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Stability Analysis of Markovian Jump System With Multi-Time-Varying Disturbances Based on Improved Interactive Convex Inequality and Positive Definite Condition

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ABSTRACT Time-varying disturbances not only exist in the external delay of Markovian jump system but also may exist in the transfer rate of internal probability transfer. This paper put forward an improved interactive convex inequality method for Markovian jump system (MJS) with multi-time-varying disturbances: time-varying delays and time-varying transition rates of probability transfer. On this basis, an improved Lyapunov function is quoted to more approximate the actual system, and the optimization of positive definite conditions of the matrix in Lyapunov functional is discussed, the specific positive definite condition is improved. At the end of this paper, the effectiveness of the above methods is verified by several simulations.

INDEX TERMS Markovian jump parameters, time-varying delay, time-varying transition rates, improved interactive convex inequality.

I. INTRODUCTION

In the actual system, some factors such as the change of working environment, the failure of parts and components will lead to the sudden change of system structure [1]. These mutations not only reduce the performance of the system, but also lead to instability of the system. Markovian jump system (MJS) is attracting attention because it can effectively describe such structural mutations. MJS was first proposed by Krasovskii *et al.* in the 1960s. It is a hybrid system with continuous time state variables and discrete time modal variables. In MJS, discrete modal variables are Markov processes with discrete modes in continuous time, whose modal values are derived from a finite set. As a special hybrid system, MJS has been the focus of research in the past decades [2]–[4]. For example, in economic systems [5], networked control systems [6], industrial systems [7], fault detection systems [8], etc., and has been widely used in transportation systems, manufacturing systems and power systems. Because of its many advantages and wide application background, MJS is an important factor to become the basis of this study.

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However, due to various practical complex factors, all information of probability transfer in MJS may be difficult to obtain or expensive to obtain. The probability transfer is not completely known or transfer rates may change over time. For example, in networked control system, it will cost a lot to obtain the total transfer probability due to packet loss and delay. At present, some scholars have been engaged in the related research of partial unknown probability transfer and time-varying transfer rates, and have achieved some research results [9]–[13]. On the other hand, time delays occur in most engineering fields, which will not only destroy the actual performance of the system, but also make the system unstable, and even lead to the collapse of the whole system. In summary, time-varying disturbance factors not only exist outside the MJS, but also may exist in the internal probability transfer rates, which seriously affect the stability of the MJS. Therefore, Multi-time-varying disturbance factors are the first problem studied in this paper.

In recent years, inequality research has become a hot topic in the stability analysis of systems, such as Jensen inequality [14], Wirtinger inequality [15], inequality based on free matrix [16], improved Wirtinger inequality [17], interactive convex combination method [18], etc. Among them, as mentioned in [18], the interactive convex inequality can produce a stability criterion with fewer decision variables on the basis that the conservatism will not increase, so this inequality method has been widely used. In [19], the interactive convex inequality is optimized by introducing four relaxation matrix variables. Although this method reduces the conservatism of the original interactive convex inequality, the introduction of four relaxation matrix variables will increase the computational complexity. Therefore, this paper focuses on the improved interactive convex inequality, which not only derives the less conservative stability conditions, but also reduces the number of relaxation matrix variables to reduce the computational complexity. This is the second problem studied in this paper.

In addition, Lyapunov's second law, i.e. the Lyapunov function $V(t)$ satisfies conditions $V(t) > 0$ and derivative $\mathcal{L}V(t) < 0$, are usually used to analyze the stability of MJS. In order to satisfy condition $V(t) > 0$, the traditional method usually constructs the following Lyapunov function:

$$V(t) = x^T(t)Px(t) + \int_{t-d(t)}^{t} x^T(s)Qx(s)ds + \cdots$$

In order to calculate conveniently, P, Q are positive definite symmetry by default. This method is conservative in stability analysis. How to improve the positive definite condition of Lyapunov function is the third problem studied in this paper.

In summary, based on the traditional standard MJS, this paper first considers the Multi-time-varying disturbance factors: the time-varying delay of the external transmission of the system and the time-varying probability transfer of the internal conversion of the system. Secondly, the improved interactive convex inequality is used to analyze the stability of the system. On this basis, we also discuss the optimization of positive definite conditions of the matrix in Lyapunov functional. By using the relationship within the functional matrix, we improve the specific positive definite condition.

Next, this paper will be divided into three parts. The first part introduces the mathematical modeling of MJS, probability transfer function and related lemmas. The second part introduces the important results of this paper, which are divided into the MJS Multi-time-varying stability conditions and the improvement of the positive definiteness of Lyapunov functional. The third part proves the feasibility and validity of the results of Theorem 1 and Corollary 1 through several comparative examples. 0 indicates the zero matrix. $\mathbf{R}^n$ shows the n-dimensional Euclidean space. "T" represents the matrix transposition. $\{\Omega, \digamma, \mathcal{P}\}$ represents the probability space. Ω, $\digamma$ and $\mathcal{P}$ respectively represent the sample space, σ -algebra of subsets of the sample space and probability measure on $\digamma$. $*$ represents the transposition term of diagonal symmetry.

