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Stability Analysis of Discrete-Time Switched T-S Fuzzy Systems With All Subsystems Unstable

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ABSTRACT In this paper, the stability problem of discrete-time switched nonlinear systems with all subsystems unstable is investigated. The nonlinear subsystems are represented by the Takagi–Sugeno (T-S) fuzzy models. By constructing a novel piecewise multiple Lyapunov function approach, an exponential stability condition of switched T-S fuzzy systems is first established with the bounded maximum average dwell time. A numerical example is finally provided to illustrate the effectiveness of the obtained theoretical results.

INDEX TERMS Discrete-time switched nonlinear systems, Takagi-Sugeno (T-S) fuzzy models, bounded maximum average dwell time (BMADT), piecewise multiple Lyapunov function (PMLF).

I. INTRODUCTION

The switched system is a typical hybrid system consisting of continuous-time or discrete-time subsystems and discrete switching events [1]. This class of systems has widely application backgrounds in information theory, flight control systems, power electronics and so on. Because of the numerous applications, the theory of the switched system has been developed rapidly in the past few decades. Stability problem is one of the research priorities of the switched system. In the early research work, the efforts are mainly focused on the switched systems composed entirely of stable subsystems [2]–[7]. In recent several years, some results have been achieved to deal with the case of switched systems with some unstable subsystems [8]–[14]. The main idea of these works [8]–[13] is to activate the stable subsystem long enough to compensate for the state divergence of the unstable subsystem. The paper [14] designed a quasi-alternative switching signal made the stable subsystems are not necessarily running sufficient long time. However, when all the subsystems are unstable, these methods are no longer applicable.

On the other hand, in real life, system nonlinearity exists widely in various fields such as robots, automotive, network application, and so on. So that, the stability problems and control issues for nonlinear systems have also been broadly researched [15]–[20]. In order to deal with nonlinear parts, the Takagi-Sugeno (T-S) fuzzy model has been introduced [21], which could approximate smooth nonlinear functions to any arbitrary accuracy. Besides, convex conditions of linear systems can be extended to nonlinear systems by utilizing the T-S fuzzy model. Recently, many results of switched nonlinear systems have been reached via T-S fuzzy model, which show that T-S fuzzy model provides a very efficient way to synthesize and analyze complex switched nonlinear systems. To list a few, the problem of asynchronous H_∞ control for switched nonlinear systems was addressed in [22], [23]. Robust stability problems and standard L_1-gain performance analysis of interval positive switched T-S fuzzy systems was investigated in [24]. The problems of stabilization for switched T-S fuzzy systems composed of unstable subsystems were studied in [25]–[28].

In practical application, subsystems of switched systems could be unstable due to sensor fails, equipment noise or actuator failure. To the worst case, all the subsystems may be unstable. Therefore, it has important theoretical significance and practical application value to study the switched system with all subsystems unstable. For this class of systems, the stability results can be achieved by carefully designing a state-dependent or time-dependent switching signal. The state-dependent switching strategies must depend on the the full plant state information [29], [30], which restricts the implementation. The time-dependent switching strategies are relatively easier for stability analysis. By using a discretized Lyapunov function technique, a sufficient condition

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for continuous-time switched linear systems was proposed in [31]. Switching-time-dependent time-varying discretized Lyapunov function was constructed to analyze the stability and stabilization problems for the switched linear stochastic systems in [32]. It's worth noting that the results of the aforementioned works are reached under dwell time switching. The time intervals between two adjacent switching instants are confined by a pair of upper and lower bounds. So these results may be restrictive in some circumstances. Therefore, it makes sense to extend the dwell time switching property to the average dwell time (ADT), which allows the switching activated outside the boundary. The paper [25] constructed a "decreasing-jump" piecewise Lyapunov-like function to analysis the finite-time exponential stabilization problem of switched T-S systems under ADT switching. The paper [26] studied the problems of stabilization for continuous-time switched nonlinear systems composed of unstable subsystems by using ADT switching. The paper [27] proposed a new mode-dependent average dwell time (MDADT) switching and time-scheduled multiple quadratic Lyapunov function to tackle the stabilization problems of continuous-time switched nonlinear systems. However, in order to obtain stability conditions, the time intervals of each switching are still limited by minimum dwell time boundaries in these works, which are somewhat conservative.

