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Efficient Method for Improving the Spreading Efficiency in Small-World Networks and Assortative Scale-Free Networks

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ABSTRACT Many complex systems are abstractly regarded as complex networks in the study of complicated problems. The research field of information spreading in complex networks has attracted extensive interest. Many spreading strategies have been proposed for improving the spreading efficiency in complex networks. However, the strategies differ in terms of performance in various complex networks. In this paper, a hybrid and effective method for improving the spreading efficiency in small-world networks and assortative scale-free networks is proposed. The proposed method can be applied to solve the essential problem of low spreading efficiency due to spreading to small-degree vertices. The proposed method combines two strategies: 1) a set of top small-degree vertices are specified as the initial spreaders and 2) vertices preferentially spread information to large-degree neighbors. Sixty-eight groups of Monte Carlo experiments are conducted in three real complex networks and seventeen synthetic complex networks. According to the experimental results and theoretical analysis, the proposed method is efficient for improving the spreading efficiency in small-world networks and assortative scale-free networks. Moreover, in assortative scale-free networks, the improvement in the spreading efficiency that is realized via the proposed method increases with the assortativity coefficient.

INDEX TERMS Small-world network, scale-free network, spreading efficiency, spreading strategy, assortativity.

I. INTRODUCTION

Complex networks are utilized to investigate real complex systems [1], such as food webs [2], [3], the Internet of Things (IoT) [4]–[6], computer networks [7], [8], biological networks [9]–[11], communications [12]–[14], social networks [15]–[17], brain networks [18], [19], and technological networks [20], [21]. The fundamental strategy is to consider real complex networks abstractly as graphs that are composed of edges and vertices [3], [22]. The analysis of the structures in those graphs involves many mathematical and physical problems. Fundamental information on many complex problems in real complex systems can be obtained by analyzing the graph structures. Two typical complex networks are identified in many real systems: (1) scale-free networks [8], in which the degrees of the vertices are heterogeneous, and (2) small-world networks [23], in which the degrees of the vertices are homogeneous.

In emergency, the information spreading in a group of people is expected to be fast, such as the rumors, warnings, or public opinions diffusion in a group of people. In this research field, a group of people can be abstractly regarded as a complex network which is composed of vertices and edges. The vertices express the people in the group and the edges express the communication channels in the group. To improve the information spreading efficiency in complex networks, the spreading processes in various complex networks are analyzed extensively to identify valid and efficient approaches. In addition, many spreading strategies for various complex networks are proposed by employing the features of network structures and spreading processes.

Spreading strategy of networks with different structures differ in terms of performance. Many real social systems are organized into groups of people. Those groups of people are

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formed within complex networks and referred to as communities in networks [24]. In networks that have communities, the vertices that are linked to several communities are the preferential spreaders for improving the spreading efficiency. However, in networks that do not have significant communities, this strategy is not efficient. In addition, in various real systems, vertices tend to connect to vertices that have similar degrees; such networks are referred to as assortative networks [25] and examples include social networks. In contrast, in other real systems, well-connected vertices preferentially attach to many less-well-connected vertices; such networks are referred to as disassortative networks [26] and examples include technological networks and biological networks. To improve the spreading efficiency, large-degree vertices are preferentially selected as the spreaders [27] in uncorrelated scale-free networks. However, the function of large-degree vertices in improving the spreading efficiency is unclear in assortative scale-free networks. Moreover, in homogeneous networks, the diffusion of large-degree initial spreaders is not significant, whereas it is strong in heterogeneous networks. The features of network structures significantly impact the performance of spreading strategies.

In scale-free networks, the degrees of vertices are heterogeneous. Most vertices are less-well-connected, while a small number of vertices are linked to most other vertices. Large-degree vertices are regarded as influential spreaders. When the large-degree vertices are selected as the initial spreaders in scale-free networks without degree correlations, small-degree vertices can be efficiently informed by the linked large-degree vertices. However, in assortative scale-free networks, large-degree vertices tend to connect to each other and most small-degree vertices cannot be informed efficiently if large-degree vertices are initial spreaders, which leads to lower spreading efficiency. In small-world networks, as illustrated in sub-figures 2(d) and 2(e), the degrees of most vertices are similar. As shown in sub-figures 2(d) and 2(e), a few vertices that locate in the green dashed circles have extremely small or large degrees [28]. However, the diffusion of large-degree initial spreaders is weak because that the number of them is small. Moreover, the small number of edges for spreading to those vertices that have extremely small degree results in the low efficiency of spreading to them, which causes lower spreading efficiency in the network. Therefore, the spreading efficiency can be improved by reducing the amount of time that is required for spreading to vertices that have extremely small degrees in small-world networks and assortative scale-free networks.

There are two main approaches for reducing the amount of time that is required for spreading to small-degree vertices: (1) several small-degree vertices are selected as the initial spreaders and preferentially informed at the beginning of the spreading process and (2) vertices spread the information by preferentially selecting small-degree neighbors [29]. The main difference between the two approaches is that the extremely small-degree vertices are selected in the first approach, whereas in the second approach, the small-degree vertices are simply the small-degree neighbors of vertices. Thus, according to the above analysis, the first approach is selected because low spreading efficiency is caused by spreading to several specific vertices with extremely small degrees in the network.

Moreover, the diffusion of large-degree vertices cannot be ignored in an improved spreading strategy; hence, a strategy [27] in which vertices preferentially spread information to their large-degree neighbors is also selected. In this strategy, the large-degree vertices that are informed with priority are the large-degree neighbors of vertices that could be not of large degrees in the global network. Theoretically, in homogeneous networks, this strategy is also efficient.

According to the above description, in this paper, a hybrid, efficient method for improving the spreading efficiency in small-world networks and assortative scale-free networks is proposed. The main strategy of the proposed method is (1) to select the vertices that are of extremely small degree as the initial spreaders to reduce the time that is required for spreading information to them and (2) to utilize the diffusion of large-degree neighbors of vertices in the spreading process. To evaluate the performance of the proposed method, an Information Spreading model is constructed via Multi-agent Modeling (the 2MIS model). Second, four spreading strategies are designed in the 2MIS model. Third, 68 groups of Monte Carlo experiments are conducted on 17 synthetic networks and 3 real networks.

The remainder of this paper is organized as follows: Section II describes the related work. Section III presents the theoretical analysis and experimental designs of the proposed method. Section IV presents and discusses the experimental results in 20 complex networks with 3 spreading strategies. Section V discusses the main advantages and shortcoming of the proposed method. Section VI presents several conclusions of this work.

II. RELATED WORK

Extensive spreading strategies have been proposed for improving the spreading efficiency as much as possible in complex networks. More vertices are expected to be informed in a shorter period using these improved strategies. Typically, influential vertices in complex networks are exploited to improve the spreading efficiency [30], such as vertices that have large degree centralities, high k-shell values, or high betweenness. In complex networks, information can be rapidly diffused by the influential vertices because they have advantages in terms of the network structure. Typically, the diffusion of influential vertices plays a role at the beginning [27], [31] or in the spreading process, namely, the influential vertices could be the initial spreaders or preferential spreading targets in the spreading process.

According to Jalili and Perc [30] and Ma *et al.* [32], influential initial spreaders have an important effect on the spreading efficiency. An efficient method for identifying influential initial spreaders based on dense groups was proposed. The influential initial spreaders are from various dense groups in which vertices have more internal connections than external ones. That means, different initial spreaders affect the spreading efficiency in different dense groups. According to Kitsak *et al.* [33], the vertices that have high k-shell values can strongly impact the spreading efficiency and should be selected as initial spreaders. Zhou *et al.* [34] presented an approach for ranking the vertices according to their collective influence and posited that the vertices that have a large collective influence should be selected as the initial spreaders to improve the spreading efficiency.

