Received March 29, 2019, accepted April 19, 2019, date of publication April 22, 2019, date of current version May 1, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2912704

New Design of Constellation and Bit Mapping for Dual Mode OFDM-IM

KEE-HOON KIM[1] AND HOSUNG PARK[2]
[1]Department of Electronic Engineering, Soonchunhyang University, Asan 31538, South Korea
[2]School of Electronics and Computer Engineering, Chonnam National University, Gwangju 61186, South Korea

Corresponding author: Hosung Park (hpark1@jnu.ac.kr)

This work was supported in part by the Soonchunhyang University Research Fund, and in part by the National Research Foundation of Korea (NRF) through the Korea Government, Ministry of Science, ICT and Future Planning (MSIP), under Grant NRF-2018R1C1B5030493.

ABSTRACT Dual mode orthogonal frequency division multiplexing with index modulation (DM-OFDM-IM) is recently proposed, where the subcarriers within each subblock are divided into two groups, which are modulated by two distinguishable signal constellations. That is, the DM-OFDM-IM activates all subcarriers to convey data and eliminate the limits of spectrum efficiency in OFDM with index modulation (OFDM-IM). However, the conventional constellation pair in the DM-OFDM-IM is naively designed and thus induces much power increase. Also, there has been no discussion on symbol-to-bit mapping in the constellation pair. In this paper, a new constellation pair design and the corresponding bit mapping for the DM-OFDM-IM is proposed based on the average bit error probability (ABEP). Since the optimization problem of minimizing the ABEP is difficult to solve, we propose a suboptimal constellation pair through three steps; First, the shape of the constellation pair is design based on only the pairwise error probability (PEP) term in the ABEP. Second, a good bit mapping for the constellation pair is proposed. Third, we further modify the constellation pair by considering the ABEP. The simulation results show that the proposed constellation pair with the bit mapping substantially enhances the bit error rate (BER) performance of the DM-OFDM-IM compared with the conventional DM-OFDM-IM system.

INDEX TERMS Orthogonal frequency division multiplexing (OFDM), index modulation (IM), constellation, average bit error probability (ABEP), pairwise error probability (PEP).

I. INTRODUCTION

In recent years, multicarrier transmission has become an attractive technique in many wireless standards to meet the increasing demand for high data rate communication systems. One of the most popular multicarrier techniques, orthogonal frequency division multiplexing (OFDM), has developed into a widely-used scheme for wideband digital communication. The major advantage of OFDM over single-carrier schemes is its ability to cope with frequency-selective fading channel with only one-tap equalizer. Therefore, OFDM has become an integral part of IEEE 802.16 standards [1]. Many attempts to further improve the classical OFDM system have been made.

The concept of index modulation (IM) originates from spatial modulation technique [2]–[4] in multiple-input multiple-output (MIMO) systems. The spatial modulation utilizes the indices of antennas to convey extra information during the transmission. IM was introduced into OFDM systems as subcarrier index modulation (SIM) in [5], where additional bits can be transmitted through subcarrier indices. Spectral efficiency and energy efficiency can be improved at the same time because it exploits another dimension for transmission. Motivated by this work, several OFDM with index modulation (OFDM-IM) schemes have been proposed [6]–[9], where the additional bits are transmitted by the subcarrier activation pattern. In [10], the compressed sensing is combined with OFDM-IM to decrease the complexity of the detector. Since a portion of subcarriers are deactivated and unused during the transmission, it results in the decrease of the overall spectral efficiency.

More recently, dual mode OFDM with index modulation (DM-OFDM-IM) has been introduced in [11]. The DM-OFDM-IM system modulates all subcarriers using a pair of disjoint constellations and implicitly transmits additional bits through the subcarrier indices pattern of each constellation. In such fashion, DM-OFDM-IM has the potential to further increase the data transmission rate. Similar

The associate editor coordinating the review of this manuscript and approving it for publication was Pallab K. Choudhury.

to OFDM-IM, a generalized version of DM-OFDM-IM was proposed in [12]. Also, a low complexity maximum likelihood (ML) detector is proposed for DM-OFDM-IM in [13] and how to define the mapping between index bits and subcarrier indices patterns is discussed in [14]. In [15], zero-padded tri-mode OFDM-IM is proposed, where only a fraction of subcarriers are modulated by two distinguishable constellation alphabets, while the others remain empty. However, the shape of the pair of constellations used in [11] and [12] is naively designed, which induces a huge power increase. Also, symbol-to-bit mapping in the constellation pair, which will be simply denoted as *bit mapping* in this paper, was not discussed. To the authors' best knowledge, the problems about how to jointly design the shape of the constellation pair and its bit mapping has not been discussed in the literature, which has unnegligible effects on the bit error rate (BER) performance of DM-OFDM-IM systems.

In this paper, well-designed new constellation pair and bit mapping for DM-OFDM-IM are proposed. The proposed constellation pair is designed based on the average bit error probability (ABEP) analysis. Since it is difficult to solve the problem of minimizing ABEP, we obtain a suboptimal constellation pair through sequential three steps. As a result, the DM-OFDM-IM using the proposed constellation pair shows better BER performance than the DM-OFDM-IM using the conventional constellation pair in [11] and [12], which is verified by simulations.

The rest of this paper is organized as follows. Section II describes the system model of DM-OFDM-IM and the conventional constellation pair. Section III presents the proposed constellation pair based on ABEP analysis, where three steps to obtain the proposed constellation pair are presented in its subsections. In Section IV, the simulation results are given to evaluate the benefit of the proposed constellation pair and bit mapping for DM-OFDM-IM. Finally, our conclusions are drawn in Section V.

Notations: Vectors are denoted by boldface letters as $\mathbf{X}$ and its α-th element is denoted by $X(\alpha)$. $\mathcal{CN}(0,\sigma^2)$ represents the distribution of a circularly symmetric complex Gaussian (CSCG) random variable with zero mean and variance σ^2. The Euclidean norm of a vector $\mathbf{X}$ is denoted by $\|\mathbf{X}\|_2$. Also, $\hat{I}$ and $\hat{\mathbf{X}}$ represent the estimates of a set I and a vector $\mathbf{X}$, respectively. Two disjoint constellations will be named simply A and B.