Similar to the idea of analyzing the switched system composed of stable and unstable subsystems, the paper [33] firstly proposed a new BMADT and divergence time concept to research the stability problem of switched linear systems. The definition of BMADT is more general than the traditional maximum average dwell time (MADT) and dwell time. The results under BMADT in [33] removed the minimum dwell time limitation relative to the ADT conditions in [25]–[27]. However, the switching frequency was limited by the compensation lower bound of BMADT. On the other hand, the results cannot apply to switched nonlinear systems directly as the stability conditions are nonconvex. To the best of our knowledge, the research on switched T-S fuzzy systems under BMADT condition has not been explored yet.

In this paper, we first defined a time span with fixed compensation bounds for each switching based on the BMADT method. If the dwell time belongs to this time span, the switching is considered to be "stable-switching", and if it is not, it is considered to be "unstable-switching". Besides, a novel piecewise multiple Lyapunov function (PMLF) was constructed with interpolation technique. In paper [25]–[27], the value of the Lyapunov function is decreasing at the switching instant and either increasing or decreasing during two successive switching instants. But the value of our Lyapunov function is always less than the value at last switching instant in the defined time span, which meets the exponential decay condition. So, the state divergence made by the "unstable-switching" will be compensated by the "stable-switching". Finally, stability conditions of the switched T-S systems under BMADT switching are derived in terms of linear matrix inequalities.

The main contributions are list as follows: (1) The BMADT approach is firstly applied in switched nonlinear systems for stability analysis. (2) New stability conditions of discrete-time switched T-S fuzzy systems with all subsystems unstable are derived by constructing a novel PMLF. (3) The stability conditions have removed the minimum dwell time limitation of ADT conditions and switching frequency limitation of BMADT conditions in [33]. (4) Larger MADT can be obtained as the upper compensation bound increasing, which can get less conservative results.

The rest of this paper is organized as follows. Section II gives the system descriptions and definition. The stability analysis and proofs are given in Section III. A numerical example is provided to demonstrate the feasibility and effectiveness of our results in Section IV. Section V gives the conclusions.

Notations: The notations used are fairly standard. $P > 0 (\geq 0)$ denotes a positive definite (semi-positive definite) matrix P. The notation $\|\cdot\|$ denotes the Euclidean norm for vectors. $\mathbb{R}^n$ represents the n-dimensional Euclidean space. $\mathbb{N}$ and $\mathbb{N}^+$ denote the set of non-negative and positive integers. The superscript "T" stands for matrix transpose.

II. SYSTEM DESCRIPTIONS AND PRELIMINARIES

Following class of discrete-time switched nonlinear system are considered in this paper:

$$x(k+1) = f_{\sigma(k)}(x(k), k), \ x(k_0) = x_0, \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the state vector, and x_0 and k_0 denote the initial state and initial time. $\sigma(k)$ is switching signal which takes values in the finite set $\mathcal{S} = \{1, 2, ..., M\}$, $M \in \mathbb{N}^+$ is the number of subsystems. $f_{\sigma(k)}(\cdot)$ are nonlinear function. For a switching sequence $k_0 < k_1 < \cdots < k_p < \cdots$, while k_p is the switching time, $\Delta_p = k_p - k_{p-1}$ is the corresponding dwell time of p-th switching.

In this paper, we describe the switched nonlinear system by T-S fuzzy model, and the i-th fuzzy subsystem is represented as follows:

Model Rule m for subsystem i: IF $z_{i1}(k)$ is M_{im1} and $\cdots$ and $z_{il}(k)$ is M_{iml}, THEN

$$x(k+1) = A_{im}x(k), \ i \in \mathcal{S}, \ m \in \mathcal{R} = \{1, 2, ..., r\}, \tag{2}$$

where $z_{i1}(k), z_{i1}(k), \cdots, z_{il}(k)$ are the premise variables, A_{ij} is a real matrix with appropriate dimension, M_{iml} is the fuzzy set and r is the number of model rules. Through fuzzy blending, the final outputs of the i-th fuzzy subsystem can be inferred as follows:

$$x(k+1) = \sum_{m=1}^{r} h_{im}(z_i(k)) A_{im} x(k), \tag{3}$$

where $h_{im}(z_i(k))$ are the normalized membership functions and

$$h_{im}(z_i(k)) = \frac{\prod_{n=1}^{l} M_{imn}(z_i(k))}{\sum_{m=1}^{r} \prod_{n=1}^{l} M_{imn}(z_i(k))} \geq 0, \tag{4}$$

$$\sum_{m=1}^{r} h_{im}(z_i(k)) = 1. \tag{5}$$

For convenience, we use h_{im} to represent $h_{im}(z_i(k))$ in the rest of the paper.

