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A Model Predictive Control Performance Monitoring and Grading Strategy Based on Improved Slow Feature Analysis

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ABSTRACT Slow feature analysis (SFA) has been adopted for control performance monitoring (CPM) recently. However, due to the selection criterion of the dominant slow features (SFs) and the performance monitoring statistics, the traditional SFA-based CPM method has certain limitations in monitoring model predictive control (MPC) performance and fails to distinguish the direction of performance change, i.e., whether the performance becomes better or worse. In order to solve the above problems, an MPC performance monitoring and grading strategy based on improved SFA is proposed in this paper. First, a new criterion for selecting dominant SFs is proposed. On this basis, two combined monitoring indices are built to monitor steady-state and dynamic characteristics of MPC systems, respectively. Besides, an SFA-based predictable performance assessment index is proposed to indicate the direction of performance change. Finally, a performance grading strategy based on improved SFA is established to classify current MPC performance to four levels. Two simulation examples demonstrate the effectiveness and superiority of the proposed method.

INDEX TERMS Performance monitoring, performance grading, model predictive control, slow feature analysis, predictable performance assessment index.

I. INTRODUCTION

Process control is a pivotal integral part of modern process industries, and a large number of process loops operate under the different controllers to satisfy various control requirements. In order to control processes with constraints, multi-interacting variables and complex dynamics, model predictive control (MPC) has been developed and widely applied to a range of complex industrial processes such as automotive, medicine, aerospace, refining and petrochemical industries [1]. Due to the advanced nature of MPC algorithm, it can save resources and energy, increase production safety, improve product quality, and ultimately maximize the economic profit of the factory [2], [3]. Although most MPC controllers operate well because of controller tuning at the

commissioning stage, their control performance deteriorates after a period of production due to various factors, including plant process model-mismatch, disturbance fluctuation, variation of raw material property, fault of sensor/actuator, and change of constraint sets [4], [5]. This directly affects the safety, efficiency, and product quality of plants. Therefore, the control performance assessment (CPA) or control performance monitoring (CPM) technology has attracted wide attention from academia to industry over the past thirty years [6].

The purpose of CPA/CPM is to detect performance degradation by analyzing routine closed-loop operating data, which is a precondition for improving control performance. The research on CPA/CPM can be traced back to the work of Harris in 1989, in which the minimum variance control (MVC) benchmark was proposed for univariate processes [7]. Since then, further studies on CPA/CPM have

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been extended to performance assessment of multiple-input multiple-output processes, feedback/forward controllers, cascade controllers, and MPC controllers [8]–[11]. Several monographs [12]–[14] and review articles [15]–[19], [21] have been published to summarize the academic developments and industrial applications of CPA/CPM in recent years. In general, CPA/CPM technologies are mainly divided into three categories: theoretical optimal benchmarks, user-specified benchmarks, and historical benchmarks. As the earliest proposed performance benchmark, theoretical optimal benchmarks, such as the MVC benchmark [7] and the linear-quadratic Gaussian (LQG) control benchmark [22], rely on sufficiently accurate prior knowledge of the process model. However, it is often difficult to accurately establish the process and disturbance model in practical industrial applications. Furthermore, for MPC systems, the performance bound in the theoretical case is difficult to achieve in the industrial case due to the restrictions of the controller structure and the existence of model-mismatch. By contrast, user-specified benchmarks, such as internal model control (IMC) benchmark [23] and achievable proportional-integral-derivative benchmark [24], can avoid the ideal assumptions about the process and be more realistic because of taking into account the user's demand preferences and the constraints on controller structure. These methods may be useful in many situations. However, there are no general guidelines for how to select a custom performance reference for such methods. Which performance reference scheme is most suitable is still determined by experienced users or control engineers [14].

As data-driven methods, historical benchmarks do not require process/model knowledge and can be calculated using historical operational data with the desired control performance. Therefore, they are suitable for evaluating or monitoring the control performance of various types and scales, including MPC. Inspired by the MVC benchmark, Yu and Qin [25], [26] defined statistical covariance-based performance index, which utilizes only historical output data. After that, several improved covariance-based methods were proposed. Li *et al.* [27] proposed a method based on dissimilarity analysis to detect changes in the distribution characteristics of controlled variables (CVs). Yan *et al.* [28] conducted a hypothesis test on the equality of the output covariance matrices to monitor control performance, and this method can make full use of the information in the entire covariance matrices. The aforementioned covariance-based CPM methods only fasten on the information in CVs of the process and ignore the potential information in other process variables such as manipulated variables (MVs) and disturbance variables. Tian *et al.* [4] suggested that all types of process variables are provided to indicate current control performance and built a 2-norm based covariance index to assess MPC performance. Shang *et al.* [29] constructed a partial least squares (PLS) cross-product matrix which contains the information of MVs and CVs and this cross-product matrix can extract the maximum dissimilarity information between the current

matrix and the benchmark matrix to assess MPC performance. In addition, multivariate statistical methods, such as principal component analysis (PCA) [30], [31] and PLS [32], are introduced to assess MPC performance, respectively. However, a limitation of traditional historical benchmark methods is that only the steady-state information of process data is retained and the temporal dynamic information is not considered.

Recently, a novel multivariate statistical technology, slow feature analysis (SFA) [33], was proposed to separate temporal slow features (SFs) from process variables and was first used for nonlinear process fault diagnosis [34]. In recent years, SFA-based methods have been successfully adopted in monitoring industrial processes [35]–[37] and control performance [38]. In order to monitor the distributions of both the steady-state and temporal dynamic characteristics of process variables simultaneously, two pairs of monitoring statistics based on the SFA were designed innovatively by Shang *et al.* [35]. Then, an SFA method based on dynamic statistics of CVs is used to capture the changes of process dynamics caused by the feedback compensation of the controller and to monitor control performance [38]. However, The existing SFA-based performance monitoring method has the following problems when they are used for MPC performance monitoring. Firstly, for traditional SFA-based CPM method, slower SFs are selected as the dominant SFs and the faster SFs are ignored. Nevertheless, when control performance changes, abnormal process dynamics may cover both increased and decreased temporal variations [39], and the abnormal information may exist in the slower SF or in the faster SF. Therefore, the traditional criterion for selecting dominant SFs may lead to poor performance. Secondly, traditional SFA-based CPM method uses dynamic information of CVs to monitor performance and ignores the steady-state information and other valuable process variables, this makes it difficult to achieve satisfactory results for monitoring MPC performance. Furthermore, although the traditional SFA-based method can detect performance change, it cannot distinguish the direction of performance change, i.e., whether the performance becomes better or worse [38]. Finally, as an important prerequisite for performing performance maintenance and improved work, the CPA method should have the capability of performance grading. However, to date, there have been few reports on control performance grading (CPG) studies.

In response to these problems, an MPC monitoring and grading strategy based on improved SFA is established. A new selection criterion is proposed based on an expression degree of original variables on SFs and the selected dominant SFs can more fully express the information of original variables. The expression degree is designed by the mapping relationship between the lag-1 autocorrelations of SFs and the lag-1 autocorrelations of original variables, which is derived from the objective function of SFA. Aiming at the problem that the steady-state information of process variables is ignored in performance monitoring, two combined

monitoring indices are established to monitor the steady-state and dynamic variations of process variables in MPC performance. In addition, an SFA-based predictable performance assessment index (PPAI) is constructed by the generalized eigenvalue corresponding to the slowest SF and used to indicate the direction of MPC performance change. The PPAI is based on the intrinsic relationship between the predictability of the process closed-loop output and the control performance [39], [40], so it has the ability to assess whether the MPC performance becomes better or worse. Finally, in order to achieve the purpose of grading the current MPC performance, a performance grading strategy is proposed by incorporating the SFA-based monitoring indices and the SFA-based PPAI. The performance grading strategy can classify current MPC performance into four levels: improved performance, normal performance, degraded performance, and the worst performance.

The remainder of the article is presented as follows. Section II introduces the basic SFA algorithm and performance monitoring statistics based on the statistical properties of SFA. In section III, the MPC performance monitoring and grading strategy based on improved SFA is presented. Case studies and discussions are provided in section IV. Finally, the conclusion is drawn in section V.

II. TRADITIONAL PERFORMANCE MONITORING METHOD BASED ON SLOW FEATURE ANALYSIS

A. SLOW FEATURE ANALYSIS

The idea of slow feature analysis is to seek transformation functions of which output signals vary as slowly as possible [33]. For m -dimensional temporal input series $\mathbf{x}(t) = [x_1(t), \dots, x_m(t)]^T$, the objective of SFA is to find a transformation function $g_j(\mathbf{x})(j = 1, \dots, m)$ that generates the output slow feature series $s_j(t) = g_j(\mathbf{x}(t))$ such that

$$\Delta(s_j) = \langle \dot{s}_j^2 \rangle_t \quad (1)$$

is minimized under the constraints

$$\langle s_j \rangle_t = 0, \quad (2)$$

$$\langle s_j^2 \rangle_t = 1, \quad (3)$$

$$\forall i \neq j, \quad \langle s_i s_j \rangle_t = 0, \quad (4)$$

where $\dot{s}_j(t) = s_j(t) - s_j(t-1)$ represents the first-order derivative of s_j , and $\Delta(s_j)$ denotes a measure of the slowness of s_j . The symbol $\langle \cdot \rangle_t$ is the temporal average and can be defined as

$$\langle f(t) \rangle_t = \frac{1}{t_1 - t_0} \int_{t_0}^{t_1} f(t) dt. \quad (5)$$

Here, the primary objective in (1) is to minimize the temporal variations of SFs. The rationale of constraints in (2) and (3) can be interpreted as each SF has zero mean and unit variance in order to avoid the trivial solution $s_j \equiv \text{const}$. The constraint in (4) ensures that SFs are independent of each other and contain different information. Furthermore, $\{s_j\}_{j=1}^m$ can be

arranged in descending order such that the first SF is the slowest, the second SF is the second slowest, etc.

