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Wideband Direction-of-Arrival Estimation With Arbitrary Array via Coherent Annihilating

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ABSTRACT In this paper, a new wideband DOA estimation method for arbitrary array application is proposed. The arbitrary array manifold is approximately decomposed under the concept of manifold separation technique (MST) and a linear combination of the equal interval sampling complex exponentials (EISCE) can be synthesized. Since the EISCEs at all frequencies contain the same DOA information, they can be annihilated by the same spatial annihilating filter. So, the coherent annihilating scheme is proposed, and all narrowband components are coherently combined to construct an optimization problem under the structural total least squares framework. During the optimization problem construction, the general solution method is proposed to enable the model approximation error in MST to be reduced to a negligible level. Finally, the optimization problem is solved through the multiple measurement vectors structural total least norm (MMV-STLN) approach and the DOAs are estimated from the reconstructed spatial annihilating filter. The new method is free of frequency focusing and is capable of handling both incoherent and coherent signals. The simulation results verify that the proposed method surpasses the existing methods largely in resolution and estimation performance under the low SNR, snapshot deficient, and closely spaced sources scenarios.

INDEX TERMS Coherent annihilating, general solution method, low SNR and snapshot deficient scenarios, multiple measurement vectors structural total least norm (MMV-STLN), wideband direction-of-arrival (DOA) estimation.

I. INTRODUCTION

The direction-of-arrival estimation which is one of the key research topics of the array signal processing finds its wide applications in the military and civilian fields, such as radar, wireless communication, radio surveillance, and microphone array [1]–[3].

Due to the wide frequency bandwidth of the communication signals, speech signals, most radar signals, etc., the conventional narrowand DOA estimation method can not work or can not take full advantage of the signal information. So the wideband DOA estimation technique aimed at the wideband signal arouses great interest. For the array geometry, the regular array (e.g., uniform linear array (ULA) or uniform circular array (UCA)) is preferred since it has more applicable algorithms. However, the regular array may degrade into an irregular array owing to the array imperfections [4]. Besides, for certain applications, the irregular array will be an unavoidable choice [5], [6] or a better choice [7], [8] in contrast to the regular array. Therefore, it is believed that the wideband DOA estimation with the arbitrary array has its practical value and deserves our attention.

The classical MUSIC-based wideband DOA estimation methods can be directly applied to the arbitrary array. They consist of the incoherent subspace method (ISM) [9] and the coherent subspace method (CSM) [10]–[13]. The ISM first divides the wideband array output into several narrowband ones and then apply the narrowband MUSIC to each narrowband component independently. Finally, the results from all the narrowband components are averaged to output the final DOA estimation. The ISM generally does not perform well due to the incoherent processing. Unlike ISM, the CSM adopts the coherent processing. It uses the frequency focusing technique to map the sample covariance matrices at different

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frequency bins to the same reference frequency bin and then the mapped covariance matrices are averaged to generate a new covariance matrix on which narrowband MUSIC can be run. The CSM can get better performance than the ISM. According to the construction method of the focusing matrix, different CSMs are proposed, such as CSSM [10], RSS [11], TCT [12], WAVES [13]. However, all these methods need the pre-estimation of the DOAs to construct the focusing matrix and the final DOA estimation performance is sensitive to the DOA pre-estimation error. The TOPS is a new wideband algorithm which is free of pre-estimation of the DOAs [14]. It is based on rank deficiency of a matrix formed with the noise subspaces and the frequency-aligned signal subspaces at the true DOAs. Nevertheless, the spurious peaks in the TOPS pseudospectrum are difficult to remove. In addition, all the above methods need to perform spatial scanning for the arbitrary array application (TOPS needs to do singular value decomposition (SVD) at each spatial grid), which will increase the computational burden.

It is known that some fast DOA estimation algorithms (e.g., Root-MUSIC [15] which belongs to the subspace based method and IQML [16] which belongs to the maximum likelihood (ML) based method) which are free of spatial scanning are only applicable to the ULA. A logical idea is to transform the arbitrary array into the ULA. In [17], for wideband application the arbitrary array steering vectors at different frequency bins are interpolated into the ULA steering vectors with the same mathematical expressions by the least squares method. So the interpolated covariance matrices can be coherently combined and Root-MUSIC can be adopted owing to the ULA. However, this interpolation is not accurate enough and is only valid in a small spatial sector [18]. Another more promising array transformation method is commonly called the manifold separation technique (MST) [19] which originates from the wavefield modeling formalism developed for array processing [20]–[23]. The MST approximately decomposes the arbitrary array steering vector into the product of the array sampling matrix and the coefficient vector composed of the Fourier bases which has the same Vandermonde structure as the ULA steering vector. The approximation error can be reduced to a negligible level by increasing the order of Fourier basis. In [20] and [22], the MST is adopted. The array sampling matrices at difference frequencies are mapped to the reference frequency by the least squares to enable the coherent processing and the Root-MUSIC is utilized to output the DOA estimates. Unfortunately, flat array sampling matrix (more columns than rows) which is usually necessary for smaller approximation error will bring mapping error and lead to a biased DOA estimation. Furthermore, as we know, the performance of the subspace based methods (e.g., MUSIC and Root-MUSIC) is rather poor under the low SNR and snapshot deficient scenarios [24]. In [25], a IQML-like wideband DOA estimation method for arbitrary array called FRIDA is proposed. In this method, the MST is also used and multi-band information is coherently combined to form an optimization problem via the finite rate of innovation (FRI) principle [26]. The optimization problem is solved by the IQML algorithm. However, the tall array sampling matrix constraint and the covariance domain processing in this method result in performance degradation [27]. Besides, the IQML can only provide a suboptimal solution [28], [29].

In this paper, we propose a new wideband DOA estimation method for arbitrary array application based on coherent annihilating. In Section II, the wideband signal formulation and the MST for the arbitrary array is introduced. With MST, the EISCE can be synthesized by approximately decomposing the arbitrary array steering vector into the product of the array sampling matrix and the coefficient vector. In Section III, first, the annihilating concept is introduced. Since the EISCEs at all frequencies contain the same DOA information, they can be annihilated by the same spatial annihilating filter. So, then the coherent annihilating scheme is proposed, and all narrowband components are coherently combined to construct an optimization problem in the STLS sense. During the optimization problem construction, the general solution method is proposed to enable the model approximation error in MST to be reduced to a negligible level. Finally, the spatial annihilating filter is reconstructed by solving the optimization problem through MMV-STLN approach. After the annihilating filter is reconstructed, the DOAs can be obtained by finding the roots of the polynomial formed from the annihilating filter coefficients. The new method is free of frequency focusing, and is capable of handling both incoherent and coherent signals. In Section IV, the performance of the new method is verified by comprehensive numerical simulations and is compared with the existing counterparts. Section V concludes the whole paper.