Gao *et al.* [31] proposed an efficient strategy that is based on contact memory in uncorrelated networks. Vertices preferentially spread the information of neighbors that have fewer accumulated contacts in this strategy. At the beginning of the information spreading process, large-degree neighbors are preferentially informed, while at the last phase, small-degree neighbors are preferentially selected as targets. Theoretically, in this strategy, large-degree neighbors are utilized to diffuse information quickly at the beginning because of their high levels of influence. Moreover, the time consumption for spreading to small-degree vertices is reduced at the last phase of spreading. Liu *et al.* [35] and Pei and Makse [36] posited that the large-degree neighbors can help diffuse information rapidly in the spreading process. Moreover, they have demonstrated that information can be efficiently diffused when all vertices preferentially spread information to the large-degree neighbors. In local structures, large-degree neighbors are regarded as influential vertices that can spread information efficiently.

Traditionally, influential vertices are preferentially selected as the influential initial spreaders or spreading targets for improving the spreading efficiency. However, the time consumption for spreading to small-degree vertices always results in a lower spreading efficiency. Yang *et al.* [37], [38] considered the important role of small-degree neighbors in the spreading process. The strategy of giving priority to small-degree vertices can increase the total infection density based on a classical contact model with both the accepted and specified probabilities in the uncorrelated scale-free network. Moreover, Gao *et al.* [29] proposed an efficient method for improving the spreading efficiency in assortative scale-free networks: The small-degree neighbors are regarded as the preferentially diffused targets, while the initial spreaders are selected randomly. Yang *et al.* [38] and Zhou *et al.* [39] demonstrated that the preference on small-degree vertices is more efficient than that on large-degree vertices in the spreading process.

According to the description above, influential vertices are typically specified as the initial spreaders to optimize the spreading potential over the whole network. However, because of the heterogeneity of complex networks, the importance of small-degree neighbors in the spreading process attracts more attention. The fundamental reason that the small-degree vertices are important for the spreading efficiency is that they are less well connected. Their few connections lead to low efficiency in informing them. The vertices that have extremely small degrees are vital objects for reducing the spreading time, especially in assortative heterogeneous or homogeneous networks, in which the function of the influential initial spreaders is weaker and the vertices of extremely small degree are more important for the spreading efficiency. Theoretically, for the local structures of vertices, if there is no negative influence from small-degree vertices, the strategy of vertices spreading information according to a list that prioritizes large-degree neighbors is efficient. Thus, the strategy of specifying vertices that are of extremely small degree as the initial spreaders can be combined with the strategy of vertices preferentially spreading information to large-degree neighbors. This hybrid strategy could more efficiently spread information in assortative heterogeneous and homogeneous networks.

III. PROPOSED METHOD

A. ESSENTIAL STRATEGIES OF THE PROPOSED METHOD

The proposed method is composed of two strategies: (1) a set of vertices of extremely small degree are selected as initial spreaders at the beginning of the spreading process and (2) vertices preferentially spread information to large-degree neighbors in the spreading process.

The information spreading process is similar to epidemic infection in the SI model [40], [41]. In a heterogeneous network, $i_k(t)$ denotes the density of the informed vertices that have degree k at time step t. Then,

$$\frac{i_k(t)}{dt} = \lambda_k i_k(t)(1 - i_k(t))\Theta_k(t) \tag{1}$$

where λ_k denotes the spreading probability of the vertices that have degree k and the probability $\Theta_k(t)$ that a vertex with the degree k is linked with an informed vertex, which is expressed as follows:

$$\Theta_k(t) = \sum_l P(l|k) i_l(t) \tag{2}$$

where $P(l|k)$ denotes the probability that a vertex that has degree k is linked with a vertex that has degree l. The following also holds [2], [3]:

$$\Theta_k(t) = \frac{\sum_k i_k(t)k}{\sum_k N_k k} \tag{3}$$

where N_k denotes the number of vertices that have degree k, namely, $N_k = NP(k)$; N denotes the network size; and $P(k)$ denotes the degree distribution of the network. It follows that $\frac{i_k(t)}{dt} = f(k)$. Hence, $\frac{i_k(t)}{dt}$ is a function of degree k and the correlation is positive. In an assortative scale-free network, when large-degree vertices are selected as the influential initial spreaders, small-degree (k_s) vertices cannot be informed rapidly because of the huge difference between the degrees of the small-degree vertices and the informed vertices. This causes $\Theta_{k_s}(t)$ to be smaller, which results in a lower value of $\frac{i_{k_s}(t)}{dt}$. Moreover, the number of small-degree vertices is large, which leads to a lower spreading efficiency for the whole network. Hence, the lower degree k leads to a lower growth rate of $i_k(t)$. To mitigate the impact of this feature, the vertices that have extremely small degree should be selected as the initial spreaders. This approach could result in an increase in $\Theta_{k_s}(t)$, which would lead to an increase in $\frac{i_{k_s}(t)}{dt}$. Meanwhile, due to the positive correlation between the degree k and $\frac{i_k(t)}{dt}$, the growth of $i_k(t)$ is faster when the degree k is larger, which could facilitate the growth of $i(t)$ on the whole network. Thus, this approach, in which vertices preferentially spread information to large-degree neighbors, could be an excellent choice.

For small-world networks, most vertices have similar degrees. However, a few vertices could have extremely small or large degrees. Compared with large-degree vertices, small-degree vertices are more difficult to be informed in the spreading process because of their few connections. The spreading process would be efficient if the vertices of extremely small degree have been informed at the beginning. Similar to scale-free networks, the strategy of vertices spreading information preferentially to large-degree neighbors could also be efficient in small-world networks. Moreover, because of the small difference between among the degrees of most vertices in small-world networks, this hybrid strategy could be efficient in both uncorrelated and correlated small-world networks.

B. INFORMATION SPREADING MODEL THAT IS BASED ON MULTI-AGENT MODELLING (2MIS MODEL)

In this paper, an **I**nformation **S**preading model, which is similar to the SI model [40], [41], is constructed based on **M**ulti-agent **M**odeling (2MIS model), in which the individuals differ from one another. There are four assumptions in the 2MIS model:

(1) At the beginning of the spreading process, all vertices are uninformed and a set of specified vertices are informed initially; they are the initial spreaders. In this paper, according to the list of vertices, which are sorted in ascending order of degree, a set of top small-degree vertices are selected as the initial spreaders at the beginning of the spreading process.

(2) In the spreading process, a vertex is in the informed state immediately after it has been informed. Moreover, since people cannot diffuse information to neighbors immediately, a delay time is specified before a vertex spreads the information. The delay time differs among vertices and follows a uniform distribution from 1 to 5 seconds.

(3) In practice, time is consumed when a vertex spreads information to one of its neighbors. For example, when a person sends a message to his friend, the process occurs over a period of time. Thus, in the spreading process, a waiting time is specified for when a vertex spreads information to another vertex. To improve the efficiency and reliability of the simulations, the waiting time is set to 1 second. Hence, the spreading of information from one vertex to another takes 1 second. Moreover, vertices spread information based on spreading probabilities that follow a Uniform distribution from 0 to 1.

(4) In the 2MIS model, each vertex can spread information once to all its neighbors to improve the efficiency of the simulations. Hence, no vertex is informed twice by the same informed vertex. Moreover, each vertex spreads information to all its neighbors. This design is beneficial for examining the efficiency of the spreading strategy.

According to the previous description, the spreading process is summarized as follows: At time step t, $i(t)$ vertices are informed. $s(t)$ denotes the number of uninformed vertices at time step t and $i(0)$ denotes the number of initial spreaders. Then,

$$\frac{i(t)}{dt} = \sum_{i(t)}^{j} i_j(t) = \lambda_j(t)s_j(t) - \sigma(t) \tag{4}$$

where $i(t)$ denotes the number of informed vertices in the complex network at time step t, j denotes an informed vertex, $\lambda_j(t)$ denotes the spreading probability of vertex j at time step t, $i_j(t)$ denotes the number of informed neighbors of vertex j, $s_j(t)$ denotes the number of uninformed neighbors of vertex j, and $\sigma(t)$ denotes the number of repeated informed neighbors that are repeatedly counted at time step t. Among vertices and time steps, $\lambda_j(t)$ differs. Moreover, according to the design of the 2MIS model, $t = \alpha(t) + \beta(t)$, where $\alpha(t)$ denotes the delay time of the vertices before spreading information, $\beta(t)$ denotes the waiting time before spreading information to neighbors, $\beta(t) = 1$ and $\alpha(t)$ ranges from 1 to 5.