For linear cases, the linear mapping from $\mathbf{x}(t)$ to SF $s_j(t)$ is written as

$$s_j(t) = \mathbf{w}_j^T \mathbf{x}(t) = \sum_{i=1}^m w_{ji} x_i(t), \quad (6)$$

and this can be further concisely formulated as

$$\mathbf{s}(t) = \mathbf{W} \mathbf{x}(t), \quad (7)$$

where $\mathbf{w}_j(j = 1, \dots, m)$ denotes coefficient vector, and $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_m]^T$ is the coefficient matrix. Then the optimization problem in (1) is equivalent to the following generalized eigenvalue problem [33]:

$$\mathbf{A} \mathbf{W} = \mathbf{B} \mathbf{W} \mathbf{\Omega}, \quad (8)$$

where $\mathbf{A} = \langle \dot{\mathbf{x}} \dot{\mathbf{x}}^T \rangle_t$ denotes the covariance matrix of the first-order derivative of \mathbf{x} , and $\mathbf{B} = \langle \mathbf{x} \mathbf{x}^T \rangle_t$ denotes the covariance matrix of \mathbf{x} . $\mathbf{\Omega} = \text{diag}[\omega_1, \dots, \omega_m]$ is a diagonal matrix of generalized eigenvalues

$$\omega_j = \Delta(s_j) = \langle \dot{s}_j^2 \rangle_t. \quad (9)$$

The SF corresponding to the smallest generalized eigenvalue has the greatest slowness.

B. STATISTICAL PROPERTIES FOR SLOW FEATURE ANALYSIS

The optimization problem (1) can also be solved by singular value decomposition (SVD), and the statistical properties of SFA can be further clarified. On the basis of the statistical properties, the performance monitoring statistics are finally constructed. Therefore, this intuitive SVD solution is shown in the next analysis. First, the covariance matrix \mathbf{B} is decomposed by SVD method:

$$\mathbf{B} = \mathbf{U} \mathbf{\Lambda} \mathbf{U}^T. \quad (10)$$

In order to remove crosscorrelations, \mathbf{x} can be sphered as follows:

$$\mathbf{z} = \mathbf{\Lambda}^{-1/2} \mathbf{U}^T \mathbf{x} \quad (11)$$

and $\langle \mathbf{z} \mathbf{z}^T \rangle_t = \mathbf{I}$. Then the optimization problem (1) is transformed into finding a matrix \mathbf{L} that satisfies

$$\mathbf{s} = \mathbf{L} \mathbf{z} \quad (12)$$

and

$$\langle \mathbf{s} \mathbf{s}^T \rangle_t = \mathbf{I}, \quad (13)$$

Substituting (12) into (13) leads to

$$\mathbf{L} \mathbf{L}^T = \mathbf{I}, \quad (14)$$

which indicates that $\mathbf{L} = [\mathbf{l}_1, \dots, \mathbf{l}_m]^T$ is an orthogonal matrix.

In the next step, the SVD is performed on the covariance matrix of the first-order derivative of \mathbf{z} :

$$\langle \dot{\mathbf{z}} \dot{\mathbf{z}}^T \rangle_t = \mathbf{L}^T \mathbf{\Omega} \mathbf{L}. \quad (15)$$

By (12) and (15), it is obtained that $\langle \dot{s}s^T \rangle_t = \mathbf{\Omega}$, and the slowness of the SF s_j is derived from $\Delta(s_j) = \omega_j$, where ω_j is the j -th diagonal element of the matrix $\mathbf{\Omega}$.

Finally, \mathbf{W} can be computed as:

$$\mathbf{W} = \mathbf{L}\mathbf{\Lambda}^{-1/2}\mathbf{U}^T. \quad (16)$$

Based on the above analysis, SFs have the following statistical characteristics [35]:

$$\langle s \rangle_t = \mathbf{0}, \quad \langle ss^T \rangle_t = \mathbf{I}, \quad (17)$$

$$\langle \dot{s} \rangle_t = \mathbf{0}, \quad \langle \dot{s}\dot{s}^T \rangle_t = \mathbf{\Omega}, \quad (18)$$

that is,

$$\langle x \rangle_t = \mathbf{0}, \quad \langle xx^T \rangle_t = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^T, \quad (19)$$

$$\langle \dot{x} \rangle_t = \mathbf{0}, \quad \langle \dot{x}\dot{x}^T \rangle_t = \mathbf{U}\mathbf{\Lambda}^{1/2T}\mathbf{L}\mathbf{\Omega}\mathbf{L}\mathbf{\Lambda}^{1/2}\mathbf{U}^T, \quad (20)$$

From (19) and (20), it can be seen that the SFA algorithm presents process data information in two dimensions: the steady-state distribution $P(x)$ by (19) and the temporal distribution $P(\dot{x})$ by (20). Compared with multivariate statistical methods such as PCA, SFA can provide more abundant monitoring information.

C. PERFORMANCE MONITORING STATISTICS BASED ON SLOW FEATURE ANALYSIS

In the conventional SFA-based CPM method, only the CVs are used to build the SFA model and all SFs are partitioned into dominant subspace s_d and residual subspace s_e . The SFs that are slower than input average slowness $\Delta(x_i)$ are specified as the dominant SFs, whereas the others in residual subspace are faster than $\Delta(x_i)$ and appear to manifest fast noise-like behaviors [35]. Input average slowness $\Delta(x_i)$ is expressed as

$$\Delta(x_i) = \sum_{j=1}^m a_{ij}\Delta(s_j), \quad \sum_{j=1}^m a_{ij} = 1, \quad a_{ij} \geq 0. \quad (21)$$

Then based on the goal of monitoring process dynamics and the statistical properties of s_d and s_e , performance monitoring statistics in traditional SFA-based CPM method are defined as [38]

$$S_d^2 = \dot{s}_d^T \mathbf{\Omega}_d^{-1} \dot{s}_d \quad (22)$$

$$S_e^2 = \dot{s}_e^T \mathbf{\Omega}_e^{-1} \dot{s}_e \quad (23)$$

where $\mathbf{\Omega}_d$ and $\mathbf{\Omega}_e$ are diagonal matrices composed of generalized eigenvalues corresponding to SFs in s_d and s_e .

The traditional SFA-based CPM method is based on dynamic statistics of the process closed-loop output and is used to capture the changes of process dynamics caused by the feedback compensation of the controller. It is effective for monitoring the performance of regular control loops. Whereas, as an advanced controller, the MPC performs online model modifications to compensate for model-mismatch and external interference, which makes it difficult to obtain satisfactory results using only dynamic statistics of CVs to monitor MPC performance.

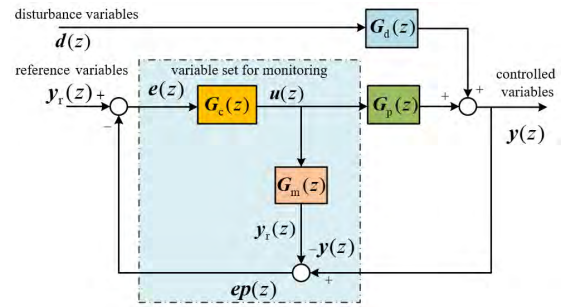


FIGURE 1. Schematic diagram of the IMC structure for MPC.

III. AN MPC PERFORMANCE MONITORING AND GRADING STRATEGY WITH IMPROVED SLOW FEATURE ANALYSIS

In this section, firstly, the selection and the data preprocessing of the monitored variables for MPC systems are introduced. Then, performance monitoring indices are established based on a new criterion for selecting dominant SFs. Besides, a performance grading strategy is proposed after building the predictable performance assessment index. Finally, a performance monitoring and grading strategy is demonstrated.

A. SELECTION AND PREPROCESSING OF THE MONITORED VARIABLES FOR MPC

With the spread of industrial measurement technology and the development of distributed control system (DCS) technology, it is easy to acquire and store rich plant process data. These data include valuable information on processes that is not fully utilized. Therefore, it is necessary to select and mine the monitored variables when applying CPM technology.

Based on the information of the current process model, MPC calculates the future control actions by minimizing the quadratic objective function

$$J(k) = E\{(y(k+1) - y_r(k+1))^T \mathbf{Q}(y(k+1) - y_r(k+1)) + \Delta u^T(k) \mathbf{R} \Delta u(k)\} \quad (24)$$

where $y(k+1)$ and $y_r(k+1)$ represent the output variables and reference variables, respectively, $\Delta u(k)$ denotes the increment of MVs, \mathbf{Q} and \mathbf{R} are weighting matrices. From (24), it can be seen that $J(k)$ is related to both the covariance matrix of CVs and the covariance matrix of MVs. Thus, both $y(k)$ and $u(k)$ are selected into the monitored variable set. In addition, according to the IMC structure for MPC in Fig. 1, the model prediction error can be derived as [4]:

$$ep(z) = (\mathbf{I} + (\mathbf{G}_p(z) - \mathbf{G}_m(z))\mathbf{G}_c(z))^{-1}(\mathbf{G}_p(z) - \mathbf{G}_m(z))\mathbf{G}_c(z)y_r(z) + (\mathbf{I} + (\mathbf{G}_p(z) - \mathbf{G}_m(z))\mathbf{G}_c(z))^{-1}\mathbf{G}_d(z)d(z) \quad (25)$$

where z denotes the Z-transform variable. $\mathbf{G}_p(z)$, $\mathbf{G}_m(z)$, $\mathbf{G}_c(z)$, and $\mathbf{G}_d(z)$ are the transfer functions of the plant, process model, controller, and disturbance, respectively. From (25), we can see that both process model-mismatch and disturbance can affect the model prediction error. Therefore,

the model prediction error contains performance deterioration information of MPC due to process and/or disturbance variations. Based on the above analysis, model prediction error also should be selected into the monitored variable set for performance monitoring. Finally, the following variable set

$$\mathbf{x} = [\mathbf{u}, \mathbf{y}, \mathbf{ep}] \in \mathbf{R}^{N \times m} \quad (26)$$

is used for monitoring MPC performance.

Due to the inherent physical dynamics of the chemical process and the feedback actions of the controller, the current process data is related to past process data in a period of time. With this in mind, each input vector is augmented by using d lag samples and the data matrix is stacked as follows

$$\mathbf{X}_d = \begin{bmatrix} \mathbf{x}(t)^\top & \mathbf{x}(t-1)^\top & \cdots & \mathbf{x}(t-d)^\top \\ \mathbf{x}(t+1) & \mathbf{x}(t)^\top & \cdots & \mathbf{x}(t-d+1)^\top \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{x}(t+N-1) & \mathbf{x}(t+N-2)^\top & \cdots & \mathbf{x}(t+N-d-1)^\top \end{bmatrix} \quad (27)$$

where N is the number of samples of the monitored variables. Thus, the input dimension of the SFA model is expanded as $m_s = m(d+1)$.

