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Consensus Constructing in Large-Scale Group Decision Making With Multi-Granular Probabilistic 2-Tuple Fuzzy Linguistic Preference Relations

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ABSTRACT As the number of participants involves in decisions getting complex and the heterogeneity could be produced among decision makers, a large-scale group decision making (LGDM) method with consensus constructing need to be considered. In order to demonstrate the complex relationship and reduce heterogeneity among decision makers, a consensus process of LGDM is proposed in this paper, in which multi-granular probabilistic fuzzy linguistic preference relations (MGPFLPRs) are used to represent sub-group's preferences information. First, mathematical programming is proposed to deal with MGPFLPR based on expected multiplicative consistency and obtain the priority weight vector. Second, collective priority weights of alternative are obtained by fusing sub-group's priority weights of alternative based on the weighted averaging operator. Then, an automatic iteration consensus reaching algorithm is implemented for the purpose of reaching a consensus in LGDM with MGPFLPRs. Finally, an emergency decision problem is applied to demonstrate the effectiveness of the proposed method.

INDEX TERMS Probabilistic linguistic preference relation, multi-granular linguistic term sets, large-scale group decision making, expected multiplicative consistency, emergency decision.

I. INTRODUCTION

As the decision-making environment becomes more complex and diverse, a large number of decision makers (DMs) who are the strong support of the network environment and societal demands should take part in the decision making process [1]–[3]. As a result, some methods are proposed to resolve the large-scale group decision making (LGDM) problems. The methods can be classified into four frameworks: clustering-based LGDM [4], [5], consensus reaching process (CRP) in LGDM [6]–[12], LGDM methods [13]–[18], and LGDM support systems [19].

For complex and multi-faceted problems, DMs may give vague or uncertain knowledge about alternative instead of exact numerical values. Consequently, linguistic term sets [20], [21] or fuzzy numbers [22]–[24] could be used to express preferences information of DMs in LGDM process. A variety of linguistic models, including fuzzy number-based model, 2-tuple linguistic, and virtual linguistic, have been introduced to deal with practical problems [25]–[29]. Then, there are more and more LGDM methods which are researched based on linguistic information [13]–[18]. Liu *et al.* [13] presented a two-layer weight determination model based on linguistic information. Liu *et al.* [14] proposed a weight value determination method for multiple groups based on subjective and objective information. Zhang *et al.* [15] proposed a LGDM model based on multi-granular linguistic distribution information. Song and Li [16] presented a LGDM model with incomplete multi-granular probabilistic linguistic term sets. Gou *et al.* [17] developed a CRP for LGDM with double hierarchy hesitant fuzzy linguistic preference relations. Wu *et al.* [18] used linguistic principal component analysis and fuzzy equivalence clustering based on linguistic information to solve LGDM problems.

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Existing related studies have made significant contributions to LGDM method researches. However, the linguistic information-based LGDM methods mentioned above have some drawbacks. For [13], [14], [17], [18], they only provide a single linguistic item set with DMs to express preferences information which does not conform to the characteristics of large group decision makers. In LGDM processes, DMs come from different departments and fields, where different decision makers have different educational background, knowledge, and experience, i.e., therefore it appears insufficient in respect of merely a single linguistic item set [30]. In this way, it is more appropriate for heterogeneous DMs to employ multi-granular linguistic information expressing evaluation values [31]–[33].

Consensus is a significant issue widely considered in group decision making problems [34]–[36]. Furthermore, how to implement CRP in LGDM is also an important issue. While, previous studies based on multi-granular linguistic information [15], [16] have not yet been considered CRP, which reduces the acceptability of decision results. According to whether DMs participate in the consensus process, the consensus models may be classified as interactive CRP and automatic CRP. For interactive CRP, it's time consuming to supervise and modify opinions, which might not only increase the CRP's discussion rounds for LGDM, but also lead to a result that some DMs may lose their motivation and then eventually give up the discussion process [37]. The automatic CRP has the advantage that no DMs need to participate in the consensus process and rapidly approach the consensus goal. Therefore, it is realistic in LGDM problems to carry out automatic iteration CRP. Especially for emergency decision making within a limited amount of time, it is more suitable to implement automatic iteration CRP.

The first and foremost step during any LGDM problems is to gather assessment information from decision makers. Pang, Wang and Xu [38] propose a novel linguistic-based presentation tool called probabilistic linguistic term set (PLTS), which consists of two parts: the linguistic terms, and corresponding probabilities. It provides a powerful tool to indicate the preferences of the whole group or a subgroup in an LGDM problem. For example, ten consumers are invited to rate the control of a car based on linguistic terms set {somewhat good, good, very good}. Six consumers think the control level is 'somewhat good', three consumers think the control level is 'good', and one consumer thinks the control level is 'very good'. Thus, the PLTS {somewhat good (0.6), good (0.3), very good (0.1)} can be applied to represent the control of a car. Meanwhile, the preference information obtained by making pairwise between any two alternatives is more clearly to reflect the relationship between two alternatives [17]. Therefore, it is very suitable and significant to apply probabilistic fuzzy linguistic preference relations with PLTSs based on multi-granular linguistic term sets into LGDM.

Based on the above motivations, the aims of this paper are to construct a multi-granular linguistic-based consensus model in LGDM and establish simultaneously an automatic iteration CRP. Firstly, fuzzy linguistic preference relation (FLPR) is applied to represent preference information of DMs over alternative, and then MGPFLPRs are obtained through the statistic calculating to represent subgroup preferences information. An expected multiplicative consistency of the probabilistic fuzzy linguistic preference relation is defined and a mathematical programming model for MGPFLPRs is presented to get priority weights of alternative. Furthermore, sub-group priority weights are fused and the collective priority weights of alternative are obtained. For the PLPRs which are not reached the satisfactory consensus level, a self-iterating CRP algorithm is implemented to obtain the consensus with satisfying level.

The main contributions of this paper on LGDM problems can be summed up as follows:

1) A mathematical programming model is proposed to deal with MGPFLPR based on expected multiplicative consistency, which avoids uniform translation of linguistic granularity and reserves more original judgment information.
2) A self-iterating consensus reaching process, which not only is easy to implement but also fits the characteristics of LGDM problems, is constructed for MGPFLPRs.
3) The proposed approach is applied to emergency rescue plan selection, which has shown broad application in solving the practical decision-making problems.

The rest of this paper is organized as follows. Section II reviews some basic knowledge of linguistic information, FLPR, PLPR, and expected multiplicative consistency of PLPR. In Section III, a consensus process with a mathematical programming model and a CRP algorithm are presented to deal with MGPFLPRs in LGDM. In Section ĉô, an emergency decision-making problem is solved by the proposed method, and a comparison analysis between our method and the previous research is also given in the section. Concluding remarks are given in Section V.

II. PRELIMINARIES

In this section, we review the related knowledge about linguistic information, multi-granular linguistic information and 2-tuple FLPR and PLPR.

A. LINGUISTIC INFORMATION

To facilitate the assessment using linguistic for DMs, a linguistic term set should be determined beforehand. The widely used linguistic term set has following characteristics: a) the odd value with granularity; b) its membership functions is symmetrical and uniformly distributed; c) the midterm of linguistic term set represents "indifference", with the remainder of the linguistic terms symmetrically and uniformly being placed on either side of it. Let $S = \left\{s_0, s_1, \cdots, s_g\right\}$ be a linguistic term set with odd granularity $g + 1$. Moreover, the term set S should satisfy the following features [25]: 1) A negation operator: $Neg\,(s_i) = s_{g-i}$; 2) An order: $s_i \geq s_j$ if $i \geq j$.

The 2-Tuple linguistic expressive model was presented [25] and it may improve the accuracy and interpretability of linguistic computational models.