II. SYSTEM DESCRIPTION AND PROBLEM ANALYSIS

Consider the following MJS with time-varying delay:

$$\begin{cases} \dot{x}(t) = A(r(t))x(t) + A_d(r(t))x(t-d(t)) \\ x(t) = \Phi(t), \quad t \in [-d, 0] \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $A(\cdot)$ and $A_d(\cdot)$ are arbitrary real constant matrices with suitable dimensions, $r(t)$ expressed in value on the limited state space $S = \{1, \cdots, M\}$ and $\{\Omega, \digamma, \mathcal{P}\}$ defines even steady Markov chain. The condition transfer rate matrix $\Pi(t) = (\mu_{ij})_{N\times N}$ indicated with:

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \mu_{ij}\Delta + o(\Delta); & j \neq i, \\ 1 + \mu_{ii}\Delta + o(\Delta); & j = i \end{cases}$$

where $\mu_{ij} \geq 0$, if $j \neq i$, $\mu_{ii} = -\sum_{j=1, j\neq i}^{N} \mu_{ij}$. In MJS (1), the time delay $d(t)$ satisfies : $0 \leq d(t) \leq d$ and $d_m \leq \dot{d}(t) \leq d_M$. $\Phi(t)$ is an initial condition.

Lemma 1 [20]: Let $\mathcal{R}_1, \mathcal{R}_2 \in \mathbf{R}^m$ be real symmetric positive definite matrices and $\varpi_1, \varpi_2 \in \mathbf{R}^m$ and a scalar $\alpha \in (0, 1)$. For any $Y_1, Y_2 \in \mathbf{R}^m$, the following inequality is willfully tenable

$$\begin{aligned} \mathbb{F}(\alpha) &= \frac{1}{\alpha}\varpi_1^T\mathcal{R}_1\varpi_1 + \frac{1}{1-\alpha}\varpi_2^T\mathcal{R}_2\varpi_2 \\ \mathbb{F}(\alpha) &\geq \varpi_1^T[\mathcal{R}_1 + (1-\alpha)(\mathcal{R}_1 - Y_1\mathcal{R}_2^{-1}Y_1^T)]\varpi_1 \\ &\quad + \varpi_2^T[\mathcal{R}_2 + \alpha(\mathcal{R}_2 - Y_2^T\mathcal{R}_1^{-1}Y_2)]\varpi_2 \\ &\quad + 2\varpi_1^T[\alpha Y_1 + (1-\alpha)Y_2]\varpi_2 \end{aligned}$$

Remark 1: In [18], $\mathbb{F}(\alpha) \geq \varpi_1^T\mathcal{R}_1\varpi_1 + \varpi_2^T\mathcal{R}_2\varpi_2 + 2\varpi_1^TS\varpi_1$. In [19], the degree of freedom of $\mathcal{R}_1$, $\mathcal{R}_2$ and S are improved by introducing four relaxation matrix variables X_1, X_2, Y_1, Y_2 and $\begin{bmatrix} \mathcal{R}_1 - X_1 & Y_1 \\ * & \mathcal{R}_2 \end{bmatrix} \geq 0$ $\begin{bmatrix} \mathcal{R}_1 & Y_2 \\ * & \mathcal{R}_2 - X_2 \end{bmatrix} \geq 0$, but introducing additional relaxation matrix variables will increase the computational complexity. Lemma 1 eliminates a part of relaxation matrix variables X_1 and X_2 on the basis of [19], which not only reduces the conservativeness, but also reduces the computational complexity. It is easy to show that Lemma 1 is greater than [18] and [19].

Lemma 2 [13]: When the conversion rate matrix with $M-\Pi(t)$ is located at the vertices of a bounded domain $\mathcal{D}$, domain $\mathcal{D}_1$ is made up of the following expression:

$$\mathcal{D}_1 = \left\{ \begin{array}{l} \Pi(r(t))|\Pi(r(t)) = \sum_{l=1}^{M} r_l(t)\Pi^{(l)} \\ \sum_{l=1}^{M} r_l(t) = 1, \quad r_l(t) \geq 0 \end{array} \right\}, \tag{2}$$

where $\Pi^{(l)}(l = 1, 2, \cdots, M)$ expresses polyhedron apex, $r(t)$ is the parameter vector, its speed change hypothesis for has also known. Therefore $\dot{r}(t)$ belongs to

$$\mathcal{D}_2 = \left\{ -v_l \leq \dot{r}_l(t) \leq v_l, v_l \geq 0, l = 1, 2, \cdots, M-1 \right\} \tag{3}$$

Remark 2: We can get $\sum_{l=1}^{M} r_l(t) = 1$ is equivalent to $\sum_{l=1}^{M-1} \dot{r}_l(t) + \dot{r}_M(t) = 0$. So $\dot{r}_M(t)$ is expressed by $|\dot{r}_M(t)| \leq \sum_{l=1}^{M-1} v_l$.

Lemma 3 [21]: Regarding any $n \times n$ constant matrix $R > 0$ and the vector quantity function $\omega : [r_1, r_2] \to \mathbf{R}^n$, $r_2 > r_1$, causes following integral inequality definition to be correct

$$\begin{aligned} &\int_{r_1}^{r_2} \dot{\omega}^T(s)R\dot{\omega}(s)ds \\ &\quad \geq \frac{1}{r_2 - r_1}[(\omega(r_2) - \omega(r_1))^T R(\omega(r_2) - \omega(r_1)) \\ &\quad + 3\varrho_1^T R\varrho_1 + 5\varrho_2^T R\varrho_2] \end{aligned}$$

where

$$\begin{cases} \varrho_1 = \omega(r_2) + \omega(r_1) - \dfrac{2}{r_2 - r_1}\displaystyle\int_{r_1}^{r_2} \omega(s)ds \\ \varrho_2 = \omega(r_2) - \omega(r_1) - \dfrac{6}{r_2 - r_1}\displaystyle\int_{r_1}^{r_2} \omega(s)ds \\ \qquad + \dfrac{12}{(r_2 - r_1)^2}\displaystyle\int_{r_1}^{r_2} (r_2 - s)\omega(s)ds \end{cases}$$

