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Resource Allocation in Heterogeneous Cognitive Radio Network With Non-Orthogonal Multiple Access

WOPING XU, RUNHE QIU, AND XUE-QIN JIANG, (Member, IEEE)
College of Information Sciences and Technology, Donghua University, Shanghai 201620, China
Engineering Research Center of Digitized Textile & Apparel Technology, Ministry of Education, Donghua University, Shanghai 201620, China
Corresponding author: Runhe Qiu (qiurh@dhu.edu.cn)

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ABSTRACT In this paper, we study resource allocation problems for a two-tier cognitive heterogeneous network in interweave spectrum sharing mode. Secondary users (SUs) in small cells (SCs) opportunistically access the licensed spectrum resources. Non-orthogonal multiple access (NOMA) is used to boost the number of accessible SUs sharing the limited and dynamic licensed spectrum holes. Practically, there exists a tradeoff: an SC can increase its instantaneous sum throughput by accessing more idle bandwidth, which creates higher liability due to the dynamics of licensed spectrum and contention among the multiple SCs. Aiming to maximize the sum throughput of second-tier SCs network, we formulate a mixed integer non-linear programming problem with the constraints of the available idle bandwidth, the successive interference cancellation complexity, the transmission power budget, and the minimum data requirements. To efficiently solve this problem, we decompose the original optimization problem into bandwidth resource allocation subproblem, SUs clustering subproblem, and power allocation subproblem. Based on the scale of SCs network and the activities of licensed spectrum, we introduce an optimal bandwidth configuration to maximize the average sum throughput of SCs. By analyzing the derivation of the achievable rate expression of a NOMA-enabled SU, we develop a novel SUs clustering algorithm which can improve the throughput of a cluster by grouping SUs with more distinctive channel conditions. With the results of SUs clustering, we propose power allocation within a NOMA cluster by using Karush-Kuhn-Tucker optimality conditions. Furthermore, we perform power allocation across NOMA clusters by using the difference of convex programming. The simulation results validate the performance of the proposed resource allocation algorithms.

INDEX TERMS Cognitive heterogeneous network, interweave spectrum sharing mode, kmeans clustering, non-orthogonal multiple access (NOMA), resource allocation.

I. INTRODUCTION

Recently, the demands of the forthcoming fifth-generation (5G) communication system are expected to achieve 1,000 times the system capacity and 10 times the spectral efficiency of fourth-generation (4G) networks [1]. It is easy to understand that the conventional mobile network architecture is not able to support the increasing requirements of the communication system. To keep pace with the explosive growth of mobile data, one of the promising solutions is to deploy traffic offloading with the contemporary mobile networks [2]. The heterogeneous network is regarded as a promising and convenient traffic offloading technology approach where small cells (SCs) overlays with the macro cell (MC) to improve the capacity of the whole network [3].

However, the additional deployment of SCs in a heterogeneous network inevitably brings the challenges in terms of interference management and resource allocation. The heterogeneous cognitive network, whose infrastructures are cognitive radio (CR) technologies enabled, is known as a promising option for interference mitigation and radio resource efficiency improvement [4]. In a heterogeneous cognitive network, the users in MCs are considered as the primary users (PUs) who has the higher transmission priority, while the

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users in the SCs are considered as the secondary users (SUs) with lower transmission priority. The cognitive SCs may share the spectrum with primary MC by three paradigms, the interweave mode, the overlay mode, and the underlay mode. In interweave, the SUs can access to the licensed channel if it is not occupied by PUs. In overlay mode, SUs provide cooperation for the PUs to obtain access to the licensed spectrum. Regarding to the underlay mode, SUs are allowed to access the licensed spectrum coexisted with PUs under the tolerable interference constraint. Among these three paradigms, the interweave sharing mode is known to be an interference limited spectrum sharing mode since SUs are allowed to use the licensed channels which are not occupied by PUs. Especially for a primary system which uses a rather static spectrum, i.e., TV bands, the interweave spectrum sharing approach is viable and spectrum-efficient [5]. Therefore, the interweave spectrum sharing mode is adopted in this paper. However, in interweave spectrum sharing mode, one of the major challenges is to efficiently improve the spectrum reuse due to the dynamics of PUs [6].

Besides CR, non-orthogonal multiple access (NOMA) is another promising technology to enhance the spectrum reuse and connectivity density [7] [8]. Comparing with the conventional orthogonal multiple access (OMA) techniques, NOMA allows multiple users to multiplex on non-orthogonal resources, such as frequency channels or spreading codes [9]. NOMA can provide higher spectrum efficiency by supporting multiple users on the same resource block. The existing NOMA schemes can be classified into two categories, namely, power-domain and code-domain NOMA. In this paper, we focus on the power domain NOMA which serves multiple users at different power levels and employs successive interference cancellation (SIC) to detect the desired signals.

Integrating NOMA into CR network has a huge potential to further improve the spectrum efficiency by boosting the number of users to be served [10]. Recently, the authors in [11]–[16] discussed the performance of CR with NOMA and demonstrated that CR-NOMA can achieve a better spectrum efficiency compared to CR with conventional OMA. In [11], the CR-inspired NOMA has been proposed, and the impact of users paring has been discussed. The authors in this paper regarded the NOMA as a special case of CR system, where the user with strong channel condition viewed as an SU squeezed into the spectrum owned by the user with poor channel condition viewed as a PU. The authors in [12] discussed the application of NOMA in underlay CR network with stochastic geometry model and evaluated the outage probability of NOMA users. In [13], authors studied a joint antenna selection problem in MIMO CR-inspired NOMA network. In [14], the authors studied the optimal sensing duration and power allocation for underlay cognitive NOMA-OFDM system. In [15], the security of CR NOMA network was studied. The authors of this paper proposed an artificial noise-aided cooperative jamming scheme to improve the security of both primary and secondary network. In [16], authors proposed a new cooperative transmission scheme to exploit the spatial diversity in multicast CR-NOMA system.

Different from the works mentioned above, in this paper, we discuss the resource allocation problem in NOMA cognitive heterogeneous network in the interweave mode. To the best of our knowledge, this topic has not been discussed. The main contributions of our work are summarized as follows:

- We formulate the resource allocation problem in NOMA cognitive heterogeneous network in the interweave mode. We introduce an optimal channel reconfiguration scheme to improve the spectrum efficiency for the proposed cognitive heterogeneous network with limited idle spectrum resources.
- In multiple NOMA users scenario, we formulate a novel NOMA SUs clustering algorithm to improve the sum throughput of SUs multiplexing over the same spectrum band by analyzing the derivation of a NOMA-enabled SU's sum throughput equation.
- With SUs clustering results, we derive optimal power allocation which maximizes the sum throughput within a NOMA cluster by using Karush-Kuhn-Tucker (KKT) optimality condition. A difference of convex (DC) programming based power allocation algorithm is proposed to allocate power across NOMA clusters and further enhance the system sum throughput.

The rest of the paper is organized as follows. In Section II, we establish the system model for two-tier heterogeneous NOMA cognitive network and formulate the optimization problem. The optimal bandwidth configuration is discussed in Section III. Section IV presents the SUs clustering algorithm. The power allocation problem is described in section V. Simulation results in VI are evaluated to assess the performance of the proposed allocation. Finally, Section VII draws the conclusion.

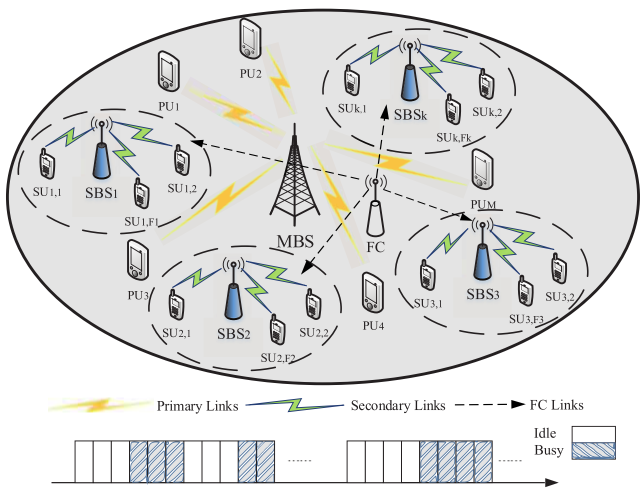

FIGURE 1. System Model.

