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# DOA Estimation Using Single or Dual Reception Channels Based on Cyclostationarity

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**ABSTRACT** This paper addresses the problem of direction-of-arrival (DOA) estimation for cyclostationary signals using less reception channel in comparison with the number of sensors. The system is considered to be realized by using the reception channel(s) to receive the signal reached each sensor in turn. Such simplification results in a cost reduction of the system and the effect of inconsistency among different channels are removed. However, non-synchronized sampling also makes the traditional DOA estimation algorithms ineffective. To cope with the problem, the signal models for DOA estimation using a single channel and dual channels are formulated based on signal cyclostationary, and the single channel cyclic MUSIC (SC-Cyclic-MUSIC) algorithm and the dual channels cyclic MUSIC (DC-Cyclic-MUSIC) algorithm are proposed correspondingly. This paper shows that both SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm work well when the switching interval is known, and a satisfied performance can also be obtained by DC-Cyclic-MUSIC algorithm when the uncertainty of the switching interval exists. Moreover, the proposed estimators are suitable for both narrow-band and wide-band signals. The computer simulations are provided to verify the effectiveness of the proposed algorithms.

**INDEX TERMS** Direction-of-arrival (DOA) estimation, cyclostationarity, single channel cyclic MUSIC algorithm, dual channels cyclic MUSIC algorithm.

## I. INTRODUCTION

Direction-of-arrival (DOA) estimation of signals has been a hot area of research over the past decades in the field of communications monitoring, sonar, radar and so on. Since 1980s, a great number of outstanding algorithms (e.g., MUSIC, ESPRIT, MODE) have been proposed in this field. MUSIC [1] is one of the most representative technique that has been extensively studied and widely applied in many fields.

In recent years, the cyclostationarity has been drawn great attention in the communication of signal processing [2], [3], such as blind source extraction [4], filtering [5], beamforming [6]. It has been shown that most of the man-made signals, e.g. amplitude modulation, frequency modulation and phase modulation signals are cyclostationary, and this property can be utilized in improving the performance

of DOA estimation [7]. Gardner first proposed MUSIC-Like CYCCOR METHOD and ESPRIT-Like CYCCOR METHOD exploiting the cyclostationarity in 1988, by replacing the covariance matrix with the cyclic correlation matrix [8]. Schell and Gardner researched on the signal-selective properties of Cyclic-MUSIC algorithm and derived the bias and mean-squared error of Cyclic-MUSIC algorithm under the assumption that the signals were complex, possibly non-Gaussian, and cyclostationary [9], and then proposed two advanced algorithms, Multi-Cyclic-MUSIC algorithm and Adaptive-Cyclic-MUSIC algorithm [10]. The Multi-Cyclic-MUSIC algorithm can simultaneously estimate the directions of arrival of signals having different cycle frequencies instead of sequentially processing each separate cycle frequency. The Adaptive-Cyclic-MUSIC algorithm reduces the sensitivity of Cyclic-MUSIC algorithm to error of the cycle frequency. Chargé proposed a root-MUSIC-like algorithm based on cyclostationarity [11]. Pokrajac *et al.* proposed frequency domain Cyclic-MUSIC algorithm by exploitation

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the cyclostationarity of signals [12], [13]. The proposed algorithm doesn't require any knowledge of the optimal time lag parameter which appeared in time domain Cyclic-MUSIC algorithms. Liu Z. proposed unitary Cyclic ESPRIT-like algorithm by constructing the forward backward cyclic auto-correlation matrix using Hermitian mapping. This approach not only substantially reduces the computational complexity via real-valued decomposition, but also outperforms the forward backward spatial smoothing algorithm for uncorrelated or correlated sources. However, All of above algorithms are only suitable for narrow-band signal. Xu and Kailath proposed a spectral correlation algorithm which was suitable for estimating DOA of wide-band signal via exploitation of cyclostationarity by combination of temporal and spatial processing [14]. Yan and Fan proposed two cyclic-MUSIC-like DOA estimation algorithms for wide-band cyclostationary signals, the averaged cyclic MUSIC (ACM) algorithm and the extended wideband cyclic MUSIC (EWCM)algorithm [15]. The new methods have more relaxed requirements on the signals than in [14]. Liu *et al.* expended the algorithm by exploiting conjugate matrix information [16]. The improved cyclic cross correlation MUSIC algorithm has better signal detection performance when the source signals existed coherent source signal. Song and Liu explored the coarray interpolation techniques with cyclostationary signals [17], [18]. These methods integrate the cyclostationarity with the spatial domain and increase the degrees-of-freedom(DOFs) of algorithm.

All of methods above estimate the DOAs by utilizing the orthogonality between the array manifold and the noise subspace of the array covariance matrix or cyclic covariance matrix. To get the covariance matrix, the number of reception channels has to be equivalent to the number of sensors, and synchronous sampling for different reception channels is needed to reserve the direction information of incoming signals. But, for a large number of reception channels, it is difficult to ensure the consistency among all channels. Because of the channel inconsistency, the DOA estimation performance of the multi-channel system will degrade [19], [20]. Moreover, the cost of the multi-channel receiver is very high.

To remove the effect of channel inconsistency, a type single channel DOA estimation methods based on MUSIC algorithm were proposed. The most popular method used single channel to receive the signal of different sensors in turn. Based on the data collected by single channel, these methods recovered the array covariance matrix by using perturbation techniques [21]–[23], phase compensation [24]–[26], or interpolation [27], and so on, and then estimated the DOAs of incoming signals using MUSIC algorithm. But the performance of these algorithms degrades because of the switch interval and switch interval error [28]. Moreover, these algorithms can only be applied to narrow-band signal.

In this paper, we proposed a cyclic MUSIC algorithm for single channel DOA estimation based on cyclostationarity [29]. The algorithm is not only suitable for narrow-band signal, but also for wide-band signal. The performance of

algorithm is not affected by switch interval but will degrades if the switch interval is not known exactly. To overcome this shortage, we propose a cyclic MUSIC algorithm using dual channels which works well even if the switch interval with error.

