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Finite-Time Trajectory Tracking Control for Overhead Crane Systems Subject to Unknown Disturbances

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ABSTRACT In this paper, a finite-time trajectory tracking control method for overhead crane systems with unknown disturbances is proposed. The controller is designed in the framework of observer-based control design, therefore, it works well even in the presence of internal and external disturbances. Prior knowledge of the system parameters, including the payload mass, the trolley mass, the cable length, and the friction-related coefficients, is not required for the designed controller. Moreover, the proposed controller can prove the state convergence on the sliding surface without any approximations to the original dynamic equations. More precisely, to estimate unknown disturbances, a terminal sliding mode observer is designed. And then, a finite-time trajectory tracking controller is synthesized by using the estimated information. As shown by Lyapunov techniques, the designed controller guarantees the finite-time tracking result. The simulation results are provided to illustrate the superior control performance and strong robustness of the proposed finite-time trajectory tracking control method.

INDEX TERMS Underactuated overhead crane, terminal sliding mode observer, Lyapunov techniques, finite time, trajectory tracking control, robustness.

I. INTRODUCTION

Cranes are machines that are utilized to transport heavy payloads or hazardous materials from one place to another. These actions commonly take place in industries such as factories, construction, marine industries and harbors. Based on the degrees of freedom that the support mechanism offers at the suspension point, cranes can be divided into overhead or bridge cranes, gantry cranes, boom cranes, and rotary (tower) cranes [1]–[3]. Regardless of different types of cranes, the underactuated characteristic is the fundamental nature of cranes. Underactuated systems have fewer actuators than degrees of freedom; therefore, they have the merits of lower costs, simpler structure than fully-actuated systems [4]–[7]. As a result, control of crane systems is becoming an interesting research field. Overhead cranes are the most widely used ones in all types of cranes, and many approaches have been proposed [8]–[27].

Until now, the control problem of overhead cranes is still a fairly open topic. On one hand, overhead cranes always suffer from unknown disturbances induced by uncertain trolley mass, uncertain payload mass, uncertain cable length, uncertain friction, and external disturbances [28]–[31]. Those uncertainties are difficult to be predicted in advance, leading to severe control performance (payload swing suppression effects and trolley positioning efficiency) degradation. Therefore, crane control methods should consider unknown disturbances. It is very challenging for the control of general overhead crane systems.

Sliding-mode control and adaptive control methods seem to be effective to tackle unknown disturbances [32]–[34]. More precisely, these traditional first-order SMC methods [35]–[38] have already been introduced to solve the problems of positioning and anti-sway, obtaining good control results. However, traditional first-order SMC methods have discontinuous variable structures, which might bring potential damage to the actuating devices and suffer from undesirable chattering effects. To avoid chattering, second-order [39], [40] and high-order [41] SMC methods have

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been provided. Nevertheless, they are only applicable to systems with relative degree being not more than 2, and can only deal with matched disturbances. Therefore, many disturbance observer-based SMC methods are proposed to reject the high-order mismatched disturbance [42]–[44]. In addition to these SMC controllers, several adaptive controllers [24], [25], [35], [45]–[48] are proposed to overhead crane systems. The control methods above can merely provide at best asymptotic stability, which is insufficient for high precision transportation tasks. Moreover, the overhead crane is known to be differentially flat [49]. Therefore, by using differential-flatness technique, Zhang *et al.* [50] achieve the simultaneous motion regulation and payload swing suppression and elimination within finite time. However, the abovementioned control methods assume that the unknown disturbances satisfy the "linearity-in-parameters" condition.

Inspired by [51], to solve the existing problems, we propose a finite-time trajectory tracking controller based on a terminal sliding mode observer. In particular, an observer is used to estimate the unknown disturbances. And then, by using the estimated information, the finite-time trajectory tracking controller is proposed. The stability and convergence of the closed-loop system are proven by Lyapunov techniques. Some simulation results are used to illustrate the superior performance of the proposed finite-time trajectory tracking controller.

The main contribution of this paper can be concluded as follows:

1) For underactuated overhead cranes, this paper derives the *first SMC* method which can achieve finite-time convergence.
2) The designed controller does *not*need the exact values of the system parameters (e.g., trolley/payload masses, cable length, friction parameters) thanks to the observer for unknown disturbances; it only needs the nominal values.
3) In comparison with most existing control methods, the proposed controller shows superior control performance, especially swing elimination.

This paper is organized in the following manner. In Section 2, overhead crane model is briefly introduced. In Section 3, a terminal sliding mode observer and a finite-time trajectory tracking controller without payload-swing feedback are designed. Simulation results are exhibited in Section 4. Section 5 gives some concluding remarks.

II. OVERHEAD CRANE DYNAMICS

An underactuated overhead crane system shown in Figure 1 is considered, whose dynamics are described as follows [4], [12], [19], [22]:

$$\left(M+m_p\right)\ddot{x}+m_pl\ddot{\theta}\cos\theta-m_pl\dot{\theta}^2\sin\theta+f_{rx}+h=F, \tag{1}$$

$$m_pl\ddot{x}\cos\theta+m_pl^2\ddot{\theta}+m_pgl\sin\theta=0, \tag{2}$$

where M denotes the trolley mass, m_p is the payload mass, l and g represent the cable length and the gravitational constant, respectively, h is the unknown disturbances, $x(t)$ and $\theta(t)$ are the trolley displacement and the payload swing, respectively, F denotes the control force imposed on the trolley, and f_{rx} stands for the friction.

