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Current Distribution and Input Impedance of a Horizontal Linear Antenna in the Presence of a Layered Region

HUI RAN ZENG, TONG HE, LE LI, AND KAI LI
College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, China

Corresponding author: Kai Li (kaili@zju.edu.cn)

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ABSTRACT In this paper, the current distribution and an input impedance of a horizontal linear antenna in the presence of a layered region are investigated both analytically and numerically. In particular, we demonstrate the case of a four-layered region. The wave-number and characteristic impedance of an eccentrically insulated antenna are obtained first. Subsequently, the analytical formula for the current distribution on the antenna in a four-layered region is derived by applying a limiting process. The computational results show that the upper and lower dielectric layers have different effects on the current distribution and input impedance of the antenna. It is also found that the imaginary part of the input impedance as a function of antenna length has two zero points, where the first and second zero points correspond to a half-wavelength antenna (series resonance) and a full-wavelength antenna (parallel resonance), respectively. It is indicated that when the thickness of the dielectric layers or the antenna height changes, variations on the resonance length of the antenna are observed by examining the imaginary part of the input impedance.

INDEX TERMS Current distribution, horizontal linear antenna, layered media.

I. INTRODUCTION

Horizontal linear antennas on or near the boundary between two different dielectrics are investigated widely because of their useful applications in communication systems, over-the-horizon radars, geophysical explorations, and so on [1]–[6]. In early studies of a linear antenna over a lossy half-space, a transmission line model formed by a horizontal wire and its geometric image in the high conducting earth was used by Busch [7]. Unfortunately, in analyzing the wave number and characteristic impedance, the electrical properties of the earth are not taken into account. Afterwards, some progress is carried out by Carson [8], and it is noted that either the conductivity of the earth was large enough or the operating frequency was adequately low. Namely, the earth was taken as a good conductor.

In early investigations of the input impedance of linear antennas above a lossy half-space, the field formulas of Sommerfeld type integrals have been used as a starting point to obtain the expressions for the input impedance of dipoles at different heights above an imperfect earth [9], [10]. Furthermore, in Wait's works [11], a modal equation for the traveling wave in an infinite conductor parallel to a dissipative half-space was derived. Nevertheless, the solutions of the wave number by Wait remained only in a modal equation form, which required further estimations. Later on, an essential approach for solving the traveling wave produced by an antenna over a lossy half-space was proposed by King *et al.* [12]. The formulas of the current distribution and input impedance are obtained from the theory of a cylindrical antenna with an eccentric dielectric coating [13]. Subsequent works were also carried by King and Shen to extend the analysis to the case where the antenna was covered by a dielectric layer [14], [15]. It is noted that a limiting process was used in the work by King *et al.*, which is addressed simply as follows [12]–[15]: By extending the radius of outermost layer to infinity while maintaining a constant distance between the antenna and the outer layer, the wave number of an eccentric insulated antenna was transformed into the form that is equivalent to the wave number of an antenna in a half-space.

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Most existing works only considered the cases of a horizontal antenna in a two-layered and three-layered region with the current and impedance computed by numerical methods. For example, Pocklington's integral equation was solved by the method of moments (MoM), and the input impedance and radiation fields of a wire antenna on a single dielectric layer [16], [17]. The case of double dielectric layers is analyzed and calculated in [18]. Moreover, in much of the research associated with the layered region, the current distribution is assumed, and many of these current assumptions are not sufficient [19]. Investigations on the current distribution and input impedance of a horizontal antenna in a four-layered region with analytical methods have not been found in available works.

Due to the complexity of the covering layer, the underlying medium has a considerable influence on the input impedance of a surface antenna [20], [21]. If the effects of the dielectric layers are not carefully considered, the performance of the antenna may be compromised. In the present study, by extending King's theory to the case of a three-layered insulated antenna, we will attempt to treat the problem of a horizontal linear antenna in the presence of a four-layered region analytically and numerically. Also, we extend the theory to the case of n-layered region. The wave number of an eccentrically insulated antenna with three insulating layers has been investigated. Then, the analytical formulas for the current distribution and input impedance in a four-layered region can be obtained by using a limiting process.

II. FORMULATION OF THE PROBLEM

A. GEOMETRY AND NOTATIONS

The physical model of an eccentrically insulated antenna with three insulators in cylindrical coordinates (r, θ, z) is illustrated in Fig. 1. The center-driven cylindrical antenna with radius a is isolated from the outmost Region 4 by three cylindrical dielectrics. The radii for Regions 1, 2, and 3 are denoted by b, c, d, respectively. All regions are assumed to be nonmagnetic, i.e. $\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_0$. With a harmonic time dependency $\exp(-i\omega t)$, the wave number for Region j is written as

$$k_j = \omega\left[\mu\left(\varepsilon_j + i\frac{\sigma_j}{\omega}\right)\right]^{1/2} = \omega(\mu\widetilde{\varepsilon}_j)^{1/2} \tag{1}$$

where $j = 1, 2, 3, 4$. The complex permittivity is denoted by $\widetilde{\varepsilon}_j = \varepsilon_j + i\sigma_j/\omega$, $\omega = 2\pi f$ is the angular frequency, and ε_j and σ_j are the permittivity and conductivity of Region j, respectively.