C. ALGORITHM FOR GENERATING SYNTHETIC CORRELATED NETWORKS

In this paper, in addition to four real correlated complex networks, synthetic correlated complex networks are generated via the rewriting strategy of Xulvi-Brunet and Sokolov [42]. The steps of the procedure are listed as follows:

(1) Two edges in a basic complex network are randomly selected.

(2) The four degrees of the four vertices that are linked by the two selected edges are compared to generate a list of the vertices that are sorted in descending order of degree.

(3) The selected two edges are removed from the edge list of the complex network; see **Figure 1**. (i) When an assortative complex network is expected to be generated, a new edge links the first and second vertices in the sorted vertex list. Another new edge links the third and fourth vertices in the sorted vertex list. (ii) In contrast, when a disassortative complex network is expected to be generated, a new edge links the first and fourth vertices in the sorted vertex list. Another new edge links the second and third vertices in the sorted vertex list.

The ***P***earson ***C***oefficient (*PC*), which was proposed by Newman [43], is applied in this paper to measure the degree correlation of a complex network. If $PC > 0$, the complex network is assortative; if $PC = 0$, the complex network is uncorrelated; and if $PC < 0$, the complex network is

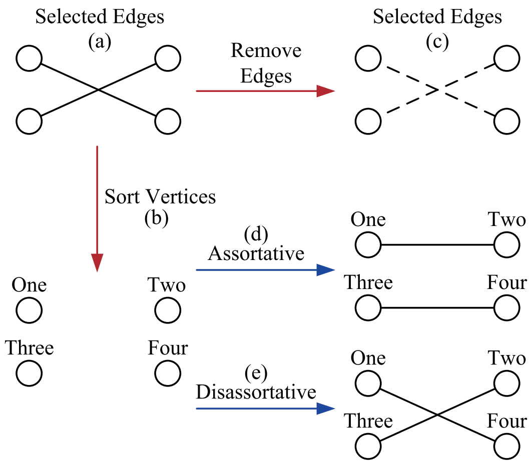


FIGURE 1. Illustrations of the rewriting procedure for generating correlated complex networks. (a) Two edges are selected randomly. (b) The four vertices connected by the two edges are sorted by degree in ascending order. The numbers behind the circles are the sorted sequence. (c) The two randomly selected edges are removed from the current complex network. (d) Assortative complex networks would be generated when the four vertices are connected in sequence. (e) Disassortative complex networks would be generated when the four vertices are connected crosswise.

disassortative. The Pearson coefficient is expressed as

$$PC = \frac{M^{-1}\sum_{e_{ij}\in E} k_i k_j - [M^{-1}\sum_{e_{ij}\in E} \frac{1}{2}(k_i+k_j)]^2}{M^{-1}\sum_{e_{ij}\in E} \frac{1}{2}(k_i^2+k_j^2) - [M^{-1}\sum_{e_{ij}\in E} \frac{1}{2}(k_i+k_j)]^2} \tag{5}$$

where k_i and k_j are the degrees of the vertices, namely, v_i and v_j, that are linked by e_{ij}; M denotes the total number of edges; and E denotes the set of all edges in the complex network.

The above rewriting procedure is repeated until a complex network that has the expected degree correlation is generated. After rewriting the basic complex network, an expected correlated complex network is created.

D. EXPERIMENTAL DESIGN

To evaluate the performance of our proposed method, 68 experiments are conducted. In those experiments, 4 spreading strategies and 20 complex networks are used. The four strategies are described as follows:

(1) **LLS**: a set of top **L**arge-degree vertices are selected as the initial spreaders and vertices spread information according to a list from **L**arge-degree neighbors to **S**mall-degree neighbors;

(2) **SLS**: a set of top **S**mall-degree vertices are selected as the initial spreaders and vertices spread information according to a list from **L**arge-degree neighbors to **S**mall-degree vertices;

(3) **SR**: a set of top **S**mall-degree vertices in a list that is sorted in ascending order of degree are selected as the initial spreaders and vertices spread information **R**andomly;

(4) **RSL**: a set of vertices are **R**andomly selected as the initial spreaders and vertices spread information according to a list from **S**mall-degree neighbors to **L**arge-degree vertices.

Typically, influential vertices are employed to improve the spreading efficiency; hence, LLS is utilized as a comparison spreading strategy in our experiments. The performance of the strategy of selecting a set of top small-degree vertices as the initial spreaders can be evaluated by comparing the spreading efficiencies that are realized using LLS and SLS. Moreover, the performance of the strategy of vertices preferentially spreading the information to large-degree neighbors can be evaluated by comparing the spreading efficiencies that are realized using SLS and SR. In addition, RSL, which was proposed by Gao *et al.* [29], is utilized as a comparison strategy for evaluating the performance of the proposed hybrid strategy, namely, SLS.

In this paper, 17 synthetic networks and 3 real networks are used to verify the performance of spreading strategies. To preferably compare the performance of various spreading strategies, we generate 6 synthetic scale-free networks and 11 synthetic small-world networks with different assortativity. The network sizes of all synthetic networks are the same for comparing the impact of various assortativity on the spreading efficiency.

We first generate a synthetic scale-free network (network size is 1000) and a synthetic small-world network (network size is 1000) without degree correlations by using the software Anylogic [44]. Anylogic is a multi-agent modeling platform. Scale-free networks are generated in Anylogic by exploiting the **B**arabási-**A**lbert (BA) algorithm [8]; and small-world networks are generated in Anylogic by exploiting the **W**atts-**S**trogatz (WS) algorithm [45]. The **B**arabási-**A**lbert (BA) algorithm and **W**atts-**S**trogatz (WS) algorithm are the typical and traditional methods for generating scale-free and small-world networks. Then, on the basis of the generated scale-free and small-world networks without degree correlations by Anylogic, we generate the scale-free and small-world networks with different assortativity by using the rewriting algorithm which is described in **Subsection III-C**. Please Note that the rewriting algorithm cannot change the network structure of the originally generated scale-free and small-world networks. That means, all synthetic scale-free networks have the same size and the same degree distribution; all synthetic small-world networks also have the same size and the same degree distribution.

As listed in **Table 1**, the real scale-free network with $PC = 0.36$ was generated using email data from a large European research institution. This email network represents communications between institution members. The real scale-free network with $PC = 0.91$ is composed of friends from Facebook. This Facebook data was collected from survey participants using the Facebook App. The related network data of the two real scale-free networks can be found at the Stanford Large Network Dataset Collection (url: https://snap.stanford.edu/data/). The real small-world network is a friendship network in a student group. This friendship data was collected by asking each student to list

TABLE 1. Information on twenty complex networks.

Complex Network		Vertices	Edges	Pearson Coefficient	Strategies
Scale-free Network	Real	1005	25571	0.36 [46], [47]	LLS SLS SR RSL
		4039	88234	0.91 [48]	
	Synthetic	1000	1996	0.00	
		1000	1996	0.10	
		1000	1996	0.20	
		1000	1996	0.30	
		1000	1996	0.40	
		1000	1996	0.50	
Small-world Network	Real	2540	12969	0.63 [49]	LLS SLS SR
	Synthetic	1000	1996	0.00	
		1000	1996	0.10	
		1000	1996	0.20	
		1000	1996	0.30	
		1000	1996	0.40	
		1000	1996	0.50	
		1000	1996	-0.10	
		1000	1996	-0.20	
		1000	1996	-0.30	
		1000	1996	-0.40	
		1000	1996	-0.50	