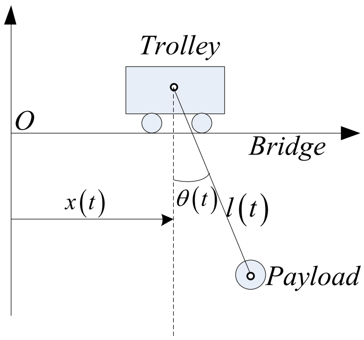

FIGURE 1. Schematic illustration of an overhead crane.

Remark 1: The unknown disturbances h are induced by the uncertain trolley mass ΔM, uncertain payload mass Δm_p, uncertain cable length Δl, uncertain friction Δf_{rx}, external disturbances d. M, m_p, l, f_{rx} in this paper stand for the nominal values of the trolley mass, the payload mass, the cable length, the friction, respectively.

The following assumption is made to facilitate the tracking control law design and stability analysis.

Assumption 1: The unknown disturbances h are described by a differentiable function with time derivative u. Although the input u is unknown, the amplitude of the signal u is bounded by a known constant $\rho \in \mathbf{R}^+$, i.e. $|u| \leq \rho$.

III. MAIN RESULTS

A. TERMINAL SLIDING MODE OBSERVER DESIGN FOR UNKNOWN DISTURBANCES h

To guarantee high-performance control, the unknown disturbances h in the overhead crane dynamics should be identified firstly and then compensated [52]. In this subsection, a terminal sliding mode observer is designed for unknown disturbances h.

Define an auxiliary function $Q=\left(M+m_p\right)\dot{x}$. Taking the time derivative of Q, we are led to the following results:

$$\dot{Q}=F-f_{rx}+m_pl\dot{\theta}^2\sin\theta-m_pl\ddot{\theta}\cos\theta-h. \tag{3}$$

Let us introduce an auxiliary function E as $f_{rx}-m_pl\dot{\theta}^2\sin\theta+m_pl\ddot{\theta}\cos\theta$, then, (3) can be written as

$$\dot{Q}=F-E-h, \tag{4}$$

as $t\geq T_0$.

To facilitate the following observer design, a new state $\chi(t)$ is constructed accordingly as follows:

$$\chi\left(t\right)=k_{02}\int_{T_0}^{t}\left(F-E-\chi\left(\tau\right)\right)d\tau-k_{02}Q, \tag{5}$$

where $k_{02}\in\mathbf{R}^+$ stands for a positive observer gain.

After taking the time derivative (5), it can be calculated as

$$\dot{\chi}\left(t\right)=-k_{02}\chi\left(t\right)+k_{02}h. \tag{6}$$

Then, the problem of estimating the unknown disturbances h can be formulated as that of estimating the state of a linear system driven by $\chi(t)$ and an unknown input, where $\chi(t)$ is measurable/calculable. Introduce two variables γ_1 and γ_2 as $\gamma_1 = \chi(t)$ and $\gamma_2 = h$, respectively, along with Assumption 1, (6) can be written as

$$\dot{\gamma}_1 = -k_{02}\gamma_1 + k_{02}\gamma_2, \tag{7}$$

$$\dot{\gamma}_2 = u. \tag{8}$$

To estimate the unknown disturbances h/γ_2, the following terminal sliding mode observer is defined:

$$\begin{aligned}\dot{\hat{\gamma}}_1 &= -k_{02}\hat{\gamma}_1 + k_{02}\hat{\gamma}_2 - \gamma_v - l_2 e_1\\ &= -k_{02}\hat{\gamma}_1 + k_{02}\hat{\gamma}_2 - l_1 sgn(e_1) - l_2 e_1,\end{aligned} \tag{9}$$

$$\begin{aligned}\dot{\hat{\gamma}}_2 &= -l_3 e_1 - l_4 \lfloor \gamma_v \rceil^{\frac{p_2}{q_2}} - l_5 sgn(\gamma_v)\\ &= -l_3 e_1 - l_4 l_1^{\frac{p_2}{q_2}} sgn(e_1) - l_5 sgn(e_1)\end{aligned} \tag{10}$$

with $\hat{\gamma}_1$ and $\hat{\gamma}_2$ being the estimates of γ_1 and γ_2, respectively, $l_1, l_2, l_3, l_4, l_5 \in \mathbf{R}^+$ representing positive observer gains, $q_2, p_2 \in \mathbf{R}^+$ denoting odd numbers with $q_2 > p_2$, $\gamma_v = l_1 sgn(e_1)$, $e_1 = \hat{\gamma}_1 - \gamma_1$, $e_2 = \hat{\gamma}_2 - \gamma_2$, $\lfloor \gamma_v \rceil^{\frac{p_2}{q_2}} = |\gamma_v|^{\frac{p_2}{q_2}} sgn(\gamma_v)$.

The observer error vector $\boldsymbol{e}$ is defined as $\boldsymbol{e} = [e_1 e_2]^T$. Then, from (7)-(10), the dynamics of the observer error vector $\boldsymbol{e}$ are obtained as

$$\begin{aligned}\dot{e}_1 &= -k_{02}e_1 + k_{02}e_2 - \gamma_v - l_2 e_1\\ &= -k_{02}e_1 + k_{02}e_2 - l_1 sgn(e_1) - l_2 e_1,\end{aligned} \tag{11}$$

$$\begin{aligned}\dot{e}_2 &= -l_3 e_1 - l_4 \lfloor \gamma_v \rceil^{\frac{p_2}{q_2}} - l_5 sgn(\gamma_v) - u\\ &= -l_3 e_1 - l_4 l_1^{\frac{p_2}{q_2}} sgn(e_1) - l_5 sgn(e_1) - u.\end{aligned} \tag{12}$$

Lemma 1: Under the developed terminal sliding mode observer (9) and (10), the observer error vector $\boldsymbol{e}$ in the observer error system (11) and (12) is uniformly ultimately bounded.