II. DUAL MODE OFDM-IM AND THE CONVENTIONAL CONSTELLATION PAIR

A. DUAL MODE OFDM-IM

The DM-OFDM-IM transmitter [11] is illustrated in Fig. 1. First, m incoming bits are partitioned by a bit splitter into G groups, $\mathbf{b}^1, \cdots, \mathbf{b}^G$, where each consists of p bits as

$$\mathbf{b}^g = [b^g(1)\; b^g(2)\; \cdots\; b^g(p)], \quad g = 1, \cdots, G \tag{1}$$

and $p = m/G$. Each bit stream $\mathbf{b}^g$ is fed into an index selector and two different constellation mappers for generating a DM-OFDM-IM block $\mathbf{X}^g$ of length $n = N/G$, where N is the

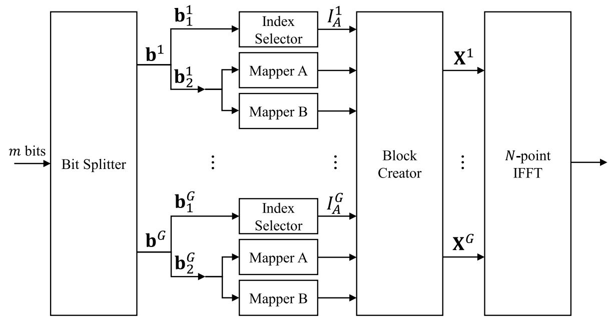

FIGURE 1. A block diagram of the DM-OFDM-IM transmitter.

size of inverse fast Fourier transform (IFFT). In contrast to the existing OFDM-IM, whereby only part of the subcarriers are actively modulated, in the DM-OFDM-IM scheme all the subcarriers are modulated in each block, which leads to spectral efficiency enhancement. That is, k out of n subcarriers are modulated by M_A-ary modulation from the first constellation $\mathcal{M}_A$ and the remaining $n-k$ subcarriers are modulated by M_B-ary modulation from the second constellation $\mathcal{M}_B$ in each block. We denote the constellation pair as $(\mathcal{M}_A, \mathcal{M}_B)$. Clearly, we have to design the constellation pair $(\mathcal{M}_A, \mathcal{M}_B)$ satisfying $\mathcal{M}_A \cap \mathcal{M}_B = \varnothing$. Note that the OFDM-IM can be viewed as the DM-OFDM-IM with $\mathcal{M}_B = \{0\}$. M_A and M_B can be different or the same [11]. However, in this paper we assume

$$M_A = M_B = M \tag{2}$$

which is reasonable in a practical sense. Actually, the previous work in [11] and [12] also focused on the case of (2).

In particular, we represent the bit stream $\mathbf{b}^g$ in (1) as

$$\mathbf{b}^g = [\mathbf{b}_1^g\; \mathbf{b}_2^g] \tag{3}$$

where

$$\begin{aligned} \mathbf{b}_1^g &= [b^g(1)\; \cdots\; b^g(p_1)] \\ \mathbf{b}_2^g &= [b^g(p_1+1)\; \cdots\; b^g(p_1+p_2)] \end{aligned} \tag{4}$$

whose sizes are p_1 and p_2, respectively and thus $p = p_1 + p_2$. The first bit stream $\mathbf{b}_1^g$ in (3) is utilized by the index selector as in Fig. 1. It determines the indices of the subcarriers modulated by $\mathcal{M}_A$ in the g-th block. We denote the corresponding index set as

$$I_A^g = \{i_{A,1}^g, \cdots, i_{A,k}^g\} \tag{5}$$

where $i_{A,j}^g \in \{1, \cdots, n\}$ and $j = 1, \cdots, k$. Then, the indices of the subcarriers modulated by $\mathcal{M}_B$ in the g-th block is uniquely determined by I_A^g. We denote the corresponding index set as

$$I_B^g = \{i_{B,1}^g, \cdots, i_{B,n-k}^g\} \tag{6}$$

where $i_{B,u}^g \in \{1, \cdots, n\}$ and $u = 1, \cdots, n-k$. The number of possible patterns of I_A^g is $\binom{n}{k}$ and thus p_1 is

$$p_1 = \left\lfloor \log_2 \binom{n}{k} \right\rfloor. \tag{7}$$

The second bit stream $\mathbf{b}_2^g$ in (3), whose size is $p_2 = n\log_2 M$, is represented by n bit substreams with size $\log_2 M$ as

$$\mathbf{b}_2^g = [\mathbf{b}_{2,1}^g \; \mathbf{b}_{2,2}^g \; \cdots \; \mathbf{b}_{2,n}^g] \tag{8}$$

where

$$\mathbf{b}_{2,\alpha}^g = [b^g(p_1+(\alpha-1)\log_2 M+1) \; \cdots \; b^g(p_1+\alpha \log_2 M)] \tag{9}$$

and $\alpha = 1, \cdots, n$. Then the bit substream $\mathbf{b}_{2,\alpha}^g$ with size $\log_2 M$ can be mapped into a symbol from either $\mathcal{M}_A$ or $\mathcal{M}_B$ because of (2).

Considering the subcarrier indices pattern I_A^g, the g-th block $\mathbf{X}^g = [X^g(1) \; X^g(2) \; \cdots \; X^g(n)]$ can be generated by, $\alpha = 1, \cdots, n$,

$$X^g(\alpha) = \begin{cases} \mathcal{M}_A(\mathbf{b}_{2,\alpha}^g) & \alpha \in I_A^g \\ \mathcal{M}_B(\mathbf{b}_{2,\alpha}^g) & \alpha \in I_B^g \end{cases} \tag{10}$$

where $\mathcal{M}_A(\cdot)$ and $\mathcal{M}_B(\cdot)$ mean the bit-to-symbol mapper using $\mathcal{M}_A$ and $\mathcal{M}_B$, respectively. After forming all G blocks, they are concatenated to generate the DM-OFDM-IM symbol sequence as

$$\bar{\mathbf{X}} = [\mathbf{X}^1 \; \mathbf{X}^2 \; \cdots \; \mathbf{X}^G]. \tag{11}$$

Then, it is transformed into the DM-OFDM-IM signal sequence in time domain by N-point IFFT and transmitted.

