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Design of Mapping Matrix for Arithmetic Bit-Interleaved Coded Modulation

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ABSTRACT Arithmetic bit-interleaved coded modulation (A-BICM) is a coded rate compatible modulation (RCM). It achieves progressive mapping to strike a trade-off between high signal-to-noise ratio (SNR) and low SNR. However, the weights for parity bits are random and not sufficient, which decreases the performance significantly. In order to address the problem, this paper proposes a novel weight mapping scheme for A-BICM. The proposed scheme aims to design the degree and the weight distribution in the mapping matrix to decrease the error probability of parity bits. Through analysis, we have found that the proposed scheme is beneficial to increase the minimum Euclidean distance of symbols. Through simulation, the proposed scheme is verified to increase the throughput of transmission and decrease the demapping complexity at the same time. Both analysis and simulation results show that the proposed scheme outperforms the original A-BICM.

INDEX TERMS Rate compatible modulation, arithmetic bit-interleaved coded modulation, mapping matrix, throughput, complexity.

I. INTRODUCTION

In wireless communication one of key research is to transmit information effectively and reliably over a time-varying channel. Adaptive transmission techniques are effective approach to improve bandwidth utilization. Many adaptive schemes have been widely researched like adaptive coding and modulation (ACM) [1], [2], Raptor codes [3], Spinal codes [4] and analog fountain codes [5]. Rate compatible modulation (RCM) as a novel modulation technology [6], [7] do not require channel information at the transmitter and achieves seamless rate adaptation. In many scenarios, such as deep space communication and smart grid [8], [9], time delay is an important metric. What's more, the transmission time delay of RCM is small compared to ACM. RCM achieves significant throughput gain in moderate-to-high signal-to-noise ratio (SNR). How to improve the throughput performance of RCM in low SNR is a necessary research.

Since Shannon's landmark paper [10], finding coding scheme to approach channel capacity has been a central research. Under time-varying channel, effectiveness and reliability of information transmission are inherently contradictory. According to Shannon theory, combining higher-order modulation and channel coding is an effective way to increase the throughput.

In the past few years, many coding and modulation schemes have been proposed. Multilevel coding as a pioneer work is firstly proposed in [11]. In 1982, trellis-coded modulation (TCM) was proposed by Ungerboeck [12]. TCM which maximizes Euclidean distance of codeword achieves performance improvement under addictive white Gaussian noise (AWGN) channels, but the improvement under the fading channel is not obvious. The reason is that code performance depends on minimum Euclidean distance in AWGN and code diversity in fading channels.

In 1992, Zehavi proposed bit-interleaved coded modulation (BICM) by introducing a bitwise interleaver at the transmitter based on TCM. BICM in [13], [14] which add a bitwise interleaver between coding and modulation to maximize the dispersion of codewords and get great performance in fading channel. BICM is an error control technique that combines error correcting coding and modulation as a whole. Meanwhile, BICM achieves diversity gain and code gain.

Inspired by the above techniques, Wu *et al*. proposed arithmetic bit interleaved coded modulation (A-BICM) [15] by cascading systematic low-density parity check (LDPC) codes and RCM. A large mapping matrix is designed for the specific implementation of A-BICM. Compared to original RCM, A-BICM achieves greater throughput gain at low SNR.

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However, A-BICM does not take the weights corresponding to parity bits into the mapping matrix design. The weight distribution of mapping matrix corresponding to the parity bits is random, which make the energy of parity bits is not average. Furthermore, the weights corresponding to parity bits are less, which makes the energy of the parity bits carried by symbols is low. These two problems decrease the performance significantly.

In this paper, a step is taken towards the design of mapping matrix of A-BICM by using the proposed scheme. Due to the reason that the energy of information bits is much larger than the energy of parity bits, parity bits have a much greater impact on the error probability of the entire bits. Through analysis, we draw a conclusion that increasing the number of weights corresponding to the parity bits is beneficial to increase the minimum Euclidean distance of codewords. A novel mapping scheme is proposed to increase the degree of mapping matrix corresponding to the parity bits and the weights corresponding to parity bits are more average. Furthermore, we choose a simpler weight set for the proposed scheme, which decrease the computation complexity of demapping.

The rest of the paper is organized as follows. Section II reviews the related work. Section III presents the proposed weight mapping scheme. Section IV analyzes the influence of the proposed scheme. Performance evaluations are provided in Section V. In the Section VI, conclusion of the paper is made.

II. NOTATION AND RELATED WORK

In this section, we will first provide the notation, and then show the related work on original RCM and original A-BICM.

A. NOTATION

In this paper, $\boldsymbol{b}$ denotes a vector of source bits and the length of $\boldsymbol{b}$ is n. $\boldsymbol{b}$ is encoded with a systematic LDPC code and $\boldsymbol{x}$ denotes the coded sequence. The length of $\boldsymbol{x}$ is $(n+p)$. $\boldsymbol{u}$ is a symbol vector generated by RCM and the length of $\boldsymbol{u}$ is $(n+p)$. $\boldsymbol{r}$ is the received signal through addictive white Gaussian (AWGN) channel and the length of $\boldsymbol{r}$ is $(n+p)$. $\boldsymbol{e}$ is the vector of AWGN channel noise with mean zero and variance σ^2 and the length of $\boldsymbol{e}$ is $(n+p)$. $\boldsymbol{G}$ denotes the mapping matrix of RCM and the size of $\boldsymbol{G}$ is $n \times (n+p)$. $\boldsymbol{G_a}$ denotes the mapping matrix of A-BICM and the size of $\boldsymbol{G_a}$ is $(n+m)\times(n+p)$. $\boldsymbol{G_n}$ denotes the mapping matrix of the proposed scheme and the size of $\boldsymbol{G_n}$ is also $(n+m)\times(n+p)$.