The axis of the four cylinders with radii a, b, c, d are separated from each other by D_{ab}, D_{bc}, D_{cd}, and D_{ad}, respectively. It is assumed that the current distribution on the antenna is equivalent to a line current located at $r = x_0$ and $\theta = 0°$, which is yet to be determined. Since all cylinders have the same eccentricity, by applying a conformal mapping method, the four cylinders can be conformally mapped to four coaxial cylinders, with their common axis located at a distance of x_0 from the center of Region 3.

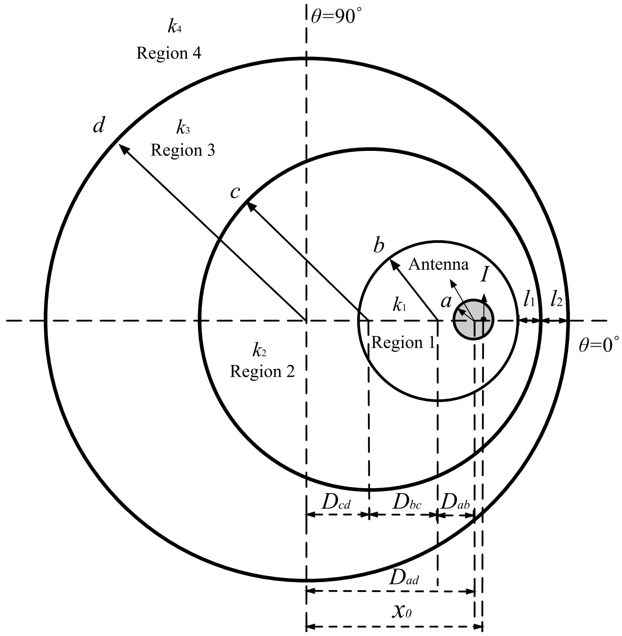

FIGURE 1. Geometry and notations of an insulated antenna with three eccentric insulation layers.

B. SOLUTIONS FOR THE ELECTROMAGNETIC FIELDS OF EACH REGION

With a Fourier transform employed, the current distribution on a finite or infinite dipole with eccentric coating is derived readily [13]. Based on the work, we will attempt to treat analytically three-layered insulated antenna. The components $\widetilde{E}_z$ of each region can be written in the following forms:

$$\begin{aligned}\widetilde{E}_{z1}(r,\theta,\zeta) = {} & \frac{i\widetilde{I}(\zeta)(k_1^2-\zeta^2)}{2\pi\omega\widetilde{\varepsilon}_1}\Omega(r,\theta) + \frac{i\widetilde{I}(\zeta)\omega\mu}{2\pi k_4 d}\Delta(r,\theta) \\ & + \frac{i\widetilde{I}(\zeta)}{2\pi\omega}\left\{\left[\frac{(k_2^2-\zeta^2)}{\widetilde{\varepsilon}_2} - \frac{(k_1^2-\zeta^2)}{\widetilde{\varepsilon}_1}\right]\Omega_{bc} \right. \\ & \left. + \left[\frac{(k_3^2-\zeta^2)}{\widetilde{\varepsilon}_3} - \frac{(k_2^2-\zeta^2)}{\widetilde{\varepsilon}_2}\right]\Omega_{cd}\right\}\end{aligned} \tag{2}$$

$$\begin{aligned}\widetilde{E}_{z2}(r,\theta,\zeta) = {} & \frac{i\widetilde{I}(\zeta)(k_2^2-\zeta^2)}{2\pi\omega\widetilde{\varepsilon}_2}\Omega(r,\theta) + \frac{i\widetilde{I}(\zeta)\omega\mu}{2\pi k_4 d}\Delta(r,\theta) \\ & + \frac{i\widetilde{I}(\zeta)}{2\pi\omega}\left[\frac{(k_3^2-\zeta^2)}{\widetilde{\varepsilon}_3} - \frac{(k_2^2-\zeta^2)}{\widetilde{\varepsilon}_2}\right]\Omega_{cd}\end{aligned} \tag{3}$$

$$\widetilde{E}_{z3}(r,\theta,\zeta) = \frac{i\widetilde{I}(\zeta)(k_3^2-\zeta^2)}{2\pi\omega\widetilde{\varepsilon}_3}\Omega(r,\theta) + \frac{i\widetilde{I}(\zeta)\omega\mu}{2\pi k_4 d}\Delta(r,\theta) \tag{4}$$

$$\begin{aligned}\widetilde{E}_{z4}(r,\theta,\zeta) = {} & \frac{i\widetilde{I}(\zeta)\omega\mu}{2\pi k_4 d}\left[\frac{H_0^{(1)}(k_4 r)}{H_1^{(1)}(k_4 d)} \right. \\ & \left. + 2\sum_{m=1}^{\infty}\left(\frac{x_0}{d}\right)^m \frac{H_m^{(1)}(k_4 r)}{H_{m+1}^{(1)}(k_4 d)}\cos(m\theta)\right]\end{aligned} \tag{5}$$

where $\widetilde{I}(\zeta)$ denotes the Fourier transform of the current function $I_z(z)$, and ζ is the transform variable with respect to z, and

$$\Omega(r,\theta) = \frac{1}{2}\ln\left[\frac{(rx_0)^2 - 2d^2 rx_0\cos\theta + d^4}{d^2(r^2 - 2x_0 r\cos\theta + x_0^2)}\right] \tag{6}$$

$$\Delta(r,\theta) = \frac{H_0^{(1)}(k_4 d)}{H_1^{(1)}(k_4 d)} + 2\sum_{m=1}^{\infty}\left(\frac{rx_0}{d^2}\right)^m \frac{H_m^{(1)}(k_4 d)}{H_{m+1}^{(1)}(k_4 d)}\cos(m\theta). \tag{7}$$

It is noted that the constants Ω_{bc} and Ω_{cd} are the values of $\Omega(r,\theta)$ on a circle of radius b or c, respectively.