At the receiver, the received symbol sequence in frequency domain for the g-th block is

$$\mathbf{Y}^g = \mathbf{X}^g\mathbf{H}^g + \mathbf{W}^g \tag{12}$$

where $\mathbf{Y}^g = [Y^g(1) \; Y^g(2) \; \cdots \; Y^g(n)]$ is the received symbol sequence, $\mathbf{H}^g = \mathrm{diag}([H^g(1) \; H^g(2) \; \cdots \; H^g(n)])$ is the $n \times n$ matrix whose diagonal elements are channel frequency response (CFR), $\mathbf{W}^g = [W^g(1) \; W^g(2) \; \cdots \; W^g(n)]$ is the additive white Gaussian noise (AWGN) for g-th block. Also, $W^g(\alpha) \sim \mathcal{CN}(0, N_0), \alpha = 1, \cdots, n$. Note that the performance within different blocks are identical and it is sufficient to investigate a single block to determine the overall system performance [8].

The optimal ML detector for the g-th group is

$$\hat{\mathbf{X}}^g = \arg\min_{\mathbf{X}^g} \|\mathbf{Y}^g - \mathbf{X}^g\mathbf{H}^g\|_2 \tag{13}$$

where $\mathbf{X}^g$ is the possible realizations formed by (10) with possible I_A^g. Clearly, the number of possible realizations of $\mathbf{X}^g$ is $2^{p_1}M^n$. From $\hat{\mathbf{X}}^g = [\hat{X}^g(1) \; \hat{X}^g(2) \; \cdots \; \hat{X}^g(n)]$, $\hat{I}_A^g$ is determined. Then, $\hat{\mathbf{b}}_1^g$ is easily obtained from $\hat{I}_A^g$ and $\hat{\mathbf{b}}_2^g$ can be obtained by, $\alpha = 1, \cdots, n$,

$$\hat{\mathbf{b}}_{2,\alpha}^g = \begin{cases} \mathcal{M}_A^{-1}(\hat{X}^g(\alpha)) & \alpha \in \hat{I}_A^g \\ \mathcal{M}_B^{-1}(\hat{X}^g(\alpha)) & \alpha \in \hat{I}_B^g. \end{cases} \tag{14}$$

It can be seen from (13) that the computational complexity of the ML detector in terms of complex multiplications is $\mathcal{O}(2^{p_1}M^n)$. Therefore, the ML detector is impractical to implement for large p_1, n, and modulation order M, due to its exponentially increasing complexity. In [13], a low complexity ML detector is proposed and it requires complex multiplications on the order of $\mathcal{O}(2nM)$, which can be implemented practically. Therefore, we use this low complexity ML detector in simulations.

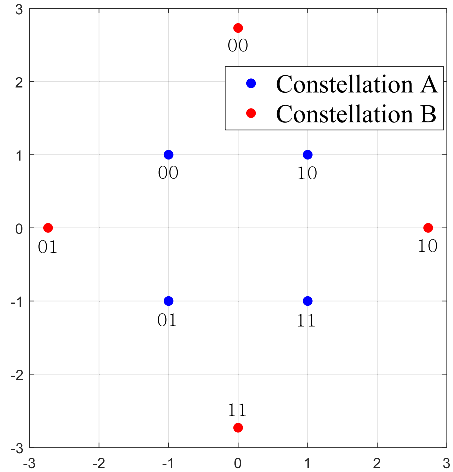

FIGURE 2. The conventional constellation pair $\mathcal{M}_A^{\mathrm{Conv},4}$ (constellation A) and $\mathcal{M}_B^{\mathrm{Conv},4}$ (constellation B) when $M = 4$.

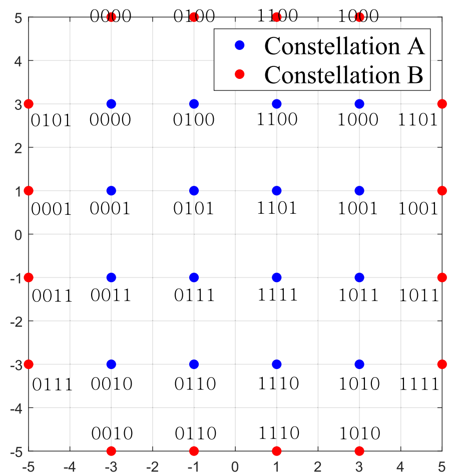

FIGURE 3. The conventional constellation pair $\mathcal{M}_A^{\mathrm{Conv},16}$ (constellation A) and $\mathcal{M}_B^{\mathrm{Conv},16}$ (constellation B) when $M = 16$.

B. THE CONVENTIONAL SIGNAL CONSTELLATION PAIR

In [11] and [12], the two constellation pairs for $M = 4$ and $M = 16$ are proposed based on the classical constellations, quadrature phase shift keying (QPSK) and 16 quadrature amplitude modulation (16QAM), respectively. Specifically, for $M = 4$, $\mathcal{M}_A^{\mathrm{Conv},4} = \{\pm 1 \pm j\}$ and $\mathcal{M}_B^{\mathrm{Conv},4} = \{-(1+\sqrt{3}), -(1+\sqrt{3})j, (1+\sqrt{3}), (1+\sqrt{3})j\}$. Also, the conventional constellation pair for $M = 16$ is $\mathcal{M}_A^{\mathrm{Conv},16} = \{\pm 1 \pm j, \pm 1 \pm 3j, \pm 3 \pm j, \pm 3 \pm 3j\}$ and $\mathcal{M}_B^{\mathrm{Conv},16} = \{-3 \pm 5j, -1 \pm 5j, +1 \pm 5j, +3 \pm 5j, \pm 5 - 3j, \pm 5 - 1j, \pm 5 + 1j, \pm 5 + 3j\}$. Figs. 2 and 3 show the two conventional constellation pairs $(\mathcal{M}_A^{\mathrm{Conv},4}, \mathcal{M}_B^{\mathrm{Conv},4})$ and $(\mathcal{M}_A^{\mathrm{Conv},16}, \mathcal{M}_B^{\mathrm{Conv},16})$, respectively.

