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Velocity-Level Tri-Criteria Optimization Scheme for Different Complex Path Tracking of Redundant Manipulators

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ABSTRACT In order to remedy the position deviation of the joint angles and ensure the smooth motion trajectory of the end-effector at velocity level, a velocity-level tri-criteria optimization (VLTCO) scheme of redundant manipulator's is proposed. The proposed VLTCO scheme combines the minimum velocity norm (MVN), the repetitive motion planning (RMP), and the infinity-norm velocity minimization (INVM) through the weighting factors, which guarantees the joint velocity to be near zero after finishing the redundant manipulators task. At the same time, the scheme considers the joint-angle and joint-velocity physical limits, which can keep the joint angle and joint velocity within their given range and prevent the occurrence of high joint velocity during the task duration. Finally, the VLTCO scheme is reformulated as one general quadratic program (QP) problem. The QP problem is then solved by a linear-variational-inequality-based primal–dual neural network solver (LVI-PDNN). The validity and advantages of the VLTCO scheme are substantiated by the simulation results and the physical experiment based on the JACO 2 redundant manipulator.

INDEX TERMS Velocity-level tri-criteria optimization (VLTCO), complex path tracking, joint-velocity physical limits, quadratic program (QP), physical experiment, redundant manipulators.

I. INTRODUCTION

In recent years, robot manipulators have widely used in numerous application fields, such as space exploration [1], [2], undersea exploration [3], search and rescue in complex environments [4], household services [5], [6], [7] and so on. Robot manipulators are divided into redundant manipulators and non-redundant manipulators. Compared with non-redundant manipulators, redundant manipulators have more degrees of freedom (DOF) than required for operational tasks, which make the redundant manipulators work highly flexibly and reliably in some complex environments. Due to the redundancy of the redundant manipulators, there may be multiple solutions or no solutions to joint

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variables when redundant manipulators perform a given task. The traditional solution of redundant manipulators [8] is based on the pseudo-inverse method, which consists of a minimal norm and a homogeneous solution. This method has caused the joint-angle deviation (after the end-effector completes a task, the motion of the end-effector is closed, but the trajectory of the joint angle vector in the joint space is not closed). Also, this method cannot consider the problem of inequality constraints, and it takes a lot of time in the calculation process [9], [10]. To overcome the disadvantage, quadratic programming (QP) methods are proposed in recent years. Many optimization schemes based on quadratic programming have been presented. In reference [11], the optimization scheme was tactfully unified and expressed as QP problems subject to equality, inequality and bound constraint. The resultant of the QP problems can be solved

by many efficient algorithms, such as dual neural networks (DNN) [12], [13], RZND [36], primal-dual neural networks (PDNN) [14], linear-variational-inequality based primal-dual neural network (LVI-PDNN) [15]. In order to solve non-repetitive motion problem of redundant manipulators and position deviation of the joint-angles during the task duration, researchers have shown that the above problem can be solved in more favorably manner via online optimization schemes [16], [17].

Online motion planning considers multiple optimization goals, multiple physical constraints and it has been widely used in redundant manipulators motion planning [18], [37]. Some scholars used different optimization scheme to achieve online motion planning. Such as: the novel minimum-velocity-norm (MVN) [19], the a novel minimum-acceleration-norm (MAN) [27], effective repetitive motion planning (RMP) [20], [21], the infinity-norm velocity minimization (INVM) [22], the infinity-norm torque-minimization (INTM) [23] and Zhang and Zhang [34] have proposed acceleration-level drift-free (ALDF) scheme and developed two recurrent neural networks to solve the challenging resultant QP problem. However, in the actual application process, single-criteria optimization can not fully meet the needs in complex conditions. For this reason, the optimization scheme of bi-criteria was proposed. Such as: Zhang *et al*. [33] presented bi-criteria neural optimization scheme to diminish discontinuity points arising in the infinity-norm velocity minimization scheme. This scheme combines the minimum infinity-norm and the minimum two-norm solutions via a weighting factor.

Other optimization motion planning were proposed by many researchers. For example: English *et al.* [24] developed a single framework for implementing of velocity control on kinematically redundant manipulators. Kaluarachchi *et al*. [25] designed a lightweight tendon drive redundant manipulator using a single motor. Zhao *et al* [26] proposed ant colony optimization algorithm for motion planning of hyper-redundant robot manipulators. Zhang *et al*. [28] have investigated, developed and compared six different methods to solve repetitive motion planning scheme and drawn a strong conclusion about performances of these six methods. To overcome the shortcomings of traditional Jacobian-matrix-pseudo-inverse (JMPI) method, Chen *et al*. [35] proposed a novel Jacobian-matrix-adaption (JMA) method for the tracking control of robot manipulators via the zeroing dynamics.

