bring us the benefit of coding gain, except for $\rho_R = 1$ where the full diversity is achieved. The diversity orders for the different value of ρ_R from lowest toward highest (i.e. 0, 0.9, 0.95, 0.995), are equal to 2 and the diversity order when $\rho_R = 1$ is equal to 5 which also confirm our derived expression in (52). It should be mentioned that for the reason of weak turbulence conditions, pointing errors and IM/DD technique, the effect of RF link becomes dominant.

Figure 7 shows the analytical expression for the OP of fixed gain relaying in (36) for outdated CSI case with direct link, against the global average SNR (i.e. $\bar{\gamma} = \bar{\gamma}_{SQ} = d_1\mu_r$ with $d_1 = 1$). Weak, moderate and strong turbulence conditions and non-zero boresight pointing error are supposed. The fading severity of the source-relay and source-destination links are respectively set to $m_R = 2$ and $m_D = 1$ with $\rho_R = 0.9$. It is observed that the simulation results are in excellent agreement with the derived expression in (36) indicating its accuracy. There is no error floor as expected from analytical derivation. The diversity orders from lowest toward highest outage curves are equal to 1.466, 1.596, 1.93, 2.19 and 2.27 which are in coincidence with the diversity order expression in (52).

Figure 8 demonstrates the analytical expression for the OP of the cognitive scenario given in (36) and OP of the non-cognitive scenario given in (18) with outdated CSI as a function of $\bar{\gamma}$. Different location of PU is considered of cognitive scenario. We suppose a two-dimensional plane ($[x, y]$) for the location of S and P where the channel mean power of each link is proportional to the inverse of fourth power of their distance. The location of S is considered $[0, 0]$ and three different locations $[0.1, 0.1]$, $[0.2, 0.2]$ and $[0.3, 0.3]$ are considered for P (A unit in this plane is a Kilometer). Weak turbulence conditions and IM/DD technique with zero boresight are considered for the FSO link. We consider $N = 5$ available relays and best relay selection (i.e. $k = 5$). Moreover, the fading severity of h_k and h_0 are considered $m_R = 2$ and $m_D = 1$, respectively and $\rho_R = 0.707$. Again, at high SNRs, the effect of RF link becomes dominant. As mentioned in section V, regardless of being cognitive or non-cognitive, the diversity order is given in (52). For the reason of outdated CSI, the diversity order is $G_d = 3$. As expected from our analysis, furthering the PU from source reduces the OP and its curve moves toward the non-cognitive curve.

Figure 9 illustrates the OP versus correlation factor ρ_R. We consider weak turbulence conditions and IM/DD technique with zero boresight displacement. The fading severity of the RF links are assumed $m_R = 3$ and $m_D = 2$. The average electrical SNR of the FSO link and average SNR of the RF link are supposed $\bar{\gamma}_{SQ} = 10$dB and $\mu_r = 30$dB, respectively. As expected, by increasing the value of k, the OP decreases. It can be observed that, while we intentionally select the worst existed relay (i.e. $k = 1$) based on perfect

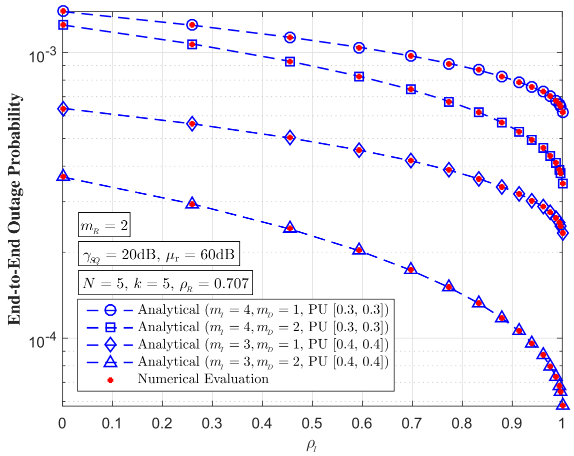

FIGURE 10. Outage probability versus (ρ_I) under strong turbulence conditions for fixed gain relaying with IM/DD technique and non-zero boresight displacement.

knowledge of CSI (i.e. $\rho_R = 1$), we definitely select the worst existed relay and as a result we have the worst OP performance. However, with decreasing correlation factor ρ_R in this scenario, the OP performance improves since selecting the worst existed relay is not granted.

Finally, we present the OP of mixed RF-FSO system with respect to ρ_I in Fig. 10. The SNRs of the RF and the FSO links are set to $\overline{\gamma}_{SQ} = 20$dB and $\mu_r = 60$dB, respectively and $\rho_R = 0.707$. We assume strong turbulence conditions and IM/DD technique with non-zero boresight displacement. We calculate corresponding value of α_I based on different values of ρ_I for the fix interference probability $P_I = 0.1$ and using numerical methods. For different values of ρ_I from 0 to 1, for $m_I = 4$, the value of α_I is varied from 0.38 to 1 and for $m_I = 3$, the value of α_I is varied from 0.33 to 1.

An interesting point is, when we do not have any information of interference link CSI (i.e. $\rho_I = 0$), there is no need to decreases transmission power to zero. With setting the power of source to 0.38 Q and 0.32 Q respectively for $m_I = 4$ and $m_I = 3$, the interference probability constraint (i.e. $P_I = 0.1$) is guaranteed. As expected, by increasing ρ_I, the value of α_I increases and OP performance improves.

As can be seen, there is a non-linear relationship between OP and ρ_I. When $\rho_I > 0.8$, the OP curve's slope with respect to ρ_I is non-linear and more than $\rho_I < 0.8$, which means that additional overhead data which leads to increasing the value of ρ_I, significantly improves the outage performance.

VII. CONCLUSION

In this paper, we have investigated the performance of cooperative asymmetric dual-hop RF-FSO relay network with direct link and selection combining for cognitive and non-cognitive scenarios assuming the availability the imperfect CSI for partial relay selection and underlay power restriction and non-zero boresight pointing error for the FSO links. The closed-form expressions for the outage probability (OP), average bit-error rate and ergodic capacity of the non-cognitive scenario and OP of the cognitive scenario were derived which complement and generalized several previous results in the literature. The asymptotic expressions of OP and diversity order of this network were obtained for both perfect CSI and outdated CSI cases. It has been shown that, the diversity gains are available and their exact values depend on RF links and FSO turbulence parameters, pointing error and detection method in the destination, regardless of interference channel parameter of PU. We prove when CSI of the RF link is outdated, increasing the number of available relays in relay selection does not increases diversity. Despite the existence of correlation between dual-hop mixed RF-FSO system and direct link, the full diversity were archived. More importantly, it has been shown increasing number of relays does not necessarily enhance the performance and it has been limited by the atmospheric turbulence conditions and pointing error of the FSO link.

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