III. THE PROPOSED SIGNAL CONSTELLATION PAIR AND BIT MAPPING FOR DM-OFDM-IM

Without loss of generality, we omit the group index g for convenience hereafter. As we mentioned, it is sufficient to investigate a single block to determine the overall system performance. Then, the conditional ABEP in a single block of the DM-OFDM-IM can be evaluated by

$$P_b(\mathbf{H}) \simeq \frac{1}{pn_{\mathbf{X}}} \sum_{\mathbf{X}} \sum_{\hat{\mathbf{X}}} \Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right) e(\mathbf{X}, \hat{\mathbf{X}}) \tag{15}$$

where $n_{\mathbf{X}}$ is the number of the possible realizations of $\mathbf{X}$, $\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right)$ is the conditional pairwise error probability (PEP), and $e(\mathbf{X}, \hat{\mathbf{X}})$ represents the number of bit errors for the corresponding pairwise error event.

The shape of the constellation pair changes the values of $\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right)$ and the bit mapping in the constellation pair changes the values of $e(\mathbf{X}, \hat{\mathbf{X}})$ in (15), but the values of p and $n_{\mathbf{X}}$ are independent from the constellation pair. Therefore, the optimal constellation pair, which minimizes the objective function in (15), can be obtained by jointly considering both $\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right)$ and $e(\mathbf{X}, \hat{\mathbf{X}})$ of possible constellation pairs, which is too complex to solve. Therefore, we will obtain a suboptimal constellation pair and its bit mapping through sequential three steps.

A. STEP 1: DESIGN THE CONSTELLATION PAIR BASED ON PEP

In this subsection, we first design the shape of the constellation pair based on only the conditional PEP term in (15). The well-known conditional PEP expression for the model in (12) is given in [16] as

$$\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right) = Q\left(\frac{\delta}{\sqrt{2N_0}}\right) = Q\left(\frac{\delta\sqrt{SNR}}{\sqrt{2E_b}}\right) \tag{16}$$

where $\delta = \|\mathbf{XH} - \hat{\mathbf{X}}\mathbf{H}\|_2$, $SNR = E_b/N_0$, and E_b is the energy per bit value. It is clear that in (16), for a fixed SNR, $\delta/\sqrt{E_b}$ is the metric determining the conditional PEP.

In DM-OFDM-IM (or OFDM-IM), the error event can be classified into two types. One is the *symbol demodulation error*, where the modulated symbols are erroneously estimated when I_A is correctly estimated. The other type is the *index demodulation error*, where I_A is erroneously estimated. Clearly, the index demodulation error affects not only errors in $\mathbf{b}_1$ but also errors in $\mathbf{b}_2$, which is error propagation. Since the worst case pairwise error event dominates the system performance, let us consider the worst case scenario for each error type.

First, the value of δ in the worst case of the symbol demodulation error, denoted as δ_1, becomes

$$\delta_1 = \min\Bigg(\min_{s_A \neq \hat{s}_A, s_A, \hat{s}_A \in \mathcal{M}_A} |H(\alpha_0)||s_A - \hat{s}_A|, \min_{s_B \neq \hat{s}_B, s_B, \hat{s}_B \in \mathcal{M}_B} |H(\alpha_1)||s_B - \hat{s}_B| \Bigg) \tag{17}$$

where $\alpha_0 \neq \alpha_1$ and $\alpha_0, \alpha_1 = 1, \cdots, n$. Since $|H(\alpha_0)|$ and $|H(\alpha_1)|$ have the same distribution, the desirable design criterion is

$$\min_{s_A \neq \hat{s}_A, s_A, \hat{s}_A \in \mathcal{M}_A} |s_A - \hat{s}_A| = \min_{s_B \neq \hat{s}_B, s_B, \hat{s}_B \in \mathcal{M}_B} |s_B - \hat{s}_B| \tag{18}$$

which means the typical criterion that the minimum Euclidean distances of two distinct signal points in constellations $\mathcal{M}_A$ and $\mathcal{M}_B$ have to be the same.

Second, the value of δ in the worst case of the index demodulation error, denoted as δ_2, becomes

$$\delta_2 = \min_{s_A \in \mathcal{M}_A, s_B \in \mathcal{M}_B} \sqrt{|H(\alpha_0)|^2 + |H(\alpha_1)|^2} |s_A - s_B|. \tag{19}$$

Therefore, $\min_{s_A \in \mathcal{M}_A, s_B \in \mathcal{M}_B} |s_A - s_B|$ has to be large if E_b is fixed. It can be seen that the diversity order of this error event is two.

Let us investigate the values of δ_1 and δ_2 of the conventional constellation pair. In Figs. 2 and 3, it is easily seen that

$$\begin{aligned} \delta_1^{\text{Conv}} &= 2|H(\alpha_0)| \\ \delta_2^{\text{Conv}} &= 2\sqrt{|H(\alpha_0)|^2 + |H(\alpha_1)|^2} \end{aligned} \tag{20}$$

for both modulation orders $M = 4$ and $M = 16$. Also, the energy per bit values in Figs. 2 and 3 are $E_b^{\text{Conv},4} \simeq 1.8928$ and $E_b^{\text{Conv},16} \simeq 4.444$, respectively.

Based on the PEP analysis, we propose the constellations for $M = 4$ as

$$\begin{aligned} \mathcal{M}_A^{\text{Prop1},4} &= \mathcal{M}_A^{\text{Conv},4} \oplus (0.5 + 0.5j) \\ \mathcal{M}_B^{\text{Prop1},4} &= \mathcal{M}_A^{\text{Conv},4} \oplus (-0.5 - 0.5j) \end{aligned} \tag{21}$$

where $\oplus$ is the element-wise addition. For $M = 16$, we propose the constellations as

$$\begin{aligned} \mathcal{M}_A^{\text{Prop1},16} &= \mathcal{M}_A^{\text{Conv},16} \oplus (0.5 + 0.5j) \\ \mathcal{M}_B^{\text{Prop1},16} &= \mathcal{M}_A^{\text{Conv},16} \oplus (-0.5 - 0.5j). \end{aligned} \tag{22}$$

Figs. 4 and 5 show the proposed constellation pair $(\mathcal{M}_A^{\text{Prop1},4}, \mathcal{M}_B^{\text{Prop1},4})$ and $(\mathcal{M}_A^{\text{Prop1},16}, \mathcal{M}_B^{\text{Prop1},16})$, respectively. The reason for the bit mapping structure in Figs. 4 and 5 will be presented in the next subsection. Here we focus on the values of δ_1 and δ_2. In Figs. 4 and 5, it is seen that

$$\begin{aligned} \delta_1^{\text{Prop1}} &= 2|H(\alpha_0)| \\ \delta_2^{\text{Prop1}} &= \sqrt{2}\sqrt{|H(\alpha_0)|^2 + |H(\alpha_1)|^2} \end{aligned} \tag{23}$$

for both modulation orders $M = 4$ and $M = 16$. The energy per bit values in Figs. 4 and 5 are $E_b^{\text{Prop1},4} = 1$ and $E_b^{\text{Prop1},16} \simeq 2.3333$, respectively.