Definition 1 [25]: Suppose $\beta \in [0, g]$ be the result of a symbolic aggregation operation in a linguistic term set $S = \{s_0, s_1, \cdots, s_g\}$. Then, the equivalent information to β in the 2-tuple is secured by means of the following function:

$$\Delta : [0, g] \rightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \text{ with } \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-0.5, 0.5) \end{cases} \tag{1}$$

where round is the rounding operation.

Definition 2 [25]: Suppose $S = \{s_0, s_1, \cdots, s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple, then there exists a function Δ^{-1} which transforms a 2-tuple into its equivalent numerical value $\beta \in [0, g]$. The function Δ^{-1} is defined as follows:

$$\Delta^{-1} : S \times [-0.5, 0.5) \rightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{2}$$

According to definition 1 and definition 2, a linguistic label can be transformed into a two-tuple linguistic by adding a zero as a symbolic translation, i.e., $\Delta(s_i) = (s_i, 0)$.

In addition, the comparison of two-tuple linguistic was given in [25]. For two 2-tuples: (s_i, α) and (s_j, β), then

(1) If $i < j$, then (s_i, α) is smaller than (s_j, β).

(2) If $i = j$, then

(a) if $\alpha = \beta$, then (s_i, α) and (s_j, β) represent the same information;

(b) If $\alpha < \beta$, then (s_i, α) is smaller than (s_j, β).

B. MULTI-GRANULAR LINGUISTIC INFORMATION

When large amounts of DMs are taken part in a decision making process, different DMs showcase different levels of uncertainty regarding the items. It is natural that linguistic term sets with multi-granularity could be used to provide their preferences about the alternative. If a decision maker has the capacity to deliver precise information, he/she may use a finer granularity linguistic term set. Contrarily, the decision maker is likely to use a coarse granularity linguistic term set [26].

In this paper, LGDM problems based on multi-granular linguistic term sets could be handled where DMs $d_k\,(k = 1, 2, \cdots, l)$ may express their linguistic preference relations on the set of alternatives $X = \{x_1, x_2, \cdots, x_n\}$ based on multi-granular linguistic term sets. Let $S^{g(1)}, S^{g(2)}, \cdots, S^{g(r)}$ be multi-granular linguistic term sets to be used by the DMs, where $S^{g(h)} = \left\{s_0^{g(h)}, s_1^{g(h)}, \cdots, s_{g(h)-1}^{g(h)}\right\}$, $h \in \{1, 2, \cdots, r\}$ is a linguistic term set with a granularity of $g(h)$.

C. 2-TUPLE FLPR AND MULTIPLICATIVE CONSISTENCY OF FLPR

Definition 3 [39]: The $B = (b_{ij})_{n\times n}$ is called a 2-tuple fuzzy LPR, if the following conditions hold for all i, j

$$\Delta^{-1}(b_{ij}) + \Delta^{-1}(b_{ji}) = g, \quad \Delta^{-1}(b_{ii}) = \frac{g}{2}. \tag{3}$$

where $S = \{s_0, s_1, \cdots, s_g\}$ is a given linguistic term set.

Jin *et al.* [40] proposed the definition of multiplicative consistent linguistic preference relation based on the multiplicative consistency of fuzzy preference relation as follows:

Definition 4 [40]: For an FLPR $B = (b_{ij})_{n\times n}$, there exists a weight vector $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ with $\sum_{i=1}^n \omega_i = 1$, $\omega_i \geq 0$ and satisfies the following equation

$$\Delta^{-1}(b_{ij}) = g\frac{\omega_i}{\omega_i + \omega_j}, \quad \forall i, \ j \in \{1, 2, \cdots, n\}. \tag{4}$$

then $B = (b_{ij})_{n\times n}$ could be defined a multiplicative consistent FLPR.

D. PLTS, PLPR AND EXPECTED MULTIPLICATIVE CONSISTENCY OF PLPR

Based on research [38], the definition of probabilistic linguistic term set (PLTS) with 2-tuple linguistic is proposed as follows:

Definition 5 [16]: The definition of PLTS can be given as follows:

$$L(p) = \left\{ L^{(k)}\left(p^{(k)}\right) \middle| L^{(k)} \in S, p^{(k)} \geq 0, k = 1, 2, \cdots, \#L(p), \sum_{k=1}^{\#L(p)} p^{(k)} \leq 1 \right\}. \tag{5}$$

where $L^{(k)}\left(p^{(k)}\right)$ denotes the 2-tuple linguistic $L^{(k)}$ associated with the probability $p^{(k)}$, whereas $\#L(p)$ suggests the number of all different 2-tuple linguistic terms in $L(p)$.

On the basis of the probabilistic linguistic preference relation (PLPR) proposed by Zhang *et al.* [41], we give the definition of PLPR based on 2-tuple linguistic as follows:

Definition 6: Let PLPR be represented by a matrix $B = (L_{ij}(p))_{n\times n} \subset X \times X, i, j = 1, 2, \cdots, n$. $L_{ij}(p) = \left\{L_{ij,k}(p_{ij,k}) \middle| k = 1, 2, \cdots, \#L_{ij}\right\}$ are PLTSs based on the given linguistic scale set $S = \{s_0, s_1, \cdots, s_g\}$, where $p_{ij,k} \geq 0, \sum_{k=1}^{\#L_{ij}} p_{ij,k} = 1$, and $\#L_{ij}(p)$ is the number of linguistic terms in $L_{ij}(p)$. $\#L_{ij}$ is expressed as the preference degrees of the alternative x_i over x_j and satisfies the following conditions:

$$p_{ij,k} = p_{ji,k}, \quad \Delta^{-1}(L_{ij,k}) + \Delta^{-1}(L_{ji,k}) = g,$$
$$L_{ii}(p) = \left\{s_{g_2}(1)\right\}, \quad \#L_{ij} = \#L_{ji} \tag{6}$$
$$L_{ij,k} < L_{ij,k+1}, \quad L_{ji,k} > L_{ji,k+1}, \quad i > j \tag{7}$$

where $L_{ij,k}$ and $p_{ij,k}$ are the kth 2-tuple linguistic term and the occurrence probability of the kth linguistic term in $L_{ij}(p)$, respectively.

Remark 1: The PLTS based on 2-tuple linguistic can be used to represent the linguistic assessment of a group. Assume that risk of an investment program is assessed by five DMs using a linguistic term set $S^5 = \{s_0^5, s_1^5, \cdots, s_4^5\}$. If the assessments of the five DMs are $s_1^5, s_2^5, s_1^5, s_3^5, s_2^5$, then the overall assessment could be denoted as a PLTS $\{s_1^5(0.4), s_2^5(0.4), s_3^5(0.2)\}$.

Definition 7: Let $L(p) = \{L_k(p_k) | k = 1, 2, \cdots, \#L(p)\}$ be a PLTS, its expected value can be defined as follows:

$$E(L(p)) = \bar{e} = \sum_{k=1}^{\#L(p)} \Delta^{-1}(L_k) \cdot p_k, \tag{8}$$

where $\#L(p)$ is the number of possible elements in $L(p)$.

Zhou and Xu [42] proposed the expected consistency of probabilistic hesitant fuzzy preference relations to establish probability calculation method. Inspired by [42], the expected multiplicative consistency of the PLPR is presented based on the multiplicative consistency of FLPR as follows:

Definition 8: Let alternative $X = \{x_1, x_2, \cdots, x_n\}$ be a set of alternative, $H = (L_{ij}(p))_{n\times n} \subset X \times X, i, j = 1, 2, \cdots, n$ is a PLPR, where $L_{ij}(p) = \{L_{ij,k}(p_{ij,k}) | k = 1, 2, \cdots, \#L_{ij}\}$ is a PLTS expressed as the preference degrees of alternative x_i over x_j, then H is the expected multiplicative consistency if $\bar{e}_{ij} \cdot \bar{e}_{jk} \cdot \bar{e}_{ki} = \bar{e}_{ik} \cdot \bar{e}_{kj} \cdot \bar{e}_{ji}, \forall i, j, k \in N$, which can be expressed as follows:

$$\bar{e}_{ij} = \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k} = g\frac{\omega_i}{\omega_i + \omega_j}, \quad i, j = 1, 2, \cdots, n. \tag{9}$$

where $k = 1, 2, \cdots, \#L_{ij}$ and $\#L_{ij}$ is the number of possible linguistic terms in L_{ij}.