The remainder of this paper is organized as follows. In section II, array model, cyclostationarity and MUSIC algorithm are introduced. In section III, we introduce the existing single channel MUSIC algorithm and analyse its performance. To improve the performance, a single channel cyclic MUSIC algorithm is proposed. In section IV, the dual channels cyclic MUSIC algorithm is proposed. In section V, the performance of the proposed algorithm is simulated and the results are compared with the existing MUSIC algorithms.

Throughout this paper, the following notations are used. The superscriptions  $*$ ,  $T$  and  $H$  denote the conjugate, transpose and Hermitian transpose, respectively. The notations  $E\{\cdot\}$  and  $\odot$  stand for the expectation and Hadamard product, respectively.  $\mathbf{D}(\mathbf{x})$  represents the diagonal matrix with  $\mathbf{x}$  in its main diagonal.  $\mathbf{I}_M$ ,  $\mathbf{1}_{m,n}$  and  $\mathbf{0}$  denote the  $M \times M$  identify matrix, the  $m \times n$  matrix of 1s and the matrix of 0s with a appropriate dimension unless otherwise specified, respectively.

## II. BACKGROUND

### A. CYCLOSTATIONARITY

For a continuous stochastic signal  $s(t)$ , if its mean and autocorrelation function are periodic with same period, signal  $s(t)$  is called generalized cyclostationarity signal. Its mean and autocorrelation function can be expressed as [30]

$$E[s(t)] = E[s(t + mT_0)], \quad (1)$$

$$R_s(t, \tau) = R_s(t + mT_0, \tau), \quad (2)$$

where  $T_0$  is the period of mean and autocorrelation function,  $m$  is an integer,  $\tau$  is the lag parameter. For periodic autocorrelation function  $R_s(t, \tau)$ , it can be expressed in Fourier series as

$$R_s(t, \tau) = \sum_{\alpha=-\infty}^{\infty} R_s^\alpha(\tau) e^{j2\pi\alpha t}, \forall \alpha \neq 0, \quad (3)$$

$$R_s^\alpha(\tau) = \left\langle s(t + \tau/2) s^*(t - \tau/2) e^{-j2\pi\alpha t} \right\rangle_t, \quad (4)$$

where  $R_s^\alpha(\tau)$  is the cyclic autocorrelation function of  $s(t)$ ,  $\langle \bullet \rangle_t$  is averaging operation of  $t$ , and  $\alpha$  is cycle frequency.

More generally, a continuous stochastic signal  $s(t)$  is said to exhibit cyclostationarity if there exists a non-zero cycle frequency  $\alpha$  for which the cyclic autocorrelation function  $R_s^\alpha(\tau)$  is not identically zero. Manmade signal is generally obtained by using a transmitted signal to modulate a parameter of periodic signals, and the modulation will generate the cyclostationarity [30].

*Property 1:* if the cyclic autocorrelation function of  $s(t)$  is  $R_s^\alpha(\tau)$ , then the cyclic autocorrelation function of  $y(t) = s(t + t_0)$  is  $R_s^\alpha(\tau) e^{j2\pi\alpha t_0}$ .

Proof:

$$\begin{aligned}
 R_y^\alpha(\tau) &= \left\langle s(t+t_0+\tau/2)s(t+t_0-\tau/2)e^{-j2\pi\alpha t} \right\rangle_t \\
 &= \left\langle s(t+t_0+\tau/2)s(t+t_0-\tau/2)e^{-j2\pi\alpha(t+t_0)} e^{j2\pi\alpha t_0} \right\rangle_t \\
 &= R_s^\alpha(\tau)e^{j2\pi\alpha t_0}
 \end{aligned} \tag{5}$$

**B. ARRAY MODEL**

Suppose that there is an array antenna with  $M$  array sensors, and  $K$  far-field signals  $s_1(t), s_2(t), \dots, s_K(t)$  impinge on the array from the directions  $\theta_1, \theta_2, \dots, \theta_K$  respectively. The received signal of the  $m$ th sensor can be expressed as

$$x_m(t) = \sum_{k=1}^K s_k(t + \tau_m(\theta_k)) + e_m(t), \tag{6}$$

where  $s_k(t)$  is the incident signal from the direction  $\theta_k$ ,  $e_m(t)$  is the random noise received by the  $m$ th sensor, and  $\tau_m(\theta_k)$  is the delay of  $s_k(t)$  reached the  $m$ th sensor.

**C. MUSIC ALGORITHM**

MUSIC algorithm is first proposed by Schmidt in 1986, which is a matrix decomposition method based on feature space. The method estimates the DOAs by utilizing the orthogonality between the array manifold and the noise subspace of the array covariance matrix.

Assume that  $s_k(t)$  is narrow-band signal with center frequency  $f$ , we get

$$s_k(t + \tau_m(\theta_k)) \approx s_k(t) \exp(j2\pi f \tau_m(\theta_k)). \tag{7}$$

Then the receive data of  $M$ -sensor array can be expressed as

$$\mathbf{x}(t) = [x_1(t), x_2(t), \dots, x_M(t)]^T = \mathbf{A}s(t) + \mathbf{e}(t). \tag{8}$$

where

$$\begin{aligned}
 \mathbf{A} &= [\mathbf{a}(\theta_1), \dots, \mathbf{a}(\theta_k), \dots, \mathbf{a}(\theta_K)], \\
 \mathbf{a}(\theta_k) &= [e^{j2\pi f \tau_1(\theta_k)}, e^{j2\pi f \tau_2(\theta_k)}, \dots, e^{j2\pi f \tau_M(\theta_k)}]^T, \\
 \mathbf{s}(t) &= [s_1(t), s_2(t), \dots, s_K(t)]^T, \\
 \mathbf{e}(t) &= [e_1(t), e_2(t), \dots, e_M(t)]^T.
 \end{aligned}$$