Definition 1: [9] *Given a switching signal $\sigma(k)$, the switched system (1) is said to be globally exponentially stable (GES) if there exist scalars $\lambda > 0, 0 < \mu < 1$, such that the solution of the system satisfies $\|x(k)\| \leq \lambda\mu^{k-k_0}\|x(k_0)\|, \ \forall k \geq k_0$ for any initial condition $x(k_0)$.*

Definition 2: Given a switching signal $\sigma(k)$ with the switching sequence $\{k_p\}$ and $\underline{k} \leq \overline{k} \in \mathbb{N}$, for any $k \in [k_p, k_{p+1}-1]$, we define divergence time $T_{\underline{k},\overline{k}}(k)$ as

$$T_{\underline{k},\overline{k}}(k) = \sum_{s=0}^{p-1} T^s_{\underline{k},\overline{k}}(k_{s+1}) + T^p_{\underline{k},\overline{k}}(k), \tag{6}$$

where

$$T^s_{\underline{k},\overline{k}}(k) = \begin{cases} k - k_s, & k < k_s + \underline{k}, \\ 0, & k_s + \underline{k} \leq k \leq k_s + \overline{k}, \\ k - k_s - \overline{k}, & k > k_s + \overline{k}. \end{cases} \tag{7}$$

$\underline{k}$ and $\overline{k}$ are the lower and the upper compensation bounds, respectively.

Remark 1: The divergence time describes the time span that outside the compensation bounds $[\underline{k}, \overline{k}]$. For each switching, the total divergence time will only increase when the dwell time is less than $\underline{k}$ or larger than $\overline{k}$.

Definition 3: [33] *For finite time $[k_0, k]$, denote $N(k)$ as the switching number, if there exist $\underline{k} \leq \overline{k} \in \mathbb{N}$ and positive constant $\tau_{\underline{k},\overline{k}}$ satisfies*

$$N(k) \geq \frac{k - k_0}{\tau_{\underline{k},\overline{k}} + \overline{k}} - N_0, \tag{8}$$

$$T_{\underline{k},\overline{k}}(k) \leq \frac{\tau_{\underline{k},\overline{k}} \cdot (k - k_0)}{\tau_{\underline{k},\overline{k}} + \overline{k}} + T_0, \tag{9}$$

then we said $\tau_{\underline{k},\overline{k}}$ is the bounded maximum average dwell time (BMADT) of the switching signal. N_0 and T_0 are corresponding slack variables.

Remark 2: The BMADT describes the relationship between divergence time and switching number. If we choose $T_{\underline{k},\overline{k}}(k_p) = 0, \ \forall p \in \mathbb{N}$, the definition of BMADT will be reduced to dwell time in paper [31], [32]. When $\underline{k} = \overline{k} = 0$, then total divergence time $T_{0,0}(k) = k - k_0$ is the total running time. Accordingly, $\tau_{0,0}$ is the MADT defined in paper [25], [26]. For the discrete-time switched system, if we set $\underline{k} = 1$, the MADT is $\tau_{MADT} = \tau_{1,\overline{k}} + \overline{k}$.

III. STABILITY ANALYSIS

In this section, we consider the stability problem for discrete-time switched T-S fuzzy systems with all subsystems unstable under BMADT switching signal.

Lemma 1: [34] *Given two matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times n}$, and symmetric positive definite matrix $P \in \mathbb{R}^{m\times m}$, then*

$$A^T PB + B^T PA \leq A^T PA + B^T PB. \tag{10}$$

Lemma 2: For the discrete-time switched nonlinear system (1). Given constants $\alpha > 1$, $0 < \beta < 1$ and $1 \leq \underline{k} \leq \overline{k} \in \mathbb{N}^+$, if there exist nonnegative function $V_{\sigma(k)}(x(k)) : \mathbb{R}^N \to \mathbb{R}, \forall\sigma(k) = i \in \mathcal{S}$ and two positive scalars γ_1 and γ_2 such that

$$\gamma_1(\|x(k)\|)^2 \leq V_i(x(k)) \leq \gamma_2(\|x(k)\|)^2, \tag{11}$$

$$V_i(x(k+1)) \leq \alpha V_i(x(k)), \ k \in [k_p, k_p + \underline{k} - 1] \cup [k_p + \overline{k}, k_{p+1} - 1], \tag{12}$$

$$V_i(x(k+1)) \leq V_i(x(k)), \ k \in [k_p + \underline{k}, k_p + \overline{k} - 1], \tag{13}$$