Then, compared to the conventional constellation pair, we have

$$\begin{aligned} \frac{\delta_1^{\text{Conv}}}{\sqrt{E_b^{\text{Conv},4}}} &< \frac{\delta_1^{\text{Prop1}}}{\sqrt{E_b^{\text{Prop1},4}}} \\ \frac{\delta_1^{\text{Conv}}}{\sqrt{E_b^{\text{Conv},16}}} &< \frac{\delta_1^{\text{Prop1}}}{\sqrt{E_b^{\text{Prop1},16}}} \end{aligned} \tag{24}$$

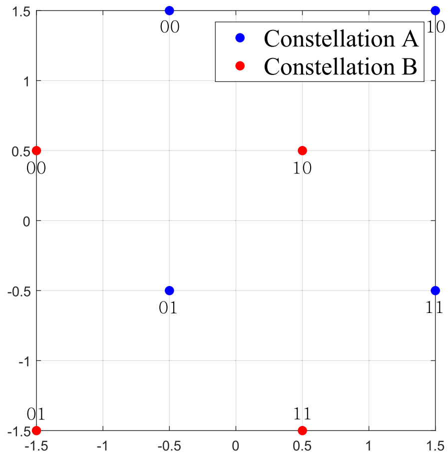

FIGURE 4. The proposed constellation pair $\mathcal{M}_A^{\mathrm{Prop1,4}}$ (constellation A) and $\mathcal{M}_B^{\mathrm{Prop1,4}}$ (constellation B) when $M = 4$.

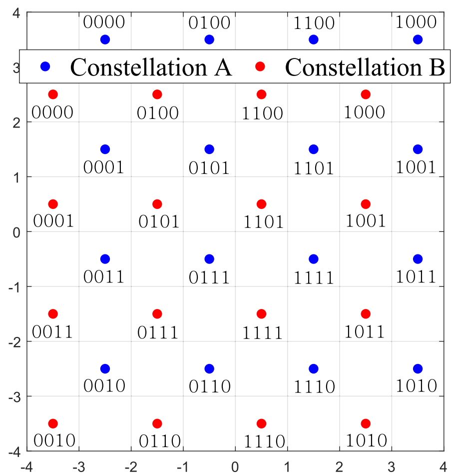

FIGURE 5. The proposed constellation pair $\mathcal{M}_A^{\mathrm{Prop1,16}}$ (constellation A) and $\mathcal{M}_B^{\mathrm{Prop1,16}}$ (constellation B) when $M = 16$.

which means the proposed constellation pair gives lower conditional PEP values for the symbol demodulation error type, and

$$\frac{\delta_2^{\mathrm{Conv}}}{\sqrt{E_b^{\mathrm{Conv},4}}} \simeq \frac{\delta_2^{\mathrm{Prop1}}}{\sqrt{E_b^{\mathrm{Prop1},4}}}$$
$$\frac{\delta_2^{\mathrm{Conv}}}{\sqrt{E_b^{\mathrm{Conv},16}}} \simeq \frac{\delta_2^{\mathrm{Prop1}}}{\sqrt{E_b^{\mathrm{Prop1},16}}} \qquad (25)$$

which means the proposed constellation pair gives similar conditional PEP values for the index demodulation error type. Since the symbol demodulation error dominates the system performance in the high SNR regime because of its single diversity order [8], the proposed constellation pair can give better symbol error performance in the high SNR regime than that of the conventional constellation pair.

It is remarked that the proposed constellation pair has the same values of δ_1 and δ_2 as those of the OFDM-IM, where classical QPSK or 16QAM is used for active subcarriers in the OFDM-IM system.

B. STEP 2: PROPOSED BIT MAPPING STRUCTURE

Here, we will explain the reason for using the bit mapping structure in Figs. 4 and 5. First, a simple example is given to have an insight on DM-OFDM-IM. Let us consider the case $n = 4, k = 2$, and $M = 16$. Also, we assume the following situations;

$$\begin{aligned} I_A &= \{1, 3\} \\ \mathbf{b}_1 &= [10] \\ \mathbf{b}_2 &= [1011\ 0000\ 1111\ 0111]. \end{aligned} \qquad (26)$$

Then, (10) gives us

$$\mathbf{X} = [\mathcal{M}_A(1011)\ \mathcal{M}_B(0000)\ \mathcal{M}_A(1111)\ \mathcal{M}_B(0111)]. \qquad (27)$$

We assume that the subcarrier indices pattern is erroneously detected at the receiver as

$$\hat{I}_A = \{1, 2\}. \qquad (28)$$

For convenience, we also assume that there is no symbol demodulation error. Then, from (14), we have

$$\hat{\mathbf{b}}_2 = [1011\ \mathcal{M}_A^{-1}(\mathcal{M}_B(0000))\ \mathcal{M}_B^{-1}(\mathcal{M}_A(1111))\ 0111]. \qquad (29)$$

From the above example, to mitigate the error propagation due to the index demodulation error event, the bit difference between the input bit stream and the output bit stream of $\mathcal{M}_B^{-1}(\mathcal{M}_A(\cdot))$ or $\mathcal{M}_A^{-1}(\mathcal{M}_B(\cdot))$ has to be small. Also, it is desirable that the symbols in each constellation are mapped to the bits using Gray mapping in order to cope with the symbol demodulation error events occurring in each constellation. To satisfy the two requirements, we propose using the same Gray mapping structure as in Figs. 4 and 5. Note that the bit mapping in the conventional constellation pair in Figs. 2 and 3 is designed in a similar way for a fair comparison in simulations even though the previous work in [11] contains no mention of bit mapping.