B. ORIGINAL RCM

RCM is essentially a bit-to-symbol mapping. $\boldsymbol{x} = \{x_1, x_2, \ldots, x_{n+p}\}$ is a coded sequence. Let $\boldsymbol{w} = \{w_1, w_2, \ldots, w_L\}$ be the weight set and $\boldsymbol{w}$ is a multiset. It should be emphasized that the weights are allowed to appear more than once. The transmitter randomly selects L coded bits to generate one modulation symbol u_i by arithmetic summation. L is called the degree of the mapping matrix.

$$u_i = \sum_{l=1}^{L} w_l x_{i_l}, \tag{1}$$

where $w_l \in \boldsymbol{w}$ and the subscript i_l is the index of bit that is weighted by w_l in modulation symbol u_i. When the number of transmitted symbols is $(n+p)$, (1) can be rewritten into a matrix form:

$$\boldsymbol{u} = \boldsymbol{G}\boldsymbol{x}, \tag{2}$$

where $\boldsymbol{u} = \{u_1, u_2, \ldots, u_i, \ldots, u_{n+p}\}$. Due to the certain weight set $\boldsymbol{w}$, the discrete modulation alphabet is also defined as:

$$\chi = \left\{ \sum_{l=1}^{L} w_l x_{i_l} | x \in \{0,1\}^L, w_l \in \boldsymbol{w} \right\}. \tag{3}$$

As an adaptive transmission technology, it is similar to LT code [16]. The transmitter will continuously transmit the symbols and stops transmitting until an acknowledgment (ACK) which represents successful decoding is received. At the receiver, the received signal $\boldsymbol{r} = \{r_1, r_2, \ldots, r_i, \ldots, r_{n+p}\}$ is given as:

$$\boldsymbol{r} = \boldsymbol{G}\boldsymbol{x} + \boldsymbol{e}, \tag{4}$$

where $\boldsymbol{e} = \{e_1, e_2, \ldots, e_i, \ldots, e_{n+p}\}$. The maximum a posteriori probability (MAP) estimation is able to demodulate the transmitted bits. The process of estimation is to find the optimal solution for the following problem:

$$\hat{\boldsymbol{x}} = argmax\ p(\boldsymbol{x}|\boldsymbol{r}). \tag{5}$$

In order to solve Equation (5), a belief propagation (BP) decoding algorithm was proposed in [6] which borrowed a similar idea in [17]. To lower the computational complexity, many improved decoding algorithms [7], [18]–[20] have been proposed.

C. ORIGINAL A-BICM

In [21], [22], superposition mapping (SM) has been studied. The SM is divided into equal-power allocation (EPA; Type I sigma-mapping), unequal-power allocation (UPA) and group power allocation (GPA; UPA and GPA are Type II sigma-mapping). SM has been thought to outperform BICM with phase shift keying (PSK) modulation or quadrature amplitude modulation (QAM). Mentioned in [15], RCM is essentially an interleaving sigma-mapping. Therefore, RCM is able to replace the process of interleaving and modulation.

Through many experiments [6], RCM can get great throughput performance at high SNR region. However, it is not ideal at middle or low SNR region. A-BICM is essentially a joint design of LDPC codes and sigma-mapping. The original model of A-BICM is shown in Fig. 1.

A-BICM chooses a systematic LDPC code as the error correcting code. The rate of systematic LDPC codes is

FIGURE 1. Conceptual block diagram of A-BICM.

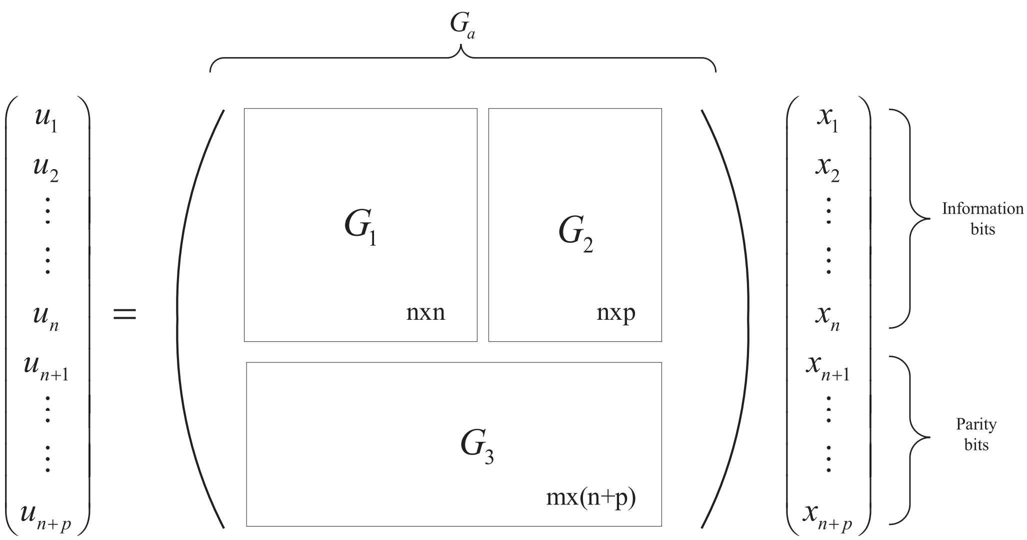

FIGURE 2. The mapping process of A-BICM in matrix form.