Lemma 4 [22]: Regarding quadratic function $\mathbb{L}(s) = a_2 s^2 + a_1 s + a_0$, in which $a_i \in \mathbf{R}(i = 0, 1, 2)$, if the inequalities are tenable $\mathbb{L}(0) < 0$, $\mathbb{L}(h) < 0$, $-h^2 a_2 + \mathbb{L}(0) < 0$, for $\forall s \in [0, h]$, we may obtain

$$\mathbb{L}(s) < 0$$

Remark 3: Combined with the above lemmas, the following conditions should be satisfied for the stability analysis of MJS (1) with Multi-time-varying disturbances: (a)Construct suitable Lyapunov functional $V(t)$. (b)$V(t) > 0$. (c)Derivative $\mathcal{L}V(t) < 0$. Next, Theorem 1 gives the sufficient condition for the feasibility of stability analysis, and Corollary 1 improves the feasibility condition (b). Finally, the feasibility and validity of methods studied in this paper are verified by comparing the upper bounds of time delay.

III. MAIN RESULTS

Theorem 1: Given scalars $d > 0$ and $\dot{d}(t)$, $d_\triangle$ denotes any constant in the interval $\dot{d}(t) \in [d_m, d_M]$, for any delay $d(t)$, the MJS (1) is stochastically stable, if there exist $\mathbf{R}^n$ symmetry matrices $P_i^{(l)} > 0$, $Q_1 > 0$, $Q_2 > 0$, $R > 0$, $Z_1 > 0$, $Z_2 > 0$, matrices Y_1 and Y_2 of suitable dimension, for all $i = 1, \cdots, N$, such that the following LMIs hold:

$$\mathcal{J}_i^{(ls)} + \mathcal{J}_i^{(sl)} < 0 \tag{4}$$

$$\mathcal{M}_i^{(ls)} + \mathcal{M}_i^{(sl)} < 0 \tag{5}$$

$$\mathcal{N}_i^{(ls)} + \mathcal{N}_i^{(sl)} < 0, \quad 1 \le l \le s \le M \tag{6}$$

where

$$\mathcal{J}_i^{(ls)} = \begin{bmatrix} \Omega_{11}^{ls}(0, d_\triangle) & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ * & -R & 0 & 0 \\ * & * & -3R & 0 \\ * & * & * & -5R \end{bmatrix}$$

$$\mathcal{M}_i^{(ls)} = \begin{bmatrix} \Omega_{11}^{ls}(d, d_\triangle) & \nabla_{12} & \nabla_{13} & \nabla_{14} \\ * & -R & 0 & 0 \\ * & * & -3R & 0 \\ * & * & * & -5R \end{bmatrix}$$

$$\mathcal{N}_i^{(ls)} = \begin{bmatrix} \Omega_{11}^{ls}(0, d_\triangle) - d^2\eth_1(d_\triangle) & \Delta_{12} & \Delta_{13} & \Delta_{14} \\ * & -R & 0 & 0 \\ * & * & -3R & 0 \\ * & * & * & -5R \end{bmatrix}$$

$$\Delta_{12} = (Y_{211}\sigma_1)^T + (Y_{212}\sigma_2)^T + (Y_{213}\sigma_3)^T \tag{7}$$

$$\Delta_{13} = (Y_{221}\sigma_1)^T + (Y_{222}\sigma_2)^T + (Y_{223}\sigma_3)^T \tag{8}$$

$$\Delta_{14} = (Y_{231}\sigma_1)^T + (Y_{232}\sigma_2)^T + (Y_{233}\sigma_3)^T \tag{9}$$

$$\nabla_{12} = \varsigma_1^T Y_{111} + \varsigma_2^T Y_{121} + \varsigma_3^T Y_{131} \tag{10}$$

$$\nabla_{13} = \varsigma_1^T Y_{112} + \varsigma_2^T Y_{122} + \varsigma_3^T Y_{132} \tag{11}$$

$$\nabla_{14} = \varsigma_1^T Y_{113} + \varsigma_2^T Y_{123} + \varsigma_3^T Y_{133} \tag{12}$$

$$\sigma_1 = e_1 - e_2, \sigma_2 = e_1 + e_2 - 2e_6,$$
$$\sigma_3 = e_1 - e_2 - 6e_6 + 12e_7$$
$$\varsigma_1 = e_2 - e_3, \varsigma_2 = e_2 + e_3 - 2e_4,$$
$$\varsigma_3 = e_2 - e_3 - 6e_4 + 12e_5$$

$$Y_1 = \begin{bmatrix} Y_{111} & Y_{112} & Y_{113} \\ Y_{121} & Y_{122} & Y_{123} \\ Y_{131} & Y_{132} & Y_{133} \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} Y_{211} & Y_{212} & Y_{213} \\ Y_{221} & Y_{222} & Y_{223} \\ Y_{231} & Y_{232} & Y_{233} \end{bmatrix}$$

and

$$\begin{aligned} &\Omega_{11}^{ls}(d(t), \dot{d}(t)) \\ &\quad = \Omega_{11}(d(t), \dot{d}(t)) + \Upsilon^{(ls)} \\ &\Upsilon^{(ls)} \\ &\quad = (A_i e_1 + A_{di} e_2)^T P_i^{(l)} e_1 + e_1^T P_i^{(l)} (A_i e_1 \\ &\qquad + A_{di} e_2) + e_1^T \{\sum_{j=1}^{N} \mu_{ij}^{(l)} P_j^{(s)} + \sum_{n=1}^{M-1} \pm (P_i^{(n)} - P_i^{(M)})\} e_1 \end{aligned}$$