Notations used in the paper are introduced as follows. $[\cdot]^T$, $[\cdot]^H$ and $[\cdot]^+$ are denoted as the transpose, conjugate transpose and pseudo-inverse operator, respectively. $\|\cdot\|_2$ and $\|\cdot\|_F$ denote the $\ell 2$ norm and the Frobenius norm, respectively. $*$ and $\otimes$ are the convolution, and Kronecker product operator respectively. vec$[\cdot]$ is the operator that builds a column vector by stacking the column vectors of a matrix below one another, and $\boldsymbol{I}_N$ is the $N \times N$ identity matrix. $\mathbf{0}_N$ represents a zero vector with N elements. a_j denotes the jth element of $\boldsymbol{a}$, $\boldsymbol{A}_{i\cdot}$ denotes the ith row of $\boldsymbol{A}$, $\boldsymbol{A}_{\cdot j}$ denotes the jth column of $\boldsymbol{A}$ and $\boldsymbol{A}_{i,j}$ denotes the element at ith row and jth column of $\boldsymbol{A}$.

II. WIDEBAND SIGNAL FORMULATION AND ARBITRARY ARRAY MANIFOLD SEPARATION

Assume an M-element arbitrary array is located in the x-y plane of the Cartesian coordinate system and the Cartesian coordinates of the mth sensor are $[\rho_m \cos(\phi_m), \rho_m \sin(\phi_m)]$ where $[\rho_m, \phi_m]$ are its corresponding polar coordinates. K far-field wideband signals in the x-y plane from directions $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_K]$ are received by the array. Setting the origin as the reference point, we can write the output of the mth sensor at time t as

$$y_m(t) = \sum_{k=1}^{K} s_k[t - \tau_m(\theta_k)] + \epsilon_m(t) \tag{1}$$

where $s_k(t)$ is the kth signal at reference point, $\tau_m(\theta_k)$ is the propagation delay of the kth signal from DOA θ_k between the mth sensor and the reference point, and $\epsilon_m(t)$ is the additive Gaussian noise of the mth sensor which is temporally and spatially white. Since the signals are wideband, the delay $\tau_m(\theta_k)$ can not be directly converted into phase. The general method is to transform the digital wideband array outputs into several narrowband components through the discrete Fourier transformation (DFT). The time domain array outputs are divided into N segments, and for each segment the DFT is performed to obtain J narrowband components. After frequency decomposition, the array output can be written in a vector form as

$$\boldsymbol{y}_j[n] = \boldsymbol{A}_j(\boldsymbol{\theta})\boldsymbol{s}_j[n] + \boldsymbol{\epsilon}_j[n] \tag{2}$$

where $j = 1, 2, \ldots, J$ is the frequency index, $n = 1, 2, \ldots, N$ is the snapshot index, $\boldsymbol{y}_j[n]$ is the array output at frequency f_j and snapshpt n, $\boldsymbol{A}_j(\boldsymbol{\theta}) = [\boldsymbol{a}_j(\theta_1), \boldsymbol{a}_j(\theta_2), \ldots, \boldsymbol{a}_j(\theta_K)]$ is the array manifold at frequency f_j, $\boldsymbol{a}_j(\theta_k)$ is the array steering vector, $\boldsymbol{s}_j[n]$ is the signal vector, and $\boldsymbol{\epsilon}_j[n]$ is the noise vector. Now the problem is to estimate the DOAs accurately with the data set $\boldsymbol{y}_j[n], j = 1, 2, \ldots, J, n = 1, 2, \ldots, N$.

Since the array geometry is known, the array steering vector has its analytical expression. $\left[a_j(\theta_k)\right]_m$ which is the mth element of $\boldsymbol{a}_j(\theta_k)$ can be written as $\left[a_j(\theta_k)\right]_m = \exp(\mathrm{j}\kappa_j\rho_m\cos(\theta_k - \phi_m))$ where $\kappa_j = 2\pi f_j/c$ and c is the signal propagation velocity. According to the Jacobi-Anger expansion, we have

$$\begin{aligned}\left[a_j(\theta_k)\right]_m &= \exp[\mathrm{j}\kappa_j\rho_m\cos(\theta_k - \phi_m)] \\ &= \sum_{q=-\infty}^{+\infty} \mathrm{j}^q J_q(\kappa_j\rho_m)\exp[\mathrm{j}q(\theta_k - \phi_m)] \\ &\approx \sum_{q=-Q}^{+Q} \underbrace{\mathrm{j}^q J_q(\kappa_j\rho_m)\exp(-\mathrm{j}q\phi_m)}_{G_{m,q+Q+1}(f_j)}\underbrace{\exp(\mathrm{j}q\theta_k)}_{d_{q+Q+1}(\theta_k)} \\ &\approx \left[\boldsymbol{G}_j\right]_{m\cdot}\boldsymbol{d}(\theta_k)\end{aligned} \tag{3}$$

where $\boldsymbol{G}_j \in \mathbb{C}^{M\times(2Q+1)}$ is called the array sampling matrix, $\boldsymbol{d}(\theta_k)$ which is a vector composed of the Fourier bases is called the coefficient vector [21], Q is called the order of the Fourier basis, and $J_q(\cdot)$ is the qth order Bessel function of the first kind. Equation (3) is based on the concept of MST. The original array response is approximately decomposed under the finite Fourier bases. The decomposed coefficient vector $\boldsymbol{d}(\theta_k)$ now is similar to the steering vector of the ULA and is unrelated to the frequency, which will enable the coherent processing of the multi-band components. Since the Bessel function $J_q(\kappa_j\rho_m)$ decays super-exponentially as $q \to \infty$ beyond $|q| = \kappa_j\rho_m$ [21], the model approximation error in (3) can be reduced to a negligible level by increasing Q to a large enough value.

Substituting (3) into (2), we have

$$\begin{aligned}\boldsymbol{y}_j[n] &= \boldsymbol{G}_j\boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{s}_j[n] + \boldsymbol{\epsilon}_j[n] \\ &= \boldsymbol{G}_j\boldsymbol{x}_j[n] + \boldsymbol{\epsilon}_j[n]\end{aligned} \tag{4}$$

where $\boldsymbol{D}(\boldsymbol{\theta}) = [\boldsymbol{d}(\theta_1), \boldsymbol{d}(\theta_2), \ldots, \boldsymbol{d}(\theta_K)]$ and $\boldsymbol{x}_j[n]$ is similar to the noiseless ULA output. Here we replace the approximately equal sign with the equal sign in (4) since we will assign a large enough value to Q.

III. COHERENT ANNIHILATING FOR WIDEBAND DOA ESTIMATION

A. COHERENT ANNIHILATING

As $\boldsymbol{x}_j[n] = \boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{s}_j[n]$ in (4), we say $\boldsymbol{x}_j[n]$ is a linear combination of the equal interval sampling complex exponentials (EISCE) and the combination coefficients are the elements in $\boldsymbol{s}_n(f_j)$. So $\boldsymbol{x}_j[n]$ can be annihilated by a filter $\boldsymbol{h} = [h_1, h_2, \ldots, h_{K+1}]^T$ [27], [30], [31]. The annihilating relation can be written as

$$\boldsymbol{x}_j[n] * \boldsymbol{h} = \boldsymbol{0}, \quad n = 1, \ldots, N,\ j = 1, \ldots, J \tag{5}$$

and the one side Z-transform of $\boldsymbol{h}$ is

$$\sum_{k=1}^{K+1} h_k z^{-(k-1)} = \prod_{k=1}^{K}\left[1 - \frac{\exp(\mathrm{j}\theta_k)}{z}\right]. \tag{6}$$

Since elements of $\boldsymbol{x}_j[n]$ are sampled in the spatial domain, we call $\boldsymbol{h}$ the spatial annihilating filter. If we find the filter $\boldsymbol{h}$ which satisfies (5), the DOAs are able to be obtained by finding the roots of (6). In (5), the annihilating relation is valid at any frequencies and any snapshots with the same filter $\boldsymbol{h}$, which will be the theory foundation of the coherent annihilating.