C. WAVE NUMBER AND CHARACTERISTIC IMPEDANCE OF A THREE-LAYERED ECCENTRICALLY INSULATED ANTENNA

At the outer surface of the antenna, the electric field component $\widetilde{E}_{z1}$ is written in the following form

$$\widetilde{E}_{z1}(D_{ad},0,\zeta) \tag{8}$$

$$= \left(\frac{i\widetilde{I}\Omega_{ad}}{2\pi\omega\widetilde{\varepsilon}_1}\right)\left\{k_1^2 + \left(\frac{k_1^2}{k_4 d\,\Omega_{ad}}\right)\Delta(D_{ad},0)\cdot T^2 - \zeta^2\right\} \tag{9}$$

where

$$\Omega_{ad} = \cosh^{-1}\left(\frac{a^2+d^2-D_{ad}^2}{2ad}\right) \tag{10}$$

$$T^2 = \left[\frac{\Omega_{ad}}{\Omega_{ab}+\Omega_{cd}+(\widetilde{\varepsilon}_1/\widetilde{\varepsilon}_2)\Omega_{bc}+\widetilde{\varepsilon}_e\Omega_{cd}}\right] \tag{11}$$

$$\widetilde{\varepsilon}_e = \widetilde{\varepsilon}_1(\widetilde{\varepsilon}_2-\widetilde{\varepsilon}_3)/\widetilde{\varepsilon}_2\widetilde{\varepsilon}_3. \tag{12}$$

Similarly, the rest three ones Ω_{ab}, Ω_{bc}, and Ω_{cd} can be expressed as follows:

$$\Omega_{mn} = \cosh^{-1}\left(\frac{m^2+n^2-D_{mn}^2}{2mn}\right), \quad mn = ab, bc, cd. \tag{13}$$

Generally, an antenna can be regarded as a good conductor. If the feeding gap is small enough, then at the feeding point a delta function generator will maintain the electric field $\widetilde{E}_{z1} = -V_0^e$, which a feeding voltage is V_0^e. By substituting the relation into (9), $\widetilde{I}(\zeta)$ is determined readily. Thus, by taking the inverse Fourier transform, the current distribution on an antenna of length $2h$ with three eccentric dielectric coatings shown in Fig. 1 can be expressed in the form of

$$I_z(z) = \frac{-iV_0^e}{2Z_c}\frac{\sin k_L(h-|z|)}{\cos k_L h} \tag{14}$$

where

$$k_L = k_1\left[1+\frac{\Delta(D_{ad},0)}{k_4 d\,\Omega_{ad}}\right]^{1/2}\cdot T \tag{15}$$

$$Z_c = \frac{k_L}{2\pi\omega\widetilde{\varepsilon}_1}\left[\frac{\Omega_{ab}\widetilde{\varepsilon}_2\widetilde{\varepsilon}_3+\Omega_{bc}\widetilde{\varepsilon}_1\widetilde{\varepsilon}_3+\Omega_{cd}\widetilde{\varepsilon}_t}{\widetilde{\varepsilon}_2\widetilde{\varepsilon}_3}\right] \tag{16}$$

$$\widetilde{\varepsilon}_t = \widetilde{\varepsilon}_1\widetilde{\varepsilon}_2-\widetilde{\varepsilon}_1\widetilde{\varepsilon}_3+\widetilde{\varepsilon}_2\widetilde{\varepsilon}_3. \tag{17}$$

It is clear that once the wave number k_L and characteristic impedance Z_c of the four-layered region are obtained, the current distribution will be determined readily.

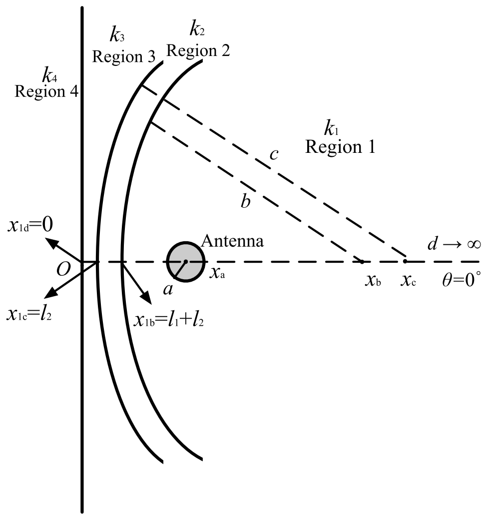

FIGURE 2. Approximate equivalent model of a three-layered eccentrically insulated antenna when the outer boundary radius *d* between regions 3 and 4 becomes infinite.

D. DETERMINATION OF THE CURRENT ON THE ANTENNA IN A FOUR-LAYERED REGION

In order to deal with a horizontal linear antenna in a four-layered region, a limiting process is applied to d (radius of Region 3), i.e. let $d\to\infty$. In this case, Region 4 becomes a half-space as illustrated in Fig. 2.

