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Emergency Alternative Selection Based on an E-IFWA Approach

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ABSTRACT With the increasing occurrence frequency of emergency events, how to select the most desirable alternative has been as one of the major issues in emergency management. In this paper, a new method incorporating an extension entropy, Best-Worst method and Intuitionistic fuzzy weighted averaging operator (E-IFWA) is proposed to manage emergency alternative selection. E-IFWA method uses intuitionistic fuzzy number (IFN) to represent incomplete information (fuzzy information and missing information), which can describe the preference of decision-makers more clearly due to its more options. Extension intuitionistic fuzzy entropy is proposed to determine objective weight, and the Best-Worst method (BWM) is adapted to determine subjective weight, hence the objective and subjective combined weight of decision-makers and criteria are considered in this paper. The experiments including a simple example and a case study compared with the existing method illustrate that E-IFWA method is effective and can get a more reasonable result in emergency management.

INDEX TERMS Decision making, emergency alternative selection, fuzzy sets, intuitionistic fuzzy number, weight determination, fuzzy entropy, best-worst method.

I. INTRODUCTION

Emergency management has received great attention in recent years for frequent disasters and man-made catastrophic events, such as the earthquake and nuclear leak event of Fukushina, Japan in 2011, and the outbreak of Ebola virus in West Africa in 2014. These disasters have not only imposed severe suffering to human's lives and property, but also affected the stability of society. Hitherto, emergency management has becoming an important issue in the field of decision making [1]–[4].

Emergency management is the discipline and profession of applying science, technology, planning and management to deal with extreme events that can produce extensive damage [5], which is often conceptualized as the issue of a complex multi-objective optimization and has been extensively studied in a broad range of literature [6]–[9]. One of the important tasks in emergency management is Emergency Alternative Evaluation and Selection (EAES), in which the object is to evaluate emergency alternatives balancing within a number of criteria and opinions from different decision-makers. Based on the assessments of decision-makers for alternatives under different criteria, the best candidate or the ranking of alternatives should be determined. Hence to some extent, EAES can be described by ordinary multi-criteria decision making (MCDM) model [10]–[14].

Several methods have been proposed to solve emergency alternative selection problems. For example, Zhao *et al.* [15] presented a hybrid emergency decision-making method, integrating fuzzy analytic hierarchy process (FAHP) described by linguistic terms with enhanced weighted ordered weighted averaging (WOWA) operator, which was applied in unattended train operation metro system. Ju *et al.* [16] introduced a new framework of incorporating ANP, DEMATEL and TL-TOPSIS to deal with emergency management. Ju and Huang [17] presented a hybrid method combining DS/AHP with extended TOPSIS to determine the preference ranking of emergency alternatives.

Nevertheless, an inevitable process in decision-making problem is linguistic evaluation by experts to describe the performance of alternatives on different criteria/factors [18]–[21]. An urgent problem is that crisp numbers are not suitable

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enough for experts (decision-makers) to depict their complicated judgement. For instance, their assessments could be uncertain and vague, thus how to represent fuzzy information is of severe importance. What's more, the situation of missing information should also be taken into consideration. These two cases (fuzzy information and missing information) are collectively referred as incomplete information in this paper. In addition, another critical issue is the subjectivity of expert evaluation in weight determination of criteria and decision-makers. In general, experts should evaluate the relative performance of alternatives under different criteria using their experience and professional knowledge, which brings the challenge of scientificity and objectivity in weight determination. In summary, two critical problems have to be considered in emergency alternative selection problems.

- The representation of incomplete information in evaluation process.
- The subjectivity of expert evaluation in weight determination.

Many theories have been proposed to deal with incomplete information, like Z numbers [22], [23], D numbers [24]–[26] and three-way decisions [27]. To well address the above two issues, Intuitionistic fuzzy sets (IFS) [28] is applied in emergency management, which can deal with the former issue. IFS, which is a generalization of fuzzy sets, is an useful data representation model [29], [30]. Compared with fuzzy sets, except for the membership degree and non-membership degree, hesitation degree is also considered in IFS. In this case, IFS can give more options to decision-makers for describing their attitude more clearly. Therefore, it has superiority in flexibility and practicability when dealing with uncertainty and fuzziness. The tremendous popularity of IFS has brought to life many applications in different fields [31], [32]. For example, Tian *et al.* [33] solved green supplier selection problems using improved TOPSIS and Best-Worst method under intuitionistic fuzzy environment. In [34], authors used IFS for dealing with uncertain data in wireless sensor networks.

Further process about IFS is associated with intuitionistic fuzzy entropy, which is a measure of fuzziness (or uncertainty) related to an IFS and thus is introduced to address the latter issue. Entropy represents the measure of the disorder of a system and has two kinds of meanings in information science: (1) entropy represents the amount of information [35], [36]; (2) entropy can measure the uncertainty of information [37]. Due to its profound physical meaning, an important application of entropy is about weight determination [38]. Correspondingly, there are two methods in weight determination by entropy: using the amount of information and the uncertainty of information. We consider that the amount of useful information is the decisive metric. The larger amount of information doesn't mean more useful information, while the larger uncertainty of information means less useful information. Hence, it is more reasonable to determine weight by the uncertainty of information. The physical meaning can be interpreted as below: the larger the value of entropy is, the more uncertainty the information has, then the more other information it needs, the smaller the weight is. Intuitionistic fuzzy entropy has been extensively studied in the literature, general references include [39]–[42].