C. STEP 3: FURTHER OPTIMIZATION BASED ON ABEP

In this subsection, we further optimize the shape of the proposed constellation pairs in Figs. 4 and 5 by jointly considering the values of $\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right)$ and $e(\mathbf{X}, \hat{\mathbf{X}})$ in (15).

Let us consider the case of $M = 4$ first with a simple example of index demodulation error events. If we use $(\mathcal{M}_A^{\mathrm{Prop1,4}}, \mathcal{M}_B^{\mathrm{Prop1,4}})$ in Fig. 4, the error events having the same $\Pr\left(\mathbf{X} \to \hat{\mathbf{X}}|\mathbf{H}\right)$ incurs the different numbers of bit errors. For example, one case with

$$\begin{aligned} \mathbf{X} &= [(1.5 + 1.5j)\ (0.5 + 0.5j)\ (1.5 + 1.5j)\ (0.5 + 0.5j)] \\ \hat{\mathbf{X}} &= [(1.5 + 1.5j)\ (1.5 + 1.5j)\ (0.5 + 0.5j)\ (0.5 + 0.5j)] \end{aligned} \qquad (30)$$

and the other case with

$$\mathbf{X} = [(-0.5 - 0.5j)\ (0.5 + 0.5j)\ (-0.5 - 0.5j)\ (0.5 + 0.5j)]$$
$$\hat{\mathbf{X}} = [(-0.5 - 0.5j)\ (-0.5 - 0.5j)\ (0.5 + 0.5j)\ (0.5 + 0.5j)] \tag{31}$$

have the same δ and thus the same value of $\Pr\left(\mathbf{X} \rightarrow \hat{\mathbf{X}}|\mathbf{H}\right)$, but the former gives zero bit errors and the latter gives four bit errors when we only consider the bit errors in $\mathbf{b_2}$. (They have the same number of bit errors in $\mathbf{b_1}$ because the differences of subcarrier indices pattern between $\mathbf{X}$ and $\hat{\mathbf{X}}$ for two cases are the same.) Intuitionally, it is undesirable characteristic for minimizing ABEP in (15). Therefore, it is desirable to modify the shape of the constellation pair in order that the error event having the larger value of $\Pr\left(\mathbf{X} \rightarrow \hat{\mathbf{X}}|\mathbf{H}\right)$ incurs the smaller number of bit errors and vice versa.

We simply modify the proposed constellation pair in Fig. 4 by making two constellations come closer to each other. By doing this, the symbols from two constellations having the same bit representation can come closer to each other while the symbols having the much different bit representations can be apart from each other. Since it is hard to theoretically determine how close two constellations are, we performed massive simulations and obtained the further optimized constellation pair for $M = 4$ as

$$\mathcal{M}_A^{\text{Prop2,4}} = \mathcal{M}_A^{\text{Conv,4}} \oplus (0.3 + 0.3j)$$
$$\mathcal{M}_B^{\text{Prop2,4}} = \mathcal{M}_A^{\text{Conv,4}} \oplus (-0.3 - 0.3j) \tag{32}$$

which are depicted in Fig. 6.

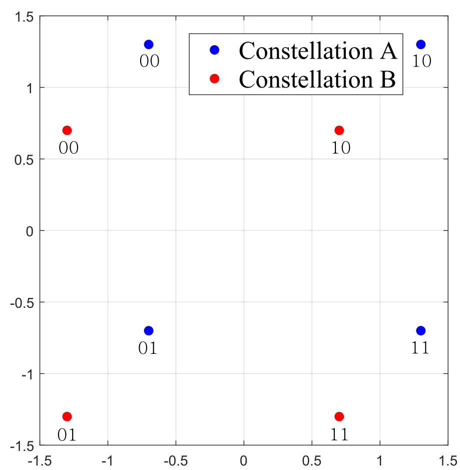

FIGURE 6. The further optimized proposed constellation pair $\mathcal{M}_A^{\text{Prop2,4}}$ (constellation A) and $\mathcal{M}_B^{\text{Prop2,4}}$ (constellation B) when $M = 4$.

By using $(\mathcal{M}_A^{\text{Prop2,4}}, \mathcal{M}_B^{\text{Prop2,4}})$ in Fig. 6, the bit error performance due to symbol demodulation error events can be clearly enhanced. It is remarked that the shape of each signal constellation in $(\mathcal{M}_A^{\text{Prop2,4}}, \mathcal{M}_B^{\text{Prop2,4}})$ in Fig. 6 is the same as each signal constellation in $(\mathcal{M}_A^{\text{Prop1,4}}, \mathcal{M}_B^{\text{Prop1,4}})$ in Fig. 4, which means δ in (16) and $e(\mathbf{X}, \hat{\mathbf{X}})$ corresponding to all symbol demodulation error events remain the same. However, E_b in (16) is decreased by this modification. The energy per bit value in Fig. 6 is $E_b^{\text{Prop2,4}} \simeq 0.872$. Therefore, we expect, in the high SNR regime, the further optimized constellation pair $(\mathcal{M}_A^{\text{Prop2,4}}, \mathcal{M}_B^{\text{Prop2,4}})$ gives the better BER performance than $(\mathcal{M}_A^{\text{Prop1,4}}, \mathcal{M}_B^{\text{Prop1,4}})$ without the further optimization.

For $M = 16$, we obtained the further optimized proposed constellation pair by simulations as

$$\mathcal{M}_A^{\text{Prop2,16}} = \mathcal{M}_A^{\text{Conv,16}} \oplus (0.3 + 0.3j)$$
$$\mathcal{M}_B^{\text{Prop2,16}} = \mathcal{M}_A^{\text{Conv,16}} \oplus (-0.3 - 0.3j) \tag{33}$$

and they are depicted in Fig. 7. The energy per bit value in Fig. 7 is $E_b^{\text{Prop2,16}} \simeq 2.2622$ and the same analysis as the case of $M = 4$ can be applied.

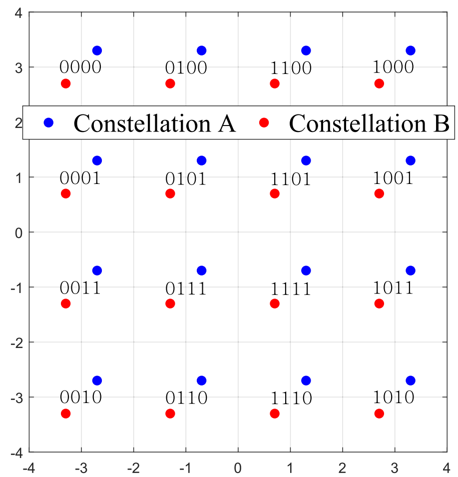

FIGURE 7. The further optimized proposed constellation pair $\mathcal{M}_A^{\text{Prop2,16}}$ (constellation A) and $\mathcal{M}_B^{\text{Prop2,16}}$ (constellation B) when $M = 16$.