B. SELECTION CRITERION OF DOMINANT SLOW FEATURES

In traditional SFA method, the SFs that are slower than $\Delta(x_j)$ are considered more important and selected as the dominant SFs, while SFs with faster slowness are considered to be noise-like behaviors and ignored. However, since performance information is complicated, the SFs with slower slowness may not change significantly when control performance changes, that is, the information reflecting the performance change may be mainly divided into the residual subspace or evenly divided into two subspaces. This would lead to missed detection of deterioration in control performance. Because the traditional criterion for selecting dominant SFs ignores the expression degree of original process variables, the expression information of some process variables on the dominant SFs may be too small [42]. Therefore, a more reasonable selection criterion of dominant SFs is proposed in this subsection.

In the case of discrete input signals, the objective function (1) can be further deduced as [43]:

$$\begin{aligned} \min_{w_j} \Delta(s_j) &= \langle \dot{s}_j^2 \rangle_t \\ &= \langle (s_j(t+1) - s_j(t))^2 \rangle_t \\ &= \langle s_j(t+1)s_j(t+1) \rangle_t + \langle s_j(t)s_j(t) \rangle_t \\ &\quad - \langle s_j(t)s_j(t+1) \rangle_t - \langle s_j(t+1)s_j(t) \rangle_t \\ &= 2\langle s_j(t)^2 \rangle_t - 2\langle s_j(t+1)s_j(t) \rangle_t \\ &= 2 - 2\langle s_j(t+1)s_j(t) \rangle_t \\ &= 2 - 2R_j^s(1) \end{aligned} \quad (28)$$

where $R_j^s(1)$ is lag-1 autocorrelation of output signal s_j and $R_j^s(1) \in [-1, 1]$. Therefore, the objective function (28) can

be equivalent to:

$$\max_{w_j} \bar{\Delta}(s_j) = 1 - \frac{1}{2} \Delta(s_j) = R_j^s(1), \quad (29)$$

From (29), it can be seen that the goal of SFA can also be interpreted as finding mapping functions of which output signals with the largest lag-1 autocorrelation, that is to say, the slowest SF has the largest lag-1 autocorrelation.

According to the linear transformation (6), the correlation function of SFs are derived by

$$\begin{aligned} R_{pq}^s(\tau) &= \langle s_p(t+\tau)s_q(t) \rangle_t \\ &= \langle \mathbf{w}_p^\top \mathbf{x}(t+\tau) \mathbf{w}_q^\top \mathbf{x}(t) \rangle_t \\ &= \langle (\sum_{j=1}^m w_{pj}x_j(t+\tau)) (\sum_{i=1}^m w_{qi}x_i(t)) \rangle_t \\ &= \sum_{p=1}^m \sum_{q=1}^m w_{pj}w_{qi} \langle x_j(t+\tau)x_i(t) \rangle_t \\ &= \sum_{i=1}^m w_{pi}w_{qi}R_i^x(\tau) \quad 1 \leq p \leq m, 1 \leq q \leq m, \end{aligned} \quad (30)$$

where $R_{pq}^s(\tau)$ denotes the lag- τ correlation between s_p and s_q , and $R_i^x(\tau)$ denotes the lag- τ autocorrelation of the i -th input signal x_i . Let $p = q = j$, $\tau = 1$, eq.(30) can be rewritten into:

$$\begin{aligned} R_j^s(1) &= \sum_{i=1}^m w_{ji}^2 R_i^x(1) \\ &= w_{j1}^2 R_1^x(1) + w_{j2}^2 R_2^x(1) + \dots + w_{jm}^2 R_m^x(1). \end{aligned} \quad (31)$$

where $R_j^s(1)$ denotes the lag-1 autocorrelation of the j -th output SF s_j .

Based on (31), it shows that lag-1 autocorrelation of s_j is a linear mapping of lag-1 autocorrelations of input variables x_i ($i = 1, \dots, m$), and mapping weights are the squares of the elements in the coefficient matrix. According to (29) and (31), the target of SFA algorithm is to extract and accumulate the lag-1 autocorrelation information of original input variables, this is different from the PCA algorithm, whose idea is to extract the variance information of original input variables.

For an original variable x_i , the greater the value of weight coefficient w_{ji}^2 , the more information is expressed on SF s_j . The weights $\{w_{1i}^2, w_{2i}^2, \dots, w_{mi}^2\}$ of the lag-1 autocorrelation $R_i^x(1)$ in all SFs are different, it is possible that the weights in these dominant SFs are small. In this assuming case, if the original variable x_i contains much information on performance change, the SFA-based statistics may not reflect the abnormal information because the information in the variable x_i is less expressed. Therefore, it is essential to centralize the information of original process variables into a specific low-dimensional space as evenly as possible.

In light of (31), an information expression weight matrix can be obtained as

$$\tilde{\mathbf{W}} = [\tilde{w}_{1T}, \tilde{w}_{2T}, \dots, \tilde{w}_{mT}], \quad (32)$$

$$\tilde{w}_i = [w_{1i}^2, w_{2i}^2, \dots, w_{mi}^2]. \quad (33)$$

Since every element is positive in \tilde{W} , the coefficient w_{ji}^2 can be seen as an effective measure of the expression degree of the i -th original variable x_i on the j -th SFs $_j$.

According to the above analysis, for each original variable x_i , the SF $s_{j_{\max}^i}$ corresponding to the largest weight in \tilde{w}_i is selected as the dominant SF, thereby ensuring that the dominant SF has the maximum expression of the original variable x_i . j_{\max}^i is obtained as follows

$$j_{\max}^i = \arg \max \{w_{ji}^2\}_{j=1}^m, \quad i = 1, \dots, m. \quad (34)$$

Considering that the same SF may be the maximum expression of different original variables, ignoring repetition, the dominant subspace can be a low-dimensional space of the original variable space. For the selected dominant subspace, the cumulative expression information for each original variable can be calculated as

$$Cum_w = [cum_1, \dots, cum_m], \quad (35)$$

where, $cum_1 = \sum_{i=1}^m \{w_{j_{\max}^1}^2\} / \sum_{i=1}^m \{w_{j_1}^2\}, \dots, cum_m = \sum_{i=1}^m \{w_{j_{\max}^m}^2\} / \sum_{i=1}^m \{w_{j_m}^2\}$.

The feature information in the dominant SFs balances every original variable, which solves the problem of insufficient information expression of a certain original variable. The degree of information balance of the original variables can be measured by calculating the variance of the cum_i :

$$\sigma_{Cum}^2 = \sum_{i=1}^m (cum_i - \mu_{Cum})^2 / m. \quad (36)$$

where $\mu_{Cum} = \sum_{i=1}^m cum_i / m$. The smaller the value of σ_{Cum}^2 , the more balanced the dominant SFs can express all the original variables, and the performance monitoring is more efficient and reliable.

C. TWO COMBINED PERFORMANCE MONITORING INDICES

In this subsection, two combined performance monitoring indices are built to monitor steady-state and dynamic changes in MPC performance, respectively. Based on the criterion proposed in the previous subsection, SFs can be divided into dominant subspace $\bar{s}_d = [s_{j_1}, \dots, s_{j_m}]^T \in \mathbf{R}^{M_d}$ and residual subspace $\bar{s}_e = \{s_j | s_j \in s \text{ and } s_j \notin s_d\}$, $\bar{s}_e \in \mathbf{R}^{M_e}$, where M_d is the number of dominant SFs, and M_e is the number of residual SFs. Then, on the basis of the statistical properties of \bar{s}_d and \bar{s}_e , the first pair of performance monitoring statistics for monitoring steady variations of $x(t)$ can be defined as:

$$\bar{T}^2 = \bar{s}_d^T \bar{s}_d \quad (37)$$

$$\bar{T}_e^2 = \bar{s}_e^T \bar{s}_e \quad (38)$$

Meantime, the second pair of performance monitoring statistics for monitoring dynamic variations of $x(t)$ is designed as:

$$\bar{S}^2 = \dot{\bar{s}}_d^T \Omega_d^{-1} \dot{\bar{s}}_d \quad (39)$$

$$\bar{S}_e^2 = \dot{\bar{s}}_e^T \Omega_e^{-1} \dot{\bar{s}}_e \quad (40)$$

where, Ω_d and Ω_e are diagonal matrices composed of generalized eigenvalues corresponding to SFs in \bar{s}_d and \bar{s}_e .

In traditional SFA-based CPM method, only monitoring statistics S^2 and S_e^2 are used to focus on the impact of process dynamics of CVs. However, this method has its limitation when used for monitoring MPC performance due to the compensation of online model modification of MPC. In order to better utilize the steady-state and dynamic information of the process, we combine the two pairs of performance monitoring statistics into two monitoring indices as follow

$$\varphi = \frac{\bar{T}^2}{\bar{T}_\alpha^2} + \frac{\bar{T}_e^2}{\bar{T}_\alpha^2} \quad (41)$$

$$\psi = \frac{\bar{S}^2}{\bar{S}_\alpha^2} + \frac{\bar{S}_e^2}{\bar{S}_\alpha^2} \quad (42)$$

where, \bar{T}_α^2 and $\bar{\delta}_\alpha^2$ indicate the confidence limits of \bar{T}^2 and \bar{T}_e^2 with a confidence limit of α , \bar{S}_α^2 and $\bar{\xi}_\alpha^2$ are the confidence limits of \bar{S}^2 and \bar{S}_e^2 with a confidence limit of α . These two combined indices in (41) and (42) are constructed with different physical meanings. The statistic φ measures the consistency of the data point x with the steady-state distribution $P(x)$, and the statistic ψ evaluates the consistency with the temporal distribution $P(\dot{x})$. To inspect if performance changes, the corresponding confidence limits of monitoring statistics φ and ψ are needed.