Since each frequency components has N snapshots, (4) can be written in the multiple measurement vectors (MMV) model which is

$$\boldsymbol{Y}_j = \boldsymbol{G}_j\boldsymbol{X}_j + \boldsymbol{E}_j \tag{7}$$

where $\boldsymbol{Y}_j$, $\boldsymbol{X}_j$ and $\boldsymbol{E}_j$ are the MMV representations of $\boldsymbol{y}_j[n]$, $\boldsymbol{x}_j[n]$ and $\boldsymbol{\epsilon}_j[n]$, respectively (e.g., $\boldsymbol{Y}_j = \left[\boldsymbol{y}_j[1], \boldsymbol{y}_j[2], \ldots, \boldsymbol{y}_j[N]\right]$). Since $\boldsymbol{X}_j = \boldsymbol{D}(\boldsymbol{\theta})\boldsymbol{S}_j$ where $\boldsymbol{S}_j \in \mathbb{C}^{K\times N}$ is the MMV representation of $\boldsymbol{s}_n(f_j)$, we have $\mathrm{rank}[\boldsymbol{X}_j] \leqslant K$ (If there are coherent sources or $N < K$, $\mathrm{rank}[\boldsymbol{X}_j] < K$). So, even though each column of $\boldsymbol{X}_j$ satisfies the annihilating relation in (5), at least $N - K$ columns do not provide new information because there are at most K independent columns in $\boldsymbol{X}_j$ (i.e., there exist redundant snapshots.). Thus, we can compress the data size by performing the principal component analysis (PCA) on $\boldsymbol{Y}_j$ without information loss. The PCA is peformed via the singular value decomposition (SVD). Assume the SVD of $\boldsymbol{Y}_j$ is $\boldsymbol{Y}_j = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{V}^H$, and then the compressed $\boldsymbol{Y}_j$ is [32]

$$\boldsymbol{Y}_j' = \boldsymbol{Y}_j\boldsymbol{V}\boldsymbol{F} \tag{8}$$

where $\boldsymbol{F} = [\boldsymbol{I}_K, \boldsymbol{0}]^T \in \mathbb{R}^{N\times K}$ if $N > K$, otherwise $\boldsymbol{F} = \boldsymbol{I}_N$. Substituting (7) into (8), we have

$$\boldsymbol{Y}_j' = \boldsymbol{G}_j\boldsymbol{X}_j' + \boldsymbol{E}_j' \tag{9}$$

where $X'_j = X_j VF = D(\theta)S_j VF = D(\theta)S'_j$ and $E'_j = E_j VF$. Now the number of columns of X'_j becomes K (Assume $N \geq K$) and each column of X'_j still satisfies the annihilating relation in (5) since it is still a linear combination of the EISCE.

The annihilating relation in (5) can be expressed in a matrix form which is

$$L\left[x_j[n]\right] h = 0 \tag{10}$$

where $L[\cdot]$ is a matrix operator and $L[\beta]$ where $\beta = x_j[n]$ is a Toeplitz matrix written as

$$L[\beta] = \begin{bmatrix} \beta_{K+1} & \beta_K & \cdots & \beta_1 \\ \beta_{K+2} & \beta_{K+1} & \cdots & \beta_2 \\ \vdots & \vdots & \ddots & \vdots \\ \beta_{2Q+1} & \beta_{2Q} & \cdots & \beta_{2Q-K+1} \end{bmatrix}. \tag{11}$$

Since $x_j[n]$ is a single measurement vector (SMV), we call (10) the SMV annihilating. For the X'_j whose columns all satisfy (10), the annihilating relation can be written as

$$\bar{L}[X'_j] h = 0 \tag{12}$$

where $\bar{L}[\cdot]$ is a column-wise operator of $L[\cdot]$ and $\bar{L}[B] = \left[L^T[B_{\cdot 1}], L^T[B_{\cdot 2}], \ldots, L^T[B_{\cdot K}]\right]^T$ where $B = X'_j$. Since X'_j is MMV, we call (10) the MMV annihilating.

For the wideband DOA estimation, the frequency components $X'_j, j = 1, 2, \ldots, J$ all satisfy (12). Define $X' = [X'_1, X'_2, \ldots, X'_J]$, and then the wideband version of (12) can be written as

$$\bar{L}[X'] h = \begin{bmatrix} \bar{L}[X'_1] \\ \bar{L}[X'_2] \\ \vdots \\ \bar{L}[X'_J] \end{bmatrix} h = 0. \tag{13}$$

Here we call (13) the wideband annihilating since it contains all the frequency components, or the coherent annihilating since all signal information which contains information among snapshots and among frequencies is coherently combined with a single annihilating filter.

B. OPTIMIZATION PROBLEM CONSTRUCTION UNDER STLS FRAMEWORK

Now the problem is we do not have access to X'_j, so the next step is to extract it from (9). If $M \geq 2Q+1$ (i.e., G_j is a tall matrix), then $G^+ G = I_{2Q+1}$ and we have $X'_j = G^+_j(Y'_j - E'_j)$. However, small Q will make the model approximation error in (3) non-negligible. Thus in general, for high performance MST based DOA estimation $M < 2Q+1$ is required. Under this underdetermined condition, we can express X'_j by its general solution which is

$$X'_j = G^+_j(Y'_j - E'_j) + (I_{2Q+1} - G^+_j G_j) W_j \tag{14}$$

where W_j is a certain unknown matrix.

By defining

$$\Sigma_j = G^+_j E'_j - (I_{2Q+1} - G^+_j G_j) W_j \tag{15}$$

and substituting (14) into (12), we have

$$\left\{\bar{L}[G^+_j Y'_j] - \bar{L}[\Sigma_j]\right\} h = 0 \tag{16}$$

Combine the J frequency components and define

$$Z = \left[G^+_1 Y'_1 \; G^+_2 Y'_2 \; \cdots \; G^+_J Y'_J\right], \tag{17}$$

$\Sigma = [\Sigma_1, \Sigma_2, \ldots, \Sigma_J]$ and then (16) turns into a wideband version which is

$$\left\{\bar{L}[Z] - \bar{L}[\Sigma]\right\} h = 0 \tag{18}$$

In the least squares (LS) sense, to reconstruct h, we need to minimize $\|\bar{L}[\Sigma]\|_F^2$ under the constraint of (18). However, $\bar{L}[\Sigma]$ is the mapped noise instead of the original noise. Multiplying both sides of (15) by G_j we have $G_j \Sigma_j = E'_j$. This is because $G_j G^+_j = I_M$ due to the assumed $M < 2Q+1$. Then, define a new matrix variable Σ' as

$$\begin{aligned} \Sigma' &= \left[G_1\Sigma_1 \quad G_2\Sigma_2 \quad \cdots \quad G_J\Sigma_J\right] \\ &= \left[E'_1 \quad E'_2 \quad \cdots \quad E'_J\right]. \end{aligned} \tag{19}$$