IV. SIMULATION RESULTS

To evaluate the benefit of the proposed constellation pair, we simulate DM-OFDM-IM systems over a Rayleigh frequency selective fading channel, where we use $H^g(\alpha) \sim \mathcal{CN}(0, 1)$ for all g's and α's. In simulations of DM-OFDM-IM systems, we use $N = 128$, $n = 4$, and $k = 2$. Based on the results in [14], the subcarrier indices patterns $I_A \in \{\{1, 2\}, \{3, 4\}, \{1, 3\}, \{2, 4\}\}$ are used, which correspond to the bit streams $\mathbf{b}_1 \in \{00, 11, 10, 01\}$ and thus $p_1 = 2$. The low complexity ML detection in [13] is used in all simulations. In figures, signal-to-noise ratio (SNR) means E_b/N_0. Also, we assume the perfect channel estimation at the receiver.

Fig. 8 shows the BER performances of the DM-OFDM-IM systems with $M = 4$ and OFDM-IM as benchmark. The legend means the follows; Conv. constellation, Prop. constellation 1, and Prop. constellation 2 denote DM-OFDM-IM with $(\mathcal{M}_A^{\text{Conv,4}}, \mathcal{M}_B^{\text{Conv,4}})$, $(\mathcal{M}_A^{\text{Prop1,4}}, \mathcal{M}_B^{\text{Prop1,4}})$, and $(\mathcal{M}_A^{\text{Prop2,4}}, \mathcal{M}_B^{\text{Prop2,4}})$, respectively. Also, OFDM-IM denotes the

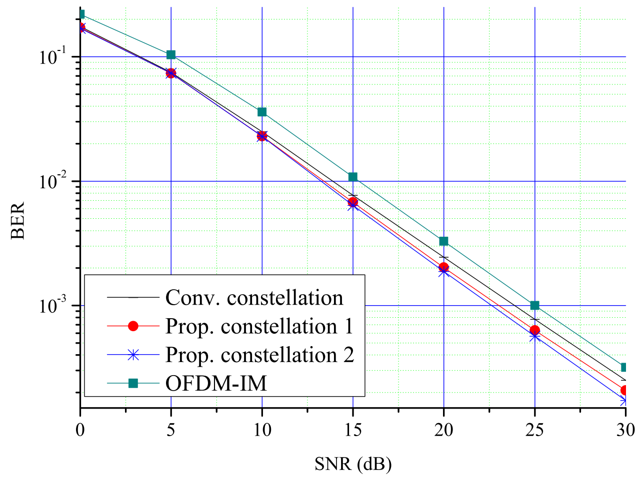

FIGURE 8. The BER performance comparison of DM-OFDM-IM when $M = 4$ and OFDM-IM. The Convolution. constellation means DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Conv},4}, \mathcal{M}_B^{\mathrm{Conv},4})$. The Prop. constellation 1 and 2 mean DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Prop1},4}, \mathcal{M}_B^{\mathrm{Prop1},4})$ and $(\mathcal{M}_A^{\mathrm{Prop2},4}, \mathcal{M}_B^{\mathrm{Prop2},4})$, respectively.

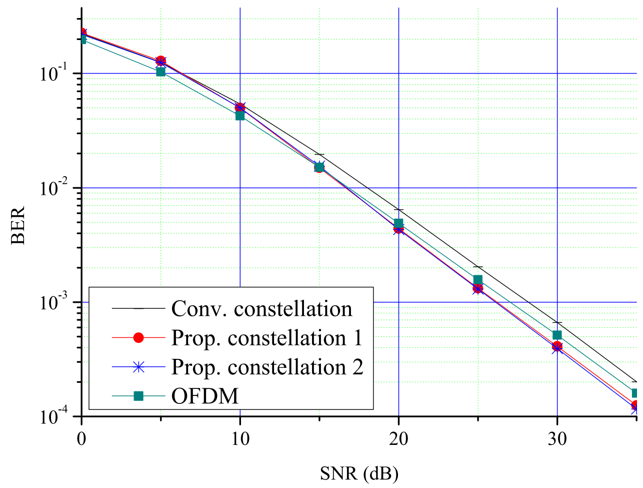

FIGURE 9. The BER performance comparison of DM-OFDM-IM when $M = 16$ and OFDM. The Conv. constellation means DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Conv},16}, \mathcal{M}_B^{\mathrm{Conv},16})$. The Prop. constellation 1 and 2 mean DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Prop1},16}, \mathcal{M}_B^{\mathrm{Prop1},16})$ and $(\mathcal{M}_A^{\mathrm{Prop2},16}, \mathcal{M}_B^{\mathrm{Prop2},16})$, respectively.

OFDM-IM system in [8], where $N = 128$, $n = 4$, and $k = 2$ are used and two active subcarriers are modulated by 16QAM. Note that the OFDM-IM has the same spectral efficiency as the DM-OFDM-IM cases with 2.5 bits/s/Hz. It is shown that two proposed constellation pairs give better BER performance than the conventional constellation pair in the high SNR regime as we expected. Also, DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Prop2},4}, \mathcal{M}_B^{\mathrm{Prop2},4})$ gives the best BER performance because it is further optimized by Step 3 based on ABEP.

Fig. 9 shows the BER performances of the DM-OFDM-IM systems with $M = 16$ and classical OFDM as benchmark. The legend means the follows; Conv. constellation, Prop. constellation 1, and Prop. constellation 2 denote DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Conv},16}, \mathcal{M}_B^{\mathrm{Conv},16})$, $(\mathcal{M}_A^{\mathrm{Prop1},16}, \mathcal{M}_B^{\mathrm{Prop1},16})$, and $(\mathcal{M}_A^{\mathrm{Prop2},16}, \mathcal{M}_B^{\mathrm{Prop2},16})$, respectively. Also, OFDM denotes the classical OFDM system, where the number of subcarriers is 128 and all subcarriers are modulated by 16QAM. Note that the DM-OFDM-IM systems have the spectral efficiency 4.5 bits/s/Hz and the classical OFDM has the spectral efficiency 4 bits/s/Hz.