III. CONSENSUS PROCESS WITH MULTI-GRANULAR PROBABILISTIC FLPRS (MGPFLPRS)

In this section, a consensus process in LGDM is presented based on MGPFLPRs. Firstly, a mathematical programming model is proposed to deal with MGPFLPR based on expected multiplicative consistency and obtain the priority weight vector in Subsection A. Then, weighted averaging (WA) operator is used to obtain collective priority weight vector in Subsection B. In Subsection C, a CRP algorithm is established for LGDM. Finally, a step by step procedure of the GDM model with MGPFLPRs is constructed in Subsection D.

A. A MATHEMATICAL PROGRAMMING MODEL TO DEAL WITH MGPFLPRs

In order to obtain weight vector of alternatives, a mathematical programming model can be developed based on the expectant consistency of PLPR as follows:

$$\begin{aligned} \min \varepsilon_{ij} &= \left|(\bar{e}_{ij} - g)\,\omega_i + \bar{e}_{ij}\omega_j\right| \\ &= \left|\left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k} - g\right)\omega_i \right. \\ &\quad \left. + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\omega_j\right| \\ s.t. &\begin{cases} \sum_{i=1}^{n} \omega_i = 1, \quad \omega_i \geq 0 \\ i, \qquad\qquad j = 1, 2, \cdots, n \end{cases} \end{aligned} \tag{10}$$

In order to simplify the model (10), Theorem 1 is presented as follows:

Theorem 1: For the model (10), the following relationship is established

$$\begin{aligned} &\left|\left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ji,k}) \cdot p_{ji,k} - g\right)\omega_j + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ji,k}) \cdot p_{ji,k}\omega_i\right| \\ &\quad = \left|\left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k} - g\right)\omega_i \right. \\ &\qquad \left. + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\omega_j\right|. \end{aligned}$$

Proof: Based on the properties of the PLPR, we have $\Delta^{-1}(L_{ji,k}) = g - \Delta^{-1}(L_{ij,k}), p_{ji,k} = p_{ij,k}$.

Then,

$$\begin{aligned} &\left|\left(\sum_{k=1}^{\#L_{ji}} \Delta^{-1}(L_{ji,k}) \cdot p_{ji,k} - g\right)\omega_j + \sum_{k=1}^{\#L_{ji}} \Delta^{-1}(L_{ji,k}) \cdot p_{ji,k}\omega_i\right| \\ &= \left|\left(\sum_{k=1}^{\#L_{ij}} \left(g - \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\right) - g\right)\omega_j \right. \\ &\quad \left. + \sum_{k=1}^{\#L_{ij}} \left(g - \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\right)\omega_i\right| \\ &= \left| - \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\omega_j \right. \\ &\quad \left. + \sum_{k=1}^{\#L_{ij}} \left(g - \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\right)\omega_i\right| \\ &= \left|\left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k} - g\right)\omega_i \right. \\ &\quad \left. + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}(L_{ij,k}) \cdot p_{ij,k}\omega_j\right| \end{aligned}$$

which completes the proof of Theorem 1. □

According to Theorem 1, the model (10) is simplified to the following model (11).

$$\min \varepsilon_{ij} = \left| \left(\bar{e}_{ij} - g\right) \omega_i + \bar{e}_{ij}\omega_j \right| = \left| \left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k} - g \right) \omega_i + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k}\omega_j \right|$$

$$s.t. \begin{cases} \sum_{i=1}^{n} \omega_i = 1, & \omega_i \geq 0 \\ i, & j = 1, 2, \cdots, n, \quad i < j. \end{cases} \tag{11}$$

Moreover, the model (11) can be solved by the following mathematical programming

$$\min \ f = \sum_{i=1}^{n-1} \sum_{j=2, j>i}^{n} \left(t_{ij}d_{ij}^{+} + m_{ij}d_{ij}^{-} \right)$$

$$s.t. \begin{cases} \left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k} - g \right) \omega_i \\ \quad + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k}\omega_j - t_{ij}d_{ij}^{+} + m_{ij}d_{ij}^{-} = 0 \\ d_{ij}^{+}, \quad d_{ij}^{-} \geq 0 \\ \sum_{i=1}^{n} \omega_i = 1, \quad \omega_i \geq 0 \\ i, \quad j = 1, 2, \cdots, n, \quad i < j. \end{cases} \tag{12}$$

where d_{ij}^{+} and d_{ij}^{-} are the positive and negative deviations with respect to the goal ε_{ij}, respectively. t_{ij} and m_{ij} are the weights corresponding to d_{ij}^{+} and d_{ij}^{-}, respectively.

We assume that all goals $\varepsilon_{ij}\,(i, j = 1, 2, \cdots, n)$ are fair and then $t_{ij} = m_{ij} = 1\,(i, j = 1, 2, \cdots, n, i < j)$. Thus, the model (12) can be translated into the following model

$$\min \ f = \sum_{i=1}^{n-1} \sum_{j=2, j>i}^{n} \left(d_{ij}^{+} + d_{ij}^{-} \right)$$

$$s.t. \begin{cases} \left(\sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k} - g \right) \omega_i \\ \quad + \sum_{k=1}^{\#L_{ij}} \Delta^{-1}\left(L_{ij,k}\right) \cdot p_{ij,k}\omega_j - d_{ij}^{+} + d_{ij}^{-} = 0 \\ d_{ij}^{+}, \quad d_{ij}^{-} \geq 0 \\ \sum_{i=1}^{n} \omega_i = 1, \quad \omega_i \geq 0 \\ i, \quad j = 1, 2, \cdots, n, \quad i < j \end{cases} \tag{13}$$

Remark 2: It is note that the priority weights can be directly derived by model (13). More important, the model (13) can be used to deal with MGPFLPRs, that is to say, in order to deal with PLPR based on linguistic term set $S^{g(h)} = \left\{ s_0^{g(h)}, s_1^{g(h)}, \cdots, s_{g(h)-1}^{g(h)} \right\}$, the g in model (13) is replaced by $g(h) - 1$.

In the following, an example is solved by means of the model (13).

Example 1: Let H be a PLPR based on a given linguistic term set $S^9 = \left\{ s_0^9, s_1^9, \cdots, s_8^9 \right\}$ and H is given as follows, as shown at the top of the next page.

Based on model (13), a mathematical programming is obtained as follows

$$\min \ f = d_{12}^{+} + d_{12}^{-} + d_{13}^{+} + d_{13}^{-} + d_{14}^{+} + d_{14}^{-} + d_{23}^{+} + d_{23}^{-} + d_{24}^{+} + d_{24}^{-} + d_{34}^{+} + d_{34}^{-}$$

$$s.t. \begin{cases} -6\omega_1 + 2\omega_2 - d_{12}^{+} + d_{12}^{-} = 0 \\ -2.4\omega_1 + 5.6\omega_3 - d_{13}^{+} + d_{13}^{-} = 0 \\ -3\omega_1 + 5\omega_4 - d_{14}^{+} + d_{14}^{-} = 0 \\ -5.4\omega_2 + 2.6\omega_3 - d_{23}^{+} + d_{23}^{-} = 0 \\ -5\omega_2 + 3\omega_4 - d_{24}^{+} + d_{24}^{-} = 0 \\ -5.5\omega_3 + 2.5\omega_4 - d_{34}^{+} + d_{34}^{-} = 0 \\ \omega_1 + \omega_2 + \omega_3 + \omega_4 = 1 \\ d_{12}^{+}, d_{12}^{-}, d_{13}^{+}, d_{13}^{-}, d_{14}^{+}, d_{14}^{-}, d_{23}^{+}, d_{23}^{-}, d_{24}^{+}, d_{24}^{-}, \\ \qquad d_{34}^{+}, d_{34}^{-} \geq 0 \\ \omega_1, \omega_2, \omega_3, \omega_4 \geq 0 \end{cases}$$

Using MATLAB, the optimal solutions are determined as follows

$$\omega = (0.341, 0.192, 0.146, 0.321)^T.$$

Hence, the ranking of alternative is $x_1 \succ x_4 \succ x_2 \succ x_3$.