$$V_i(x(k_p + \underline{k})) \leq \frac{1}{\alpha^{\underline{k}-1}} V_i(x(k_p + \underline{k} - 1)), \tag{14}$$

$$V_{\sigma(k_p)}(x(k_p)) \leq \beta V_{\sigma(k_{p-1})}(x(k_p)), \tag{15}$$

then the switched nonlinear system (1) is GES for any switching signal with BMADT satisfies

$$\tau_{\underline{k},\overline{k}} < -\frac{\ln\beta}{\ln\alpha}. \tag{16}$$

Proof 1: Consider the situation that $k \in [k_p + \underline{k}, k_p + \overline{k}]$, we have

$$\begin{aligned} &V_{\sigma(k)}(x(k)) \\ &\quad \leq V_{\sigma(k)}(x(k-1)) \\ &\quad \cdots \\ &\quad \leq V_{\sigma(k)}(x(k_p + \underline{k})) \\ &\quad \leq \frac{1}{\alpha^{\underline{k}-1}} V_{\sigma(k)}(x(k_p + \underline{k} - 1)) \\ &\quad \leq \frac{1}{\alpha^{\underline{k}-1}} \cdot \alpha V_{\sigma(k)}(x(k_p + \underline{k} - 2)) \\ &\quad \cdots \\ &\quad \leq \frac{1}{\alpha^{\underline{k}-1}} \cdot \alpha^{\underline{k}-1} V_{\sigma(k)}(x(k_p)) \\ &\quad = V_{\sigma(k)}(x(k_p)) \\ &\quad \leq \beta V_{\sigma(k_{p-1})}(x(k_p)). \end{aligned} \tag{17}$$

Without loss of generality, we suppose $\forall k \in [k_p, k_{p+1} - 1]$, $p \in \mathbb{N}^+$, $k > k_p + \overline{k}$, $\Delta_p = k_p - k_{p-1} < \underline{k}$, we have

$$\begin{aligned} &V_{\sigma(k)}(x(k)) \\ &\quad \leq \alpha V_{\sigma(k_p)}(x(k-1)) \\ &\quad \cdots \\ &\quad \leq \alpha^{k-k_p-\overline{k}} V_{\sigma(k_p)}(x(k_p + \overline{k})) \\ &\quad \leq \beta\alpha^{T^p_{\underline{k},\overline{k}}(k)} V_{\sigma(k_{p-1})}(x(k_p)) \\ &\quad \leq \beta\alpha^{T^p_{\underline{k},\overline{k}}(k)} \cdot \alpha V_{\sigma(k_{p-1})}(x(k_p - 1)) \\ &\quad \cdots \\ &\quad \leq \beta\alpha^{T^p_{\underline{k},\overline{k}}(k) + k_p - k_{p-1}} V_{\sigma(k_{p-1})}(x(k_{p-1})) \\ &\quad \leq \beta^2\alpha^{T^p_{\underline{k},\overline{k}}(k) + T^{p-1}_{\underline{k},\overline{k}}(k_p)} V_{\sigma(k_{p-2})}(x(k_{p-1})) \\ &\quad \cdots \end{aligned}$$

$$
\begin{aligned}
&\leq \beta^{N(k)}\alpha^{T^{p}_{\underline{k},\overline{k}}(k)+\sum_{s=0}^{p-1}T^{s}_{\underline{k},\overline{k}}(k_{s+1})}V_{\sigma(k_0)}(x(k_0))\\
&\leq \beta^{N(k)}\alpha^{T_{\underline{k},\overline{k}}(k)}V_{\sigma(k_0)}(x(k_0))\\
&\leq \beta^{\frac{k-k_0}{\tau_{\underline{k},\overline{k}}+\overline{k}}-N_0}\alpha^{\frac{\tau_{\underline{k},\overline{k}}(k-k_0)}{\tau_{\underline{k},\overline{k}}+\overline{k}}+T_0}V_{\sigma(k_0)}(x(k_0))\\
&\leq \frac{\alpha^{T_0}}{\beta^{N_0}}(\beta\alpha^{\tau_{\underline{k},\overline{k}}})^{\frac{k-k_0}{\tau_{\underline{k},\overline{k}}+\overline{k}}}V_{\sigma(k_0)}(x(k_0)).
\end{aligned}
\tag{18}
$$