The normalized function Ω could be estimated with the limit $d\to\infty$. We write

$$\Omega_{ad} = \cosh^{-1}\left(\frac{a^2+d^2-D_{ad}^2}{2ad}\right) = \cosh^{-1}\frac{L}{a} \tag{18}$$

$$\Omega_{cd} = \cosh^{-1}\left(\frac{c^2+d^2-D_{cd}^2}{2cd}\right) = \cosh^{-1}\frac{I}{K} \tag{19}$$

$$\Omega_{ab} = \cosh^{-1}\left(\frac{a^2+b^2-D_{ab}^2}{2ab}\right) = \cosh^{-1}\left[\frac{L\cdot J-Q}{a\cdot P}\right] \tag{20}$$

$$\Omega_{bc} = \cosh^{-1}\left(\frac{b^2+c^2-D_{bc}^2}{2bc}\right) = \cosh^{-1}\left[\frac{I\cdot J-2l_2\cdot Q}{K\cdot P}\right] \tag{21}$$

where $I = L^2-a^2+l_2^2$, $J = L^2-a^2+(l_1+l_2)^2$, $K = L^2-a^2-l_2^2$, $P = L^2-a^2-(l_1+l_2)^2$, $Q = 2(L^2-a^2)(l_1+l_2)$. As illustrated in Fig. 3, these parameters are represented by the radius of the antenna a, the thickness of the two dielectric layers l_1 and l_2, and the distance L from the plane boundary between the lower dielectric layer and the half-space to the center of the antenna.

As an examination on (18)-(21), we use (11), i.e

$$T^2 = \frac{\Omega_{ad}}{\Omega_{ab}+\Omega_{cd}+(\widetilde{\varepsilon}_1/\widetilde{\varepsilon}_2)\Omega_{bc}+\widetilde{\varepsilon}_e\Omega_{cd}}. \tag{22}$$

It follows that when $\widetilde{\varepsilon}_1 = \widetilde{\varepsilon}_2 = \widetilde{\varepsilon}_3$, $T^2 = 1$.

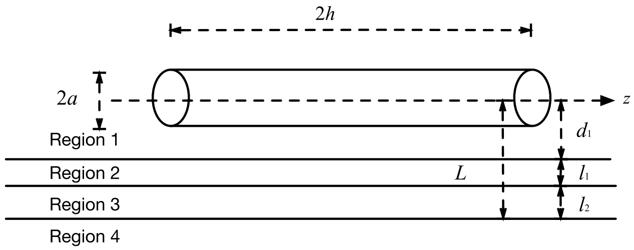

FIGURE 3. A horizontal linear antenna in the presence of a four-layered region.

By substitutions of (18)-(22) into (31), we have

$$k_L = k_1\left\{1+\frac{2}{\Omega_{ad}}\left[\frac{1}{X^2}-\frac{K_1(X)}{X}+\frac{i\pi I_1(X)}{2X}-i\left(\frac{X}{3}+\frac{X^3}{45}+\frac{X^5}{1575}+\frac{X^7}{99225}+\ldots\right)\right]\right\}^{1/2}\cdot T \quad (23)$$

where $X = k_4(L+\sqrt{L^2-a^2})$. I_1 and K_1 are the first-order modified Bessel functions of the first kind and second kind, respectively.

After obtaining the complex wave number k_L, the characteristic impedance Z_c is determined by substituting (23) into (16), and then the analytical formula of the current distribution is obtained. The formulas for k_L and Z_c can apply to the structure shown in Fig. 3. The effect of the presence of a four-layered region can be seen in the expression of k_L, which depends on the height of the antenna and the constitutive properties of the medium on which it is placed.

Once the current distribution of the antenna is determined, the input impedance Z_{in} can also be obtained readily. We write

$$Z_{\text{in}} = \frac{V_0^e}{I_z(0)}. \quad (24)$$

By now, the analytical formulas for the current distribution and input impedance of a horizontal linear antenna in the presence of a four-layered region have been derived. Next, the corresponding computations and discussions will be carried out.

E. EXTENSION TO THE CASE OF N-LAYER REGION

The center-driven cylindrical antenna with radii d_1 is isolated from the outmost Region n by $n-1$ cylindrical dielectrics. The radii for Region 1, 2, $\cdots$, $n-1$ are denoted by d_2, d_3, $\cdots$, d_n, respectively. The wave number for Region j are represented by

$$k_j = \omega(\mu\tilde{\varepsilon}_j)^{1/2} \quad (25)$$

where $j = 1, 2, 3, \cdots, n$. The axis of the n-layered cylinders with radii $d_1, d_2, d_3, \cdots, d_n$ are separated from each other by $D_{d_1d_2}, D_{d_2d_3}, \cdots, D_{d_{n-2}d_{n-1}}$, and $D_{d_{n-1}d_n}$, respectively. Since the electric field in Region 1 are most relevant for the determination of the current distribution on the antenna, we take the electric field in Region 1 as an example, the component $\tilde{E}_{z1}$ can be written in the following forms:

$$\begin{aligned}\tilde{E}_{z1}(r,\theta,\zeta) &= \frac{i\tilde{I}(\zeta)(k_1^2-\zeta^2)}{2\pi\omega\tilde{\varepsilon}_1}\Omega(r,\theta)+\frac{i\tilde{I}(\zeta)\omega\mu}{2\pi k_n d_n}\Delta(r,\theta)\\ &+\frac{i\tilde{I}(\zeta)}{2\pi\omega}\left\{\left[\frac{(k_{n-1}^2-\zeta^2)}{\tilde{\varepsilon}_{n-1}}-\frac{(k_{n-2}^2-\zeta^2)}{\tilde{\varepsilon}_{n-2}}\right]\Omega_{d_{n-1}d_n}\right.\\ &+\left[\frac{(k_{n-2}^2-\zeta^2)}{\tilde{\varepsilon}_{n-2}}-\frac{(k_{n-3}^2-\zeta^2)}{\tilde{\varepsilon}_{n-3}}\right]\Omega_{d_{n-2}d_{n-1}}\\ &\left.\cdots+\left[\frac{(k_2^2-\zeta^2)}{\tilde{\varepsilon}_2}-\frac{(k_1^2-\zeta^2)}{\tilde{\varepsilon}_1}\right]\Omega_{d_2d_3}\right\}\end{aligned} \quad (26)$$

In order to determine the wave number and characteristic impedance for the antenna, the electric field component $E_{\text{ôŔřĆ}z1}$ at the outer surface of the antenna is derived, which could be written in the following form

$$\tilde{E}_{z1}(D_{d_1d_n},0,\zeta) = \left(\frac{i\tilde{I}\Omega_{d_1d_n}}{2\pi\omega\tilde{\varepsilon}_1}\right)\left\{k_1^2+\left(\frac{k_1^2}{k_nd_n\cdot\Omega_{d_1d_n}}\right)\Delta(D_{d_1d_n},0)\cdot T^2-\zeta^2\right\} \quad (27)$$

where

$$T^2 = \frac{\Omega_{d_1d_n}}{\Lambda} \quad (28)$$

$$\begin{aligned}\Lambda &= \Omega_{d_1d_2}+\Omega_{d_3d_4}+\cdots+\Omega_{d_{n-2}d_{n-1}}+\Omega_{d_{n-1}d_n}\\ &+\frac{\tilde{\varepsilon}_1}{\tilde{\varepsilon}_2}\Omega_{d_2d_3}+\frac{\tilde{\varepsilon}_1(\tilde{\varepsilon}_2-\tilde{\varepsilon}_3)}{\tilde{\varepsilon}_2\tilde{\varepsilon}_3}\Omega_{d_3d_4}+\frac{\tilde{\varepsilon}_1(\tilde{\varepsilon}_3-\tilde{\varepsilon}_4)}{\tilde{\varepsilon}_3\tilde{\varepsilon}_4}\Omega_{d_4d_5}\\ &+\cdots+\frac{\tilde{\varepsilon}_1(\tilde{\varepsilon}_{n-2}-\tilde{\varepsilon}_{n-1})}{\tilde{\varepsilon}_{n-2}\tilde{\varepsilon}_{n-1}}\Omega_{d_{n-1}d_n}\quad (n\geq 5)\end{aligned} \quad (29)$$

$$\Omega_{d_id_j} = \cosh^{-1}\left(\frac{d_i^2+d_j^2-D_{d_id_j}^2}{2d_id_j}\right)\quad (i,j=1,2,\cdots n) \quad (30)$$

Similarly, the current on the antenna of length $2h$ with $n-1$ eccentric dielectric coatings has the same distribution except the wave number, characteristics impedance are different. They are

$$k_L = k_1\left[1+\frac{\Delta(D_{d_1d_n},0)}{k_nd_n\Omega_{d_1d_n}}\right]^{1/2}\cdot T1 \quad (31)$$

$$Z_c = \frac{k_L}{2\pi\omega\tilde{\varepsilon}_1}\cdot F/\prod_{i=2}^{n-1}\tilde{\varepsilon}_i \quad (32)$$

$$\begin{aligned}F &= \prod_{i=2}^{n-1}\tilde{\varepsilon}_i\sum_{i=1,j=2}^{n-1,n}\Omega_{d_id_j}+\tilde{\varepsilon}_1\prod_{i=3}^{n-1}\tilde{\varepsilon}_i\Omega_{d_2d_3}\\ &+\tilde{\varepsilon}_1(\tilde{\varepsilon}_2-\tilde{\varepsilon}_3)\prod_{i=4}^{n-1}\tilde{\varepsilon}_i\Omega_{d_3d_4}\\ &+\cdots+\tilde{\varepsilon}_1(\tilde{\varepsilon}_{n-2}-\tilde{\varepsilon}_{n-1})\prod_{i=2}^{n-3}\tilde{\varepsilon}_i\Omega_{d_{n-1}d_n}\quad (n\geq 5)\end{aligned} \quad (33)$$

and where $\prod$ is denoted as a cumulative symbol.

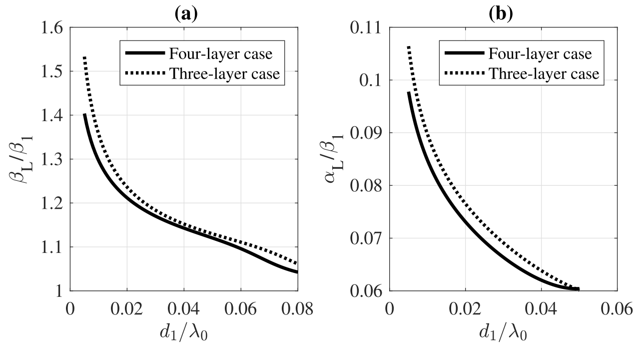

FIGURE 4. Normalized phase and attenuation constants for horizontal antenna in the presence of a three- and four-layered region. (a) Phase constants. (b) Attenuation constants.