II. SYSTEM MODEL AND PROBLEM FORMULATION

A. SYSTEM MODEL

As shown in Fig. 1, consider a NOMA downlink two-tier heterogeneous cognitive network, where K SCs are overlaid with a single MC. In the MC, a macro base station (MBS) is located in the center of the circular region with radius R_m as its coverage. Similarly, a small cell base station (SBS) is located in the center of each SC with radius R_s as its coverage. All K SBSs are cognitive techniques enabled and NOMA techniques enabled. Denote the index set of base station by $\mathcal{BS} = \{k|0 \le k \le K\}$ where $k = 0$ and $1 \le k \le K$ represent the MBS and K SBSs, respectively. There are M PUs and F_k NOMA-enabled SUs uniformly distributed within the MC and the kth SC, respectively. Denote the index set of M PUs by $\mathcal{M} = \{m|1 \le m \le M\}$ and the index set of F_k SUs by $\mathcal{F}_k = \{f|1 \le f \le F_k\}$. There are S orthogonal licensed channels in the system and the unit bandwidth is w. The index set of the S channels is denoted by $\mathcal{S} = \{s|1 \le s \le S\}$. At any time, each licensed channel is either idle or busy state. Denote the idle probability of the sth channel as $P_s \in \{P_1, \ldots, P_S\}$, which is calculated by

$$P_s = \frac{E\left(T_s^I\right)}{E\left(T_s^I\right) + E\left(T_s^B\right)}, \tag{1}$$

where $E\left(T_s^I\right)$ and $E\left(T_s^B\right)$ denote the average idle and busy time of the sth channel, respectively. In second-tier SC network, there exists a fusion center (FC) that gathers the channel state information and performs the central resource allocation actions. We also assume the sensing results obtained at FC are perfect [17].

Due to the limited number of idle channels, the total number of SUs in second-tier SC network is usually larger than the number of SUs who acquire the transmission opportunities. Without loss of generality, we assume the round-robin scheduling scheme for all SUs in the second-tier SCs network. Considering the implementation complexity of SIC at an SU receiver, we assume that F_c SUs can be multiplexed over an idle spectrum block. Define SUs who are multiplexed over the same channel form a NOMA cluster. Denote the set of NOMA clusters of kth SC as $\mathcal{C}_k$. A NOMA cluster of kth SC $\mathcal{C}_k^n \subseteq \mathcal{C}_k$, $1 \le n \le N$, $\cup_{n=1}^N \mathcal{C}_k^n = \mathcal{C}_k \subseteq \mathcal{F}_k$, and $\cap_{n=1}^N \mathcal{C}_k^n = \emptyset$. Besides, the bandwidth (in channels number) allocated to $\mathcal{C}_k^n$ is denoted by ω_k^n, where $1 \le \omega_k^n \le S$.

The normalized channel gain between the kth SBS and the ith SU of kth SC is denoted by $h_{k,i}$, which accounts for both distance-based path-loss and shadowing. The transmission power from kth SBS to the ith SU of the kth SC is denoted by $p_{k,i}$. The combination symbols transmitted from the kth SBS to an SU of a NOMA cluster are expressed as

$$x_{k,n} = \sum_{i \in \mathcal{C}_k^n} x_{k,i}\sqrt{p_{k,i}}, \tag{2}$$

where $x_{k,i}$ is the modulated symbol transmitted from the kth SBS to the ith SU and $E\left[\left|x_{k,i}\right|^2\right] = 1$. The received signal at ith SU is

$$\begin{aligned} y_{k,n} &= h_{k,i}x_{k,n} + z_0 \\ &= h_{k,i}x_{k,i}\sqrt{p_{k,i}} \\ &\quad + \sum_{j \ne i, j \in \mathcal{C}_k^n} h_{k,i}x_{k,j}\sqrt{p_{k,j}} + z_0, \end{aligned} \tag{3}$$

where $z_0 \sim \mathcal{CN}(0, \delta_0^2)$ denotes the additive white Gaussian noise (AWGN). The received signal-to-interference-plus-noise ratio (SINR) of the ith SU of NOMA cluster $\mathcal{C}_k^n$ can be represented as

$$\gamma_{k,i} = \frac{p_{k,i}\left|H_{k,i}\right|^2}{\omega_k^n + I_{k,i}}, \tag{4}$$

where $\left|H_{k,i}\right|^2 \triangleq \left|h_{k,i}\right|^2/\delta_0^2$ is the channel response normalized by noise (CRNN) of the ith SU.

$$I_{k,i} = \sum_{j \ne i, j \in \mathcal{C}_k^n} \left|H_{k,i}\right|^2 p_{k,j}$$

is denoted as the interference which the ith SU receives from the other SUs in the same NOMA cluster. Without loss of generality, all SUs of the kth SC are sorted in descending CRNNs order by

$$\left|H_{k,1}\right|^2 \ge \left|H_{k,2}\right|^2 \ge \ldots \left|H_{k,F_k}\right|^2. \tag{5}$$

According to the downlink NOMA principle, the ith SU is able to decode signals of the jth SU for $i > j$ and remove them from its own signal, but treats the signals of j'th SU for $j' > i$ as a interference. Define a indicator variable $\alpha_{k,i}^n$ as follows:

$$\alpha_{k,i}^n = \begin{cases} 1, & i \in \mathcal{C}_k^n \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

Based on Shannon's capacity formula, the data rate of the ith SU in NOMA cluster $\mathcal{C}_k^n$ can be expressed as

$$R_{k,i} = \alpha_{k,i}^n \omega_k^n \log_2\left(1 + \bar{\gamma}_{k,i}\right), \tag{7}$$

where

$$\bar{\gamma}_{k,i} = \frac{p_{k,i}\left|H_{k,i}\right|^2}{\omega_k^n + \sum\limits_{\substack{j=1,\\ j<i}}^{i-1} \alpha_{k,j}^n p_{k,j}\left|H_{k,i}\right|^2}. \tag{8}$$

B. PROBLEM FORMULATION

The optimal resource allocation problem for throughput maximization in the SCs network can be formulated as

$$\begin{aligned} R_t &= \sum_{k=1}^{K}\sum_{n=1}^{N}\sum_{i=1}^{F_k} \alpha_{k,i}^n \omega_k^n \log_2\left(1 + \bar{\gamma}_{k,i}\right) \\ &= \sum_{k \in K}\sum_{\mathcal{C}_k^n \in \mathcal{C}_k}\sum_{i \in \mathcal{C}_{k,n}} R_{k,i}. \end{aligned} \tag{9}$$

Accordingly, the resource allocation is performed under the following constraints:

- The bandwidth constraints: The total number of idle channels is no more than the total number of licensed channels. Thus, we have

$$\sum_{k=1}^{K}\sum_{n=1}^{N} \omega_k^n \le S \quad \forall k, n, \tag{10}$$

and

$$1 \leq \omega_k^n \leq S, \quad \forall k, n. \tag{11}$$

- SUs clustering constraints: An SU of the kth SC is grouped into only one NOMA cluster. The entire set of $\mathcal{C}_k$ is a subset of the set $\mathcal{F}_k$. In addition, the set of NOMA clusters are disjoint. Thus, we have

$$\sum_{n=1}^{N} \alpha_{k,i}^n \leq 1, \quad \forall k, i \tag{12}$$

and

$$\cup_{n=1}^{N} \mathcal{C}_k^n = \mathcal{C}_k \subseteq \mathcal{F}_k, \quad \cap_{n=1}^{N} \mathcal{C}_k^n = \emptyset, \forall k. \tag{13}$$

- SIC implementation complexity constraint: Due to the implementation complexity of SIC, at most F_c SUs can be non-orthogonal multiplexed over an idle spectrum block. Thus, we have

$$\left|\mathcal{C}_k^n\right| \leq F_c, \quad \forall k, n, \tag{14}$$

where $\left|\mathcal{C}_k^n\right|$ denotes the size of NOMA cluster $\mathcal{C}_k^n$.
- Total power budget of an SBS:

$$0 \leq \sum_{n=1}^{N} \sum_{i=1}^{F_k} \alpha_{k,i}^n p_{k,i} \leq p_k, \quad \forall k, \tag{15}$$

where p_k denotes the power budget of the kth SBS.
- SU's minimum data rate requirement constraint: The minimum data rate requirement for individual SU should be guarantee. Thus, we have:

$$\sum_{n=1}^{N} \alpha_{k,i}^n R_{k,i} \geq R_{k,i}^{\min}, \quad \forall k, i, \tag{16}$$

where $R_{k,i}^{\min}$ denotes the minimum data rate requirement of the ith SU in the kth SC.