Another research related to IFS is aggregation operators, which takes a critical role during the combination of information process. The most common types of operators include power aggregation operators [43], order-weighted operators [44], [45], Bonferroni mean operators [46], [47] and so on. These operators have its own characteristics and have been applied to IFS and its extended forms. Especially, these operators can be utilized to deal with multi-criteria decision making problems, like in [48], Jana *et al.* developed a model based on bipolar fuzzy Dombi aggregation operators for selection of investment alternatives. Nie *et al.* [49] proposed a Pythagorean fuzzy MCDM approach using partitioned normalized weighted Bonferroni mean operator.

Based on the above analysis, a new method based on entropy, best-worst method (BWM) and intuitionistic fuzzy weighted averaging operator (IFWA), which is abbreviated as E-IFWA, is proposed in this paper to solve emergency alternative selection problem. Firstly we adopt Intuitionistic fuzzy number (IFN) to represent linguistic assessment of decision-makers, which can address the representation of incomplete information well. Multiple decision matrices for experts are then constructed. Secondly a new intuitionistic fuzzy entropy is put forward, which can be regarded as an extension of Shannon entropy. Under the concept, the objective weight of decision-makers can be determined while another method BWM is adapted to determine subjective weight, then the combined weight will be applied in IFWA to obtain the weighted decision matrix. Thirdly, after calculating the combined weight of criteria using the extension entropy and BWM, the IFNs of different criteria are aggregated to get the final fused evaluation of alternatives. Finally, the score and accuracy functions of IFN are regarded as comparison rules to rank alternatives. A case analysis illustrates the efficiency of E-IFWA through comparison of existing method.

The rest of this paper is organized as follows. Section II introduces related concepts. Section III presents the proposed method E-IFWA and an simple example. A case study of emergency alternative selection is illustrated in Section IV. Section V ends the paper with the conclusion.

II. PRELIMINARIES

A. INTUITIONISTIC FUZZY SETS

Intuitionistic fuzzy set (IFS), which was firstly introduced by Atanassov [28], is an generalization of the classic fuzzy set [50]. IFS is an effective method to solve the problem under uncertain environment [51]–[53]. The concept of IFS is defined as follows.

Definition 1: Let X be a finite set, an IFS G in X is described by

$$G = \{\langle x, \mu_G(x), \nu_G(x)\rangle | x \in X\} \tag{1}$$

where $0 \leq \mu_G(x) + \nu_G(x) \leq 1$, $x \in$ X. $\mu_G(x) \in [0,1]$ and $\nu_G(x) \in [0,1]$ denote the degree of membership and non-membership of the element x to G respectively. Additionally, $\pi_G(x) = 1-\mu_G(x)-\nu_G(x)$ is called the hesitation degree of $x \in G$, representing the degree of hesitancy of x to G. If the value of $\pi_G(x)$ is small, then the information about x is more certain and vice versa [54].

For an IFS, the pair $(\mu_G(x), \nu_G(x))$ is called an intuitionistic fuzzy number (IFN), an IFN can be simply denoted as $\alpha = (\mu, \nu)$, where $\mu, \nu \in [0, 1]$, $\mu+\nu \leq 1$ and $\mu+\nu+\pi = 1$. Some definitions about IFN are presented in [55].

Definition 2: Let $\alpha = (\mu, \nu)$ be an IFN. The score function of α is defined as:

$$S(\alpha) = \mu - \nu \tag{2}$$

where $S(\alpha) \in [-1, 1]$. The larger $S(\alpha)$ is, the greater α is.

Definition 3: Let $\alpha = (\mu, \nu)$ be an IFN. The accuracy function of α is defined as:

$$H(\alpha) = \mu + \nu \tag{3}$$

where $H(\alpha) \in [0,1]$. The larger $H(\alpha)$ is, the greater α is.

Based on the score function and the accuracy function, Xu and Yager proposed a comparison method of IFNs [55].

Definition 4: For any two IFNs α_1 and α_2, the ordering relationship is established as follows:

$$\begin{aligned} &If \quad S(\alpha_1) > S(\alpha_2), \quad then \quad \alpha_1 > \alpha_2 \\ &If \quad S(\alpha_1) = S(\alpha_2), \quad and \\ &\quad (a) \quad If \quad H(\alpha_1) > H(\alpha_2), \quad then \quad \alpha_1 > \alpha_2; \\ &\quad (b) \quad If \quad H(\alpha_1) = H(\alpha_2), \quad then \quad \alpha_1 = \alpha_2. \end{aligned} \tag{4}$$

B. THE IFWA OPERATOR

In order to solve MCDM problems more flexibly, based on the conception of IFN, Xu [44] presented the intuitionistic fuzzy weighted averaging (IFWA) operator. It is defined as follows:

Definition 5: For a collection of IFNs $\{\alpha_1, \ldots, \alpha_i, \ldots, \alpha_n\}$, $\alpha_i = (\mu_i, \nu_i)$, the operator of IFWA is:

$$IFWA(\alpha_1, \alpha_2, \ldots, \alpha_n) = (1-\prod_{i=1}^{n}(1-\mu_i)^{w_i}, \prod_{i=1}^{n}(\nu_i)^{w_i}) \tag{5}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of α, $w_i \in [0, 1]$ and $\sum_{i=1}^{n} w_i = 1$.

C. SHANNON ENTROPY

Shannon entropy [56] is an efficient tool to measure the amount of information. The value of entropy of a message is directly related to its uncertainty. The larger the value of entropy, the more uncertainty the message has. Due to the powerful function of measurement, Shannon entropy has been applied widely in many fields, like engineering [57], mathematics [58], [59] and physics [60], [61]. Shannon entropy is calculated as follows:

$$H_S = -\sum_{i=1}^{n} p_i \, log_b \, p_i \tag{6}$$

where H_S is the value of Shannon entropy. n is the amount of basic states in a state space, p_i is the appearing probability of state i satisfying $\sum_{i=1}^{n} p_i = 1$ and b is base of logarithm. When $b = 2$, the unit of Shannon entropy is bit.