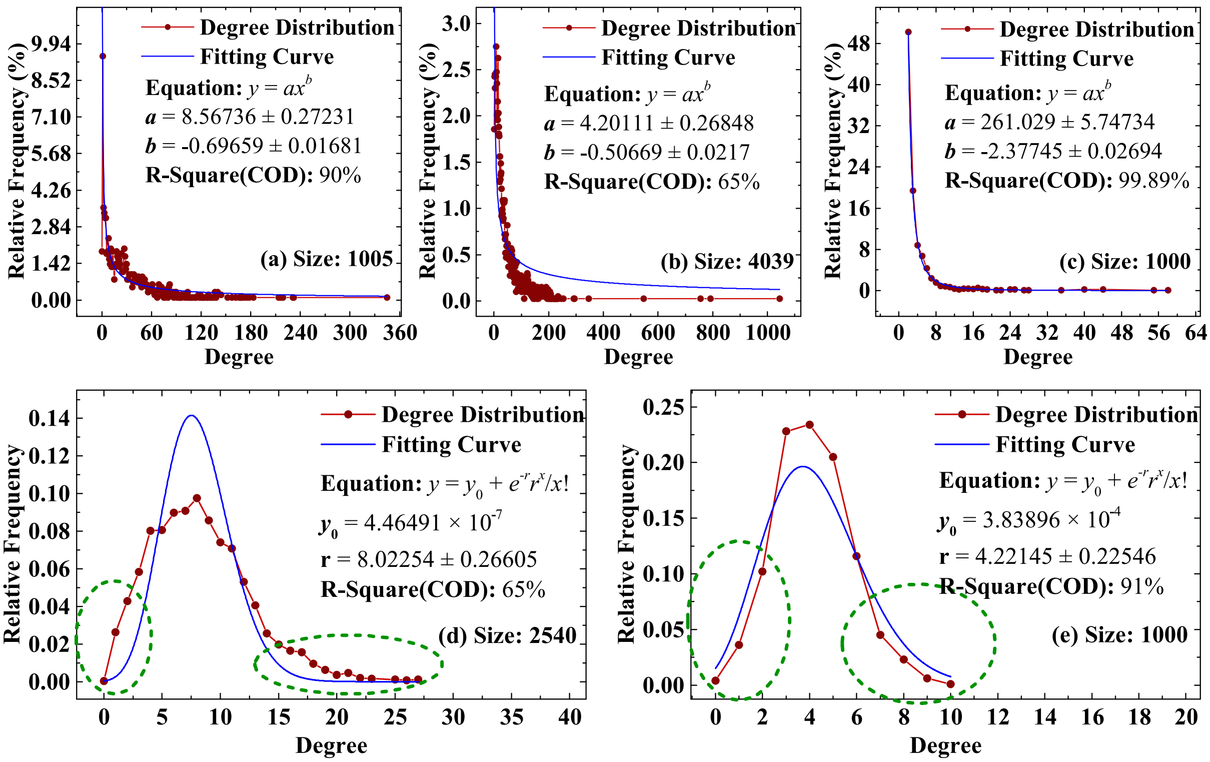
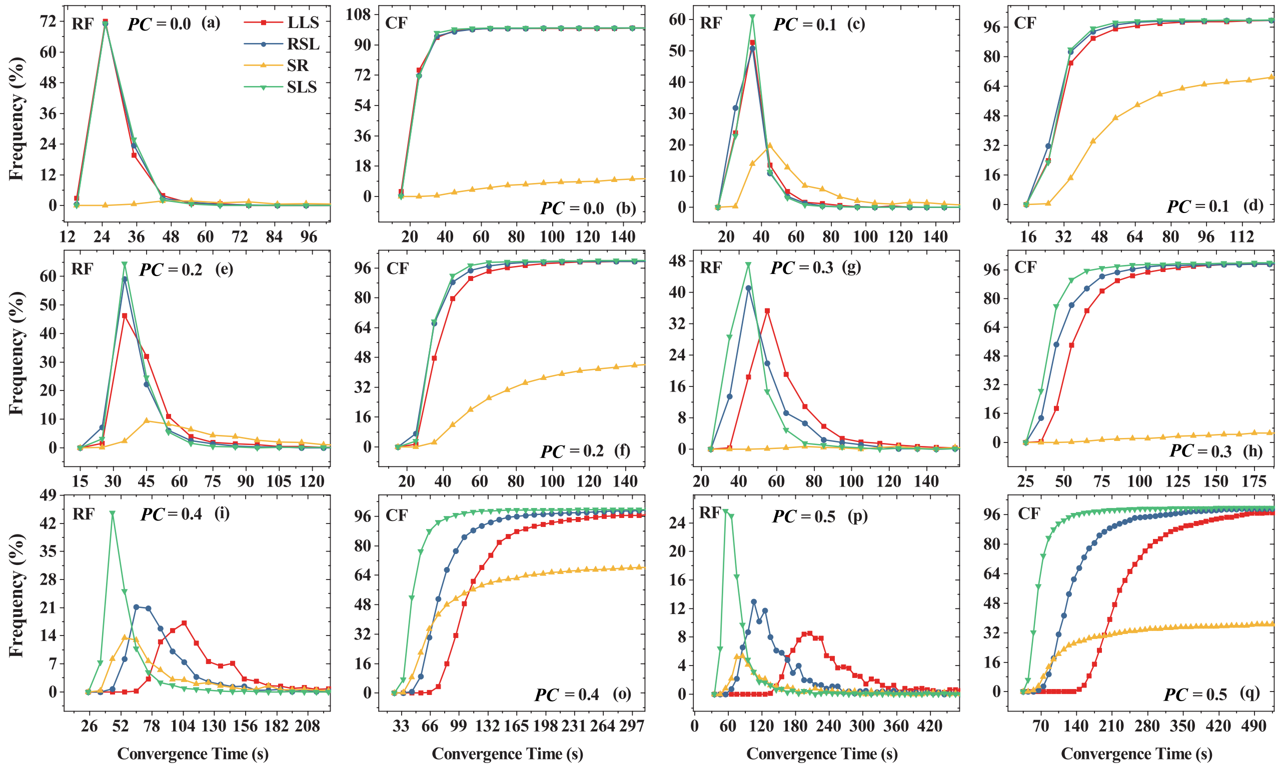


FIGURE 2. The degree distributions and corresponding fitting curves of the 17 synthetic and 3 real networks.

his 5 best female and his 5 best male friends. The related network data of the real small-world network can be found at KONECT (url: http://konect.uni-koblenz.de/networks/).

The degree distributions of the 17 synthetic and the 3 real networks are listed in **Figure 2**. Moreover, because that the degree distributions of scale-free networks follow the Power-law distribution, and the degree distributions of small-world networks follow the bell-shaped distribution, to examine the scale-free and small-world properties of those 20 networks, the degree distributions of the 6 synthetic and the 2 real scale-free networks are fitted according to the Power-law function (see **Equation 6**) which is the typical degree distribution function of the BA scale-free network. And the degree distributions of the 11 synthetic and 1 real small-world networks are fitted according to the Poisson function (see **Equation 7**) which is the typical degree distribution function of the WS small-world network.



RF: the relative frequency distributions of convergence times.
CF: the cumulative frequency distributions of convergence times.
Red line: the frequency distributions of LLS. Blue line: the frequency distributions of RSL.
Yellow line: the frequency distributions of SR.
Green line: the frequency distributions of SLS.
LLS: a set of top large-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
SLS: a set of top small-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
SR: a set of top small-degree vertices are selected as the initial spreaders and vertices spread information randomly.
RSL: a set of vertices are randomly selected as the initial spreaders and vertices preferentially spread information to the small-degree neighbours.
***PC*:** the Pearson Coefficient of the current complex network.

FIGURE 3. Relative and cumulative frequency distributions of convergence times in the six synthetic scale-free networks.