Assumption 2: During this process, we assume that the estimates of γ_1 and γ_2 as are chosen as $\hat{\gamma}_1(0) = \gamma_1(0)$ and $\hat{\gamma}_2(0) = \gamma_2(0)$, respectively.

Proof: Consider the following positive definite function as:

$$V_{O2} = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2. \tag{13}$$

Taking the time derivative of (13), along with the observer error vector $\boldsymbol{e}$ dynamics (11) and (12) yields

$$\begin{aligned}\dot{V}_{O2}(t) &= -k_{02}e_1^2 + k_{02}e_1e_2 - l_2 e_1^2 - l_3 e_1 e_2 - e_1\gamma_v\\ &\quad - l_4 e_2 \lfloor \gamma_v \rceil^{\frac{p_2}{q_2}} - l_5 e_2 sgn(\gamma_v) - e_2 u\\ &\leq -\boldsymbol{e}^T\boldsymbol{\beta}\boldsymbol{e} - l_1|e_1| + \left(l_4 l_1^{\frac{p_2}{q_2}} + l_5 + \rho\right)\|\boldsymbol{e}\|,\end{aligned} \tag{14}$$

where

$$\boldsymbol{\beta} = \begin{bmatrix} k_{02}+l_2 & -k_{02}\\ l_3 & 0\end{bmatrix}.$$

Due to the fact that k_{02}, l_2, and l_3 are positive, it can be obtained that $\boldsymbol{\beta}$ is positive definite. Therefore, the minimum eigenvalue of $\boldsymbol{\beta}$ denoted by $\lambda_{\min}$ is positive.

Consequently, the following results can be obtained:

$$\begin{aligned}\dot{V}_{O2}(t) &\leq -\lambda_{\min}\|\boldsymbol{e}\|^2 + \left(l_4 l_1^{\frac{p_2}{q_2}} + l_5 + \rho\right)\|\boldsymbol{e}\|\\ &= -\|\boldsymbol{e}\|\left[\lambda_{\min}\|\boldsymbol{e}\| - \left(l_4 l_1^{\frac{p_2}{q_2}} + l_5 + \rho\right)\right].\end{aligned} \tag{15}$$

To guarantee $\dot{V}_{O2} < 0$ for $\|\boldsymbol{e}\| \neq 0$, the following terms in (15) should be positive, i.e.:

$$\|\boldsymbol{e}\| > \frac{l_4 l_1^{\frac{p_2}{q_2}} + l_5 + \rho}{\lambda_{\min}} = \xi. \tag{16}$$

In other words, $\dot{V}_{O2}$ is negative for $\|\boldsymbol{e}\| \neq 0$ when $\boldsymbol{e}$ is outside the compact set $D = \{\boldsymbol{e} : \|\boldsymbol{e}\| \leq \xi\}$. It is implied that V_{O2} decreases monotonically. Obviously, the decrease of V_{O2} eventually drives $\boldsymbol{e}$ into the set D, and then it will always confine in D. That just means the set D is attractive. Under Assumption 2, $\|\boldsymbol{e}(T_0)\| = 0$ can be obtained. Then, following the Lyapunov theory and the LaSalle extension [53], all the states as $t \geq T_0$ are confined in D, which also proves the uniformly ultimately boundedness of $\boldsymbol{e}$.

Theorem 1: Consider the observer error dynamics in (11) and (12) obtained from the linear system (7) and (8) and the terminal sliding mode observer in (9) and (10). Choose the observer gains l_1, l_2, l_3, l_4, and l_5 such that

$$l_1 \geq \max\left\{(k_{02}l_5 + k_{02}\rho + \lambda_{\min}\varepsilon_0)^{\frac{p_2}{q_2}}, \left(\frac{1+k_{02}l_4}{\lambda_{\min}}\right)^{\frac{q_2}{q_2-p_2}}\right\}, \tag{17}$$

$$l_5 - \rho > 0. \tag{18}$$

where $\varepsilon_0 \in \mathbf{R}^+$ denotes a positive constant. Then, the unknown disturbances h are exactly estimated by $\hat{\gamma}_2(t)$ within finite time.

Proof: Two parts are included to prove Theorem 1.

1) Finite-time convergence of e_1:

The following Lyapunov function candidate is considered:

$$V_{O3}(t) = \frac{1}{2}e_1^2. \tag{19}$$

After differentiating (19) with respect to time and then substituting (11) into the resulting equation, we yield

$$\begin{aligned}\dot{V}_{O3}(t) &= e_1(-k_{02}e_1 + k_{02}e_2 - \gamma_v - l_2 e_1)\\ &\leq -(k_{02}+l_2)e_1^2 - (l_1 - k_{02}\|e_2\|)|e_1|\\ &\leq -(l_1 - k_{02}\|e\|)|e_1|.\end{aligned} \tag{20}$$

To guarantee $\dot{V}_{O3}(t) < -\varepsilon_0|e_1| < 0$ for $|e_1| \neq 0$, we select

$$\begin{aligned}&(l_1 - k_{02}\|\boldsymbol{e}\|) > \varepsilon_0\\ &\Rightarrow \lambda_{\min}l_1 > k_{02}l_4 l_1^{\frac{p_2}{q_2}} + k_{02}l_5 + k_{02}\rho + \lambda_{\min}\varepsilon_0\\ &\Rightarrow l_1^{\frac{p_2}{q_2}}\left(\lambda_{\min}l_1^{\frac{q_2-p_2}{q_2}} - k_{02}l_4\right) > k_{02}l_5 + k_{02}\rho + \lambda_{\min}\varepsilon_0,\end{aligned} \tag{21}$$

where the conclusions of (16) have been utilized.