$n/(n+p)$ and the length of the LDPC codes is $n+p$. Its specific implementation is based on the design of mapping matrix (shown in Fig. 2). A-BICM design a mapping matrix $\boldsymbol{G_a}$ to finish the concatenation of the systematic LDPC codes and RCM. $\boldsymbol{G_1}$ and $\boldsymbol{G_3}$ are original RCM mapping matrix (the detail of construction method can be found in [6]), where the size of $\boldsymbol{G_1}$ is $n \times n$ and the size of $\boldsymbol{G_3}$ is $m \times (n+p)$. G_2 is just an all-zero matrix and its size is $n \times p$. The design guarantees that the parity bits of the LDPC codes will work when the SNR is low. In other words, LDPC codes will not work at high SNR.

It should be noted that the weights of $\boldsymbol{G_3}$ corresponding to the parity bits is random. The joint matrix construction method of A-BICM destroys the characteristics of the weight average distribution. However, the property is a crucial factor in the construction of RCM mapping matrix. Moreover, the weights corresponding to parity bits are less. In fact, the values of weight represent how much energy will be delivered to bits. Therefore, the energy of the parity bit is less. We will redesign the mapping matrix to not only satisfy the properties of the original RCM mapping matrix but also increase the energy of parity bits.

III. PROPOSED WEIGHT MAPPING SCHEME

In the proposed mapping scheme, a construction method is proposed for $\boldsymbol{G_3}$. In original A-BICM, $\{\pm1, \pm2, \pm4, \pm4, \pm8\}$ is the weight set of $\boldsymbol{G_1}$ and $\boldsymbol{G_3}$. As for the proposed scheme, we will choose a simpler weight set $\{\pm1, \pm2, \pm4, \pm8\}$ for $\boldsymbol{G_3}$ rather than $\{\pm1, \pm2, \pm4, \pm4, \pm8\}$ and $\boldsymbol{G_1}$ uses the same weight set as A-BICM. Assume the degree of $\boldsymbol{G_1}$ is $L^{(1)}$ and the degree of $\boldsymbol{G_3}$ is $L^{(2)}$. Therefore, in the proposed mapping scheme, the degree of $\boldsymbol{G_1}$ is 10 ($L^{(1)} = 10$) the degree of $\boldsymbol{G_3}$ is 8 ($L^{(2)} = 8$). The simpler weight set will reduce the complexity of demapping.

Our construction method is based on two principles: 1. The new mapping matrix should follow the original restrictions of RCM mapping matrix [6]. 2. The part of mapping matrix corresponding to the parity bits should be follow the restriction of RCM mapping matrix.

Based on the two principles, constructing a novel mapping matrix is divided into the following three steps. Firstly, We divided the $\boldsymbol{G_3}$ into two matrices denoted as $\boldsymbol{G_{31}}$ and $\boldsymbol{G_{32}}$ as Fig. 3. The sizes of $\boldsymbol{G_{31}}$ and $\boldsymbol{G_{32}}$ are $m \times n$ and $m \times p$ respectively. The degree of $\boldsymbol{G_{31}}$ and $\boldsymbol{G_{32}}$ are denoted as L_1 and L_2, respectively. Therefore, $L^{(2)} = L_1 + L_2$. In order to satisfy the property of the RCM mapping matrix, L_1 and L_2 are always even number.

$\boldsymbol{G_{31}}$ and $\boldsymbol{G_{32}}$ are composed of some basic matrices. The size of base matrix $\boldsymbol{A}(k)$ that constitutes $\boldsymbol{G_{31}}$ is $p/L_2 \times 2n/L_1$, where k is the variable. $\boldsymbol{A}(k)$ is shown as (6) (at the bottom of next page).

According to the weight set, the value of k is 1, 2, 4 or 8. Therefore, the base matrix of $\boldsymbol{G_{31}}$ are $\{\boldsymbol{A}(1), \boldsymbol{A}(2), \boldsymbol{A}(4), \boldsymbol{A}(8)\}$. The base matrix of $\boldsymbol{G_{32}}$ is $\boldsymbol{B}(q)$ where q is the variable and the size of $\boldsymbol{B}(q)$ is $p/L_2 \times 2p/L_2$. $\boldsymbol{B}(q)$ is shown as

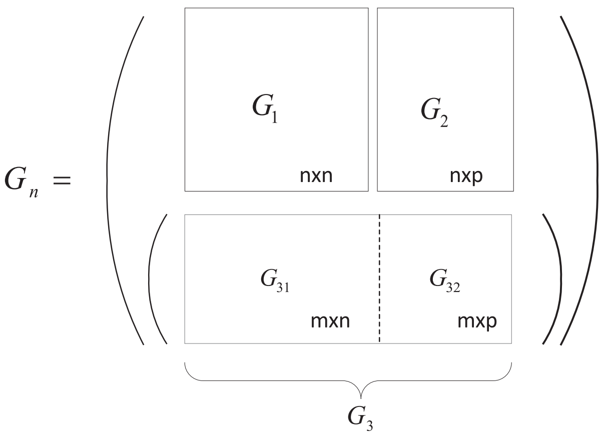

FIGURE 3. **Mapping matrix of the proposed scheme.**

follow:

$$\boldsymbol{B}(q)=\begin{pmatrix} q & -q & & & & & \\ & & q & -q & & & \\ & & & & \ddots & & \\ & & & & & q & -q \end{pmatrix}_{\frac{p}{L_2}\times\frac{2p}{L_2}}. \tag{7}$$