Under normal circumstances, since the array signal and noise are independent, data covariance matrix can be expressed as

$$\mathbf{R} = E [\mathbf{x}(t)\mathbf{x}^H(t)] = \mathbf{A}\mathbf{R}_s\mathbf{A}^H + \sigma^2\mathbf{I}_M, \tag{9}$$

where  $\mathbf{R}_s$  is the covariance matrix of  $s(t)$ ,  $\sigma^2$  is the variance of noise. Assume that sources are incoherent, the covariance matrix  $\mathbf{R}$  is positive definite. Decomposition of  $\mathbf{R}$  by EVD, we get

$$\mathbf{R} = \mathbf{U}_s \Sigma_s \mathbf{U}_s^H + \mathbf{U}_n \Sigma_G \mathbf{U}_n^H, \tag{10}$$

where the diagonal matrices  $\Sigma_s$  and  $\Sigma_G$  consist of the  $K$  largest and the  $M - K$  smallest eigenvalues of  $\mathbf{R}$ , respectively,  $\mathbf{U}_s \in \mathbb{C}^{M \times K}$  spans the signal subspace whose columns are the eigenvectors corresponding to the  $K$  largest eigenvalues, and  $\mathbf{U}_n \in \mathbb{C}^{M \times (M-K)}$  spans the noise subspace whose

columns are the eigenvectors corresponding to the  $M - K$  smallest eigenvalues.

Under ideal conditions signal subspace and the data space span the same space, that is, steering vectors are orthogonal to the noise subspace as follows:

$$\mathbf{a}(\theta)\mathbf{U}_n = \mathbf{0}. \tag{11}$$

Considering the actual receiving data is finite,  $\mathbf{a}(\theta)$  and  $\mathbf{U}_n$  can't be completely orthogonal. So there required the minimum optimization search

$$\theta_{MUSIC} = \arg \min \mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta). \tag{12}$$

Therefore, the spectral estimation formula of MUSIC algorithm is as follows

$$\mathbf{p}(\theta) = 1 / \mathbf{a}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{a}(\theta). \tag{13}$$

The DOAs are obtained by finding the  $K$  maximum extreme points of  $\mathbf{p}(\theta)$ . The directions associated with the extreme points are the DOAs of incoming signals.

**III. SINGLE CHANNEL MUSIC ALGORITHM**

MUSIC algorithm is usually used in multi-channel system, but the DOA estimation performance of multi-channel system degrades because of channel inconsistency [19]. Many papers have discussed how to use single reception channel to remove the effect of channel inconsistency. The single receiver architecture is shown in Fig.1. In this section, we introduce two single channel MUSIC algorithms and conduct comparisons.

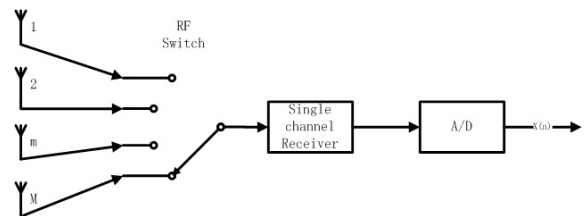


FIGURE 1. The single channel receiver architecture.

**A. THE EXISTING SINGLE CHANNEL MUSIC ALGORITHM**

Considering the single receiver architecture shown in Fig.1, the single channel receiver sequentially receives the array outputs at a regular period  $T_s$  and samples one snapshot of each sensor in turn. The received signal of the  $m$ th sensor can be expressed as in (6). The sampled data  $x(n)$  is given by

$$x(n) = x_m(nT_s) = \sum_{k=1}^K s_k(nT_s + \tau_m(\theta_k)) + e_m(nT_s). \tag{14}$$

where  $m = n \bmod M$ . The receive data of the  $m$ th sensor can be expressed as

$$\begin{aligned}
 x_m(n) &= x((n-1)M + m) \\
 &= \sum_{k=1}^K s_k((n-1)MT_s + mT_s + \tau_m(\theta_k)) \\
 &\quad + e_m((n-1)MT_s + mT_s).
 \end{aligned} \tag{15}$$

Assume that  $s_k(t)$  is narrow-band signal with center frequency  $f$ , we get

$$s_k((n-1)MT_s + mT_s + \tau_m(\theta_k)) \approx \exp(j2\pi f(mT_s + \tau_m(\theta_k)))s_k((n-1)MT_s), \quad (16)$$

so

$$x_m(n) \approx \exp(j2\pi f n MT_s) \sum_{k=1}^K \exp(j2\pi f \tau_m(\theta_k))s_k((n-1)MT_s) + e_m((n-1)MT_s + mT_s). \quad (17)$$

We construct the receive data matrix  $\mathbf{x}(n)$  of the array

$$\mathbf{x}(n) = [x_1(n), \dots, x_M(n)]^T = \Psi(f)\mathbf{A}\mathbf{s}(n) + \mathbf{e}(n), \quad (18)$$

where

$$\begin{aligned} \Psi(f) &= \mathbf{D}(e^{j2\pi f T_s}, \dots, e^{j2\pi f n T_s}, \dots, e^{j2\pi f M T_s}), \\ \mathbf{s}(n) &= [s_1((n-1)MT_s), \dots, s_K((n-1)MT_s)]^T, \\ \mathbf{e}(n) &= [e_1((n-1)MT_s), \dots, e_M((n-1)MT_s)]^T. \end{aligned}$$

Set

$$\mathbf{B} = \Psi(f)\mathbf{A}, \quad (19)$$

and we get

$$\mathbf{x}(n) = \mathbf{B}\mathbf{s}(n) + \mathbf{e}(n). \quad (20)$$

Equation (20) is similar as (8), so we can use MUSIC algorithm to estimate the DOAs of sources. We called the algorithm as Single Channel MUSIC (SC-MUSIC) algorithm. The spectral estimation formula of SC-MUSIC algorithm is as follows

$$\mathbf{p}(\theta) = 1/\mathbf{b}^H(\theta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{b}(\theta), \quad (21)$$

where  $\mathbf{b}(\theta) = \Psi(f)\mathbf{a}(\theta)$ . The DOAs are obtained by finding the  $K$  maximum extreme points of  $\mathbf{p}(\theta)$ . The directions associated with the extreme points are the DOAs of incoming signals.

## B. DISCUSSION

There are some factors which impact the performance of the SC-MUSIC algorithm.

First, to get data matrix with  $N$  snapshots, the algorithm has to do  $N \times M$  times switch. When  $N$  and (or) $M$  are large, the switch times are not acceptable.