To guarantee (21) holds, we choose

$$\begin{cases} l_1^{\frac{p_2}{q_2}} > k_{02}l_5 + k_{02}\rho + \lambda_{\min}\varepsilon_0, \\ \lambda_{\min} l_1^{\frac{q_2-p_2}{q_2}} - k_{02}l_4 > 1. \end{cases} \tag{22}$$

Solving (22), it is obtained that

$$l_1 \geq \max\left\{(k_{02}l_5 + k_{02}\rho + \lambda_{\min}\varepsilon_0)^{\frac{q_2}{p_2}}, \left(\frac{1+k_{02}l_4}{\lambda_{\min}}\right)^{\frac{q_2}{q_2-p_2}}\right\}. \tag{23}$$

Then,

$$\begin{aligned} \dot{V}_{O3}(t) &< -\varepsilon_0 |e_1| \\ &= -\varepsilon_0 2^{\frac{1}{2}} V_{O3}^{\frac{1}{2}}(t) \\ &< 0, \end{aligned} \tag{24}$$

is assured for $|e_1| \neq 0$.

Integrating (24) with respect to time, it is easy to see that

$$\begin{aligned} &\int_{T_0}^{t} V_{O3}^{-\frac{1}{2}}(t)\dot{V}_{O3}(t)\,dt < -2^{\frac{1}{2}}\varepsilon_0 t \\ &\qquad\Downarrow \\ &2V_{O3}^{\frac{1}{2}}(t) < 2V_{O3}^{\frac{1}{2}}(0) - 2^{\frac{1}{2}}\varepsilon_0 t. \end{aligned} \tag{25}$$

From (25), it is known that by T_1:

$$T_1 = \frac{|e_1(0)|}{\varepsilon_0}, \tag{26}$$

$e_1 \equiv 0 \Rightarrow \dot{e}_1 = 0$. After T_1, from (11) and the conclusions of $e_1 = 0$ and $\dot{e}_1 = 0$, it can be obtained that

$$(\gamma_v)_{t \geq T_1} = k_{02}e_2. \tag{27}$$

2) Finite-time convergence of e_2:

By substituting the conclusions of (27), $(e_1)_{t\geq T_1} = 0$, and $(\dot{e}_1)_{t\geq T_1} = 0$ into (12), it is obtained that

$$\begin{aligned} \dot{e}_2 &= -l_4 k_{02}^{\frac{p_2}{q_2}} e_2^{\frac{p_2}{q_2}} sgn(k_{02}e_2) - l_5 sgn(k_{02}e_2) - u \\ &= -l_4 k_{02}^{\frac{p_2}{q_2}} e_2^{\frac{p_2}{q_2}} sgn(e_2) - l_5 sgn(e_2) - u, \end{aligned} \tag{28}$$

as $t \geq T_1$.

To accomplish the proof, the following positive definite scalar function is considered:

$$V_{O4}(t) = \frac{1}{2}e_2^2. \tag{29}$$

Taking the time derivative of (29) and substituting (28) into the resulting equation yields

$$\begin{aligned} \dot{V}_{O4}(t) &= e_2\dot{e}_2 \\ &= e_2\left(-l_4 k_{02}^{\frac{p_2}{q_2}} e_2^{\frac{p_2}{q_2}} sgn(e_2) - l_5 sgn(e_2) - u\right) \\ &\leq -l_4 k_{02}^{\frac{p_2}{q_2}} |e_2|^{\frac{p_2+q_2}{q_2}} - (l_5 - \rho)|e_2| \\ &\leq -l_4 k_{02}^{\frac{p_2}{q_2}} 2^{\frac{p_2+q_2}{2q_2}} V_{O4}^{\frac{p_2+q_2}{2q_2}}, \end{aligned} \tag{30}$$

where (18) is used.

Solving (29), one has

$$\begin{aligned} &\int_{T_1}^{t} V_{O4}^{-\frac{p_2+q_2}{2q_2}} \dot{V}_{O4}(t)\,dt \leq -l_4 k_{02}^{\frac{p_2}{q_2}} 2^{\frac{p_2+q_2}{2q_2}}(t - T_1) \\ &\qquad\Downarrow \\ &\frac{2q_2}{q_2 - p_2} V_{O4}^{\frac{q_2-p_2}{2q_2}}(t) \leq \left(\begin{aligned} &\tfrac{2q_2}{q_2-p_2} V_{O4}^{\frac{q_2-p_2}{2q_2}}(T_1) \\ &-l_4 k_{02}^{\frac{p_2}{q_2}} 2^{\frac{q_2+p_2}{2q_2}}(t - T_1) \end{aligned}\right). \end{aligned} \tag{31}$$

From (31), it can be obtained that by the time T_2:

$$T_2 = T_1 + \frac{2q_2 V_{O4}^{\frac{q_2-p_2}{2q_2}}(T_1)}{l_4 k_{02}^{\frac{p_2}{q_2}} 2^{\frac{p_2+q_2}{2q_2}}(q_2 - p_2)}, \tag{32}$$

$|e_2| = 0$. In other words, the unknown disturbances h are exactly estimated by $\hat{\gamma}_2(t)$ within finite time T_2.