B. FUSE SUB-GROUP PRIORITY WEIGHT VECTORS TO OBTAIN COLLECTIVE PRIORITY WEIGHT VECTOR

In order to obtain the collective priority weight vector, sub-group priority weight vectors $\omega_k\,(k = 1, 2, \cdots, r)$ can be combined into collective priority weight vector ω_c based on the DMs' importance with the help of WA operator.

$$WA\left(\omega_1, \omega_1, \cdots, \omega_r\right) = \sum_{k=1}^{r} u_k\omega_k. \tag{14}$$

where $u_k\,(k = 1, 2, \cdots, r)$ is the weight vector about $\omega_k\,(k = 1, 2, \cdots, r)$, with $u_k > 0$, $\sum_{k=1}^{r} u_k = 1$.

DMs' importance is existent in two categories, equal importance and unequal importance. In the case of decision makers with unequal weights, it can be weighted according to the position, experience and prestige of the decision maker. That is to say, decision-makers with high positions, rich experience and high prestige should be given high weight. In the following, a proposition is put forward to determine the weight of sub-group's PLPRs from the linguistic term set $S^{g(h)}$, $h \in \{1, 2, \cdots, r\}$ in both cases.

(1) DMs $d_k\,(k = 1, 2, \cdots, l)$ possess equal importance. In the same manner, the weight of the DMs D_q who put to

$$H = \begin{bmatrix} \{s_4^9(1)\} & \{s_2(1)\} & \{s_5^9(0.7), s_7^9(0.3)\} & \{s_5^9(1)\} \\ \{s_6(1)\} & \{s_4^9(1)\} & \{s_2^9(0.4), s_3^9(0.6)\} & \{s_3^9(1)\} \\ \{s_2^9(0.7), s_1^9(0.3)\} & \{s_6^9(0.4), s_5^9(0.6)\} & \{s_4^9(1)\} & \{s_2^9(0.6), s_3^9(0.4)\} \\ s_3^9(1) & s_3^9(1) & s_6^9(0.6), s_5^9(0.4) & s_4^9(1) \end{bmatrix}$$

use the linguistic term set $S^{g(h)}$, $h \in \{1, 2, \cdots, r\}$ is received as

$$u_q = \frac{D_q}{l} \tag{15}$$

(2) DMs d_k $(k = 1, 2, \cdots, l)$ possess unequal importance. Let $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_l)^T$ suggests the weighting vector of the DMs d_k $(k = 1, 2, \cdots, l)$, where $0 \le \lambda_k \le 1$, and $\sum_{k=1}^{l} \lambda_k = 1$. The weight of the DMs D_q who put to utilization the linguistic term set $S^{g(h)}$, $h \in \{1, 2, \cdots, r\}$ is received as

$$u_q = \sum\nolimits_{d_k \in D_q} \lambda_k \tag{16}$$

Based on the above analysis, the weight of sub-group's PLPR $H_k \in S^{g(h)}$ $(k = 1, 2, \cdots, r)$ is the weight of the DMs D_q who put to utilization the linguistic term sets $S^{g(h)}$, $h \in \{1, 2, \cdots, r\}$.

Example 2: Suppose that there are 12 DMs with equal importance to evaluate alternative using FLPR. They use linguistic term set S^5, S^7, S^9 for assessment are 2, 6, and 4, respectively. Then, the sub-group PLPRs are obtained as $H_1 \in S^5, H_2 \in S^7, H_3 \in S^9$ and the weights of H_1, H_2, H_3 are 1/6, 1/2, and 1/3, respectively.

C. CONSENSUS REACHING PROCESS ALGORITHM

As shown in [43], [44], CRP algorithm based on direct approach has two desirable properties: (1) it can avoid internal inconsistency with MGFLPRs, and (2) it is satisfied with the Pareto principle in social choice theory. Inspired by the direct approach, we adopt the direct approach to manage CRP in this study.

(1) Consensus index

Definition 9: Let ω_k $(k = 1, 2, \cdots, r)$ and ω_c be sub-group priority weight vector and collective priority weight vector, respectively. The sub-group consensus index (SGCI) of $H_k \in S^{g(h)}$ $(k = 1, 2, \cdots, r)$ is defined as follows:

$$SGCI\,(H_k) = 1 - \sqrt{\sum\nolimits_{i=1}^{n} \left(\omega_i^{(k)} - \omega_i^{(c)}\right)^2} \tag{17}$$

If $SGCI\,(H_k) = 1$ $(k = 1, 2, \cdots, r)$, all decision makers agree with the group; otherwise, the larger $SGCI\,(H_k)$ is, the higher the consensus index.

1) GROUP CONSENSUS-REACHING PROCESS

The goal of the feedback adjustment is to help DMs improve the consensus level by the adjustment suggestions [45]–[48]. In this study, an automatic iteration adjustment strategy is proposed based on the sub-group original preference information and the adjusted collective preference information to regulate the $H_k \in S^{g(h)}$ $(k = 1, 2, \cdots, r)$. Sub-group's preference relations can be revised to improve the group consensus level. According to the different granular linguistic structures, an automatic iteration adjustment mechanism should be presented as follows:

For sub-group's $H_k \in S^{g(h)} = \left\{s_0^{g(h)}, s_1^{g(h)}, \cdots, s_{g(h)-1}^{g(h)}\right\}$, the adjustment proposals are given as follows:

$$\begin{cases} \overline{h}_{ij,k} = \frac{h_{ij,k}+h_{ij,c}^{g(h)}}{2}, & i < j \\ \overline{h}_{ij,k} = s_{g(h)-1/2}, & i = j \\ \overline{h}_{ij,k} = \Delta\left((g\,(h) - 1) - \Delta^{-1}\left(\overline{h}_{ij,k}\right)\right), & i > j \end{cases} \tag{18}$$

where $h_{ij,c}^{g(h)} = \Delta\left((g\,(h) - 1)\,\frac{\omega_{i,c}}{\omega_{i,c}+\omega_{j,c}}\right)$.

Furthermore, based on the adjustment proposals, an automatic iteration consensus-reaching Algorithm 1 is constructed as follows:

Algorithm 1 An Automatic Iteration Consensus Reaching Process.

Input: Sub-group's PLPRs $H_1, H_2, \cdots, H_r$, the weight vector u_k of H_k $(k = 1, 2, \cdots, r)$, the thresholds $\overline{SGCI}$ and the maximum iterations $t_{\max} \ge 1$.

Output: Adjusted Sub-group's PLPRs $H_1^{(t)}, H_2^{(t)}, \cdots, H_r^{(t)}$, $SGCI\left(H_k^{(t)}\right)$ $(k = 1, 2, \cdots, r)$, $\omega^* = \omega_c^{(t)}$ and the iterations t.

Step 1: Set $t = 0$ and $H_k^{(0)} = H_k$ $(k = 1, 2, \cdots, r)$.

Step 2: Obtain sub-group's priority weight vector $\omega_1^{(t)}, \omega_2^{(t)}, \cdots, \omega_r^{(t)}$ using the model (13).