D. BEST-WORST METHOD

Best-worst method, abbreviated as BWM, was proposed by Jafar Rezaei to solve MCDM [62], [63]. According to BWM, the best and the worst items are first identified by the expert, then pairwise comparisons are conducted between each of these two items and the other items, finally by formulating and solving a maximin problem, the weight of different items can be determined by BWM. A consistency ratio is computed for BWM to check the reliability of comparisons. Compared with the traditional AHP, it requires less comparison data and produces more consistent comparisons [62]. Owing to the superiority of BWM, it has been used in many applications like FMEA [64] and cloud service selection [65]. Following is to describe the steps of BWM.

For a set of items $\{t_1, t_2, \ldots, t_l\}$, after selecting the best and the worst items by experts, they determine the preferences of the best item over all the other items by using a number from 1 to 9 (1 means equally important and 9 signifies extremely important), the result is presented as a 'best-to-others' vector as follows:

$$U_{BO} = (u_{B1}, u_{B2}, \ldots, u_{Bl})$$

where u_{Bj} indicates the preference of the best item B over item j, and $u_{BB} = 1$. Similarly, the preference of all the items over the worst item is presented as a 'others-to-worst' vector.

$$V_{OW} = (v_{1W}, v_{2W}, \ldots, v_{lW})^T$$

where v_{jW} indicates the preference of item j over the worst item W, and $v_{WW} = 1$. To derive the optional weight $(w_1^*, w_2^*, \ldots, w_l^*)$, BWM introduces an optional linear programming model of ε as follows:

$$\begin{aligned} &\textbf{min } \varepsilon \\ &\textbf{s.t.} \begin{cases} |\frac{w_B}{w_j} - u_{Bj}| \leq \varepsilon, & \text{for all } j \\ |\frac{w_j}{w_W} - v_{jW}| \leq \varepsilon, & \text{for all } j \\ \sum_j w_j = 1 \\ w_j \geq 0, & \text{for all } j \end{cases} \end{aligned} \tag{7}$$

Besides, BWM introduces the concept of consistency ratio (CR) to measure the consistency of comparison, which is calculated as follows:

$$CR = \frac{\varepsilon^*}{Consistency \quad Index} \tag{8}$$

wherein the value of Consistency Index (CI) corresponds to u_{BW} (the preference of the best item over the worst item), as shown in Table 1. ε^* is the optional solution of ε in Eq. (7). CR $\in [0,1]$, the smaller CR is, the more consistent the comparison vector is. Generally, CR ≤ 0.1 shows that the obtained vector is acceptable [33].

TABLE 1. Consistency index [62].

u_{BW}	1	2	3	4	5	6	7	8	9
CI	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

III. THE E-IFWA METHOD

A. THE EXTENSION ENTROPY

Before describing the proposed method, the definition of a new Intuitionistic fuzzy entropy (the extension entropy) is introduced here, which is applied to determine objective weight. The inspiration is from the relationship between IFN and Dempster-Shafer evidence theory (DST). DST [66], [67] is a powerful data representation tool and is widely used in many applications such as decision making [68]–[70], information fusion [71]–[74] and uncertainty modeling [75]–[79]. In the mathematical framework of DST, a finite non-empty set Ω is called a frame of discernment (FOD), which means all possible answers to a problem. A subset A of FOD is called a proposition, the confidence degree assigned to each proposition is called basic probability assignment (BPA), i.e., a BPA is a mapping m: $2^{\Omega} \rightarrow [0, 1]$, satisfying $\sum_{A\in 2^{\Omega}} m(A) = 1$.

According to existing works [80], [81], intuitionistic fuzzy values can be handled in the framework of DST. When analyzing any situation in context of IFS, we deal with the following three hypotheses: $x \in G$, $x \notin G$ and the situation when both the hypotheses $x \in G$, $x \notin G$ cannot be rejected (the case of hesitation). In the spirit of DST, three hypotheses may correspond to three propositions under the frame of discernment $\Omega = \{Y, N\}$. m(Y) means the probability of $x \in G$, i.e., as the membership degree of $x \in G$: $\mu_G(x) = m(\text{Y})$. Similarly, $\nu_G(x) = m(N)$. Since m(Y, N) is usually treated as vagueness, a natural assumption is $m(Y, N) = \pi_G(x)$. The triplet $\mu_G(x)$, $\nu_G(x)$, $\pi_G(x)$ represents a correct basic assignment function. In this context, combined with the concept of Shannon entropy, a new Intuitionistic fuzzy entropy is presented.