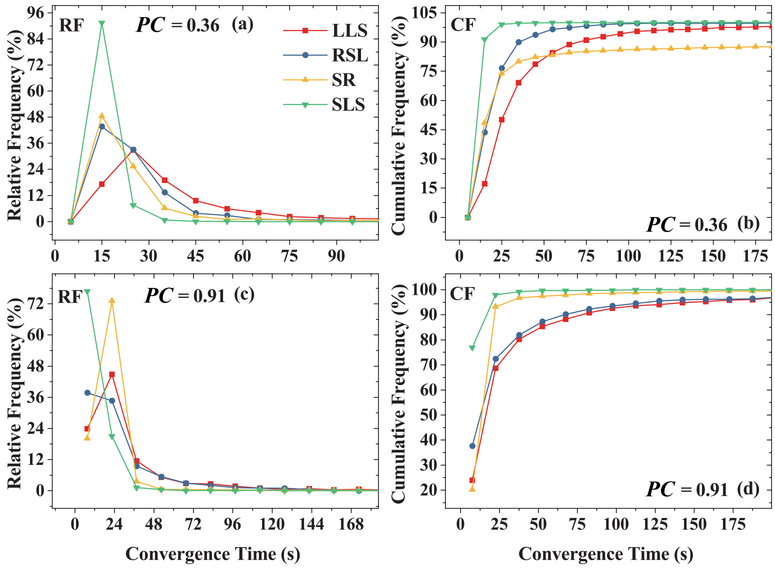
It can be observed that, the Goodness of Fit in sub-figures 2(a) and 2(c) is larger than 90%, that means, the real network with 1005 and the 6 synthetic scale-free networks are typically BA scale-free networks. However, the Goodness of Fit in sub-figure 2(b) is 65%, that means, the real network with 4039 could be not the typical BA scale-free network. But the degree distribution of the real network with 4039 also follows the Power-law distribution. This is, the real network with 4039 is a scale-free network. The Goodness of Fit in sub-figure 2(d) and 2(e) is 65% and 91% respectively. That means, the synthetic 11 small-world networks are typical WS small-world networks. The real network with 2540 could be not the typical WS small-world network. But the degree distribution of the real network with 2540 follows the bell-shaped distribution. This is, the real network with 2540 is a small-world network.

$$y = ax^b \tag{6}$$

$$y = y_0 + \frac{e^{-r} r^x}{x!} \tag{7}$$

All four spreading strategies are applied in eight scale-free networks: two real assortative scale-free networks, a synthetic uncorrelated scale-free network, and five synthetic assortative scale-free networks. The LLS, SLS, and SR strategies are exploited in twelve small-world networks: a real correlated small-world network, a synthetic uncorrelated small-world network, and ten synthetic correlated small-world networks. The *PC*s and the employed spreading strategies for twenty complex networks are listed in **Table 1**.

A spreading process simulation has converged when the number of informed vertices no longer increases. The convergence time is recorded for evaluating the spreading efficiency of the simulation. To reduce the impact of randomness, Monte Carlo experiments are conducted. Moreover, a random number generator is created for generating random numbers in the range of 0 to 1000 (excluding 1000) in a Monte Carlo experiment. There are 1000 repeated simulations in each Monte Carlo experiment. Therefore, 1000 convergence times of the repeated 1000 simulations are generated in each Monte Carlo



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***PC*:** the Pearson Coefficient of the current complex network.

FIGURE 4. Relative and cumulative frequency distributions of convergence times in the two real scale-free networks.

experiment. According to the previous description, Sixty-eight Monte Carlo experiments are reported in this paper.

IV. RESULTS

To compare the spreading strategies in various complex networks in terms of spreading efficiency, the relative frequency distribution and cumulative frequency distribution of the 1000 convergence times in each Monte Carlo experiment are generated.

A. SPREADING EFFICIENCY IN SCALE-FREE ASSORTATIVE NETWORKS

Thirty-two Monte Carlo experiments are conducted in six synthetic scale-free networks and two real scale-free networks. The **R**elative **F**requency distribution (RF) and **C**umulative **F**requency distribution (CF) of the 1000 convergence times in each Monte Carlo experiment are calculated and presented in **Figures 3** $\sim$ **4**.

The performance of the strategy of selecting a set of vertices of extremely **S**mall degree as the initial spreaders ("S-" for short) can be evaluated by comparing LLS and SLS in terms of efficiency. The performance of the strategy of vertices preferentially spreading information to the **L**arge-degree neighbors ("-LS" for short) can be evaluated by comparing SR and SLS in terms of efficiency. The improvement that is realized via the proposed method, namely, SLS, can be evaluated by comparing the previous method, namely, RSL, and SLS in terms of efficiency.

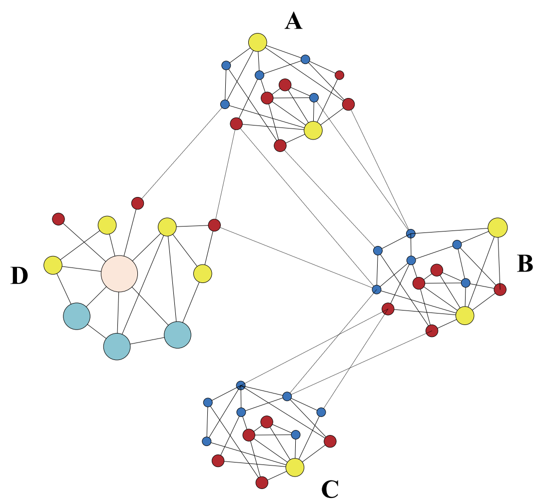
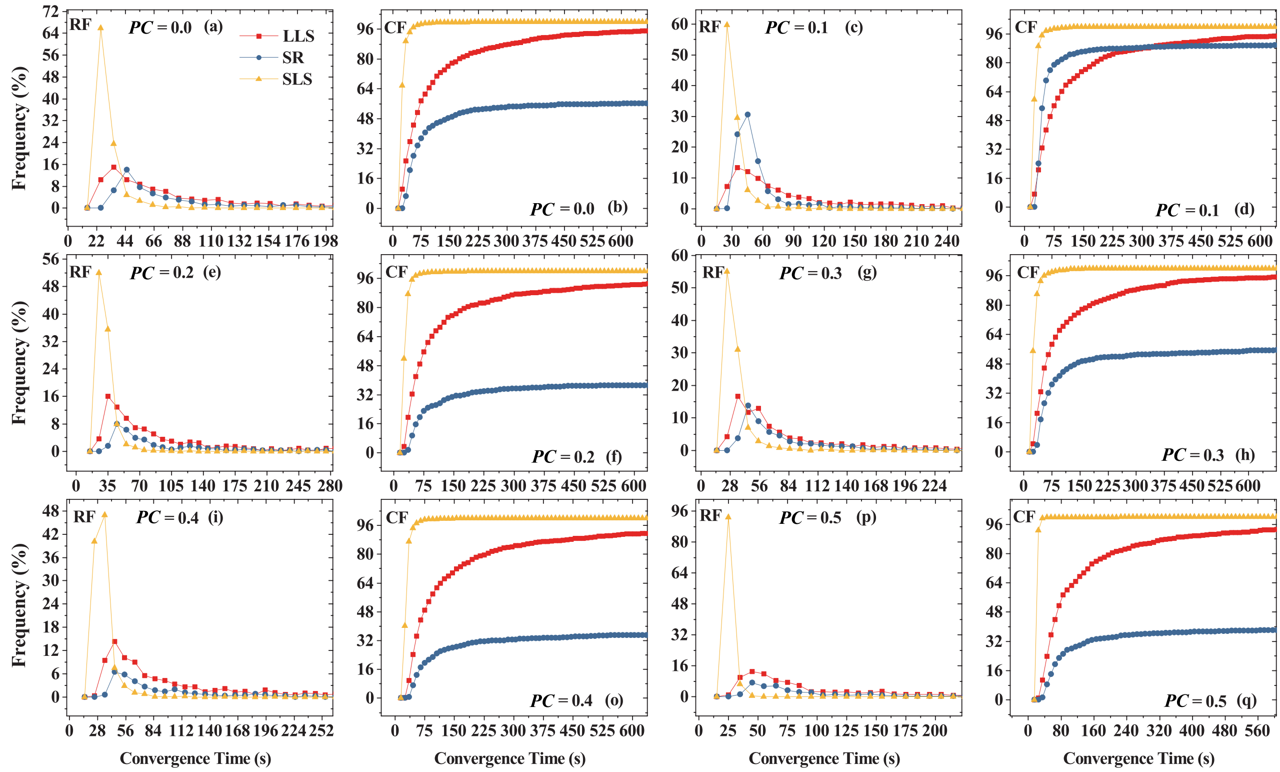


FIGURE 5. Illustration of the community structure in the assortative scale-free network. The vertices with large degrees have large diameters. The vertices with small degrees have small diameters. The number of communities composed of small-degree vertices is larger than that composed of large-degree vertices. A, B, and C are the communities composed of small-degree vertices. D is the community composed of large-degree vertices.