Let $\lambda = \frac{\gamma_2}{\gamma_1}\cdot\frac{\alpha^{T_0}}{\beta^{N_0}}$, $\mu = (\beta\alpha^{\tau_{\underline{k},\overline{k}}})^{\frac{1}{\tau_{\underline{k},\overline{k}}+\overline{k}}}$, *then with (11)(16) we have* $\lambda > 0$, $\mu = \exp\{\frac{\ln\beta+\tau_{\underline{k},\overline{k}}\ln\alpha}{\tau_{\underline{k},\overline{k}}+\overline{k}}\} < 1$,

$$
\begin{aligned}
\|x(k)\|^2 &\leq \frac{1}{\gamma_1}V_{\sigma(k)}(x(k))\\
&\leq \frac{1}{\gamma_2}\lambda\mu^{k-k_0}V_{\sigma(k_0)}(x(k_0))\\
&\leq \lambda\mu^{k-k_0}\|x(k_0)\|^2.
\end{aligned}
\tag{19}
$$

so $\|x(k)\| \leq \sqrt{\lambda}\sqrt{\mu}^{k-k_0}\|x(k_0)\|$ *satisfied Definition 1, the switched nonlinear system (1) is GES with BMADT satisfies (16). This completes the proof.*

For $(p+1)$-th switching, we define the time span $[k_p+\underline{k}, k_p+\overline{k}]$ with compensation bounds $[\underline{k},\overline{k}]$. When the corresponding dwell time belongs to the time span, the switching is considered as the "stable-switching", otherwise it is the "unstable-switching". From the proof process (17) it can be seen that the value of Lyapunov function decay exponentially when the switching occurred in the "stable-switching" time span. In the divergence time span, the value of Lyapunov function can either increase or decay. Therefore, the state divergence can be absorbed by the "stable-switching".

Remark 3: The existing stability conditions of switched systems with all subsystems unstable under ADT switching are usually limited by a minimum dwell time condition [25]–[27]. The dwell time of each switching should not be less than a lower bound τ_{min}. *The proposed BMADT approach eliminates this limitation.*

Remark 4: In paper [33], the switching signal must satisfying the condition $N_{\underline{k}}(k) \leq \epsilon N(k)$. ϵ *is a constant belong to the span [0,1).* $N_{\underline{k}}(k)$ *is the switching number when dwell time is less than* $\underline{k}$. *The constructed Lyapunov function for switched nonlinear system in this paper eliminates this restriction.*

Then, the stability result of switched T-S fuzzy system (3) can be given in following theorem.

Theorem 1: Consider switched T-S fuzzy system (3). Given constants $\alpha > 1, 0 < \beta < 1, 1 \leq \underline{k} \leq \overline{k} \in \mathbb{N}^+$, *and* $m \in \mathcal{R}$, *if there exist matrices* $P_{i,f} > 0, f \in [0,\ldots,\overline{k}], \forall(i,j)\in\mathcal{S}\times\mathcal{S}, i\neq j$, *such that*

$$
\begin{bmatrix} -\alpha P_{i,f} & A_{im}^T P_{i,f+1}\\ P_{i,f+1}A_{im} & -P_{i,f+1}\end{bmatrix} < 0,\ f\in[0,\ldots,\underline{k}-2], \tag{20}
$$

$$
\begin{bmatrix} -P_{i,f} & A_{im}^T P_{i,f+1}\\ P_{i,f+1}A_{im} & -P_{i,f+1}\end{bmatrix} < 0,\ f\in[\underline{k},\ldots,\overline{k}-1], \tag{21}
$$

$$
\begin{bmatrix} -\alpha P_{i,\overline{k}} & A_{im}^T P_{i,\overline{k}}\\ P_{i,\overline{k}}A_{im} & -P_{i,\overline{k}}\end{bmatrix} < 0, \tag{22}
$$

$$
\begin{bmatrix} -\alpha^{1-\underline{k}} P_{i,\underline{k}-1} & A_{im}^T P_{i,\underline{k}}\\ P_{i,\underline{k}}A_{im} & -P_{i,\underline{k}}\end{bmatrix} < 0, \tag{23}
$$

$$
P_{i,0}-\beta P_{j,f} \leq 0,\ f\in[1,\ldots,\overline{k}], \tag{24}
$$

then the switched T-S fuzzy system (3) is GES for any switching signal with BMADT satisfies

$$
\tau_{\underline{k},\overline{k}} < -\frac{\ln\beta}{\ln\alpha}. \tag{25}
$$

Proof 2: Constructing a piecewise multiple Lyapunov function for discrete-time switched T-S fuzzy system (3) as follows:

$$
V_i(k) = x^T(k)P_i(k)x(k),\ \forall i\in\mathcal{S}, \tag{26}
$$

where

$$
P_i(k) = \begin{cases} P_{i,f},\ f=k-k_p,\ k\in[k_p,k_p+\overline{k}],\\ P_{i,\overline{k}},\ k\in[k_p+\overline{k}+1,k_{p+1}-1].\end{cases} \tag{27}
$$