Definition 6: Let $B = (\mu_B(x), \nu_B(x))$ be an IFN in the universe of discourse X, then the entropy measure is defined as follows.

$$H_E(B) = -\sum p(x)\log\frac{p(x)}{e^{|x|-1}} \tag{9}$$

where $x = \{\{Y\}, \{N\}, \{Y, N\}\}$, p(x) is the degree of the element x to B. That is, $p(\{Y\})$ is equal to $\mu_B(x)$, $p(\{N\})$ is equal to $\nu_B(x)$, $p(\{Y, N\})$ is equal to $\pi_B(x)$. $|x|$ is the number of elements in x, i.e., $|x| = 1$ or 2. Through further deduction we can obtain the following equation.

$$\begin{aligned} H_E(B) &= -\sum p(x)\log\frac{p(x)}{e^{|x|-1}} \\ &= -(p(\{Y\})\log(\frac{p(\{Y\})}{e^{1-1}}) + p(\{N\})\log(\frac{p(\{N\})}{e^{1-1}}) \\ &\quad + p(\{Y, N\})\log\frac{p(\{Y, N\})}{e^{2-1}}) \\ &= -(\mu_B(x)\log(\frac{\mu_B(x)}{e^{1-1}}) + \nu_B(x)\log(\frac{\nu_B(x)}{e^{1-1}}) \\ &\quad + \pi_B(x)\log\frac{\pi_B(x)}{e^{2-1}}) \\ &= -(\mu_B(x)\log\mu_B(x) + \nu_B(x)\log\nu_B(x) \\ &\quad + \pi_B(x)\log\frac{\pi_B(x)}{e}) \end{aligned} \tag{10}$$

Example 3.1. Assume an IFN M$=(0.7, 0.3)$, $\pi_M(x) = 0$. The associated Shannon entropy $H_S(M)$ and the extension entropy $H_E(M)$ are calculated as follows.

$$H_S(M) = -(0.7 * log_2 0.7 + 0.3 * log_2 0.3) = 0.8813$$
$$H_E(M) = -(0.7 * log_2 0.7 + 0.3 * log_2 0.3) = 0.8813$$

Example 3.2. Assume an IFN N$=(0.7, 0.2)$, $\pi_N(x) = 0.1$. The extension entropy $H_E(N)$ is computed as follows. Notably, Shannon entropy cann't measure the IFN N because $0.7 + 0.2 \neq 1$.

$$H_E(N) = -(0.7 * log_2 0.7 + 0.2 * log_2 0.2 + 0.1 * log_2(\frac{0.1}{e})) = 1.3013$$

Clearly, Example 3.1 shows that when the hesitant degree is equal to 0, the result of Shannon entropy and the extension entropy are identical. What's more, compared with IFN M in Examples 3.1 and IFN N in Example 3.2, $\pi_M(x) = 0$ while $\pi_N(x) = 0.1$, it obviously shows that there exists more uncertainty or fuzziness in IFN N, more information is needed to make a judgement, so the entropy value of N should be larger than value of M, which coincides with the calculation result in the new proposed entropy, $H_E(M) = 0.8813 < H_E(N) = 1.3013$.

Definition 7: Assume E_i refers to the entropy of the ith element, $1 \leq i \leq$ n. After normalizing using Eq. (11), the weight of n elements is obtained using Eq. (12).

$$e_i = \frac{E_i}{\sum E_i} \tag{11}$$

$$w_i = \frac{1 - e_i}{\sum(1 - e_i)} \tag{12}$$

B. THE PROCEDURE OF E-IFWA

In this section, based on the proposed fuzzy entropy (defined in Section III-A), BWM and IFWA, a new method called E-IFWA is proposed to deal with emergency alternative selection problem.

Suppose that there n decision-makers $(D_1, D_2, \ldots, D_n)$ in emergency management for the assessment of m alternatives $(A_1, A_2, \ldots, A_m)$ in terms of k criteria $(C_1, C_2, \ldots, C_k)$. Decision-makers (experts) can firstly use natural language phrases, that is, five linguistic terms including "Very Low (VL)", "Low (L)", "Moderate (M)", "High (H)" and "Very High (VL)", to determine the relative performance of alternatives regarding each criteria. After mapping linguistic terms into corresponding IFNs in Table 2, the decision matrix D^h is constructed and denoted as $D^h = [d_{ij}^h]$, where $d_{ij}^h = (\mu_{ij}^h, \nu_{ij}^h)$ indicates the hth expert's evaluation on the performance of

TABLE 2. Transformation rules from linguistic variables to IFNs.

Linguistic terms	IFN
Very Low / Very Bad (VL / VB)	(0.10, 0.85)
Low / Bad (L / B)	(0.30, 0.65)
Moderate (M)	(0.50, 0.50)
High / Good (H / G)	(0.75, 0.20)
Very High / Very Good(VH / VG)	(0.90, 0.05)

ith alternative under jth criteria, where $h \in \{1, 2, \ldots, n\}$, $i \in \{1, 2, \ldots, m\}, j \in \{1, 2, \ldots, k\}$.

$$D^h = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_k \end{array} \\ \begin{bmatrix} d^h_{11} & d^h_{12} & \cdots & d^h_{1k} \\ d^h_{21} & d^h_{22} & \cdots & d^h_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ d^h_{m1} & d^h_{m2} & \cdots & d^h_{mk} \end{bmatrix} \end{array}$$

As there are many experts involved in the evaluation process and the importance of each expert should be taken into consideration, BWM is used to calculate the subjective weight and the proposed entropy is used to compute the objective weight. Suppose the subjective weight is represented as $w_s = \{w_{s1}, w_{s2}, \ldots, w_{sn}\}$, the objective weight is represented as $w_o = \{w_{o1}, w_{o2}, \ldots, w_{on}\}$, then the combined weight is calculated as follows:

$$w = \frac{w_{si} * w_{oi}}{\sum_{i=1}^{n} w_{si} * w_{oi}} \tag{13}$$

Next we should combine n decision matrix (D^1, $D^2, \ldots, D^n$) into a weighted decision matrix (D) using IFWA operator. $D = [d_{ij}]$, where $d_{ij} = (\mu_{ij}, \nu_{ij})$ indicates a comprehensive evaluation of ith alternative under jth criteria. The specific fusion formula is shown in Eq. (14), w_k is the combined weight of experts calculated in Eq. (13) and $\sum_{k=1}^{n} w_k = 1$.