Accordingly, the optimization problem can be formulated as

$$\begin{aligned}
\mathbb{OP} \quad & \max_{\alpha_{k,i}^n, \omega_k^n, p_{k,i}} \sum_{k \in K} \sum_{\mathcal{C}_k^n \in \mathcal{C}_k} \sum_{i \in \mathcal{C}_k^n} R_{k,i} \\
& s.t. \; C1 : \alpha_{k,i}^n \in \{0, 1\}, \quad \forall k, i, n, \\
& \quad C2 : \sum_{k=1}^{K} \sum_{n=1}^{N} \omega_k^n \leq S, \\
& \quad C3 : 1 \leq \omega_k^n \leq S, \quad \forall k, n, \\
& \quad C4 : \sum_{n=1}^{N} \alpha_{k,i}^n \leq 1, \quad \forall k, i, \\
& \quad C5 : \cup_{n=1}^{N} \mathcal{C}_k^n \subseteq \mathcal{F}, \quad \cap_{n=1}^{N} \mathcal{C}_k^n = \emptyset, \forall k, \\
& \quad C6 : \left|\mathcal{C}_k^n\right| \leq F_c, \quad \forall k, n, \\
& \quad C7 : 0 \leq \sum_{n=1}^{N} \sum_{i=1}^{F_c} \alpha_{k,i}^n p_{k,i} \leq p_k, \quad \forall k, \\
& \quad C8 : \sum_{n=1}^{N} \alpha_{k,i}^n R_{k,i} \geq R_{k,i}^{\min}, \quad \forall k, i,
\end{aligned}$$

where C1 is the indicator variable; C2 and C3 are the bandwidth constraints according to (10) and (11); C4 and C5 are the SUs clustering constraint according to (12)and (13); C6 is the SIC implementation complexity constraint for SUs based on (14); C7 ensures the power budget constraint at an SBS based on (15); C8 guarantees the minimum data rate requirements for SUs according to (16). Since this optimization problem is a mixed integer non-linear programming problem and NP-hard, it is challenging to obtain optimal solutions within polynomial time. Owing to the considerable complexity of global optimum solution, we decouple $\mathbb{OP}$ to three subproblem $\mathbb{OP}_1$, $\mathbb{OP}_2$ and $\mathbb{OP}_3$ to separately optimize the bandwidth allocation, SUs clustering and power allocation.

III. OPTIMAL BANDWIDTH CONFIGURATION

In this section, we discuss the optimal bandwidth configuration subproblem for limited bandwidth assignment in the second-tier SC network. With the given SUs clustering results and power allocation, the optimal bandwidth configuration subproblem can be formulated as

$$\begin{aligned}
\mathbb{OP}_1 \; & \max_{\omega_{k,n}} \sum_{k \in K} \sum_{\mathcal{C}_k^n \in \mathcal{C}_k} \sum_{i \in \mathcal{C}_k^n} R_{k,i} \\
& s.t. \; C2, C3, C8.
\end{aligned} \tag{17}$$

To solve $\mathbb{OP}_1$, the major challenge is to efficiently utilize the limited idle bandwidths to K SCs according to the licensed channel state activities. There exists a tradeoff in subproblem $\mathbb{OP}_1$: an SC can occupy larger bandwidths to increase its sum throughput, but the contention among multiple SCs creates higher accessing overhead for larger bandwidths. Based on the study on SUs' optimal bandwidth selection in [6], we introduce an optimal bandwidth configuration by formulating the average throughput for an SC of the second-tier SCs network.

Since each SC has the same behavior based on the proposed system model, each SC has equal idle spectrum access opportunities. Therefore, we solve the $\mathbb{OP}_1$ by finding the optimal bandwidth that maximizes the average sum throughput of an SC. According to the optimal bandwidth selection model in [6], we have the average sum throughput equation for K SCs

$$R_k(\Omega) = \frac{K_{avg}(\Omega)}{K} \sum_{\mathcal{C}_k^n \in \mathcal{C}_k} \sum_{i \in \mathcal{C}_k^n} R_{k,i} \tag{18}$$

where $\Omega = \sum_{n=1}^{N} \omega_k^n$ is the number of idle channels allocated to the kth SC and $R_k(\Omega)$ is the average throughput of the kth SC. $K_{avg}(\Omega)$ is the average number of SCs that the system can support, which depends on Ω. We denote $K_{\max}$ and $K_{\min}$ as the upper bound and lower bound of $K_{avg}(\Omega)$, respectively. $K_{\max}$ depends on both the number of SCs K and the maximum number of SCs that S licensed channels can support given Ω, $\lfloor \frac{S}{\Omega} \rfloor$. Therefore, $K_{\max} = \min\left(K, \lfloor \frac{S}{\Omega} \rfloor\right)$. We also note that the system can support $K_{\min}$ SCs when the number of idle channels is less than $(K_{\min} + 1)\,\Omega$ but greater than $K_{\max}\Omega$. Therefore, the average number of SCs $K_{avg}(\Omega)$ can be calculated by

$$K_{avg}(\Omega) = \sum_{K_{\min}=0}^{K_{\max}-1} \sum_{j=K_{\min}\Omega}^{(K_{\min}-1)\Omega-1} K_{\min}\bar{P}_S(j) + \sum_{j=K_{\max}N}^{S} K_{\max}\bar{P}_S(j), \quad (19)$$

where $\bar{P}_S(j) = \binom{S}{j}(P_s)^j(1-P_s)^{S-j}$ is the probability that j channels are idle. Therefore, the optimal bandwidth configuration $\Omega^\star$ for an SC can be calculated by

$$\Omega^\star = \max\left(\arg\max \frac{K_{avg}(\Omega)}{K}\right). \quad (20)$$

IV. SUB-OPTIMAL SUs CLUSTERING SCHEME

In this section, we investigate the SUs clustering scheme for grouping SUs into several NOMA clusters in each SC. With the given optimal bandwidth configuration result, the optimization problem $\mathbb{OP}$ shows that the SUs clustering and power allocation are coupled with each other in terms of throughput optimization. Therefore, we formulate the SUs clustering subproblem by assuming equal power is allocated to each NOMA cluster, in which each power of an SU is determined by fractional transmission power allocation (FTPA). The FTPA is widely used in the orthogonal frequency division multiple access (OFDMA) system and NOMA system due to its low computational complexity [18]. In the FTPA scheme, the transmit power of ith SU which is a cluster member of $\mathcal{C}_k^n$ is allocated based on its channel gain which is given as

$$p_{k,i} = \frac{p_k\left(\left|H_{k,i}\right|^2\right)^{-\theta}}{N\sum_{i\in\mathcal{C}_k^n}\left(\left|H_{k,i}\right|^2\right)^{-\theta}}, \quad (21)$$

where θ $(0 \le \theta \le 1)$ is the decay factor. With the optimal bandwidth configuration result and the given power allocation, the SUs clustering subproblem can be formulated as

$$\mathbb{OP}_2 \max_{\alpha_{k,n}} \sum_{k\in K}\sum_{\mathcal{C}_k^n\in\mathcal{C}_k}\sum_{i\in\mathcal{C}_k^n} R_{k,i} \quad \text{s.t. } C1, C4, C5, C6, C8. \quad (22)$$

To solve the subproblem $\mathbb{OP}_2$, we propose a sub-optimal SUs clustering scheme. The proposed scheme exploits the impact of the CRNNs gaps between any two SUs of an SC on a NOMA cluster's throughput and aims to increase the sum throughput of a NOMA cluster.

