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Parameter and Tolerance Economic Design for Multivariate Quality Characteristics Based on the Modified Process Capability Index With Individual Observations

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ABSTRACT The process capability index (PCI) is widely used in an on-line quality control stage for measuring and controlling the quality level of a production process. The calculation of PCI requires a large number of samples, but in the off-line quality control stage, a certain production process in off-line quality control stage only has a few individual observations. From the perspective of quality loss and tolerance cost, this paper proposes a parameter and tolerance economic design approach for multivariate quality characteristics based on the modified PCI with individual observations. The response surface models of mean and variance are constructed using individual observations, and exponential models are fitted according to the tolerance cost data of design variables. A modified PCI is proposed with the consideration of three types of quality characteristics. The optimal design variables and tolerances are obtained by a comprehensive optimization model that is constructed based on the proposed PCI. An example of an isobutylene-isoprene rubber (IIR) inner tube is used to (i) demonstrate the implementation of our proposed approach, (ii) improve the PCI value and reflect the sensitivity of the deviation between process mean and specification, and (iii) reduce the risk of increasing cost of quality caused by replicated experimental design and some other unknown reasons.

INDEX TERMS Parameter, tolerance, economic design, multivariate quality characteristics, process capability index, individual observations.

I. INTRODUCTION

Juran predicted that the 21st century would be the century of quality [1]. Product quality is not only the lifeline of enterprise, but also the key to winning customers worldwide. In quality management, cost of quality (CoQ) was first introduced by Feigenbaum and extensively studied as a component of efforts to improve quality and reduce costs [2]. A widely used CoQ approach is the prevention-appraisal-failure (PAF) model of Feigenbaum, which was also adopted by the American Society for Quality [3]. The PAF model uses the preventive cost, appraisal cost and failure cost to describe the total quality costs [4]–[6]. According to the PAF model, we know that parameter design and tolerance design are important sources of preventive and appraisal cost. Therefore, the question of how to improve the technology of parameter design and tolerance design is significant to improving the quality level and reducing the total cost of quality.

In the area of modern quality engineering and management, variation is considered as the main cause of quality problems. Although variation cannot be eliminated completely, its impact on a production process can be controlled

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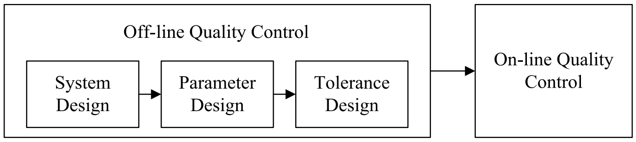

FIGURE 1. Relationship between off-line quality control and on-line quality control.

using statistical tools and approaches [7]. Dr. Taguchi formed the theory of on-line quality control and off-line quality control by applying statistical tools and economic approaches. Taguchi introduced the three design approaches of system design, parameter design and tolerance design, which are widely used in off-line control [8]. The relationship between off-line quality control and on-line quality control is illustrated in Figure 1. From Figure 1, we find that off-line quality control and on-line quality control are implemented in sequence. When the tolerance design constraint cannot be satisfied, the system and parameter must be redesigned. This type of improvement process always causes higher prevention cost and appraisal cost due to replicated experimental design. Therefore, it is beneficial to improve the optimal quality level and reduce the CoQ by applying the technology of on-line quality control to parameter design and tolerance design.

Parameter design is the key component of off-line quality control. The basic principle of parameter design is to choose the optimal parameter sets that reduce the influence of noise variables and make the products more robust and reliable. Many scholars have studied parameter design [9]–[11]. System design and parameter design are used to obtain the optimal design variables, and tolerance design decides whether the tolerance range of design parameters should be adjusted from the perspective of economy. In engineering practice, Dr. Taguchi's 'three designs' are conducted in sequence. Bisgaard and Ankenman [12] noted that the optimal sets of design variables were obtained under the assumption of a specific range of tolerance. However, in tolerance design, the adjustment of tolerance means that the optimal sets from parameter design are not actually optimal. Therefore, it is necessary to implement parameter design and tolerance design simultaneously. The related literature can be consulted [13]–[20], but previous research only solves the design problems of off-line quality control stage and does not consider the indices of the on-line quality control stage. Therefore, it is worthwhile to introduce the indices of on-line quality control into parameter and tolerance design.

Statistical process control is an important on-line quality control technology that consists of process capability analysis and process control [21]. Process capability analysis focuses on evaluating the ability of process that makes products or services to meet given specifications. Such an ability is usually measured by process capability indices [22], [23]. Certain scholars suggested that process capability can be estimated when PCI is introduced into the parameter and tolerance design stages [24]–[26]. In previous studies, the approach to solution of parameter and tolerance parallel design for multivariate quality characteristics is described as follows. First, sample the experiment repeatedly and construct a response surface model. Second, calculate the PCI for a single quality characteristic. Third, establish a multivariate process capability index (MPCI), and finally, treat it as an optimization function and obtain the optimal design variables using algorithms. However, those studies overlooked three problems: (i) normally, the calculation of process capability index requires a large amount of samples, but only individual observations can be collected due to unreplicated experiments, as documented in the literature [27]–[31]; (ii) these approaches primarily aimed to fix only the-nominal-the-best type (NTB-type) quality characteristic but cannot be used on the-larger-the-best type (LTB-type) and the-smaller-the-best type (STB-type) quality characteristics; (iii) these approaches considered the deviance of process mean from the target value but ignored the difference between process mean and specification. This paper proposes a new approach to solve these problems.

In this paper, we propose a parameter and tolerance economic design approach based on the modified PCI for multivariate quality characteristics with individual observations. First, the response surface models for process mean and variance are constructed in terms of design variables and tolerances. Second, the tolerance cost models are fitted with the tolerance cost data of design variables. Third, according to the types of quality characteristics and the deviance of process mean from the specification, a modified PCI based on quality loss and tolerance cost is proposed. Finally, a comprehensive optimization model based on the modified PCI is constructed.

The remainder of this article is organized as follows. Section 2 supplies a review of parameter design and tolerance design, Section 3 introduces the process capability index and quality loss function, and Section 4 proposes the parameter and tolerance economic design approach for multivariate quality characteristics based on a modified PCI with individual observations. Section 5 describes an example to illustrate the feasibility of the proposed approach and compares it with other approaches, and Section 6 presents the conclusions.

II. LITERATURE REVIEW

Dr. Taguchi initially proposed a parameter design approach based on inner-outer arrays and signal-noise ratio. Many scholars subsequently researched the Taguchi parameter design approach [32], [33]. The response surface method (RSM) was first proposed by Box and Wilson [34] and introduced in parameter design by Shoemarker *et al.* [35]. This approach explored the relationship between design variables and outputs using experiments, built a response surface model to establish an objective function, and finally obtained optimal design variables using an optimization method [36], [37]. Vining and Myers [38] noted that the deviation of mean from the target and the variance both cause variation and proposed the dual response surface method (DRSM). With the difference in customer demand for products, multiple quality characteristics often must be considered in the optimization design of a product or process. Common statistical approaches for multivariate quality characteristics are the quality loss function approach, Mahalanobis distance approach, and desirability function approach [39]–[44].