RF: the relative frequency distributions of convergence times.
CF: the cumulative frequency distributions of convergence times.
Red line: the frequency distributions of LLS.
Blue line: the frequency distributions of SR.
Yellow line: the frequency distributions of SLS.
LLS: a set of top large-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
SR: a set of top small-degree vertices are selected as the initial spreaders and vertices spread information randomly.
SLS: a set of top small-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
***PC*:** the Pearson Coefficient of the current complex network.

FIGURE 6. Relative and cumulative frequency distributions of convergence times in the assortative small-world networks.

The performance evaluation results of the four hybrid strategies in scale-free networks with various *PC*s are presented in the following table.

According to the list of the four hybrid strategies that is sorted according to performance, strategies S- and -LS perform well.

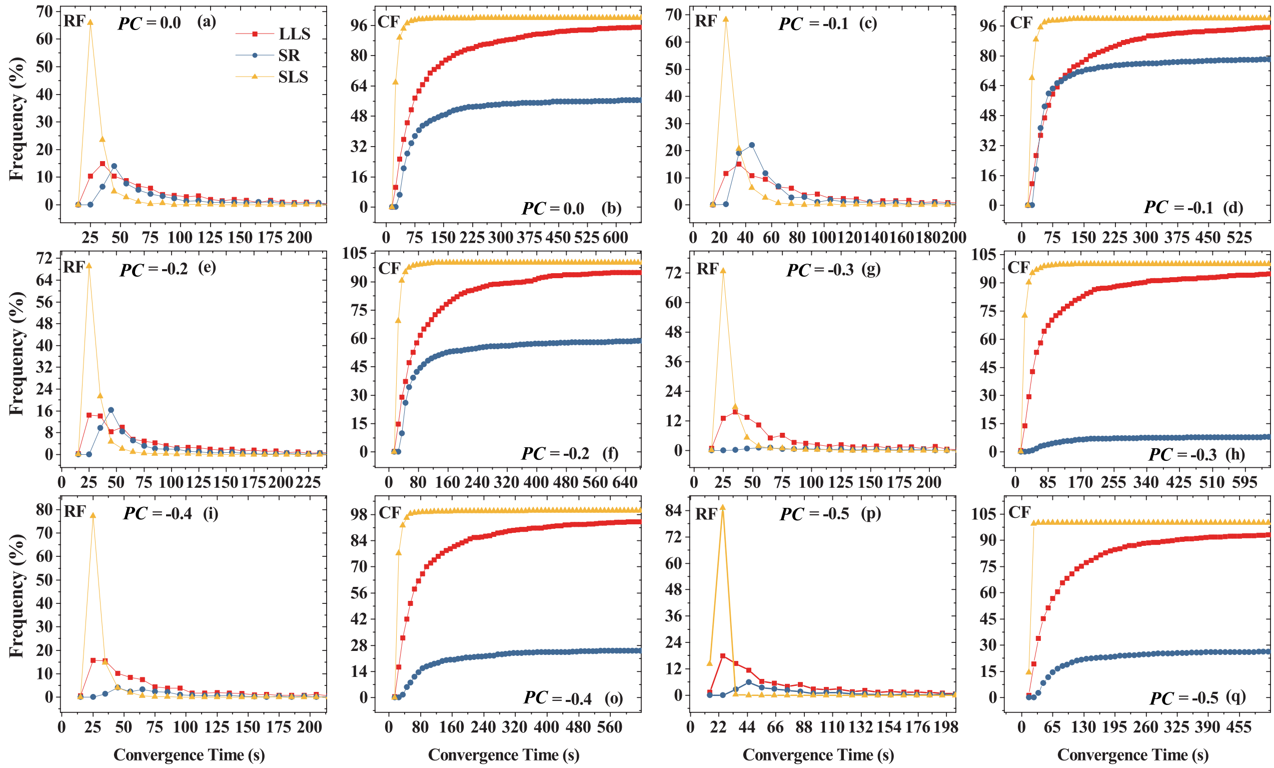
Based on the performance rankings in **Table 2**, the proposed method, namely, SLS, is always efficient in the scale-free networks, even in the uncorrelated scale-free network, and the improvement that is realized by SLS becomes more substantial as *PC* value increases. Hence, the small-degree vertices play a more important role in scale-free networks that have higher *PC* values.

Moreover, RSL always outperforms LLS in terms of efficiency in the assortative scale-free networks; hence, the conclusion of Gao *et al.* [29] is valid. The relatively weak performance of LLS demonstrates that the influential vertices are not efficient in improving the spreading efficiency in assortative scale-free networks.

TABLE 2. Performances evaluation results for the four hybrid strategies in the eight scale-free networks. "=" denotes that performances of the two strategies are almost the same, ">" denotes that the performance of the former is higher than that of the latter, "– >" denotes that the performance of the former is slightly higher than that of the latter, "+ >" denotes that the performance of the former is significantly higher than that of the latter, and "≈" denotes that the performance advantages of the two strategies differ among the metrics.

Scale-Free Network	*PC*	Performance Rankings
Synthetic	0.0	SLS = RSL = LLS +> SR
	0.1	SLS -> RSL -> LLS +> SR
	0.2	SLS -> RSL > LLS +> SR
	0.3	SLS > RSL +> LLS +> SR
	0.4	SLS +> RSL +> LLS +> SR
	0.5	SLS +> RSL +> LLS +> SR
Real	0.36	SLS +> RSL +> LLS ≈ SR
	0.91	SLS +> SR +> RSL +> LLS

In addition, the performance of SR is not stable in the two real scale-free networks. As shown in **Figures 4(c)** and **4(d)**, in the real network with $PC = 0.91$, the performance of SR is more efficient than that of LLS and RSL. This could be



RF: the relative frequency distributions of convergence times.
CF: the cumulative frequency distributions of convergence times.
Red line: the frequency distributions of LLS.
Blue line: the frequency distributions of SR.
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LLS: a set of top large-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
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SLS: a set of top small-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
***PC*:** the Pearson Coefficient of the current complex network.

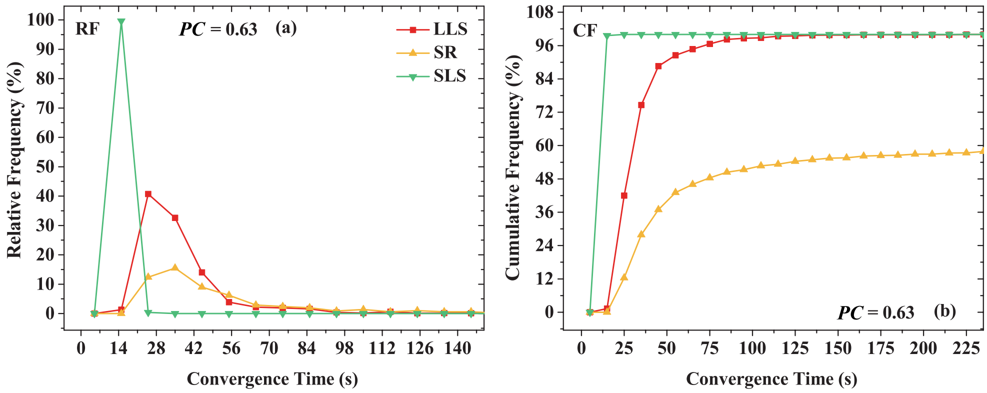
FIGURE 7. Relative and cumulative frequency distributions of convergence times in the disassortative small-world networks.

caused by the extremely high *PC* of the real network, which leads to a more efficient performance of "S-". As shown in **Figure 4(a)**, in the real network with $PC = 0.36$, the corresponding convergence time in peak when using SR is smaller than that when using LLS. However, as illustrated in **Figure 4(b)**, the cumulative frequency of convergence times increases more slowly when using SR than that when using LLS in the later stage. This further verifies the importance role of the strategy "-LS" on improving the spreading efficiency although that the performance of "S-" is also efficient.