B. FINITE-TIME TRAJECTORY TRACKING CONTROLLER

To accomplish the trajectory tracking control method design, the expression of the estimated trolley acceleration is defined as

$$\begin{aligned} \ddot{\hat{x}} &= k_{03}\lfloor e_3 \rceil^{\frac{p_3}{q_3}} + \ddot{x}_d + \delta_0 e_3 \\ &= k_{03}|e_3|^{\frac{p_3}{q_3}} sgn(e_3) + \ddot{x}_d + \delta_0 e_3, \end{aligned} \tag{33}$$

where $k_{03}, \delta_0 \in \mathbf{R}^+$ denote positive gains, $p_3, q_3 \in \mathbf{R}^+$ are positive odd numbers such that $p_3 < q_3$, $e_3 = \dot{x}_d - \dot{x}$ represents the trolley velocity tracking error, $\ddot{\hat{x}}$ stands for the second time derivative of $\hat{x}$, $\dot{x}_d$ denotes the desired trolley velocity trajectory, $\lfloor e_3 \rceil^{\frac{p_3}{q_3}} = |e_3|^{\frac{p_3}{q_3}} sgn(e_3)$.

Thus, the finite-time tracking control law without payload-swing feedback is designed as follows:

$$\begin{aligned} F &= -k_{04}\lfloor e_4 \rceil^{\frac{p_3}{q_3}} + \hat{\gamma}_2 + e_3 + (m_p + M)\ddot{\hat{x}} + E \\ &= -k_{04}|e_4|^{\frac{p_3}{q_3}} sgn(e_4) + \hat{\gamma}_2 + e_3 + (m_p + M)\ddot{\hat{x}} + E, \end{aligned} \tag{34}$$

where $e_4 = \ddot{x} - \ddot{\hat{x}}$ represents the estimated error of the trolley acceleration, $k_{04} \in \mathbf{R}^+$ is a positive control gain, $\lfloor e_4 \rceil^{\frac{p_3}{q_3}} = |e_4|^{\frac{p_3}{q_3}} sgn(e_4)$.

Theorem 2: The proposed tracking control method (34) as well as the terminal sliding mode observer (9)-(10) guarantee that the trolley converges to the desired trajectory in finite time.

Proof: From the definition of e_3 and e_4, it is obtained that

$$\dot{e}_3 = \ddot{x}_d - \ddot{x} = \ddot{x}_d - \ddot{\hat{x}} - e_4. \tag{35}$$

In consequence, it follows from (33) and (35) that

$$\dot{e}_3 = -k_{03}\lfloor e_3 \rceil^{\frac{p_3}{q_3}} - e_4 - \delta_0 e_3. \tag{36}$$

On the other hand, it is easily concluded from (1) that

$$\begin{aligned} \dot{e}_4 &= \dddot{x} - \dddot{\hat{x}} \\ &= \frac{1}{(m_p + M)}\begin{pmatrix} F - f_{rx} + m_p l\dot{\theta}^2 \sin\theta \\ -m_p l\ddot{\theta}\cos\theta - h \end{pmatrix} - \dddot{\hat{x}}. \end{aligned} \tag{37}$$

To prove Theorem 2, the following positive definite Lyapunov function candidate for the overhead crane systems is chosen:

$$V(t) = \frac{e_3^2}{2} + \frac{(m_p + M) e_4^2}{2}. \quad (38)$$

Differentiating (38), and inserting (34), (35) as well as (37), we have

$$\begin{aligned}\dot{V}(t) &= e_3\dot{e}_3 + e_4 (m_p + M) \dot{e}_4 \\ &= e_3 \left\{-k_{03} \lfloor e_3 \rceil^{\frac{p_3}{q_3}} - e_4 - \delta_0 e_3\right\} - e_4 (m_p + M) \ddot{\hat{x}} \\ &\quad + e_4 \left(F - f_{rx} + m_p l\dot{\theta}^2 \sin\theta - m_p l\ddot{\theta}\cos\theta - h\right) \\ &= -k_{03} |e_3|^{\frac{p_3+q_3}{q_3}} - k_{04} \|e_4\|^{\frac{p_3+q_3}{q_3}} - \delta_0 e_3^2 + e_4 (\hat{\gamma}_2 - h) \\ &\quad + e_4 \left(E - f_{rx} + m_p l\dot{\theta}^2 \sin\theta - m_p l\ddot{\theta}\cos\theta\right). \quad (39)\end{aligned}$$

According to the definition of and from Theorem 1 that $h = \hat{\gamma}_2$ for all $t \geq T_2$. As a result, it leads (39) to

$$\dot{V} \leq -k_{03} |e_3|^{\frac{p_3+q_3}{q_3}} - k_{04} |e_4|^{\frac{p_3+q_3}{q_3}}, t \geq T_2 \quad (40)$$

Hence, after finite time T_2, (40) can be simplified as:

$$\dot{V} \leq -\bar{k} V^{\frac{p_3+q_3}{2q_3}}, t \geq T_2, \quad (41)$$

where

$$\bar{k} = \min\left\{k_{03} 2^{\frac{p_3+q_3}{2q_3}}, k_{04} \left(\frac{2}{m_p + M}\right)^{\frac{p_3+q_3}{2q_3}}\right\}.$$

Solving (34), it is obtained that

$$\int_{T_2}^{t} V^{-\frac{p_3}{+} q_3 2q_3} \dot{V} dt \leq -\bar{k}(t - T_2)$$
$$\Downarrow$$
$$\frac{2q_3}{q_3 - p_3} V^{\frac{q_3-p_3}{2q_3}}(t) \leq \frac{2q_3}{q_3 - p_3} V^{\frac{q_3-p_3}{2q_3}}(T_2) - \bar{k}(t - T_2). \quad (42)$$

Thus, from (42), it can be obtained that $e_3 \equiv 0$ and $e_4 \equiv 0$ for all $t \geq T_3$ where

$$T_3 = T_2 + \frac{2q_3 V^{\frac{q_3-p_3}{2q_3}}(T_2)}{(q_3 - p_3)\bar{k}}, \quad (43)$$

indicating the trolley positioning error e_3 converges to 0 within finite time T_3.