A. THE NUMBER OF NOMA CLUSTERS IN AN SC

According to the proposed system model, the number of clusters relies on the optimal bandwidths configuration result in Section III. To boost the number of available SUs in second-tier SC network, we assume that at most one idle channel can be allocated to a NOMA cluster, which means $\omega_k^n = 1, \forall k, n$. Therefore, the number of NOMA clusters $N = \Omega^\star$ in an SC.

B. THE CRNNs-GAP BASED SUs CLUSTERING SCHEME

In this subsection, we first discuss the impact of SUs clustering on the sum throughput of an SC by analyzing the derivation of a NOMA cluster's sum throughput equation.

$$\begin{aligned}
R_{k,n} &= \omega_k^n \sum_{i=1}^{F_c} \log_2\left(1 + \frac{p_{k,i}\left|H_{k,i}\right|^2}{\omega_k^n + \sum_{j=1}^{i-1} p_{k,j}\left|H_{k,i}\right|^2}\right) \\
&= \omega_k^n \log_2\left\{\left(\frac{p_{k,1}\left|H_{k,1}\right|^2 + \omega_k^n}{\omega_k^n}\right)\left(\frac{(p_{k,1}+p_{k,2})\left|H_{k,2}\right|^2 + \omega_k^n}{p_{k,1}\left|H_{k,2}\right|^2 + \omega_k^n}\right)\cdots\left(\frac{\sum_{j=1}^{F_c} p_{k,j}\left|H_{k,F_c}\right|^2 + \omega_k^n}{\sum_{j=1}^{F_c-1} p_{k,j}\left|H_{k,F_c}\right|^2 + \omega_k^n}\right)\right\} \\
&= \omega_k^n \log_2\left\{\left(\frac{\sum_{j=1}^{F_c} p_{k,j}\left|H_{k,F_c}\right|^2 + \omega_k^n}{\omega_k^n}\right)\left(\frac{p_{k,1}\left|H_{k,1}\right|^2 + \omega_k^n}{p_{k,1}\left|H_{k,2}\right|^2 + \omega_k^n}\right)\cdots\left(\frac{\sum_{j=1}^{F_c-1} p_{k,j}\left|H_{k,(F_c-1)}\right|^2 + \omega_k^n}{\sum_{j=1}^{F_c-1} p_{k,j}\left|H_{k,F_c}\right|^2 + \omega_k^n}\right)\right\} \\
&= \omega_k^n \log_2\left\{\left(\frac{\sum_{j=1}^{F_c} p_{k,j}\left|H_{k,F_c}\right|^2 + \omega_k^n}{\omega_k^n}\right)\prod_{i=2}^{F_c}\left(\frac{\sum_{j=1}^{i-1} p_{k,j}\left|H_{k,(i-1)}\right|^2 + \omega_k^n}{\sum_{j=1}^{i-1} p_{k,j}\left|H_{k,i}\right|^2 + \omega_k^n}\right)\right\}
\end{aligned} \quad (23)$$

Then, we introduce the sub-optimal SUs clustering scheme which is inspired by kmeans clustering [19].

For the sake of analysis, we firstly group the first F_c SUs ordered by CRNNs in (5) into a NOMA cluster. According to the achievable rate expression of the ith SU of the kth SC in (7), the sum throughput of this NOMA cluster can be represented by (23), as shown at the bottom of the previous page, where $R_{k,n}$ is the sum throughput of NOMA cluster $\mathcal{C}_k^n$. We can observe in (23) that the sum throughput of a NOMA cluster $R_{k,n}$ can be increased by selecting SUs with larger CRNNs gaps between each others into the same NOMA cluster. That is, with a given power allocation, the sum throughput of a NOMA cluster can be increased by selecting its cluster member whose CRNNs are more distinctive. Based on the aforementioned analysis, we propose a sub-optimal SUs clustering scheme based on CRNNs gap. The procedure of the proposed SUs clustering scheme is as follows.

The FC firstly constructs a CRNNs gap graph $G_k(V, E, H)$ for the kth SC, where $v_i \in V$ denotes the vertex set whose elements represent SUs, $(v_i, v_j) \in E$ denotes the edges set whose elements represent the edge between two vertices, and $\left||H_{k,i}|^2 - |H_{k,j}|^2\right| = h_{k,(i,j)} \in H$ denotes the CRNNs gap set whose elements represent the weights for associated edges. With the CRNNs gap graph $G_k(V, E, H)$, we formulate a CRNNs gap maximization problem for SUs NOMA clustering as following:

$$\begin{aligned} \max & \sum_{\substack{v_i \in \mathcal{C}_k^u, v_j \in \mathcal{C}_k^{u'} \\ i \neq j}} h_{k,(i,j)} \\ s.t.\ & \cup_{u=1}^{F_c} \mathcal{C}_k^u = V, \\ & \mathcal{C}_k^u \cap \mathcal{C}_k^{u'} = \emptyset, \\ & u, u' \in \{1, 2, \ldots, N\}, \forall k. \end{aligned} \tag{24}$$

Since (24) is an NP-hard problem which is intractable, we propose a sub-optimal SUs clustering scheme which consists of SUs pre-clustering and NOMA SUs clustering. The SUs pre-clustering algorithm is proposed to group SUs into F_c disjoint pre-clusters based on kmeans clustering. Denotes the pre-clusters set as $\left\{\widehat{\mathcal{C}}_k^1, \widehat{\mathcal{C}}_k^2, \ldots, \widehat{\mathcal{C}}_k^{F_c}\right\}$. The proposed SUs pre-clustering algorithm groups the SUs whose CRNNs gaps are relatively small into the same pre-cluster. In other words, the CRNNs gaps of SUs in different pre-clusters are larger compared with the CRNNs gaps of SUs in the same pre-cluster. Then, based on the SUs pre-clustering results, we obtain the SUs NOMA clustering results by grouping SUs from different pre-clusters into the same NOMA cluster.

The procedure of SUs pre-clustering is initialized by setting up the CRNNs gap graph. There are F_c SUs of the kth SC being set as pre-cluster centers (pre-CCs), which are denoted by $\{c_{k,1}, c_{k,2}, \ldots, c_{k,F_c}\}$, and other SUs being set as the per-cluster members (CMs). To maximize the optimal problem (24), the sth SU belongs to uth per-cluster whose CC is $c_{k,u}$ if $h_{k,(s,u)} \leq h_{k,(s,u')}$. After that, we obtain F_c disjoint pre-clusters of kth SC. Algorithm 1 on this page describes the detailed description of the proposed SUs pre-clustering algorithm. The description of the parameters is given in table 1 on this page.

