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A Precoding Scheme for Eliminating Data Identification Problem in Single Carrier System Using Data-Dependent Superimposed Training

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ABSTRACT In single carrier (SC) systems using a data-dependent superimposed training (DDST) scheme, the data-induced interference that occurs during channel estimation is eliminated, but at the expense of data distortion. This distortion causes a data identification problem (DIP), in which the data sequence cannot be uniquely identified at the receiver end. Accordingly, the present study proposes a precoding scheme which solves the DIP in SC systems using DDST. Five design criteria are derived for the precoding matrix in order to eliminate DIP, achieve frequency diversity, maintain a low peak-to-average power ratio (PAPR), and minimize the receiver complexity. In addition, a low-complexity detection method is proposed for reducing the search space of the detection process. The simulation results show that the proposed precoding scheme successfully eliminates the DIP in traditional DDST schemes while maintaining the same PAPR as that of the traditional SC systems based on DDST.

INDEX TERMS Cyclic prefixed single carrier (CP-SC) system, data-dependent superimposed training (DDST), data identification problem (DIP), precoding.

I. INTRODUCTION

Cyclic prefixed single carrier (CP-SC) systems are a promising technique for realizing wideband wireless communication systems [1]–[5]. In frequency-selective fading channels, the use of a cyclic prefix (CP) not only eliminates intersymbol interference (ISI), but also converts the linear convolution between the channel and the transmitted signal into a circular convolution [6]. As in orthogonal frequency division multiplexing (OFDM) systems, each frequency bin (subcarrier) in a CP-SC system experiences a flat fading channel. Moreover, the channel effects can be compensated using a one-tap equalizer. Therefore, SC systems with frequency-domain equalization (SC-FDE) are an attractive solution for broadband wireless communications.

SC transmission has several advantages for practical wireless communications, including a low peak-to-average power ratio (PAPR) [3] and good frequency diversity. However, the performance of CP-SC is fundamentally dependent on the accuracy of the channel estimates. In general, channel estimation is achieved using pilot symbols. Therefore, the problem of optimizing the pilot allocation in CP-SC systems has attracted great attention in recent years [7]–[10]. In [7], [8], the authors proposed a pilot-aided CP-SC system, in which a known block is added as a suffix to each symbol block before performing cyclic prefixing. Moreover, the authors in [11]–[15] presented a superimposed training (ST) scheme, in which the pilot symbols are superimposed directly on the data symbols in order to reduce the transmission bandwidth.

However, while ST methods have a high bandwidth efficiency, the unknown data symbols induce substantial interference, and hence the accuracy of the channel estimates is severely degraded. To address this problem, the authors in [16] proposed a data-dependent superimposed training (DDST) scheme, in which a data-dependent

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sequence is superimposed on the original data sequence in order to eliminate the data-induced interference. However, in doing so, DDST induces a distortion of the data sequence. Various methods have been proposed at the receiver end for suppressing this distortion and improving the bit error rate (BER) performance as a result [16], [17]. However, these methods only partially resolve the problem since there exists a certain probability that the transmitter will transform two different input sequences into the same output sequence. Given this occurrence, the data cannot be uniquely recovered at the receiver end, and hence a data identification problem (DIP) occurs.

This study proposes a precoding scheme for eliminating the DIP in CP-SC systems based on DDST. The study commences by describing the transmitter and receiver architectures. A low-complexity detection (LCD) scheme is then proposed. In addition, five design criteria for the precoding matrix are introduced. For example, it is shown that the precoding matrix should have the form of the Kronecker product of a sub-matrix and an identity matrix. Secondly, it is demonstrated that all of the column vectors of the precoding matrix should have ideal periodic autocorrelation functions (PACFs) in order to maximize the frequency diversity. Thirdly, it is shown that the precoding matrix should have the form of a diagonal matrix with elements of equal magnitude in order to achieve a low PAPR of the transmitted signal. Fourthly, the precoding matrix should be unitary such that the modulated symbols can be recovered at the receiver end by simply multiplying the Hermitian of the precoding matrix. Finally, the criterion for the precoding matrix to avoid DIP is derived. It is shown that the first four design criteria can be combined into a single criterion, namely the precoding matrix should have the form of the Kronecker product of an identity matrix and a diagonal matrix with equal-magnitude diagonal elements. The study additionally presents a design criterion for the phase of the diagonal elements which reduces the search space of the detection process and avoids DIP. Utilizing the proposed criteria, a precoding matrix is derived for the case of M-QAM modulation. The feasibility of the proposed design is demonstrated by means of numerical simulations.

The remainder of this paper is organized as follows. Section II presents the transmitter architecture of a precoded CP-SC system using the DDST scheme. Section III describes the receiver architecture and proposes a low-complexity detection scheme. Section IV derives the design criteria required to achieve frequency diversity and a low PAPR. Section V analyzes the DIP and derives the necessary conditions for avoiding its occurrence. Section VI presents the proposed design criteria for the phases of the diagonal elements of the precoding matrix. Section VII presents and discusses the simulation results. Finally, Section VIII provides some brief concluding remarks.

Notation: Throughout the remainder of this study, bold letters denote matrices or column vectors; $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^T$ denote conjugate, transpose, and Hermitian transpose operations, respectively; and $\otimes$ is the Kronecker product. In addition, $\tilde{\cdot}$ is the principle submatrix. $\|\cdot\|$ indicates the Euclidean norm of a vector; and $\lfloor\cdot\rceil$ gives the closest constellation point. $(\cdot)_{\mathrm{mod}\,N}$ is the modulo N operation and $E[\cdot]$ is the expectation operation. I_N, 0_N, and 1_N denote the $N \times N$ identity matrix, the $N \times 1$ all-zero matrix, and the $N \times 1$ all-one matrix, respectively. Finally, $\mathbf{F}_N$ is the fast Fourier transform (FFT) matrix with the (p, q)th entry given by $\frac{1}{\sqrt{N}} \exp\left(\frac{-j2\pi pq}{N}\right)$. $x(p)$ denotes the pth entry of the vector $\mathbf{x}$ and $[\cdot]_{p,q}$ denotes the (p, q)th entry of a matrix.

II. TRANSMITTER ARCHITECTURE

In CP-SC systems utilizing the DDST scheme, a data-dependent sequence is superimposed onto the data sequence in the time domain in order to remove the frequency components of the sub-carriers reserved for the pilot signals. In this way, the frequency-domain pilot signals are free from data-induced interference at the receiver end, and hence the accuracy of the channel estimates is improved. However, in removing some of the frequency components, DDST induces a distortion of the data sequences at the transmitter. In addition, there exists a certain probability that the transmitter will transform two different input sequences into the same output sequence, and hence the data cannot be uniquely recovered at the receiver. In other words, a data identification problem (DIP) occurs. Accordingly, the present study proposes a precoding scheme for eliminating DIP in SC systems based on DDST while maintaining an equivalent PAPR and frequency diversity to that of a traditional SC-DDST system.

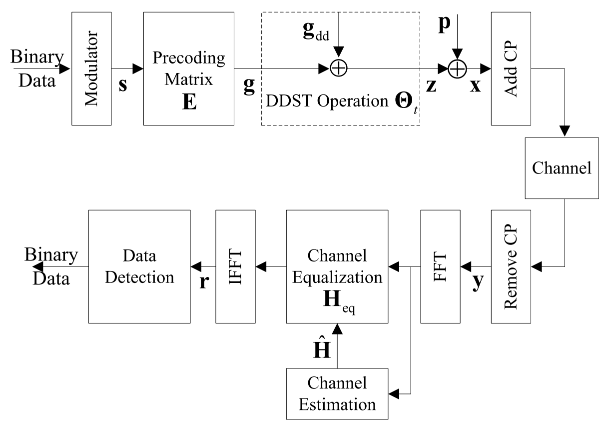


FIGURE 1. Architecture of precoded CP-SC system using DDST.