$$\begin{aligned} &\Omega_{11}(d(t), \dot{d}(t)) \\ &\quad = (A_i e_1 + A_{di} e_2)^T Q_1 (A_i e_1 + A_{di} e_2) \\ &\qquad + 2e_1^T Q_1 e_1 + e_2^T Q_1 e_2 + e_3^T Q_1 e_3 + d^2(t) e_6^T Z_1 e_6 \\ &\qquad + d^2 (A_i e_1 + A_{di} e_2)^T R (A_i e_1 + A_{di} e_2) - e_9^T Q_2 e_9 \\ &\qquad - 2e_3^T Q_2 e_3 - e_1^T Q_2 e_1 - e_2^T Q_2 e_2 \\ &\qquad - (1 - \dot{d}(t))[e_8^T Q_1 e_8 + 2e_2^T Q_1 e_2 + e_1^T Q_1 e_1 + e_3^T Q_1 e_3] \\ &\qquad - (2 - \alpha)[\varsigma_1^T R \varsigma_1 + 3\varsigma_2^T R \varsigma_2 + 5\varsigma_3^T R \varsigma_3] \\ &\qquad + (1 - \dot{d}(t))[e_8^T Q_2 e_8 + 2e_2^T Q_2 e_2 + e_1^T Q_2 e_1 + e_3^T Q_2 e_3 \\ &\qquad + (d - d(t))^2 e_4^T Z_2 e_4] + Sym\{d(t)(A_i e_1 + A_{di} e_2)^T Q_1 e_1 \\ &\qquad + d(t)(1 - \dot{d}(t)) e_8^T Q_1 e_2 + d(t) e_9^T Q_1 e_3 \\ &\qquad + d^2(t)(\dot{d}(t) - 1) e_2^T Z_1 e_7\} + Sym\{(d - d(t)) \\ &\qquad \times (A_i e_1 + A_{di} e_2)^T Q_2 e_1 + (d - d(t))(1 - \dot{d}(t)) e_8^T Q_2 e_2 \\ &\qquad + (d - d(t)) e_9^T Q_2 e_3 - (d - d(t))^2 e_3^T Z_2 e_5\} \\ &\qquad - (1 + \alpha)[\sigma_1^T R \sigma_1 + 3\sigma_2^T R \sigma_2) + 5\sigma_3^T R \sigma_3] \\ &\qquad - Sym\{\varsigma_1^T \alpha Y_1 \sigma_1 + \varsigma_2^T \alpha Y_1 \sigma_2 + \varsigma_3^T \alpha Y_1 \sigma_3 \\ &\qquad + \varsigma_1^T (1 - \alpha) Y_2 \sigma_1 + \varsigma_2^T (1 - \alpha) Y_2 \sigma_2 + \varsigma_3^T (1 - \alpha) Y_2 \sigma_3\} \end{aligned} \tag{13}$$

$$\begin{aligned} &\eth_1(\dot{d}(t)) \\ &\quad = e_6^T Z_1 e_6 + (1 - \dot{d}(t)) e_4^T Z_2 e_4 \\ &\qquad + Sym\{(\dot{d}(t) - 1) e_2^T Z_1 e_7 - e_3^T Z_2 e_5\} \end{aligned} \tag{14}$$

with $e_i (i = 1, 2, \cdots, 9)$ being the ith $n \times 9n$ block-row vectors of the $9n \times 9n$ identity matrix.

Proof: We introduce an improved Lyapunov-Krasovskii functional: $V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t)$

where

$$V_1(t) = x^T(t) P(r(t)) x(t)$$

$$V_2(t) = \int_{t-d(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds$$

$$
\begin{aligned}
&+\int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds\\
&+\int_{t-d(t)}^{t} x^T(t)Q_1x(t)ds\\
&\times\int_{t-d(t)}^{t} x^T(t-d(t))Q_1x(t-d(t))ds\\
&+\int_{t-d(t)}^{t} x^T(t-d)Q_1x(t-d)ds\\
V_3(t) =& \int_{t-d}^{t-d(t)} \dot{x}^T(s)Q_2\dot{x}(s)ds\\
&+\int_{t-d}^{t-d(t)} x^T(s)Q_2x(s)ds\\
&+\int_{t-d}^{t-d(t)} x^T(t)Q_2x(t)ds\\
&+\int_{t-d}^{t-d(t)} x^T(t-d(t))Q_2x(t-d(t))ds\\
&+\int_{t-d}^{t-d(t)} x^T(t-d)Q_2x(t-d)ds\\
V_4(t) =& \int_{t-d(t)}^{t}(\int_{t-d(t)}^{s} x^T(\theta)d\theta)Z_1(\int_{t-d(t)}^{s} x(\theta)d\theta)ds\\
&+\int_{t-d}^{t-d(t)}(\int_{t-d}^{s} x^T(\theta)d\theta)Z_2(\int_{t-d}^{s} x(\theta)d\theta)ds\\
V_5(t) =& d\int_{t-d}^{t}\int_{\theta}^{t} \dot{x}^T(s)R\dot{x}(s)dsd\theta
\end{aligned}
$$