$$\begin{aligned} d_{ij} = (\mu_{ij}, \nu_{ij}) &= IFWA_w(d^1_{ij}, d^2_{ij}, \cdots, d^n_{ij}) \\ &= (1 - \prod_{k=1}^{n}(1 - \mu^k_{ij})^{w_k}, \prod_{k=1}^{n}(\nu^k_{ij})^{w_k}) \end{aligned}$$

$$D = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \cdots & C_k \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1k} \\ d_{21} & d_{22} & \cdots & d_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mk} \end{bmatrix} \end{array} \tag{14}$$

Simultaneously we should consider the weight of criteria in the same way of computing experts' weight. The criteria which has a larger weight will give more influence to the selection of alternatives. Based on the subjective and objective combined weight of criteria, IFNs of each criteria in weighted decision matrix D are combined to get the final fused evaluation of an alternative, which is represented as P_i, $i \in \{1, \cdots, m\}$. The specific fusion formula is shown in Eq. (15), q_j is the combined weight of criteria and $\sum_{j=1}^{k} q_j = 1$.

$$\begin{aligned} P_i = (\mu_i, \nu_i) &= IFWA_q(d_{i1}, d_{i2}, \cdots, d_{ik}) \\ &= (1 - \prod_{j=1}^{k}(1 - \mu_{ij})^{q_j}, \prod_{j=1}^{k}(\nu_{ij})^{q_j}) \end{aligned} \tag{15}$$

According to Eqs.(2-3), we compute the score function ($S_i = \mu_i$ - ν_i) and the accuracy function ($H_i = \mu_i + \nu_i$) of P_i. The priority sequence of alternatives is determined by the comparison of any two IFNs P_i and P_j (see Definition 4), $i, j = 1, \cdots, m$. The detailed rules are shown as follows:

(1) $S_i > S_j$, the ith alternative is better than the jth alternative, denotes by $i \succ j$;

(2) $S_i < S_j$, the ith alternative is worse than the jth alternative, denotes by $i \prec j$;

(3) $S_i = S_j \cap H_i = H_j$, the ith alternative has no difference to the jth alternative, denotes by $i \sim j$;

(4) $S_i = S_j \cap H_i > H_j$, the ith alternative is better than the jth alternative, denotes by $i \succ j$;

(5) $S_i = S_j \cap H_i < H_j$, the ith alternative is worse than the jth alternative, denotes by $i \prec j$;

Based on the above analysis, the proposed method E-IFWA can be listed in six steps. The processing of the model is illustrated in Fig. 1.

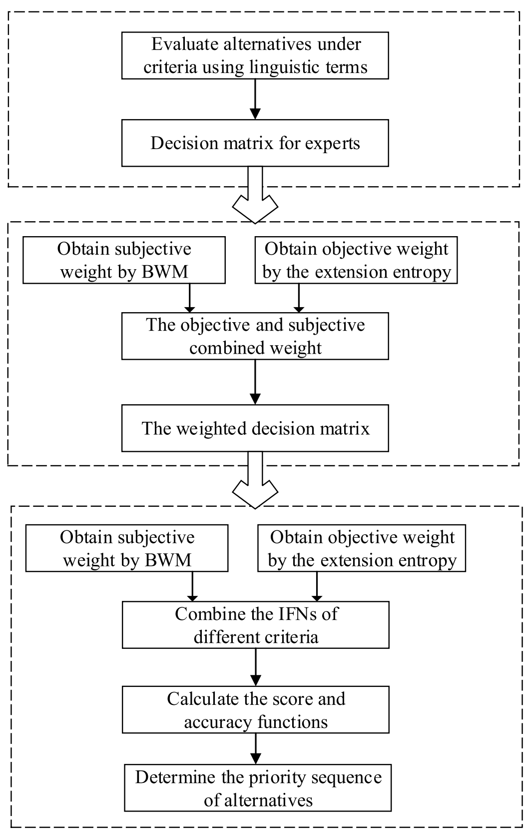

FIGURE 1. The processing of E-IFWA.

Step 1 Construct decision matrix D^h for experts' evaluation constituted by IFNs.

Step 2 Weight decision-makers based on the extension entropy and BWM.
With Eq.(10) to calculate each IFN's entropy, we can compute the sum of the entropy for every element d_{ij}^h in decision matrix D^h, represented as E_h^1, hitherto E_h^1 is the information entropy of the hth decision-maker. Using Eqs.(11-12), the objective weight of decision-makers can be determined. Incorporating with the subjective weight by BWM, we can obtain the combined weight using Eq. (13).

Step 3 Obtain the weighted decision matrix D using Eq. (14).

Step 4 Weight the criteria based on the extension entropy and BWM.
With Eq.(10) to calculate each IFN's entropy, we can compute the sum of the entropy for every element d_{ij} in the jth column of D, represented as E_j^2, hitherto E_j^2 is the information entropy of the jth criteria. Using Eqs.(11-12), the objective weight of criteria can be determined. Similarly, with the subjective weight by BWM, we can obtain the combined weight using Eq. (13).

Step 5 Combine the IFNs of different criteria to get the final fused evaluation of alternative using Eq. (15).

Step 6 Determine the priority sequence of alternatives according to the accuracy and score functions.