Figure 1 presents a block diagram of the proposed precoded CP-SC system, in which both the DDST scheme and frequency-domain equalization (FDE) are employed. An assumption is made that the pilots are located in equally-spaced frequency bins, $n = rQ+t$, where $r = 0, 1, \ldots, K-1$, t is the initial pilot index, Q is the pilot spacing, and K is the total number of frequency-domain pilot bins. In addition, it is assumed that the energy of the pilots is uniformly distributed over the K frequency bins. It is noted that K should be larger than or equal to the channel length L in order to ensure an accurate channel estimation.

In the proposed precoding scheme, a series of N modulated symbols are collected to form a CP-SC data block, i.e.,

$$\mathbf{s} = [s(0)\ \ s(1)\ \ \cdots\ s(N-1)]^T. \tag{1}$$

The modulated data symbols have a zero mean and are mutually independent. In addition, the input data are assumed to be M-QAM modulated, where M is the modulation order. Furthermore, the precoded signal vector is given by

$$\mathbf{g} = \mathbf{E} \cdot \mathbf{s}, \tag{2}$$

where $\mathbf{E}$ is an $N \times N$ precoding matrix. In the DDST scheme, a data-dependent sequence is superimposed onto the precoded sequence $\mathbf{g}$ to eliminate data-induced interference at the pilot frequency bins. The data-dependent sequence has the form

$$\mathbf{g}_{\mathrm{dd}} = -\mathbf{F}_N^H \mathbf{T}_t \mathbf{F}_N \mathbf{g} \equiv -\mathbf{J}_t \mathbf{g}, \tag{3}$$

where $\mathbf{J}_t \equiv \mathbf{F}_N^H \mathbf{T}_t \mathbf{F}_N$ and $\mathbf{T}_t$ is an $N \times N$ diagonal matrix with the diagonal elements given by

$$[\mathbf{T}_t]_{n,n} = \begin{cases} 1, & n = rQ + t,\ r = 0, 1, \ldots, K-1, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

It can be shown that

$$\mathbf{J}_t = \tilde{\mathbf{J}}_t \otimes \mathbf{I}_K, \tag{5}$$

where $\mathbf{I}_K$ is a $K \times K$ identity matrix, and $\tilde{\boldsymbol{J}}_t$ is a $Q \times Q$ matrix with the form (6), as shown at the bottom of this page. Note that $\tilde{\mathbf{J}}_t$ is the principal sub-matrix of $\mathbf{J}_t$ [18]. It can be shown that $\tilde{\mathbf{J}}_t = \mathbf{F}_Q^H \tilde{\mathbf{T}}_t \mathbf{F}_Q$, where $\tilde{\mathbf{T}}_t$ is a $Q \times Q$ matrix in which the (t, t) th element is 1 and the remaining elements are all equal to zero.

The superimposed signal $\mathbf{z} \equiv \mathbf{g} + \mathbf{g}_{\mathrm{dd}}$ removes the data components in the frequency bins of the pilot signals but results in data distortion, i.e.,

$$\begin{aligned} \mathbf{z} &\equiv \mathbf{g} + \mathbf{g}_{\mathrm{dd}} = \mathbf{E}\mathbf{s} - \mathbf{J}_t \mathbf{E}\mathbf{s} \\ &= (\mathbf{I}_N - \mathbf{J}_t)\, \mathbf{E}\mathbf{s} \equiv \boldsymbol{\Theta}_t \mathbf{E}\mathbf{s}, \end{aligned} \tag{7}$$

where

$$\begin{aligned} \boldsymbol{\Theta}_t &\equiv \mathbf{I}_N - \mathbf{J}_t = \mathbf{I}_Q \otimes \mathbf{I}_K - \tilde{\mathbf{J}}_t \otimes \mathbf{I}_K \\ &= \left(\mathbf{I}_Q - \tilde{\mathbf{J}}_t\right) \otimes \mathbf{I}_K \equiv \tilde{\boldsymbol{\Theta}}_t \otimes \mathbf{I}_K, \end{aligned} \tag{8}$$

in which the $Q \times Q$ matrix $\tilde{\boldsymbol{\Theta}}_t$ is given by (9), as shown at the bottom of this page. It was shown in [16] that the computational complexity of the DDST operation $\boldsymbol{\Theta}_t$ can be significantly reduced by using the Kronecker product operation, i.e., $\boldsymbol{\Theta}_t = \tilde{\boldsymbol{\Theta}}_t \otimes \mathbf{I}_K$. In this study, this property is exploited to reduce the computational complexity of the precoding matrix design. More specifically, the sequences $\mathbf{z}$ and $\mathbf{g}$ of length N are partitioned into K independent subgroups of length Q, i.e.,

$$\tilde{\mathbf{z}}_k = \tilde{\boldsymbol{\Theta}}_t \tilde{\mathbf{g}}_k,\quad k = 0, 1, \ldots, K-1, \tag{10}$$

where

$$\tilde{\mathbf{z}}_k \equiv [z(k)\ z(k+K)\ \cdots\ z(k+(Q-1)K)]^T \tag{11}$$

and

$$\tilde{\mathbf{g}}_k \equiv [s(k)\ g(k+K)\ \cdots\ g(k+(Q-1)K)]^T. \tag{12}$$

To reduce the computational complexity incurred in obtaining the superimposed signal $\mathbf{z}$ from the original data sequence $\mathbf{s}$, the first criterion for the precoding matrix design is specified as follows:

$$(\textbf{Criterion 1})\qquad \mathbf{E} = \tilde{\mathbf{E}} \otimes \mathbf{I}_K, \tag{13}$$

$$\tilde{\mathbf{J}}_t \equiv \frac{1}{Q} \begin{bmatrix} 1 & e^{\frac{j2\pi t(Q-1)}{Q}} & e^{\frac{j2\pi t(Q-2)}{Q}} & \cdots & e^{\frac{j2\pi t}{Q}} \\ e^{\frac{j2\pi t}{Q}} & 1 & e^{\frac{j2\pi t(Q-1)}{Q}} & \cdots & e^{\frac{j2\pi 2t}{Q}} \\ e^{\frac{j2\pi 2t}{Q}} & e^{\frac{j2\pi t}{Q}} & 1 & \cdots & e^{\frac{j2\pi 3t}{Q}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e^{\frac{j2\pi t(Q-1)}{Q}} & e^{\frac{j2\pi t(Q-2)}{Q}} & e^{\frac{j2\pi t(Q-3)}{Q}} & \cdots & 1 \end{bmatrix}. \tag{6}$$

$$\tilde{\boldsymbol{\Theta}}_t \equiv \left(\mathbf{I}_Q - \tilde{\mathbf{J}}_t\right) = \frac{1}{Q} \begin{bmatrix} Q-1 & -e^{\frac{j2\pi t(Q-1)}{Q}} & \cdots & -e^{\frac{j2\pi t}{Q}} \\ -e^{\frac{j2\pi t}{Q}} & Q-1 & \cdots & -e^{\frac{j2\pi 2t}{Q}} \\ \vdots & \vdots & \ddots & \vdots \\ -e^{\frac{j2\pi t(Q-1)}{Q}} & -e^{\frac{j2\pi t(Q-2)}{Q}} & \cdots & Q-1 \end{bmatrix}. \tag{9}$$