Traditional methods are based on statistical mechanisms to determine confidence limits and require certain specified assumptions, for example, the process variables need to follow a Gaussian distribution. Due to the complexity of the industrial process, it is difficult to ensure that the process variables meet a specific distribution assumption. In order to resolve this limitation, a nonparametric empirical density estimation technique, data-driven kernel density estimation (KDE) method was used to calculate confidence limits [44].

Given the samples $\{f_1, f_2, \dots, f_n\}$, a univariate kernel density estimator is defined by

$$\hat{P}(f) = \frac{1}{nh} \sum_{i=1}^n \ker\left\{\frac{f - f_i}{h}\right\}, \quad (43)$$

where \ker is the kernel function which is a symmetric density function and h is the bandwidth of the estimator. In this work, a widely used kernel function, the Gaussian kernel is selected [45]. The confidence limits of the indices φ and ψ can be obtained using the KDE method through the following three steps. First, the values of indices \bar{T}^2 , \bar{T}_e^2 and \bar{S}^2 , \bar{S}_e^2 in the benchmark period are calculated. Second, the density functions of those four indices in the benchmark period are estimated by using the KDE method, and the confidence limits \bar{T}_α^2 , $\bar{\delta}_\alpha^2$ and \bar{S}_α^2 , $\bar{\xi}_\alpha^2$ can be obtained by calculating the point occupying the area percentage of the density functions. Third, the values of indices φ and ψ in the benchmark period are computed. Lastly, the density functions of φ and ψ can be estimated, and the confidence limits CL_φ and CL_ψ are obtained.

After determining the confidence limits CL_φ and CL_ψ , the corresponding performance monitoring strategy can be

summarized as follows. If ψ return normal when φ violates its confidence limit, it indicates that current steady-state deviates from design operating point. This situation occurs because the compensation of the MPC controller counteracts the effects of model-mismatch or external disturbances on the process. Therefore, although the control performance has changed at this time, it is still within the adjustment range of the MPC controller, which makes the process dynamic not greatly affected. If φ goes beyond the confidence limit when a steady deviation is detected, it indicates that the process dynamics are affected, so the MPC performance is severely deteriorated and further maintenance actions should be taken. Two monitoring indices are used simultaneously to provide alert information.

D. MPC PERFORMANCE GRADING STRATEGY

For a multi-loop process control system, it is necessary to grade the control performance status to different levels. On the one hand, from a security perspective, the state with the worst performance degradation should be first concerned and maintained to prevent the control system from moving toward a fault. On the other hand, from an economic point of view, prioritizing the worst performance state can reduce the workload of performance maintenance and achieve greater economic returns. However, there are few studies on control performance grading. In this subsection, a performance grading strategy is presented to divide the current MPC performance state into four levels: improved performance, normal performance, degraded performance, and the worst performance. The worst performance status requires the highest priority for maintenance. When performing performance grading, not only performance monitoring information for detecting performance changes but also information for indicating the direction of performance change is required. The traditional SFA-based CPM method cannot give the direction of performance change [38]. To solve this problem, an SFA-based predictable performance assessment index is proposed.

As pointed out in ref. [40], if the predictability of an MPC controller increases, the performance of this controller is worse, and vice versa. From the equivalent objective function of SFA in (29), it can be seen that the slowest SF has the largest lag-1 autocorrelation. By substituting (9) into (29),

$$\omega_j = 2 - 2R_j^s(1), \quad (44)$$

where $R_j^s(1) \in [-1, 1]$, and $\omega_j \in [0, 4]$. For a time series signal, if the lag-1 autocorrelation is large, it means that the current value has a strong correlation with the value of the previous moment, i.e. the current value is significantly affected by the value of the previous moment. In this case, the accuracy of predicting the current value based on the value of the previous moment is high, that is, the time series has high predictability. From (44), we can get the following conclusions, if the value of $R_j^s(1)$ is close to -1 or 1 , ω_j is close to 1 and 4 respectively, and the predictability of s_j is

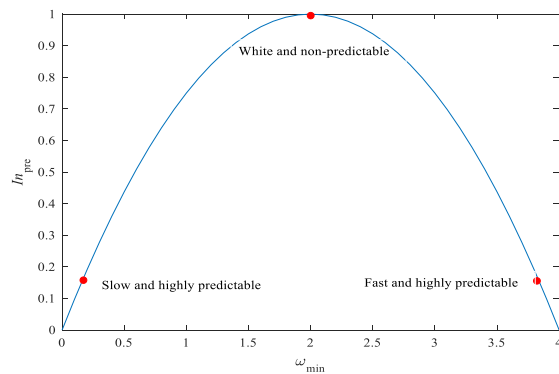


FIGURE 2. The relationship between generalized eigenvalues ω_{\min} and predictable index $I_{n_{\text{pre}}}$.

high. Conversely, if the value of $R_j^s(1)$ is close to 0 , ω_j is close to 2 , and the predictability of s_j is low.

It has been proved by (31) that if lag-1 autocorrelation of every original variable is close to 0 , the lag-1 autocorrelation of s_j is close to 0 . That is to say, if the overall predictability of original variables is low, the predictability of s_j will also be low, and vice versa. Thus, the predictability of original variables can be reflected by the distance between the generalized eigenvalue corresponding to the slowest SF and the value 2 . According to this connection, the SFA-based predictable index is constructed as

$$I_{n_{\text{pre}}} = 1 - \left(\frac{\omega_{\min} - 2}{2} \right)^2, \quad (45)$$

where ω_{\min} is the generalized eigenvalue corresponding to the slowest SF. If $I_{n_{\text{pre}}}$ is close to 1 , the original variables is non-predictable, and this means control performance is good. If $I_{n_{\text{pre}}}$ is close to 0 , the original variables is highly predictable, and this means control performance is poor. This relationship is shown in Fig.2.

According to (9) and (45), the SFA-based predictable index reveals one-step predictable information of the original variables and is used to performance assessment. This is different from the static and dynamic information of the original variables extracted by (41) and (42) for performance monitoring. In order to use the historical benchmark data, we further designed the predictable performance assessment index (PPAI) based on historical data:

$$\eta_{\text{pre}} = \frac{I_{n_{\text{pre}}}^{\text{act}}}{I_{n_{\text{pre}}}^{\text{ben}}} \quad (46)$$

where $I_{n_{\text{pre}}}^{\text{ben}}$ and $I_{n_{\text{pre}}}^{\text{act}}$ are predictable indices of the benchmark data and the current monitored data, respectively. If η_{pre} is less than 1 significantly, the current monitored performance is deteriorated. If η_{pre} is close to 1 , the current performance is as good as that of the benchmark. If η_{pre} is larger than 1 significantly, the current performance is better than the benchmark performance. In this paper, the 3σ criterion is used to determine whether PPAI is significantly greater than 1 or less than 1 . In order to obtain the mean and standard deviation of PPAI, a series of PPAIs under the benchmark

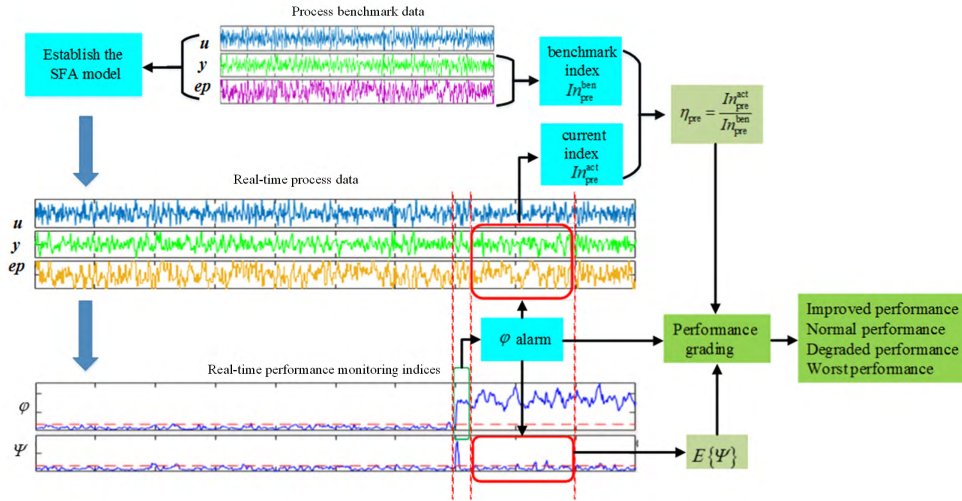


FIGURE 3. Schematic diagram of performance monitoring and grading strategy for MPC based on SFA.

data are calculated using the sliding window technique [29]. The length of the sliding window can be determined using the benchmark data. If the length of the sliding window is too large, the fluctuation of PPAI is small, and the response to the dynamic change of the process is slow. If the length of the sliding window is too small, the fluctuation of PPAI is large, which may cause false alarms due to inaccuracy or sensitivity. The selection of the window length should be a compromise between the fluctuation and the dynamic response of PPAI.

For a general controller, only the process closed-loop output data y is used to calculate the PPAI. Whereas, for the MPC controller, (25) shows that the prediction error ep also contains predictable information. Therefore, the PPAI is computed by using the variable set \tilde{x}

$$\tilde{x} = [y, ep] \in \mathbf{R}^{N \times m_p} \quad (47)$$

and this variable set data is augmented by using matrix stacking techniques with d -lag samples in (27).

By using performance monitoring indices and PPAI index, the SFA-based performance grading strategy can rank the current MPC performance as follows: if both performance monitoring indices φ and ψ do not exceed their confidence limits, current MPC performance is unchanged and graded as normal performance. If only the index φ exceeds the confidence limit and the value of PPAI is greater than 1 significantly, it means current MPC performance changes towards better direction and is graded as improved performance. If only the index φ exceeds the confidence limit and the value of PPAI is less than 1 significantly, it means the dynamic of the MPC system has not been significantly affected and current MPC performance is graded as degraded performance. If both performance monitoring indices φ and ψ exceed their confidence limits and the value of PPAI is less than 1 significantly, it means that the dynamic of the MPC system has been significantly affected and current MPC performance is graded as the worst performance.