Algorithm 1 Kmeans Based SUs Pre-Clustering Algorithm

Construct the CRNNs gap graph $G_k(V, E, H)$.
Initialize $U = F_c$ CCs for SUs of the kth SC to be assigned, $c_k = \{c_{k,1}, c_{k,2}, \ldots, c_{k,F_c}\}$.
Initialize the parameter It, D, $d_{\widehat{\mathcal{C}}_k^u}$.
for every cluster u **do**

$$\widehat{\mathcal{C}}_k^u := \arg\min \left\| v_s - c_{k,u} \right\|^2$$

    for every CC $c_{k,u}$ **do**

$$c_{k,u} := \frac{\sum_{i=1}^{U} 1\left\{\widehat{\mathcal{C}}_k^i = u\right\} v_s}{\sum_{i=1}^{U} 1\left\{\widehat{\mathcal{C}}_k^i = u\right\}}$$

        Update

$$\bar{d}_{\widehat{\mathcal{C}}_k^u} = \frac{1}{\left|\widehat{\mathcal{C}}_k^u\right|} \sum_{n \in \widehat{\mathcal{C}}_k^u} h_{k,(n,c_{k,u})}$$

$$\bar{D} = \frac{1}{|U|} \sum_{u \in U} \bar{d}_{\widehat{\mathcal{C}}_k^u}$$

    end for
end for
Map SUs into clusters.
while $it < It$ **do**
    Update c_k, $\widehat{\mathcal{C}}_k^u$ $\bar{d}_{\widehat{\mathcal{C}}_k^u}$,
    $it = it + 1$
end while
Return the SUs pre-clusters $\left\{\widehat{\mathcal{C}}_k^1, \widehat{\mathcal{C}}_k^2, \ldots, \widehat{\mathcal{C}}_k^{F_c}\right\}$ and the CCs $c_k = \{c_{k,1}, c_{k,2}, \ldots, c_{k,F_c}\}$.

TABLE 1. Parameter description.

It	Maximal iterations
U	Pre-cluster number
D	Average $h_{k,(i,j)}$ of all SUs to be assigned
$d_{\widehat{\mathcal{C}}_k^u}$	Average CRNNs gap between CMs and CC in pre-cluster $\widehat{\mathcal{C}}_k^u$

Then, based on the pre-clustering results, we proposed a sorting based NOMA SUs clustering algorithm. Due to the optimal bandwidth configuration results and the SIC implementation complexity, there are F_cN SUs can obtain transmission opportunities in an SC. Therefore, the FC selects N SUs from each SUs per-cluster by round robin scheduling scheme and then, sorts these selected SUs by their CRNNs.

Algorithm 2 Sorting Based NOMA SUs Clustering Algorithm

1: Select N SUs from each pre-cluster by round robin scheduling scheme, respectively.
2: **for** every pre-cluster $\widehat{\mathcal{C}}_k^u$ **do**
3: Sort SUs of $\widehat{\mathcal{C}}_k^u$ by CRNNs:
4: $\left|H_{k,1}^{\widehat{\mathcal{C}}_k^u}\right|^2 \leq \left|H_{k,2}^{\widehat{\mathcal{C}}_k^u}\right|^2 \leq \ldots \leq \left|H_{k,N}^{\widehat{\mathcal{C}}_k^u}\right|^2$.
5: Group the SU with CRNNs order index n into nth NOMA cluster:
6: $\mathcal{C}_k^n = \left\{\left|H_{k,n}^{\widehat{\mathcal{C}}_k^1}\right|^2, \left|H_{k,n}^{\widehat{\mathcal{C}}_k^2}\right|^2, \ldots, \left|H_{k,n}^{\widehat{\mathcal{C}}_k^{F_c}}\right|^2\right\}$.
7: **end for**
8: **Return** NOMA clusters $\{\mathcal{C}_k^1, \mathcal{C}_k^2, \ldots, \mathcal{C}_k^N\}$.

Denote $\left|H_{k,i}^{\widehat{\mathcal{C}}_k^u}\right|^2$ as the CRNN of an SU in pre-cluster $\widehat{\mathcal{C}}_k^u$. According to the CRNNs orders of pre-clusters, the proposed NOMA SUs clustering algorithm groups the SUs with same index of a pre-cluster into a NOMA cluster. The detailed description of the sorting based NOMA SUs clustering algorithm is described in Algorithm 2.

C. COMPLEXITY ANALYSIS

The asymptotic complexity of the proposed scheme is analyzed in this subsection. In Algorithm 1, for K SCs, a worst-case complexity can be calculated as $O(\sum_{i=1}^{K} F_k F_c I_t)$, where I_t denotes the iterations, F_k is the number of SUs to be clustered in the kth SUs, tand F_c denotes the proposed clusters number of each SC [19]. In Algorithm 2, a worst-case complexity can be calculated as $O\left(F_c N^2\right)$, where N is the number of SUs to be sorted. Therefore, the total complexity of the proposed SUs clustering scheme is $O\left(\sum_{i=1}^{K} F_k F_c I_t + F_c N^2\right)$. Compared with the exhaustive search for SUs clustering, which has a worst-case complexity of $O(\sum_{k=1}^{K} (F_k)^{F_c})$, the proposed SUs clustering scheme has a much lower complexity.

V. POWER ALLOCATION

To further improve the throughput of the SCs network, we consider power allocations algorithm in this section. According to the optimal bandwidth configuration results and NOMA clustering results, the power allocation subproblem can be represented as

$$\mathbb{OP}_3 \max_{p_{k,i}} \sum_{k\in K} \sum_{\mathcal{C}_k^n \in \mathcal{C}_k} \sum_{i \in \mathcal{C}_k^n} R_{k,i}$$
$$s.t.\ C7, C8. \tag{25}$$

Since the power allocations are independent across the SCs, we simplify the $\boldsymbol{OP_3}$ to the power allocation problem for an SC with the given bandwidth configuration result and SUs NOMA clustering results. Considering N given NOMA clusters of the kth SC, the power allocation vector of these NOMA clusters is denoted by $\boldsymbol{p}_k = \left[\hat{p}_k^1, \hat{p}_k^2, \ldots, \hat{p}_k^n, \ldots, \hat{p}_k^N\right]^\top$, where $\hat{p}_k^n$ represents total power allocated to NOMA cluster $\mathcal{C}_k^n$. In NOMA cluster $\mathcal{C}_k^n$, the CRNNs order of SUs are sorted by

$$\left|H_{k,1}^n\right|^2 \geq \left|H_{k,2}^n\right|^2 \ldots \geq \left|H_{k,i}^n\right|^2 \ldots \geq \left|H_{k,F_c}^n\right|^2.$$

The minimum data rate requirements of these SUs are denoted by $\bar{R}_{k,1}^n, \bar{R}_{k,2}^n, \ldots, \bar{R}_{k,i}^n, \ldots \bar{R}_{k,F_c}^n$. Denote the power proportional factor matrix of SUs in kth SC as $\boldsymbol{\beta}_k = \left[\boldsymbol{\beta}_k^n, \boldsymbol{\beta}_k^2, \ldots, \boldsymbol{\beta}_k^N\right]$, where $\boldsymbol{\beta}_k^n = [\beta_{k,1}^n, \beta_{k,2}^n, \ldots, \beta_{k,F_c}^n]^\top$ represents the power proportional factor vector of NOMA cluster $\mathcal{C}_k^n$ and $\beta_{k,i}^n \in (0, 1)$. Therefore, $\mathbb{OP}_3$ is simplified to

$$\mathbb{OP}'_3 \max_{\boldsymbol{\beta}_k, \boldsymbol{p}_k} \sum_{n=1}^{N} \sum_{i=1}^{F_c} \log_2\left(1 + \frac{\beta_{k,i}^n \hat{p}_{k,n} \left|H_{k,i}^n\right|^2}{1 + \sum_{j=1}^{i-1} \beta_{k,j}^n \hat{p}_{k,n} \left|H_{k,i}\right|^2}\right)$$
$$s.t.\ C1' : \beta_{k,i}^n \in (0, 1)\,, \forall i, n, k$$
$$C2' : \sum_{i=1}^{F_c} \beta_{k,i}^n \leq 1, \forall n, k$$
$$C3' : \sum_{n=1}^{N} \hat{p}_k^n \leq p_k, \forall k$$
$$C4' : \log_2\left(1 + \frac{\beta_{k,i}^n \hat{p}_{k,n} \left|H_{k,i}^n\right|^2}{1 + \sum_{j=1}^{i-1} \beta_{k,j}^n \hat{p}_{k,n} \left|H_{k,i}^n\right|^2}\right) \leq \bar{R}_{k,i}^n,$$
$$\forall i, n, k,$$

where $i \in \{1, \ldots, F_c\}$, $n \in \{1, \ldots, N\}$, and $k \in \{1, \ldots, K\}$. C1' and C2' are the power proportional factor constraints for SUs within a NOMA cluster; C3' guarantees the total transmit power constraint of the kth SBS according to C7; C4' ensures the minimum data rate requirements according to the C8. In the following subsections, we derive closed-form expressions for the optimal power allocations within a NOMA cluster by using KKT optimality conditions. After that, we perform power allocation across the NOMA clusters by using DC programming.