C. A SIMPLE EXAMPLE

Assume two decision matrix D^1, D^2 are given below. Mention that the symbol "$*$" means that the expert can't make a decision or the information is missing.

$$D^1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} VH & H & H \\ * & M & M \\ M & M & M \end{bmatrix} \end{array}$$

$$D^2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} H & L & L \\ VH & M & * \\ VH & M & H \end{bmatrix} \end{array}$$

According to the transformation rule in Table 2, we can obtain IFN decision matrices. The "$*$" corresponds to the IFN (0.00, 0.00).

$$D^1 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (0.90, 0.05) & (0.75, 0.20) & (0.75, 0.20) \\ (0.00, 0.00) & (0.50, 0.50) & (0.50, 0.50) \\ (0.50, 0.50) & (0.50, 0.50) & (0.50, 0.50) \end{bmatrix} \end{array}$$

$$D^2 = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} (0.75, 0.20) & (0.30, 0.65) & (0.30, 0.65) \\ (0.90, 0.05) & (0.50, 0.50) & (0.00, 0.00) \\ (0.90, 0.05) & (0.50, 0.50) & (0.75, 0.20) \end{bmatrix} \end{array}$$

In like manner of Example 3.1, the value of each IFN's entropy can be derived according to Eq.(10). The sum of all IFNs' entropy in D^1 is the corresponding fuzzy entropy of the first expert, represented as E_1^1. Also we can get E_2^1.

$$E_1^1 = 9.2116 \quad E_2^1 = 9.2793$$

Based on Eqs.(11-12), the objective weight of two experts are:

$$e_1 = \frac{9.2116}{9.2116 + 9.2793} = 0.4982$$
$$e_2 = \frac{9.2793}{9.2116 + 9.2793} = 0.5018$$
$$w_{o1} = \frac{1 - e_1}{1 - e_1 + 1 - e_2} = 0.5018$$
$$w_{o2} = \frac{1 - e_2}{1 - e_1 + 1 - e_2} = 0.4982$$

Assume the subjective weight of two experts using BWM is 0.5 respectively, i.e., $w_{s1} = w_{s2} = 0.5$. According to Eq. (13), the combined weight of two experts are computed as $w_1 = 0.5018, w_2 = 0.4982$.

Taking the IFNs $d_{11}^1 = (0.90, 0.05)$ in D^1 and $d_{11}^2 = (0.75, 0.20)$ in D^2 as an example to illustrate the process of IFWA.

$$\begin{aligned} d_{11} &= IFWA_w(d_{11}^1, d_{11}^2) = w_1 d_{11}^1 + w_2 d_{11}^2 \\ &= (1 - (1 - 0.90)^{0.5018} * (1 - 0.75)^{0.4982}, \\ &\quad (0.05)^{0.5018} * 0.20^{0.4982}) \\ &= (0.8422, 0.0997) \end{aligned}$$

Similarly, we can obtain the weighted decision matrix D of D^1 and D^2 as shown at the bottom of the next page:
Based on D, the fuzzy entropy and associated objective weight of each criteria are shown in the first two columns of Table 3.

TABLE 3. Example of the weight of criteria.

	fuzzy entropy E_i^2	objective weight q_{oi}	subjective weight q_{si}	combined weight q_i
C_1	4.0113	0.3213	0.0714	0.0671
C_2	3.9347	0.3247	0.3387	0.3218
C_3	3.2772	0.3540	0.5899	0.6111

Following gives the procedure of BWM to compute the subjective weight of criteria. Assume that after the discussion of two experts, they identify C_3 and C_1 as the best and worst criteria respectively, the two vectors are given as $U_{BO} = (8, 2, 1)$ and $V_{OW} = (1, 5, 8)^T$. According to Eq. (7), BWM establishes the following model:

$$\min \varepsilon$$
$$\text{s.t.} \begin{cases} |\frac{w_3}{w_1} - 8| \le \varepsilon \\ |\frac{w_3}{w_2} - 2| \le \varepsilon \\ |\frac{w_2}{w_1} - 5| \le \varepsilon \\ w_1 + w_2 + w_3 = 1 \\ w_1, w_2, w_3 \ge 0 \end{cases}$$

Solve this model, we have: $w_1^* = 0.0714, w_2^* = 0.3387, w_3^* = 0.5589$, and $\varepsilon^* = 0.26$. As $u_{BW} = u_{31} = 8$, $CR = 0.26/4.47 = 0.058$, which implies a very good consistency. Hence the combined weight of three criteria is shown in the last column of Table 3.

In a similar, using IFWA operator, we can obtain a comprehensive evaluation IFN for each alternative. For example, the final fused IFN P_1 for alternative A_1 is calculated as follows:

$$\begin{aligned} P_1 &= IFWA_q(d_{11}, d_{12}, d_{13}) = q_1 d_{11} + q_2 d_{12} + q_3 d_{13} \\ &= (1 - (0.8422)^{0.0671} * (0.5825)^{0.3218} * (0.5825)^{0.6111}, \\ &\quad (0.0997)^{0.0671} * (0.3598)^{0.3218} * (0.3598)^{0.6111}) \\ &= (0.6089, 0.3301) \end{aligned}$$

Finally, according to the accuracy and score functions, we can make comparison of any two IFNs and determine the priority sequence of alternatives. In Table 4, $S_2 > S_1 > S_3$, so the ranking of alternatives is $A_1 \succ A_2 \succ A_3$.

TABLE 4. Example of the ranking of alternatives.

Alternatives	P_i	S_i	H_i	ranking
A_1	(0.6089, 0.3301)	0.2788	0.9389	2
A_2	(0.4011, 0.0000)	0.4011	0.4011	1
A_3	(0.6163, 0.3503)	0.2661	0.9666	3

TABLE 5. Decision matrix from three decision makers [17].