In Fig. 9, it is shown that the two proposed constellation pairs give better BER performance than the conventional constellation pair in the high SNR regime as we expected. Also, DM-OFDM-IM with $(\mathcal{M}_A^{\mathrm{Prop2},16}, \mathcal{M}_B^{\mathrm{Prop2},16})$ gives the best BER performance, but the benefit of the further optimization in Step 3 is small, different from the case of $M = 4$. This is because the benefit mainly comes from the reduced E_b as we mentioned in Subsection III-C, but the reduction of E_b is small for $M = 16$. (We have $E_b^{\mathrm{Prop1},4}/E_b^{\mathrm{Prop2},4} = 0.595$ dB and $E_b^{\mathrm{Prop1},16}/E_b^{\mathrm{Prop2},16} = 0.134$ dB.) Also, the DM-OFDM-IM system with the proposed constellation pair gives better BER performance than the classical OFDM system in the high SNR regime even though the DM-OFDM-IM has the higher spectral efficiency.

V. CONCLUSIONS

In this paper, the well-designed constellation pair with the good bit mapping structure for DM-OFDM-IM is proposed based on the ABEP analysis. Since it is hard to obtain the optimal constellation pair minimizing the ABEP, the suboptimal constellation pair is proposed through the sequential three steps. The simulation results show that using the proposed constellation pair substantially enhances the BER performance of the DM-OFDM-IM system compared to using the conventional constellation pair.

REFERENCES

[1] J. G. Andrews *et al.*, "What will 5G be?" *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[2] R. Y. Mesleh, H. Haas, S. Sinanovic, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, no. 4, pp. 2228–2241, Jul. 2008.

[3] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: Challenges, opportunities, and implementation," *Proc. IEEE*, vol. 102, no. 1, pp. 56–103, Jan. 2014.

[4] P. Yang, M. Di Renzo, Y. Xiao, S. Li, and L. Hanzo, "Design guidelines for spatial modulation," *IEEE Commun. Surveys Tuts.*, vol. 17, no. 1, pp. 6–26, 1st Quart., 2015.

[5] R. Abu-Alhiga and H. Haas, "Subcarrier-index modulation OFDM," in *Proc. IEEE 20th Int. Symp. Pers., Indoor Mobile Radio Commun.*, Sep. 2009, pp. 177–181.

[6] D. Tsonev, S. Sinanovic, and H. Haas, "Enhanced subcarrier index modulation (SIM) OFDM," in *Proc. IEEE GLOBECOM Workshops (GC Wkshps)*, Dec. 2011, pp. 728–732.

[7] R. Fan, Y. J. Yu, and Y. L. Guan, "Orthogonal frequency division multiplexing with generalized index modulation," in *Proc. IEEE Global Commun. Conf. (GLOBECOM)*, Dec. 2014, pp. 3880–3885.

[8] E. Başar, Ü. Aygölü, E. Panayırcı, and H. V. Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Trans. Signal Process.*, vol. 61, no. 22, pp. 5536–5549, Nov. 2013.

[9] T. Mao, Q. Wang, Z. Wang, and S. Chen, "Novel index modulation techniques: A survey," *IEEE Commun. Surveys Tuts.*, vol. 21, no. 1, pp. 315–348, 1st Quart., 2018.

[10] S. Lu, I. A. Hemadeh, M. El-Hajjar, and L. Hanzo, "Compressed-sensing-aided space-time frequency index modulation," *IEEE Trans. Veh. Technol.*, vol. 67, no. 7, pp. 6259–6271, Jul. 2018.

[11] T. Mao, Z. Wang, Q. Wang, S. Chen, and L. Hanzo, "Dual-mode index modulation aided OFDM," *IEEE Access*, vol. 5, pp. 50–60, 2017.

[12] T. Mao, Q. Wang, and Z. Wang, "Generalized dual-mode index modulation aided OFDM," *IEEE Commun. Lett.*, vol. 21, no. 4, pp. 761–764, Apr. 2017.

[13] X. Zhang, H. Bie, Q. Ye, C. Lei, and X. Tang, "Dual-mode index modulation aided OFDM with constellation power allocation and low-complexity detector design," *IEEE Access*, vol. 5, pp. 23871–23880, 2017.

[14] X. Li, H. Wang, N. Guan, and W. Lai, "A dual-mode index modulation scheme with Gray-coded pairwise index mapping," *IEEE Commun. Lett.*, vol. 22, no. 8, pp. 1580–1583, Aug. 2018.

[15] T. Mao, Q. Wang, J. Quan, and Z. Wang, "Zero-padded orthogonal frequency division multiplexing with index modulation using multiple constellation alphabets," *IEEE Access*, vol. 5, pp. 21168–21178, 2017.

[16] H. Jafarkhani, *Space-Time Coding: Theory and Practice*. Cambridge, U.K.: Cambridge Univ. Press, 2005.

KEE-HOON KIM received the B.S., M.S., and Ph.D. degrees in electrical engineering and computer science from Seoul National University, Seoul, South Korea, in 2008, 2010, and 2015, respectively. He was a Senior Engineer with Samsung Electronics, from 2015 to 2017. He is currently an Assistant Professor with the Department of Electronic Engineering, Soonchunhyang University, Asan, South Korea. His areas of research interests include compressed sensing, OFDM systems, and communication theory.

HOSUNG PARK received the B.S., M.S., and Ph.D. degrees in electrical engineering from Seoul National University, Seoul, South Korea, in 2007, 2009, and 2013, respectively. He was a Postdoctoral Researcher with the Institute for New Media and Communications, Seoul National University, in 2013, and the Qualcomm Institute, the California Institute for Telecommunications and Information Technology, and the University of California at San Diego, San Diego, CA, USA, from 2013 to 2015. He has been an Associate Professor with the School of Electronics and Computer Engineering, Chonnam National University, Gwangju, South Korea, since 2015. His research interests include channel codes for communications systems, coding for memory/storage, coding for distributed storage, communication signal processing, compressed sensing, and network information theory.