In order to reduce the joint-angles deviation of at the joint-velocity level and achieve repeated motion planning. Considering multiple optimization design, the velocity level tri-criteria optimization (VLTCO) scheme is proposed. This schemes consider the joint-angle and joint-velocity physical limits, which guarantees the joint-velocity within in their bound constraints. The proposed scheme can reduce the joint position deviation and ensure smooth trajectory of the end-effector. The VLTCO scheme is reformation reformulated as one standard QP problem. The online

FIGURE 1. The block diagram of the VLTCO scheme for redundant manipulators motion planning.

linear-variational-inequalities-based primal-dual neural network (LVI-PDNN) is used to solve the above QP problem. The feasibility and advantages of the proposed scheme are verified by computer simulations of three different complex path-tracking tasks. The physical experiment (i.e., Chinese character "ping" cyclic path) based on the JACO 2 redundant manipulator further proves the effectiveness of the VLTCO scheme. The block diagram of the proposed scheme corresponding to VLTCO method is shown in Fig. [1.](#page-1-0)

Before ending the introduction of this paper, it is necessary to introduce the main contributions of this paper.

- A VLTCO scheme is proposed to remedy the position deviation of joint-angles at the velocity-level. The proposed VLTCO scheme combines the MVN, the RMP and the INVM scheme through the weighting factors.
- The joint-angle and joint-velocity physical limits are taken into account during the operation of the redundant manipulators.
- Computer simulations by different path-tracking tasks (i.e., four-leaf clover pattern, pentagram and Chinese character ''ping'' pattern) are given to illustrate the validity and feasibility of the proposed scheme.
- The physical experiment based on the $JACO²$ redundant manipulator (i.e. tracking the Chinese character ''ping'' path) is further verified the effectiveness of the proposed VLTCO scheme.

II. SCHEME PROPOSED AND QUADRATIC PROGRAM REFORMATION

The forward kinematics equation of the redundant manipulators can be expressed as follows:

$$
r(t) = f(\theta(t))
$$
 (2.1)

where $r(t) \in R^m$ and $\theta(t) \in R^n$ in the Cartesian space represent the end-effector position vector and the join-space vector, respectively. The *n* and *m* denote the dimensionality of joint space and the dimensionality of task space, respectively. In this paper, we use the $JACO²$ redundant manipulator, which is a six-DOF robot manipulator, and the $n = 6$, $m = 3$.

For nonlinearity and the redundancy of the equation [\(2.1\)](#page-1-1), it is usually difficult to be solved. So the above problem of redundancy resolution can be considered at the joint-velocity level:

$$
J(\theta(t))\dot{\theta}(t) = \dot{r}(t) \tag{2.2}
$$

where $\dot{r}(t)$ and $\dot{\theta}(t)$ are the first-order derivative of $r(t)$ and $\theta(t)$, respectively. The $J(\theta(t))$ is the Jacobian matrix and defined as $J(\theta(t)) = \partial f(\theta(t))/\partial \theta(t)$.

A. SCHEME PROPOSED

The proposed VLTCO scheme combines minimum velocity norm (MVN), the repetitive motion planning (RMP) and infinity norm velocity minimization (INVM) through the weighting factor. The proposed scheme can be designed as:

$$
\min. \varphi \|\dot{\theta}\|_2^2/2 + \tau \|\dot{\theta} + C\|_2^2/2 + (1 - \varphi - \tau) \|\dot{\theta}\|_\infty^2/2
$$
\n(2.3)

where $\|\cdot\|_2$ and $\|\cdot\|_{\infty}$ denote the two-norm and an infinite norm of a vector, respectively. *C* is defined as $\mu[\theta - \theta(0)]$. $\varphi \in (0, 1)$, $\tau \in (0, 1)$ and $(1 - \varphi - \tau) \in (0, 1)$ are weighting factor which are used respectively to adjust the weights of MVN, RMP and INVM. Considering the physical limits of the joint-angle and joint-velocity of the redundant manipulators, the VLTCO scheme of the redundant manipulators can be expressed as follows:

min.
$$
\varphi \|\dot{\theta}\|_2^2/2 + \tau \|\dot{\theta} + C\|_2^2/2 + (1 - \varphi - \tau)\|\dot{\theta}\|_\infty^2/2
$$
 (2.4)

$$
\text{s.t.} \quad J(\theta)\dot{\theta} = \dot{r} \tag{2.5}
$$

$$
\theta^- \leqslant \theta \leqslant \theta^+\tag{2.6}
$$

$$
\dot{\theta}^- \leqslant \dot{\theta} \leqslant \dot{\theta}^+ \tag{2.7}
$$

with $C = \mu(\theta - \theta(0))$

where θ^- and θ^+ , $\dot{\theta}^-$ and $\dot{\theta}^+$ denote the lower and upper limits of the joint-angle, joint-velocity vectors respectively. The parameter μ is a positive design parameter and is used to scale the magnitude of the manipulator response to the joint displacement $[\theta - \theta(0)]$. $\theta(0)$ denote the initial value of the joint angle vector $\theta(t)$.

To achieve the purposed of minimizing $\|\dot{\theta} + C\|_2^2/2$ and inspired by the gradient-dynamics method [32] [31], we can define the first scalar-valued error function as follow:

$$
e_1(t) = \theta(t) - \theta(0) \tag{2.8}
$$

Following the effective approach of Zhang *et al.* [32] to force the error function converge to zero, we can simply set:

$$
\dot{e}_1(t) = -\mu e_1(t) = -\mu[\theta(t) - \theta(0)] \tag{2.9}
$$

where design parameter μ is designed to adjust the magnitude of manipulators response to the joint drift *e*¹ to zero.

It is worth pointing out that, forcing the $\dot{e}_1(t) = -\mu e_1(t)$ can guarantees that $e_1(t)$ converges to zero. Theoretically, when $t \rightarrow \infty$, $e_1(t) = \exp(-\mu t)(e_1(0)) \rightarrow 0$ with design parameter $\mu > 0$. Through the above analysis, the design parameters μ should be as larger as possible theoretically. However, considering the physical conditions such as joint-angle limits of redundant manipulators and the computational performance, the design parameter μ is set as $\mu = 5$ which can meet the accuracy requirement without causing too much burden on the computers and redundant arms.

B. QP PROBLEM REFORMULATION

The traditional pseudo-inversion method is difficult to handle the inequality of the above equation $(2.6)-(2.7)$ $(2.6)-(2.7)$ $(2.6)-(2.7)$, and thus the joint physical limits of the redundant manipulators can not be considered. Since almost all the redundant manipulators are physically constrained by mechanical constraints (i.e., joint-angle limits, joint-velocity limits, force limits and so on). It is feasible to consider the redundancy resolution scheme [\(2.4\)](#page-2-0)-[\(2.7\)](#page-2-0).

Firstly, the first and second terms in equation [\(2.4\)](#page-2-0) represent the MVN and the RMP schemes, respectively. The MVN and RMP criteria can be expressed as follows:

$$
\varphi \|\dot{\theta}\|_2^2/2 = \varphi \dot{\theta}^{\mathrm{T}} I \dot{\theta}/2 \tag{2.10}
$$

$$
\tau \|\dot{\theta} + C\|_2^2 / 2 = \tau (\dot{\theta}^T I \dot{\theta} + \Lambda \dot{\theta}) / 2 \tag{2.11}
$$

where the superscript T denotes the transpose of a vector (or a matrix), $I \in R^{n+m}$ denotes an identity matrix and $\Lambda = 2\mu$.

Secondary, the third terms in the equation [\(2.4\)](#page-2-0) represent the INVM criterion, the INVM criterion $((1 - \varphi - \tau)\|\ddot{\theta}\|_{\infty}^2/2)$ can be rewritten equivalently as follows:

min.
$$
(1 - \varphi - \tau)p^2/2
$$
 (2.12)

$$
\text{s.t.} \qquad \begin{bmatrix} I & -I_{\nu} \\ -I & -I_{\nu} \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ p \end{bmatrix} \leqslant \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \qquad (2.13)
$$

where $p = ||\dot{\theta}||_{\infty}$, I_{ν} is a vector whose all elements are one $(i.e., I_v = [1, 1, ..., 1]^T \in R^{n+m}).$