So Σ' is the original noise. Now, in the structural total least squares (STLS) sense, we can minimize $\|\Sigma'\|_F^2$. For $\|\Sigma'\|_F^2$, we have

$$\begin{aligned} \|\Sigma'\|_F^2 &= \|\text{vec}[\Sigma']\|_2^2 \\ &= \|Gv\|_2^2 \end{aligned} \tag{20}$$

where G is

$$G = \begin{bmatrix} I_K \otimes G_1 & & \\ & \ddots & \\ & & I_K \otimes G_J \end{bmatrix} \tag{21}$$

and v is

$$v = \text{vec}\,[\Sigma] = \text{vec}\left\{\left[\Sigma_1 \; \Sigma_2 \; \cdots \; \Sigma_J\right]\right\} \tag{22}$$

As we see, in (18), $\bar{L}[\Sigma]$ is a vertical stack of Toeplitz matrices, then we have

$$\begin{aligned} \bar{L}[\Sigma] h &= \text{vec}\left\{[I_K \otimes R(h)]\left[\text{vec}[\Sigma_1] \cdots \text{vec}[\Sigma_J]\right]\right\} \\ &= [I_{KJ} \otimes R(h)]\, v \end{aligned} \tag{23}$$

where $R(h) \in \mathbb{C}^{(2Q+1-K)\times(2Q+1)}$ is

$$R(h) = \begin{bmatrix} h_{K+1} & \cdots & h_1 & 0 & \cdots & 0 \\ 0 & h_{K+1} & \cdots & h_1 & \cdots & \vdots \\ \vdots & \ddots & \ddots & \cdots & \ddots & 0 \\ 0 & \cdots & 0 & h_{K+1} & \cdots & h_1 \end{bmatrix}. \tag{24}$$

So, substituting (23) into (18) and considering (20), we are able to construct an optimization problem under the STLS framework. It is given by

$$\begin{aligned} \min_{h,v} \quad & \|Gv\|_2^2 \\ \text{s. t.} \quad & \bar{L}[Z] h - [I_{KJ} \otimes R(h)]\, v = 0 \\ & \omega^H h = 1 \end{aligned} \tag{25}$$

where $\boldsymbol{\omega}^H \boldsymbol{h} = 1$ is used to ensure the uniqueness of $\boldsymbol{h}$ and $\boldsymbol{\omega}$ is a constant vector which later will be assigned $\boldsymbol{\omega} = \boldsymbol{h}^{(0)}$. $\boldsymbol{h}^{(0)}$ is the initialization of $\boldsymbol{h}$ in the following iterative calculation.

Since the constructions of the cost function and the constraint in (25) are mainly based on using the general solution of an underdetermined system, we call this construction procedure the general solution method.

C. FILTER RECONSTRUCTION FOR WIDEBAND DOA ESTIMATION

Now we need to reconstruct the filter $\boldsymbol{h}$ by solving the optimization problem in (25). Although the first constraint in (25) is nonlinear with $\boldsymbol{h}$ and there is another unknown vector variable $\boldsymbol{\nu}$ in the optimization problem, (25) is able to be perfectly solved by the MMV-STLN approach [30] which is a variation of the classical STLN approach [33]–[35].

In the MMV-SLTN approach, the iterative calculation is required and in the $(i+1)$th iteration the first constraint in (25) is linearized around the last iteration solution. Define $\Delta\boldsymbol{h}$ as a small change in $\boldsymbol{h}^{(i)}$, $\Delta\boldsymbol{\nu}$ as a small change in $\boldsymbol{\nu}^{(i)}$ and let $\boldsymbol{r}(\boldsymbol{h}^{(i)}, \boldsymbol{\nu}^{(i)}) = \bar{\boldsymbol{L}}[\boldsymbol{Z}]\boldsymbol{h}^{(i)} - [\boldsymbol{I}_{KJ} \otimes \boldsymbol{R}(\boldsymbol{h}^{(i)})]\boldsymbol{\nu}^{(i)}$. Here, the superscript (i) indicates the iteration index. Then given the last iteration solution $\boldsymbol{h}^{(i)}$ and $\boldsymbol{\nu}^{(i)}$, we have

$$\begin{aligned} \boldsymbol{r}(\boldsymbol{h}^{(i+1)}, \boldsymbol{\nu}^{(i+1)}) &= \boldsymbol{r}(\boldsymbol{h}^{(i)} + \Delta\boldsymbol{h}, \boldsymbol{\nu}^{(i)} + \Delta\boldsymbol{\nu}) \\ &= \boldsymbol{r}(\boldsymbol{h}^{(i)}, \boldsymbol{\nu}^{(i)}) + \boldsymbol{J}^{(i)}\left[\Delta\boldsymbol{h}^T, \Delta\boldsymbol{\nu}^T\right]^T \end{aligned} \quad (26)$$

where $\boldsymbol{J}^{(i)} = \left[\bar{\boldsymbol{L}}[\boldsymbol{Z}] - \bar{\boldsymbol{L}}[\boldsymbol{\Sigma}^{(i)}], -\boldsymbol{I}_{KJ} \otimes \boldsymbol{R}(\boldsymbol{h}^{(i)})\right]$ is the Jacobian of $\boldsymbol{r}(\boldsymbol{h}, \boldsymbol{\nu})$ with respect to $[\boldsymbol{h}, \boldsymbol{\nu}]$ and $\boldsymbol{\Sigma}^{(i)}$ can be constructed from $\boldsymbol{\nu}^{(i)}$ according to (22). So, in the $(i+1)$th iteration, the unknowns become the $\Delta\boldsymbol{h}$ and $\Delta\boldsymbol{\nu}$ and the optimization problem in (25) can be transformed into a standard linear equality constrained LS problem which is

$$\begin{aligned} \min_{\Delta\boldsymbol{h}, \Delta\boldsymbol{\nu}} \quad & \left\| \boldsymbol{\Lambda} \begin{bmatrix} \Delta\boldsymbol{h} \\ \Delta\boldsymbol{\nu} \end{bmatrix} - \boldsymbol{\mu}^{(i)} \right\|_2^2 \\ \text{s. t.} \quad & \begin{bmatrix} \boldsymbol{J}^{(i)} \\ \boldsymbol{\omega}' \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{h} \\ \Delta\boldsymbol{\nu} \end{bmatrix} = \begin{bmatrix} -\boldsymbol{r}(\boldsymbol{h}^{(i)}, \boldsymbol{\nu}^{(i)}) \\ 1 - \boldsymbol{\omega}^H \boldsymbol{h} \end{bmatrix} \end{aligned} \quad (27)$$

where $\boldsymbol{\Lambda} \in \mathbb{C}^{(MKJ+K+1)\times((2Q+1)KJ+K+1)}$ is a block diagonal matrix with the first block being a $(K+1)$-by-$(K+1)$ zero matrix and the rest block being $\boldsymbol{G}$, $\boldsymbol{\mu}^{(i)} = [\boldsymbol{0}_{K+1}^T, (\boldsymbol{G}\boldsymbol{\nu}^{(i)})^T]^T$, and $\boldsymbol{\omega}' = [\boldsymbol{\omega}^H, \boldsymbol{0}_{(2Q+1)KJ}^T]$. The standard method for solving (27) is the Lagrangian multipliers and there are also the off-the-shelf robust and efficient constrained linear least-squares solvers, such as *cgglse* function in the LAPACK library and *lsqlin* function in the Matlab optimization toolbox.