C. SELECTION OF THE DESIRED TROLLEY TRAJECTORY

As stated previously, the controller given in (34) guarantees that the position of the trolley x tracks the desired trajectory x_d in finite time T_3. However, it cannot regulate the payload swing angle θ to 0. Therefore, to suppress and eliminate the payload swing, the desired trolley trajectory x_d is selected as [54]

$$x_d = \underbrace{\frac{p_d}{2} + \frac{k_v^2}{4k_a} \ln\left(\frac{\cosh(2k_a t/k_v - \varepsilon)}{\cosh(2k_a t/k_v - \varepsilon - 2p_d k_a/k_v^2)}\right)}_{x_{d1}} + , \underbrace{\kappa \int_0^t \theta dt}_{x_{d2}}, \quad (44)$$

where $p_d \in \mathbf{R}^+$ stands for the trolley target position, $k_a, k_v \in \mathbf{R}^+$ represent the maximum trolley acceleration and velocity, respectively, $\varepsilon \in \mathbf{R}^+$ is a positive parameter introduced to regulate the initial acceleration; $\kappa > 1.0754$ represents a positive control gain. The desired trolley trajectory consists of two parts: (i) a positioning reference trajectory component x_{d1} to guide the trolley to the desired position and (ii) a swing-eliminating component x_{d2} to effectively eliminate the payload swing without affecting the trolley position. The trajectory x_d is smooth and uniformly continuous with the following properties [54]:

Property 1: it guarantees that the payload swing angle, the angular velocity, and the angular acceleration converge to 0 asymptotically in the sense that:

$$\lim_{t\to\infty} \left[\theta\ \dot{\theta}\ \ddot{\theta}\right]^T = [0\ 0\ 0]^T. \quad (45)$$

Property 2: x_d ensures that the trolley arrives at the target position while the corresponding velocity and acceleration converge to 0 in the sense that

$$\lim_{t\to\infty} [x_d\ \dot{x}_d\ \ddot{x}_d]^T = [p_d\ 0\ 0]^T. \quad (46)$$

IV. NUMERICAL SIMULATION RESULTS

In this section, the control performance of the proposed finite-time tracking controller based on a terminal sliding mode observer is illustrated. The simulation study is divided into two scenarios. More precisely, in the first scenario, the performance of the proposed controller is validated by comparing it with the LQR controller [8], the enhanced coupling (EC) controller [12], and the PD controller [31]. It should be pointed out that the LQR controller, EC controller, and PD controller are designed without considering the unknown disturbances. Therefore, the unknown disturbances h are set to 0 in scenario 1. In the second scenario, the robustness of the proposed controller with respect to unknown disturbances is verified and we compare it with the motion planning-based adaptive (MPA) controller [24].

For completeness, the expressions of the LQR controller, the enhanced coupling controller, the PD controller and the motion planning-based adaptive controller are given as follows:

1) LQR controller:

$$F = -k_1 \varepsilon_x - k_2 \dot{x} - k_3 \theta - k_4 \dot{\theta} + f_{rx}, \quad (47)$$

with $k_1, k_2 \in \mathbf{R}^+$, $k_3, k_4 \in \mathbf{R}^-$ being control gains.

2) EC controller:

$$F = -k_p \left(\int_0^t \xi_x dt - p_{dx}\right) - k_\xi \xi_x + \lambda (M + m_p) \dot{\theta} + f_{rx}, \quad (48)$$

where $k_p, k_\xi, \lambda \in \mathbf{R}^+$ denote positive control gains. The expression of the auxiliary function ξ_x is given as follows:

$$\xi_x = \dot{x} - \lambda\theta. \quad (49)$$

3) PD controller:

$$F = -k_p x - k_d \dot{x} + f_{rx}, \quad (50)$$

where $k_p, k_d \in \mathbf{R}^+$ stand for positive control gains.

4) MPA controller:

$$F = -Y^T \hat{\omega} - k_p r - k_d \dot{r}, \tag{51}$$

with k_p, $k_d \in \mathbf{R}^+$ standing for positive control gains, $r = x - x_d$ denoting trolley trajectory tracking error, $\hat{\omega}$ being the online estimation of ω, which is generated by the following update law:

$$\dot{\hat{\omega}} = \Gamma Y \dot{r}, \tag{52}$$

where Γ represents a diagonal, positive definite, update gain matrix.

Scenario 1: Control performance validation with exact parameter information: In this scenario, the actual and nominal values of overhead crane plants are equal. The crane plant parameters are set as

$$M = 7kg, \quad m_p = 1kg, \ l = 0.6m, \ h = 0.$$

The friction has the following form:

$$f_{rx} = 4.4 \tanh\left(\frac{\dot{x}}{0.01}\right) + 0.5 \left|\dot{x}\right| \dot{x}.$$

The desired trolley location is selected as

$$p_d = 1m.$$

The parameters for the desired trolley trajectory (44) are chosen as

$$k_a = 0.5, \quad k_v = 0.5, \ \varepsilon = 2, \ \kappa = 4.$$

The control gains for the proposed controller, LQR controller, EC controller, and PD controller are given in Tab. 1, which are obtained by trial and error.