where $\tilde{\mathbf{E}}$ is a $Q \times Q$ matrix. As a result, $\mathbf{z}$ can be rewritten as

$$\mathbf{z} = \left(\tilde{\boldsymbol{\Theta}}_t \otimes \mathbf{I}_K\right) \cdot \left(\tilde{\mathbf{E}} \otimes \mathbf{I}_K\right) \cdot \mathbf{s} = \left(\tilde{\boldsymbol{\Theta}}_t \tilde{\mathbf{E}} \otimes \mathbf{I}_K\right) \cdot \mathbf{s}, \quad (14)$$

and $\tilde{\mathbf{z}}_k$ is obtained as

$$\tilde{\mathbf{z}}_k = \tilde{\boldsymbol{\Theta}}_t \tilde{\mathbf{E}} \tilde{\mathbf{s}}_k, \quad k = 0, 1, \ldots, K-1, \quad (15)$$

where

$$\tilde{\mathbf{s}}_k \equiv [s(k)\; s(k+K)\; \cdots\; s(k+(Q-1)K)]^T. \quad (16)$$

The pilot sequence $\mathbf{p}$ is then superimposed onto the data sequence in the time domain, i.e.,

$$\mathbf{x} = \mathbf{z} + \mathbf{p}, \quad (17)$$

where the equivalent frequency-domain pilot signals are located in equally-spaced frequency bins. Before transmission, the CP is added to the front of the data sequence to prevent inter block interference (IBI). In the present study, the channel is assumed to be quasi-stationary, i.e., the channel remains static over an entire data block. The discrete-time channel impulse response is thus given by

$$\mathbf{h} = [h(0)\; h(1)\; \ldots h(L-1)]^T. \quad (18)$$

Each channel tap $h(l)$, $0 \leq l \leq L-1$, is assumed to be an independent complex Gaussian distributed random variable with zero mean and a variance $\sigma^2_{h(l)}$.

III. RECEIVER ARCHITECTURE AND THE PROPOSED LCD SCHEME

A. RECEIVER ARCHITECTURE

Assume that perfect synchronization is achieved at the receiver end. After removing the CP, the received signal block can therefore be expressed as

$$\mathbf{y} = \mathbf{H}_C \mathbf{x} + \mathbf{n} = \mathbf{H}_C (\boldsymbol{\Theta}_t \mathbf{E} \mathbf{s} + \mathbf{p}) + \mathbf{n}, \quad (19)$$

where $\mathbf{n}$ is a vector denoting additive white Gaussian noise (AWGN) with zero mean and a variance σ^2_n. In addition, $\mathbf{H}_C$ is defined as an $N \times N$ circulant matrix with the first column given by $[\mathbf{h}^T\; \mathbf{0}^T_{(N-L)}]^T$, where $\mathbf{h}$ is given in (18) and $\mathbf{0}_{(N-L)}$ is a zero column vector of length $N-L$. Notably, the discrete Fourier transform (DFT) of $\boldsymbol{\Theta}_t \mathbf{E} \mathbf{s}$ is equal to zero in the pilot frequency bins. Therefore, the unknown data vector $\mathbf{s}$ has no impact on the channel estimation process.

In the proposed precoded CP-SC system, the frequency-selective fading channel effect is compensated at the receiver end using frequency-domain equalization (FDE). In the FDE approach, an FFT operation is firstly applied to transform the received signal $\mathbf{y}$ to the frequency domain. Since the pilots are known at the receiver node, channel estimation can be easily accomplished. The matrix $(\mathbf{I}_N - \mathbf{T}_t)$ is then applied to remove the pilot frequency bins and one-tap channel equalization is performed at the remaining bins, where the channel coefficients $\hat{H}_k$, $k = 0, 1, \cdots, N-1$, are obtained from the channel estimation process. Following an IFFT operation, the equalized signal is obtained as

$$\mathbf{r} = \mathbf{F}^H_N \mathbf{H}_{\text{eq}} (\mathbf{I}_N - \mathbf{T}_t) \mathbf{F}_N \mathbf{y}, \quad (20)$$

where $\mathbf{H}_{\text{eq}}$ is a diagonal matrix with the (k, k)th element given by $\frac{\hat{H}^*_k}{\left|\hat{H}_k\right|^2 + \sigma^2_n}$ [19]. Finally, the binary data sequence is obtained by means of an inverse precoding operation followed by data detection.

Data detection is generally performed using the maximum-likelihood (ML) criterion. However, such an approach requires an exhaustive search among all the M^N possible signal vectors, i.e.,

$$\hat{\mathbf{s}}_{\text{ML}} = \arg \min_{\mathbf{s} \in \Omega^{N \times 1}} \|\mathbf{r} - \boldsymbol{\Theta}_t \mathbf{E} \mathbf{s}\|, \quad (21)$$

where $\Omega^{N \times 1}$ denotes the set of N-dimensional modulated symbol vectors. Thus, while ML detection yields an excellent result, it incurs a high computational cost. Various techniques have been developed to solve (21) in a computationally efficient manner by dividing the large system into a number of small systems [20]. In particular, the implementation cost can be reduced by applying a subgroup detector (SD) to the partitioned sequences given in (15), i.e.,

$$\hat{\tilde{\mathbf{s}}}_{k,\text{SD}} = \arg \min_{\tilde{\mathbf{s}}_k \in \Omega^{Q \times 1}} \left\| \tilde{\mathbf{r}}_k - \tilde{\boldsymbol{\Theta}}_t \tilde{\mathbf{E}} \tilde{\mathbf{s}}_k \right\|, \quad (22)$$

where $\tilde{\mathbf{r}}_k$ is the kth subgroup of $\mathbf{r}$. As for the case shown in (15), the kth subgroup $\tilde{\mathbf{r}}_k$ only depends on the Q elements of $\tilde{\mathbf{s}}_k$ rather than all the N elements of $\mathbf{s}$. Furthermore, since the data sequence of size N has been partitioned into K subgroups of size Q, the SD is required to perform only K iterations of searches among the M^Q possible signal vectors in each subgroup. In other words, compared to ML detection, which requires a search to be made among all M^N possible signal vectors, the SD scheme has a much lower complexity; particularly when N is large.

B. THE PROPOSED LCD SCHEME

To further reduce the computational complexity of the proposed scheme, this section presents a low-complexity detection (LCD) scheme, in which the search space of the candidate signals is dramatically decreased. As described above, in the DDST scheme, a data-dependent sequence is superimposed on the data sequence in the time domain to remove the frequency components of the sub-carriers reserved for the pilot signals, i.e., $n = rQ + t$, $r = 0, 1, \ldots, K-1$, For the kth subgroup, (15) can be rewritten as

$$\begin{aligned}\tilde{\mathbf{z}}_k &= \tilde{\boldsymbol{\Theta}}_t \tilde{\mathbf{E}} \tilde{\mathbf{s}}_k = \left(\mathbf{I}_Q - \tilde{\mathbf{J}}_t\right) \tilde{\mathbf{E}} \tilde{\mathbf{s}}_k \\ &= \mathbf{F}^H_Q \left(\mathbf{I}_Q - \tilde{\mathbf{T}}_t\right) \mathbf{F}_Q \tilde{\mathbf{E}} \tilde{\mathbf{s}}_k, \end{aligned} \quad (23)$$

where the third equality holds since $\tilde{\mathbf{J}}_t = \mathbf{F}^H_Q \tilde{\mathbf{T}}_t \mathbf{F}_Q$ and $\mathbf{I}_Q = \mathbf{F}^H_Q \mathbf{I}_Q \mathbf{F}_Q$. From (23), $\tilde{\mathbf{z}}_k$ is obtained from $\tilde{\mathbf{E}} \tilde{\mathbf{s}}_k$ by removing the t-th element of the frequency bins of size Q. Inspired by this characteristic, the LCD scheme presented in this section consists of two steps, namely (1) the removed signal components are first estimated, and (2) the estimated results are used to recover the original signal and symbol-by-symbol detection is performed. The motivation and detailed operations of the proposed LCD are further elaborated in the following.