By deriving the above functions, we can get

$$
\begin{aligned}
\mathcal{L}V_1(t) =& 2\dot{x}(t)P_i(r(t))x(t)\\
&+x^T(t)\sum_{j=1}^{N}\mu_{ij}(r(t))P_j(r(t))x(t)\\
&+x^T(t)(\frac{dP_i(r(t))}{dt})x(t)\\
\mathcal{L}V_2(t) =& \dot{x}^T(t)Q_1\dot{x}(t)-\dot{x}^T(t-d(t))Q_1\dot{x}(t-d(t))\\
&+x^T(t)Q_1x(t)-x^T(t-d(t))Q_1x(t-d(t))\\
&+\int_{t-d(t)}^{t} 2\dot{x}^T(t)Q_1x(t)ds\\
&+\int_{t-d(t)}^{t} 2\dot{x}^T(t-d(t))Q_1x(t-d(t))ds\\
&+\int_{t-d(t)}^{t} 2\dot{x}^T(t-d)Q_1x(t-d)ds\\
\mathcal{L}V_3(t) =& \dot{x}^T(t-d(t))Q_2\dot{x}(t-d(t))\\
&-\dot{x}^T(t-d)Q_2\dot{x}^T(t-d)\\
&+x^T(t-d(t))Q_2x(t-d(t))\\
&-x^T(t-d)Q_2x(t-d)\\
&+\int_{t-d}^{t-d(t)} 2\dot{x}^T(t)Q_2x(t)ds\\
&+\int_{t-d}^{t-d(t)} 2\dot{x}^T(t-d(t))Q_2x(t-d(t))ds\\
&+\int_{t-d}^{t-d(t)} 2\dot{x}^T(t-d)Q_2x(t-d)ds\\
\mathcal{L}V_4(t) =& \int_{t-d(t)}^{t} 2(\int_{t-d(t)}^{s} x^T(\theta)d\theta)'Z_1(\int_{t-d(t)}^{s} x(\theta)d\theta)ds\\
&+\int_{t-d}^{t-d(t)} 2(\int_{t-d}^{s} x^T(\theta)d\theta)'Z_2(\int_{t-d}^{s} x(\theta)d\theta)ds\\
\mathcal{L}V_5(t) =& d^2\dot{x}^T(t)R\dot{x}(t)-d\int_{t-d}^{t}\dot{x}^T(s)R\dot{x}(s)ds
\end{aligned}
$$

Dividing second integral terms in $\mathcal{L}V_5(t)$

$$
\begin{aligned}
d&\int_{t-d}^{t}\dot{x}^T(s)R\dot{x}(s)ds\\
&= d\int_{t-d}^{t-d(t)}\dot{x}^T(s)R\dot{x}(s)ds + d\int_{t-d(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds
\end{aligned}
$$

Using Lemma 3, we have

$$
\begin{aligned}
d&\int_{t-d}^{t-d(t)}\dot{x}^T(s)R\dot{x}(s)ds\\
&\geq \frac{1}{\alpha}\xi^T(t)[\varsigma_1^TR\varsigma_1+3\varsigma_2^TR\varsigma_2+5\varsigma_3^TR\varsigma_3]\xi(t)\\
&\times d\int_{t-d(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds\\
&\geq \frac{1}{1-\alpha}\xi^T(t)[\sigma_1^TR\sigma_1+3\sigma_2^TR\sigma_2+5\sigma_3^TR\sigma_3]\xi(t)
\end{aligned}
$$

Using Lemma 1 and Schur complement, we have $\mathcal{L}V(t) \leq \xi^T(t)\Theta(t)\xi(t)$

where $\Theta(t)$, as shown at the top of the next page.

$\Omega_{11}(d(t),\dot{d}(t))$, ∇_{12}, ∇_{13}, ∇_{14}, Δ_{12}, Δ_{13}, Δ_{14} are shown in (13), (7)-(12) and

$$
\begin{aligned}
\xi&(t)\\
&= col\{x(t), x(t-d(t)), x(t-d), \int_{t-d}^{t-d(t)}\frac{x(s)}{d-d(t)}ds,\\
&\int_{t-d}^{t-d(t)}\frac{(t-d(t)-s)x(s)}{(d-d(t))^2}ds, \int_{t-d(t)}^{t}\frac{x(s)}{d(t)}ds,\\
&\int_{t-d(t)}^{t}\frac{(t-s)x(s)}{d^2(t)}ds, \dot{x}(t-d(t)), \dot{x}(t-d)\}\\
\Upsilon&(r(t))\\
&= Sym\{(A_ie_1+A_{di}e_2)^TP_i(r(t))e_1\}\\
&+e_1^T\{\sum_{j=1}^{N}\mu_{ij}(r(t))P_j(r(t))+\frac{dP_i(r(t))}{dt}\}e_1
\end{aligned}
$$

Note that

$$
\Theta(t) = d^2(t)\eth_1(\dot{d}(t)) + d(t)\eth_2(\dot{d}(t)) + \eth_3(\dot{d}(t))
$$

$\eth_1(\dot{d}(t))$ is defined in (14), and $\eth_2(\dot{d}(t))$, $\eth_3(\dot{d}(t))$ are the appropriate matrices. We define

$$
\mathcal{N}^T\Theta(t)\mathcal{N} = a_2d^2(t) + a_1d(t) + a_0
$$

Applying lemma 4, $\mathcal{N}^T\Theta(t)\mathcal{N} < 0$ for $d(t) \in [0, d]$ and $\dot{d}(t) \in [d_m, d_M]$, if the following hold

$\mathcal{J}(r(t))$

$$\Theta(t)=\begin{bmatrix}\Omega_{11}(d(t),\dot d(t))+\Upsilon(r(t)) & \nabla_{12} & \nabla_{13} & \nabla_{14} & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ * & (1-\alpha)R & 0 & 0 & 0 & 0 & 0\\ * & * & 3(1-\alpha)R & 0 & 0 & 0 & 0\\ * & * & * & 5(1-\alpha)R & 0 & 0 & 0\\ * & * & * & * & \alpha R & 0 & 0\\ * & * & * & * & * & 3\alpha R & 0\\ * & * & * & * & * & * & 5\alpha R\end{bmatrix}$$

$$=\begin{bmatrix}\Omega_{11}(0,d_\Delta)+\Upsilon(r(t)) & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}<0$$

$\mathcal{M}(r(t))$

$$=\begin{bmatrix}\Omega_{11}(d,d_\Delta)+\Upsilon(r(t)) & \nabla_{12} & \nabla_{13} & \nabla_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}<0$$

$\mathcal{N}(r(t))$

$$=\begin{bmatrix}\Omega^{ls}_{11}(0,d_\Delta)-d^2\eth_1(d_\Delta)+\Upsilon(r(t)) & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}<0 \quad (15)$$

Let $\mathcal{N}=\xi(t)$, then $\mathcal{L}V(t)\le\xi^T(t)\Theta(t)\xi(t)<0$ for $d(t)\in[0,d]$ and $\dot d(t)\in[d_m,d_M]$.