After (27) is solved, we obtain the estimated $\hat{\Delta\boldsymbol{h}}$ and $\hat{\Delta\boldsymbol{\nu}}$. Then update rule for $\boldsymbol{h}^{(i+1)}$ and $\boldsymbol{\nu}^{(i+1)}$ is given by

$$\begin{cases} \boldsymbol{h}^{(i+1)} & \leftarrow \boldsymbol{h}^{(i)} + \hat{\Delta\boldsymbol{h}} \\ \boldsymbol{\nu}^{(i+1)} & \leftarrow \boldsymbol{\nu}^{(i)} + \hat{\Delta\boldsymbol{\nu}} \end{cases} \quad (28)$$

The iterative calculation is terminated when the maximum number of iterations I is reached or the iteration arrives the convergence point which is assumed as

$$\frac{\|\boldsymbol{h}^{(i+1)} - \boldsymbol{h}^{(i)}\|_2}{\|\boldsymbol{h}^{(i)}\|_2} \leq \zeta \quad (29)$$

where ζ is the iteration termination threshold which is a small number.

After the iteration converges, we obtain the reconstructed annihilating filter $\hat{\boldsymbol{h}} = [\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_{K+1}]^T$, and according to (6) the final estimated DOAs $\hat{\boldsymbol{\theta}} = [\hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_K]^T$ can be derived as

$$\hat{\boldsymbol{\theta}} = \text{angle}\left[\text{root}\left(\sum_{k=1}^{K+1} \hat{h}_k z^{-(k-1)} = 0\right)\right] \quad (30)$$

For the iterative calculation in (28), we need the initializations which are $\boldsymbol{h}^{(0)}$ and $\boldsymbol{\nu}^{(0)}$. Since multiple local minima exist in the cost function of (25) because of the non-convexity, we need to choose initializations close to the optimal solution. From (6), we find that $\boldsymbol{h}$ can be estimated by performing the inverse one-sided Z-transform on a polynomial constituted from the pre-estimated DOAs, and the pre-estimated DOAs can be coarsely achieved via the simple conventional beamforming (CBF) (or the Capon Beamformer). Assuming $\tilde{\theta}_k, k = 1, 2, \ldots, \hat{K}$ is the pre-estimated DOAs and $\hat{K}$ is the number of peaks of the pre-estimated spatial spectrum, we derive $\boldsymbol{h}^{(0)}$ as

$$\boldsymbol{h}^{(0)} = \mathcal{Z}^{-1}\left\{\prod_{k=1}^{\hat{K}}\left[1 - \frac{\exp(\mathrm{j}\tilde{\theta}_k)}{z}\right]\right\}. \quad (31)$$

The true number of sources K is assumed to be known or estimated by classical source number estimator such as the Akaike information criterion (AIC) method [36] or the minimum description length (MDL) method [37]. The closely-spaced sources may make $\hat{K} < K$, resulting in $K - \hat{K}$ zeros in $\boldsymbol{h}^{(0)}$ according to (31). In the simulation part, we will see this has no influence on the resolution of the method. For $\boldsymbol{\nu}$, its initialization can be written as

$$\boldsymbol{\nu}^{(0)} = \text{vec}\left[\boldsymbol{Z} - \boldsymbol{D}(\tilde{\boldsymbol{\theta}}')\boldsymbol{Z}'\right] \quad (32)$$

and

$$\boldsymbol{Z}' = \left[\boldsymbol{A}(\tilde{\boldsymbol{\theta}}')_1^+ \boldsymbol{Y}_1' \quad \boldsymbol{A}(\tilde{\boldsymbol{\theta}}')_2^+ \boldsymbol{Y}_2' \quad \cdots \quad \boldsymbol{A}(\tilde{\boldsymbol{\theta}}')_J^+ \boldsymbol{Y}_J'\right], \quad (33)$$

where $\tilde{\boldsymbol{\theta}}'$ consisting of $\{\tilde{\theta}_k - \theta_{beam}/4, \tilde{\theta}_k, \tilde{\theta}_k + \theta_{beam}/4\}_{k=1}^{\hat{K}}$ is the extended coarse DOA estimation obtained by CBF or Capon Beamformer and θ_{beam} is the array beamwidth [11]. The detail derivation steps of (32) are shown in Appendix V.

Now the steps of the proposed wideband DOA estimation method can be summarized as follows:

1) Perform DFT on each of the N segments of the array output to form the MMV data $\boldsymbol{Y}_j, j = 1, 2, \ldots, J$, and then perform PCA on $\boldsymbol{Y}_j$ to produce the compressed $\boldsymbol{Y}_j', j = 1, 2, \ldots, J$.
2) Obtain the initializations $\boldsymbol{h}^{(0)}$ and $\boldsymbol{\nu}^{(0)}$ via (31) and (32).

3) Start the iteration calculation, and in each iteration solve the stand linear equality constrained LS problem in (27) and update $\boldsymbol{h}$ and $\boldsymbol{v}$ through (28).
4) Estimate the DOAs using (30) after the convergence of the iteration.

D. SOME DISCUSSIONS

In the proposed method, the arbitrary array steering vector $\boldsymbol{a}_j(\theta)$ is approximated by the product of the array sampling matrix $\boldsymbol{G}_j$ and the coefficient vector $\boldsymbol{d}(\theta)$. The model approximation error is controlled by the Fourier basis order Q. Larger Q brings smaller approximation error, but will lead to higher computational complexity. As the analytical formula for determining the suitable Q is hard to derive [21], here we give a empirical criterion. First, find the smallest circle enclosing all of the array elements, which can be done by the efficient Welzl's algorithm [38]. The radius of the circle is denoted by $\rho_{\max}$ and the origin of the circle is set as the array reference point. Then, according to the Bessel function in (3), the empirical criterion for determining the suitable Q is

$$Q = \frac{4\pi\rho_{\max}}{\lambda_h} \tag{34}$$

where λ_h is the wavelength of the highest frequency f_J within the wide bandwidth. This criterion will ensure the model approximation error can be safely neglected and the DOA estimation performance will not deteriorate in general signal condition. We will show this in the simulation part.

The convergence of the iteration calculation in the proposed method is the same as convergence of the STLN approach. As it is shown in [33], the STLN is essentially a Gauss-Newton method. In general, the Gauss-Newton method will converge to a local minimum when the residual is small. This condition is satisfied in many applications and in our simulations the proposed method converges in all situations. Since the optimization problem in (25) is not convex, convergence to a global minimum is not guaranteed. However, in the numerical simulation we will find only when the error in the DOA pre-estimation is very large may the iteration occasionally converge to a wrong solution.