It's similar as k that the value of q is taken as 1, 2, 4 and 8. Therefore, $\{\boldsymbol{B}(1), \boldsymbol{B}(2), \boldsymbol{B}(4), \boldsymbol{B}(8)\}$ are the base matrix that constitutes $\boldsymbol{G_{32}}$.

Secondly, randomly permute the columns of these basic matrices denoted as $\pi(\boldsymbol{A}(1))$, $\pi(\boldsymbol{A}(2))$, $\pi(\boldsymbol{A}(4))$, $\pi(\boldsymbol{A}(8))$, $\pi(\boldsymbol{B}(1))$, $\pi(\boldsymbol{B}(2))$, $\pi(\boldsymbol{B}(4))$, $\pi(\boldsymbol{B}(8))$, where $\pi(\cdot)$ represents the process of random permutation of columns of a matrix and $\pi(\cdot)$ is different each time.

Thirdly, let $\boldsymbol{G_{01}}$ denote the submatrix of $\boldsymbol{G_{31}}$ and let $\boldsymbol{G_{02}}$ denote the submatrix of $\boldsymbol{G_{32}}$. By stacking different number of $\boldsymbol{G_{01}}$ or $\boldsymbol{G_{02}}$, we can get $\boldsymbol{G_{31}}$ and $\boldsymbol{G_{32}}$ respectively. When $L_1 = 4$ and $L_2 = 4$, the construction method of $\boldsymbol{G_{01}}$ and $\boldsymbol{G_{02}}$ is showed as (8) and (9), respectively.

$$\boldsymbol{G_{01}}=\begin{bmatrix} \pi(\boldsymbol{A}(4)) & \pi(\boldsymbol{A}(2)) \\ \pi(\boldsymbol{A}(1)) & \pi(\boldsymbol{A}(8)) \\ \pi(\boldsymbol{A}(8)) & \pi(\boldsymbol{A}(1)) \\ \pi(\boldsymbol{A}(2)) & \pi(\boldsymbol{A}(4)) \end{bmatrix} \tag{8}$$

$$\boldsymbol{G_{02}}=\begin{bmatrix} \pi(\boldsymbol{B}(8)) & \pi(\boldsymbol{B}(1)) \\ \pi(\boldsymbol{B}(2)) & \pi(\boldsymbol{B}(4)) \\ \pi(\boldsymbol{B}(4)) & \pi(\boldsymbol{B}(2)) \\ \pi(\boldsymbol{B}(1)) & \pi(\boldsymbol{B}(8)) \end{bmatrix} \tag{9}$$

Under the condition of $L_2 = 6$ or $L_2 = 8$, the construction method is almost the same as $L_2 = 4$. The only difference is that the number of base matrix of $\boldsymbol{G_{02}}$ will increase and the number of base matrix of $\boldsymbol{G_{01}}$ will decrease. In this case we can find that $\boldsymbol{G_{01}}$ is an all-zero matrix when $L_2 = 8$.

The design guarantees that the weights of $\boldsymbol{G_{32}}$ are not random and $\boldsymbol{G_{32}}$ satisfy the property of RCM mapping matrix. The more weights corresponding to the parity bits guarantees that symbols will carry more energy of the parity bits. In the analysis of the next section, we will show that increasing the degree of $\boldsymbol{G_{32}}$ is beneficial to increase the minimum Euclidean distance of codewords.

IV. ANALYSIS ON THE PROPOSED WEIGHT MAPPING SCHEME

In the section, we will analyze the influence of weights of $\boldsymbol{G_{32}}$. Assume $\boldsymbol{w^{(1)}} = \left\{w_1^{(1)}, w_2^{(1)}, \cdots, w_{l_1}^{(1)}\right\}$ is the weight set of $\boldsymbol{G_1}$ and $\boldsymbol{w^{(2)}} = \left\{w_1^{(2)}, w_2^{(2)}, \cdots, w_{l_2}^{(2)}\right\}$ is the weight set of $\boldsymbol{G_3}$. According to Equation (1), we get the energy of transmission symbol. $E_u^{(1)}$ and $E_u^{(2)}$ denote the energy of symbols generated by $\boldsymbol{G_1}$ and $\boldsymbol{G_3}$, respectively.

$$\begin{cases} E_u^{(2)} = C \cdot \sum\limits_{l=1}^{L^{(2)}} \left(w_l^{(2)}\right)^2 x_{i_l} \\ E_u^{(1)} = C \cdot \sum\limits_{l=1}^{L^{(1)}} \left(w_l^{(1)}\right)^2 x_{i_l}, \end{cases} \tag{10}$$

where C is a power normalization factor. It can be seen that the weight values corresponding to the bits is actually the energy given to the bits. The weight set that we choose for the mapping matrix is symmetric and the information bits "0" or "1" has the equal probability 0.5. Let $E_e^{(1)}$ and $E_e^{(2)}$ denote the average energy of symbols generated by $\boldsymbol{G_1}$ and $\boldsymbol{G_3}$, respectively.