Second, the algorithm has assumed  $s_k((n-1)MT_s + mT_s + \tau_m(\theta_k)) \approx \exp(j2\pi f(mT_s + \tau_m(\theta_k)))s_k((n-1)MT_s)$ . The assumption is true only for narrow-band signal. However, as the bandwidth of signal increases, the approximation error becomes large, that is, the  $\Psi(f)$ , and  $\mathbf{A}$  are not exact. So the DOA estimation error will increase quickly because of the approximation error.

Third, the algorithm assumed the switch interval from one sensor to another only contains the sample time. But in practice, the switch interval also must contain channel delay which maybe many thousands times  $T_s$ . Thus is  $x(n) = s_k((n-1)M(T_s + t_d(n)) + m(T_s + t_d(n)) + \tau_m(\theta_k)) + e((n-1)M(T_s + t_d(n)))$ .  $t_d(n)$  is channel delay of the  $n$ th switch. Because of the performance of device, there are

another random errors among all of  $t_d(n)$  which will introduce extra uncertain phases. The extra uncertain phases will enlarge the error of DOA estimation as the channel inconsistency does in multiple channels system.

To improve the performance of the SC-MUSIC algorithm, we have to reduce the switch times and get more exact  $\Psi(f)$ , and  $\mathbf{A}$  which are not affected by bandwidth and switch interval.

## C. THE IMPROVED SINGLE CHANNEL MUSIC ALGORITHM

In this algorithm, to reduce the switch times, we also use one reception channel to sample each sensor in turn, but sample  $N$  snapshots of one sensor each time. There only needs  $M$  times switch. The sampled signal  $x(n)$  can be expressed as

$$\begin{aligned} x(n) &= x_m(t + t_m + iT_s) \\ &= \sum_{k=1}^K s_k(t + t_m + iT_s + \tau_m(\theta_k)) + e_m(t + t_m + iT_s), \end{aligned} \quad (22)$$

where  $m = n \bmod N$ ,  $i = n - mN$ . The sample data of the array can be expressed as

$$\mathbf{X}(t) = [\mathbf{x}_1(t + t_1), \dots, \mathbf{x}_m(t + t_m), \dots, \mathbf{x}_M(t + t_M)]^T, \quad (23)$$

where

$$\begin{aligned} \mathbf{x}_m(t + t_m) &= [x_m(t + t_m), x_m(t + t_m + T_s), \\ &\dots, x_m(t + t_m + (N-1)T_s)], \end{aligned} \quad (24)$$

$t_m$  is the delay of  $m$ th sensor which contains  $(m-1)NT_s$  and  $(m-1)$  times switch interval.

Assuming that  $s_k(t)$  is the cyclostationary signal with the cyclic frequency  $\alpha$ , The cyclic autocorrelation function  $R_{x_m}^\alpha(\tau)$  of  $x_m(t)$  in (6) can be expressed as

$$\begin{aligned} R_{x_m}^\alpha(\tau) &= \left\langle x_m(t + \tau/2)x_m^*(t - \tau/2)e^{-j2\pi\alpha t} \right\rangle \\ &= \left\langle \sum_{k=1}^K \sum_{i=1}^K s_k(t + \tau_m(\theta_k) + \tau/2)s_i^*(t + \tau_m(\theta_i) - \tau/2)e^{-j2\pi\alpha t} \right\rangle \\ &\quad + \left\langle \sum_{k=1}^K s_k(t + \tau_m(\theta_k) + \tau/2)e_m^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle \\ &\quad + \left\langle e_m(t + \tau_m(\theta_k) + \tau/2) \sum_{k=1}^K s_k^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle \\ &\quad + \left\langle e_m(t + \tau_m(\theta_k) + \tau/2)e_m^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle. \end{aligned}$$

Assuming that the signal  $s_k(t)$  is uncorrelated with each other at the cyclic frequency  $\alpha$  and  $e_m(t)$  is white Gaussian noise, we get

$$\begin{aligned} \left\langle s_k(t + \tau_m(\theta_k) + \tau/2)s_i^*(t + \tau_m(\theta_i) - \tau/2)e^{-j2\pi\alpha t} \right\rangle &= 0, \\ \left\langle s_k(t + \tau_m(\theta_k) + \tau/2)e_m^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle &= 0, \\ \left\langle e_m(t + \tau_m(\theta_k) + \tau/2)s_k^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle &= 0, \\ \left\langle e_m(t + \tau_m(\theta_k) + \tau/2)e_m^*(t + \tau_m(\theta_k) - \tau/2)e^{-j2\pi\alpha t} \right\rangle &= 0. \end{aligned}$$

Then  $R_{x_m}^\alpha(\tau)$  can be simplified as

$$\begin{aligned} R_{x_m}^\alpha(\tau) &= \left\langle \sum_{k=1}^K s_k(t + \tau_m(\theta_k) + \tau/2) s_k^*(t + \tau_m(\theta_k) - \tau/2) e^{-j2\pi\alpha t} \right\rangle \\ &= \sum_{k=1}^K R_{s_k}^\alpha(\tau) e^{j2\pi\alpha\tau_m(\theta_k)}. \end{aligned}$$

As shown in property 1, The cyclic autocorrelation function  $R_{x_m}^\alpha(t + t_m, \tau)$  of  $\mathbf{x}_m(t + t_m)$  can be expressed as

$$R_{x_m}^\alpha(t + t_m, \tau) = e^{j2\pi\alpha t_m} \sum_{k=1}^K R_{s_k}^\alpha(\tau) e^{j2\pi\alpha\tau_m(\theta_k)}. \quad (25)$$