	Performance	C_1	C_2	C_3	C_4
DM_1	A_1	VH	H	H	VH
	A_2	*	M	M	*
	A_3	M	**M**	M	M
	A_4	VH	H	H	H
	A_5	H	M	M	H
DM_2	A_1	H	VH	**VH**	H
	A_2	L	M	M	L
	A_3	L	*	H	*
	A_4	H	VH	H	M
	A_5	H	M	*	H
DM_3	A_1	VH	H	VH	H
	A_2	M	*	M	M
	A_3	L	L	L	H
	A_4	M	VH	VH	M
	A_5	M	L	M	H

TABLE 6. IFN decision matrix from three decision makers.

	Performance	C_1	C_2	C_3	C_4
DM_1 (D^1)	A_1	(0.90, 0.05)	(0.75, 0.20)	(0.75, 0.20)	(0.90, 0.05)
	A_2	(0.00, 0.00)	(0.50, 0.50)	(0.50, 0.50)	(0.00, 0.00)
	A_3	(0.50, 0.50)	**(0.50, 0.50)**	(0.50, 0.50)	(0.50, 0.50)
	A_4	(0.90, 0.05)	(0.75, 0.20)	(0.75, 0.20)	(0.75, 0.20)
	A_5	(0.75, 0.20)	(0.50, 0.50)	(0.50, 0.50)	(0.75, 0.20)
DM_2 (D^2)	A_1	(0.75, 0.20)	(0.90, 0.05)	**(0.90, 0.05)**	(0.75, 0.20)
	A_2	(0.30, 0.65)	(0.50, 0.50)	(0.50, 0.50)	(0.30, 0.65)
	A_3	(0.30, 0.65)	(0.00, 0.00)	(0.75, 0.20)	(0.00, 0.00)
	A_4	(0.75, 0.20)	(0.90, 0.05)	(0.75, 0.20)	(0.50, 0.50)
	A_5	(0.75, 0.20)	(0.50, 0.50)	(0.00, 0.00)	(0.75, 0.20)
DM_3 (D^3)	A_1	(0.90, 0.05)	(0.75, 0.20)	(0.90, 0.05)	(0.75, 0.20)
	A_2	(0.50, 0.50)	(0.00, 0.00)	(0.50, 0.50)	(0.50, 0.50)
	A_3	(0.30, 0.65)	(0.30, 0.65)	(0.30, 0.65)	(0.75, 0.20)
	A_4	(0.50, 0.50)	(0.90, 0.05)	(0.90, 0.05)	(0.50, 0.50)
	A_5	(0.50, 0.50)	(0.30, 0.65)	(0.50, 0.50)	(0.75, 0.20)

IV. APPLICATION IN EMERGENCY ALTERNATIVE SELECTION

In this section, a case study of emergency alternative evaluation and selection is illustrated based on the proposed model.

Step 1 Construct decision matrix for experts' evaluation. Based on [17], four criteria "preparing capacity (C_1)", "rescuing capacity (C_2)", "recovering capacity (C_3)" and "responding time (C_4)" are identified as evaluation criteria to measure the selection of emergency alternatives. Three decision-makers (DM_1, DM_2, DM_3) are invited to give their assessment over five alternatives (A_1, A_2, A_3, A_4, A_5) using linguistic terms shown in Table 5. Then according to the mapping rules from linguistic terms to IFNs in Table 2, we can obtain the corresponding IFN decision matrix as shown in Table 6.

Step 2 Weight decision-makers based on the extension entropy and BWM. According to Table 6, firstly we can calculate the associated fuzzy entropy regarding experts using Eq.(10), then the objective weight of three experts can be obtained via Eqs.(11–12).

$$E_1^1 = 20.2560, \quad E_2^1 = 21.3386, \; E_3^1 = 20.1159$$
$$w_{o1} = 0.3359, \quad w_{o2} = 0.3271, \; w_{o3} = 0.3370$$

Since experts with different background should have different weight, we use BWM to measure the subjective weight. The expert leader can determine the best and worst experts and give the preferences fairly, so the subjective weight of experts can be determined. For simplicity, we assume that the subjective weight is $w_{s1} = w_{s2} = w_{s3} = 0.333$. The combined weight of experts can be determined as $w_1 = 0.3359, w_2 = 0.3271, w_3 = 0.3370$.

Step 3 Construct weighted decision matrix D for criteria based on IFWA. With the combined weight of three decision-makers in Step 2, according to Eq. (14), the weighted decision matrix D can be calculated as shown at the bottom of the next page:

Step 4 Weight criteria using the extension entropy and BWM. According to Eqs. (10–12), the fuzzy entropy and objective weight of criteria can be calculated and are shown in the first two columns of Table 7. Assume after discussion of three experts, they identify C_2 and C_4 as the best and worst criteria respectively, the two vectors are given as $U_{BO} = (3, 1, 1, 5)$ and $V_{OW} = (2, 5, 4, 1)^T$. Establishing the optional model and solve it,

$$D = \begin{bmatrix} (0.8422, 0.0997) & (0.5825, 0.3598) & (0.5825, 0.3598) \\ (0.6824, 0.0000) & (0.5000, 0.5000) & (0.2938, 0.0000) \\ (0.7757, 0.1588) & (0.5000, 0.5000) & (0.6460, 0.3168) \end{bmatrix}$$

the optional weight can be computed as $w^* = (0.150, 0.418, 0.349, 0.0083)$ and $\varepsilon^* = 0.2$. CR $= 0.2/2.3 = 0.087 < 0.1$. The combined weight is shown in last column of Table 7.