$$\begin{cases} E_e^{(2)} = \frac{1}{2} \cdot C \cdot \sum\limits_{l=1}^{L^{(2)}} \left(w_l^{(2)}\right)^2 \\ E_e^{(1)} = \frac{1}{2} \cdot C \cdot \sum\limits_{l=1}^{L^{(1)}} \left(w_l^{(1)}\right)^2. \end{cases} \tag{11}$$

According to the mapping matrix $\boldsymbol{G_n}$, the symbols generated by $\boldsymbol{G_1}$ and $\boldsymbol{G_3}$ both carry the energy of the information bits. The energy of these bits depends on the number of the transmitted symbols. Let E_s denote the average energy of information bits. When the number of the transmitted

$$\boldsymbol{A}(k)=\left(\begin{pmatrix} k & -k & & & & \\ & & \ddots & & & \\ & & & k & -k & \\ & & & & & k \; -k \end{pmatrix}_{\frac{p}{L_2}\times\frac{2p}{L_2}} \begin{matrix} 0 & \cdots & \cdots & 0 \\ \vdots & \ddots & & 0 \\ \vdots & & \ddots & 0 \\ 0 & 0 & 0 & 0 \end{matrix}\right)_{\frac{p}{L_2}\times\frac{2n}{L_1}}. \tag{6}$$

symbols reaches $(n+p)$, E_s is expressed as follow:

$$E_s = \frac{1}{2}\cdot C\cdot\sum_{l=1}^{L^{(1)}}(w_l^{(1)})^2 + \frac{L_1}{2L^{(2)}}\cdot C\cdot\sum_{l=1}^{L^{(2)}}(w_l^{(2)})^2 = E_e^{(1)} + \frac{L_1}{L^{(2)}}\cdot E_e^{(2)}. \quad (12)$$

According to (9), the absolute value of the weights of each column of $\boldsymbol{G_{32}}$ are the same. Therefore, the energy of each parity bit is treated as equal. The average energy of each parity bit is denoted as E_p. When the number of the transmitted symbols reaches $(n+p)$, E_p is expressed as follow:

$$E_p = \frac{L_2}{2L^{(2)}}\cdot C\cdot\sum_{l=1}^{L^{(2)}}(w_l^{(2)})^2 = \frac{L_2}{L^{(2)}}\cdot E_e^{(2)}. \quad (13)$$

It can be found from (12) and (13) that the energy of information bits is far greater than the energy of parity bits. Let η denote the energy ratio of information bits to parity bits. When the transmitted symbols reaches $(n+p)$, η can be expressed as follow:

$$\eta = \frac{E_s}{E_p} = \frac{E_e^{(1)} + \frac{L_1}{L^{(2)}}\cdot E_e^{(2)}}{\frac{L_2}{L^{(2)}}\cdot E_e^{(2)}}. \quad (14)$$

It should be noted that the value of η will increase when the number of transmitted symbols is less. When the number of symbols is n, $\eta\to\infty$. Through analysis, it is known that η is very large, which shows that the energy of information bits is much larger than the energy of the parity bits. Therefore, the error probability of parity bits is much larger than that of information bits. A-BICM cascades LDPC codes, which improve the performance at low SNR compared to RCM. The key factor to improve the performance at low SNR is to use parity bits to correct error information bits. However, parity bits are hard to correct the information bits when the error probability of parity bits is large. Therefore, it is crucial to increase the energy of parity bits in the transmission process. (13) shows that increasing L_2 is a good way to improve the energy of parity bits.

The analysis is not limited to the aspect of energy. The proposed scheme is also beneficial to increase the minimum Euclidean distance of codewords. $\boldsymbol{u}$ is the transmitted symbol vector. Through AWGN channel, the received signal vector is $\boldsymbol{r}$, where $\boldsymbol{r}=\boldsymbol{u}+\boldsymbol{e}$, and $\boldsymbol{e}$ represents noise vector. According to Maximum Likelihood(ML) criterion, the optimal estimation $\hat{\boldsymbol{x}}$ is given as follow:

$$\hat{\boldsymbol{x}} = \arg\min_{x\in\{0,1\}^N} \|\boldsymbol{r}-\boldsymbol{G}\boldsymbol{x}\|. \quad (15)$$

Let $\boldsymbol{x}'$ denote the wrong bits vector after demapping. The following conditions should be satisfied.

$$\left\|\boldsymbol{r}-\boldsymbol{G}\boldsymbol{x}'\right\|^2 < \|\boldsymbol{r}-\boldsymbol{G}\boldsymbol{x}\|^2. \quad (16)$$

The error probability can be found in [5] expressed as p_e.