Set

$$\mathbf{r}_{\mathbf{X}}^\alpha(\tau) = [R_{x_1}^\alpha(t + t_1, \tau), \dots, R_{x_m}^\alpha(t + t_m, \tau), \dots, R_{x_M}^\alpha(t + t_M, \tau)]^T, \quad (26)$$

$$\mathbf{r}_{\mathbf{s}}^\alpha(\tau) = [R_{s_1}^\alpha(\tau), \dots, R_{s_k}^\alpha(\tau), \dots, R_{s_K}^\alpha(\tau)]^T, \quad (27)$$

then we get

$$\mathbf{r}_{\mathbf{X}}^\alpha(\tau) = \Psi(\alpha) \mathbf{A} \mathbf{r}_{\mathbf{s}}^\alpha(\tau), \quad (28)$$

where

$$\begin{aligned} \mathbf{a}(\theta_k) &= [e^{j2\pi\alpha\tau_1(\theta_k)}, e^{j2\pi\alpha\tau_m(\theta_k)}, \dots, e^{j2\pi\alpha\tau_M(\theta_k)}]^T, \\ \Psi(\alpha) &= \mathbf{D}(e^{j2\pi\alpha t_1}, e^{j2\pi\alpha t_m}, \dots, e^{j2\pi\alpha t_M}). \end{aligned}$$

Here we can regard  $\mathbf{A}$  as the steering vector of the antenna array at the cyclic frequency  $\alpha$ . Equation(28) is called as the array model in cyclic frequency domain which is similar with time-domain model.

For  $\tau = T_s, 2T_s, \dots, NT_s$ , the new data matrix can be expressed as

$$\mathbf{R}_{\mathbf{X}}(\alpha) = [\mathbf{r}_{\mathbf{X}}^\alpha(T_s), \mathbf{r}_{\mathbf{X}}^\alpha(2T_s), \dots, \mathbf{r}_{\mathbf{X}}^\alpha(NT_s)]. \quad (29)$$

Then we apply MUSIC algorithm to estimate the DOAs of the sources. We called the algorithm as Single Channel Cyclic MUSIC (SC-Cyclic-MUSIC) algorithm. The process is expressed as follow:

*Step 1:* Calculate the cyclic autocorrelation function  $R_{x_m}^\alpha(t + t_m, \tau)$  of the  $\mathbf{x}_m(t + t_m)$ .

*Step 2:* Form the new data matrix  $\mathbf{R}_{\mathbf{X}}(\alpha)$  and calculate correlation function  $\mathbf{C}(\alpha)$  of  $\mathbf{R}_{\mathbf{X}}(\alpha)$ .

$$\mathbf{C}(\alpha) = \mathbf{R}_{\mathbf{X}}(\alpha) \bullet (\mathbf{R}_{\mathbf{X}}(\alpha))^H \quad (30)$$

*Step 3:* Attain its eigenvectors by eigenvalue decomposition (EVD) of  $\mathbf{C}(\alpha)$ . It can be expressed as

$$\mathbf{C}(\alpha) = [\mathbf{S}_\alpha \mathbf{G}_\alpha] \begin{bmatrix} \sum \mathbf{S}_\alpha \mathbf{0} \\ \mathbf{0} \sum \mathbf{G}_\alpha \end{bmatrix} [\mathbf{S}_\alpha \mathbf{G}_\alpha]^H \quad (31)$$

where the diagonal matrices  $\Sigma_{\mathbf{S}_\alpha}$  and  $\Sigma_{\mathbf{G}_\alpha}$  consist of the  $K$  largest and the  $M - K$  smallest eigenvalues of  $\mathbf{C}(\alpha)$ , respectively,  $\mathbf{S}_\alpha \in \mathbb{C}^{M \times K}$  spans the signal subspace whose columns are the eigenvectors corresponding to the  $K$  largest eigenvalues, and  $\mathbf{G}_\alpha \in \mathbb{C}^{M \times (M-K)}$  spans the noise subspace

whose columns are the eigenvectors corresponding to the  $M - K$  smallest eigenvalues.

*Step 4:* Calculate pseudo-spectral  $\mathbf{p}(\theta)$ .

$$\mathbf{p}(\theta) = \frac{\mathbf{a}^H(\theta) \mathbf{a}(\theta)}{\Psi^H(\alpha) \mathbf{a}^H(\theta) \mathbf{G}_\alpha \mathbf{G}_\alpha^H \mathbf{a}(\theta) \Psi(\alpha)} \quad (32)$$

The DOAs are obtained by finding the  $K$  maximum extreme points of  $\mathbf{p}(\theta)$ . The directions associated with the extreme points are the DOAs of incoming signals.

### D. ANALYSIS

Compared with SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm only needs  $M$  times switch to get data matrix  $\mathbf{X}(t)$  with  $N$  snapshots. This is more valuable in practical application. Moreover, the algorithm just exploits the time shift property of cyclic autocorrelation function, and has no assumption or approximation on the signal, so the  $\Psi(\alpha)$  and  $\mathbf{A}$  are not affect by signal bandwidth. The  $\Psi(\alpha)$  and  $\mathbf{A}$  are exact for both narrow-band signal and wide-band signal when the switch interval is known exactly. But the switch interval can be not known exactly in practice, so the  $\Psi(\alpha)$  will exist an error which will degrade the performance of SC-Cyclic-MUSIC algorithm.

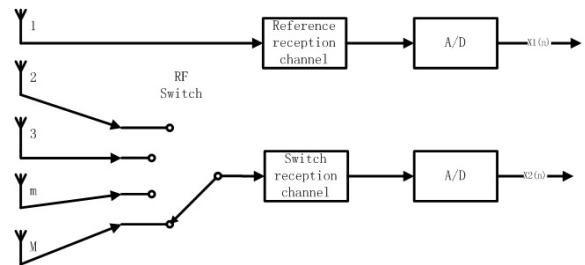


FIGURE 2. The Dual channels receiver architecture.

### IV. DUAL CHANNELS MUSIC ALGORITHM

To reduce the effect of switch interval error, we proposed a dual channels MUSIC algorithm using two reception channel to receive the signal of array sensors. One reception channel which is called reference channel receives the signal of the reference sensor, and another reception channel which is called switch channel receives the signal of the rest sensors in turn. Two channels sample at the same time and sample  $N$  snapshots each turn. The architecture of Dual channels receiver is shown in Fig.2.