E. MPC PERFORMANCE MONITORING AND GRADING STRATEGY

The SFA-based performance monitoring and grading strategy include an offline modeling stage and an online monitoring and grading stage, as shown in Fig. 3 and Fig. 4. The detailed performance monitoring and grading procedure can be described in detail as follows:

The offline modeling stage:

- 1) Select appropriate benchmark data for MPC systems from historical process data contained in the DCS database.
- 2) Based on the benchmark data and variable set $x = [u, y, ep] \in \mathbf{R}^{N \times m}$, obtain the extended data X and build the SFA model.
- 3) Select dominant subspace \bar{s}_d and residual subspace \bar{s}_e by the new criterion of selecting dominant SFs proposed in (34).
- 4) Calculate the performance monitoring indices φ and ψ of the benchmark data using (41) and (42), and determine the confidence limits CL_φ and CL_ψ using (43).
- 5) Calculate the SFA-based predictable index In_{pre}^{ben} using the stacked data of variable set $\tilde{x} = [y, ep] \in \mathbf{R}^{N \times m_p}$, and calculate a series of PPAI under the benchmark data by sliding window technology [29]. Then, obtain the mean $\bar{\eta}_{pre}^{ben}$ and standard deviation σ of PPAI.

The online monitoring and grading stage:

- 1) Current monitored data x_{new} is collected from the MPC process.
- 2) Use the data x_{new} to obtain the extended data X_{new} and compute the current SFs based on built SFA model.
- 3) Calculate the current performance monitoring indices φ and ψ using (41) and (42).
- 4) If both performance monitoring indices φ and ψ are within their confidence limits, current

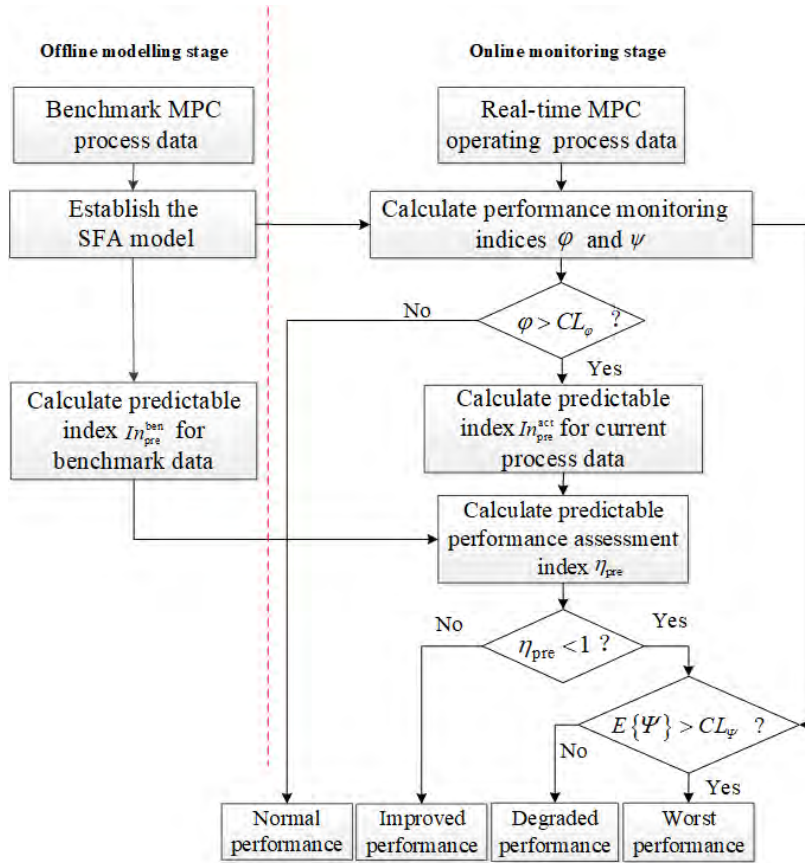


FIGURE 4. SFA-based performance monitoring and grading strategy flow chart.

MPC performance does not change and is graded as normal performance.

- 5) If steady-state monitoring index ϕ exceeds the confidence limit, then collect real-time process data \bar{x}_{new} of a sliding window after the current sample time to calculate the current predictable index I_{pre}^{act} , and calculate the mean of ψ in the same window. Then continue to step 6).
- 6) Calculate the current PPAI η_{pre}^{act} . If $\eta_{pre}^{act} > \bar{\eta}_{pre}^{ben} + 3\sigma$, current MPC performance is better than benchmark performance and graded as improved performance. If $\eta_{pre}^{act} < \bar{\eta}_{pre}^{ben} - 3\sigma$, continue to step 7)
- 7) When ϕ exceeds the confidence limit and $\eta_{pre}^{act} < \bar{\eta}_{pre}^{ben} - 3\sigma$, if the average value of ψ is within the confidence limit, current MPC performance is deteriorated and graded as degraded performance. If the average value of ψ exceeds the confidence limit, the dynamic of the MPC system has been significantly affected. Current MPC performance is seriously degraded and graded as the worst performance.

IV. CASE STUDIES

In this section, two MPC control processes are used to evaluate the proposed SFA-based performance monitoring and

grading method. The first one is a numerical simulation process, the other one is a well-known benchmark Wood-Berry distillation column. Performance monitoring and grading method proposed in this paper is compared with the traditional SFA-based CPM method.

A. A NUMERICAL SIMULATION PROCESS

In this section, two experimental cases are implemented on a 2×2 numerical simulation process, which is used in ref. [6] and ref. [27]. This process is given by

$$y(k) = G_p u(k) + G_d a(k), \quad (48)$$

where the process transfer function G_p and disturbance transfer function G_d are given by

$$G_p = \begin{bmatrix} q^{-1} & K_{12}q^{-1} \\ \frac{1-0.4q^{-1}}{1-0.1q^{-1}} & \frac{1-0.4q^{-1}}{1-0.8q^{-1}} \end{bmatrix}, \quad (49)$$

$$G_d = \begin{bmatrix} 1 & -0.6 \\ \frac{1-0.5q^{-1}}{0.5} & \frac{1-0.5q^{-1}}{1} \\ \frac{1-0.5q^{-1}}{1-0.5q^{-1}} & \frac{1-0.5q^{-1}}{1-0.5q^{-1}} \end{bmatrix}. \quad (50)$$

The noise $a(k)$ follows a standard Gaussian distribution with the covariance $\Sigma_a = \text{diag}\{0.01, 0.01\}$. The process is

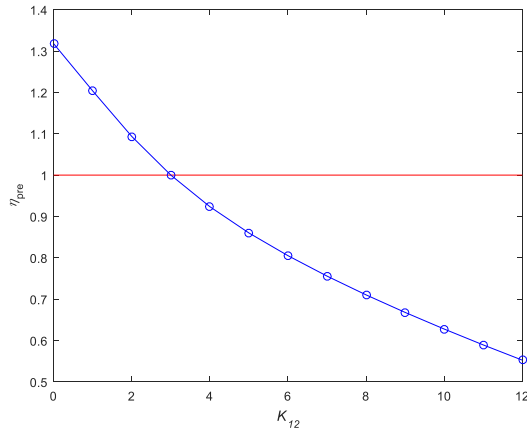


FIGURE 5. The trajectory of the predictable performance assessment index.

controlled by an MPC controller and the controller parameters are $P = 20$, $M = 1$, $Q = I$, and $R = 0.1I$. The set points of y_1 and y_2 are set to 1 and 0.5, respectively. The parameter K_{12} is constant and can be adjusted to alter the process model. When $K_{12} = 0$, the MPC control system operates in optimal control state. In order to verify that the proposed PPAI can indicate the direction of performance change, 2000 samples with model parameter $K_{12} = 3$ are set as performance benchmark data. The stacked data \tilde{X} of variable set $\tilde{x} = [y, ep]$ is with one lagged sample and used to calculate the benchmark predictable index m_{pre}^{ben} .

Firstly, the relationship between the PPAI value η_{pre} and the MPC performance is shown in this case study. As K_{12} increases from 0 to 12, the performance of the MPC system will change. 2000 samples are collected to calculate the predictable index m_{pre}^{act} and the PPAI η_{pre} . Fig. 5 shows the trajectory of the PPAI η_{pre} while K_{12} varies from 0 to 12. When $K_{12} < 3$, the value of PPAI is greater than 1, this means the current MPC performance is better than the benchmark performance and can be graded as improved performance. When $K_{12} > 3$, the value of PPAI is less than 1, this means the current MPC performance is worse than the benchmark performance and can be graded as degraded performance or the worst performance. When $K_{12} = 3$, the value of PPAI is equal to 1, this means the current MPC performance is the same as the benchmark performance and can be graded as normal performance. Thus, the PPAI η_{pre} is effective in assessing the direction of performance change.

Then, the proposed CPM method is compared with the traditional SFA-based CPM method. 2000 samples under the benchmark period are used to build the SFA model for performance monitoring. The significance level of performance monitoring indices is chosen as $\alpha = 0.95$ and the extended monitored data X is with one lagged sample throughout this article. For the CPM method proposed in this paper, the information expression weight matrix \tilde{W} in (32) is calculated as

Based on the proposed criterion, the dominant SFs are determined as $\tilde{s}_d = [s_1, s_3, s_5, s_6, s_7, s_9, s_{10}, s_{11}, s_{12}]^T$. The cumulative expression information for each original

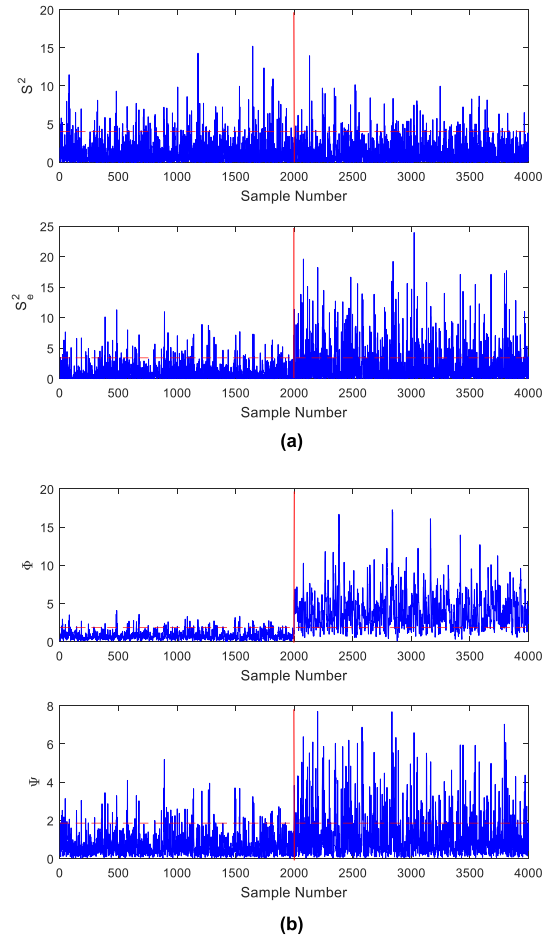


FIGURE 6. Monitoring results of the numerical example. (a) SFA based on dynamic statistics of CVs. (b) SFA method based on combined indices.

variable can be calculated as Cum_1 . Cum_1 and the variance of the values in Cum_1 are listed in Table 1. The dominant SFs selected by the traditional selection criterion are $s_d = [s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9]^T$, and the cumulative expression information Cum_2 is obtained. Cum_2 and the variance of the values in Cum_2 are also listed in Table 1.