For convenience, let α_k denote the removed signal component of the t-th frequency bin, i.e.,

$$\alpha_k = \mathbf{f}_{Q,t}\tilde{\mathbf{E}}\tilde{\mathbf{s}}_k, \tag{24}$$

where $\mathbf{f}_{Q,t}$ is the t-th row of $\mathbf{F}_Q$. The removed signal components can be obtained via the frequency-domain compensation operation $\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right)$. In the absence of noise disturbance, the modulated signal $\tilde{\mathbf{s}}_k$ in the time-domain can then be obtained by performing $\tilde{\mathbf{E}}^{-1}\mathbf{F}^H\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right)$. The resultant signal is thus basically one of the original constellation points of the adopted modulation scheme, i.e.,(25), as shown at the bottom of this page, where $\lfloor\cdot\rceil$ returns the closest constellation point.

In the first step of the proposed LCD scheme, the optimal value of α_k is obtained by means of an exhaustive search, i.e.,(26), as shown at the bottom of this page, where Ξ denotes the set of all the possible values of α_k. The main concept of (26) is to minimize the distance of (25) caused by the noise disturbance. Having obtained the estimated result $\hat{\alpha}_k$, the removed signal components are recovered and symbol-by-symbol detection of $\tilde{\mathbf{s}}_k$ is performed following an inverse precoding operation $\tilde{\mathbf{E}}^{-1}$, i.e.,

$$\hat{\tilde{\mathbf{s}}}_k = \left\lfloor \tilde{\mathbf{E}}^{-1}\tilde{\mathbf{r}}_k + \alpha_k\tilde{\mathbf{E}}^{-1}\left(\mathbf{f}_{Q,t}\right)^H \right\rceil. \tag{27}$$

In summary, the proposed LCD scheme consists of the following steps:

- **Step 1:** An exhaustive search is performed using (26) to find the optimal value of α_k.
- **Step 2:** A symbol-by-symbol detection is performed using (27).

It is noted that the search space of α_k in (26) is basically determined by $\tilde{\mathbf{s}}_k$, pilot spacing Q, and precoding matrix $\tilde{\mathbf{E}}$, as demonstrated in (24). As the size of $\tilde{\mathbf{s}}_k$ is $Q \times 1$ and th modulation order is M, the search space of α_k is at most M^Q. However, different $\tilde{\mathbf{s}}_k$ may have the same value of α_k because of the use of precoding matrix $\tilde{\mathbf{E}}$. Thus, the actual search space of α_k in the proposed LCD scheme can be much lower than that in the SD scheme, i.e., M^Q. In practice, the actual number of candidate signals depends on the precoding matrix $\tilde{\mathbf{E}}$, the adopted modulation scheme, and the pilot spacing Q. In addition, all the possible values of α_k and the vectors $\alpha_k\tilde{\mathbf{E}}^{-1}\left(\mathbf{f}_{Q,t}\right)^H$ can be obtained off-line. Therefore, the proposed LCD scheme is suitable for real-time operation.

IV. DESIGN CRITERIA OF PRECODING MATRIX

This section addresses three main aspects of the precoding matrix design.

A. FREQUENCY DIVERSITY

SC transmission has the advantage of frequency diversity [3]. That is, the data symbol energy is equally distributed over the full frequency domain. In implementing the proposed precoding system, the same frequency diversity should be maintained. To facilitate analysis, let the notation $\boldsymbol{\Psi} \equiv \mathbf{F}_N\mathbf{E}$ be introduced, where $\boldsymbol{\Psi}$ is the DFT of the precoding matrix $\mathbf{E}$. To obtain the maximum frequency diversity, all of the elements of $\boldsymbol{\Psi}$ should have the same magnitude $1/\sqrt{N}$ [19]. Let $\boldsymbol{\Psi}_m$ and $\mathbf{E}_m$ denote the mth column of $\boldsymbol{\Psi}$ and $\mathbf{E}$, respectively, i.e., $\boldsymbol{\Psi}_m \equiv \mathbf{F}_N\mathbf{E}_m$. We further define the element-wsie multiplication of $\boldsymbol{\Psi}_m$ and $\boldsymbol{\Psi}_m^*$ as $\mathbf{R}_{\Psi\Psi} \equiv \boldsymbol{\Psi}_m \odot \boldsymbol{\Psi}_m^*$, where the nth element of $\mathbf{R}_{\Psi\Psi}$ is given by

$$R_{\Psi\Psi}(n) = [\boldsymbol{\Psi}]_{n,m}[\boldsymbol{\Psi}]_{n,m}^* = \left|[\boldsymbol{\Psi}]_{n,m}\right|^2. \tag{28}$$

Since all of the elements of $\boldsymbol{\Psi}$ have the same magnitude, i.e., $\left|[\boldsymbol{\Psi}]_{n,m}\right| = 1/\sqrt{N}$, it is easily shown that

$$\boldsymbol{\Psi}_m \odot \boldsymbol{\Psi}_m^* = \frac{1}{N}\mathbf{1}_Q. \tag{29}$$

Performing IDFT on both sides of (29) yields

$$\sum_{n=0}^{N-1}\mathbf{E}_m(n)\,\mathbf{E}_m\left((n+\tau)_N\right) = \delta(\tau), \tag{30}$$

where $\delta(n)$ is the delta function. (30) implies that each of the column vectors $\mathbf{E}_m$ has an ideal periodic autocorrelation function (PACF). In other words, each column vector $\mathbf{E}_m$ is a perfect sequence [21]–[23]. Therefore, the second design criterion for the precoding matrix can be stated as

(Criterion 2)

All the column vectors of $\mathbf{E}$ have ideal PACFs. (31)

$$\left\| \begin{array}{l} \tilde{\mathbf{E}}^{-1}\mathbf{F}_Q^H\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right) \\ -\left\lfloor \tilde{\mathbf{E}}^{-1}\mathbf{F}_Q^H\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right)\right\rceil \end{array} \right\| = 0, \quad \sigma_{\mathrm{n}}^2 = 0, \tag{25}$$

$$\begin{aligned} \hat{\alpha}_k &= \arg\min_{\alpha_k\in\Xi} \left\| \begin{array}{l} \tilde{\mathbf{E}}^{-1}\mathbf{F}_Q^H\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right) \\ -\left\lfloor \tilde{\mathbf{E}}^{-1}\mathbf{F}_Q^H\left(\mathbf{F}_Q\tilde{\mathbf{r}}_k + \left[\mathbf{0}_{1\times t}, \alpha_k, \mathbf{0}_{1\times(Q-t-1)}\right]^T\right)\right\rceil \end{array} \right\| \\ &= \arg\min_{\alpha_k\in\Xi} \left\| \tilde{\mathbf{E}}^{-1}\tilde{\mathbf{r}}_k + \alpha_k\tilde{\mathbf{E}}^{-1}\left(\mathbf{f}_{Q,t}\right)^H - \left\lfloor \tilde{\mathbf{E}}^{-1}\tilde{\mathbf{r}}_k + \alpha_k\tilde{\mathbf{E}}^{-1}\left(\mathbf{f}_{Q,t}\right)^H \right\rceil \right\|, \end{aligned} \tag{26}$$