Thirdly, because the proposed VLTCO scheme is solved at the joint-velocity level (i.e., $\dot{\theta}$). The range of the joint physical limits $[\theta^-, \theta^+]$ in the equation [\(2.6\)](#page-2-0) can be converted to the following bound constraint based on the corresponding velocity [11]:

$$
\kappa(\theta^{-} - \theta) \le \dot{\theta} \le \kappa(\theta^{+} - \theta) \tag{2.14}
$$

where the parameters $\kappa > 0$ is designed to define the feasible region of θ . It should be noted that the parameter κ needs to ensure that the feasible region of converted by the joint limits [\(2.14\)](#page-2-1) is greater than or equal to its original region given by (2.7) . Thus the bound constraints $(2.6)-(2.7)$ $(2.6)-(2.7)$ can be combined into a unified bound constraint whose lower limit and upper limit can be expressed as:

$$
\zeta^{-} = \max\{\kappa(\theta_i^{-} - \theta_i), \dot{\theta}_i^{-}\}\tag{2.15}
$$

$$
\zeta^+ = \min\{\kappa(\theta_i^+ - \theta_i), \dot{\theta}_i^+\}\tag{2.16}
$$

Fourthly, in order to transform the VLTCO solution into a standard quadratic programming, the decision variable *x* is defined $x = [\dot{\theta}^T, p]^T \in R^{n+1}$. The unified QP form can be expressed as follows:

$$
\min. \; x^{\mathrm{T}} Dx / 2 + g^{\mathrm{T}} x \tag{2.17}
$$

$$
A x = b \tag{2.18}
$$

 $Ex \le d$ (2.19)

$$
x^- \le x \le x^+ \tag{2.20}
$$

$$
D = \begin{bmatrix} (\varphi + \tau)I & 0 \\ 0 & (1 - \varphi - \tau) \end{bmatrix} \in R^{(n+1)\times(n+1)}
$$

\n
$$
g = \begin{bmatrix} \tau \Lambda(\theta - \theta(0)) \\ 0 \end{bmatrix} \in R^{(n+1)}, A = \begin{bmatrix} J & 0 \end{bmatrix} \in R^{m\times(n+1)}
$$

\n
$$
b = \dot{r} \in R^m, E = \begin{bmatrix} I & -I_v \\ -I & -I_v \end{bmatrix} \in R^{2n\times(n+1)}, d = 0 \in R^{2n}
$$

\n
$$
x^- = \begin{bmatrix} \zeta^- \\ 0 \end{bmatrix} \in R^{n+1}, x^+ = \begin{bmatrix} \zeta^+ \\ \varpi \end{bmatrix} \in R^{n+1}
$$

where ϖ is a large constant, which is used to replace $+\infty$ numerically.

C. QP SOLUTION

In this section, the LVI-PDNN solver [21] is used for solving the QP [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2) problem.

Firstly, QP problem [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2) should first be converted to a linear variational inequalities (LVI) problem. That is equivalent to finding a primal-dual equilibrium matrix *y* [∗] ∈ $\Omega := \Omega = \{y | y^- \leq y \leq y^+ \} \subset R^{2(3n+m+1)}$ which satisfies the inequalities as below

$$
(y - y^*)^T (My^* + q) \ge 0, \forall y \in \Omega \tag{2.21}
$$

where the primal-dual decision variable vector $y \in R^{3n+m+1}$ and its lower and upper bounds is defined as:

$$
y = \begin{bmatrix} x \\ u \\ v \end{bmatrix}, y^- = \begin{bmatrix} x^- \\ -I_v \varpi \\ 0 \end{bmatrix}, y^+ = \begin{bmatrix} x^+ \\ I_v \varpi \\ I_v \varpi \end{bmatrix}
$$

where decision vector $u \in R^m$ and $v \in R^{2n}$ are defined for equality constraint [\(2.18\)](#page-2-2) and inequality constraint [\(2.19\)](#page-2-2), respectively. In addition, the augmented matrix *M* ∈ $R^{(3n+m+1)} \times (3n+m+1)$ and vector $q \text{ ∈ } R^{(3n+m+1)}$ are defined as

$$
M = \begin{bmatrix} D & -A^T & E^T \\ A & 0 & 0 \\ -E & 0 & 0 \end{bmatrix}, q = \begin{bmatrix} g \\ -b \\ d \end{bmatrix}
$$