Step 3: Compute the collective priority weight vector $\omega_c^{(t)}$ corresponding to $\omega_1^{(t)}, \omega_2^{(t)}, \cdots, \omega_r^{(t)}$ by means of WA operator.

Step 4: Count the $SGCI\left(H_k^{(t)}\right)$ $(k = 1, 2, \cdots, r)$ according to Eq. (17). If $SGCI\left(H_k^{(t)}\right) \ge \overline{SGCI}$ $(k = 1, 2, \cdots, r)$ or $t \ge t_{\max}$, go to Step 6; Or else, find the PLPR $H_k^{(t)}$ that satisfies $SGCI\left(H_k^{(t)}\right) < \overline{SGCI}$ and proceed to Step 5.

Step 5: Ascertain the position of the elements $d_{i_\tau j_\tau,k}^{(t)}$ for expert e_k satisfying $SGCI\left(H_k^{(t)}\right) < \overline{SGCI}$, where $d_{i_\tau j_\tau,k}^{(t)} = \max\limits_{(i,j)} \left|\Delta^{-1}\left(h_{ij,k}^{(t)}\right) - \Delta^{-1}\left(h_{ij,c}^{(t)}\right)\right|$, and then adjust sub-group's PLPRs according to Eq. (18). Set $t = t + 1$ and go to Step 2.

Step 6: Let $\overline{H}_k = H_k^{(t)}$ $(k = 1, 2, \cdots, r)$ and $\omega^* = \omega_c^{(t)}$. Output $SGCI\left(\overline{H}_k\right)$ $(k = 1, 2, \cdots, r)$, $\omega^* = \omega_c^{(t)}$ and iterations t.

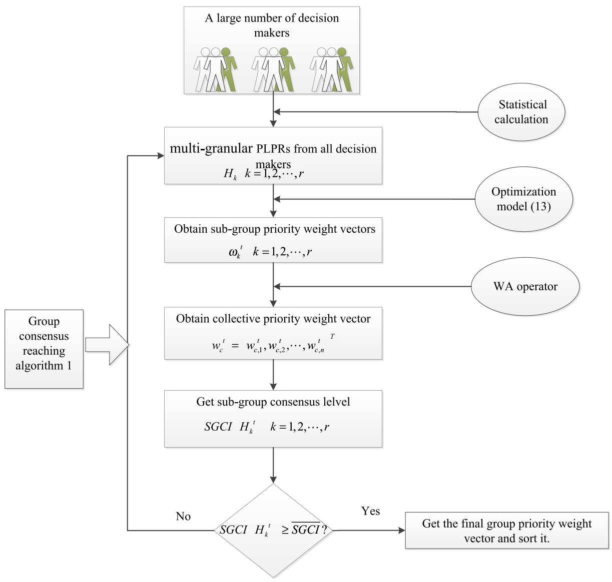

FIGURE 1. Procedure of the consensus model with MGPFLPRs.

D. A STEP BY STEP PROCEDURE OF THE CONSENSUS MODEL WITH MGPFLPRs

Assume that there are n alternative $x_1, x_2, \cdots, x_n$. H_k $(k = 1, 2, \cdots, r)$ indicate sub-group's PLPRs based on the given linguistic term sets $S^{g(1)}, S^{g(2)}, \cdots, S^{g(r)}$, respectively. The flow chart of proposed consensus model in LGDM is illustrated by Fig. 1, and its procedure is specifically shown as follows:

Step 1: Each DM provides assessed linguistic terms between two alternative by pared comparison method based on given linguistic scale sets $S^{g(1)}, S^{g(2)}, \cdots, S^{g(r)}$ by means of their experience and expertise knowledge, and then obtain sub-group preferences information, i.e., the sub-group's PLPRs H_k $(k = 1, 2, \cdots, r)$.

Step 2: Obtain the sub-group's priority weight vectors for H_k $(k = 1, 2, \cdots, r)$ by means of model (13), respectively.

Step 3: Fuse sub-group's priority weight vectors to collective priority weight vector based on the DMs' importance with the help of WA operator.

Step 4: Achieve group consensus with satisfying index by means of Algorithm 1.

Step 5: Ranking for alternative x_i $(i = 1, 2, \cdots, n)$ according to the final collective priority weight vector.

Step 6: End.

IV. CASE STUDY AND COMPARATIVE ANALYSIS

In this section, an emergency case of mine accident rescue is solved by the proposed model and then a comparative analysis is made on relevant literature.

A. EMERGENCY RESCUE PLAN SELECTION

A crumpling accident took place in Pingyi coal mine in the city of Linyi in Shandong Province of China at 7:56 a.m. on December 25, 2015. An aggregate of four miners could come out of the well whereas twenty five got trapped underground. The municipal government called for a quick meeting having emergency DMs d_k $(k = 1, 2, \cdots, 40)$ including medical experts, fire soldiers, mine representatives and geological experts to decide for the best rescue alternative in short time spans out of four available alternative x_i $(i = 1, 2, 3, 4)$ as follows:

(a) Mining rescue channel and escape path in the manner of roadway drivage underground (x_1);

(b) Restore the wellbore and take mine cars down into the mine (x_2);

(c) Putting to use partial blasting and arranging mining machines (x_3);

(d) Sending huge mechanical tools and deep-hole drilling machines above mine (x_4).

The DMs have the liability for ensure people's safety in a short span of time so that the DMs are under the high nervousness. Under the scenario, it is hard to utilize straightforward value to present their preference for DMs. Natural linguistic is the natural shape of preference display. Hence, it is suitable using FLPR to describe their preference for DMs under intricate as well as unsure scenarios. The multi-granularity linguistic term sets used by DMs are S^5, S^7, S^9, as shown at the top of the next page.

Step 1: The evaluating performance of alternative is obtained by 40 DMs based on a paired comparison technology through means of a questionnaire. Each DM provides her/his preference information with x_i over x_j $(i, j = 1, 2, 3, 4)$ using FLPR, which represents the preferences of the DM over each pair of alternatives (x_i, x_j). Through statistical calculations, the sub-group PLPRs of alternatives x_i $(i = 1, 2, 3, 4)$ with multi-granularity linguistic term sets S^5, S^7, S^9 are shown in Table 2-4 (See appendix), respectively.

Step 2: The sub-group's priority weight vectors are obtained based on H_k $(k = 1, 2, 3)$ from the model (13).

$$\omega_1^{(0)} = (0.3171, 0.2561, 0.1707, 0.2561)^T;$$
$$\omega_2^{(0)} = (0.289, 0.2666, 0.1142, 0.3302)^T;$$
$$\omega_3^{(0)} = (0.248, 0.2232, 0.1567, 0.372)^T.$$

Step 3: Group consensus reaching process

As DMs with equal importance choosing S^5, S^7 and S^9 for the assessment of alternatives are 10, 20, and 10 respectively, the weighting vector $u = (0.25, 0.5, 0.25)^T$ is collected by Eq. (15). Assembly of the three decision matrices is carried out with the help of the WA operator taking into account the bases of weighting vector $u = (0.25, 0.5, 0.25)^T$, the original collective preference vector is $\omega_c^{(0)} = (0.2858, 0.2531, 0.139, 0.3221)^T$ and consensus indices of each sub-group are as follows:

$$SGCI_1 = 0.92, SGCI_2 = 0.97, SGCI_3 = 0.93.$$

According to the practical problems, the DMs agree to setup $\overline{SGCI} = 0.94$. Then Algorithm 1 is applied to adjust the original MGPFLPRs. Since $SGCI_k < \overline{SGCI}$ $(k = 1, 3)$, we need to find the position of elements $d^{(0)}_{i_\tau j_\tau,k}$ $(k = 1, 3)$, where $d^{(t)}_{i_\tau j_\tau,k} = \max_{(i,j)} \left|\Delta^{-1}\left(p^{(t)}_{ij,k}\right) - \Delta^{-1}\left(p^{(t)}_{ij,c}\right)\right|$. With regard to $H^{(0)}_1$, as $d^{(0)}_{12,1} = d^{(0)}_{21,1} = \max_{(i,j)} \left|\Delta^{-1}\left(h^{(0)}_{ij,1}\right) - \Delta^{-1}\left(h^{(0)}_{ij,c}\right)\right| = 0.48$, $h^{(0)}_{12,1}, h^{(0)}_{21,1}$ need be adjusted to $h^{(0)}_{12,1} = \left(s^5_2, 0.4\right)(1), h^{(0)}_{21,1} = \left(s^5_2, -0.4\right)(1)$ according to $H^{(1)}_{1,c}$, as shown at the bottom of the this page. where $H^{(1)}_{1,c}$, as shown at the bottom of the this page. With regard to $H^{(0)}_3$ the same procedure is implemented to obtain the adjusted PLPR $H^{(1)}_3$ according to the $H^{(1)}_{3,c}$ as follows, where $H^{(0)}_3$, as shown at the bottom of the this page. Let $t = 1$, then go to Step 2.