$$p_e = Q(\frac{1}{2\sigma}\sqrt{\left\|\boldsymbol{G}\boldsymbol{x}-\boldsymbol{G}\boldsymbol{x}'\right\|^2/E_u}), \quad (17)$$

where $Q(x) = \frac{1}{2\sigma}\int_x^\infty e^{-x^2/2}dx$ and E_u represents the energy of symbols. According to (17), we can find that minimum Euclidean distance of codewords determines the error probability. When only one error place is appeared, the minimum Euclidean distance is give in [7]. The average energy of information bits is different from that of parity bits. Let d_1 denote the minimum Euclidean distance when the error is appeared in information bits. d_1^2 is shown as follow:

$$d_1^2 = \min_{x\in\{0,1\}^n}\frac{1}{E_e^{(1)}}\left\|\sum_{i=1}^{n}(\boldsymbol{G_1}[i]+\boldsymbol{G_{31}}[i])\left(x_i-x_i'\right)\right\|^2, \quad (18)$$

where $\boldsymbol{G_1}[i]$ and $\boldsymbol{G_{31}}[i]$ represents the i^{th} column of $\boldsymbol{G_1}$ and $\boldsymbol{G_{31}}$, respectively. According to [23], (18) can be simplified as follow:

$$d_1^2 = \frac{2mL_1}{n}\cdot C\cdot\frac{1}{L^{(2)}}\sum_{j=1}^{L^{(2)}}(w_j^{(2)})^2/E_e^{(2)} + \frac{2nL^{(1)}}{n}\cdot C\cdot\frac{1}{L^{(1)}}\sum_{j=1}^{L^{(1)}}(w_j^{(1)})^2/E_e^{(1)} = \frac{4mL_1}{nL^{(2)}}+4. \quad (19)$$

Let d_2 denote the minimum Euclidean distance when the error is appeared in parity bits. d_2^2 is shown as follow:

$$d_2^2 = \min_{x\in\{0,1\}^p}\frac{1}{E_e^{(2)}}\left\|\sum_{i=1}^{p}\boldsymbol{G_{32}}[i]\left(x_i-x_i'\right)\right\|^2, \quad (20)$$

where $\boldsymbol{G_{32}}[i]$ represents the i^{th} column of $\boldsymbol{G_{32}}$. (20) can be simplified into the following form:

$$d_2^2 = \frac{2mL_2}{p}\cdot C\cdot\frac{1}{L^{(2)}}\sum_{j=1}^{L^{(2)}}(w_j^{(2)})^2/E_e^{(2)} = \frac{4mL_2}{pL^{(2)}}. \quad (21)$$

In the actual design of mapping matrix, m is generally larger than p. Therefore, $d_2^2 < 4 < d_1^2$. The minimum Euclidean distance of codewords is represented as d_{min}. According to the above analysis, we can draw the conclusion.

$$d_{min}^2 = d_2^2 = \frac{4mL_2}{pL^{(2)}}. \quad (22)$$

(22) shows that d_{min} is related to the degree of $\boldsymbol{G_{32}}$ and the degree of $\boldsymbol{G_3}$. Increasing the value of L_2 is beneficial to increase d_{min}. (22) also verifies the proposed mapping scheme. Firstly, the proposed scheme increases the position of non-zero elements in each row of $\boldsymbol{G_{32}}$ compared to A-BICM. Secondly, a more simple weight set is used for $\boldsymbol{G_3}$. Through analysis, the proposed scheme is able to decrease the minimum Euclidean distance of codewords.

V. PERFORMANCE EVALUATION

In this section, we mainly compare the performance of bit error rate (BER) and throughput between the existing scheme and the proposed scheme. Then, the complexity of different schemes also have been analyzed.