B. PAPR

In the proposed precoded CP-SC system, the PAPR of the transmitted signal is defined as

$$\mathrm{PAPR}\left(\mathbf{z}\right)=\frac{\max\limits_{0\le n\le L_oN-1}\mid z\left[n\right]\mid^2}{E\left[\mid z\left[n\right]\mid^2\right]},\tag{32}$$

where L_0 is the oversampling factor and is assumed to have a value of $L_0 = 4$ in the present study. Obviously, the adopted precoding matrix substantially influences the PAPR and should be properly designed. For the proposed CP-SC system to have a comparable PAPR to that of the conventional DDST-based scheme, the precoding matrix should be a diagonal matrix in which all of the elements have an equal magnitude. That is,

$$\begin{aligned}&(\textbf{Criterion 3})\\&\mathbf{E}\text{ should be a diagonal matrix with elements}\\&\text{of equal magnitude.}\end{aligned}\tag{33}$$

It is noted that if the precoding matrix meets Criterion 3, i.e., (33), it also fulfills Criterion 2, i.e., (31).

C. UNITARY MATRIX

From (26), the precoding matrix should be unitary, i.e., $\mathbf{E}^{-1} = \mathbf{E}^H$, such that the modulated symbols can be recovered at the receiver end by simply multiplying the Hermitian of the precoding matrix. Therefore, the fourth design criterion of the precoding matrix is given as

$$(\textbf{Criterion 4})\qquad \mathbf{E}\text{ is an unitary matrix.}\tag{34}$$

D. A BRIEF SUMMARY OF THE PROCODING MATRIX DESIGN CRITERIA

It can be easily shown that criteria 1, 2, 3 and 4 are all simultaneously satisfied if the precoding matrix $\mathbf{E}$ has the following form:

$$(\textbf{Criterion I/Criteria 1, 2, 3, 4})\quad \mathbf{E}=\tilde{\mathbf{E}}\otimes\mathbf{I}_K,\tag{35}$$

where $\tilde{\mathbf{E}}$ is a diagonal matrix of size $Q\times Q$ in which the qth diagonal element has the form

$$\tilde{\mathbf{E}}_{q,q}=\exp\left(j\theta_q\right),\quad \theta_q\in[0,2\pi].\tag{36}$$

V. ANALYSIS OF DIP FOR PRECODED CP-SC SYSTEM USING FDE AND DDST SCHEME

The DIP phenomenon occurs when two arbitrary input signals $\tilde{\mathbf{s}}_k$ and $\tilde{\mathbf{s}}_k'$ have the same output, i.e.,

$$\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\tilde{\mathbf{s}}_k=\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\tilde{\mathbf{s}}_k',\tag{37}$$

or equivalently,

$$\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\mathbf{d}\equiv\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\left(\tilde{\mathbf{s}}_k-\tilde{\mathbf{s}}_k'\right)\equiv\tilde{\mathbf{\Lambda}}\left(\tilde{\mathbf{s}}_k-\tilde{\mathbf{s}}_k'\right)=\mathbf{0}_Q,\tag{38}$$

where $\mathbf{d}\equiv\left(\tilde{\mathbf{s}}_k-\tilde{\mathbf{s}}_k'\right)$ and $\tilde{\mathbf{\Lambda}}\equiv\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}$. (38) indicates that the DIP occurs when the vector $\mathbf{d}$ belongs to the null space of $\tilde{\mathbf{\Lambda}}$, i.e.,

$$\mathbf{d}\in\mathrm{Null}\left(\tilde{\mathbf{\Lambda}}\right),\tag{39}$$

where $\mathrm{Null}\left(\tilde{\mathbf{\Lambda}}\right)$ is the null space of the matrix $\tilde{\mathbf{\Lambda}}$. Let's denote $\mathbf{v}_{\text{null}}$ as the null vector of $\tilde{\mathbf{\Lambda}}$, i.e., $\tilde{\mathbf{\Lambda}}\mathbf{v}_{\text{null}}=\mathbf{0}_Q$. It is shown in [18] that $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}\mathbf{v}_{\text{null}}=\mathbf{0}_Q$. Therefore, the null vectors of $\tilde{\mathbf{\Lambda}}$ are also the null vectors of $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}$. Because $\tilde{\mathbf{\Lambda}}\equiv\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}$, it follows that

$$\begin{aligned}\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}&=\tilde{\mathbf{E}}^H\tilde{\mathbf{\Theta}}_t^H\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\\&=\tilde{\mathbf{E}}^H\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}\\&=\tilde{\mathbf{E}}^H\mathbf{F}_Q^H\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}\\&=\left(\tilde{\mathbf{E}}^H\mathbf{F}_Q^H\right)\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\left(\tilde{\mathbf{E}}^H\mathbf{F}_Q^H\right)^H,\end{aligned}\tag{40}$$

where the second equality holds since $\mathbf{\Theta}_t\mathbf{\Theta}_t = \mathbf{\Theta}_t$ and $\mathbf{\Theta}_t^H = \mathbf{\Theta}_t$ [16]. From (40), the following results can be obtained:

- Since the precoding matrix $\mathbf{E}$ (as well as $\tilde{\mathbf{E}}$) is designed to be an unitary matrix (Criterion 4), $\tilde{\mathbf{E}}^H\mathbf{F}_Q^H$ is also an unitary matrix, In other words, the matrix $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}$ can be decomposed to the unitary-diagonal-unitary matrices form.
- It can be seen from (23) that $\tilde{\mathbf{T}}_t$ has only one zero-valued diagonal element. Therefore, the rank of $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}$ is $Q-1$ and $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}$ has only one null vector.

Since the t-th diagonal element of $\tilde{\mathbf{T}}_t$ is the only zero element, it can be shown that the null vector of $\tilde{\mathbf{\Lambda}}^H\tilde{\mathbf{\Lambda}}$ is the t-th column of $\tilde{\mathbf{E}}^H\mathbf{F}_Q^H$, which in turn is the null vector of $\tilde{\mathbf{\Lambda}}$, i.e.,

$$\mathbf{v}_{\text{null}}=\left(\mathbf{f}_{Q,t}\tilde{\mathbf{E}}\right)^H.\tag{41}$$

As a result, the precoding matrix should be designed such that $\left(\tilde{\mathbf{s}}_k-\tilde{\mathbf{s}}_k'\right)\notin\mathrm{Null}\left(\tilde{\mathbf{\Lambda}}\right)$, i.e.,

$$\begin{aligned}&(\textbf{Criterion II/Criterion 5})\\&\left(\tilde{\mathbf{s}}_k-\tilde{\mathbf{s}}_k'\right)\notin\mathrm{span}\left(\left(\mathbf{f}_{Q,t}\tilde{\mathbf{E}}\right)^H\right).\end{aligned}\tag{42}$$

Satisfying (42) guarantees that the precoded data sequences are unique. Hence, the DIP is eliminated.

VI. PHASE DESIGN OF DIAGONAL ELEMENTS TO REDUCE SEARCH SPACE AND AVOID DIP

Any matrices which fulfill both Criterion I and Criterion II are appropriate candidates for the proposed precoding scheme. However, as demonstrated in the following, if the phases of the diagonal elements are properly selected, then it is possible not only to avoid the DIP, but also to significantly reduce the search space.