The sample data of the reference channel can be expressed as

$$\mathbf{X}_1(t) = [\mathbf{x}_1(t + t_1 + \varepsilon_{t_1}), \dots, \mathbf{x}_1(t + t_{m-1} + \varepsilon_{t_{m-1}}), \dots, \mathbf{x}_1(t + t_{M-1} + \varepsilon_{t_{M-1}})]^T, \quad (33)$$

where

$$\begin{aligned} \mathbf{x}_1(t + t_{m-1} + \varepsilon_{t_{m-1}}) &= [x_1(t + t_{m-1} + \varepsilon_{t_{m-1}}), \dots, \\ & \quad x_1(t + t_{m-1} + \varepsilon_{t_{m-1}} + nT_s), \dots, \\ & \quad x_1(t + t_{m-1} + \varepsilon_{t_{m-1}} + (N - 1)T_s)]. \end{aligned} \quad (34)$$

The sample data of the switch channel can be expressed as

$$\mathbf{X}_2(t) = [\mathbf{x}_2(t + t_1 + \varepsilon_{t_1}), \dots, \mathbf{x}_m(t + t_{m-1} + \varepsilon_{t_{m-1}}), \dots, \mathbf{x}_M(t + t_{M-1} + \varepsilon_{t_{M-1}})]^T, \quad (35)$$

where

$$\begin{aligned} \mathbf{x}_m(t + t_{m-1} + \varepsilon_{t_{m-1}}) &= [x_m(t + t_{m-1} + \varepsilon_{t_{m-1}}), \\ &\dots, x_m(t + t_{m-1} + \varepsilon_{t_{m-1}} + nT_s), \\ &\dots, x_m(t + t_{m-1} + \varepsilon_{t_{m-1}} + (N - 1)T_s)]. \end{aligned} \quad (36)$$

Here  $t_1, t_{m-1}, t_{M-1}$  are certain switch interval.  $\varepsilon_{t_1}, \varepsilon_{t_{m-1}}, \varepsilon_{t_{M-1}}$  are the random errors of  $t_1, t_{m-1}, t_{M-1}$  correspondingly.

Using (25), (26), (27), (28), we get

$$\mathbf{r}_{\mathbf{X}_1}^\alpha(\tau) = \Psi(\alpha) \bullet \mathbf{1}_{M-1,K} \bullet \mathbf{r}_s^\alpha(\tau), \quad (37)$$

$$\mathbf{r}_{\mathbf{X}_2}^\alpha(\tau) = \Psi(\alpha) \mathbf{B} \mathbf{r}_s^\alpha(\tau). \quad (38)$$

where

$$\begin{aligned} \mathbf{B} &= [b(\theta_1), \dots, b(\theta_k), \dots, b(\theta_K)], \\ \mathbf{b}(\theta_k) &= [e^{j2\pi\alpha\tau_2(\theta_k)}, \dots, e^{j2\pi\alpha\tau_m(\theta_k)}, \dots, e^{j2\pi\alpha\tau_{M-1}(\theta_k)}]^T, \\ \Psi(\alpha) &= \mathbf{D}(1, e^{j2\pi\alpha(t_1+\varepsilon_{t_1})}, \dots, e^{j2\pi\alpha(t_{M-2}+\varepsilon_{t_{M-2}})}). \end{aligned}$$

Set

$$\begin{aligned} \mathbf{r}_{\mathbf{X}_{21}}^\alpha(\tau) &= \mathbf{r}_{\mathbf{X}_2}^\alpha(\tau) \odot (\mathbf{r}_{\mathbf{X}_1}^\alpha(\tau))^* \\ &= \mathbf{B} \mathbf{r}_s^\alpha(\tau) (\mathbf{r}_s^\alpha(\tau))^H \bullet \mathbf{1}_{M-1,1} = \mathbf{B} \mathbf{q}_s^\alpha(\tau) \end{aligned} \quad (39)$$

Equation(39) is similar with time-domain model.

For  $\tau = T_s, 2T_s, \dots, NT_s$ , the new data matrix can be expressed as

$$\mathbf{R}_{\mathbf{X}_{21}}^\alpha = [\mathbf{r}_{\mathbf{X}_{21}}^\alpha(T_s), \mathbf{r}_{\mathbf{X}_{21}}^\alpha(2T_s), \dots, \mathbf{r}_{\mathbf{X}_{21}}^\alpha(NT_s)]. \quad (40)$$

Then we apply MUSIC algorithm to estimate the DOAs of the sources. The algorithm is called as Dual Channels MUSIC (DC-Cyclic-MUSIC) algorithm. The process is expressed as follow:

*Step 1:* Calculate the cyclic autocorrelation function  $R_{x_m}^\alpha(t + t_m + \varepsilon_{t_{m-1}}, \tau)$  of the  $x_m(t + t_m + \varepsilon_{t_{m-1}})$ .

*Step 2:* Form the new data matrix  $\mathbf{R}_{\mathbf{X}_{21}}^\alpha(\alpha)$ , and calculate correlation function  $\mathbf{C}(\alpha)$  of  $\mathbf{R}_{\mathbf{X}_{21}}^\alpha(\alpha)$ .

$$\mathbf{C}(\alpha) = \mathbf{R}_{\mathbf{X}_{21}}^\alpha(\alpha) \bullet (\mathbf{R}_{\mathbf{X}_{21}}^\alpha(\alpha))^H \quad (41)$$

*Step 3:* Attain its eigenvectors by EVD of  $\mathbf{C}(\alpha)$ . It can be expressed as

$$\mathbf{C}(\alpha) = [\mathbf{S}_\alpha \mathbf{G}_\alpha] \begin{bmatrix} \sum \mathbf{S}_\alpha \mathbf{0} \\ \mathbf{0} \sum \mathbf{G}_\alpha \end{bmatrix} [\mathbf{S}_\alpha \mathbf{G}_\alpha]^H. \quad (42)$$

*Step 4:* Calculate pseudo-spectral  $\mathbf{p}(\theta)$ .

$$\mathbf{p}(\theta) = \frac{\mathbf{b}^H(\theta) \mathbf{b}(\theta)}{\mathbf{b}^H(\theta) \mathbf{G}_\alpha \mathbf{G}_\alpha^H \mathbf{b}(\theta)} \quad (43)$$

The DOAs are obtained by finding the  $K$  maximum extreme points of  $\mathbf{p}(\theta)$ . The directions associated with the extreme points are the DOAs of incoming signals.