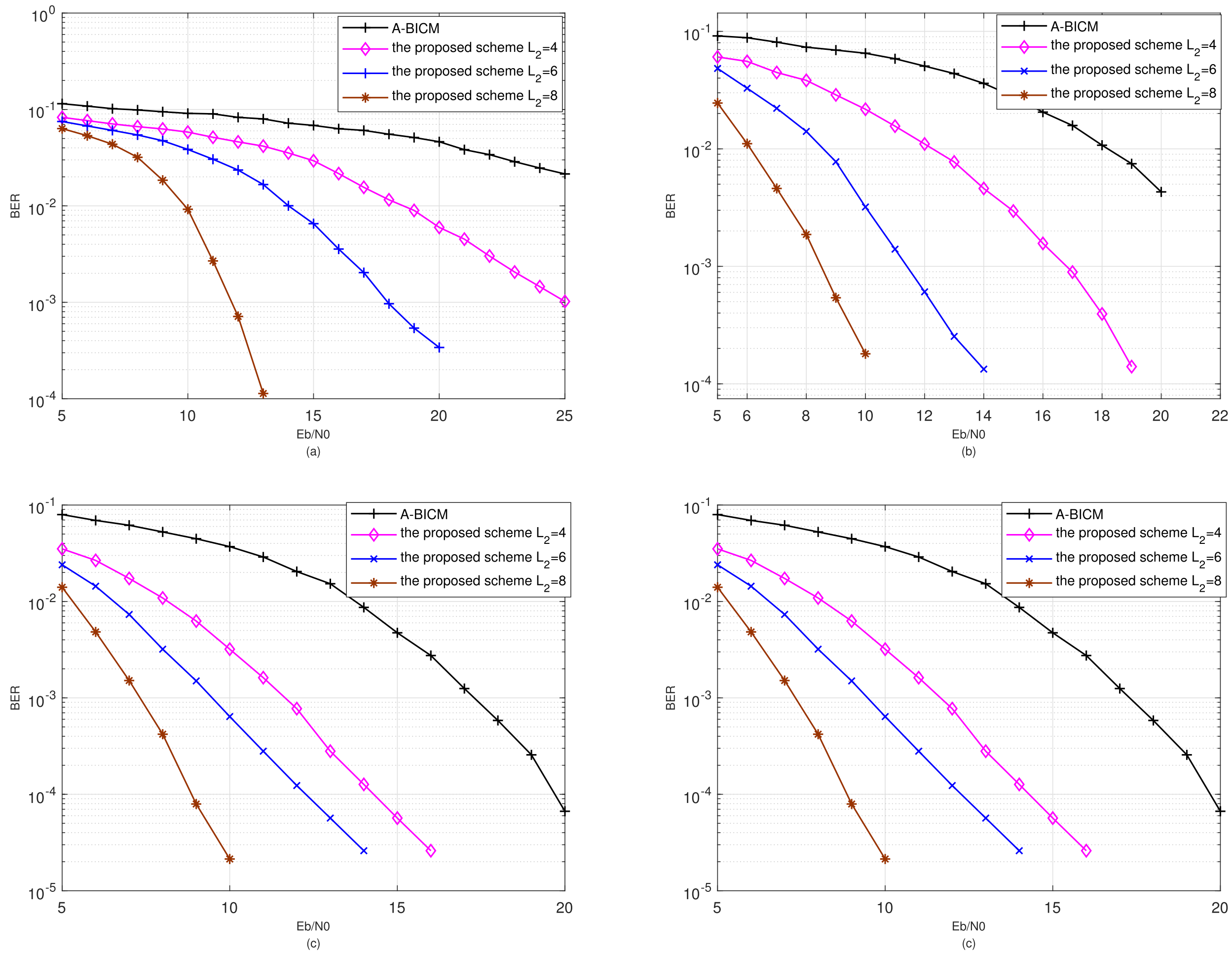

FIGURE 4. BER performances of different schemes with different transmission number of symbols. (a) M=500, (b) M=550, (c) M=600, (d) M=650.

A. SIMULATION

Simulations were carried out under AWGN channels. We implemented the proposed mapping scheme, A-BICM and RCM. The proposed scheme and A-BICM are both coded RCM. We used a rate 2/3 systematic LDPC code (600,400). Its variable node degree is 3 and its check node degree distribution is $0.03x^7 + 0.94x^8 + 0.03x^9$. In the proposed scheme, we selected $\boldsymbol{w_1} = \{\pm1, \pm2, \pm4, \pm4, \pm8\}$ as the weight set of $\boldsymbol{G_1}$ and $\boldsymbol{w_2} = \{\pm1, \pm2, \pm4, \pm8\}$ as the weight set of $\boldsymbol{G_3}$. The size of mapping matrix $\boldsymbol{G_n}$ is 800×600. In the $\boldsymbol{G_n}$, the size of $\boldsymbol{G_1}$ and $\boldsymbol{G_{31}}$ are both 400×400, and the size of $\boldsymbol{G_{32}}$ and $\boldsymbol{G_2}$ are both 400×200. The number of demapping iterations is set to be 10 and the number of decoding iteration for LDPC codes is set to be 50.

In order to evaluate the BER performance of the proposed scheme and A-BICM, we chose four situations where the number of transmitted symbols was different. Let M denote the number of transmitted symbols. The simulations when $M = 500, M = 550, M = 600$ and $M = 650$ were performed. Using the proposed scheme, three kinds of matrices are constructed, $L_2 = 4$, $L_2 = 6$ and $L_2 = 8$, respectively. BER is calculated using 10^6 bits. Fig. 4 shows the BER performance of different schemes after RCM demapping. We can see clearly from Fig. 4 the proposed scheme $L_2 = 8$ has optimal performance and the performance of A-BICM is the worst. The fact is that the proposed scheme $L_2 = 8$ corresponds to the largest number of weights for parity bits and A-BICM is the opposite. The result indicates that the error of parity bits is a leading impact.

In order to evaluate the spectrum efficiency, we compare the throughput of different schemes, including the proposed schemes, A-BICM, RCM and BICM adopting LDPC codes of different rate. In the schemes of BICM, we use the LDPC codes which are the standard of digital video broadcasting (DVB) [24]. The throughput performance of these schemes are shown in Fig. 5. Compared to the LDPC codes with conventional modulation, the proposed scheme and A-BICM are more smooth. We can find out that the proposed scheme and A-BICM all outperform the performance of RCM at low SNR. It shows that cascading LDPC codes is helpful for improving throughput at low SNR. The performance of the proposed scheme are all better than A-BICM. The throughput performance of the proposed scheme $L_2 = 8$ is optimal. It achieves the rate of 0.62 bits/s/Hz at 2 dB, while A-BICM