As for the computational complexity of the proposed method, the most time-consuming part is the iterative calculation. In each iteration, we need to solve a standard linear equality constrained LS problem in (27). According to [39], the complexity of solving (27) using the Lagrangian multipliers with the QR factorization for solving the corresponding KKT equations is $O([(L+M-K)KJ+K+2][LKJ+K+1]^2)$ where $L = 2Q+1$. Furthermore, as we can find $\boldsymbol{\Lambda}$ and $\boldsymbol{J}^{(i)}$ in (27) are both sparse matrices, the sparsity can be exploited to largely reduce the computational complexity [39]. In the simulation part, we use the more efficient Matlab's slover *lsqlin* with its sparse operation enabled to solve (27).

In addition, during the derivation of the proposed method, we have no assumptions about the correlativity between signals. So this method is supposed be able to work normally with the coherent signals, which is similar to the CSM method.

IV. SIMULATION SIMULATIONS

In this part, we carry out several simulations to demonstrate the performance of the proposed method, and compare it with the existing wideband DOA estimation methods that are workable with the arbitrary array, including the TCT [12] and WAVES [13] which both belong to the CSM method, TOPS [14], AI (array interpolation) [17], MSTI (MST based interpolation) [20], [22], and FRIDA [25]. The Cramer-Rao Lower Bound (CRLB) for wideband DOA estimation [17] is also employed as the performance reference.

In each simulation, the results are given based on the average of 500 Monte Carlo experiments. For the array geometry, an arbitrarily placed array display in Fig. 1 will be used in the each simulation. It has 8 elements and the element locations are normalized by the wavelength of the highest frequency within the wide bandwidth. The wideband signals has a relative bandwidth of 20% and are assumed to be composed of $J = 9$ narrowand components. For the methods needing spatial scanning, the spatial grid interval is set to $(10^{-\mathrm{SNR}/20-1})^{\circ}$ which is about 1/10 of the CRLB [24]. For AI, the interpolation sector is chosen to cover all DOAs with a margin of 10°, and the number of the corresponding virtual ULA elements is set as 8. For the proposed method, the iteration termination threshold and the maximum number of iteration are set as $\zeta = 10^{-8}$, $I = 100$, respectively.

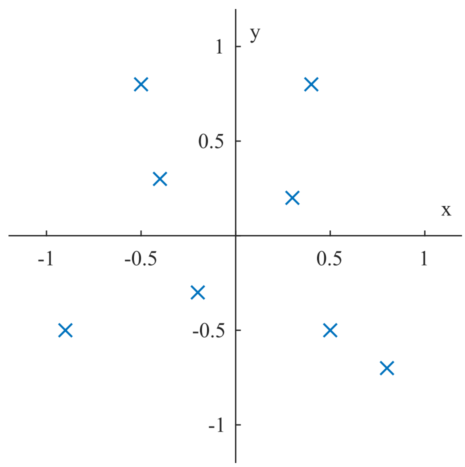

FIGURE 1. The arbitrary array geometry.

In the first simulation, we show the impact of the Fourier basis order Q on the performance of the proposed wideband DOA estimation and test the empirical criterion (34) for determining the suitable Q. To test the effect of the empirical criterion under general signal condition, we choose lenient simulation parameters, i.e. high SNR, large number of snapshots, and wide angle separation of sources. So we set SNR = 30 dB, $N = 100$ and two uncorrelated sources with the DOA being [83.2°, 110.7°]. Q is varied from 5 to 20, and the results are shown in Fig. 2. We can find small Q will lead

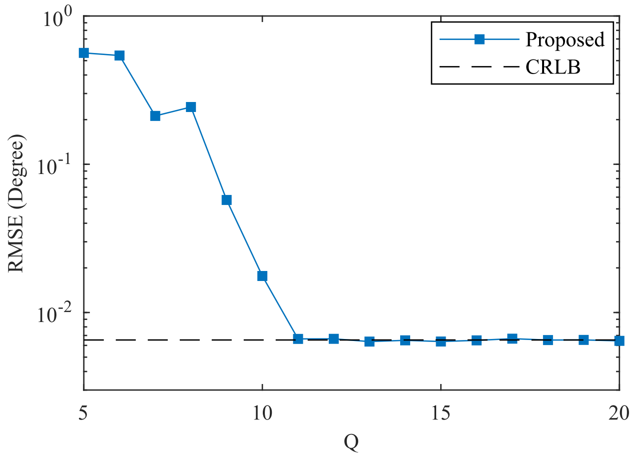

FIGURE 2. RMSE of wideband DOA estimation versus different Qs.

to inaccurate DOA estimation because of the non-negligible model approximation error, and too large Q is unnecessary because noise will bring an error floor for the DOA estimation exhibited by CRLB. The results in Fig. 2 indicates $Q = 11$ is appropriate, and the empirical criterion (34) suggests $Q = 13$ ($\rho_{max}/\lambda_h = 1.009$) for the array geometry. They are close, and empirical criterion (34) promises a more robust result. So, in the following simulations, $Q = 13$ will be used.

In the second simulation, we will test the DOA pre-estimation error on the performance of the proposed wideband DOA estimation, and compare it with the CSM methods including TCT and WAVES. To make problem simple, we only consider a single source whose DOA is 83.2°. The DOA pre-estimation error is set as a normal distributed random variable $\Delta\tilde{\theta} \sim \mathbb{N}(0, \sigma^2)$ and σ is varied from 0° to 24°. Other simulation parameters are set as SNR = 5dB, N = 20. The simulation results are shown in Fig. 3. It can be seen the proposed method is far less sensitive to the pre-estimation error than the CSM methods.

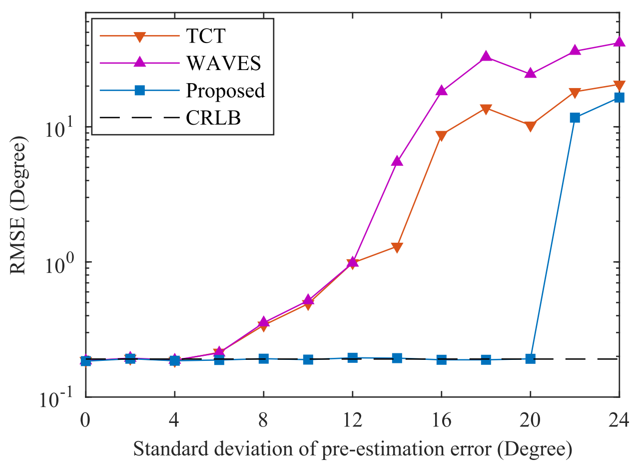

FIGURE 3. RMSE of wideband DOA estimation versus standard deviation of DOA pre-estimation error.

