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Neural Network-Based Adaptive Tracking Control for a Class of Nonlinear Singularly Perturbed Systems

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ABSTRACT In this paper, the neural network-based adaptive tracking control method is addressed for a class of multi-input affine unknown nonlinear singularly perturbed systems. Based on the Lyapunov stability theorem and by utilizing the neural networks to approximate the unknown nonlinear function, an adaptive neural network controller is constructed for the singularly perturbed nonlinear systems. Meanwhile, the proposed design method can avoid ill-conditioned numerical problems that often occur in the feedback design for singularly perturbed systems. It is proved that the proposed controller can ensure that semi-global ultimately uniformly boundedness of all the signals in the closed-loop systems while the target signals converge to a small neighborhood of the desired signal. Finally, two simulation examples are given to illustrate the theoretical results.

INDEX TERMS Adaptive control, neural networks, singularly perturbed systems, uncertain nonlinear.

I. INTRODUCTION

The many industrial systems, such as power systems, motor control systems, electronic circuit systems, robotics systems, have "slow" and "fast" dynamics due to the presence of some small parameters such as capacitances, resistances, inductances, moments of inertia, and so on [1]–[5]. It gives rise to significant difficulties in analyzing and designing the system because the small parameters can lead to high dimensionality and ill-conditioned numerical issues. In the past decades, the stability analysis and control design of singularly perturbed system have attracted great attention and have been intensively studied by many researchers [6]–[8]. For example, by using singular perturbation theory and Lyapunov stability theory, stability analysis and stabilization problem were investigated in [9]–[13]. And observer-based control [14], [15], optimal control [16]–[20], sliding-mode control [21], and H^∞ control [22], [23] also were considered. However, it is worth mentioning that adaptive tracking control approach is not considered in the existing works on unknown singularly perturbed systems. It is known that trajectory tracking plays a key role in excellent maneuverability for many industrial systems and can ensure that a predefined trajectory can be tracked with acceptable accuracy. Thus, how to solve the problem of adaptive tracking control for unknown singularly perturbed systems is important and challenging in both theory and practice.

As we all known, a good method for dealing with systems with unknown function is that the fuzzy logic systems or neural networks are accustomed to approximating the unknown functions. By utilizing the excellent approximation capabilities of neural networks, adaptive neural network control schemes were widely used to nonlinear systems with unknown functions [24], [25]. In [26], a direct adaptive control was presented for a class of strict-feedback systems with unknown functions where the neural networks were employed to approximate the unknown functions. Utilizing backstepping technique and multi-layer neural networks, the tracking control problem of a class of strict-feedback unknown nonlinear systems were addressed in [27]. In order to guarantee semi-global uniform ultimate boundedness of all the signals in the two classes of multi-input multi-output (MIMO) systems [28], [29] with unknown functions, the adaptive controllers were proposed. Based on the universal approximation property of neural networks, the adaptive control of nonlinear multi-agent

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systems with unknown functions was studied in [30], [31]. Adaptive neural network control was investigated in [32]–[34] for nonlinear systems with unknown input delay by using Lyapunov-Krasovskii functionals. Unfortunately, it is not straightforward to apply the above results to analysis and design of singularly perturbed systems with unknown functions.

The methods mentioned above show that the neural network is an effective tool to deal with highly nonlinear, uncertain, and complex systems' control issue. Due to the theoretical challenges and practical needs, some researchers pay more and more attention to singularly perturbed nonlinear systems. For instance, in [35], an H^{∞} controller was presented for a class of singularly perturbed nonlinear systems to achieve H^{∞} control performance via neural network-based control and observer design. Han *et al.* [36] first indicated dynamic multi-time-scale neural networks including both fast and slow phenomena guarantee flexibility and accuracy of nonlinear system identification efficiently. On this basis, the identification and control based on multi-layer neural networks with multi-time-scales also were proposed [37], [38]. To improve the convergence speed, the optimal bounded ellipsoid algorithm-based identification for singularly perturbed nonlinear systems with unknown functions were proposed in [39], [40]. Moreover, an approximation-based and adaptive dynamic programming method the optimal control were studied in [15], [20] for singularly perturbed nonlinear systems with unknown functions. Nevertheless, there is relatively little research on neural network-based adaptive tracking control for singularly perturbed nonlinear systems.