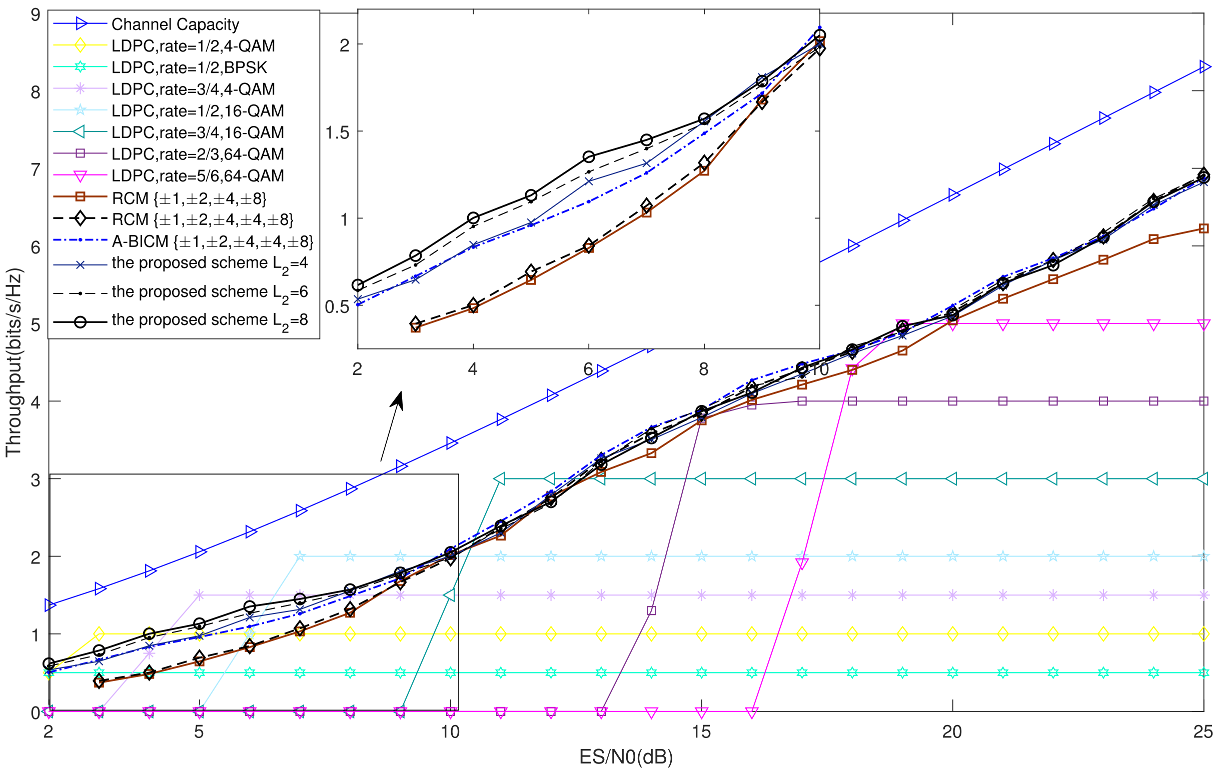

FIGURE 5. Comparison of throughput between the proposed scheme, original A-BICM, RCM and LDPC codes with conventional modulation.

TABLE 1. Comparison of computation complexity for different weight set.

Name	Weight set	L	Symbols	Multiplication	Addition
$\boldsymbol{w_1}$	$\{\pm1, \pm2, \pm4, \pm8\}$	8	31	3472	3360
$\boldsymbol{w_2}$	$\{\pm1, \pm2, \pm4, \pm4, \pm8\}$	10	39	7020	6840

is 0.5 bits/s/Hz. Therefore, the proposed scheme $L_2 = 8$ achieve 24% throughput gain at 2dB. Within the range of SNR 2-6dB, the proposed scheme $L_2 = 8$ achieves at least 14.2% throughput gain and the throughput has an average growth of 18.6%. The simulation results verify the excellent performance of the proposed scheme.

B. ANALYSIS OF COMPLEXITY

In the transmission process, the comparison of complexity is mainly for the receiver. As for LDPC decoding, the complexity of the proposed scheme and A-BICM is the same. Therefore, we only need to compare the complexity of RCM demapping. According to [25], the complexity of RCM demapping is high and the complexity is linear in the type of symbol. However, the type of symbol is determined by the weight set. As for the proposed scheme, $\boldsymbol{G_1}$ uses the same weight set as A-BICM and $\boldsymbol{G_3}$ uses a simpler weight compared to A-BICM. Therefore, the reduction in complexity mainly comes from $\boldsymbol{G_3}$.

In the process of RCM demapping, there are two processes, symbol node process and information node process respectively. The complexity of symbol node process is much higher than the complexity of information node process. In [18], the comparison of complexity is performed among symbol node process of different schemes. It also gives the complexity computation method of symbol node process. According to the computation method, Table. 1 (shown at the middle of last page) shows the computation complexity of the two different weight set. It should be noted that $\boldsymbol{w_2}$ is twice the complexity of $\boldsymbol{w_1}$. As for the demapping of symbols generated by $\boldsymbol{G_3}$, the complexity of A-BICM is twice that of the proposed scheme.

VI. CONCLUSION

In this paper, we proposed a novel weight mapping scheme of mapping matrix for A-BICM to obtain higher throughput at low SNR and reduce the computation complexity at the same time. We also analyze the influence of the weight distribution to minimum Euclidean distance of codewords. By increasing the number of weights corresponding to parity bits, it is beneficial to increase the minimum Euclidean distance. The simulation results show that the proposed scheme not only achieves the performance improvement but decreases the complexity of demapping. In the future, we will further study the way of concatenation of RCM and error correcting codes.

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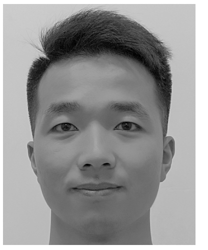

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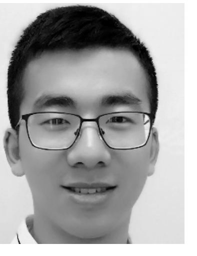

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