From the Table 1, it can be seen that more than 60% feature information of each original variable is expressed in the dominant subspace selected by the proposed criterion. While in the dominant subspace selected by the traditional criterion, the feature information of original variables x_4 and x_7 are less than 50%, in other words, the information in these two variables is not fully expressed by the dominant subspace. Therefore, the dominant subspace selected by the proposed criterion can reflect the change of the original variables in a more balanced manner. Their variance values can also verify this conclusion. The variance of the values in Cum_1 is significantly smaller than the variance of the values in Cum_2 .

Immediately after building the SFA-based performance monitoring model, the parameter K_{12} is adjusted to 8 at the 2000th sample time in the monitored period, and the performance monitoring results are shown in Fig. 6, in which the monitoring indices are plotted as the solid blue line and

$$\tilde{W} = \begin{bmatrix} 0.0000 & 0.0009 & \mathbf{0.9597} & 0.0297 & 0.0003 & 0.0015 & 0.0001 & 0.0007 & 0.0052 & 0.0001 & 0.0017 & 0.0000 \\ 0.0090 & 0.0002 & 0.0166 & 0.0378 & 0.0056 & \mathbf{0.2335} & 0.0082 & 0.0952 & 0.3815 & 0.0017 & 0.2096 & 0.0010 \\ 0.0000 & 0.0216 & 0.0000 & 0.0056 & 0.1083 & 0.0031 & 0.2149 & 0.2306 & 0.0491 & \mathbf{0.2969} & 0.0009 & 0.0690 \\ 0.0032 & 0.0867 & 0.0027 & 0.0779 & 0.1878 & 0.0845 & 0.1070 & 0.0003 & 0.0002 & 0.2413 & 0.0420 & 0.1664 \\ 0.0481 & \mathbf{0.4046} & 0.0011 & 0.0064 & 0.0833 & 0.0384 & 0.0200 & 0.1682 & 0.0167 & 0.0032 & 0.1051 & 0.1050 \\ \mathbf{0.5086} & 0.0642 & 0.0012 & 0.1104 & 0.0681 & 0.0074 & 0.0256 & 0.0904 & 0.0283 & 0.0027 & 0.0003 & 0.0930 \\ 0.0082 & 0.1227 & 0.0030 & 0.0858 & \mathbf{0.2155} & 0.2317 & 0.0373 & 0.0153 & 0.0318 & 0.0492 & 0.1886 & 0.0110 \\ 0.3126 & 0.2587 & 0.0000 & 0.0205 & 0.0292 & 0.0733 & 0.0486 & 0.0911 & 0.0040 & 0.0108 & 0.0061 & 0.1452 \\ 0.0140 & 0.0013 & 0.0000 & 0.0011 & 0.0153 & 0.0714 & 0.0003 & 0.0033 & \mathbf{0.4287} & 0.0024 & \mathbf{0.4248} & 0.0375 \\ 0.0210 & 0.0010 & 0.0001 & 0.0017 & 0.0366 & 0.0001 & \mathbf{0.2721} & \mathbf{0.2874} & 0.0543 & 0.1876 & 0.0076 & 0.1306 \\ 0.0547 & 0.0294 & 0.0000 & 0.0115 & 0.1982 & 0.0029 & 0.2603 & 0.0014 & 0.0001 & 0.2041 & 0.0071 & \mathbf{0.2305} \\ 0.0207 & 0.0088 & 0.0154 & \mathbf{0.6116} & 0.0519 & 0.2522 & 0.0057 & 0.0163 & 0.0001 & 0.0002 & 0.0062 & 0.0109 \end{bmatrix} \quad (51)$$

TABLE 1. The values and variances of the cumulative amounts of expression information for original variables.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	σ_{Cum}^2
Cum_1	0.6753	0.6544	0.9806	0.8638	0.7774	0.6087	0.8361	0.8135	0.6143	0.7462	0.7423	0.6874	0.0123
Cum_2	0.9037	0.9609	0.9845	0.3751	0.7133	0.7448	0.4620	0.6950	0.9455	0.6081	0.9792	0.6280	0.0433

the confidence limit is plotted as dashed red line. From the Fig. 6(a), it can be seen that after the 2000th sample time, the statistic S^2 almost no change and the fluctuation of the statistic S_e^2 becomes larger and the sample number of statistic S_e^2 exceeding the confidence limit increases. In Fig. 6(b), the monitoring index φ exceeds the confidence limit after the 2000th sample time and the change of the monitoring index ψ is similar to the statistic S_e^2 . From the experimental results of the two methods, it can be seen that compared with the traditional SFA-based CPM method, the method proposed in this paper has a better effect on MPC performance monitoring.

B. WOOD BERRY DISTILLATION COLUMN

In this section, further experimental studies are performed on the Wood Berry distillation column process, which separates methanol from water. This process has been used in many previous investigations for CPM, and the transfer function of the process is as below [3], [27], [29]:

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-K_{12} \times 18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix} + \begin{bmatrix} \frac{3.8e^{-8s}}{14.9s+1} \\ \frac{K_2 \times 4.9e^{-3s}}{13.2s+1} \end{bmatrix} a(s) \quad (52)$$

where y_1 and y_2 are distillate and bottom products, respectively, u_1 and u_2 are reflux and steam flow rates, respectively. a is the feed flow rate and the standard deviation is 0.003. The MPC controller parameters are $P = 50$, $M = 2$, $Q = I$, and $R = 0.1I$. The set points of y_1 and y_2 are set to 1 and

0.5, respectively, and the constraints of output variables are $-1.2 \leq y_1 \leq 1.2$, $-1.2 \leq y_2 \leq 1.2$, respectively. K_{12} and K_2 are the constants that can be adjusted to change the process model and the disturbance model, respectively. When $K_{12} = 1$ and $K_2 = 1$, MPC control system operates in optimal control state. In order to simulate four different performance levels, 3000 data samples with model parameters $K_{12} = 0.7$, $K_2 = 1$ are chosen as performance benchmark data. They are used to build the SFA-based performance monitoring model and calculate a series of the SFA-based predictable index In_{pre}^{ben} by the sliding window technology, where window width is set to 2000 samples. Then, the mean $\bar{\eta}_{pre}^{ben} = 1.0191$, standard deviation $\sigma = 0.0089$, and the confidence limits $CL_\varphi = 1.7076$ and $CL_\psi = 1.8649$ under the benchmark data. If 10 consecutive samples of the steady-state monitoring index φ exceed the confidence limit, it is considered as a performance change.

Performance monitoring and grading method proposed in this paper is compared with the traditional SFA method based on dynamic statistics of CVs. Four different cases of performance changes are studied, which are listed in Table 2. These four cases are model-mismatch (PC1, PC2), the disturbance perturbation (PC3), and the constraint saturation (PC4).

For PC1, the parameter K_{12} is adjusted to 1 at the 3000th sample time in the monitored period, thereby incurring performance change. Fig. 7 presents the monitoring results of those two approaches. We can see that the statistics S^2 and S_e^2 are within their confidence limits in Fig. 7(a) and the SFA method based on dynamic statistics fails to detect the performance change. In Fig. 7(b), at the 3031st sample time, 10 consecutive samples of the steady-state monitoring index φ exceed the confidence limit. Then collect real-time process

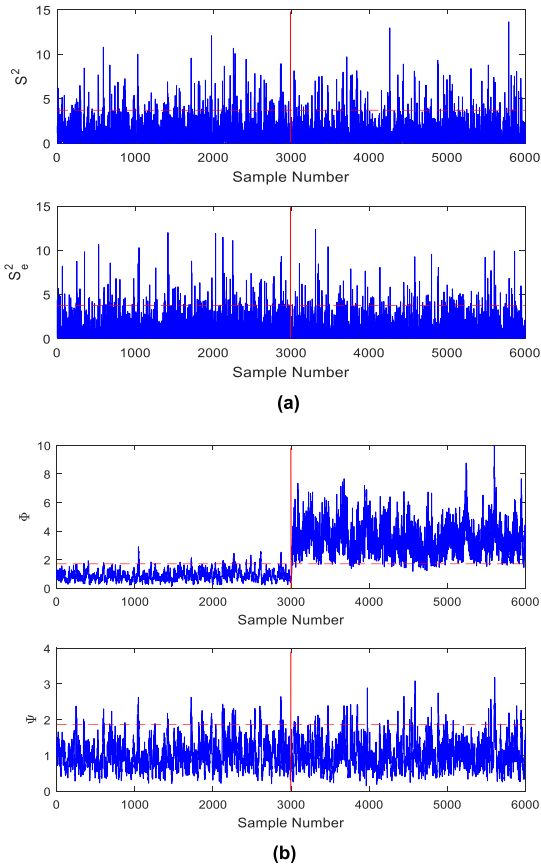


FIGURE 7. Monitoring results of wood berry distillation column of PC1. (a) SFA method based on dynamic statistics of CVs. (b) SFA based on combined indices.

TABLE 2. Parameter setting for performance change factors.