Compared with the six synthetic scale-free networks, the *PC* of the real network with $PC = 0.36$ is not the highest. However, the performance of SR in the real network with $PC = 0.36$ is more efficient than that in the six synthetic networks. This could be caused by the community structure in the real network with $PC = 0.36$. The real network with $PC = 0.36$ is of 42 communities found out by the investigators [46], [47]. Additionally, the real network is an assortative scale-free network. The vertices with large degrees are more likely to connect to the vertices with large degrees. Similarly, the vertices with small degrees tend to link with each other. In this case, small-degree vertices are more likely to be in a group while large-degree vertices group together. Considered that most of vertices are of small degrees in the scale-free network. Thus, theoretically, as illustrated in **Figure 5**, most of communities are composed of small-degree vertices. Then, information could not be diffused efficiently if the large-degree vertices are the initial spreaders. In contrast, the initial spreaders could be in many groups when the small-degree vertices are the initial spreaders. That means, the initial small-degree vertices could belong to different communities. This indicates that the initial spreaders with small degrees have the priority on improving the spreading efficiency in the assortative scale-free network with communities.

B. SPREADING EFFICIENCY IN SMALL-WORLD NETWORKS

Thirty-six Monte Carlo experiments are conducted in a real small-world network and eleven synthetic small-world



RF: the relative frequency distributions of convergence times.
CF: the cumulative frequency distributions of convergence times.
Red line: the frequency distributions of LLS.
Blue line: the frequency distributions of SR.
Yellow line: the frequency distributions of SLS.
LLS: a set of top large-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
SR: a set of top small-degree vertices are selected as the initial spreaders and vertices spread information randomly.
SLS: a set of top small-degree vertices are selected as the initial spreaders and vertices preferentially spread information to large-degree neighbours.
***PC*:** the Pearson Coefficient of the current complex network.

FIGURE 8. Relative and cumulative frequency distributions of convergence times in the real small-world networks.

TABLE 3. Performance evaluation results for the three hybrid strategies in small-world networks. ">" denotes that the performance of the former is higher than that of the latter, "+ >" denotes that the performance of the former is significantly higher than that of the latter, and "≈" denotes that the performance advantages of the two strategies differ among the metrics.

Small-world Network	*PC*	Performance Ranking
Synthetic	-0.5	SLS +> LLS +> SR
	-0.4	SLS +> LLS +> SR
	-0.3	SLS +> LLS +> SR
	-0.2	SLS +> LLS +> SR
	-0.1	SLS +> LLS > SR
	0.0	SLS +> LLS +> SR
	0.1	SLS +> LLS ≈ SR
	0.2	SLS +> LLS +> SR
	0.3	SLS +> LLS +> SR
	0.4	SLS +> LLS +> SR
	0.5	SLS +> LLS +> SR
Real	0.63	SLS +> LLS +> SR

networks. The **R**elative **F**requency distribution and **C**umulative **F**requency distribution of 1000 convergence times in each Monte Carlo experiment are calculated and presented in **Figures 6 ∼ 8**.

The performance evaluation results for the three hybrid strategies in the small-world networks with various *PC*s are presented in **Table 3**.

As illustrated in **Figures 6 ∼ 8**, the proposed method, namely, SLS, is the most efficient in both types of small-world networks with various *PC*s and the improvement that is realized by SLS is significant. For small-world networks, most of vertices have similar degrees. However, spreading to vertices that are of extremely small degree requires more time. Such vertices result in lower spreading efficiency over the whole network. The vertices that are of extremely small degree can be informed first by using SLS and the overall spreading efficiency can be improved. Because of two features of small-world networks, namely, (1) the degrees of most vertices are similar and (2) there are few extremely small-degree or large-degree vertices, the degree correlations do not significantly affect the performances of the strategies. SLS is performs efficiently in small-world networks with various *PC*s, including the uncorrelated, assortative, and disassortative small-world networks.

V. DISCUSSION

A. ADVANTAGES OF THE PROPOSED METHOD

The experimental results have demonstrated the satisfactory performance of the proposed method, namely, SLS. SLS can be employed to solve the essential problem of low spreading efficiency in complex networks: the low spreading efficiency that is caused by spreading to small-degree vertices.

Moreover, SLS can be exploited in various complex networks. According to the experimental results and theoretical analysis, SLS can be applied to improve the spreading efficiency in uncorrelated scale-free and small-world networks, assortative scale-free and small-world networks, and disassortative small-world networks.

B. SHORTCOMING OF THE PROPOSED METHOD

The proposed method cannot be used to improve the spreading efficiency in disassortative scale-free networks. In disassortative scale-free networks, vertices that have large degree are preferentially connected with small-degree vertices. Additionally, the large-degree vertices have the most connections in scale-free networks. Because of these characteristics, small-degree vertices can be informed rapidly and the impact of the informing of small-degree vertices does not affect the overall spreading efficiency in a disassortative scale-free network. The proposed method SLS is not suitable to be utilized for improving the spreading efficiency in disassortative scale-free networks.

VI. CONCLUSIONS

In this paper, an efficient method for improving the spreading efficiency in small-world networks and assortative scale-free networks is proposed. The fundamental strategies of the proposed method are as follows: (1) a set of top small-degree vertices are selected as the initial spreaders and (2) the vertices spread information based on a list that is sorted from large-degree neighbors to small-degree neighbors. To evaluate the performance of the proposed method, four spreading strategies are applied in three real complex networks and seventeen synthetic complex networks. The experimental results demonstrate that the proposed method is efficient in improving the spreading efficiency in the small-world networks and assortative scale-free networks. The performance of the proposed method increases with the increase of assortativity coefficient in the assortative scale-free networks. The proposed method can be employed to improve the spreading efficiency in various complex networks and real systems, for applications such as warning diffusion among a group of people, emergency propagation, and rumor control.

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REFERENCES

[1] D. Helbing *et al.*, "Saving human lives: What complexity science and information systems can contribute," *J. Stat. Phys.*, vol. 158, no. 3, pp. 735–781, Feb. 2015.

[2] J. A. Dunne, R. J. Williams, and N. D. Martinez, "Food-web structure and network theory: The role of connectance and size," *Proc. Nat. Acad. Sci. USA*, vol. 99, no. 20, pp. 12917–12922, Oct. 2002.

[3] M. E. J. Newman, "The structure and function of complex networks," *SIAM Rev.*, vol. 45, no. 2, pp. 167–256, Jun. 2003.

[4] S. Cuomo, P. De Michele, F. Piccialli, A. Galletti, and J. E. Jung, "IoT-based collaborative reputation system for associating visitors and artworks in a cultural scenario," *Expert Syst. Appl.*, vol. 79, pp. 101–111, Aug. 2017.

[5] S. Cuomo, P. De Michele, F. Piccialli, and A. K. Sangaiah, "Reproducing dynamics related to an Internet of Things framework: A numerical and statistical approach," *J. Parallel Distrib. Comput.*, vol. 118, no. 2, pp. 359–368, Aug. 2018.

[6] S. Cuomo, V. Di Somma, and F. Piccialli, "Remarks on a computational estimator for the barrier option pricing in an IoT scenario," *Procedia Comput. Sci.*, vol. 113, pp. 513–518, Sep. 2017.

[7] R. Albert and A.-L. Barabási, "Statistical mechanics of complex networks," *Rev. Modern Phys.*, vol. 74, no. 1, pp. 47–97, 2002.

[8] A.-L. Barabási and R. Albert, "Emergence of scaling in random networks," *Science*, vol. 286, no. 5439, pp. 509–512, 1999.

[9] S. Cuomo, P. De Michele, G. Ponti, and M. R. Posteraro, "Validation approaches for a biological model generation describing visitor behaviours in a cultural heritage scenario," in *Proc. Int. Conf. Data Manage. Technol. Appl.*, vol. 178, 2015, pp. 154–168.

[10] A.-L. Barabási and Z. N. Oltvai, "Network biology: Understanding the cell's functional organization," *Nature Rev. Genet.*, vol. 5, no. 2, pp. 101–113, 2004.

[11] M. Gosak *et al.*, "Network science of biological systems at different scales: A review," *Phys. Life Rev.*, vol. 24, pp. 118–135, Mar. 2018.