So far, we have derived the wave number and characteristic impedance of $n-1$ layered eccentrically insulated antenna. As we previously explained in section *D*, a similar limiting process can be utilized to deal with the problem of a horizontal linear antenna in a n-layered region, i.e. let $d_n \to \infty$. In this case, Region n becomes the half space we showed before. The limitation on the thickness of the layer is $k_1 l_n < 1$, where k_1 is the wave number in Region 1, and l_n is the thickness of layer n.

III. COMPUTATIONS AND DISCUSSIONS

It is clear that the analytical method can be generalized to n-layered case. In the following computations, we take a four-layered case as an example. The parameters are taken as follows: the radius of the antenna is $a = 0.0015\lambda_0$, where λ_0 denotes the wavelength in free space, the length of the antenna is $2h = \lambda_0$, the operating frequency is taken as $f = 300$ MHz, and the four regions are characterized by the relative permittivity $\varepsilon_{1r} = 1$, $\varepsilon_{2r} = 2.65$, $\varepsilon_{3r} = 8$, $\varepsilon_{4r} = 12$, conductivity $\sigma_1 = 0$ S/m, $\sigma_2 = 1 \times 10^{-4}$ S/m, $\sigma_3 = 1 \times 10^{-3}$ S/m and $\sigma_4 = 1 \times 10^{-2}$ S/m, respectively.

Typical graphs of β_L/β_1 and α_L/β_1 varying with the normalized height d_1/λ_0 are shown in Fig. 4. The properties of the three-layered case shown in Fig. 4 are $\varepsilon_{1r} = 1$, $\varepsilon_{2r} = 2.65$, $\varepsilon_{3r} = 12$, $\sigma_1 = 0$ S/m, $\sigma_2 = 1 \times 10^{-4}$ S/m, and $\sigma_3 = 1 \times 10^{-2}$ S/m. It is noted that β_L and α_L represent the phase constant and attenuation rate, respectively (i.e. $k_L = \beta_L + i\alpha_L$), while $k_1 = \beta_1$ is the wave number of Region 1 (air). It is seen that the effects of the boundary are very evident when $d_1/\lambda_0 < 0.02$, with β_L obviously larger than β_1. As d_1/λ_0 increases, β_L will gradually approach to β_1, and the influence of the boundary will be smaller and smaller. It is predicted that the antenna will be similar to an isolated antenna in the air when the height is high enough.

Moreover, it is found that both the phase constant and attenuation rate are slightly decreased when an extra dielectric layer is added.

The current distributions on the antenna with different l_1 and l_2 are shown in Fig. 5. The height of the antenna over the ground plane is taken as $d_1 = 0.01\lambda_0$. It is found that the directions of the real and imaginary components of the current are always opposite to each other in a four-layered region.

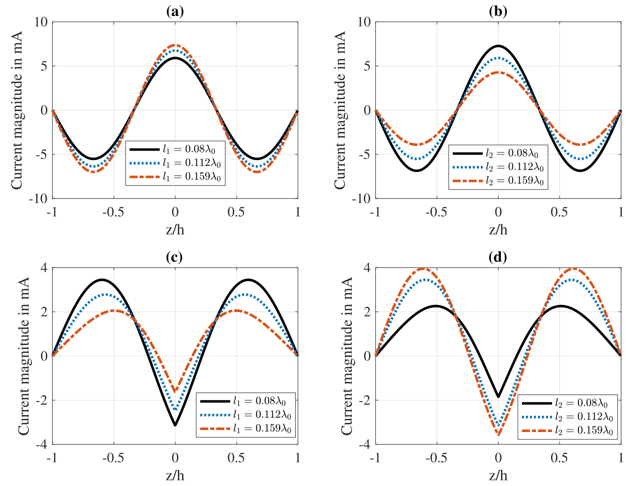

FIGURE 5. Current distributions as a function of z/h with different l_1 ((a), (c)) and different l_2 ((b), (d)).

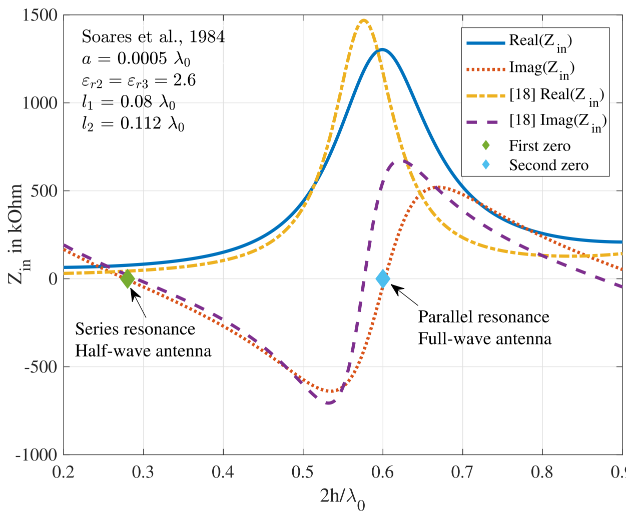

FIGURE 6. Z_{in} as a function of $2h/\lambda_0$ with $\varepsilon_{2r} = 2.6$, $l_1 = 0.08\ \lambda_0$, and $\varepsilon_{3r} = 2.6$, $l_2 = 0.112\ \lambda_0$.