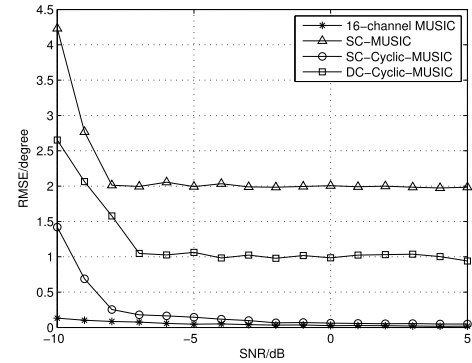


FIGURE 3. The RMSE of four algorithms in different SNR.

## V. SIMULATION

In this section, a series of numerical experiments under different conditions are conducted to examine the performance of the proposed methods. We mainly concerned how the signal bandwidth and the switch interval error affect the performance of SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm in different signal-to-noise ratio (SNR). Suppose that two BPSK modulation signals impinge on a 16-sensor uniform line array from two random directions in  $(-90^\circ \ 90^\circ)$ . The sample rate is 32MHz, so the sample period  $T_s$  is  $1/32\mu s$ . The data for every trail is 8192 samples of each sensor and  $\tau$  is  $T_s, 2T_s, \dots, 8T_s$ . The accuracy of the DOA estimate is measured by 1000 Monte Carlo runs in terms of the root mean square error (RMSE) which is defined as

$$RMSE = \sqrt{\frac{1}{1000} \sum_{i=1}^{1000} \sum_{k=1}^K (\hat{\theta}_k^{(i)} - \theta_k)^2}, \quad (44)$$

where  $\hat{\theta}_k^{(i)}$  is the estimate of for the  $i$ th trial of  $\theta_k$ , and  $K$  is the number of impinging signals. Additionally, to assess the overall reliability of all the algorithms, the Probability of Successful Detection (PSD) is defined as

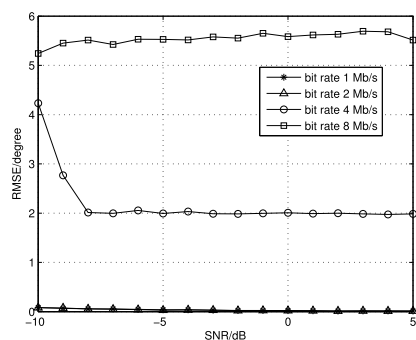
$$\text{Probability of Successful Detection} = F_s / F,$$

where  $F$  is the number of trials and is set to be 1000.  $F_s$  is the number of trials which the root mean square error is within  $3^\circ$ .

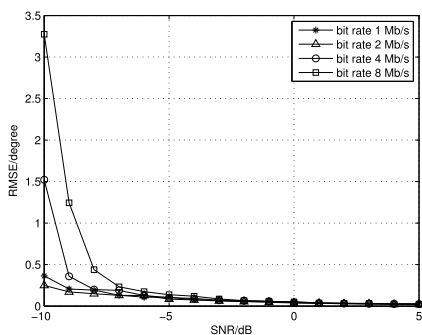
### A. DOA ESTIMATION WITH DIFFERENT BANDWIDTH

*Case 1:* This experiment examines the performance of SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm for certain bandwidth in different SNR and compared with MUSIC algorithm using 16-channel receiver. Suppose that the bit rate of two BPSK signals is 4Mb/s. The cyclic frequency is 4MHz, respectively. The SNR is set varied from  $-10$ dB to  $5$ dB in steps of  $1$ dB. For SC-MUSIC algorithm, assumed that the switch interval from one sensor to another is  $T_s$ . Fig.3 shows the RMSE of four algorithms in different SNR.

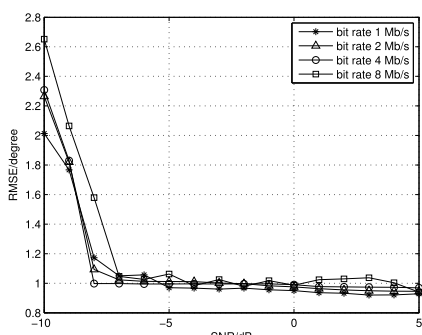
As shown in Fig.3, the RMSE of SC-Cyclic-MUSIC algorithm is similar with the classical 16-channel MUSIC algorithm, and the RMSE of DC-Cyclic-MUSIC algorithm is



(a)



(b)

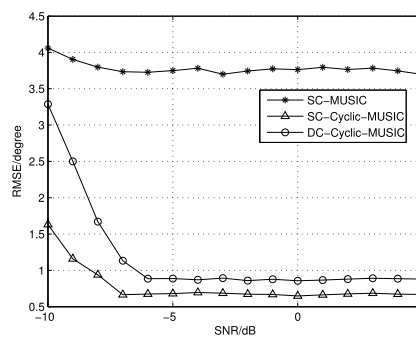


(c)

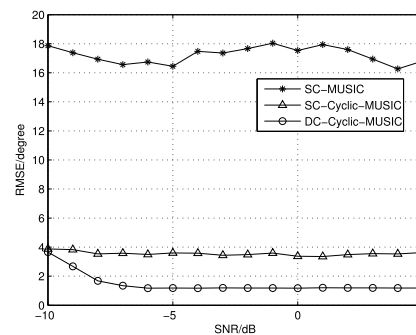
**FIGURE 4.** The RMSE of three algorithms in different bandwidth. (a) SC-MUSIC algorithm. (b) SC-Cyclic-MUSIC algorithm. (c) DC-Cyclic-MUSIC algorithm.

few larger, but the RMSE of SC-MUSIC algorithm is much larger than others.