A. PHASE DESIGN OF DIAGONAL ELEMENTS FOR REDUCING SEARCH SPACE AND AVOIDING DIP

The search space of the proposed LCD scheme is closely related to the design of the precoding matrix. This section investigates the relationship between the size of the search space and the phases of the diagonal elements in $\tilde{\mathbf{E}}$. Let α_k in (24) be rewritten as

$$\begin{aligned}\alpha_k &= \mathbf{f}_{Q,t}\tilde{\mathbf{E}}\tilde{\mathbf{s}}_k &(43)\\ &= \sum_{q=0}^{Q-1}\exp\left(\frac{j2\pi qt}{Q}\right)\exp\left(j\theta_q^{\text{prec}}\right)s\left(k+qK\right). &(44)\end{aligned}$$

For the sake of convenience, the following discussions focus arbitrarily on the case of $t = 0$. (Note, however, that the results are easily extended to the case of $t \neq 0$.) When $t = 0$, (43) can be rewritten as

$$\alpha_k|_{t=0} = \sum_{q=0}^{Q-1}\exp\left(j\theta_q^{\text{prec}}\right)s\left(k+qK\right). \tag{45}$$

It can be seen from (45) that α_k is equal to the sum of the Q phase-rotated modulation symbols. Let the number of possible distinct values of α_k be denoted as $N(\alpha_k)$. Clearly, $N(\alpha_k)$ is determined by both θ_q^{prec} and the adopted modulation scheme. The worst value of $N(\alpha_k)$, which corresponds to the largest search space, is equal to M^Q. However, if θ_q^{prec} is properly designed, $N(\alpha_k)$ can be dramatically decreased, as explained in the following.

Consider the simplest case in which the data are BPSK modulated and each subgroup contains $Q = 2$ data symbols. If $\theta_0^{\text{prec}} = \theta_1^{\text{prec}} = 0$ are adopted, it can be seen from (45) that α_k only has $N(\alpha_k) = 3$ possible values (rather than $2^2 = 4$), i.e., $\alpha_k \in \{-2, 0, 2\}$. Furthermore, if each subgroup has $Q = 4$ symbols and $\theta_q^{\text{prec}} = 0, q = 0, 1, 2, 3$, α_k has only $N(\alpha_k) = 5$ possible values (rather than $2^5 = 32$) and $\alpha_k \in \{-4, -2, 0, 2, 4\}$. If $\theta_q^{\text{prec}} = \theta$, $q = 0, 1, 2, 3$, are adopted, where θ is any arbitrary phase, α_k also has only $N(\alpha_k) = 5$ possible values and $\alpha_k \in e^{j\theta} \cdot \{-4, -2, 0, 2, 4\}$. Notably, the same result is obtained for any $\theta_q^{\text{prec}} \in \{\theta, \theta + \pi\}$.

Let the phase rotation θ_q^{prec}, $q = 0, 1, \ldots, Q-1$, be partitioned into U sets, where in the same set $\theta_q^{\text{prec}} \in \{\theta_u, \theta_u + \pi\}$, $u = 0, 1, \ldots, U-1$, $\theta_u \neq \theta_{u'}$ for $u \neq u'$. In addition, let $N(\theta_u)$ denote the number of θ_q^{prec}, $q = 0, 1, \ldots, Q-1$, within the same uth set. It therefore follows that

$$\sum_{u=0}^{U-1} N\left(\theta_u\right) = Q. \tag{46}$$

It can be shown that the number of distinct α_k is given by

$$N\left(\alpha_k\right)|_{\text{BPSK}} \leq \prod_{u=0}^{U-1}\left(N\left(\theta_u\right)+1\right). \tag{47}$$

(Note that the proof is tedious and is omitted here for brevity.)

The discussions above can be easily extended to the QPSK and M-QAM modulation schemes, where the various phase rotations belong to the same set if $\theta_q^{\text{prec}} \in \left\{\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}\right\}$. For QPSK modulation, the size of search space $N(\alpha_k)$ is upper bounded by:

$$N\left(\alpha_k\right)|_{\text{QPSK}} \leq \prod_{u=0}^{U-1}\left(N\left(\theta_u\right)+1\right)^2. \tag{48}$$

Meanwhile, for M-QAM modulation, the size of search space $N(\alpha_k)$ is upper bounded by:

$$N\left(\alpha_k\right)|_{M\text{-QAM}} \leq \prod_{u=0}^{U-1}\left(\left(\sqrt{M}-1\right)N\left(\theta_u\right)+1\right)^2. \tag{49}$$

When all the Q phase rotations θ_q^{prec} belong to the same set, the size of the search space $N(\alpha_k)$ is reduced to the minimum value. However, in this case, the precoded signals result in DIP. Therefore, the Q phase rotations θ_q^{prec} should belong to at least two different sets.

B. PROPOSED PRECODING MATRIX DESIGN

From all the discussions above, the proposed precoding matrix should have the following form:

$$\mathbf{E}^{\text{Pro}} = \tilde{\mathbf{E}}^{\text{Pro}} \otimes \mathbf{I}_K, \tag{50}$$

where $\tilde{\mathbf{E}}^{\text{Pro}}$ is a diagonal matrix of size $Q \times Q$ with the qth diagonal element given by:

$$\tilde{\mathbf{E}}_{q,q}^{\text{Pro}} = \exp\left(j\theta_q^{\text{Pro}}\right), \tag{51}$$

in which $\theta_q^{\text{Pro}} \in [0, 2\pi]$ and θ_q^{Pro}, $q = 0, 1, \ldots, Q-1$, should belong to at least two different sets.

Example of Precoding Matrix: In practice, there exist numerous precoding matrices which fulfill the proposed design criteria. For example, one possible design of the diagonal matrix $\tilde{\mathbf{E}}^{\text{Pro}}$ for M-QAM modulation is given by:

$$\begin{aligned}\tilde{\mathbf{E}}_{q,q}^{\text{Pro}} &= \exp\left(j\theta_q^{\text{Pro}}\right)\\ &= \begin{cases}1, & q = 0, 1, \cdots, Q-2,\\ \exp\left(\dfrac{j\pi}{4}\right), & q = Q-1.\end{cases}\end{aligned} \tag{52}$$

It is shown in the Appendix that the precoding matrix given in (52) successfully avoids the DIP.

VII. SIMULATION RESULTS

The performance of the proposed precoding scheme was evaluated by means of numerical simulations given a CP-SC system with a block length of $N = 256$ and a CP of length 32. In performing the simulations, the pilot sequence $\mathbf{p}$ was assigned a frequency-domain period of $K = 8$ and the precoding matrix was obtained according to (52) with $Q = 8$. The simulations assumed a multipath Rayleigh fading channel environment with an exponentially-decayed power delay coefficient of $\mu = -0.2$. The channel length was assumed to be $L = 32$ and MMSE channel equalization was performed at the receiver. Finally, perfect synchronization was assumed.