Going through 3 rounds of adjustment, the Algorithm 1 terminated. H_1, H_2 and H_3 are adjusted 3, 0, and 2 times, respectively. In addition, the final sub-group consensus index of H_k $(k = 1, 2, 3)$ and final collective priority weight vector are as follows:

$$\begin{aligned}
&SGCI\left(H^{(3)}_1\right)\\
&\quad = 0.94, SGCI\left(H^{(0)}_2\right) = 0.96, SGCI\left(H^{(2)}_3\right)\\
&\quad = 0.96\omega^* = \omega^3_c = (0.3, 0.2358, 0.1327, 0.3243)^T.
\end{aligned}$$

Furthermore, the adjustment variations of PLPRs for H_1 and H_3 is shown in Table 5 (See appendix).

TABLE 1. **The results of comparison between the proposed method and previous studies.**

Algorithms	Method	Consensus reach	Ranking
The proposed method	*Optimization*	*Yes*	$x_4 \succ x_1 \succ x_2 \succ x_3$
Operator-based approach	ELH approach	No	$x_1 \succ x_4 \succ x_2 \succ x_3$

Step 4: Rank alternative x_i $(i = 1, 2, 3, 4)$ based on the final priority weight vector

According to the final priority weight vector $\omega^* = (0.3, 0.2358, 0.1327, 0.3243)^T$, the ranking result of alternative is $x_4 \succ x_1 \succ x_2 \succ x_3$, i.e., Sending huge mechanical tools and deep-hole drilling machines above mine is the best rescue plan.

B. DISCUSSION

1) COMPARED WITH THE METHODS BASED ON OPERATOR

In what follows, we use traditional processing methods based on ELH approach [49] and DAWA (weighted averaging operator of linguistic distribution assessments) operator [50] to deal with the above emergency decision problem as follows:

$$S^5 = \left\{s^5_0 : poor, s^5_1 : slightly\ poor, s^5_2 : fair, s^5_3 : slightly\ good, s^5_4 : good\right\}$$

$$S^7 = \left\{\begin{array}{l} s^7_0 : very\ poor, s^7_1 : poor, s^7_2 : slightly\ poor, s^7_3 : fair, \\ \quad s^7_4 : slightly\ good, s^7_5 : good, s^7_6 : very\ good \end{array}\right\}$$

$$S^9 = \left\{\begin{array}{l} s^9_0 : extremly\ poor, s^9_1 : very\ poor, s^9_2 : poor, s^9_3 : slightly\ poor, s^9_4 : fair, \\ \quad s^9_5 : slightly\ good, s^9_6 : good, s^9_7 : very\ good, s^9_8 : extremly\ good \end{array}\right\}$$

$$H^{(1)}_1 = \begin{bmatrix} \{s^5_2(1)\} & \{(s^5_2, 0.4)(1)\} & \{s^5_2(0.4), s^5_3(0.6)\} & \{s^5_2(0.8), s^5_3(0.2)\} \\ \backslash & \{s^5_2(1)\} & \{s^5_1(0.3), s^5_3(0.7)\} & \{s^5_1(0.5), s^5_3(0.5)\} \\ \backslash & \backslash & \{s^5_2(1)\} & \{s^5_1(0.2), s^5_2(0.8)\} \\ \backslash & \backslash & \backslash & \{s^5_2(1)\} \end{bmatrix}$$

$$H^{(1)}_{1,c} = \begin{bmatrix} \{s^5_2(1)\} & \{(s^5_2, 0.12)(1)\} & \{(s^5_3, -0.31)(1)\} & \{(s^5_2, -0.12)(1)\} \\ \backslash & \{s^5_2(1)\} & \{(s^5_3, -0.42)(1)\} & \{(s^5_2, -0.24)(1)\} \\ \backslash & \backslash & \{s^5_2(1)\} & \{(s^5_1, 0.21)(1)\} \\ \backslash & \backslash & \backslash & \{s^5_2(1)\} \end{bmatrix}$$

$$H^{(1)}_{3,c} = \begin{bmatrix} \{s^9_4(1)\} & \{(s^9_4, 0.24)(1)\} & \{(s^9_5, 0.38)(1)\} & \{(s^9_4, -0.24)(1)\} \\ \backslash & \{s^9_4(1)\} & \{(s^9_5, 0.16)(1)\} & \{(s^9_4, -0.48)(1)\} \\ \backslash & \backslash & \{s^9_4(1)\} & \{(s^9_2, 0.42)(1)\} \\ \backslash & \backslash & \backslash & \{s^9_4(1)\} \end{bmatrix}$$

TABLE 2. The PLPR H_1 BASED ON S^5.

	A_1	A_2	A_3	A_4
A_1	$\{s_2^5(1)\}$	$\{s_1^5(0.2), s_3^5(0.8)\}$	$\{s_2^5(0.4), s_3^5(0.6)\}$	$\{s_2^5(0.8), s_3^5(0.2)\}$
A_2	\	$\{s_2^5(1)\}$	$\{s_1^5(0.3), s_3^5(0.7)\}$	$\{s_1^5(0.5), s_3^5(0.5)\}$
A_3	\	\	$\{s_2^5(1)\}$	$\{s_1^5(0.2), s_2^5(0.8)\}$
A_4	\	\	\	$\{s_2^5(1)\}$

TABLE 3. The PLPR H_2 based on S^7.

	A_1	A_2	A_3	A_4
A_1	$\{s_3^7(1)\}$	$\{s_2^7(0.3), s_3^7(0.7)\}$	$\{s_4^7(0.7), s_5^7(0.3)\}$	$\{s_2^7(0.6), s_4^7(0.4)\}$
A_2	\	$\{s_3^7(1)\}$	$\{s_3^7(0.4), s_5^7(0.6)\}$	$\{s_1^7(0.3), s_2^7(0.7)\}$
A_3	\	\	$\{s_3^7(1)\}$	$\{s_2^7(1)\}$
A_4	\	\	\	$\{s_3^7(1)\}$

TABLE 4. The PLPR H_3 based on S^9.

	A_1	A_2	A_3	A_4
A_1	$\{s_4^9(1)\}$	$\{s_3^9(0.6), s_5^9(0.4)\}$	$\{s_4^9(0.2), s_5^9(0.4), s_6^9(0.4)\}$	$\{s_2^9(0.4), s_4^9(0.6)\}$
A_2	\	$\{s_4^9(1)\}$	$\{s_4^9(0.3), s_5^9(0.7)\}$	$\{s_2^9(0.5), s_4^9(0.5)\}$
A_3	\	\	$\{s_4^9(1)\}$	$\{s_2^9(0.4), s_3^9(0.6)\}$
A_4	\	\	\	$\{s_4^9(1)\}$

Step 1: the ELH approach [49] is used to unify multi-granular linguistic information. The transformation results of Table 2-4 are showcased in Table 6-8 (See appendix), respectively.