Tolerance design is useful for quality improvement and cost reduction. Jin *et al.* [45] researched tolerance design based on nonsymmetrical quality loss in the case in which the quality characteristic follows a non-normal distribution. Zhao *et al.* [46] constructed a service model from the perspective of quality loss and tolerance cost according to the distribution of the product life and proposed a parameter economic design approach based on the quality loss of service. Zhang *et al.* [47] considered the type of quality characteristic and solved tolerance economic design problems of hierarchical products based on the quality loss function. Moreover, selected scholars proposed other tolerance design approaches to solve problems in tolerance design [48]–[50]. These studies all assumed that the design variables are fixed prior to tolerance design. According to Bisgaard's view, engineers can only obtain the true optimal design variables and tolerances through parallel parameter and tolerance design. Kim and Cho [13] solved the parameter and tolerance economic design problem using the Taguchi quality loss cost and tolerance cost. Jeang [14] performed parallel optimization for design variables and tolerances by considering quality variation and CoQ. Moskowitz *et al.* [15] analyzed the quality loss cost and the supplier/manufacturer cost and researched parameter and tolerance design problems for multivariate quality characteristics using parametric and nonparametric approaches, respectively. Plante [16] used the quadratic model to solve parameter and tolerance parallel design problems. Wu [17] proposed a parallel design approach for multivariate dynamic quality characteristics based on sequential optimization. Park *et al.* [18] considered the impact of noise factors and solved the parameter and tolerance parallel design problem of a rechargeable battery using a mixed experimental design approach. Han and Tan [19], [20] proposed computer-aided parameter and tolerance design approaches for computer experiments, in which the means and tolerances of input characteristics are simultaneously optimized to minimize the total cost. The mentioned literature solved the design problems in off-line quality control but did not consider the indices of on-line quality control. Therefore, the question of how to combine on-line and off-line quality control approaches to solve parameter and tolerance economic design problems is a topic worthy of research.

PCI is used to measure the quality level of a production process. Plante [24] proved that it is worthwhile to apply MPCI to multiple-response optimization design of both product and process. Jeang [25] analyzed the manufacturer cost and quality loss cost and subsequently implemented parameter and tolerance parallel design using MPCI. Jeang [26] used the DSRM to build the models of mean and variations and proposed an optimal function based on the MPCI from the literature [25]. In the literature, the approaches all calculated PCI via multiple sampling experiments but did not consider the case of individual observations due to unreplicated experiments. In this paper, we propose a parameter and tolerance economic design approach based on MPCI with individual observations.

III. RELEVANT FUNCTIONS

A. PROCESS CAPABILITY INDEX

In recent decades, the PCI was an important approach used in on-line quality management. Various PCIs have been proposed from the viewpoints of product specification and quality loss [22], [23]. Juran *et al.* [1] first introduced the concept of capability ratio and proposed the index C_p to compare the evaluation of process output with the tolerance range of design. The premise of process capability analysis is that the process exists in a state of statistical control, which means that the output has a stable and predictable distribution. In this paper, we assume that the quality characteristic y is normally distributed with mean μ and variance σ^2. When the mean of the process output is consistent with the center of specification limit, the C_p is expressed as follows [51]

$$C_p = \frac{USL - LSL}{6\sigma} \tag{1}$$

where USL and LSL are the upper and lower specification limits, respectively.

However, the mean of process output does not always coincide with the center of the specification limits. To measure the process capability, Kane [52] and Chang *et al.* [53] proposed indices C_{pk} and C_{pm}, respectively.

$$C_{pk} = \min\left\{\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} = (1 - w)\, C_p \tag{2}$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{(\mu - T)^2 + \sigma^2}} \tag{3}$$

where w is the skewness, and T is the target value. When $C_{pk} < 1$, the process capability is insufficient; when $1 \leq C_{pk} < 1.5$, the process capability is normal; and when $1.5 \leq C_{pk}$, the process capability reaches the level of 6 sigma, and we can loosen the tolerance to reduce the production cost. Additionally, C_{pm} is known as the Taguchi process capability index and is used to emphasize the quality loss that results from the deviation of the quality characteristic from the design target.

Pearn [54] considered the situation in which the process mean is different from target and specification center and proposed the index C_{pmk}.

$$C_{pmk} = \frac{C_{pk}}{\sqrt{\left(\frac{\mu - T}{\sigma}\right)^2 + 1}} \tag{4}$$

To analyze the process capability of the production process for multivariate quality characteristics, Wang and Du [55] extended the PCI for a single quality characteristic to multivariate quality characteristics and constructed the

MPCI using the geometric average approach.

$$MC_p = \left[\prod_{m=1}^{v} C_{p;m}\right]^{\frac{1}{v}} \tag{5}$$

where $C_{p;m}$ is the process capability index for the m th variable. Similarly, when the mean of the output deviates from the target value, we can obtain the following multivariate process capability index.

$$MC_{pk} = \left[\prod_{m=1}^{v} C_{pk;m}\right]^{\frac{1}{v}} \tag{6}$$

where $C_{pk;m}$ is the process capability index for the m th variable when the deviation exists.

B. QUALITY LOSS FUNCTION

Quality is an abstract concept and is highly difficult to precisely define. Conceptually, it is more appealing to consider that the product has the best quality when it exactly meets the requirements and that it suffers a loss of quality when it deviates from the requirements [56]. Taguchi [57] defined quality as follows: "quality is the loss a product causes to society after being shipped, other than any losses caused by its intrinsic functions". Furthermore, Taguchi quantifies the deviations from the requirements in term of monetary units using a quadratic loss function given by

$$L(y) = k(y-T)^2 \tag{7}$$

where k is the loss coefficient that transform the quality loss to a value unit. To quantify quality loss, Taguchi proposed the expected quality loss function

$$E[L(y)] = kE(y-T)^2 = k\left[\sigma^2 + (\mu - T)^2\right] \tag{8}$$

Obviously, to reduce quality loss in the production process, the variance of process output should be reduced under the condition that the process mean is close to the design target [46].

According to the Taguchi's quality viewpoint, quality characteristics can be divided into NTB-type, LTB-type and STB-type quality characteristics. The quality loss function in formulas (8) is only applicable for the NTB-type characteristic. If the quality characteristic y is the STB-type and its value is positive, the target of y can be set to zero. Thus, the quality loss function is given by the following

$$L(y) = ky^2 \tag{9}$$

Similarly, if y is the LTB-type, $1/y$. should be the STB-type, and therefore, its quality loss function is given by the following

$$L(y) = \frac{k}{y^2} \tag{10}$$

IV. PROPOSED APPROACH

A. MEAN AND VARIANCE MODELS OF QUALITY CHARACTERISTICS

Normally, the design variables are fixed values in research on parameter design of products and processes. However, in engineering practice, the design variable is a normally distributed variable with the nominal value as its mean value [13]. This paper considers the problem of multivariate quality characteristics and constructs the models of mean and variance based on the design variables and tolerances. Suppose that y_m is the m th quality characteristic, x_i is the i th design variable, z_i is the coding value of x_i, and the response surface model can be built as follows

$$y_m(z) = \beta_{m0} + \sum_{i=1}^{n} \beta_{mi} z_i + \sum_{j=i}^{n}\sum_{i=1}^{n} \beta_{mij} z_i z_j + \varepsilon_m \tag{11}$$

where β_{m0}, β_{mi} and β_{mij} are coefficients associated with the constant, first-order and quadratic terms, respectively, and ε_m is random error and is assumed to follow a normal distribution $N\left(0, \sigma_{\varepsilon_m}^2\right)$. Applying the approach of least squares based on the experimental data at each design point, the fitted model for the transfer function is given by the following

$$\hat{y}_m(z) = \hat{\beta}_{m0} + \sum_{i=1}^{n} \hat{\beta}_{mi} z_i + \sum_{j=i}^{n}\sum_{i=1}^{n} \hat{\beta}_{mij} z_i z_j \tag{12}$$

where $\hat{\beta}_{m0}$, $\hat{\beta}_{mi}$ and $\hat{\beta}_{mij}$ are the least squares estimates of β_{m0}, β_{mi} and β_{mij}, respectively. By transforming the coded variables to the original design variables, formula (12) can be rewritten as shown

$$\hat{y}_m(x) = \hat{b}_{m0} + \sum_{i=1}^{n} \hat{b}_{mi} x_i + \sum_{j=i}^{n}\sum_{i=1}^{n} \hat{b}_{mij} x_i x_j \tag{13}$$

where $\hat{b}_{m0}$, $\hat{b}_{mi}$ and $\hat{\beta}_{mij}$ are the appropriate coefficients associated with the constant, first-order and quadratic terms, respectively.