Remark 4: For the construction of $V_4(t)$, we refer to the paper [20], which introduces an augmented Lyapunov function. This construction method is very good, which can make the function modeling more close to the real system, from which we have been greatly inspired.

Remark 5: On processing integral item $d\int_{t-d}^{t}\dot x^T(s)R\dot x(s)ds$ in $\mathcal{L}V_5(t)$, it can be seen from the proof that Lemma 3 and improved interactive convex inequality (Lemma 1) play a key role in finding the upper bound of approximation of $\mathcal{L}V(t)$.

Remark 6: The stability analysis of Theorem 1 is incomplete, because there is no specific description of $\Upsilon(r(t))$, and it cannot be converted into specific LMIs, so the obtained conditions cannot be used in Matlab LMI toolbox. To describe the information of $\Upsilon(r(t))$, by referring to Lemma 2, the expression of Lyapunov matrix $P_i(r(t))$ is shown below:

$$\mathcal{D}_1=\left\{\begin{matrix}P_i(r(t))|P_i(r(t))=\sum_{l=1}^{M}r_l(t)P_i^{(l)}\\ \sum_{l=1}^{M}r_l(t)=1,\quad r_l(t)\ge 0\end{matrix}\right\}, \quad (16)$$

where $P_i^{(l)}$ expresses respective polyhedron apex.

Based on the above discussion, the time-varying transition rates in $P_i(r(t))$ is concretely expressed as follows.

Note that

$$P_i(r(t))=\sum_{l=1}^{M}r_l(t)P_i^{(l)}\Longrightarrow$$

$$\frac{dP_i(r(t))}{dt}=\sum_{l=1}^{M}\dot r_l(t)P_i^{(l)}=\sum_{n=1}^{M-1}\dot r_n(t)(P_i^{(n)}-P_i^{(M)})$$

According to (15)

$$\mathcal{J}(r(t))=\sum_{l=1}^{M}r_l^2(t)\bar{\mathcal{J}}_i^{(ll)}+\sum_{l=1}^{M-1}\sum_{s=l+1}^{M}r_l(t)r_s(t)(\bar{\mathcal{J}}_i^{(ls)}+\bar{\mathcal{J}}_i^{(sl)})<0$$

$$\mathcal{M}(r(t))=\sum_{l=1}^{M}r_l^2(t)\bar{\mathcal{M}}_i^{(ll)}+\sum_{l=1}^{M-1}\sum_{s=l+1}^{M}r_l(t)r_s(t)(\bar{\mathcal{M}}_i^{(ls)}+\bar{\mathcal{M}}_i^{(sl)})<0$$

$$\mathcal{N}(r(t))=\sum_{l=1}^{M}r_l^2(t)\bar{\mathcal{N}}_i^{(ll)}+\sum_{l=1}^{M-1}\sum_{s=l+1}^{M}r_l(t)r_s(t)(\bar{\mathcal{N}}_i^{(ls)}+\bar{\mathcal{N}}_i^{(sl)})<0$$

where

$$\bar{\mathcal{J}}_i^{(ls)}=\begin{bmatrix}\bar\Omega^{ls}_{11}(0,d_\Delta) & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}$$

$$\bar{\mathcal{M}}_i^{(ls)}=\begin{bmatrix}\bar\Omega^{ls}_{11}(d,d_\Delta) & \nabla_{12} & \nabla_{13} & \nabla_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}$$

$$\bar{\mathcal{N}}_i^{(ls)}=\begin{bmatrix}\bar\Omega^{ls}_{11}(0,d_\Delta)-d^2\eth_1(d_\Delta) & \Delta_{12} & \Delta_{13} & \Delta_{14}\\ * & -R & 0 & 0\\ * & * & -3R & 0\\ * & * & * & -5R\end{bmatrix}$$

and

$$\begin{aligned}\bar\Omega^{ls}_{11}(d(t),\dot d(t))&=\Omega_{11}(d(t),\dot d(t))+\bar\Upsilon^{(ls)}\\ \bar\Upsilon^{(ls)}&=(A_ie_1+A_{di}e_2)^TP_i^{(l)}e_1\\ &\quad+e_1^TP_i^{(l)}(A_ie_1+A_{di}e_2)\\ &\quad+e_1^T\{\sum_{j=1}^{N}\mu_{ij}^{(l)}P_j^{(s)}+\sum_{n=1}^{M-1}\dot r_n(t)(P_i^{(n)}-P_i^{(M)})\}e_1\end{aligned}$$

As for how to deal with $\sum_{n=1}^{M-1}\dot r_n(t)(P_i^{(n)}-P_i^{(M)})$, the method in [13] has been considered. It can be obtained if LMIs (4), (5) and (6) are established, then $\bar{\mathcal{J}}_i^{(ll)}<0$, $\bar{\mathcal{M}}_i^{(ll)}<0$, $\bar{\mathcal{N}}_i^{(ll)}<0$, $\bar{\mathcal{J}}_i^{(ls)}+\bar{\mathcal{J}}_i^{(sl)}<0$, $\bar{\mathcal{M}}_i^{(ls)}+\bar{\mathcal{M}}_i^{(sl)}<0$ and $\bar{\mathcal{N}}_i^{(ls)}+\bar{\mathcal{N}}_i^{(sl)}<0$ hold. In summary, LMIs (4), (5) and (6)

TABLE 1. Different results to upper bound allowed d for Example 1.