In this paper, adaptive tracking control problem for a class of singularly perturbed nonlinear systems with unknown functions will be addressed. Compared with the published literature on singularly perturbed systems, the main works of this paper are as follows: 1) The neural network is used to approximate unknown nonlinear continuous-time functions, and ε-dependent adaptive laws are constructed to alleviate the ill-conditioned numerical problem which usually occur in the analysis and design of singularly perturbed systems. 2) An adaptive neural network trajectory tracking controller for a class of unknown multi-input affine singularly perturbed nonlinear systems with unknown functions is designed. The stability property is proved utilizing Lyapunov stability theory and tracking error is shown to converge to a small bound. Moreover, two simulation examples are given to verify the effectiveness of the control schemes.

Notation: Some mathematical notations to be used throughout this paper are given below. I denotes the identity matrix; $|\cdot|$ is the usual Euclidean norm of a vector, for instance, if y is a scalar, $|y|$ denote its absolute value, if A is a matrix, $\|A\|$ denotes the Frobenius matrix norm, i.e.

$$\|A\|^2 = \sum_{ij} \|a_{ij}\|^2 = tr\{A^T A\}$$

where $tr\{\cdot\}$ denotes the trace of a matrix. For a vector function of time $X(t)$. define

$$\|X\|_2 = (\int_0^{\infty} |X(t)|^2 \, dt)^{\frac{1}{2}}$$

and

$$\|X\|_{\infty} = \sup_{0 \le t} |X(t)|$$

we will say that $X \in L_2$ if $\|d\|_2$ is finite. Similarly, we will say that $d \in L_{\infty}$ if $\|d\|_{\infty}$ is finite.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. PROBLEM FORMULATIONS

Consider the following singularly perturbed systems

$$E(\varepsilon)\,\dot{x}(t) = f(x(t)) + g(x(t))\,u(t) \tag{1}$$

where

$$E(\varepsilon) = \begin{bmatrix} I_{(n-m)\times(n-m)} & 0 \\ 0 & \varepsilon I_m \end{bmatrix} \in R^{n\times n}$$

$$f(x) = [f_1(x), f_2(x), \ldots, f_n(x)] \in R^{n\times 1}$$

$$g(x) = \begin{bmatrix} g_{11}(x) & g_{12}(x) & \cdots & g_{1m}(x) \\ g_{21}(x) & g_{22}(x) & \cdots & g_{2m}(x) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(x) & g_{n2}(x) & \cdots & g_{nm}(x) \end{bmatrix} \in R^{n\times m}$$

$x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^{n\times 1}$ is the state vectors, $u(t) = [u_1(t), u_2(t), \cdots, u_n(t)] \in R^{m\times 1}$ is the control input, and $f_i(\cdot)$, $g_{ij}(\cdot)$, $(i = 1, 2, \cdots, n, j = 1, 2, \cdots, m)$ are unknown continuous functions, and $0 < \varepsilon \ll 1$ is the singular perturbation parameter.

Remark 1: The main characteristic of (1) is that the dynamical system possesses two time scale characteristics, that is, the system states whose velocity is associated with ε (a very small parameter) evolve faster than the other system states, it will lead the system into high dimensionality and ill-conditions often occur in the feedback design for singularly perturbed systems.

Definition 1: Consider the nonlinear system

$$\dot{x} = f(x, t)$$

where $x(t) \in R^n$ is the state vector. Its solution is said to be semi-globally uniformly ultimately bounded (SGUUB), for $x(0) \in \Omega_x$ where $\Omega_x \in R^n$ is a compact set, if there exist an constant $\epsilon > 0$ and a number $N(\epsilon, x(t_0))$ such that $\|x(t)\| < \epsilon$ for all $t > t_0 + N$.

Lemma 1([41]): Let $V(t) \ge 0$ $(t \in R^+)$ be a continuous positive function with bounded initial value $V(0)$. If $\dot{V}(t) \le -aV(t) + c$ holds, where a and c are two positive constants, then: $V(t) \le e^{-at}V(0) + \frac{c}{a}\left(1 - e^{-at}\right)$.

The objective is to design the neural network approximation-based adaptive control for system (1), such that: the system states can track the desired trajectory $x_d = [x_{d1}(t), x_{12}(t), \cdots, x_{nd}(x)]^T \in R^n$ to the desired accuracy;

all signals of the closed-loop systems remain semiglobally uniformly bounded.

Throughout this paper, the following assumptions are satisfied.

Assumption 1: The state x of the system is available for measurement.

Assumption 2: The matrix $g(x)$ is either uniformly positive definite or uniformly negative definite for all $x \in \Omega_x$, where $\Omega_x \subseteq R^n$ is a compact set.

If $g(x)$ satisfies Assumption 2 then $\underline{\sigma}\,(g(x)) \geq \underline{g} > 0, \forall x \in \Omega_x$ where $\underline{\sigma}(\cdot)$ represents the smallest singular value of the matrix inside the brackets and g^* is its lower bound. Assumption 2 guarantees that (1) is uniformly strongly controllable.