In engineering practice, design variable x_i is a random variable that varies around the nominal value Δ_i and follows a normal distribution with mean value $N\left(\Delta_i, \sigma_{x_i}^2\right)$. When x_i is given, the models of mean and variance of m th quality characteristic can be derived via Taylor expansion.

$$\hat{\mu}_m(x) = \hat{Y}_m(x) + 1/2\sum_{i=1}^{n} \hat{b}_{mii}\sigma_{x_i}^2 \tag{14}$$

$$\hat{\sigma}_m^2(x) = \sum_{i=1}^{n}\left(\hat{b}_{mi} + 2\hat{b}_{mii}x_i + \sum_{j=i+1}^{n}\hat{b}_{mij}x_j + \sum_{j=1}^{i-1}\hat{b}_{mji}x_j\right)^2 \times \sigma_{x_i}^2 + \sigma_{\varepsilon_m}^2 \tag{15}$$

Based on the popular relationship between the tolerance and variance of the design variable $t_i = 3\sigma_{x_i}$, formulas (14) and (15) can be rewritten in term of x and t_i as follows

$$\hat{\mu}_m(x,t) = \hat{Y}_m(x) + 1/18\sum_{i=1}^{n} \hat{b}_{mii} t_i^2 \tag{16}$$

$$\hat{\sigma}_m^2(x,t) = 1/9 \sum_{i=1}^{n} \left(\hat{b}_{mi} + 2\hat{b}_{mii}x_i + \sum_{j=i+1}^{n} \hat{b}_{mij}x_j + \sum_{j=1}^{i-1} \hat{b}_{mji}x_j \right) \times^2 t_i^2 + \sigma_{\varepsilon_m}^2 \quad (17)$$

If the design variables are fixed values, the response surface models for mean and variance are difficult to obtain when the sampled data of output are individual observations. Therefore, the PCI cannot be implemented in optimization of design variables. In this paper, we assume that the design variables follow a normal distribution, and we obtain the models of mean and variance to solve this problem.

B. QUALITY LOSS COST MODEL

Based on the Taylor expansion approach, we construct the model of quality loss cost $c_{L;m}$ for three types of quality characteristics.

In formula (18), as shown at the bottom of the next page, for the STB-type quality characteristic, we adopt the first three terms of Taylor expansion to accurately calculate the quality loss cost.

C. TOLERANCE COST MODEL

The aim of tolerance design is to reduce the variability of product or process by decreasing the tolerance of design parameters. Although tolerance design can improve the robustness of process and product, enterprise application requires more accurate equipment and skilled staff for implementation. All of these factors result in an increase in production cost. Therefore, in the tolerance design stage, the designer should make a comprehensive decision on tolerance and cost with consideration of their relationship. Currently, certain scholars have proposed selected models for this relationship, such as the reciprocal model and exponential model. In this paper, according to the actual condition of design variables, the exponential model is used to model the tolerance cost with the following form.

$$\hat{c}_i(t_i) = \hat{c}_0 + \hat{c}_1 t_i^{-2} \quad (19)$$

The corresponding model for the total tolerance cost of all design variables can be found as shown

$$C_M(t) = \sum_{i=1}^{n} \hat{c}_i(t_i) = \sum_{i=1}^{n} \hat{c}_{0;i} + \sum_{i=1}^{n} \hat{c}_{1;i} t_i^{-2} \quad (20)$$

D. MODIFIED PCI WITH QUALITY LOSS COST AND TOLERANCE COST

In the research on process capability analysis and tolerance design, it is often assumed that the quality characteristic is the NTB-type. Formulas (1) to (4) are applied for the NTB-type characteristic but cannot be applied to LTB-type and STB-type characteristics. Considering the type of quality characteristic, the process capability index can be expressed as follows.

$$C_{pk;m}(x,t) = \begin{cases} \dfrac{d_m - 2\left|\hat{\mu}_m(x,t) - M_m\right|}{3\hat{\sigma}_m(x,t)} & NTB-type \\ \dfrac{USL_m - \hat{\mu}_m(x,t)}{3\hat{\sigma}_m(x,t)} & LTB-type \\ \dfrac{\hat{\mu}_m(x,t) - LSL_m}{3\hat{\sigma}_m(x,t)} & STB-type \end{cases} \quad (21)$$

where $d_m = (USL_m - LSL_m)/2$ is half the length of the specification interval, and $M_m = (USL_m + LSL_m)/2$ is the mid-point of the specification interval. Hence, referring to the calculation of C_{pmk} in formula (4), the index $C_{pmk;m}$ based on the quality characteristic type can be expressed as

$$C_{pmk;m}(x,t) = \begin{cases} \dfrac{d_m - \left|\hat{\mu}_m(x,t) - M_m\right|}{3\sqrt{\hat{\sigma}_m^2(x,t) + \left[\hat{\mu}_m(x,t) - T_m\right]^2}} & NTB-type \\ \dfrac{USL_m - \hat{\mu}_m(x,t)}{3\sqrt{\hat{\sigma}_m^2(x,t) + \hat{\mu}_m^2(x,t)}} & LTB-type \\ \dfrac{\hat{\mu}_m(x,t) - LSL_m}{3\sqrt{\dfrac{1}{\hat{\mu}_m^2(x,t)}\left[1 + \dfrac{3\hat{\sigma}_m^2(x,t)}{\hat{\mu}_m^2(x,t)} - \dfrac{4\hat{\sigma}_m^3(x,t)}{\hat{\mu}_m^3(x,t)}\right]}} & STB-type \end{cases} \quad (22)$$

Obviously, the $C_{pmk;m}$ from formula (22) considers the Taguchi quality loss and emphasizes the impact of deviance of the mean from target value. However, previous researchers did not consider the tolerance cost. According to the principle from the literature [25], a modified process capability index is proposed with the quality loss cost and tolerance cost as formula (23) at the bottom of the next page.

E. COMPREHENSIVE OPTIMIZATION MODEL

This paper transforms the variation from mean and variance to quality loss cost through the Taguchi quality loss function and measures the tolerance cost. Based on these two types of cost, we obtain the modified PCI model of the quality characteristic and construct a MC_{pmkc}. Thus, the parameter and tolerance economic design is described as in the following mathematical problem.

$$\begin{aligned} \max F = MC_{pmkc} &= \prod_{m=1}^{M} C_{pmkc;m}(x,t) \\ st.\ x_i^- &\le x_i \le x_i^+ \\ y_m^- &\le y_m \le y_m^+ \\ i &= 1, \cdots, n \\ m &= 1, \cdots, M \end{aligned} \quad (24)$$

where x_i^- and x_i^+ are the lower bound and upper bound of design variable x_i, and y_m^- and y_m^+ are the constraint lower bound and upper bound for y_i, respectively. From the form of objective function (25), we know that this is a global nonlinear optimization problem and can be solved by a nonlinear optimization algorithm.