Secondly, according to the reference [21], [28], we know that the equation [\(2.21\)](#page-3-0) is equivalent to a corresponding piecewise-linear equations:

$$
P_{\Omega}(y - (My + q)) - y = 0 \tag{2.22}
$$

where $P_{\Omega}(\cdot)$: $R^{2(3n+m+1)} \to \Omega$ and the *i*th element of $P_{\Omega}(y)$ is a projection operator and defined as:

$$
\begin{cases} y_i^-, & if & y_i < y_i^-, \\ y_i, & if & y_i^- \le y_i \le y_i^+, \\ y_i^+, & if & y_i \ge y_i^+, \end{cases}
$$

Thirdly, according to the neural network design experience in [29], based on the primal dual neural networks, the equation [\(2.22\)](#page-3-1) can be further computed by the linear variational inequality based primal dual neural networks (LVI-PDNN) such that:

$$
\dot{y} = \gamma (I + M^T)(P_{\Omega}(y - (My + q)) - y) \tag{2.23}
$$

where the parameter γ is designed to adjust the convergence rate of neural network, which would affect the convergent rate of the position errors. Generally speaking, the larger of the γ , the smaller of the position error. However, the larger parameter γ , the more time the computer simulation process takes. Considering the performance of the computer and practical applications, parameter γ is set to 10⁷. Finally, we have the globally exponentially convergent of the presented LVI-PDNN when solving the QP problem [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2) in real time.

III. COMPUTER AND PHYSIC EXPERIMENT

In this section, we employ the aforementioned neural network (i.e., LVI-PDNN) to solve redundancy resolution scheme [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2). The computer simulation and physical experiment results performed on the JACO 2 redundant manipulator. The end-effector of the JACO² manipulator is expected to track three paths, i.e., four-leaf clover pattern, five-point star and Chinese character ''ping''. The initial state in the three path tracking is $\theta(0) = [1.675, 2.79, -3.216,$ 4.187, -1.71 , -2.95]^T(rad). The joint angles are expected to return their initial joint after tracking the path. In addition, the joint-angle limits and the joint-velocity limits are shown in Table [1.](#page-3-2) Throughout this paper, the general parameters are set as: $\kappa = 20$, $\gamma = 10^7$ and $\Lambda = 20$.

TABLE 1. Joint-angle and Joint-Velocity Physical Limits.

In order to illustrate the validity of the proposed VLTCO scheme in remedying the joint-angle drift at the velocitylevel, we give computer simulation results of the single criteria (MVN) scheme and the bi-criteria (MVN-INVM) scheme to track the four-leaf clover pattern. Further more, another two path-tracking results (i.e., Five-point star and the Chinese character ''ping'') are illustrated based on the VLTCO scheme.

A. ''FOUR-LEAF CLOVER PATERN'' PATH-TRACKING EXAMPLES

In this paper, three different simulation results of $JACO²$ redundant manipulator following four-leaf clover pattern are presented. The motion task duration is $T_{all} = 5 \times T = 5s$.

Figs. [\(2\)](#page-4-0)-[\(4\)](#page-5-0) show the simulation results of the $JACO²$ manipulator tracking the four-leaf clover pattern in the 3-D workspace using different optimization scheme (MVN $(\varphi = 1 \text{ and } \tau = 0)$, MVN-INVM $(\varphi = 0.2 \text{ and } \tau = 0)$, VLTCO ($\varphi = 0.2$ and $\tau = 0.4$)).

Firstly, as can be seen in the Fig. [2\(](#page-4-0)a), the four-leaf clover path tracking task is performed well by the $JACO²$ manipulator, which can be further verified from small end-effector

TABLE 2. Three Joint-Angle Drift (θ_L (4) – θ_L (0) (RAD)) at the Velocity Level when the End-effector of the Redundant Manipulator Tracking the Four-leaf-Clover Pendant.

Joint $\theta(5) - \theta(0)$		H o	θ_3		ワミ	
$\alpha = 1, \beta = 0$ (MVN)	-1.13×10^{-1}	5.25×10^{-2}	-8.70×10^{-2}	-3.00×10^{-1}	-4.47×10^{-2}	4.837×10^{-2}
$\alpha = 0.2, \beta = 0$ (MVN-INVM)	-2.84×10^{-2}	2.36×10^{-3}	-6.73×10^{-2}	-5.60×10^{-2}	2.5×10^{-2}	4.837×10^{-2}
$\alpha = 0.2, \beta = 0.4$ (VLTCO)	-2.44×10^{-4}	-1.2×10^{-4}	-1.12×10^{-3}	-1.77×10^{-3}	-9.10×10^{-4}	7.79×10^{-4}

TABLE 3. Two different joint-Angle Drift (θ(i) − θ(0) (RAD)) at the Velocity Level when the End-effector of the Redundant Manipulator Tracking the different path by VLTCO scheme (with $\varphi = 0.2$ and $\tau = 0.4$).