A pre-estimation error of $\sigma = 6°$ will make CSM methods deviate from the CRLB, while the correspond value for the proposed method is $\sigma = 22°$. Then, fixing $\sigma = 24°$, we place the probability distributions of final DOA estimation results in Fig. 4 where the red dash line indicates the true DOA. Fig. 4 show the impact of pre-estimation error on the proposed method is different from it on the CSM methods. The pre-estimation error makes the focusing matrices of the CSM method inaccurate, and brings model error, which will reduce the successful estimation probability of the DOA. However, for the proposed method, the pre-estimation error is only related to the initialization of the iteration, and only large enough pre-estimation error can occasionally make the iteration converges to a wrong point. So we see the DOA successful estimation probability of the proposed method is only very slightly affected under large pre-estimation error. Then, we also plot the convergence curve of the proposed method in each Monte Carlo experiments in Fig. 5 under the pre-estimation error of $\sigma = 0°$ and $\sigma = 16°$. The vertical axis in Fig. 5 is the indication of the convergence calculated from left side of (29). We can find the proposed method usually converges quite quickly (For iteration termination threshold $\zeta = 10^{-8}$, it takes less than 10 iterations to reach the convergence) and the large pre-estimation error only slightly increases the number of iterations.

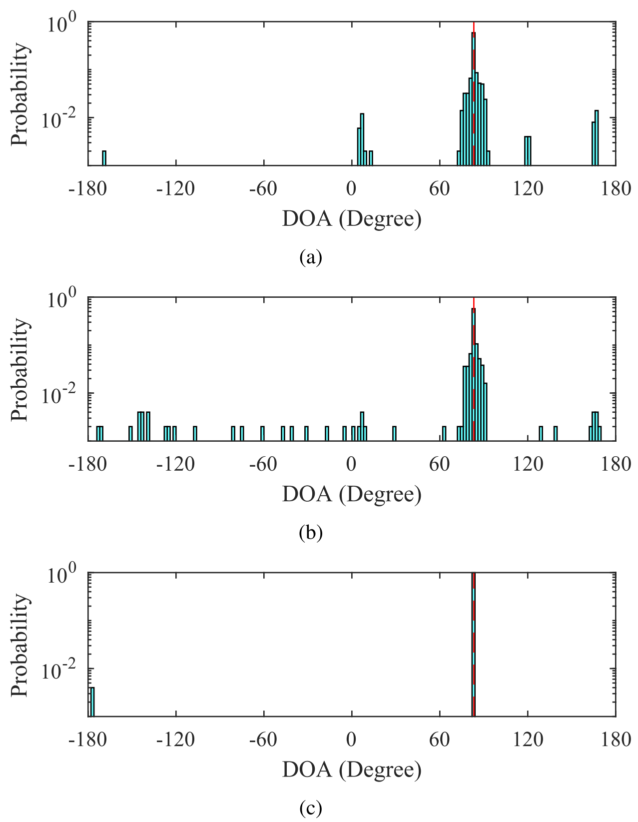

FIGURE 4. Probability distribution of DOA estimation results under $\sigma = 24°$. (a) TCT, (b) WAVES, (c) The Proposed method.

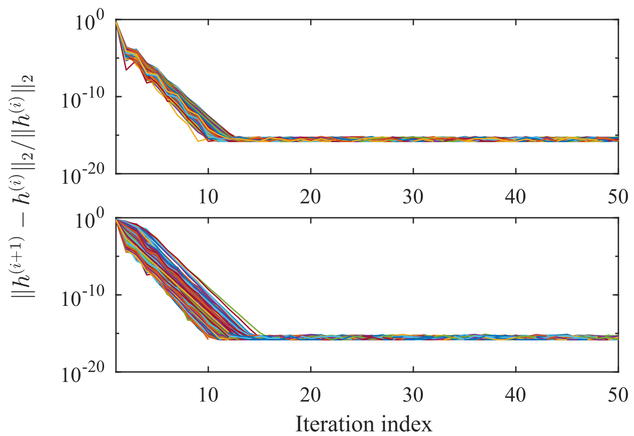

FIGURE 5. Convergence pattern of the proposed method, Top: $\sigma = 0°$, Bottom: $\sigma = 16°$.

In the third simulation, we test the wideband DOA estimation performance under different SNRs. Two uncorrelated sources from DOA $[83.2°, 93.7°]$ are considered and $N = 20$ snapshots are collected. The SNR is varied from -6 dB to 26 dB, and the simulation results are exhibited in Fig. 6. The results show that the proposed method owns the lowest SNR thresholds to achieve super-resolution, and its RMSE is smaller than all the other methods. TCT and WAVES can approximately arrive the CRLB with a higher SNR threshold. There exists an obvious error floor in MSTI and FRIDA. For MSTI it comes from the array sampling matrix mapping error, and for FRIDA it comes from the model residual error because of the covariance domain processing. The TOPS and AI have similar performance, and they only have acceptable performance under moderately high and high SNR.

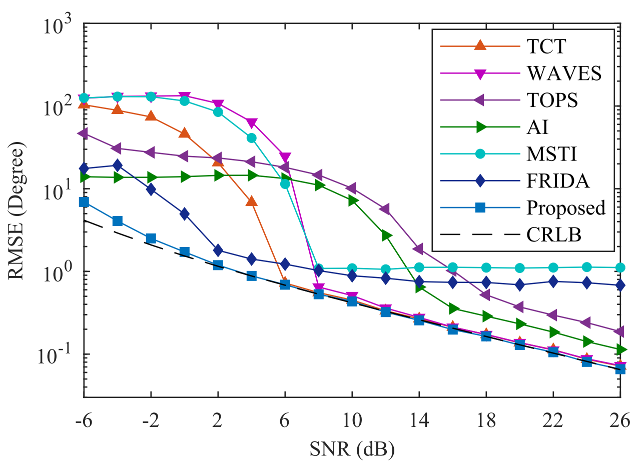

FIGURE 6. RMSE of wideband DOA estimation versus SNR.

In the fourth simulation, we test the wideband DOA estimation performance under different number of snapshots. The number of snapshots are varied from 1 to 70. The other simulation parameters are the same with the previous simulation except for fixing SNR = 5 dB. The simulation results are shown in Fig. 7. The results indicate that the proposed method requires only 4 snapshots to achieve super-resolution and has not very bad performance even under single snapshot scenario. TCT, WAVES, MSTI and FRIDA need 19, 34, 37 and 10 snapshots respectively to separate the two sources. Obvious biasedness still can be found in the results of MSTI and FRIDA. TOPS and AI are not able to separate the two sources in the considered scenarios.

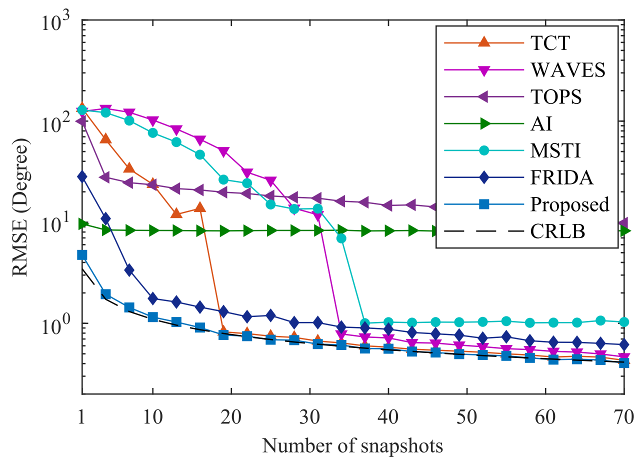

FIGURE 7. RMSE of wideband DOA estimation versus number of snapshots.