F. FRAMEWORK AND PROCEDURE

In this paper, considering the types of quality characteristics and the deviance of process mean from the specification center, we propose an economic optimization design approach that addresses the parameters and tolerances for multivariate quality characteristics. The framework and procedure are illustrated in Figure 2. This approach solves the problem with the pattern of design-experiment-modeling-optimization. This approach contains six steps described as follows.

Step 1: According to the actual case, we ascertain the types of quality characteristics and the levels of design variables. We form the experiment plan by analyzing the constraints in experiments. If the number of design variables are greater than three, a central-composite design approach is used; otherwise, a factorial design approach is used [58]. The corresponding data are collected via experiments.

Step 2: The response surface model of every quality characteristic is fitted using the experimental data from step 1. Considering the distributions of design variables, the response surface models of mean and variance of every quality characteristic are constructed based on the design variables and tolerances.

Step 3: According to the range of the tolerance from step 1, a tolerance cost experiment is implemented, and the data are collected.

Step 4: The tolerance cost models are fitted with the tolerance cost data from step 3.

Step 5: Based on the model of quality loss cost and tolerance cost, the modified PCI with tolerance cost model is obtained and is used to construct the MPCI.

Step 6: The MPCI is treated as an objective function. By maximizing the objective function, the optimal design variables and tolerances are obtained via a nonlinear optimization algorithm.

TABLE 1. Design variables and their levels in the IIR inner tube experiment.

Order	Design Variables	Levels				
		-1.682	-1	0	1	1.682
1	Semi-reinforcing furnace black (x_1)	20	24.05	30	35.95	40
2	Sulfur (x_2)	0.8	1.04	1.4	1.76	2.0
3	TMTD (x_3)	0.8	1.08	1.5	1.92	2.2

V. EXAMPLE

A. EXPERIMENT ON IIR INNER TUBE

The production process for IIR inner tube involves multiple design variables and quality characteristics, and the tolerances have a highly significant impact on the manufacturing cost [59], [60]. Therefore, it is necessary that the parameters and tolerances are designed in parallel from an economical perspective.

Step 1: The semi-reinforcing furnace black (x_1), sulfur (x_2), and TMTD (x_3) are the design variables, and the tear strength (y_1) and stretching strength (y_2) are the quality characteristics. This paper studied the design variables and their tolerances and calculated their optimal values. The two quality characteristics are both LTB-type characteristics and must satisfy the constraints $y_1 \geq 15$ and $y_2 \geq 55$, respectively. The ranges of the design variables are $20 \leq x_1 \leq 40$, $0.8 \leq x_2 \leq 2$ and $0.8 \leq x_3 \leq 2.2$. In practice, the design variables follow a normal distribution with the nominal value as their mean. Therefore, it is assumed that $x_1 \sim N\left(\Delta_1, \sigma_{x1}^2\right)$, $x_2 \sim N\left(\Delta_2, \sigma_{x2}^2\right)$ and $x_3 \sim N\left(\Delta_3, \sigma_{x3}^2\right)$ in which Δ_1, Δ_2 and Δ_3 are the nominal values of x_1, x_2 and x_3, respectively, and z_1, z_2 and z_3 are the coded variables of x_1, x_2 and x_3. The levels of design variables in this experiment are illustrated in Table 1.

According to the actual condition, a central composite design approach is implemented, and the corresponding data are collected as shown in Table 2.

$$c_{L;m} = \begin{cases} k_m E\left[\hat{y}_m(x) - T_m\right]^2 = k_m\left\{\hat{\sigma}_m^2(x,t) + \left[\hat{\mu}_m(x,t) - T_m\right]^2\right\} & NTB-type \\ k_m E\left[\hat{y}_m^2(x)\right] = k_m\left[\hat{\sigma}_m^2(x,t) + \hat{\mu}_m^2(x,t)\right] & LTB-type \\ k_m E\left[\dfrac{1}{\hat{y}_m^2(x)}\right] \approx \dfrac{k_m}{\hat{\mu}_m^2(x,t)}\left[1 + \dfrac{3\hat{\sigma}_m^2(x,t)}{\hat{\mu}_m^2(x,t)} - \dfrac{4\hat{\sigma}_m^3(x,t)}{\hat{\mu}_m^3(x,t)}\right] & STB-type \end{cases} \tag{18}$$

$$C_{pmkc;m}(x,t) = \begin{cases} \dfrac{d_m - \left|\hat{\mu}_m(x,t) - M_m\right|}{3\sqrt{\left\{\hat{\sigma}_m^2(x,t) + \left[\hat{\mu}_m(x,t) - T_m\right]^2\right\} + \dfrac{C_M(t)}{k_m}}} & NTB-type \\ \dfrac{USL_m - \hat{\mu}_m(x,t)}{3\sqrt{\left[\hat{\sigma}_m^2(x,t) + \hat{\mu}_m^2(x,t)\right] + \dfrac{C_M(t)}{k_m}}} & LTB-type \\ \dfrac{\hat{\mu}_m(x,t) - LSL_m}{3\sqrt{\dfrac{1}{\hat{\mu}_m^2(x,t)}\left[1 + \dfrac{3\hat{\sigma}_m^2(x,t)}{\hat{\mu}_m^2(x,t)} - \dfrac{4\hat{\sigma}_m^3(x,t)}{\hat{\mu}_m^3(x,t)}\right] + \frac{C_M(t)}{k_m}}} & STB-type \end{cases} \tag{23}$$