TABLE 1. Control gains for scenario 1.

Controllers	Control gains
Proposed controller	$k_{03} = 50, k_{04} = 50, \delta_0 = 3, p_3 = 1, q_3 = 5$, Other gains are set to 0
LQR controller	$k_1 = 10, k_2 = 20, k_3 = -10, k_4 = -6$
EC controller	$k_p = 15, k_\xi = 15, \lambda = 12$
PD controller	$k_p = 15, k_d = 20$

Figs. 2-5 record the performance of the four controllers. It is seen that, all of the four controllers can drive the trolley to the desired location within about 5 s. The proposed controller suppresses and eliminates the payload swing within a smaller range (maximum swing amplitude: 1.4°, and almost no residual payload swing angle) than the LQR controller (maximum swing amplitude: 5.8°, and almost no residual payload swing angle), EC controller (maximum swing amplitude: 2.2°, and almost no residual payload swing angle), and PD controller (maximum swing amplitude: 8.2°, residual swing: 4.5°). In addition, in the process of transportation, the payload continues to swing back and forth by the LQR method and the PD method, while the payload is much more steady for the proposed control method and the EC method. It can be concluded that the designed controller achieves superior control performance in the sense of payload suppression and elimination.

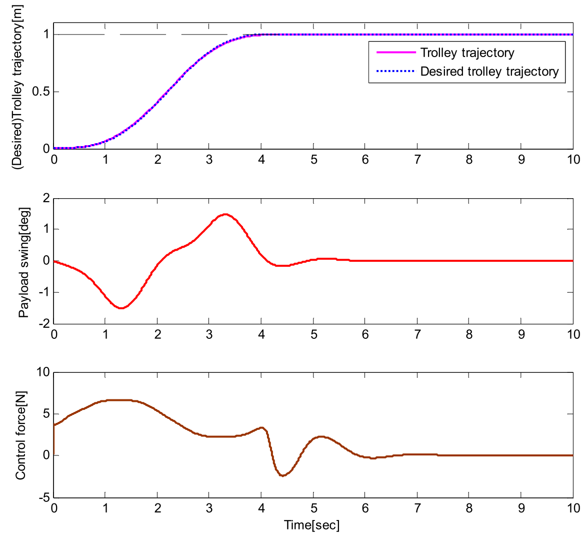

FIGURE 2. **Scenario 1: Simulation results for the proposed controller.**

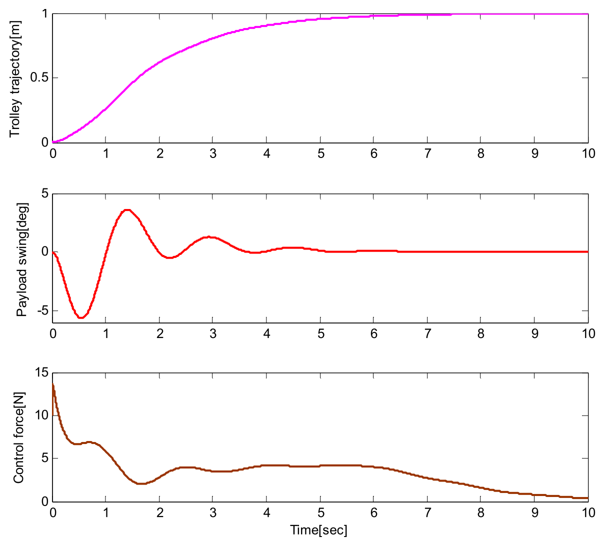

FIGURE 3. **Scenario 1: Simulation results for the LQR controller.**

Scenario 2: Control performance validation with unknown disturbances: In this scenario, the nominal crane plant parameters are selected as

$$M = 12kg, \quad m_p = 9kg, \ l = 0.7m.$$

The actual values of the trolley mass, payload mass, and cable length are 14 kg, 10 kg, 1.0 m, respectively. Therefore, it is obtained that

$$\Delta M = 2kg, \quad \Delta m_p = 1kg, \ \Delta l = 0.3m.$$

The nominal friction is set the same as it in scenario 1. The expression of the actual friction is given as

$$f_{rx} = 6.2 \tanh\left(\frac{\dot{x}}{0.01}\right) + 0.8 \left|\dot{x}\right| \dot{x}.$$

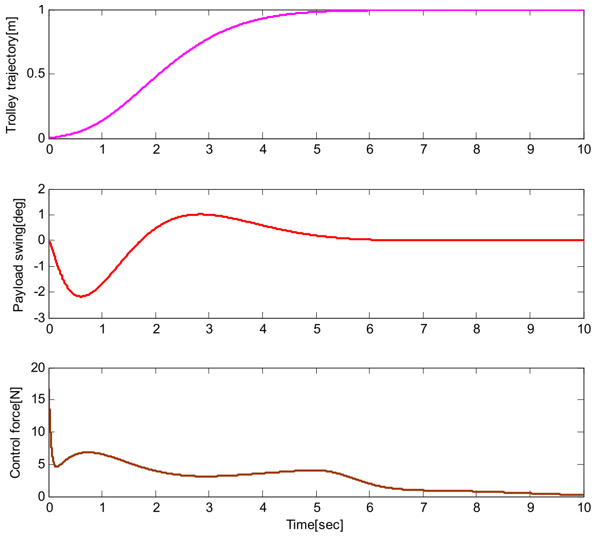

FIGURE 4. **Scenario 1: Simulation results for the enhanced coupling controller.**