TABLE 7. The weight of four criteria.

	fuzzy entropy E_i^2	objective weight q_{oi}	subjective weight q_{si}	combined weight q_i
C_1	6.0556	0.2543	0.150	0.1521
C_2	6.5853	0.2464	0.418	0.4106
C_3	5.8348	0.2563	0.349	0.3566
C_4	6.7683	0.2440	0.083	0.0807

Step 5 Combine IFNs of different criteria. Using Eq. (15), we can obtain the final fused IFNs of alternatives (Table 8).

TABLE 8. The priority sequence of alternatives.

Alternatives	P_i	S_i	H_i	ranking
A_1	(0.8420, 0.0999)	0.7420	0.9419	**1**
A_2	(0.4039, 0.0000)	0.4039	0.4039	5
A_3	(0.4291, 0.0000)	0.4291	0.4291	4
A_4	(0.8210, 0.1190)	0.7020	0.9400	**2**
A_5	(0.4992, 0.0000)	0.4992	0.4992	3

Step 6 Determine the priority sequence of alternatives. After computing the score and accuracy functions of P_i, the comparison of any two IFNs can be made and then the alternative sequence for emergency can be determined. As shown in Table 8, it is apparent that A_1 is considered as the most suitable one, the priority sequence is $A_1 \succ A_4 \succ A_5 \succ A_3 \succ A_2$.

Table 9 shows the comparison of ranking sequences with Ju et.al' method [17], it can be easily find that our result is very similar with Ju et.al's, and the difference is the ranking of A_3 and A_5. Focus on A_3 and A_5, as shown in Table 5, it is easily find that three experts almost consider that A_5 is more desirable than A_3 in terms of each criteria. Therefore common sense suggests that the ranking of A_5 should be more advanced than A_3, which coincides with the result of E-IFWA

TABLE 9. The comparison of ranking sequences.

Ranking	1	2	3	4	5
The proposed method	A_1	A_4	$\underline{A_5}$	$\underline{A_3}$	A_2
Ju et.al's method [17]	A_1	A_4	$\underline{A_3}$	$\underline{A_5}$	A_2

but is inconsistent with Ju et.al' result [17], so our result is more reasonable.

In addition, we present a sensitivity analysis by making small changes of two random IFNs. Focus on the two bold IFNs in Table 6, for the assessment on A_1 and A_3, we replace $d_{32}^1 =$ M, $d_{13}^2 =$ VH with H, M respectively. The new result of E-IFWA is shown in Table 10. Compared with the original ranking result in Table 8, A_4 becomes the best choice in replace with A_1, while the ranking of A_2, A_3 and A_5 is not changed. The reason can be explained as follows: as shown in Table 8, the score difference between A_1 and A_4 is smaller than that between A_3 and A_5, while the change degree of d_{13}^2 is larger than d_{32}^1, so S_1 is decreased, meanwhile S_4 is increased owing to the change of objective weight in experts and criteria, thus S_4 becomes larger than S_1. S_3 is improved, reducing the gap to S_5, while the ranking remains the same. In summary, it shows that E-IFWA is sensitive to the change of IFNs.

TABLE 10. The priority sequence of alternatives with modified IFNs.

Alternatives	P_i	S_i	H_i	ranking
A_1	(0.8096, 0.1305)	0.6792	0.9401	**2**
A_2	(0.4032, 0.0000)	0.4032	0.4032	5
A_3	(0.4807, 0.0000)	0.4807	0.4807	4
A_4	(0.8274, 0.1134)	0.7140	0.9408	**1**
A_5	(0.4999, 0.0000)	0.4999	0.4999	3

V. CONCLUSION

A critical problem in emergency management is how to select the most desirable alternative. In this paper, we propose a new method based on an extension entropy, BWM and Intuitionistic fuzzy weighted averaging operator (E-IFWA) to manage emergency alternative selection. Considering inherent characteristics of linguistic assessment, E-IFWA method uses IFN to represent incomplete information (fuzzy information and missing information), which can describe the preference of decision-makers more clearly due to its more options. Besides, an extension Intuitionistic fuzzy entropy is introduced in this paper, which can measure the uncertainty of IFN and then determine the importance of decision-makers and criteria. Integrating with the subjective weight by BWM, E-IFWA uses the combined weight in aggregation operation.

The experiments including a simple example and a case study compared with the existing method illustrate that the E-IFWA method is effective and can get more reasonable result in emergency management. In conclusion,

$$\begin{bmatrix} (0.8651, 0.0787) & (0.8147, 0.1271) & (0.8640, 0.0797) & (0.8162, 0.1255) \\ (0.2955, 0.0000) & (0.3684, 0.0000) & (0.5000, 0.5000) & (0.2955, 0.0000) \\ (0.3748, 0.5952) & (0.2974, 0.0000) & (0.5536, 0.4048) & (0.5034, 0.0000) \\ (0.7679, 0.1710) & (0.8640, 0.0797) & (0.8164, 0.1254) & (0.6038, 0.3675) \\ (0.6842, 0.2724) & (0.4400, 0.5462) & (0.3728, 0.0000) & (0.7500, 0.2000) \end{bmatrix}$$

the advantages of E-IFWA are shown in two aspects: one is that E-IFWA can well address the representation and management of incomplete information, the other is that E-IFWA takes comprehensive consideration of subjective and objective weight, which avoids the subjectivity of expert evaluation in weight determination. E-IFWA provides a promising way to select the most suitable emergency alternative.

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