When $k\in[k_p,k_p+\underline{k}-1]$, *with lemma (10), we can obtain*

$$
\begin{aligned}
&V_i(x(k+1))-\alpha V_i(x(k))\\
&\leq x(k+1)^T P_{i,f+1}x(k+1)-\alpha x(k)^T P_{i,f}x(k)\\
&\leq x(k)^T\{\sum_{m=1}^{r}\sum_{n=1}^{r}h_{im}h_{in}[A_{im}^T P_{i,f+1}A_{in}-\alpha P_{i,f}]\}x(k)\\
&\leq x(k)^T\{\sum_{m=1}^{r}h_{im}^2[A_{im}^T P_{i,f+1}A_{im}-\alpha P_{i,f}]\\
&\quad+\sum_{m=1}^{r}\sum_{n>m}^{r}h_{im}h_{in}[A_{im}^T P_{i,f+1}A_{in}+A_{in}^T P_{i,f+1}A_{im}\\
&\quad-2\alpha P_{i,f}]\}x(k)\\
&\leq x(k)^T\{\sum_{m=1}^{r}h_{im}^2[A_{im}^T P_{i,f+1}A_{im}-\alpha P_{i,f}]\\
&\quad+\sum_{m=1}^{r}\sum_{n>m}^{r}h_{im}h_{in}[A_{im}^T P_{i,f+1}A_{im}+A_{in}^T P_{i,f+1}A_{in}\\
&\quad-2\alpha P_{i,f}]\}x(k).
\end{aligned}
\tag{28}
$$

By using Schur complement lemma, with (20) we have

$$
A_{im}^T P_{i,f+1}A_{im}-\alpha P_{i,f} \leq 0,\ f\in[0,\ldots,\underline{k}-2]. \tag{29}
$$

With (28) and (29), we can conclude that $V_i(x(k+1))-\alpha V_i(x(k)) \leq 0$ *hold, which satisfied the condition (12). Similar to above proof process, we can proof the conditions (12), (13), (14) and (15) of Lemma 2 hold.*

By using (24), let $\sigma(k_p)=i$, $\sigma(k_{p-1})=j$ *we can proof*

$$
\begin{aligned}
&V_{\sigma(k_p)}(x(k_p))-\beta V_{\sigma(k_{p-1})}(x(k_p))\\
&= x(k_p)^T[P_{i,0}-\beta P_{j,f}]x(k_p)\\
&\leq 0.
\end{aligned}
\tag{30}
$$

Therefore, the switched T-S fuzzy system (3) is GES for any switching signal with BMADT satisfies (25) according to Lemma 2. This completes the proof.

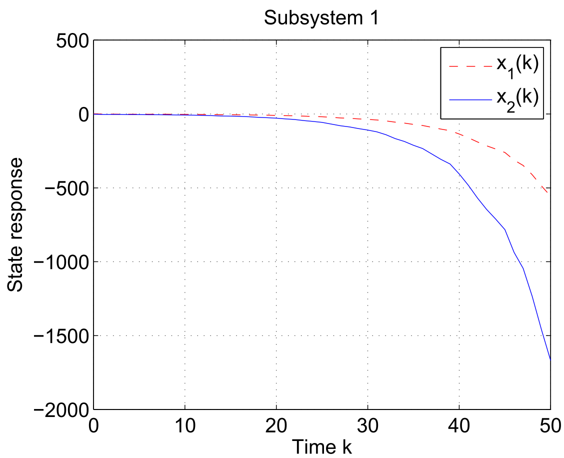

FIGURE 1. States of subsystem Ω_1.

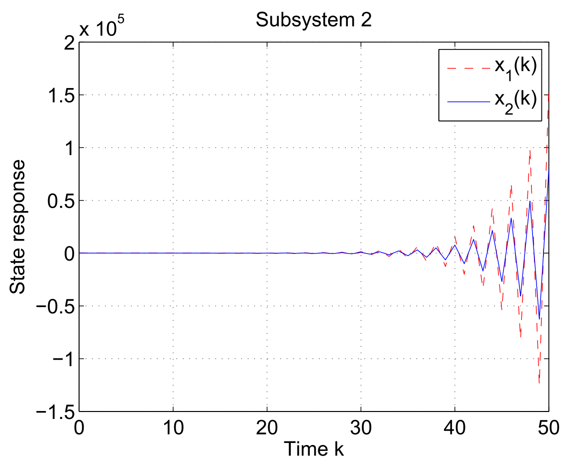

FIGURE 2. States of subsystem Ω_2.