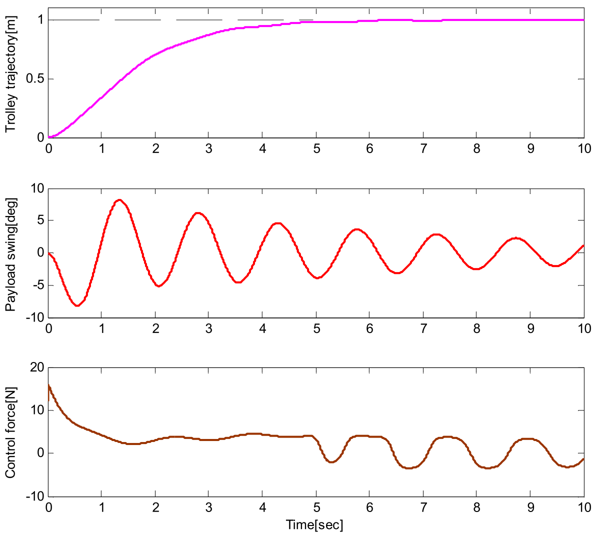

FIGURE 5. **Scenario 1: Simulation results for the PD controller.**

External sinusoid disturbances d are added to the crane dynamics, all with an amplitude of 10.

The desired trolley location is selected as

$$p_d = 1m.$$

The parameters for the desired trolley trajectory (44) are chosen as before in scenario 1.

The control gains for the proposed controller and the MPA controller are illustrated in Tab. 2.

Figs. 6-7 depict the behaviors of the proposed controller and the MPA controller in the presence of unknown disturbances. It can be seen from these figures that the tracking performance of the proposed controller is hardly affected by the unknown disturbances, whereas that of the MPA controller degrade significantly. Moreover, the maximum and residual payload swings are much less than those of the comparative

TABLE 2. **Control gains for scenario 2.**

Controllers	Control gains
Proposed controller	$k_{01}=5, p_1=1, q_1=3, b_1=10, k_{03}=50,$ $k_{04}=50, \delta_0=3, p_3=1, q_3=5, p_2=19,$ $q_2=21, l_1=l_2=l_3=l_4=l_5=0.05, k_{02}=2$
MPA controller	$k_p=300, k_d=50, \Gamma=50\mathbf{I}_5$

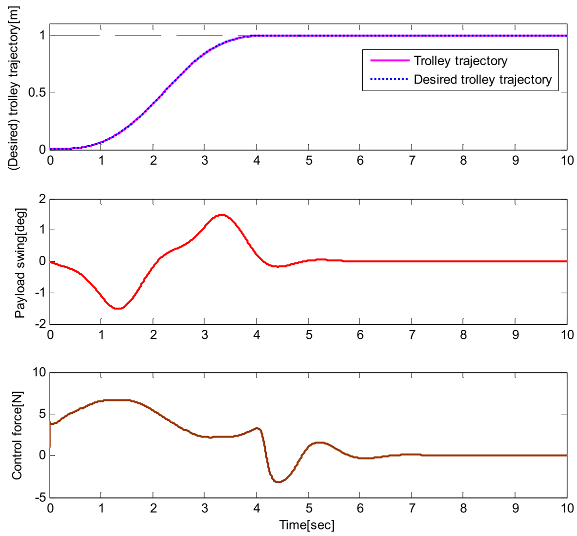

FIGURE 6. **Scenario 2: Simulation results for the proposed controller.**

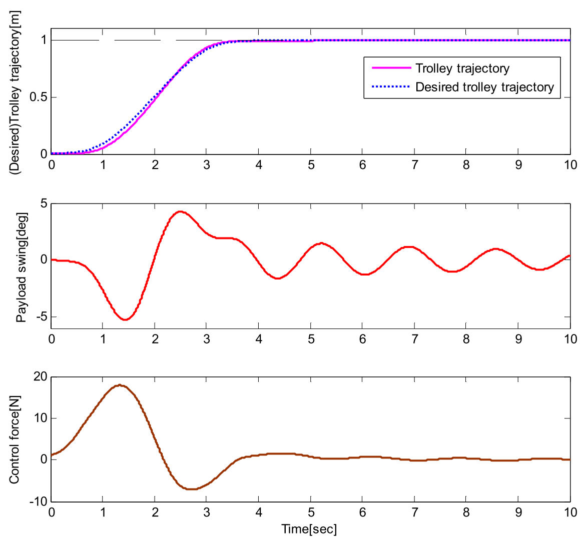

FIGURE 7. **Scenario 2: Simulation results for the MPA controller.**

controller. Those merits bring much convenience of the application of the proposed controller in practical overhead crane systems, since unknown disturbances always exist.

V. CONCLUSION

In this study, a finite-time trajectory tracking controller based on a terminal sliding mode observer is presented. The observer is used to estimate the unknown disturbances.

Therefore, unknown disturbances induced by uncertain trolley mass, uncertain payload mass, uncertain cable length, uncertain friction, as well as external disturbances are addressed. Moreover, the proposed controller can achieve finite-time convergence. Lyapunov techniques are used to prove the stability and convergence of the closed-loop system. Simulation results are illustrated to demonstrate the superior performance of the designed controller. Our future work will mainly focus on designing an observer for payload swing.

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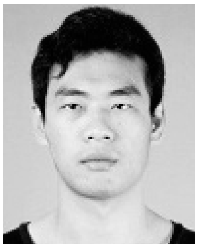

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