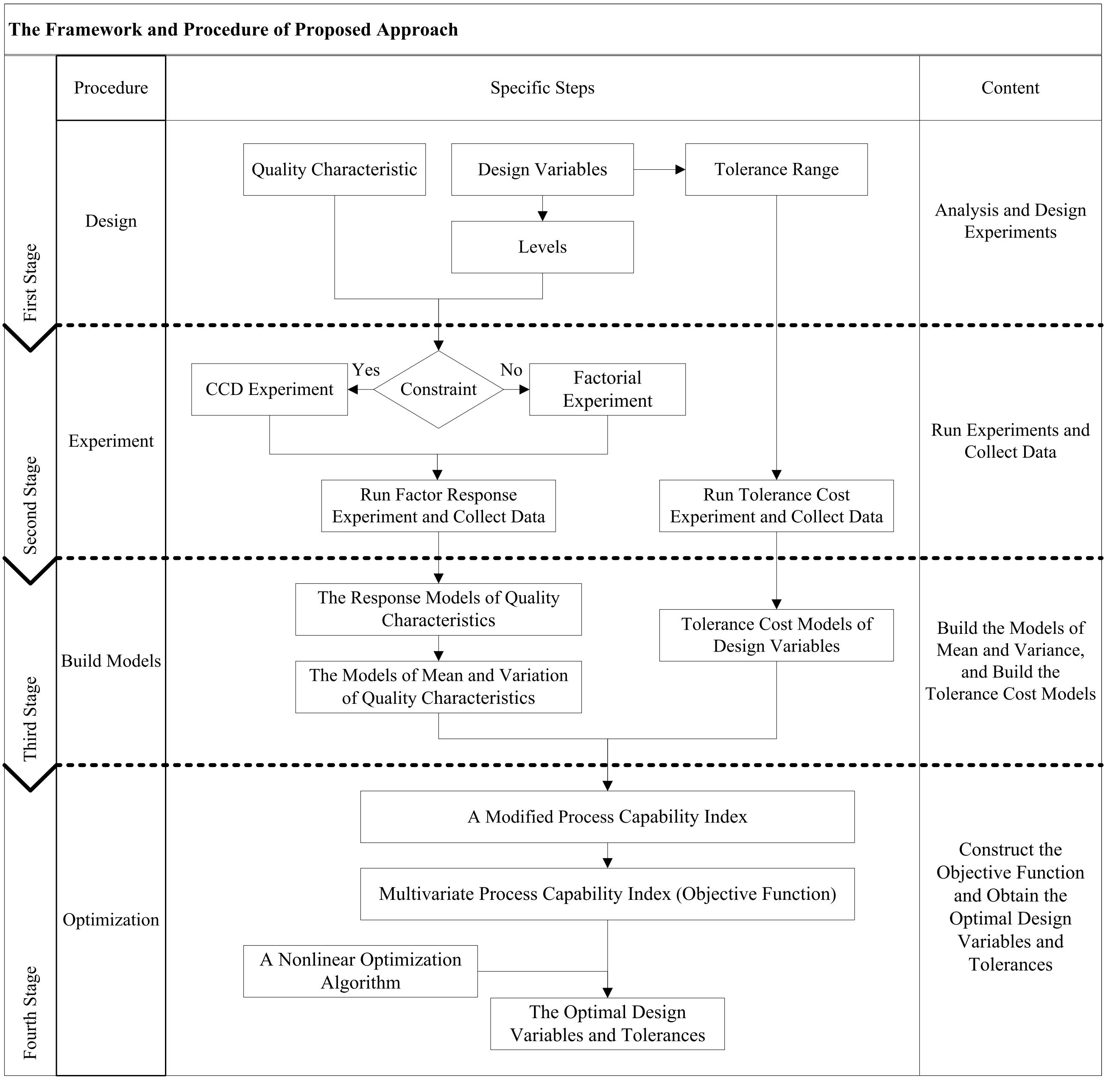

FIGURE 2. Framework and procedure for parameter and tolerance economic design.

Step 2: From the experimental data in Table 2, the response surface models for y_1 and y_2 are fitted as follows

$$\hat{y}_1(z) = 13.901 - 0.627z_1 - 0.180z_2 - 0.295z_3 - 0.365z_1^2 + 0.145z_1z_2 + 0.079z_1z_3 + 0.058z_2^2 - 0.013z_2z_3 + 0.235z_3^2$$

$$\hat{y}_2(z) = 50.063 - 0.724z_1 - 0.979z_2 - 3.922z_3 - 1.826z_1^2 + 0.790z_1z_2 + 0.923z_1z_3 + 0.118z_2^2 - 0.531z_2z_3 + 0.294z_3^2$$

By transforming the coded variables to the original variables, the response surface models can be rewritten as

$$\hat{y}_1(x) = 17.612 + 0.371x_1 - 3.700x_2 - 5.615x_3 - 0.010x_1^2 + 0.068x_1x_2 + 0.032x_1x_3 + 0.456x_2^2 - 0.088x_2x_3 + 1.357x_3^2$$

$$\hat{y}_2(x) = 55.758 + 1.897x_1 - 11.151x_2 - 20.7x_3 - 0.052x_1^2 + 0.373x_1x_2 + 0.373x_1x_3 + 0.927x_2^2 - 3.577x_2x_3 + 1.697x_3^2$$

TABLE 2. Factor response experiment data for IIR inner tube.

Order	Design Variables			Quality Characteristics	
	z_1	z_2	z_3	y_1	y_2
1	1	1	1	12.9381	44.1879
2	1	1	-1	13.3977	51.2797
3	1	-1	1	13.0368	45.6349
4	1	-1	-1	13.4433	50.6012
5	-1	1	1	13.7450	42.2110
6	-1	1	-1	14.5209	52.9933
7	-1	-1	1	14.4232	46.8187
8	-1	-1	-1	15.1460	55.4755
9	1.682	0	0	11.8112	43.6759
10	-1.682	0	0	13.9201	46.1107
11	0	1.682	0	13.7621	48.7573
12	0	-1.682	0	14.3652	52.0316
13	0	0	1.682	14.0705	44.3294
14	0	0	-1.682	15.0558	57.4533
15	0	0	0	13.9020	50.0635
16	0	0	0	13.9017	50.0633
17	0	0	0	13.9024	50.0640
18	0	0	0	13.9012	50.0628
19	0	0	0	13.8997	50.0613
20	0	0	0	13.9024	50.0639

From the above analysis, we find that the variances of estimated errors of y_1 and y_2 are 1.141×10^{-4} and 4.606×10^{-3}. We introduce the above formula and the corresponding data in formulas (14) and (15) and obtain the response surface models for mean and variance of design variables and tolerances as follows.

$$\begin{aligned}
\hat{\mu}_1(x,t) &= 17.612 - 5.556\times10^{-4} t_1^2 + 0.025 t_2^2 + 0.075 t_3^2 \\
&\quad + 0.371 x_1 - 3.700 x_2 - 5.615 x_3 - 0.010 x_1^2 \\
&\quad + 0.068 x_1 x_2 + 0.456 x_2^2 - 0.088 x_2 x_3 + 1.357 x_3^2 \\
\hat{\mu}_2(x,t) &= 55.758 - 2.889\times10^{-3} t_1^2 + 0.052 t_2^2 + 0.094 t_3^2 \\
&\quad + 1.897 x_1 - 11.151 x_2 - 20.7 x_3 - 0.052 x_1^2 \\
&\quad + 0.373 x_1 x_2 + 0.373 x_1 x_3 + 0.927 x_2^2 - 3.577 x_2 x_3 \\
&\quad + 1.697 x_3^2 \\
\hat{\sigma}_1^2(x,t) &= \frac{1}{9}(0.371 - 0.02 x_1 + 0.068 x_2 + 0.032 x_3)^2 t_1^2 \\
&\quad + \frac{1}{9}(-3.700 + 0.912 x - 0.088 x_3 + 0.068 x_1)^2 t_2^2 \\
&\quad + \frac{1}{9}(-5.615 + 2.714 x_3 - 0.032 x_1 - 0.088 x_2)^2 t_3^2 \\
&\quad + 1.141\times10^4 \\
\hat{\sigma}_2^2(x,t) &= \frac{1}{9}(1.897 - 0.104 x_1 + 0.373 x_2 + 0.373 x_3)^2 t_1^2 \\
&\quad + \frac{1}{9}(-11.151 + 1.854 x_2 - 3.577 x_3 + 0.373 x_1)^2 t_2^2 \\
&\quad + \frac{1}{9}(-20.7 + 3.394 x_3 + 0.373 x_1 - 3.577 x_2)^2 t_3^2 \\
&\quad + 4.606\times10^3
\end{aligned}$$

TABLE 3. Tolerance cost experimental data for the design variables.

Order	Design variables					
	Semi-reinforcing furnace black (x_1)		Sulfur (x_2)		TMTD (x_3)	
	Tolerance s (t_1)	Cost (c_1)	Tolerance s (t_2)	Cost (c_2)	Tolerance s (t_3)	Cost (c_3)
1	1.0	2.090	0.1	1.550	0.2	1.602
2	1.5	1.663	0.2	1.307	0.3	1.350
3	2.0	1.254	0.3	1.060	0.4	1.170
4	2.5	0.872	0.4	0.827	0.5	0.910
5	3.0	0.740	0.5	0.654	0.6	0.740
6	3.5	0.605	0.6	0.540	0.7	0.612
7	4.0	0.514	0.7	0.444	0.8	0.479
8	4.5	0.450	0.8	0,385	0.9	0.410
9	5.0	0.375	0.9	0.333	1.0	0.350

Step 3: After referring to the literature and interviewing experts, the tolerance ranges of design variables are $1.0 \le t_1 \le 5.0$, $0.1 \le t_2 \le 0.9$ and $0.2 \le t_3 \le 1.0$. According to the tolerance ranges of design variables, a tolerance cost experiment is implemented. The experimental data are shown in Table 3.