When the thickness of a certain dielectric layer changes, the overall current waveforms are similar, but the effects caused by the upper and lower dielectric layers are different. It is observed that the increase of l_1 (the upper layer) will lead to a larger real part of the current, but will cause the imaginary part of the current to decrease in the meantime. However, the above phenomenon is totally reversed when the thickness change occurs to the lower layer, i.e., an increase on l_2 (the lower layer) will cause the real part to decrease but will make the imaginary part increase.

Computations for the input impedance as a function of the normalized length $2h/\lambda_0$ are also carried out under several conditions. In order to validate our results, the comparisons on the works obtained and those by Soares *et al.* [18] are addressed in Fig. 6. The parameters are adjusted to the same ones used by Soares *et al.* [18], where $\varepsilon_{2r} = \varepsilon_{3r} = 2.6$, $l_1 = 0.08\lambda_0$, and $l_2 = 0.112\lambda_0$. It is seen that the impedances by the analytical method proposed are in good agreement with

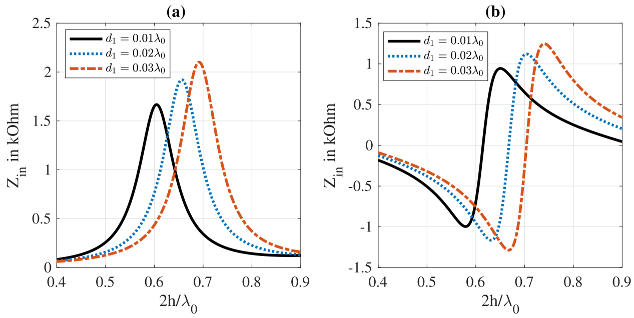

FIGURE 7. Z_{in} as a function of $2h/\lambda_0$ with $l_1 = 0.08\ \lambda_0$, $l_2 = 0.112\ \lambda_0$ at different heights ($d_1 = 0.01, 0.02, 0.03\ \lambda_0$). (a) Real part. (b) Imaginary part.

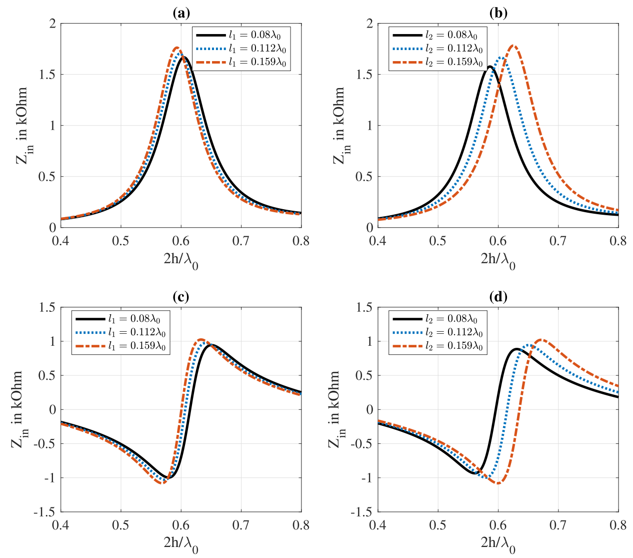

FIGURE 8. Z_{in} as a function of $2h/\lambda_0$ with different l_1 ((a), (c)) and different l_2 ((b), (d)).

the corresponding numerical results by Soares *et al*. It should be pointed out that the two results have slight differences because the horizontal antenna in Soares's model was buried in the upper dielectric layer, whereas in our model the antenna is located in the air.

From Fig. 6, there exist two zero points for the imaginary part of the input impedance, where the antenna lengths at the first and second zero points exactly correspond to a half-wavelength antenna (series resonance) and a full-wavelength antenna (parallel resonance). This finding can also be proved by Table I, where the input impedances at several specific electrical lengths are listed. It is then verified that the first zero point corresponds to a length of $2h \approx 0.3\lambda_0$ with $\beta_L h \approx \pi/2$, while the second corresponds to $2h \approx 0.6\lambda_0$ with $\beta_L h \approx \pi$. It is also found that the half-wavelength antenna has an overall minimum impedance, while the full-wavelength antenna has a maximum real part of the input impedance.

As shown in Figs. 7-9, the input impedances of the antenna are computed at different heights, thicknesses, relative permittivities, and conductivities, respectively. The antenna length ranges from $0.3\lambda_0$ to $0.8\lambda_0$. It is seen that

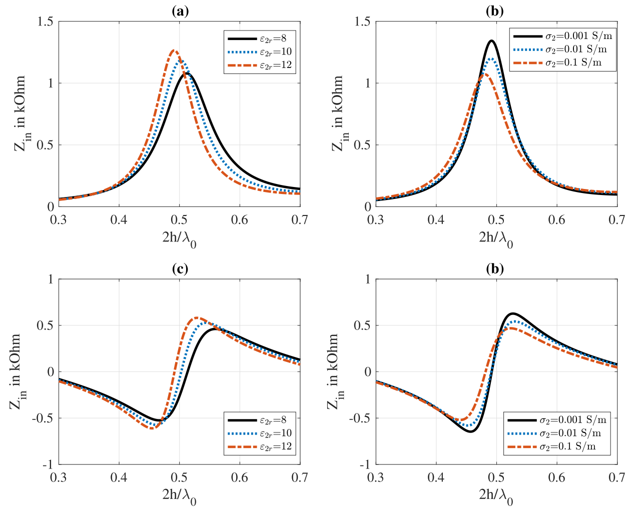

FIGURE 9. Z_{in} as a function of $2h/\lambda_0$ with different ε_{2r} ((a), (c)) and different σ_2 ((b), (d)).