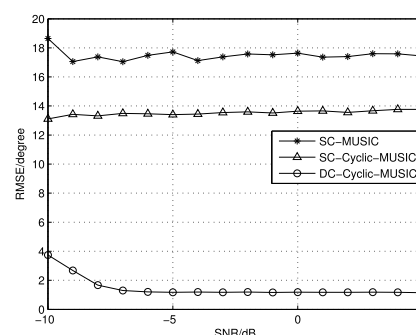
*Case 2:* This experiment examines the performance of the SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm for different bandwidth in certain SNR. Suppose that the bit rate of two BPSK signals is 1Mb/s, 2Mb/s, 4Mb/s, 8Mb/s and the SNR is set varied from  $-10\text{dB}$  to  $5\text{dB}$  in steps of  $1\text{dB}$ . For SC-MUSIC algorithm, assumed that the switch interval from one sensor to another is  $T_s$ . Fig.4 shows the RMSE of three algorithms in different bandwidth, correspondingly. As is shown in Fig.4, the RMSE of the SC-MUSIC algorithm enlarges quickly as the bit rate increases. The performance of the SC-MUSIC algorithm degrades quickly because the approximation error of  $\Psi(f)$ , and  $\mathbf{A}$  increases as the bit rate increases. For the SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC



(a)



(b)



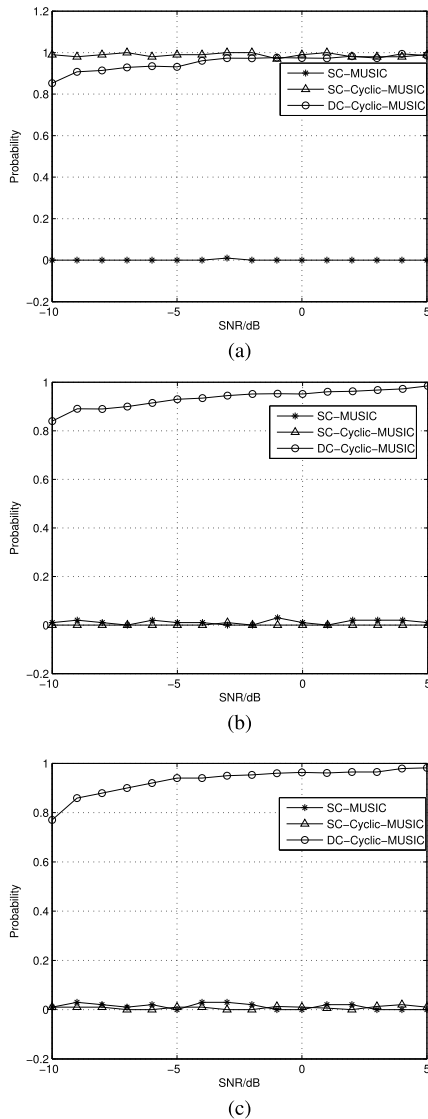
(c)

**FIGURE 5.** The RMSE of three algorithms with different switch error. (a) no switch error. (b) switch error =  $10\mu\text{s}$ . (c) switch error =  $100\mu\text{s}$ .

algorithm, there has no any approximation, so the RMSE of SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm is little affected by signal bandwidth. The RMSE of two algorithms are almost same in different bit rates.

**B. DOA ESTIMATION WITH DIFFERENT SWITCH ERROR**

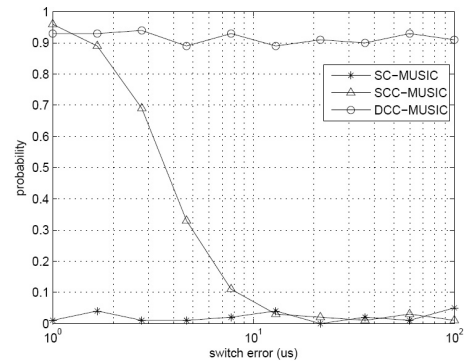
*Case 3:* This experiment examines the performance of SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC with switch error in different SNR. Suppose that the bit rate of two BPSK signals is  $4\text{Mb/s}$ . The SNR is set varied from  $-10\text{dB}$  to  $5\text{dB}$  in steps of  $1\text{dB}$  and the switch interval between two sensors is  $32\mu\text{s}$ , with no switch error,  $10\mu\text{s}$  error and  $100\mu\text{s}$  error. Fig.5 shows the RMSE of three algorithms. Figs.6 shows the Probability of Successful Detection of three algorithms. As is shown in Fig.5 and 6, the RMSE of SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm increases quickly and the



**FIGURE 6.** The probability of successful detection of three algorithms in certain switch error. (a) no switch error. (b) switch error = 10 $\mu$ s. (c) switch error = 100 $\mu$ s.

Probability of Successful Detection approaches to zero when the switch errors are 10 $\mu$ s and 100 $\mu$ s. But the performance of DC-Cyclic-MUSIC algorithm is not affected by the switch error. The RMSE and the Probability of Successful Detection of DC-Cyclic-MUSIC algorithm keeps in a reasonable range.

*Case 4:* This experiment examines the performance of SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC with different switch error in certain SNR. Suppose that the bit rate of two BPSK signals is 4Mb/s. The SNR is 10dB and the switch interval between two sensors is 32 $\mu$ s with an uncertain error. The switch error in  $\mu$ s is a Gaussian sequence with variance varied from 1 to 100 in steps of 2 dB. Fig.7 shows the Probability of Successful Detection of three algorithms. As is shown in Fig.7, the performance of SC-MUSIC algorithm and SC-Cyclic-MUSIC algorithm degrade soon when the switch error increases but the DC-Cyclic-MUSIC algorithm is not affected.



**FIGURE 7.** The probability of successful detection of three algorithm with different switch error.

## VI. CONCLUSION

Based on cyclostationarity, the signal models for DOA estimation using single channel and double channels are formulated, and SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm are proposed correspondingly. Computer simulation has proved that two algorithms have good performance when the switch interval between two sensors is known exactly. But when switch interval has an uncertain error, the performance of SC-Cyclic-MUSIC algorithm degrades soon. The DC-Cyclic-MUSIC algorithm resolves the problem of DOA estimation when the switch interval has an error. Moreover, compared with SC-MUSIC algorithm, SC-Cyclic-MUSIC algorithm and DC-Cyclic-MUSIC algorithm are not affected by signal bandwidth. They work well even if the sources are wide-band signal.

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