Step 2: The collective decision matrix is obtained by means of the DAWA operator based on weighting vector $u = (0.25, 0.5, 0.25)^T$ and presented in Table 9 (See appendix).

Step 3: Calculate the expectation of collective decision matrix $E(H_c)$ based on the Definition 5 [15], which is presented in Table 10 (See appendix).

Step 4: Computation of preference degree z_i $(i = 1, 2, 3, 4)$ based on $E(H_c)$ and ranking of alternative x_i $(i = 1, 2, 3, 4)$.

$$z_i = \Delta\left(\sum_{k=1}^{4}\frac{1}{4}\Delta^{-1}\left(E\left(h_{ik,c}\right)\right)\right).$$

According to Eq. (19), we obtain the preference degree of four alternative: $z_1 = (s_{13}, -0.04)$, $z_2 = (s_{12}, 0)$, $z_3 = (s_{11}, -0.16)$, and $z_4 = (s_{12}, 0.18)$. Therefore, the ranking is $x_1 \succ x_4 \succ x_2 \succ x_3$.

The differences between the proposed model and the operator-based approach are summarized in Table 1. It can be seen from Table 1 that the sort order of x_1 and x_4 has changed for the proposed method and operator-based approach due to different processing mechanisms. On the one hand, our method does not need to be uniform same language granularity for MGPFLPRs, which not only reduces the amount of computation but also decreases information loss as much as possible; On the other hand, our proposed model implements consensus reaching process for MGPFLPRs, while the traditional operator-based approach does not take into account this scenario. Thus, the proposed model is more reasonable and reliable for LGDM problems than the traditional operator-based approach.

2) COMPARED WITH THE CONSENSUS REACHING PROCESS [51]

The paper [51] focuses on the CRP with PLPRs. Firstly, an index for measuring the consensus degree is defined. Then, for the DMs with unacceptable consensus degree, a consensus improving process is presented based on the consensus criteria. While, this article cannot deal with the CRP with PLPRs based on multi-granular linguistic

TABLE 5. To compare the situation of PLPRS after adjusting for H_1 and H_3.

PLPRs	before the adjustment	after the adjustment
$H_1 \in S^5$	$H_1^{(0)}=\begin{bmatrix} \{s_2^5(1)\} & \{s_1^5(0.2),s_3^5(0.8)\} & \{s_2^5(0.4),s_3^5(0.6)\} & \{s_2^5(0.8),s_3^5(0.2)\} \\ \backslash & \{s_2^5(1)\} & \{s_1^5(0.3),s_3^5(0.7)\} & \{s_1^5(0.5),s_3^5(0.5)\} \\ \backslash & \backslash & \{s_2^5(1)\} & \{s_1^5(0.2),s_2^5(0.8)\} \\ \backslash & \backslash & \backslash & \{s_2^5(1)\} \end{bmatrix}$	$H_1^{(3)}=\begin{bmatrix} \{s_2^5(1)\} & \{(\mathbf{s_2^5,0.4})(\mathbf{1})\} & \{s_2^5(0.4),s_3^5(0.6)\} & \{s_2^5(0.8),s_3^5(0.2)\} \\ \backslash & \{s_2^5(1)\} & \{s_1^5(0.3),s_3^5(0.7)\} & \{s_1^5(0.5),s_3^5(0.5)\} \\ \backslash & \backslash & \{s_2^5(1)\} & \{(\mathbf{s_1^5,0.35})(\mathbf{1})\} \\ \backslash & \backslash & \backslash & \{s_2^5(1)\} \end{bmatrix}$
$H_3 \in S^9$	$H_3^{(0)}=\begin{bmatrix} \{s_4^9(1)\} & \{s_3^9(0.6),s_5^9(0.4)\} & \{s_4^9(0.2),s_5^9(0.4),s_6^9(0.4)\} & \{s_2^9(0.4),s_4^9(0.6)\} \\ \backslash & \{s_4^9(1)\} & \{s_4^9(0.3),s_5^9(0.7)\} & \{s_2^9(0.5),s_4^9(0.5)\} \\ \backslash & \backslash & \{s_4^9(1)\} & \{s_2^9(0.4),s_3^9(0.6)\} \\ \backslash & \backslash & \backslash & \{s_4^9(1)\} \end{bmatrix}$	$H_3^{(2)}=\begin{bmatrix} \{s_4^9(1)\} & \{(\mathbf{s_4^9,0.1})(\mathbf{1})\} & \{s_4^9(0.2),s_5^9(0.4),s_6^9(0.4)\} & \{(\mathbf{s_4^9,\text{-}0.5})(\mathbf{1})\} \\ \backslash & \{s_4^9(1)\} & \{s_4^9(0.3),s_5^9(0.7)\} & \{s_2^9(0.5),s_4^9(0.5)\} \\ \backslash & \backslash & \{s_4^9(1)\} & \{s_2^9(0.4),s_3^9(0.6)\} \\ \backslash & \backslash & \backslash & \{s_4^9(1)\} \end{bmatrix}$

TABLE 6. The PLPR H_1 based on S^5 is converted into $\overline{H_1}$ based on S^{25}.

	A_1	A_2	A_3	A_4
A_1	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.2),s_{18}^{25}(0.8)\}$	$\{s_{12}^{25}(0.4),s_{18}^{25}(0.6)\}$	$\{s_{12}^{25}(0.8),s_{18}^{25}(0.2)\}$
A_2	$\{s_{18}^{25}(0.2),s_6^{25}(0.8)\}$	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.3),s_{18}^{25}(0.7)\}$	$\{s_6^{25}(0.5),s_{18}^{25}(0.5)\}$
A_3	$\{s_{12}^{25}(0.4),s_6^{25}(0.6)\}$	$\{s_{18}^{25}(0.3),s_6^{25}(0.7)\}$	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.2),s_{12}^{25}(0.8)\}$
A_4	$\{s_{12}^{25}(0.8),s_6^{25}(0.2)\}$	$\{s_{18}^{25}(0.5),s_6^{25}(0.5)\}$	$\{s_{18}^{25}(0.2),s_{12}^{25}(0.8)\}$	$\{s_{12}^{25}(1)\}$

TABLE 7. The PLPR H_2 based on S^7 is converted into $\overline{H_2}$ based on S^{25}.

	A_1	A_2	A_3	A_4
A_1	$\{s_{12}^{25}(1)\}$	$\{s_8^{25}(0.3),s_{12}^{25}(0.7)\}$	$\{s_{16}^{25}(0.7),s_{20}^{25}(0.3)\}$	$\{s_8^{25}(0.6),s_{16}^{25}(0.4)\}$
A_2	$\{s_{16}^{25}(0.3),s_{12}^{25}(0.7)\}$	$\{s_{12}^{25}(1)\}$	$\{s_{12}^{25}(0.4),s_{20}^{25}(0.6)\}$	$\{s_4^{25}(0.3),s_8^{25}(0.7)\}$
A_3	$\{s_8^{25}(0.7),s_4^{25}(0.3)\}$	$\{s_{12}^{25}(0.4),s_4^{25}(0.6)\}$	$\{s_{12}^{25}(1)\}$	$\{s_8^{25}(1)\}$
A_4	$\{s_{16}^{25}(0.6),s_8^{25}(0.4)\}$	$\{s_{20}^{25}(0.3),s_{16}^{25}(0.7)\}$	$\{s_{16}^{25}(1)\}$	$\{s_{12}^{25}(1)\}$

TABLE 8. The PLPR H_3 based on S^9 is converted into $\overline{H_3}$ based on S^{25}.