Step 4: By analyzing the experimental data in Table 3, the tolerance cost models in the form of formula (19) can be found as follows

$$\begin{aligned}
\hat{c}_1(t_1) &= 0.513 + 1.794 t_1^{-2} \\
\hat{c}_2(t_2) &= 0.602 + 0.011 t_2^{-2} \\
\hat{c}_3(t_3) &= 0.540 + 0.050 t_3^{-2}
\end{aligned}$$

A total tolerance cost model can be found as follows

$$C_M(t) = 1.655 + 1.794 t_1^{-2} + 0.011 t_2^{-2} + 0.050 t_3^{-2}$$

Step 5: Because the two quality characteristics are LTB-type characteristics, we can calculate indices $C_{pmk;1}$ and $C_{pmk;2}$ as shown

$$C_{pmkc;1}(x,t) = \frac{\hat{\mu}_1(x,t) - LSL_1}{3\sqrt{\frac{k_1}{\hat{\mu}_1^2(x,t)}\left(1 + \frac{3\hat{\sigma}_1^2(x,t)}{\hat{\mu}_1^2(x,t)} - \frac{4\hat{\sigma}_1^3(x,t)}{\hat{\mu}_1^3(x,t)}\right) + C_M(t)}}$$

$$C_{pmkc;2}(x,t) = \frac{\hat{\mu}_2(x,t) - LSL_2}{3\sqrt{\frac{k_2}{\hat{\mu}_2^2(x,t)}\left(1 + \frac{3\hat{\sigma}_2^2(x,t)}{\hat{\mu}_2^2(x,t)} - \frac{4\hat{\sigma}_2^3(x,t)}{\hat{\mu}_2^3(x,t)}\right) + C_M(t)}}$$

Step 6: A comprehensive objective function is constructed as follows

Therefore, the parameter and tolerance economic design for multivariate quality characteristics is formulated as an optimization problem with a certainty variable space.

To compare the quality loss and tolerance cost, this paper assumes that the loss coefficients k_1 and k_2 in the quality loss function are 4×10^3 and 4×10^4. By maximizing the MPCI, the optimal levels of design variables and

$$F = MC_{pmkc} = \frac{\left[\hat{\mu}_1(x,t) - LSL_1\right]\left[\hat{\mu}_2(x,t) - LSL_2\right]}{9\sqrt{\left[\frac{1}{\hat{\mu}_1^2(x,t)}\left(1 + \frac{3\hat{\sigma}_1^2(x,t)}{\hat{\mu}_1^2(x,t)} - \frac{4\hat{\sigma}_1^3(x,t)}{\hat{\mu}_1^3(x,t)}\right) + \frac{C_M(t)}{k_1}\right]\left[\frac{1}{\hat{\mu}_2^2(x,t)}\left(1 + \frac{3\hat{\sigma}_2^2(x,t)}{\hat{\mu}_2^2(x,t)} - \frac{4\hat{\sigma}_2^3(x,t)}{\hat{\mu}_2^3(x,t)}\right) + \frac{C_M(t)}{k_2}\right]}}$$

$$\begin{aligned} &\max \frac{\left[\hat{\mu}_1(x,t) - LSL_1\right]\left[\hat{\mu}_2(x,t) - LSL_2\right]}{9\sqrt{\left[\frac{1}{\hat{\mu}_1^2(x,t)}\left(1 + \frac{3\hat{\sigma}_1^2(x,t)}{\hat{\mu}_1^2(x,t)} - \frac{4\hat{\sigma}_1^3(x,t)}{\hat{\mu}_1^3(x,t)}\right) + \frac{C_M(t)}{k_1}\right]\left[\frac{1}{\hat{\mu}_2^2(x,t)}\left(1 + \frac{3\hat{\sigma}_2^2(x,t)}{\hat{\mu}_2^2(x,t)} - \frac{4\hat{\sigma}_2^3(x,t)}{\hat{\mu}_2^3(x,t)}\right) + \frac{C_M(t)}{k_2}\right]}} \\ &st.\ 20 \le x_1 \le 40 \\ &\quad 0.8 \le x_2 \le 2 \\ &\quad 0.8 \le x_3 \le 2.2 \\ &\quad 1 \le t_1 \le 5 \\ &\quad 0.1 \le t_2 \le 0.9 \\ &\quad 0.2 \le t_3 \le 1.0 \\ &\quad y_1 \ge 55 \\ &\quad y_2 \ge 15 \end{aligned}$$

tolerances are calculated. This optimization problem can be viewed as a global optimization problem with a certainty variable space. The problem is that multipeak values and a large amount of computation should be solved in the optimization. The DIRECT algorithm [61]–[63] is commonly used in certainty optimization and is especially suitable for the robust parameter design problem with certainty variable spaces. In this paper, the DIRECT algorithm and the MATLAB Global Optimization Toolbox are used to solve the problem. The optimal levels of design variables $x_{optimal}$ and their tolerances $t_{optimal}$ values are (26.06, 0.80, 0.80) and (3.86, 0.90, 1.00), respectively.

B. COMPARISON

In this paper, the total cost function from literature [45] and an improved Taguchi process capability index from a previous paper [25] are used as the objective functions for comparison with the proposed approach.

Approach 1: A previous study [45] proposed a total cost function in terms of expected quality loss cost and tolerance cost. The optimal values of design variables and tolerances are obtained by minimizing the total cost function. The objective function is written as shown

$$\min F = C(x,t) = C_L(x,t) + C_M(t)$$

where $C_L(x,t) = \sum_{m=1}^{2} c_{L;m}(x,t)$ is the total expected quality loss cost in which $c_{L;m}(x,t)$ is the expected quality loss cost for the m th quality characteristic y_m.

Approach 2: The literature [25] suggested the improved Taguchi process capability index as the objective function. Previous researchers obtained the optimal design variables and tolerances by maximizing the multivariate process capability index. The objective function is written as shown

$$\max F = \prod_{m=1}^{2} C_{pmc;m} = \frac{(USL_1 - LSL_1)(USL_2 - LSL_2)}{9\sqrt{\left[c_{L;1}(x,t) + C_M(t)\right]\left[c_{L;2}(x,t) + C_M(t)\right]}}$$

where the values of USL_1, LSL_1, USL_2 and LSL_2 are 17, 15, 60 and 55, respectively.

Using approach 1 and approach 2, the optimal design variables $x_{optimal}$ are (28.51, 0.80, 0.80) and (32.65, 2.00, 0.80), and the optimal tolerance values $t_{optimal}$ are (5.00, 0.90, 0.87) and (5.00, 0.90, 0.95), respectively. Some indices are listed for comparison with the three approaches in Table 4.

Figure 3 through Figure 6 are used to compare the proposed approach with other two approaches. From Table 4, we find that the optimal levels for x_3 are all equal to 0.8 and that the optimal t_2 are all equal to 0.9 by applying these three approaches to the experiment. Therefore, we compare the quality characteristic, quality loss cost and total tolerance cost under $x_3 = 0.8$ and $t_2 = 0.9$.