FIGURE 2. JACO² manipulator motion planning of four-leaf-clover pattern path synthesized by the MVN scheme (with $\varphi = 1$ and $\tau = 0$). (a) End-effector states of JACO² manipulator. (b) End-effector positioning errors. (c) Joint-angle transients. (d) Joint-velocity transients.

position error in Fig. [2\(](#page-4-0)b) (position error less than 10×10^{-5} (m)). However, from Fig. [2\(](#page-4-0)a) we see that the end position of the joint angles do not coincide with the initial state. In other words, the manipulator dose not return to the initial position after finishing the path-tracking, which means that the position deviation of the joint-angles have happened during the task duration. The detail position deviation can be obtained from the first row of Table 2, which are less than -3.0×10^{-1} (rad). Fig. [2\(](#page-4-0)c) shows the transient states of joint angles movement, which further express the state of joint angle after following a trajectory. During executing a closed task, position deviation of joint-angles will cause accumulative error which seriously damage the manipulators. At the same time, the Fig. [2\(](#page-4-0)d) shows that $\dot{\theta}_1$ exceeds the lower joint-velocity physical limits -1.5 (rad/s) at the time t=2.23s, and in the practical application, high velocity will damage the redundant manipulators. Obviously, the MVN scheme is not efficient in remedying the position deviation of joint-angles when the redundant manipulators tack the complex path.

FIGURE 3. JACO² manipulator motion planning of four-leaf-clover pattern path synthesized by the MVN-INVM scheme (with $\varphi = 0.2$ and $\tau = 0$). (a) End-effector states of JACO² manipulator. (b) End-effector positioning errors. (c) Joint-angle transients. (d) Joint-velocity transients.

Secondly, the Figs. [3](#page-4-1) show the motion trajectory of $JACO²$ manipulator synthesized by MVN-INVM scheme (i.e., equa-tions [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2), $\varphi = 0.2$ and $\tau = 0$). The Fig. [3\(](#page-4-1)a) and (b) respectively show the end-effector state and the end-effector position errors of the JACO² manipulator, which illustrate that the task is completed well by the MVN-INVM scheme. From Fig. [3\(](#page-4-1)a) we see that position deviation of joint-angles can be effectively controlled and it can be further verified from the detail in the second row of Table [2](#page-4-2) (joint-angle error less than -6.7×10^{-2} (m)). Fig. [3\(](#page-4-1)d) shows that all the joint angle limits are with in their upper physical limits (1.5 (rad/s)) and lower physical limits (−1.5 (rad/s)), respectively. Although the MVN-INVM scheme can effectively control position deviation of joint-angles, it can not meet the requirements in some situations with high precision requirements.

Thirdly, Fig. [4](#page-5-0) shows the four-leaf clover pattern path tracking result synthesized by the proposed VLTCO scheme (i.e., equations [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2) with $\varphi = 0.2$ and $\tau = 0.4$). The four-leaf clover pattern path is tracked well which

FIGURE 4. JACO² manipulator motion planning of Four-leaf-clover pattern path synthesized by the VLTCO scheme (with $\varphi = 0.2$ and $\tau = 0.4$). (a) End-effector states of JACO 2 manipulator. (b) End-effector positioning errors. (c) Joint-angle transients. (d) Joint-velocity transients.

can be seen from Figs. [4\(](#page-5-0)a) and (b). In addition, the final state is coincided with the initial state well, which can be further illustrated from the third row of the Table [2](#page-4-2) (the joint-angle drift error less than -1.17×10^{-3} (rad)). The Figs. [4\(](#page-5-0)c) and (d) show the joint-angle and joint-velocity transient during the executing the task and the joint-angle and joint-velocity are within their physical limits, which can avoid the problem of high joint-velocity in the operation of the manipulator. This means that the position deviation of joint-angle problem is remedied well by the proposed VLTCO scheme.