Figure 2 compares the PAPR performance of various systems given the use of 16-QAM modulation. The PAPR of traditional OFDM systems is also depicted as a benchmark. Note that to obtain an improved approximation of the true PAPR in the discrete-time case, an oversampling factor of 4 is adopted. It can be seen that the traditional OFDM system

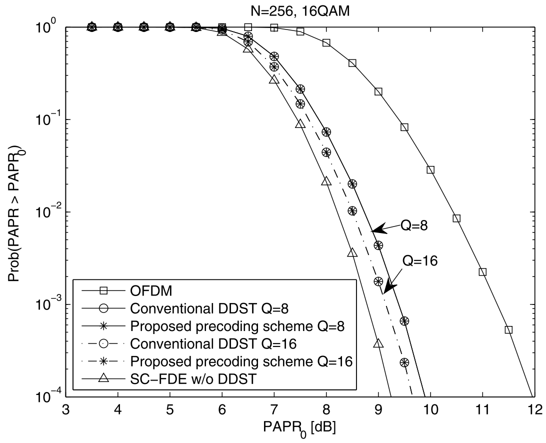


FIGURE 2. PAPR performance of various schemes for 16-QAM.

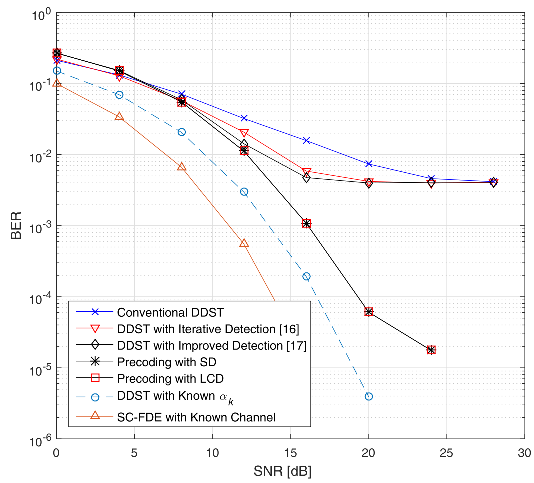


FIGURE 3. BER performance of various schemes for QPSK.

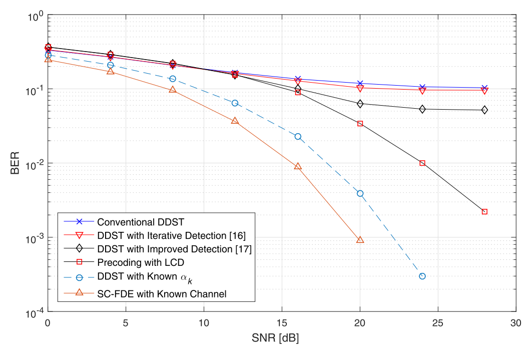


FIGURE 4. BER performance of various schemes for 16-QAM.

yields the poorest PAPR performance, while the SC-FDE system yields the best. It is also seen that the PAPR performance of the proposed precoding scheme is equivalent to that of the conventional DDST scheme.

Figures 3 and 4 show the BER performance of the various schemes given the use of QPSK and 16-QAM modulation, respectively. In Fig. 3, only the line SC-FDE with Known Channel is obtained with the assumption of the ideal channel, whereas all the rest are obtained using estimated channels. This difference is the main reason for the gap in the performance evaluation. The use of precoder only changes the transmitted data sequence without affecting the channel condition. The SC-FDE scheme with a known channel represents the ideal scenario and thus delivers the best performance. In addition, the DDST scheme with a known α_k is utilized as a benchmark. As the removed signal is known at the receiver (α_k is known), the BER performance only depends on the channel estimation error. For conventional DDST schemes, the receiver may fail to correctly identify the signal due to the DIP. Therefore, the conventional DDST scheme yields the poorest BER performance of the considered schemes and has an error floor for SNR $>$ 20dB. In [16] and [17], the data distortion is mitigated at the receiver side. However, since DIP is not eliminated, both schemes have error floors and only slightly improve conventional DDST scheme. Observing the results presented in Fig. 3 for the proposed precoding scheme implemented using either the traditional SD method or the proposed LCD method, it is seen that both schemes yield the same BER performance and successfully eliminate the error floor problem. However, it is noted that the search space of the proposed LCD scheme is much smaller than that of the SD scheme.

The simulation results presented in Fig. 4 for 16-QAM modulation are similar to those presented in Fig. 3 for QPSK. It is noted that the results for the SD scheme are omitted since the search space is too large (16^8).

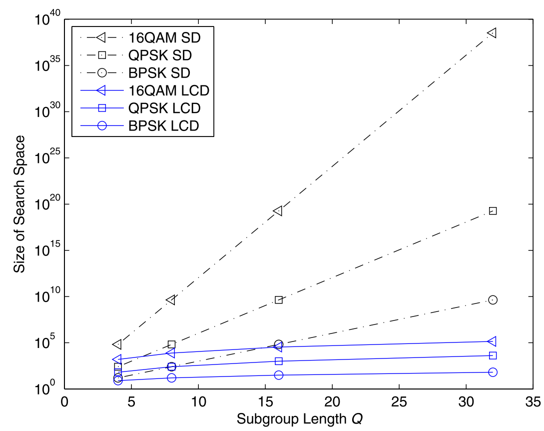


FIGURE 5. Complexity comparison of searching number in SD and LCD for various modulation schemes.

Figure 5 compares the complexity of the proposed LCD scheme with that of the conventional SD scheme for various modulation methods and subgroup lengths. For both schemes, the size of the search space increases with the modulation order. In addition, the search space increases with an increasing subgroup length Q. However, for the proposed LCD scheme, Q has only a marginal effect on the size of the search space, whereas for the SD scheme, the search space expands exponentially with increasing Q.

$$\begin{aligned} d_{min} &= \min\left(\mathbf{d}^H\left(\tilde{\mathbf{E}}^{\text{Pro}}\right)^H\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right) \\ &= \min\left(\mathbf{d}^H\left(\tilde{\mathbf{E}}^{\text{Pro}}\right)^H\mathbf{F}_Q^H\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right) \\ &= \min\left(\mathbf{d}^H\left(\tilde{\mathbf{E}}^{\text{Pro}}\right)^H\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)^H\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right) \\ &= \min\left(\left(\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right)^H\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right) \\ &= \min\left\|\left(\mathbf{I}_Q-\tilde{\mathbf{T}}_t\right)\mathbf{F}_Q\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right\|. \end{aligned} \tag{A.3}$$

VIII. CONCLUSIONS

This paper has proposed a precoding scheme for eliminating DIP in SC systems using the DDST scheme. Five design criteria have been derived for the precoding matrix. It has been shown that the precoding matrix should have the form of the Kronecker product of an identity matrix and a diagonal matrix with equal-magnitude diagonal elements. This study has also presented a phase design criterion for the diagonal elements of the precoding matrix which both avoids DIP and reduces the search space of the detection process. The simulation results have shown that the PAPR performance of the proposed precoding scheme is only marginally poorer than that of the traditional SC system, and is the same as that of the conventional DDST scheme. Moreover, the proposed precoding scheme significantly improves the BER performance of the traditional DDST scheme and eliminates the error floor.

This paper has additionally presented an LCD scheme for reducing the size of the search space in the detection process. It has been shown that the subgroup length Q has only a marginal effect on the size of the search space when the proposed LCD scheme is adopted. However, the search space expands exponentially with Q when the SD scheme is employed.