Figure 3a and Figure 3b show the response surfaces of y_1 and y_2 in terms of x_1 and x_2 when $x_3 = 0.8$. From Figure 3a, y_1 is proportional to x_1 in the early stage and decreases with x_1 after y_1 reaches its peak, whereas y_2 is inversely proportional to x_2.

Figure 3c and Figure 3d present the contour plots of y_1 and y_2 in terms of x_1 and x_2 in which q'_1, q'_2 and q'_* are the optimal design variables for approach 1, approach 2 and the proposed approach. According to the contour plots and positions of optimal design variables, the y_1 and y_2 obtained by the proposed approach when the optimal design variable is q'_* are larger than q'_1 and q'_2. Because y_1 and y_2 are LTB-type quality characteristics, the optimal design

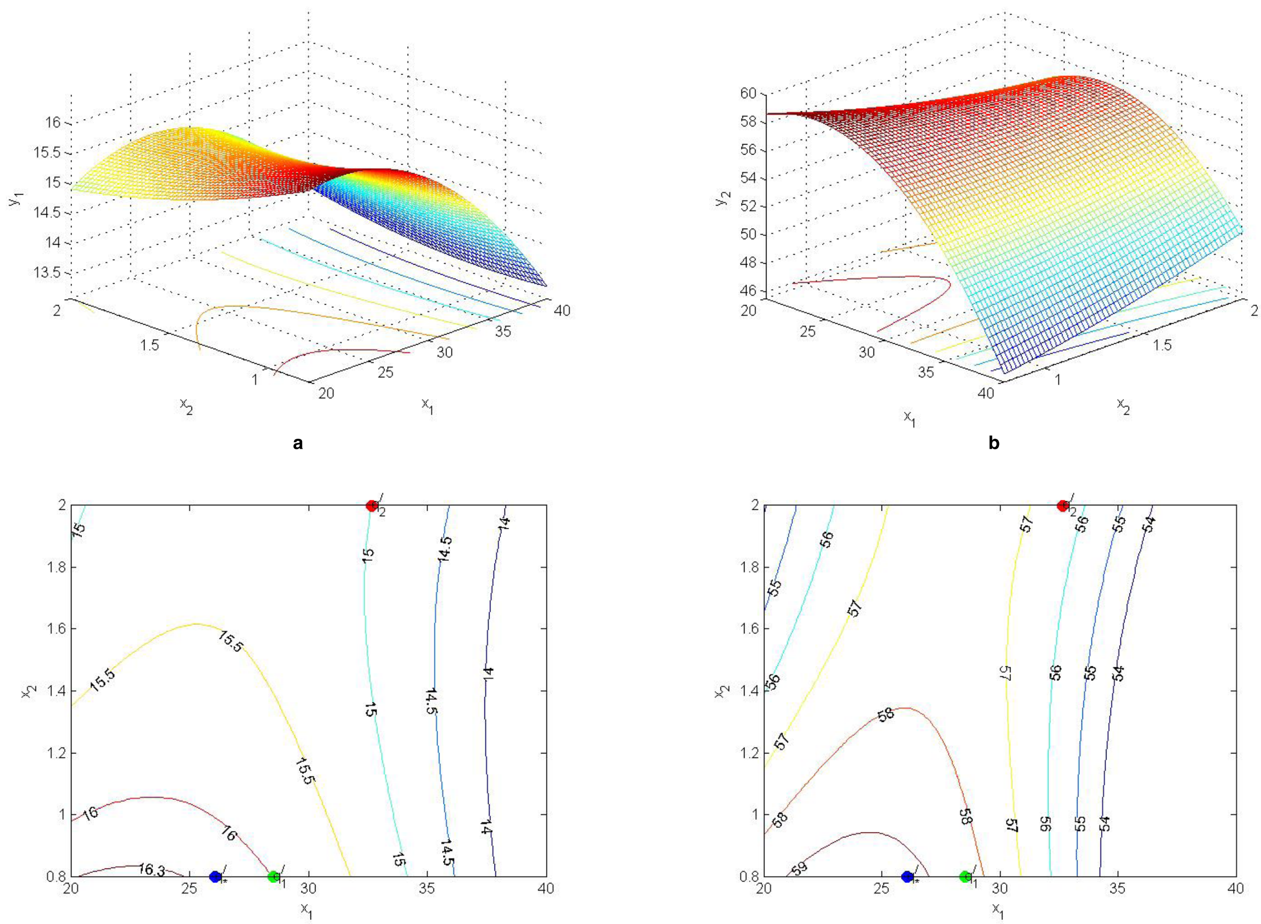

FIGURE 3. **Response surfaces and contour of y_1 and y_2 in terms of x_1 and x_2 when $x_3 = 0.8$.**

TABLE 4. **Comparison of proposed approach and approaches 1 and 2.**

	Approach 1	Approach 2	Proposed Approach
Optimal Design Variables ($x_{optimal} = (x_1, x_2, x_3)$)	(28.51, 0.80, 0.80)	(32.65, 2.00, 0.80)	(26.06, 0.80, 0.80)
Optimal Tolerances ($t_{optimal} = (t_1, t_2, t_3)$)	(5.00, 0.90, 0.87)	(5.00, 0.90, 0.95)	(3.86, 0.90, 1.00)
Quality Characteristics (y_1, y_2)	15.9938, 58.4128	15.0008, 56.4843	**16.2258**, **59.2552**
Means (μ_1, μ_2)	25.2523, 58.5399	26.7992, 56.6252	24.3672, 59.4344
Variations (σ_1, σ_2)	1.7968, 9.7154	2.2290,17.7374	2.2538, 14.5466
Quality Loss (L_1, L_2)	15.6371,11.7231	17.7759,12.5373	**15.1931,11.3922**
Total Expected Quality Loss Cost (C_L)	18.0865	18.2790	18.2586
Tolerance Cost (C_M)	1.8064	1.7957	1.8390
Total Cost (C)	19.8929	20.0747	20.0976
PCI ($C_{pmk;1}, C_{pmk;2}$)	1.1987, 0.3203	1.4445,0.1425	1.0619,0.4055
MPCI (MC_{pmk})	0.3839	0.2058	**0.4306**

variables from the proposed approach are superior to those from approaches 1 and 2.

Figure 4 displays the contour plot of quality loss function $L_1(x)$ and $L_2(x)$ in terms of x_1 and x_2 when $x_3 = 0.8$. Because y_1 and y_2 are both LTB-type characteristics, $L_1(x)$ and $L_2(x)$ are inversely proportional to y_1 and y_2. According to the position of optimal points in contour plot, the quality loss cost of proposed approach is less than those of the other two approaches.