TABLE 1. Input impedance at several electrical lengths.

$2h/\lambda_0$	$\beta_L h$	ReZ_{in}	ImZ_{in}
0.154	0.80	26.41 Ω	365.28 Ω
0.210	1.09	29.41 Ω	196.59 Ω
0.276	1.43	37.81 Ω	54.60 Ω
0.290	1.50	40.34 Ω	28.26 Ω
0.304	1.58	43.67 Ω	-1.46 Ω
0.348	1.80	56.04 Ω	-83.92 Ω
0.482	2.50	201.06 Ω	-452.22 Ω
0.604	3.13	1666.6 Ω	1.11 Ω
0.611	3.16	1636.1 Ω	199.76 Ω
0.621	3.21	1484.2 Ω	486.96 Ω

the input impedances are greatly influenced by the variation of antenna height. As the thickness of the upper dielectric layer increases, a reduction in the resonant length of the antenna can be observed by examining the zero points of the imaginary part of the impedance. Nevertheless, when the thickness of the lower layer increases, the resonance will occur at a longer antenna length. It is noticed that the peak value of input impedance increases with the other three parameters excepted for conductivities. With the increase of the conductivity of the upper layer, the impedance of the antenna decreases gradually. It is predicted that in this case, the high conductivity of the upper layer reduces the penetration depth, thereby reducing the influence of the lower layer.

IV. CONCLUSIONS

In this paper, the problem for a horizontal linear antenna in the presence of a layered region is treated analytically and numerically. Specifically, we consider the case of a four-layered region. It is initially obtained both the wave number and characteristic impedance of an eccentrically insulated antenna with three insulating layers. The analytical expression for the current distribution in a four-layered region is then derived by applying a limiting process. Besides, the analytical approach is also applicable to the case of n-layer region. Computations show that the upper and lower dielectric layers have opposite effects on the current distribution and input impedance of the antenna. It is also found that the imaginary part of the input impedance as a function of antenna length has two zero points, where the first zero point corresponds to a half-wavelength antenna (series resonance) while the second corresponds to a full-wavelength antenna (parallel resonance). Moreover, when the thickness of the upper/lower dielectric layer changes, a reduction/addition on the resonance length of the antenna can be observed by examining the imaginary part of the input impedance. For the sake of enhancing the radiation efficiency of a linear antenna in the presence of the n-layered region, the analytical method proposed in this paper is able to determine the optimal antenna parameters, so that the antenna can work with a smaller input impedance and a larger current moment.

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HUI RAN ZENG was born in Chongqing, China, in 1995. She received the B.S. degree in electronic and information engineering from the East China University of Science and Technology, Shanghai, China, in 2017. She is currently pursuing the Ph.D. degree in electromagnetic field and microwave technology with the College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, Zhejiang, China. Her current research interests include radio wave propagation and antenna theory.

TONG HE was born in Hangzhou, Zhejiang, China, in 1990. He received the B.S. degree in electrical engineering and automation from the University of Electronic Science and Technology of China (UESTC), Chengdu, Sichuan, China, in 2013, and the M.S. degree in electrical engineering from The University of Michigan-Dearborn, Dearborn, MI, USA, in 2014, respectively. He is currently pursuing the Ph.D. degree in electromagnetic field and microwave technology with the College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, Zhejiang, China.

His current research interests include radio wave propagation and antenna theory.

LE LI was born in Beijing, China, in 1994. He received the B.S. degree in electrical engineering from the Department of Information and Electronic Engineering, Zhejiang University, Hangzhou, China, in 2016, where he is currently pursuing the M.S. degree in radio wave propagation theory. His current research interest includes radio wave propagation theory and applications.

KAI LI was born in Xiao County, Anhui, China, in 1968. He received the B.S. degree in physics from Fuyang Normal University, Anhui, in 1990, the M.S. degree in radio physics from Xidian University, Xi'an, Shanxi, China, in 1994, and the Ph.D. degree in astrophysics from the Shaanxi Astronomical Observatory, the Chinese Academy of Sciences, Shaanxi, in 1998.

From 1990 to 2000, he was on the faculty of the China Research Institute of Ra-Microwave Propagation (CRIRP). From 2001 to 2002, he was a Postdoctoral Fellow with Information and Communications University (ICU), Daejeon, South Korea. From 2003 to 2005, he was a Research Fellow with the School of Electrical and Electric Engineering, Nanyang Technological University (NTU), Singapore. Since 2005, he has been a Professor with the Department of Information Science and Electronic Engineering, Zhejiang University, Hangzhou, China. His current research interests include classical electromagnetic theory and radio wave propagation.

Dr. Li is a Senior Member of the Chinese Institute of Electronics (CIE) and a member of the Chinese Institute of Space Science (CISS).