	By[25]	By[24]	By[26]	By[27]	By[23]	By [28]	By Theorem 1
d	0.84	1.23	0.40	0.73	1.33	1.87	2.14

imply that $\mathcal{J}(r(t)) < 0$, $\mathcal{M}(r(t)) < 0$ and $\mathcal{N}(r(t)) < 0$. This completes the proof.

Next, On the basis of Theorem 1, we optimize the positive definite condition and the following conclusions are drawn.

Corollary 1: Given scalars $d > 0$ and $\dot{d}(t)$, for any delay $d(t)$, we can get $V(t)$ is positive definite if there exist $\mathbf{R}^n$ symmetry matrices $P(r(t)), Q_1 > 0, Q_2 > 0, Z_1 > 0, Z_2 > 0$, $R > 0$, such that the following hold:

$$\Psi(r(t)) = P(r(t)) + d(t)Q_1 > 0$$

Proof:

$$\begin{aligned} V_1(t) &= x^T(t)P(r(t))x(t) \\ &= \frac{1}{d(t)} \int_{t-d(t)}^{t} x^T(t)P(r(t))x(t)ds \\ V_2(t) &= \int_{t-d(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \\ &\quad + \int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds \\ &\quad + \frac{1}{d(t)} \int_{t-d(t)}^{t} d(t)x^T(t)Q_1x(t)ds \\ &\quad \times \int_{t-d(t)}^{t} x^T(t-d(t))Q_1x(t-d(t))ds \\ &\quad + \int_{t-d(t)}^{t} x^T(t-d)Q_1x(t-d)ds \end{aligned}$$

Adding upper equalities

$$\begin{aligned} V_1(t) + V_2(t) = &\frac{1}{d(t)} \int_{t-d(t)}^{t} x^T(t)\Psi(r(t))x(t) \\ &+ \int_{t-d(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \\ &+ \int_{t-d(t)}^{t} x^T(s)Q_1x(s)ds \\ &+ \int_{t-d(t)}^{t} x^T(t-d(t))Q_1x(t-d(t))ds \\ &+ \int_{t-d(t)}^{t} x^T(t-d)Q_1x(t-d)ds \end{aligned}$$

$V_3(t)$, $V_4(t)$ $V_5(t)$ are shown in Theorem 1. If $\Psi(r(t)) > 0$ is established, then $V(t) > 0$ is hold base on $Q_1 > 0, Q_2 > 0$, $Z_1 > 0, Z_2 > 0, R > 0$. This completes the proof.

Remark 7: Corollary 1 eliminates the default condition of $P(r(t)) > 0$, and gives a new positive definite condition by using the internal relation of Lyapunov function matrix. If $P(r(t)) > 0$ in Theorem 1 is satisfied, the $\Psi(r(t)) > 0$ in Corollary 1 is obviously satisfied. But exchanging conditions is not feasible, it is shown that Theorem 1 is a special case of Corollary 1.

IV. NUMERICAL EXAMPLES

Four examples will be given in this section to verify the feasibility of the research methods in this paper.

Firstly, we discuss the idealization of probability transfer for MJS (1), that is, the system has no time-varying probability transfer.

Example 1: Consider MJS (1) with two modes and the matrix *parameters* [24]:

$$A_1 = \begin{bmatrix} 0.5 & -1 \\ 0 & -3 \end{bmatrix} \quad A_2 = \begin{bmatrix} -5 & 1 \\ 1 & 0.2 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} 0.5 & -0.2 \\ 0.2 & 0.3 \end{bmatrix} \quad A_{d2} = \begin{bmatrix} -0.3 & 0.5 \\ 0.4 & -0.5 \end{bmatrix}$$

with transition rates matrix

$$\Pi = \begin{bmatrix} -7 & 7 \\ 3 & -3 \end{bmatrix}$$

First, we make a special design of Theorem 1 and get rid of the time-varying transition rates in $P_i(r(t))$ and idealize it. Then, let $\dot{d}(t) = -d_m = d_M = 0$. The comparative data of the upper bound of the maximum delay in Table 1 can be obtained. The effectiveness of the improved interactive convex inequality method can be seen intuitively through Table 1.

Example 2: Consider the MJS (1) with two modes and the matrix *parameters* [29]:

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix}$$
$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix}$$
$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ 0.8436 & -1.0115 \end{bmatrix}$$

with transition rates matrix

$$\Pi = \begin{bmatrix} -0.1 & 0.1 \\ 0.8 & -0.8 \end{bmatrix}$$

We still use the Theorem 1 which is specially dealt with in Example 1. Then, get different values for $\dot{d}(t)$, $\dot{d}(t) = -d_m = d_M$, for $\dot{d}(t) \in \{0.6, 0.8, 1.6\}$. The comparative data of the upper bound of the maximum delay in Table 2 can be obtained. No matter how $\dot{d}(t)$ changes, Theorem 1 can get higher upper bound of time delay, which shows that method in this paper is less conservative.

Next, the MJS (1) with probability transition of time-varying transition rates is discussed.