This Appendix shows that the diagonal precoding matrix given in (52) successfully eliminates DIP. Let the minimum distance among all of the pairs of the modulated symbol subgroups be given as

$$d_{min} \equiv \min\left\|\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\tilde{\mathbf{s}}_k - \tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\tilde{\mathbf{s}}'_k\right\|, \quad \forall\tilde{\mathbf{s}}_k \neq \tilde{\mathbf{s}}'_k. \tag{A.1}$$

If $d_{min} \neq 0$, DIP can be avoided. The minimum distance d_{min} can be rewritten as

$$\begin{aligned} d_{min} &= \min\left\|\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\tilde{\mathbf{s}}_k - \tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\tilde{\mathbf{s}}'_k\right\| \\ &= \min\left\|\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right\| \\ &= \min\left(\left(\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right)^H\left(\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right)\right) \\ &= \min\left(\mathbf{d}^H\left(\tilde{\mathbf{E}}^{\text{Pro}}\right)^H\tilde{\mathbf{\Theta}}_t\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}\right), \end{aligned} \tag{A.2}$$

where the third equality holds since $\tilde{\mathbf{\Theta}}_t^H\tilde{\mathbf{\Theta}}_t = \tilde{\mathbf{\Theta}}_t$. (A.2) can be further written as (A.3), as shown at the top of this page.

A close inspection of (A.3) reveals that the occurrence of DIP is closely related to the energy distribution of the occupied frequency bins. For the case of $t = 0$, d_{min} in (A.3) is equal to zero when the precoded difference signal $\tilde{\mathbf{E}}^{\text{Pro}}\mathbf{d}$ allocates all its energy to the discarded frequency bin, i.e., $t = 0$, in the frequency domain. In this case, all of the time-domain signal samples have the same value and hence DIP occurs. If M-QAM modulation is adopted, the proposed precoding matrix shown in (52) rotates the last signal sample through a phase of $\pi/4$, and therefore eliminates the chance that all of the samples of the precoded signal have the same value. Consequently, DIP is successfully avoided. Similar analyses can be extended to arbitrary t. Therefore, the proposed precoding matrix successfully eliminates the DIP for M-QAM modulation.

REFERENCES

[1] J. Zeng *et al.*, "Investigation on evolving single-carrier NOMA into multi-carrier NOMA in 5G," *IEEE Access*, vol. 6, pp. 48268–48288, 2018.

[2] M. F. Bayramoglu and M. Juntti, "Equalization of frequency selective channels using multirate signal processing," *IEEE Trans. Signal Process.*, vol. 66, no. 19, pp. 5111–5126, Oct. 2018.

[3] S. Kumar, S. Majhi, and C. Yuen, "Multi-user CFOs estimation for SC-FDMA system over frequency selective fading channels," *IEEE Access*, vol. 6, pp. 43146–43156, 2018.

[4] W. Huang, Y. Huang, R. Zhao, S. He, and L. Yang, "Wideband millimeter wave communication: Single carrier based hybrid precoding with sparse optimization," *IEEE Trans. Veh. Technol.*, vol. 67, no. 10, pp. 9696–9710, Oct. 2018.

[5] J.-J. Jia, K.-C. Lai, and J.-Y. Pan, "Hybrid-domain parallel decision feedback equalization for single-carrier block transmission," *IEEE Trans. Veh. Technol.*, vol. 67, no. 2, pp. 1454–1469, Feb. 2018.

[6] X. Tu, A. Song, and X. Xu, "Prefix-free frequency domain equalization for underwater acoustic single carrier transmission," *IEEE Access*, vol. 6, pp. 2578–2588, 2018.

[7] Y. Zeng and T. S. Ng, "Pilot cyclic prefixed single carrier communication: Channel estimation and equalization," *IEEE Signal Process. Lett.*, vol. 12, no. 1, pp. 56–59, Jan. 2005.

[8] S. Adireddy, L. Tong, and H. Viswanathan, "Optimal placement of training for frequency-selective block-fading channels," *IEEE Trans. Inf. Theory*, vol. 48, no. 8, pp. 2338–2352, Aug. 2002.

[9] T. Cui and C. Tellambura, "Efficient blind receiver design for orthogonal space-time block codes," *IEEE Trans. Wireless Commun.*, vol. 6, no. 5, pp. 1890–1899, May 2007.

[10] W.-K. Ma, "Blind ML detection of orthogonal space-time block codes: Identifiability and code construction," *IEEE Trans. Signal Process.*, vol. 55, no. 7, pp. 3312–3324, Jul. 2017.

[11] S. M. S. Sadough and Z. Chamideh, "An improved variational inference approach to iterative OFDM receiver design for superimposed training-based AF relay networks," *IEEE Trans. Veh. Technol.*, vol. 67, no. 3, pp. 2243–2253, Mar. 2018.

[12] Z. He, J. Yang, X. Wang, Y. Liu, and Y. Rong, "Channel estimation of MIMO relay systems with multiple relay nodes," *IEEE Access*, vol. 5, pp. 27649–27658, 2017.

[13] W.-C. Huang, C.-P. Li, and H.-J. Li, "On the power allocation and system capacity of OFDM systems using superimposed training schemes," *IEEE Trans. Veh. Technol.*, vol. 58, no. 4, pp. 1731–1740, May 2009.

[14] W.-C. Huang, C.-P. Li, and H.-J. Li, "Optimal pilot sequence design for channel estimation in CDD-OFDM systems," *IEEE Trans. Wireless Commun.*, vol. 11, no. 11, pp. 4006–4016, Nov. 2012.

[15] W.-C. Huang, Y.-S. Yang, and C.-P. Li, "A new pilot architecture for sub-band uplink OFDMA systems," *IEEE Trans. Broadcast.*, vol. 59, no. 3, pp. 461–470, Sep. 2013.

[16] M. Ghogho, D. McLernon, E. Alameda-Hernandez, and A. Swami, "Channel estimation and symbol detection for block transmission using data-dependent superimposed training," *IEEE Signal Process. Lett.*, vol. 12, no. 3, pp. 226–229, Mar. 2005.

[17] T. Whitworth, M. Ghogho, and D. C. McLernon, "Data identifiability for data-dependent superimposed training," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Glasgow, U.K., Jun. 2007, pp. 2545–2550.

[18] R. A. Horn and C. R. Johnson, *Matrix Analysis*, 2nd ed. Cambridge, U.K.: Cambridge Univ. Press, 2012.

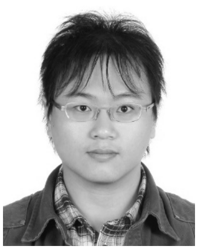
[19] Y.-P. Lin and S.-M. Phoong, "BER minimized OFDM systems with channel independent precoders," *IEEE Trans. Signal Process.*, vol. 51, no. 9, pp. 2369–2380, Sep. 2003.

[20] D. Jeong and K. Kim, "Signal detection for MIMO SC-FDMA systems exploiting block circulant channel structure," *IEEE Trans. Veh. Technol.*, vol. 65, no. 9, pp. 7774–7779, Oct. 2015.

[21] C.-P. Li, S.-H. Wang, and C.-L. Wang, "Novel low-complexity SLM schemes for PAPR reduction in OFDM systems," *IEEE Trans. Signal Process.*, vol. 58, no. 5, pp. 2916–2921, May 2010.

[22] C.-P. Li, K.-J. Chang, H.-H. Chang, and Y.-M. Chen, "Perfect sequences of odd prime length," *IEEE Signal Process. Lett.*, vol. 25, no. 7, pp. 966–969, Jul. 2018.

[23] S.-H. Wang, C.-P. Li, H.-H. Chang, and C.-D. Lee, "A systematic method for constructing sparse Gaussian integer sequences with ideal periodic autocorrelation functions," *IEEE Trans. Commun.*, vol. 64, no. 1, pp. 365–376, Jan. 2016.



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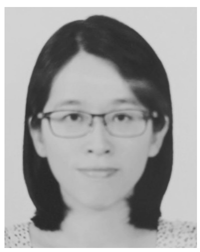

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