[12] J. W. Raymond, T. O. Olwal, and A. M. Kurien, "Cooperative communications in machine to machine (M2M): Solutions, challenges and future work," *IEEE Access*, vol. 6, pp. 9750–9766, 2018.

[13] L. Chen *et al.*, "A lightweight end-side user experience data collection system for quality evaluation of multimedia communications," *IEEE Access*, vol. 6, pp. 15408–15419, 2018.

[14] M. Perc, J. J. Jordan, D. G. Rand, Z. Wang, S. Boccaletti, and A. A. Szolnoki, "Statistical physics of human cooperation," *Phys. Rep.*, vol. 687, pp. 1–51, May 2017.

[15] Y. Liu, D. Pi, and L. Cui, "Metric learning combining with boosting for user distance measure in multiple social networks," *IEEE Access*, vol. 5, pp. 19342–19351, 2017.

[16] Z. Ning, Y. Liu, J. Zhang, and X. Wang, "Rising star forecasting based on social network analysis," *IEEE Access*, vol. 5, pp. 24229–24238, 2017.

[17] B. Podobnik, D. Horvatic, T. Lipic, M. Perc, J. M. Buldú, and H. E. Stanley, "The cost of attack in competing networks," *J. Roy. Soc. Interface*, vol. 12, no. 112, 2015, Art. no. 20150770.

[18] R. L. Buckner, J. R. Andrews-Hanna, and D. L. Schacter, "The brain's default network: Anatomy, function, and relevance to disease," in *Annals of the New York Academy of Sciences*, vol. 1124, A. Kingstone and M. B. Miller, Eds. 2008, pp. 1–38.

[19] K. W. Thee, H. Nisar, and C. S. Soh, "Graph theoretical analysis of functional brain networks in healthy subjects: Visual oddball paradigm," *IEEE Access*, vol. 6, pp. 64708–64727, 2018.

[20] G. De Prato and D. Nepelski, "Global technological collaboration network: Network analysis of international co-inventions," *J. Technol. Transfer*, vol. 39, no. 3, pp. 358–375, Jun. 2014.

[21] K.-C. Chen, M. Chiang, and H. V. Poor, "From technological networks to social networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 9, pp. 548–572, Sep. 2013.

[22] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, "Complex networks: Structure and dynamics," *Phys. Rep.*, vol. 424, nos. 4–5, pp. 175–308, Feb. 2006.

[23] A. Arenas, A. Díaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, "Synchronization in complex networks," *Phys. Rep.*, vol. 469, no. 3, pp. 93–153, Dec. 2008.

[24] M. E. J. Newman and M. Girvan, "Finding and evaluating community structure in networks," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 69, no. 2, Feb. 2004, Art. no. 026113.

[25] M. E. J. Newman, "Assortative mixing in networks," *Phys. Rev. Lett.*, vol. 89, no. 20, 2002, Art. no. 208701.

[26] C. C. Leung and H. F. Chau, "Weighted assortative and disassortative networks model," *Phys. A, Stat. Mech. Appl.*, vol. 378, no. 2, pp. 591–602, 2007.

[27] S. Wang, Y. Deng, and Y. Li, "Improving short-term information spreading efficiency in scale-free networks by specifying top large-degree vertices as the initial spreaders," *Roy. Soc. Open Sci.*, vol. 5, no. 11, 2018, Art. no. 181137.

[28] G. Zhu, C. Wang, F. Liu, L. Tang, and J. Zheng, "Age-related network topological difference based on the sleep ECG signal," *Physiolog. Meas.*, vol. 39, no. 8, Aug. 2018, Art. no. 084009.

[29] L. Gao, W. Wang, L. Pan, M. Tang, and H.-F. Zhang, "Effective information spreading based on local information in correlated networks," *Sci. Rep.*, vol. 6, Dec. 2016, Art. no. 38220.

[30] M. Jalili and M. Perc, "Information cascades in complex networks," *J. Complex Netw.*, vol. 5, no. 5, pp. 665–693, 2017.

[31] L. Gao, W. Wang, P. Shu, H. Gao, and L. A. Braunstein, "Promoting information spreading by using contact memory," *EPL (Europhys. Lett.)*, vol. 118, no. 1, 2017, Art. no. 18001.

[32] S. Ma *et al.*, "Seeking powerful information initial spreaders in online social networks: A dense group perspective," *Wireless Netw.*, vol. 24, no. 8, pp. 2973–2991, 2018.

[33] M. Kitsak *et al.*, "Identification of influential spreaders in complex networks," *Nature Phys.*, vol. 6, no. 11, pp. 888–893, 2010.

[34] M.-Y. Zhou, W.-M. Xiong, X.-Y. Wu, Y.-X. Zhang, and H. Liao, "Overlapping influence inspires the selection of multiple spreaders in complex networks," *Phys. A, Stat. Mech. Appl.*, vol. 508, pp. 76–83, Oct. 2018.

[35] Y. Liu, M. Tang, T. Zhou, and Y. Do, "Core-like groups result in invalidation of identifying super-spreader by k-shell decomposition," *Sci. Rep.*, vol. 5, May 2015, Art. no. 9602.

[36] S. Pei and H. A. Makse, "Spreading dynamics in complex networks," *J. Stat. Mech., Theory Exp.*, vol. 2013, no. 12, 2013, Art. no. P12002.

[37] J. Yang, H. Lin, and C.-X. Wu, "Preferential spreading on scale-free networks," *Phys. A, Stat. Mech. Appl.*, vol. 389, no. 18, pp. 3915–3921, 2010.

[38] R. Yang, L. Huang, and Y.-C. Lai, "Selectivity-based spreading dynamics on complex networks," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 78, no. 2, 2008, Art. no. 026111.

[39] T. Zhou, J.-G. Liu, W.-J. Bai, G. Chen, and B.-H. Wang, "Behaviors of susceptible-infected epidemics on scale-free networks with identical infectivity," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 74, no. 5, 2006, Art. no. 056109.

[40] D. Shah and T. Zaman, "Rumors in a network: Who's the culprit?" *IEEE Trans. Inf. Theory*, vol. 57, no. 8, pp. 5163–5181, Aug. 2011.

[41] Z. Wang *et al.*, "Statistical physics of vaccination," *Phys. Rep.*, vol. 664, pp. 1–113, Dec. 2016.

[42] R. Xulvi-Brunet and I. M. Sokolov, "Reshuffling scale-free networks: From random to assortative," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 70, no. 6, Dec. 2004, Art. no. 066102.

[43] M. E. J. Newman, "Mixing patterns in networks," *Phys. Rev. E, Stat. Phys. Plasmas Fluids Relat. Interdiscip. Top.*, vol. 67, no. 2, 2003, Art. no. 026126.

[44] *Anylogic Software*, The AnyLogic Company, San Jose, CA, USA, Mar. 2019.

[45] D. J. Watts and S. H. Strogatz, "Collective dynamics of 'small-world' networks," *Nature*, vol. 393, no. 6684, pp. 440–442, Jun. 1998.

[46] J. Leskovec, J. Kleinberg, and C. Faloutsos, "Graph evolution: Densification and shrinking diameters," *ACM Trans. Knowl. Discovery Data*, vol. 1, no. 1, 2007, Art. no. 2.

[47] H. Yin, A. R. Benson, J. Leskovec, and D. F. Gleich, "Local higher-order graph clustering," in *Proc. 23rd ACM SIGKDD Int. Conf. Knowl. Discovery Data Mining*, 2017, pp. 555–564.

[48] J. Leskovec and J. J. McAuley, "Learning to discover social circles in ego networks," in *Proc. Adv. Neural Inf. Process. Syst.*, vol. 1, 2012, pp. 539–547.

[49] J. Moody, "Peer influence groups: Identifying dense clusters in large networks," *Social Netw.*, vol. 23, no. 4, pp. 261–283, 2001.

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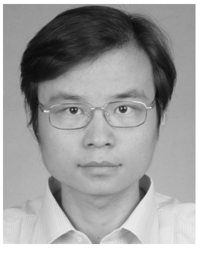

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