A. POWER ALLOCATION WITHIN A NOMA CLUSTER

In this part, we investigate the power allocation within a NOMA cluster. To obtain the power proportional factor vector of each NOMA cluster, we formulate the optimal power allocation within a NOMA cluster by assuming equal power is allocated to each NOMA cluster. Therefore, the optimal optimal allocation problem within a NOMA cluster can be expressed as

$$\max_{\boldsymbol{\beta}_k^n} \sum_{i=1}^{F_c} \log_2\left(1 + \frac{\beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2}{1 + \sum_{j=1}^{i-1} \beta_{k,j}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2}\right)$$
$$s.t.\ C1', C2', C4'. \tag{26}$$

$$\frac{\partial \mathcal{L}}{\partial \beta_{k,1}^{n}} = \frac{\hat{p}_k^n \left|H_{k,1}^n\right|^2}{\ln 2\left(1+\beta_{k,1}^n \hat{p}_k^n \left|H_{k,1}^n\right|^2\right)} - \sum_{l=2}^{F_c} \frac{\beta_{k,l}^n \hat{p}_k^n \left|H_{k,l}^n\right|^4}{\ln 2\left(1+\sum_{q=1}^{l}\beta_{k,q}^n \hat{p}_k^n \left|H_{k,l}^n\right|^2\right)\left(1+\sum_{q'=1}^{l-1}\beta_{k,q'}^n \hat{p}_k^n \left|H_{k,l}^n\right|^2\right)},$$
$$-\lambda + \mu_1 \hat{p}_k^n \left|H_{k,1}^n\right|^2 - \sum_{l=2}^{F_c} \mu_l \hat{p}_k^n \left|H_{k,l}^n\right|^2 \left(2^{\bar{R}_{k,l}^n}-1\right), \tag{29}$$

$$\frac{\partial \mathcal{L}}{\partial \beta_{k,i}^{n}} = \frac{\hat{p}_k^n \left|H_{k,i}^n\right|^2}{\ln 2\left(1+\sum_{j=1}^{i}\beta_{k,j}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2\right)} - \sum_{l=i+1}^{F_c} \frac{\beta_{k,l}^n \hat{p}_k^n \left|H_{k,l}^n\right|^4}{\ln 2\left(1+\sum_{q=1}^{l}\beta_{k,q}^n \hat{p}_k^n \left|H_{k,l}^n\right|^2\right)\left(1+\sum_{q'=1}^{l-1}\beta_{k,q'}^n \hat{p}_k^n \left|H_{k,l}^n\right|^2\right)},$$
$$-\lambda + \mu_i \hat{p}_k^n \left|H_{k,i}^n\right|^2 - \sum_{l=i+1}^{F_c} \mu_l \hat{p}_k^n \left|H_{k,l}^n\right|^2 \left(2^{\bar{R}_{k,l}^n}-1\right), \tag{30}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda^{\star}} = 1 - \sum_{i=1}^{F_c} \beta_{k,i}^n, \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_i^{\star}} = \beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2 - \left(2^{\bar{R}_{k,i}^n}-1\right)\left(\sum_{j=1}^{i-1}\beta_{k,j}\hat{p}_k^n \left|H_{k,i}^n\right|^2 + 1\right). \tag{32}$$

C4' can be converted to a liner inequality constraint with respect to $\beta_{k,i}^n$ as

$$\beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2 \geq \left(2^{\bar{R}_{k,i}^n}-1\right)\left(\sum_{j=1}^{i-1}\beta_{k,j}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2 + 1\right),$$
$$i \in \{1,\ldots,F_c\}, n \in \{1,\ldots,N\}, k \in \{1,\ldots,K\}. \tag{27}$$

According to [20], it is easy to prove that the optimal problem (26) is convex under the linear constraints C1', C2', and C4'. Therefore, we can obtain the closed-form optimal solution for it. The Lagrange function for the optimal problem (26) can be expressed as

$$\mathcal{L}\left(\boldsymbol{\beta}_k^n, \boldsymbol{\lambda}, \boldsymbol{\mu}\right) = \lambda\left(1-\sum_{i=1}^{F_c}\beta_{k,i}^n\right) + \sum_{i=1}^{F_c}\mu_i\left\{\beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2 - \left(2^{\bar{R}_{k,i}^n}-1\right)\left(\sum_{j=1}^{i-1}\beta_{k,j}\hat{p}_k^n \left|H_{k,i}^n\right|^2+1\right)\right\}$$
$$+\sum_{i=1}^{F_c}\log_2\left(1+\frac{\beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2}{1+\sum_{j=1}^{i-1}\beta_{k,j}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2}\right) \tag{28}$$

where $\boldsymbol{\lambda} = [\lambda]^{\top}$ and $\boldsymbol{\mu} = \left[\mu_1,\ldots,\mu_{F_c}\right]^{\top}$ are the lagrange multiplier vectors. Taking derivatives of (28), we can write KKT conditions on the top of this page.

Theorem 1: Closed form solution for optimal power allocation in terms of $\boldsymbol{\beta}_k^n$ for SUs within an SUs cluster is obtained as:

$$\beta_{k,1}^n = \frac{1}{\prod_{j=2}^{F_c} 2^{\bar{R}_{k,i}^n}} - \sum_{j=2}^{F_c}\frac{\left(2^{\bar{R}_{k,j}^n}-1\right)}{\left|H_{k,j}^n\right|^2\left(\prod_{l=2}^{j}2^{\bar{R}_{k,l}^n}\right)}, \tag{33}$$

and for $i = 2, 3, \ldots, F_c$,

$$\beta_{k,i}^n = \frac{\left(2^{\bar{R}_{k,i}^n}-1\right)}{\prod_{j=i}^{F_c}2^{\bar{R}_{k,j}^n}} + \frac{\left(2^{\bar{R}_{k,i}^n}-1\right)}{\left|H_{k,i}^n\right|^2 2^{\bar{R}_{k,i}^n}} - \sum_{j=i+1}^{F_c}\frac{\left(2^{\bar{R}_{k,j}^n}-1\right)}{\left|H_{k,j}^n\right|^2\left(\prod_{l=i}^{j}2^{\bar{R}_{k,l}^n}\right)}\left(2^{\bar{R}_{k,i}^n}-1\right). \tag{34}$$

Proof: see Appendix A. ■

B. POWER ALLOCATION ACROSS NOMA CLUSTERS

With the power proportional factor vector $\boldsymbol{\beta}_k^n$, the optimal problem in (26) can rewritten as

$$\max_{\boldsymbol{p}_k} \sum_{n=1}^{N}\sum_{i=1}^{F_c}\log_2\left(1+\frac{\beta_{k,i}^n \hat{p}_k^n \left|H_{k,i}^n\right|^2}{1+\sum_{j=1}^{i-1}\beta_{k,j}^n \hat{p}_k^n \left|H_{k,i}\right|^2}\right)$$
$$s.t.\ C3', C4'. \tag{35}$$

Therefore, the optimal power allocation across NOMA clusters can be expressed as

$$\max_{\boldsymbol{p}_k} \sum_{n=1}^{N}\sum_{i=1}^{F_c}\Bigg[\log_2\Bigg(1+\sum_{j=1}^{i}\beta_{k,j}^n\hat{p}_k^n|H_{k,i}^n|^2\Bigg) -\log_2\Bigg(1+\sum_{j=1}^{i-1}\beta_{k,j}^n\hat{p}_k^n|H_{k,i}^n|^2\Bigg)\Bigg]$$
$$s.t.\ C3', C4'. \tag{36}$$