IV. NUMERICAL EXAMPLE

In this section, an example is given to illustrate the main results. Consider the following switched nonlinear systems consisting of two subsystems

$$\Omega_1 = \begin{cases} x_1(k+1) = -0.48x_1(k) - 0.08\sin^2(x_1(k))x_2(k) \\ \qquad + 0.56x_2(k) + 0.1\sin^2(x_1(k))x_1(k) \\ x_2(k+1) = -0.84x_1(k) - 0.14\sin^2(x_1(k))x_2(k) \\ \qquad + 1.48x_2(k) + 0.12\sin^2(x_1(k))x_1(k) \end{cases}$$

$$\Omega_2 = \begin{cases} x_1(k+1) = -1.58x_1(k) - 0.04\sin^2(x_2(k))x_2(k) \\ \qquad + 0.56x_2(k) + 0.12\sin^2(x_2(k))x_1(k) \\ x_2(k+1) = -0.84x_1(k) - 0.02\sin^2(x_2(k))x_2(k) \\ \qquad + 0.38x_2(k) + 0.06\sin^2(x_2(k))x_1(k). \end{cases}$$

The state trajectories of subsystems Ω_1 shown in Fig. 1 and subsystems Ω_2 shown in Fig. 2 with the initial state condition $x(0) = [1, -1.5]^T$, and from the figures it can be seen that both the subsystems are unstable.

Setting $z_1(k) = \sin^2(x_1(k)), z_2(k) = \sin^2(x_2(k))$, and through the T-S fuzzy modeling method, we can obtain the fuzzy model as follow:

Subsystem 1:
Rule 1: If $z_1(k)$ is 0, then $x(k+1) = A_{11}x(k)$,
Rule 2: If $z_1(k)$ is 1, then $x(k+1) = A_{12}x(k)$.
Subsystem 2:
Rule 1: If $z_2(k)$ is 0, then $x(k+1) = A_{21}x(k)$,
Rule 2: If $z_2(k)$ is 1, then $x(k+1) = A_{22}x(k)$.

The normalized membership functions are calculated as follows:

$$h_{11}(k) = 1 - \sin^2(x_1(k)),\ h_{12}(k) = \sin^2(x_1(k)),$$
$$h_{21}(k) = 1 - \sin^2(x_2(k)),\ h_{22}(k) = \sin^2(x_2(k)).$$

and

$$A_{11} = \begin{bmatrix} -0.48 & 0.56 \\ -0.84 & 1.48 \end{bmatrix},\quad A_{12} = \begin{bmatrix} -0.38 & 0.48 \\ -0.72 & 1.34 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} -1.58 & 0.56 \\ -0.84 & 0.38 \end{bmatrix},\quad A_{22} = \begin{bmatrix} -1.46 & 0.52 \\ -0.78 & 0.36 \end{bmatrix}.$$

Through Theorem 1 we choose $\alpha = 1.69, \beta = 0.75, \underline{k} = 3, \overline{k} = 6$. By uisng the Matlab LMI toolbox, we can get feasible solution as follows:

$$P_{1,0} = \begin{bmatrix} 3.42 & -6.08 \\ -6.08 & 11.91 \end{bmatrix},\quad P_{1,1} = \begin{bmatrix} 22.09 & -12.85 \\ -12.85 & 15.31 \end{bmatrix},$$
$$P_{1,2} = \begin{bmatrix} 27.33 & -13.18 \\ -13.18 & 16.07 \end{bmatrix},\quad P_{1,3} = \begin{bmatrix} 23.17 & -8.79 \\ -8.79 & 5.63 \end{bmatrix},$$
$$P_{1,4} = \begin{bmatrix} 22.98 & -8.16 \\ -8.16 & 4.42 \end{bmatrix},\quad P_{1,5} = \begin{bmatrix} 22.88 & -7.90 \\ -7.90 & 3.70 \end{bmatrix},$$
$$P_{1,6} = \begin{bmatrix} 21.30 & -7.29 \\ -7.29 & 3.08 \end{bmatrix},\quad P_{2,0} = \begin{bmatrix} 14.20 & -4.92 \\ -4.92 & 2.01 \end{bmatrix},$$
$$P_{2,1} = \begin{bmatrix} 18.42 & -13.57 \\ -13.57 & 19.64 \end{bmatrix},\quad P_{2,2} = \begin{bmatrix} 19.25 & -15.24 \\ -15.24 & 22.93 \end{bmatrix},$$
$$P_{2,3} = \begin{bmatrix} 7.72 & -10.64 \\ -10.64 & 19.72 \end{bmatrix},\quad P_{2,4} = \begin{bmatrix} 6.55 & -10.27 \\ -10.27 & 19.62 \end{bmatrix},$$
$$P_{2,5} = \begin{bmatrix} 5.86 & -10.04 \\ -10.04 & 19.53 \end{bmatrix},\quad P_{2,6} = \begin{bmatrix} 5.23 & -9.47 \\ -9.47 & 18.61 \end{bmatrix}.$$