In the fifth simulation, we test the wideband DOA estimation performance under different angle separations between two sources. The DOA vector of the two sources is set as $[83.2°, 83.2° + \Delta\theta]$ and $\Delta\theta$ is varied from 3° to 26°. The other simulation parameters are the same with the previous simulation except for fixing SNR = 5 dB and $N = 20$. The results are shown in Fig. 8. Fig. 8 shows that the angle separation threshold of the proposed method to achieve super-resolution is 5°, which is much smaller than the 11° threshold of TCT, 13° threshold of WAVES and MSTI, and 8° threshold of FRIDA, respectively. An interesting phenomenon is that when the angle separation is larger than 15° the TCT and WAVES start to deviate from the CRLB. This is because this angle separation interval is nearly equal to the array beamwidth, which will sometimes bring large error in the DOA pre-estimation with CBF. As shown in Fig. 3, the WAVES is more sensitive to the pre-estimation error, so WAVE deviates more largely from CRLB in Fig. 8. AI are not able to separate the two sources in the considered scenarios and TOPS can only separate the sources with large angle separation.

In the last simulation, we test the wideband DOA estimation performance with correlated sources. Two sources from DOA $[83.2°, 93.7°]$ are assumed and SNR = 20 dB, $N = 20$. The correlation coefficient between signal is varied from 0.1 to 1 where correlation coefficient being 1 means two signals are coherent. The results are shown in Fig. 9. It indicates that TCT, WAVES, MSTI, AI and the proposed method are all nearly unaffected by the correlation

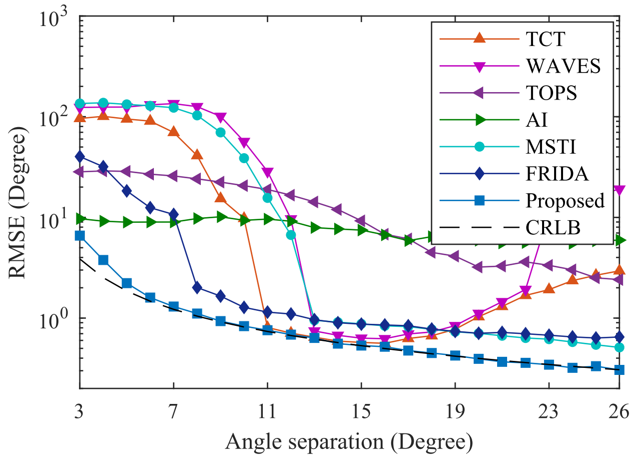

FIGURE 8. RMSE of wideband DOA estimation versus angle separation.

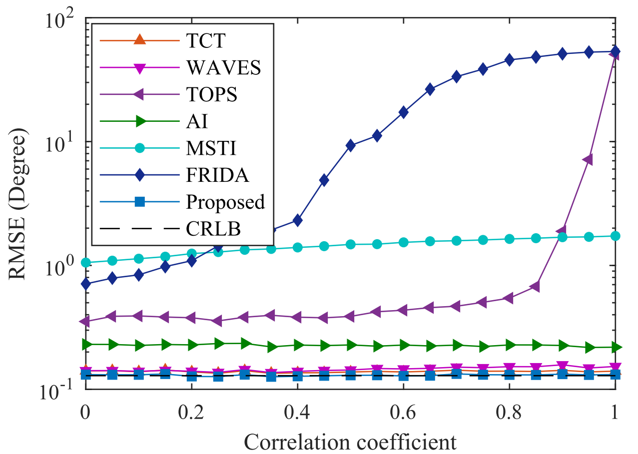

FIGURE 9. RMSE of wideband DOA estimation versus signal correlation coefficient.

between signals. For TCT, WAVES, MSTI and AI, this is beacuse of the frequency smoothing operation in them. For the proposed method, that is because the method is not based on the subspace type method and no assumptions are made about the correlation between signals. The TOPS and FRIDA are significantly affected by the correlation between signals. In addition, the proposed still owns the best estimation performance under the considered scenarios.

V. CONCLUSION

A new wideband DOA estimation method for arbitrary array application is proposed. Since the EISCEs synthesized from different frequencies via MST can be annihilated by the same spatial annihilating filter, the coherent annihilating scheme is proposed, together with the general solution method to enable the model approximation error in MST to be reduced to a negligible level. The MMV-STLN approach is used to iteratively reconstruct the annihilating filter under the STLS framework to obtain the DOAs. The new method is free of frequency focusing, capable of handling both incoherent and coherent signals, and has robust performance under the low SNR, snapshot deficient, and closely spaced sources scenarios.

APPENDIX
INITIALIZATION OF ν

According to (22), $\boldsymbol{\nu}$ is related to $\boldsymbol{\Sigma}_j, j = 1, 2, \ldots, J$. Adding (14) and (15) together, we have

$$\boldsymbol{\Sigma}_j = \boldsymbol{G}_j^{+}\boldsymbol{Y}_j' - \boldsymbol{X}_j'. \tag{35}$$

From (8), we know

$$\boldsymbol{X}_j' = \boldsymbol{D}(\tilde{\boldsymbol{\theta}}')\boldsymbol{S}_j' \tag{36}$$

where $\tilde{\boldsymbol{\theta}}'$ is the extended coarse pre-estimated DOA estimation obtained by CBF or Capon Beamformer and $\tilde{\boldsymbol{\theta}}'$ consists of $\{\tilde{\theta}_k - \theta_{beam}/4, \tilde{\theta}_k, \tilde{\theta}_k + \theta_{beam}/4\}_{k=1}^{\hat{K}}$ where θ_{beam} is the array beamwidth. Here we adopt the extended coarse DOA estimation to deal with the low resolution of the beamformer method [11]. According to (1), the least squares solution of $\boldsymbol{S}_j'$ can be written as

$$\boldsymbol{S}_j' = \boldsymbol{A}^{+}(\tilde{\boldsymbol{\theta}}')\boldsymbol{Y}_j'. \tag{37}$$

Substituting (36) and (37) into (35), we have

$$\boldsymbol{\Sigma}_j = \boldsymbol{G}_j^{+}\boldsymbol{Y}_j' - \boldsymbol{D}(\tilde{\boldsymbol{\theta}}')\boldsymbol{A}^{+}(\tilde{\boldsymbol{\theta}}')\boldsymbol{Y}_j'. \tag{38}$$

Then according to (22) and considering (17) and (33), we can write the initialization of $\boldsymbol{\nu}$ in the matrix form as (32).

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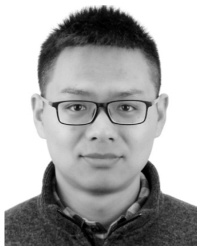

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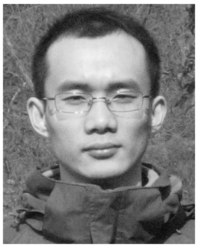

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