Case	Operation condition	parameter	variation range
PC1	Model-mismatch	K_{12}	0.7 → 1
PC2	Model-mismatch	K_{12}	0.7 → 1.8
PC3	Disturbance perturbation	K_2	1 → 2
PC4	Constraint saturation	Output constraint	1.2 → 0.97

data \tilde{x}_{new} of a window with 2000 samples after the 3031st sample time to calculate the current PPAI, $\eta_{pre} = 1.1325 > \bar{\eta}_{pre}^{ben} + 3\sigma$, and the mean of the dynamic monitoring index ψ in the same window, $E\{\psi\} = 1.0705 < CL_{\psi}$. From the result of the proposed method, we can see that the change of process model makes the MPC control system reach a new steady state without significantly changing the system dynamic characteristics. Meantime, the current PPAI value is significantly greater than that in the benchmark period, this means current control performance is better than benchmark performance. This conclusion is consistent with prior knowledge. Therefore, the current performance is classified as improved performance according to the SFA-based performance monitoring and grading strategy in Fig. 4.

For PC2, the parameter K_{12} is adjusted to 1.8 at the 3000th sample time. From the monitoring results in Fig. 8, we can see

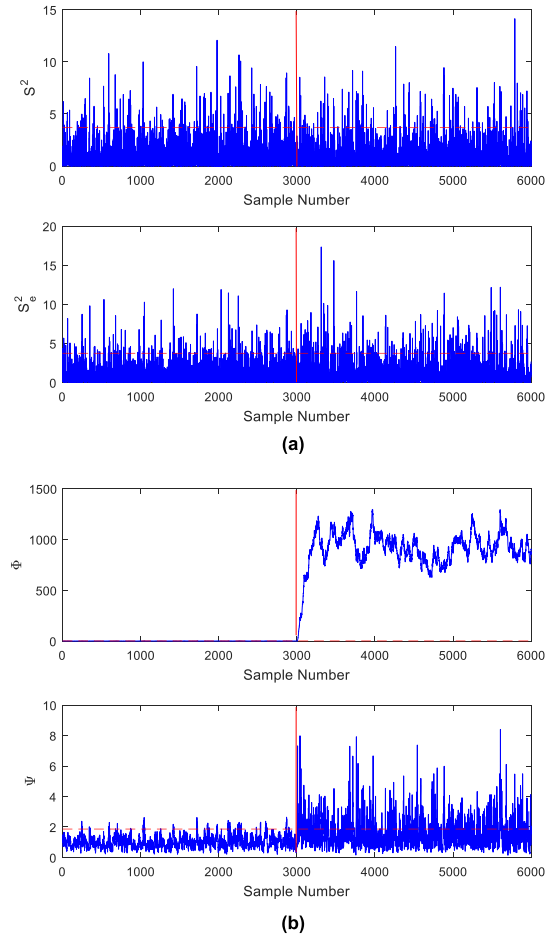


FIGURE 8. Monitoring results of wood berry distillation column of PC2. (a) SFA method based on dynamic statistics of CVs. (b) SFA based on combined indices.

that the statistics S^2 and S_e^2 do not exceed their confidence limits in Fig. 8(a) and the SFA method based on dynamic statistics fails to detect the performance change. In Fig. 8(b), 10 consecutive samples of φ are beyond the confidence limit at the 3018th sample time. Then collect real-time process data \tilde{x}_{new} of a window with 2000 samples after the 3018th sample time to calculate the current PPAI, $\eta_{pre} = 0.8276 < \bar{\eta}_{pre}^{ben} - 3\sigma$, that is, the current control performance is worse than benchmark performance. The statistic ψ fluctuates around the confidence limit, but its mean $E\{\psi\} = 1.6003 < CL_{\psi}$. According to the SFA-based performance monitoring and grading strategy, the current performance is classified as degraded performance.

For PC3, the parameter K_2 is increased to 2 at the 3000th sample time. From the monitoring results in Fig.9, the fluctuation amplitude of monitoring statistics S^2 becomes significantly larger and the performance change is detected by the traditional SFA-based CPM method. In Fig. 9(b), steady-state monitoring index φ overrun the confidence limit, and the detection time is the 3023rd sample time. Then the current PPAI $\eta_{pre}=0.8959 < \bar{\eta}_{pre}^{ben} - 3\sigma$, and $E\{\psi\} = 3.4505 > CL_{\psi}$. According to the result of the proposed method, the current performance is classified as the worst performance.

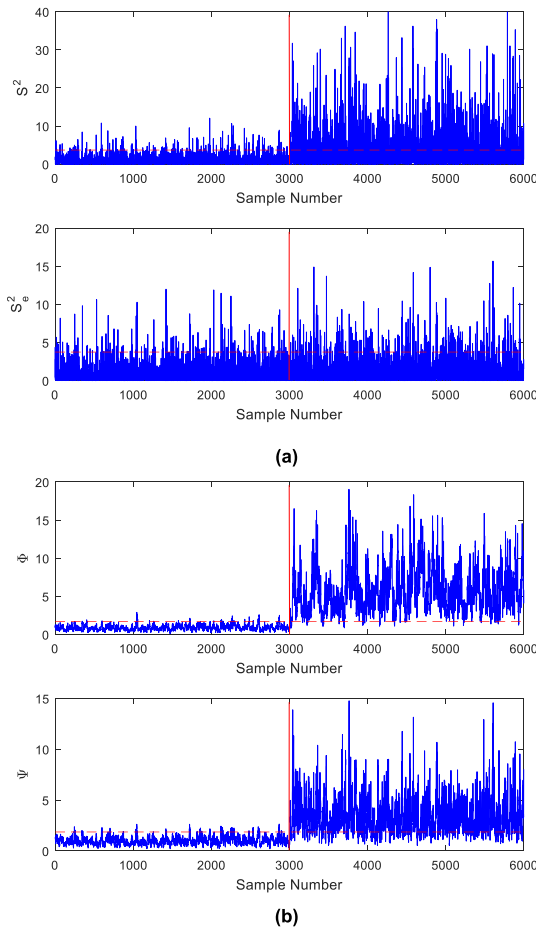


FIGURE 9. Monitoring results of wood berry distillation column of PC3. (a) SFA method based on dynamic statistics of CVs. (b) SFA based on combined indices.

TABLE 3. Performance grading results based on SFA FOR MPC.

Case	$\varphi > CL_\varphi$?	change trend of η_{pre}	$E\{\Psi\} > CL_\Psi$?	performance level
PC1	Yes	↑	No	improved performance
PC2	Yes	↓	No	degraded performance
PC3	Yes	↓	Yes	the worst performance
PC4	Yes	↓	Yes	the worst performance

For PC4, the output constraint saturation is adjusted to 0.97 at the 3000th sample time and the monitoring results are shown in Fig. 10. From the Fig. 10, we can see that the SFA method based on dynamic statistics does not detect performance change, while the proposed method significantly detects the performance change. In Fig. 10(b), 10 consecutive samples of φ are beyond the confidence limit at the 3012th sample time, the current PPAI $\eta_{pre} = 0.0766 < \bar{\eta}_{pre}^{ben} - 3\sigma$, and $E\{\psi\} = 288.8160 > CL_\psi$. According to the result of the proposed method, the current performance is classified as the worst performance.

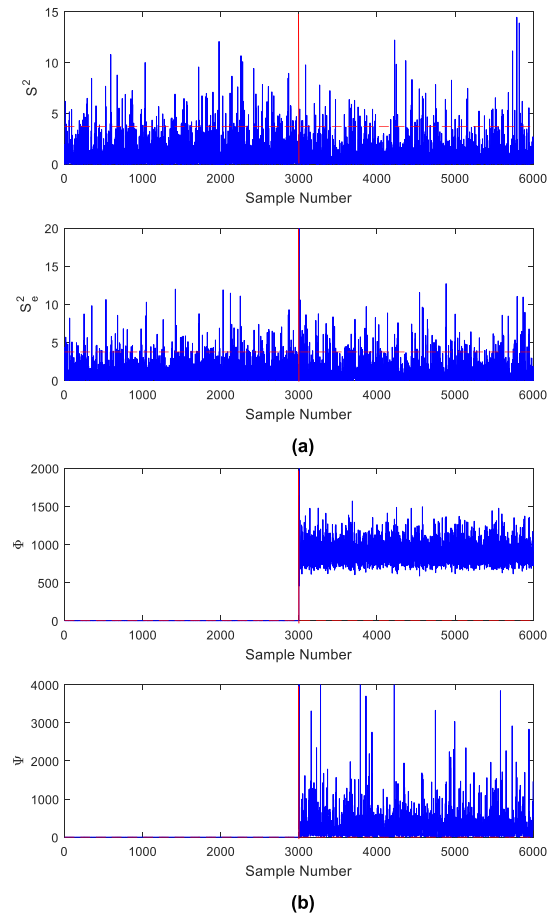


FIGURE 10. Monitoring results of wood berry distillation column of PC4. (a) SFA method based on dynamic statistics of CVs. (b) SFA based on combined indices.

From the above experiments, it can be seen that the traditional SFA method based on dynamic statistics of CVs detects the MPC performance changes in PC3, and cannot indicate the direction of performance change. The performance monitoring and grading methods proposed in this paper can effectively detect MPC performance changes in PC1-PC4, indicate the direction of performance change, and provide more information to classify the current MPC performance state to four levels, as shown in table 3.

V. CONCLUSIONS

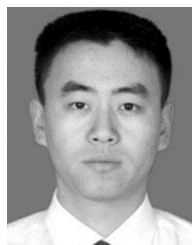
In this article, a control performance monitoring and grading strategy for MPC is established based on improved SFA. The new criterion for selecting the dominant SFs and the predictable performance assessment index are designed based on the mapping relationship between the lag-1 autocorrelations of SFs and the lag-1 autocorrelations of original variables. The dominant SFs selected using the new criterion can more comprehensively express the original variables, thus improving the monitoring effect. And then two combined monitoring indices are established to monitor steady-state and dynamic changes of MPC systems, respectively. According to the relationship between predictability and control

performance, the SFA-based predictable performance assessment index can effectively indicate the direction of change in MPC performance. Finally, the SFA-based performance grading strategy is established by combining two monitoring indices and predictable performance assessment index. The simulation results from the numerical system case study and the benchmark Wood Berry distillation column process case study demonstrate that the proposed performance monitoring method outperforms conventional SFA-based CPM method for MPC systems. In addition, it has performance grading ability, which is very useful for subsequent performance maintenance. In addition, MPC is a multi-step predictive control algorithm, but the PPAI index only reflects one-step predictable information of the original variables. In the future, the PPAI index should be extended to multi-step predictable information to better assess MPC performance.