The divergence time $T_{\underline{k},\overline{k}}(k)$ and state responses of the switched nonlinear system are shown in Fig. 3. It can be seen from Fig. 3 that the total divergence time only increases when the corresponding dwell time is less than $\underline{k}$ or larger than $\overline{k}$. There is also no boundary limit or minimum dwell time limit of dwell time for each switching. The system state can finally converge to zero under the BMADT switching signal which satisfies $\tau_{3,6} = 0.4286 < -\frac{\ln\beta}{\ln\alpha} = 0.5482$. These verify the results in Theorem 1.

For the traditional MADT switching in paper [25], [26], which without the compensation bounds, the MADT of discrete-time switched systems can be regarded as $\tau_{1,1}$. According to Remark 2, if we choose $\underline{k} = 1$, the corresponding MADT is $\tau_{MADT} = \tau_{1,\overline{k}} + \overline{k}$. Through Theorem (1), we can get the largest $\tau_{1,\overline{k}}$ and τ_{MADT} under different $\overline{k}$. The result is shown in Table 1 with fixed $\alpha = 0.69$. From the Table 1 it can be seen that τ_{MADT} increases with the increasing of upper compensation bound $\overline{k}$, i.e. larger MADT is obtained, thus less conservative results can be achieved.

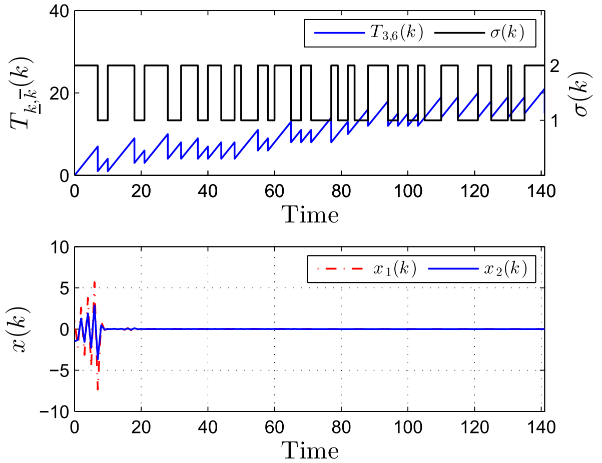

FIGURE 3. Divergence time and state trajectories.

TABLE 1. The largest MADT under different $\overline{k}$ ($\alpha = 0.69$).

$\overline{k}$	1	2	3	4	5	6
β	0.28	0.34	0.44	0.62	0.77	0.97
$\tau_{1,\overline{k}}$	2.4620	2.0559	1.5646	0.9110	0.4981	0.0580
τ_{MADT}	3.4620	4.0559	4.5646	4.9110	5.4981	6.0580

V. CONCLUSION

The stability problems of discrete-time switched nonlinear systems with all subsystems unstable are studied in this paper. The BMADT switching method is first time extent in switched nonlinear systems for stability analysis. Combining with a novel PMLF and T-S fuzzy modeling approach, the less conservative exponential stability results are derived in terms of linear matrix inequalities. Finally, an example is provided to verify the effectiveness of the results. Since the mode-dependent switching rule is more general and flexible than the mode-independent. We will consider to extend the BMADT to a mode-dependent one to further reduce conservatism. The stabilization conditions and H_∞ control problems will also be investigated in our future works.

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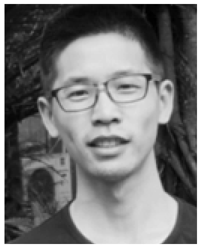

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