	A_1	A_2	A_3	A_4
A_1	$\{s_{12}^{25}(1)\}$	$\{s_9^{25}(0.6),s_{15}^{25}(0.4)\}$	$\{s_{12}^{25}(0.2),s_{15}^{25}(0.4),s_{18}^{25}(0.4)\}$	$\{s_6^{25}(0.4),s_{12}^{25}(0.6)\}$
A_2	$\{s_{15}^{25}(0.6),s_9^{25}(0.4)\}$	$\{s_{12}^{25}(1)\}$	$\{s_{12}^{25}(0.3),s_{15}^{25}(0.7)\}$	$\{s_6^{25}(0.5),s_{12}^{25}(0.5)\}$
A_3	$\{s_{12}^{25}(0.2),s_9^{25}(0.4),s_6^{25}(0.4)\}$	$\{s_{12}^{25}(0.3),s_9^{25}(0.7)\}$	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.4),s_9^{25}(0.6)\}$
A_4	$\{s_{18}^{25}(0.4),s_{12}^{25}(0.6)\}$	$\{s_{18}^{25}(0.5),s_{12}^{25}(0.5)\}$	$\{s_{18}^{25}(0.4),s_{15}^{25}(0.6)\}$	$\{s_{12}^{25}(1)\}$

term sets. In order to compare with our method, we need to do a two-step conversion: first step is to convert multi-granular linguistic information to single-granular linguistic information based on the ELH [49]. The transformation results of Table 2-4 are showcased in Table 6-8 (See appendix), respectively. Then, an asymmetric linguistic terms set $S = \{s_0, s_1, \cdots, s_\tau, \cdots, s_{2\tau-1}, s_{2\tau}\}$ used in this paper correspondingly is converted to symmetric linguistic terms set $S = \{s_{-\tau}, s_{-\tau+1}, \cdots, s_0, \cdots, s_{\tau-1}, s_\tau\}$ used in [51]. Then the CRP method [51] is applied to the above case. Due to space limitations, the main result is given as follows:

Going through 3 rounds of adjustment based on the Algorithm 1 in [51], CRP is finished. The preferences of H_1 are changed twice in position $(1, 2)$ and $(2, 4)$ by means

TABLE 9. The collective decision matrix H_c is obtained by means of the DAWA operator.

	A_1	A_2	A_3	A_4
A_1	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.05), s_8^{25}(0.15), s_9^{25}(0.15), s_{12}^{25}(0.35), s_{15}^{25}(0.1), s_{18}^{25}(0.2)\}$	$\{s_{12}^{25}(0.15), s_{15}^{25}(0.1), s_{16}^{25}(0.35), s_{18}^{25}(0.25), s_{20}^{25}(0.15)\}$	$\{s_6^{25}(0.1), s_8^{25}(0.3), s_{12}^{25}(0.35), s_{16}^{25}(0.2), s_{18}^{25}(0.05)\}$
A_2	$\{s_{18}^{25}(0.05), s_{16}^{25}(0.15), s_{15}^{25}(0.15), s_{12}^{25}(0.35), s_9^{25}(0.1), s_6^{25}(0.2)\}$	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.075), s_{12}^{25}(0.275), s_{15}^{25}(0.175), s_{18}^{25}(0.175), s_{20}^{25}(0.3)\}$	$\{s_4^{25}(0.15), s_6^{25}(0.25), s_8^{25}(0.35), s_{12}^{25}(0.125), s_{18}^{25}(0.125)\}$
A_3	$\{s_{12}^{25}(0.15), s_9^{25}(0.1), s_8^{25}(0.35), s_6^{25}(0.25), s_4^{25}(0.15)\}$	$\{s_{18}^{25}(0.075), s_{12}^{25}(0.275), s_9^{25}(0.175), s_6^{25}(0.175), s_4^{25}(0.3)\}$	$\{s_{12}^{25}(1)\}$	$\{s_6^{25}(0.15), s_8^{25}(0.5), s_9^{25}(0.15), s_{12}^{25}(0.2)\}$
A_4	$\{s_{18}^{25}(0.1), s_{16}^{25}(0.3), s_{12}^{25}(0.35), s_8^{25}(0.2), s_6^{25}(0.05)\}$	$\{s_{20}^{25}(0.15), s_{18}^{25}(0.25), s_{16}^{25}(0.35), s_{12}^{25}(0.125), s_6^{25}(0.125)\}$	$\{s_{18}^{25}(0.15), s_{16}^{25}(0.5), s_{15}^{25}(0.15), s_{12}^{25}(0.2)\}$	$\{s_{12}^{25}(1)\}$

TABLE 10. The expectation of collective decision matrix $E(H_c)$.

	A_1	A_2	A_3	A_4
A_1	$(s_{12}^{25}, 0)$	$(s_{12}^{25}, 0.15)$	$(s_{16}^{25}, 0.4)$	$(s_{11}^{25}, 0.3)$
A_2	$(s_{12}^{25}, -0.15)$	$(s_{12}^{25}, 0)$	$(s_{16}^{25}, -0.47)$	$(s_9^{25}, -0.35)$
A_3	$(s_8^{25}, -0.4)$	$(s_8^{25}, 0.47)$	$(s_{12}^{25}, 0)$	$(s_{15}^{25}, 0.35)$
A_4	$(s_{13}^{25}, -0.3)$	$(s_{15}^{25}, 0.35)$	$(s_9^{25}, -0.35)$	$(s_{12}^{25}, 0)$

of Eq. (18) in [51], respectively. The preferences of H_3 are changed once in position $(2, 4)$ by means of Eq. (18) in [51]. Finally, the ranking of alternatives is derived as $x_4 \succ x_1 \succ x_2 \succ x_3$. Moreover, it can be seen that the same ranking of alternatives are obtained between the above improving strategy and the method in this paper, which verifies the effectiveness and applicability of our method.

In addition, a comparison analysis between our method and the previous research is given for LGDM based on linguistic information. Note that different LGDM models have different purposes; therefore, there is no one model that can be deemed the best. Compared with the existing LGDM models using linguistic information [13]–[16], the novel features in our proposed approach are as follows.

- There are two advantages for using MGPFLPRs. On the one hand, the PLPRs guarantee the quality of decision information because the consistency of the FLPR makes the DMs logical and non-voluntary; On the other hand, using multi-granular linguistic terms set is aligned with the heterogeneity of multiple DMs. While, the models [13], [14] are capable enough merely for application to settle LGDM issues in a specific linguistic term set, which limits their flexibility.
- A consensus reaching algorithm is constructed based on direct consensus framework for MGPFLPRs in LGDM, while existing related studies [13]–[16] does not take into account this situation.
- Our proposed optimization model which does not need to carry out uniform granularity to deal with MGPFLPRs could obtain directly the priority weight vector of alternative. These can avoid information loss and reduce the computation complexity.
- Our proposed consensus model in LGDM to deal with MGPFLPRs is not only easy to implement and support but also to understand and use. Moreover, our model is suitable for a huge number of DMs, as the group preference PLPRs can be statistically calculated.

In brief, the suggested model delivers a new approach to managing LGDM issues together with the use of MGPFLPRs.

V. CONCLUSIONS

In this paper, a consensus model in LGDM to deal with MGPFLPRs is proposed based on mathematical programming and CRP. An automatic iteration consensus reaching algorithm based on the direct approach is constructed to achieve a level of agreement to the satisfaction level, which makes the decision results acceptable to most decision makers. Most of all, the uniform linguistic granularity is avoided by using the proposed mathematical programming. A step by step procedure of consensus process in LGDM based on the group consensus reaching algorithm has been concluded to help DMs improve the consensus level. Following the procedure, an emergency decision problem is worked out by means of the proposed model.

In future, it is also interesting to analyze LGDM problems with PLPRs based on multi-granular unbalanced linguistic terms and non-cooperative behavior in the LGDM with PLPRs.

APPENDIX

See Table 2–10.

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