TABLE 2. **Different results to upper bound allowed *d* for Example 2.**

	$\dot{d}(t) = 0.6$	$\dot{d}(t) = 0.8$	$\dot{d}(t) = 1.6$
d by Theorem 1 of [30]	0.4428	0.3795	0.3469
d by Theorem 2 of [30]	0.4492	0.4341	0.4314
d by [29]	0.4927	0.4261	0.3860
d by Theorem 1 of [23]	0.5159	0.4814	0.4789
d by Theorem 1 of [28]	0.5342	0.5147	0.5029
d by Theorem 1 of this paper	0.5642	0.5456	0.5279

Example 3: Consider MJS (1) with two modes and the matrix *parameters* [13]:

$$A_1 = \begin{bmatrix} -4.5 & 2 \\ -0.8 & -4.3 \end{bmatrix} \quad A_2 = \begin{bmatrix} -3.5 & 1.2 \\ 1.0 & -1.9 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.6 & 1.6 \\ 1.8 & -1.1 \end{bmatrix} \quad A_{d2} = \begin{bmatrix} 1.6 & 0.5 \\ -0.7 & 1.0 \end{bmatrix}$$

The supposition transition rate matrix in has in following apex polyhedron $\Pi(t) = sin^2(t)\Pi^{(1)} + cos^2(t)\Pi^{(2)}$ is time-varying:

$$\Pi^{(1)} = \begin{bmatrix} -0.8 & 0.8 \\ 0.6 & -0.6 \end{bmatrix} \quad \Pi^{(2)} = \begin{bmatrix} -0.6 & 0.6 \\ 0.8 & -0.8 \end{bmatrix}$$

Let $\dot{d}(t) = d_m = d_M = 0.6014$, the upper bound of time delay $d = 1$ in [13], $d = 1.5637$ in this paper. This shows that under the same parameters and time-varying probability transfer perturbations, higher upper bounds of time delay can be obtained in this paper. And uses Matlab LMI toolbox, to had the time-varying transition rates stochastic stable MJS (1) to give group of appropriate solutions, as follows showed:

$$P_1^{(1)} = \begin{bmatrix} 0.7678 & -3.3864 \times 10^5 \\ 3.3864 \times 10^5 & 0.4783 \end{bmatrix}$$

$$P_1^{(2)} = \begin{bmatrix} 0.7688 & 9.6048 \times 10^5 \\ -9.6048 \times 10^5 & 0.4780 \end{bmatrix}$$

$$P_2^{(1)} = \begin{bmatrix} 0.2821 & -8.8431 \times 10^5 \\ 8.8431 \times 10^5 & 0.6676 \end{bmatrix}$$

$$P_2^{(2)} = \begin{bmatrix} 0.2814 & 1.5061 \times 10^6 \\ -1.5061 \times 10^6 & 0.6679 \end{bmatrix}$$

For simulation purposes, let $x(0) = [0.1 \quad -0.1]^T$, use the above conclusion to MJS (1), Figure (a) shows probability transfer, Figure (b) represents the state response curve of given initial $x_1(t)$ and $x_2(t)$.

Finally, a set of simulations is designed to compare Theorem 1 and Corollary 1.

Example 4: We still use the system matrix and time-varying transition rates function in Example 3. Unlike the Theorem 1, Corollary 1 improves the $P(r(t)) > 0$, which is replaced by the internal relation of the matrix, thus guaranteeing $V(t) > 0$. Let $\dot{d}(t) = d_m = d_M = 0.6014$, the comparison of the upper bound of the delay can be seen in Table 3. By comparison, the result of Corollary 1 has a slight improvement, so this means Theorem 1 is a special case of Corollary 1, and Corollary 1 can get a higher upper bound of time delay.

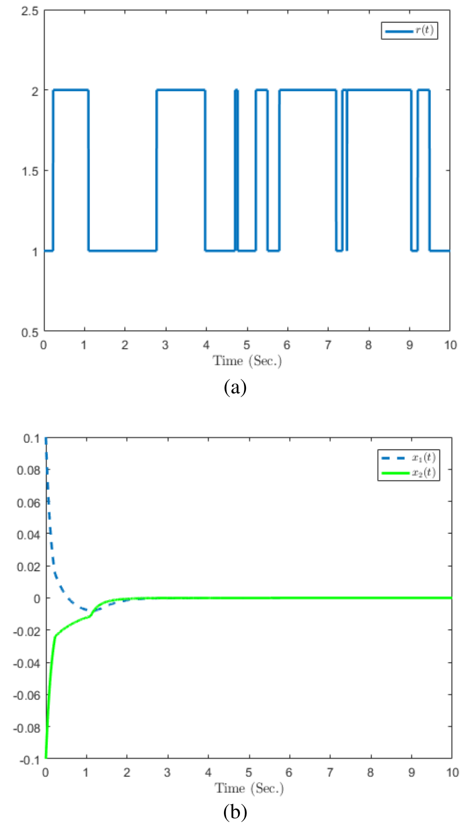

FIGURE 1. **(a) Time response of $r(t)$. (b) Time response of x_1, x_2.**

TABLE 3. **Different results to upper bound allowed *d* for Example 4.**

	By[13]	By Theorem 1	By Corollary 1
d	1	1.5637	1.5726

Remark 8: Example 1 and 2 verify that the conclusion of Theorem 1 can increase the upper bound of time delay under single time-varying disturbance. Example 3 verify that the conclusion of Theorem 1 is still valid under Multi-time-varying disturbance. Example 4 verifies that the improvement of the feasibility condition (b) by Corollary 1 is effective and can obtain a higher upper bound of time delay. In summary, these examples show that methods studied in this paper are feasible and effective.

V. CONCLUSIONS

In this paper, an improved interactive convex inequality method is proposed for MJS with multi-time-varying disturbances: time-varying delays and time-varying transition rates of probability transfer. The effectiveness of the improved inequality method can be seen through Example 1 and 2. On this basis, by using the relationship within the functional matrix, the specific positive definite condition has been improved. The effectiveness of this method is illustrated by Example 4.

In the future research work, we will study semi-Markovian jump system and uncertain transition rates, as well as related controllers and observers.

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