In order to solve the optimal problem (36) by DC programming [21], we convert the objective function in (36) to DC representation as

$$\max_{\boldsymbol{p}_k} -\{(F(\boldsymbol{p}_k))-(G(\boldsymbol{p}_k))\} = \min_{\boldsymbol{p}_k}\{(F(\boldsymbol{p}_k))-(G(\boldsymbol{p}_k))\} = \min_{\boldsymbol{p}_k} Q(\boldsymbol{p}_k)$$
$$s.t.\ C3', C4', \tag{37}$$

where

$$F(\boldsymbol{p}_k) = -\sum_{i=1}^{F_c}\sum_{n=1}^{N}\log_2\Bigg(1+\hat{p}_k^n\sum_{j=1}^{i}\beta_{k,j}^n|H_{k,i}^n|^2\Bigg)$$

and

$$G(\boldsymbol{p}_k) = -\sum_{i=1}^{F_c}\sum_{n=1}^{N}\log_2\Bigg(1+\hat{p}_k^n\sum_{j=1}^{i-1}\beta_{k,j}^n|H_{k,i}^n|^2\Bigg),$$

and both terms are convex functions and with respect to $\boldsymbol{p}_k$ because $\nabla^2 F(\boldsymbol{p}_k)$ and $\nabla^2 G(\boldsymbol{p}_k)$ are positive semi-definite matrixes. The constraint $C4'$ can be converted to the linear function of $\boldsymbol{p}_k$ which is expressed as

$$\hat{p}_k^n \geq \frac{\left(2^{\bar{R}_{k,i}^n}-1\right)}{\left\{\beta_{k,i}^n-\left(2^{\bar{R}_{k,i}^n}-1\right)\sum_{j=1}^{i-1}\beta_{k,j}\right\}\left|H_{k,i}^n\right|^2}. \tag{38}$$

Therefore, we can solve problem in (37) and find the $\boldsymbol{p}_k^\star$ by DC programming approach in Algorithm 3 on this page.

Algorithm 3 DC Programming for Power Allocation

1: Initialize $\boldsymbol{p}_k^{(0)}$, set iteration number $it = 0$. The objective function $Q(\boldsymbol{p}_k)$, the convex function $F(\boldsymbol{p}_k)$ and $G(\boldsymbol{p}_k)$.
2: **while** $\left|Q(\boldsymbol{p}_k^{(it+1)}) - Q(\boldsymbol{p}_k^{(it)})\right| > \epsilon$ **do**
3: Define convex approximation of $Q(\boldsymbol{p}_k^{(it)})$ as

$$Q^{(it)}(\boldsymbol{p}_k) = F(\boldsymbol{p}_k) - G(\boldsymbol{p}_k^{(it)}) - G^\top(\boldsymbol{p}_k^{(it)})(\boldsymbol{p}_k - \boldsymbol{p}_k^{(it)})$$

4: Solve the convex problem

$$\boldsymbol{p}_k^{(it)} = \arg\min_{C3',C4'} Q^{(it)}(\boldsymbol{p}_k)$$

5: $it \leftarrow it + 1$
6: **end while**

VI. SIMULATION RESULTS

In this section, a series of numerical experiments are presented to evaluate the performance of proposed resource allocation algorithms. For the simulations, we consider a two-tier NOMA cognitive HetNet where the K small cells are randomly distributed in the macro cell coverage areas. The coverage radius of a macro cell and a small cell are 400 m and 50 m, respectively. The unit bandwidth is $\omega = 0.2$ MHz, and $\delta_0^2 = \omega N_0$, where $N_0 = -174$ dBm/Hz is the AWGN power spectral density. We assume that the minimum rate data requirements of all SUs are equal and set as $R_{k,i}^{\min} = 1$ Kbps.

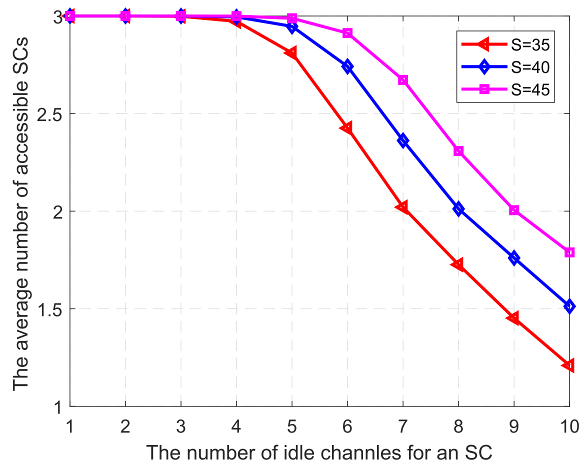

FIGURE 2. The number of idle channels for an SC versus the average number of accessible SCs with different total number of licensed channels. $K = 3$.

First, we evaluate the performance of the proposed optimal bandwidth configuration for the proposed system model. We set the number of total licensed channels as $S = 35$, 40, 45, respectively, and the ratio of a single channel idle time to its busy time as 1. The number of idle channel for an SC ranges from 1 to 10. Fig. 2 shows the average number of SCs the proposed system can support as a function of the number of idle channels an SC can obtain. Fig. 2 shows that the average number of accessible SCs decreases by the increase of the number of idle channels an SC obtains. This is because that, in the proposed scenario, an SC accessing more idle bandwidth results in higher liability which will decrease the average bandwidth for SCs. Besides, the optimal bandwidth for an SC rise by the increase of the total number of licensed channels. In Fig. 3 on the next page, we can observe that the optimal bandwidth decreases nonlinearly by the increase of the number of accessible SCs. For example, for the total number of licensed channels $S = 45$, the optimal bandwidth per SC are $\Omega^\star = 5$ for $K = 3$ and $\Omega^\star = 2$ for $K = 6$. This indicates that, the scale of the second-tier SCs network has impact on the performance of the proposed optimal bandwidth configuration.

Fig. 4 on the next page shows the pre-clustering results of the proposed pre-clustering algorithm. We set the number of SUs in an SC as $F_k = 50$. In Fig. 4, all 50 SUs are divided into

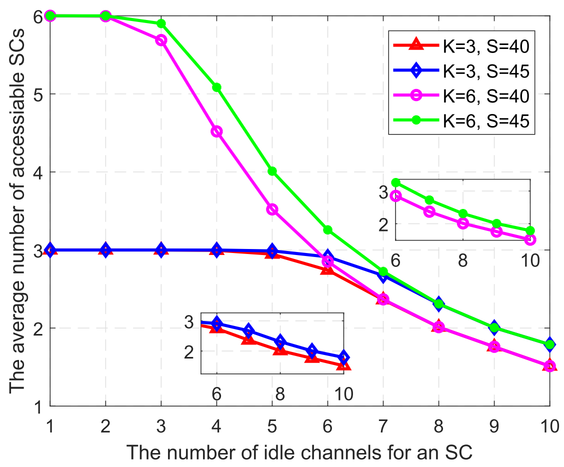

FIGURE 3. The number of idle channels for an SC versus the average number of accessible SCs with different number of SCs.

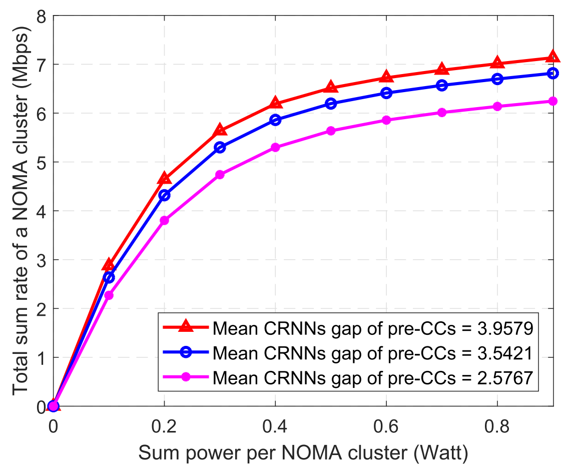

FIGURE 5. Total sum rate of a NOMA cluster versus the sum power per NOMA cluster with different mean distance of pre-cluster centers. $F_c = 4$.