REFERENCES

- [1] Y.-G. Xi, D.-W. Li, and S. Lin, "Model predictive control—Status and challenges," *Acta Automatica Sinica*, vol. 39, no. 3, pp. 222–236, Mar. 2013.
- [2] M. L. Darby and M. Nikolaou, "MPC: Current practice and challenges," *Control Eng. Pract.*, vol. 20, no. 4, pp. 328–342, Apr. 2012.
- [3] M. Kano and M. Ogawa, "The state of the art in chemical process control in Japan: Good practice and questionnaire survey," *J. Process Control*, vol. 20, no. 9, pp. 969–982, Oct. 2010.
- [4] X. Tian, G. Chen, and S. Chen, "A data-based approach for multivariate model predictive control performance monitoring," *Neurocomputing*, vol. 74, no. 4, pp. 588–597, Jan. 2011.
- [5] J. Chen, Y. Jie, and J. Mori, "Closed-loop subspace projection based state-space model-plant mismatch detection and isolation for MIMO MPC performance monitoring," in *Proc. 52nd IEEE Conf. Decision Control (CDC)*, Florence, Italy, Dec. 2013, pp. 6143–6148.
- [6] L. Shang, X. Tian, and L. Cai, "A multi-index control performance assessment method based on historical prediction error covariance," *IFAC-PapersOnLine*, vol. 50, no. 1, pp. 13892–13897, 2017.
- [7] T. J. Harris, "Assessment of control loop performance," *Can. J. Chem. Eng.*, vol. 67, no. 5, pp. 856–861, Oct. 1989.
- [8] T. J. Harris, F. Boudreau, and J. F. MacGregor, "Performance assessment of multivariable feedback controllers," *Automatica*, vol. 32, no. 11, pp. 1505–1518, 1996.
- [9] B. Huang, S. L. Shah, and R. Miller, "Feedforward plus feedback controller performance assessment of MIMO systems," *IEEE Trans. Control Syst. Technol.*, vol. 8, no. 3, pp. 580–587, May 2000.
- [10] L. Desborough and T. Harris, "Performance assessment measures for univariate feedforward/feedback control," *Can. J. Chem. Eng.*, vol. 71, no. 4, pp. 605–616, Aug. 1993.
- [11] B.-S. Ko and T. F. Edgar, "Performance assessment of cascade control loops," *AIChE J.*, vol. 46, no. 2, pp. 281–291, Feb. 2000.
- [12] B. Huang and S. L. Shah, *Performance Assessment of Control Loops: Theory and Applications*. Berlin, Germany: Springer, 1999.
- [13] B. Huang and R. Kadali, *Dynamic Modeling, Predictive Control and Performance Monitoring: A Data-Driven Subspace Approach*. Springer, 2008.
- [14] M. Jelali, *Control Performance Management in Industrial Automation*. London, U.K.: Springer, 2013.
- [15] S. J. Qin, "Control performance monitoring—A review and assessment," *Comput. Chem. Eng.*, vol. 23, no. 2, pp. 173–186, Dec. 1998.
- [16] T. J. Harris, C. T. Seppala, and L. D. Desborough, "A review of performance monitoring and assessment techniques for univariate and multivariate control systems," *J. Process Control*, vol. 9, no. 1, pp. 1–17, 1999.
- [17] M. Jelali, "An overview of control performance assessment technology and industrial applications," *Control Eng. Pract.*, vol. 14, no. 5, pp. 441–466, 2006.
- [18] S. J. Qin and J. Yu, "Recent developments in multivariable controller performance monitoring," *J. Process Control*, vol. 17, no. 3, pp. 221–227, Mar. 2007.
- [19] Y. Shardt et al., "Determining the state of a process control system: Current trends and future challenges," *Can. J. Chem. Eng.*, vol. 90, no. 2, pp. 217–245, Apr. 2012.
- [20] X. Gao, F. Yang, C. Shang, and D. Huang, "A review of control loop monitoring and diagnosis: Prospects of controller maintenance in big data era," *Chin. J. Chem. Eng.*, vol. 24, no. 8, pp. 952–962, Aug. 2016.
- [21] M. Bauer, A. Horch, L. Xie, M. Jelali, and N. Thornhill, "The current state of control loop performance monitoring—A survey of application in industry," *J. Process Control*, vol. 38, pp. 1–10, Feb. 2016.
- [22] R. Kadali and B. Huang, "Controller performance analysis with LQG benchmark obtained under closed loop conditions," *ISA Trans.*, vol. 41, no. 4, pp. 521–537, Oct. 2002.
- [23] B. Huang and S. L. Shah, "Practical issues in multivariable feedback control performance assessment," in *Proc. IFAC ADCHEM*, Banff, AB, Canada, 1997, pp. 429–434.
- [24] A. Y. Sendjaja and V. Kariwala, "Achievable PID performance using sums of squares programming," *J. Process Control*, vol. 19, no. 6, pp. 1061–1065, 2009.
- [25] J. Yu and S. J. Qin, "Statistical MIMO controller performance monitoring. Part I: Datadriven covariance benchmark," *J. Process Control*, vol. 18, nos. 3–4, pp. 277–296, Mar. 2008.
- [26] J. Yu and S. J. Qin, "Statistical MIMO controller performance monitoring. Part II: Performance diagnosis," *J. Process Control*, vol. 18, nos. 3–4, pp. 297–319, 2008.
- [27] C. Li, B. Huang, D. Zheng, and F. Qian, "Multi-input–multi-output (MIMO) control system performance monitoring based on dissimilarity analysis," *Ind. Eng. Chem. Res.*, vol. 53, no. 47, pp. 18226–18235, Nov. 2014.
- [28] Z. Yan, C.-L. Chan, and Y. Yao, "Multivariate control performance assessment and control system monitoring via hypothesis test on output covariance matrices," *Ind. Eng. Chem. Res.*, vol. 54, no. 19, pp. 5261–5272, Apr. 2015.
- [29] L.-Y. Shang, X.-M. Tian, Y.-P. Cao, and L.-F. Cai, "MPC performance monitoring and diagnosis based on dissimilarity analysis of PLS cross-product matrix," *Acta Automatica Sinica*, vol. 43, no. 2, pp. 271–279, Feb. 2017.
- [30] Q. Zhang and S. Li, "Performance monitoring and diagnosis of multivariable model predictive control using statistical analysis," *Chin. J. Chem. Eng.*, vol. 14, no. 2, pp. 207–215, Apr. 2006.
- [31] X. Deng and J. Deng, "Incipient fault detection for chemical processes using two-dimensional weighted SLKPCA," *Ind. Eng. Chem. Res.*, vol. 58, no. 6, pp. 2280–2295, Jan. 2019.
- [32] A. AlGhazzawi and B. Lennox, "Model predictive control monitoring using multivariate statistics," *J. Process Control*, vol. 19, no. 2, pp. 314–327, 2009.
- [33] L. Wiskott and T. J. Sejnowski, "Slow feature analysis: Unsupervised learning of invariances," *Neural Comput.*, vol. 14, no. 4, pp. 715–770, Apr. 2002.
- [34] X. Deng, X. Tian, and X. Hu, "Nonlinear process fault diagnosis based on slow feature analysis," in *Proc. 10th World Congr. Intell. Control Automat. (WCICA)*, Jul. 2012, pp. 3152–3156.
- [35] C. Shang, F. Yang, X. Gao, X. Huang, J. A. K. Suykens, and D. Huang, "Concurrent monitoring of operating condition deviations and process dynamics anomalies with slow feature analysis," *AIChE J.*, vol. 61, no. 11, pp. 3666–3682, May 2015.
- [36] H. Zhang, X. Tian, and X. Deng, "Batch process monitoring based on multiway global preserving kernel slow feature analysis," *IEEE Access*, vol. 5, pp. 2696–2710, 2017.
- [37] H. Zhang, X. Tian, X. Deng, and Y. Cao, "Multiphase batch process with transitions monitoring based on global preserving statistics slow feature analysis," *Neurocomputing*, vol. 293, pp. 64–86, Jun. 2018.
- [38] C. Shang, B. Huang, F. Yang, and D. Huang, "Slow feature analysis for monitoring and diagnosis of control performance," *J. Process Control*, vol. 39, pp. 21–34, Mar. 2016.
- [39] C. Zhao and B. Huang, "Control performance monitoring with temporal features and dissimilarity analysis for nonstationary dynamic processes," *IFAC-PapersOnLine*, vol. 51, no. 18, pp. 357–362, 2018.
- [40] Y. Zhao, J. Chu, H. Su, and B. Huang, "Multi-step prediction error approach for controller performance monitoring," *Control Eng. Pract.*, vol. 18, no. 1, pp. 1–12, Jan. 2010.
- [41] B. Srinivasan, T. Spinner, and R. Rengaswamy, "Control loop performance assessment using detrended fluctuation analysis (DFA)," *Automatica*, vol. 48, no. 7, pp. 1359–1363, Jul. 2012.

- [42] B. Song, Y. Ma, and H. Shi, "Improved performance of process monitoring based on selection of key principal components," *Chin. J. Chem. Eng.*, vol. 23, no. 12, pp. 1951–1957, Nov. 2015.
- [43] T. Blaschke, P. Berkes, and L. Wiskott, "What is the relation between slow feature analysis and independent component analysis?" *Neural Comput.*, vol. 18, no. 10, pp. 2495–2508, Oct. 2006.
- [44] X. Deng, X. Tian, S. Chen, and C. J. Harris, "Nonlinear process fault diagnosis based on serial principal component analysis," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 3, pp. 560–572, Mar. 2016.
- [45] L. Cai, X. Tian, and S. Chen, "Monitoring nonlinear and non-Gaussian processes using Gaussian mixture model-based weighted kernel independent component analysis," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 28, no. 1, pp. 122–135, Jan. 2017.



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