B. ''FIVE-POINT STAR'' AND CHINESE CHARACTER ''PING'' PATH-TRACKING EXAMPLES

In order to further illustrate the validity and advantage of the proposed VLTCO scheme and make a better comparison, another two path tracking (i.e., five-point star and the Chinese character ''ping'') are presented only considered the VLTCO scheme. In order to get a better comparison, all the design parameters are set the same as the four-leaf clover pattern path-tracking experiment of the VLTCO scheme except the duration.

Figs. [5](#page-5-1) and Figs. [6](#page-5-2) show the computer-simulation of the five-point star path-tracking and Chinese character ''ping'' path-tracking by the VLTCO scheme, respectively. The motion-task duration are T_{all} = 5 × T = 5s and T_{all} = $15 \times T = 15s$, respectively.

From the Fig. [5\(](#page-5-1)a) and Fig. [6\(](#page-5-2)a), we can see that the final state concise with the initial state. The end-effector position error are less than 10^{-4} through the [5\(](#page-5-1)b) and Fig. [6\(](#page-5-2)b). From the Figs. $5(c)$ $5(c)$ and (d) and Figs. $6(c)$ $6(c)$ and (d), we see that the joint-angle and joint-velocity are within their physical limits during the task period. At the same time, from the Fig. [5\(](#page-5-1)d),

FIGURE 5. JACO² manipulator motion planning of five-pointed star pattern path synthesized by the VLTCO (with $\varphi = 0.2$ and $\tau = 0.4$). (a) End-effector states of JACO² manipulator. (b) End-effector positioning errors. (c) Joint-angle transients. (d) Joint-velocity transients.

FIGURE 6. JACO² manipulator motion planning of Chinese character "ping" path synthesized by the VLTCO (with $\varphi=$ 0.2 and $\tau=$ 0.4). (a) End-effector states of JACO² manipulator. (b) End-effector positioning errors. (c) Joint-angle transients. (d) Joint-velocity transients.

the joint-angle physical limits is activated, which ensure the joint-angle velocity with in their physical limits during the task duration. The position deviation of joint-angle data can be seen through the Table [3,](#page-4-3) which show that the drift errors less than 3.309×10^{-3} (*rad*). The validity and the advantage of the proposed VLTCO scheme have been further verified through the above computer simulations.

C. PHYSICAL EXPERIMENT

In order to verify the effectiveness of the VLTCO scheme [\(2.17\)](#page-2-2)-[\(2.20\)](#page-2-2) in practical application. We perform

FIGURE 7. The physical experiment snapshots of the VLTCO scheme when the JACO² redundant manipulator tracking the Chinese character ''ping'' path.

the VLTCO scheme and the corresponding solver S-LVI-PDNN (2.23) on JACO² manipulator. The physical experiment platform is composed of computer and $JACO²$ manipulator. The joint angles data are generated by controlling velocity signal, and then read by the visual studio program and sent the data to the $JACO²$ manipulator to realize the movement of the JACO² manipulator.

The physical experiment is shown based on the $JACO²$ manipulator in this subsection, which are the end-effector tracking the Chinese character ''ping'' path. It worth pointing out that the parameters are set as before computer simulation of the path tracking based on the VLTCO scheme. The snapshot of the path-tracking physical experiment results is shown in Fig. [7.](#page-6-0) The Fig. [7](#page-6-0) shows that the $JACO²$ manipulator performs the closed path smoothly and stably during the task execution period, which shows that the end-effector pathtracking task is perform well. The physical experiment further illustrates the effectiveness, validity of the proposed VLTCO scheme.

IV. CONCLUSION

In this paper, a VLTCO scheme has been proposed to remedy the position deviation of joint-angles problem of redundant manipulators. The proposed VLTCO scheme is together with the MVN scheme, RMP scheme, INVM scheme through weight factors. It is worth pointing out that the scheme has considered the joint-angle and joint-velocity limits, which has avoided high joint-velocity during the manipulators executing the task. And then the proposed VLTCO scheme has been formulated in one unified QP problem, which have been solved by the LVI-PDNN solver. Comparative simulations have verified the effectiveness, validity of the proposed VLTCO scheme when remedying the joint-angle drift problem of the redundant manipulators. Finally, the physical experiment has further verified the advantage of the proposed scheme. The future research directions may be: ''Recurrent-Neural-Network-Based Velocity-Level Redundancy Resolution for Manipulators Subject to a Joint Acceleration Limit'', ''Adaptive Projection Neural Network for Kinematic Control of Redundant Manipulators With Unknown Physical Parameters", "motion planning problem of redundant manipulators with multi-criteria at different levels'' and so on.

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