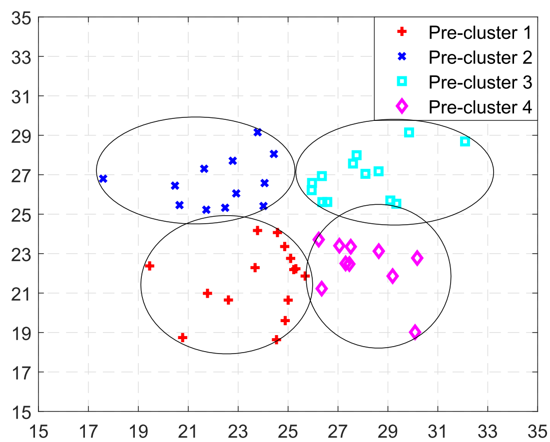

FIGURE 4. Results of kmeans based pre-clustering algorithm. $F_k = 50$ and $F_c = 4$.

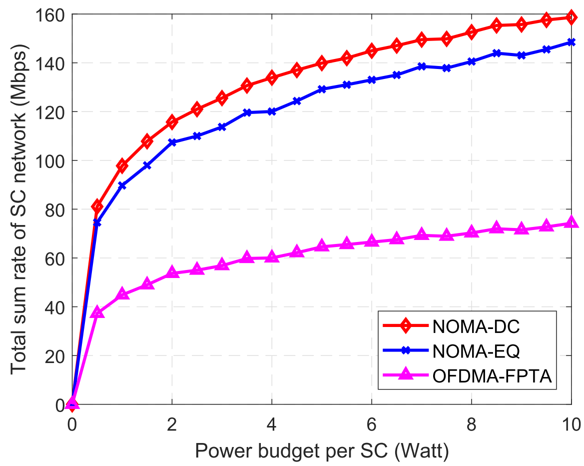

FIGURE 6. Total sum rate of small cell network versus power budget of per SC. $K = 6$, $N = \Omega^{\star} = 6$, and $F_c = 4$.

four pre-clusters according to the NOMA cluster size $F_c = 4$. We can find that our proposed pre-clustering algorithm is reasonable and convincing intuitively. We also investigate the throughput performance of a NOMA cluster by the proposed NOMA clustering algorithm and the proposed power allocation within a NOMA cluster. The sum power per NOMA cluster ranges from 0.1 W to 0.9 W. Fig. 5 on this page shows that the total sum rate of a NOMA cluster increases by the increase of the sum power per NOMA cluster. As the power grows larger, the total sum rate continues to increase, but the rate of growth becomes slower, as expected from Shannon's formula in calculating sum rate. Besides, the total sum rate of a NOMA cluster increases by the increase of the mean CRNNs gap of pre-CCs. It verifies that a NOMA cluster whose members with more distinctive CRNNs can offer a higher sum rate.

Fig. 6 on this page shows the sum rate of SC network versus the power budget per SC with different multiplex access mode and different power allocation scheme across the NOMA clusters. Note that NOMA-DC uses DC-programming based power allocation across NOMA clusters and NOMA-EQ uses the equal power allocation across NOMA clusters. It can be seen that the total sum rate of SC network increases by the increase of power budget per SC. In the proposed SC network with NOMA, our proposed spectrum allocation algorithm using DC programming for inter NOMA clusters power allocation outperforms the equal power allocation for inter NOMA clusters power allocation. Both algorithms perform better than the proposed SC network with OFDMA.

VII. CONCLUSION

In this paper, we studied optimal resource allocation in a two-tier downlink NOMA cognitive heterogeneous network in interweave spectrum sharing mode. Due to the complexity of the proposed optimization problem, we decoupled it into three subproblem. To improve the spectrum efficiency in interweave cognitive network, we introduce an optimal bandwidth configuration to secondary SC network. By analyzing the derivation of the achievable rate expression of a NOMA-enabled SU, we develop a novel SUs clustering algorithm which groups SUs with more distinctive CRNNs into the same NOMA cluster. With the results of NOMA clustering, we derived closed-form optimal power allocation within a NOMA cluster by KKT optimality conditions. To further improve the sum throughput of an SC, we applied the DC programming to approximate the non-convex inter NOMA cluster power allocation problem to convex problem which can be solved by convex optimization. Simulation results showed that the optimal bandwidth configuration is affected by the scale of the second-tier SC network. Moreover, the proposed NOMA clustering algorithm improved the throughput performance of NOMA clusters and the proposed power allocation algorithm further enhanced the throughput of SCs.

APPENDIX
PROOF OF THEOREM 1

In this part, we prove that $\mu_i > 0$ for $i \geq 2$, where μ_i is non-negative Lagrangian multiplier associate with the inequality constraint in (27). Thus, we use the following equation that holds for optimum $\boldsymbol{\beta}_k^n$,

$$\frac{\partial \mathcal{L}}{\partial \beta_{k,i}^n} = 0, \ \frac{\partial \mathcal{L}}{\partial \beta_{k,i+1}^n} = 0$$

with itself leads to KKT points of $\boldsymbol{\beta}_k^n$:

$$\begin{aligned} &\mu_{i+1}\left|H_{k,i+1}^n\right|^2 2^{\bar{R}_{k,i+1}^n} - \mu_i\left|H_{k,i}^n\right|^2 \\ &= \frac{\left(1/\ln 2\right)\left(\left|H_{k,i+1}^n\right|^2 - \left|H_{k,i}^n\right|^2\right)}{\left(1+\sum\limits_{j=1}^{i}\beta_{k,j}^n\hat{p}_{k,n}\left|H_{k,i}^n\right|^2\right)\left(1+\sum\limits_{j=1}^{i+1}\beta_{k,j}^n\hat{p}_{k,n}\left|H_{k,i+1}^n\right|^2\right)}. \end{aligned} \tag{39}$$

Since $\left|H_{k,i}^n\right|^2 \geq \left|H_{k,i+1}^n\right|^2$, the right side of (39) is always positive. Thus, $\mu_{i+1}\left|H_{k,i+1}^n\right|^2 2^{\bar{R}_{k,i+1}^n} \geq \mu_i\left|H_{k,i}^n\right|^2$. Since $\mu_1 \geq 0$ and $\mu_2\left|H_{k,2}^n\right|^2 2^{\bar{R}_{k,2}^n} \geq \mu_1\left|H_{k,1}^n\right|^2$, μ_2 is always non-negative. Therefore, we have $\mu_i \geq 0$ for $i \geq 2$.

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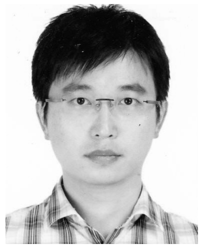
WOPING XU received the B.A.Sc. and M.A.Sc. degrees from the College of Information Sciences and Technology, Donghua University, Shanghai, China, in 2011 and 2014, respectively, where she is currently pursuing the Ph.D. degree. Her current research interests include 5G, cognitive radio networks, network energy efficiency, performance analysis for cognitive radio networks, and heterogeneous networks.

RUNHE QIU was born in Shanghai, China, in 1961. He received the B.S. degree in electrical engineering from the Shanghai University of Science and Technology, Shanghai, China, in 1983, the M.S. degree in electrical engineering from the China University of Mining and Technology, Xuzhou, in 1995, and the Ph.D. degree in physical radio engineering from East China Normal University, in 2007.

He is currently a Full Professor with the College of Information Science and Technology, Donghua University, Shanghai. He has published more than 70 papers in international conferences and academic journals at home and abroad. His current research interests include spectral estimation, intelligent detection setting and systems, and wireless broadband communication and application.



XUE-QIN JIANG received the B.S. degree in computer science from the Nanjing Institute of Technology, Nanjing, Jiangsu, China, and the M.S and Ph.D. degrees in electronics engineering from Chonbuk National University, Jeonju, South Korea. He is currently a Professor with the School of Information Science and Technology, Donghua University, Shanghai, China. He is the author/coauthor of more than 80 technical papers, several book chapters. His main research interests include LDPC codes, physical-layer security, and wireless communications.