Figure 5 shows the response surface and contour plots of the total tolerance cost C about t_1 and t_3 when $t_2 = 0.9$. From Figure 5a, the total tolerance cost decreases with

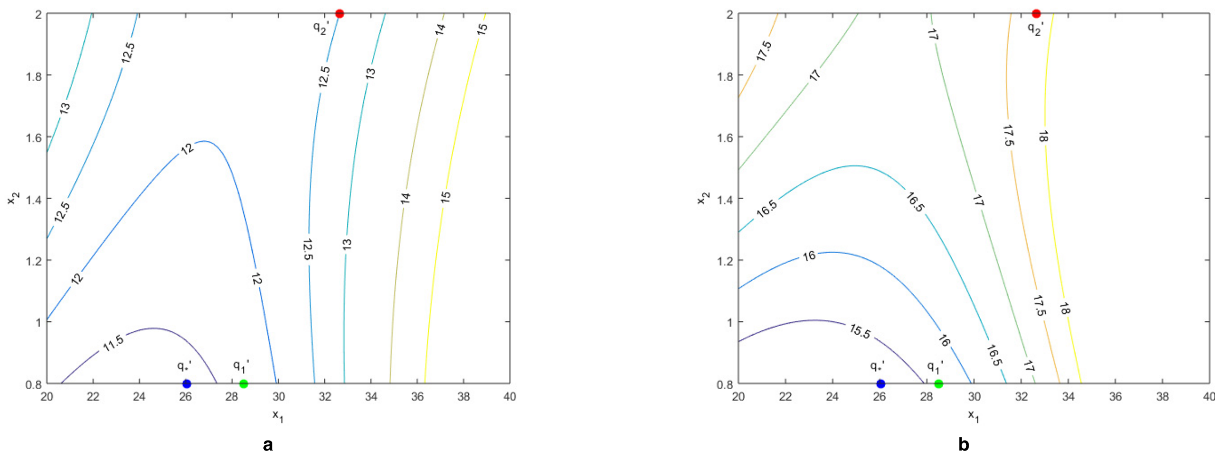

FIGURE 4. Contour of $L_1(x)$ and $L_2(x)$ in terms of x_1 and x_2 when $x_3 = 0.8$.

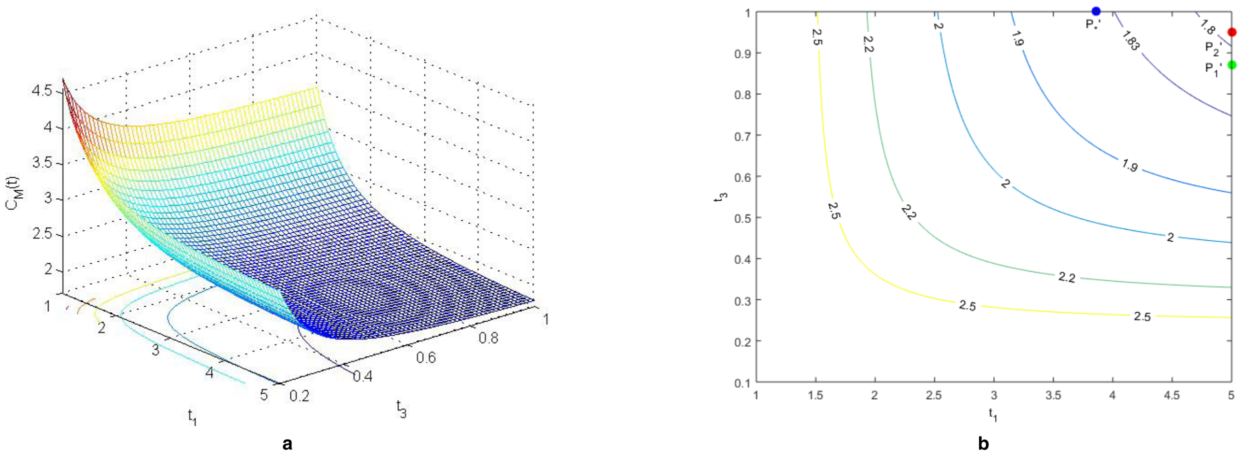

FIGURE 5. Response surface and contour of total tolerance cost $C_M(t)$ when $t_2 = 0.9$.

increasing t_1 and t_3 in the region of $t_1 \times t_2 = [1, 5] \times [0.2, 1.6]$. In Figure 5b, points p_1' p_2' and p_*' are the optimal tolerance values from approach 1, approach 2 and the proposed approach. According to the contour plots and positions of optimal tolerance points, the tolerance cost from the proposed approach is approximately similar to the counterparts obtained from approaches 1 and 2. The reason for this result is that the proposed approach reasonably increases the manufacturing cost to reduce the quality loss of production and satisfy the customer requirements.

Figure 6 illustrates the spatial position plot of quality characteristics and MC_{pmk} under the optimal design variables and tolerances. In this plot, R_1, R_2 and R_* are the optimal values of y_1, y_2 and MC_{pmk} from approach 1, approach 2 and the proposed approach, respectively. Points R_1', R_2' and R_*' are the mapping points of R_1, R_2 and R_* on the plane $MC_{pmk} = 0$. The MC_{pmk} obtained by the proposed approach shows improvements of 10.2% and 109.2% compared with approach 1 and approach 2. Obviously, the total cost from the proposed approach is close to the cost of the other

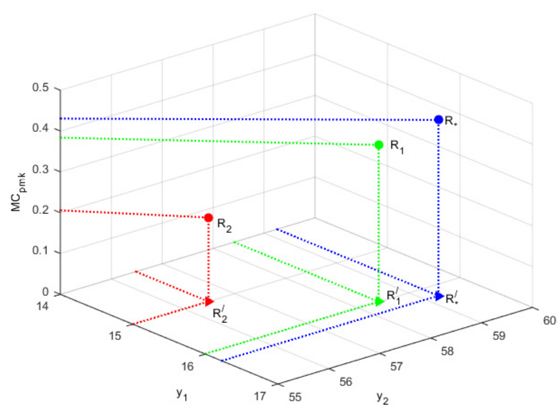

FIGURE 6. Spatial position of quality characteristics and MC_{pmk} under the optimal design variables and tolerances.

two approaches, but the PCI is better than the others. The proposed approach considers the types of quality characteristics, more sensibly reflects the deviance of mean value of quality characteristic from the target value, and emphasizes the importance of the target value. The results from the proposed approach are much better than those of the other two approaches.

It is important to note that the total cost is composed of the quality loss cost from quality characteristic variation and the tolerance cost of design variables. Approach 1 does not consider the product process capability in optimization, which means that the product process capability should be evaluated after optimization. When the process capability is less sufficient, parameter and tolerance design must be implemented again. Therefore, according to the PAF model from Feigenbaum, this situation is expected to profoundly increase the loss cost, prevention cost and appraisal cost. Approach 2 does not consider the types of quality characteristics and the impact from the deviance of target value and the center of specification. From the above analysis, ignoring the impact results in an optimization outcome that has a large difference from the design target could result in an unpredictable loss. Obviously, from the perspective of quality loss, the proposed approach improves the effectiveness and credibility of optimization result and reduces the risk of increasing the CoQ for unpredictable reasons.

VI. CONCLUSION

The procedure of parameter and tolerance economic design for multivariate quality characteristics based on the modified PCI is proposed in this paper. We analyze the different of quality characteristic types and the deviation between the specification center and the target value, and then constructed a modified PCI. In the current literature, the calculation of PCI requires a lot of data. The modified PCI proposed in this paper can be calculated with individual observations, which reduces the cost of replicated experiments. Through the process capability analysis in the product design stage, this approach improves the robustness of the optimal design variables and reduces the production cost.

This paper discusses the parameter and tolerance economic design of the experiment on IIR inner tube by comparing to the total cost function approach and Taguchi process capability index approach. The example shows that the proposed approach had better quality loss and MC_{pmk} than the other two approaches. Meanwhile, the optimal quality characteristics are LTB-type, and the proposed approach can obtain more optimal quality characteristics than the other two approaches. On the whole, the robustness and economy of the proposed approach are better than the other two approaches. It is noteworthy that Taguchi noted that the design variables can be divided into control variables and noise variables. Therefore, how to analyze and reduce the influence of noise variables on economic design of parameters and tolerances is an important subject worthy of further study.

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