Assumption 3: The desired trajectories $x_d = [x_{d1}, x_{d2}, \cdots, x_{dn}] \in R^n$ are known bounded function of time with bounded known derivatives.

Assumption 4: The function $f_i(x)$ and $g_{ij}(x)$, $i, j = 1, 2, \cdots, n$ are continuous but completely unknown.

Remark 2: The above assumptions are common assumptions in the literature and easy to be satisfied in applications [26], [33], [34].

B. NEURAL NETWORKS AND FUNCTION APPROXIMATION

It is known that neural networks are mostly used as approximation models for the unknown nonlinearities due to their inherent approximation capabilities [26], [28], [32], [41]. A class of linearly parameterized neural network used to approximate (2), the continuous function $y(x) : R^p \longrightarrow R$ is represented as follows:

$$y\,(x) = W^T Z(x) \tag{2}$$

where $y(x) \in R$ is the neural network output, the input vector of the approximator is $x = [x_1, x_2, \cdots, x_n] \in R^n$, $W \in R^{p\times 1}$ is a p-dimensional vector of updated weights, and $Z(x) = \left[z_1\,(x)\,, z_2\,(x)\,, \cdots, z_p\,(x)\right] \in R^p$ is p-dimensional vector of known continuous basis function.

Since neural networks can smoothly approximate any continuous function with arbitrary any accuracy, $y\,(x)$ over the compact set $\Omega_x \in R^n$ can be redescribed in the following form:

$$y\,(x) = W^{*T} Z\,(x) + \epsilon(x) \quad \forall x \in \Omega_x \subset R^n \tag{3}$$

where $W^* \in R^{1\times n}$ is the ideal constant weight in the output layer, $\epsilon(x) \in R^{1\times n}$ is the approximation error satisfying $\epsilon(x) \leq \overline{\epsilon}$, and $\overline{\epsilon}$ is an unknown bounded parameter. $z_i(x)$ is known smooth function, in this paper, which is chosen as the Gaussian function:

$$z_i(x) = exp\left(\frac{-(x-\varsigma_i)^T(x-\varsigma_i)}{\mu_i^2}\right), i = 1, 2, \cdots, p \tag{4}$$

where ς_i and μ_i are the center and width of the neural cell of the ith hidden layer. The optimal weight vector W^* is an "artificial" quantity required for analytical purposes. W^* is defined as the value of $\hat{W}$ that minimizes $|\epsilon(x)|$ for all $x \in \Omega_x \subset R^n$ in a compact region, i.e.

$$W^* = arg \min_{\hat{W}\in\Omega_f} \left[\sup_{x\in\Omega_x} \left|y(x) - W^T Z(x)\right|\right] \tag{5}$$

Under the optimal weight value, combining (2) and(3), we have

$$\left|y(x) - W^{*T} Z(x)\right| = |\epsilon(x)| \leq \overline{\epsilon} \tag{6}$$

In general, the ideal neural network weight W^* is unknown and needs to be estimated. In the paper, we shall consider $\hat{W}$ being the estimate of the ideal neural network weight W^*.

In this paper, $f(x) \in R^n$ and $g(x) \in R^{n\times m}$ are unknown functions so they cannot be directly used in the control law. To overcome the highly uncertain environment and provide a valid solution to our problem, we utilize the approximation capabilities of linear in the weights neural networks. Based on the neural network approximation (2), the functions $f(x)$ and $g(x)$ can be redescribed neural network linear form in the following form: $f\left(x|w_f\right) = \left[f_1(x|w_{f_1}), f_2(x|w_{f_2}), \cdots, f_n(x|w_{f_n})\right]$.

$$g(x) = \begin{bmatrix} g_{11}(x|w_{11}) & g_{12}(x|w_{12}) & \cdots & g_{1m}(x|w_{11}) \\ g_{21}(x|w_{21}) & g_{22}(x|w_{22}) & \cdots & g_{2m}(x|w_{2m}) \\ \vdots & \vdots & \ddots & \vdots \\ g_{n1}(x|w_{n1}) & g_{n2}(x|w_{n2}) & \cdots & g_{nm}(x|w_{nm}) \end{bmatrix}$$

Denote $W_f = [w_{f_1}, w_{f_2}, \cdots, w_{f_n}]^T$, $Z_f\,(x) = [z_1(x), z_2(x), \cdots, z_l(x)]^T \in R^{l\times 1}$, and

$$W_g = \begin{bmatrix} w_{11,g}^T & w_{12,g}^T & \cdots & w_{1m,g}^T \\ w_{21,g}^T & w_{22_g}^T & \cdots & w_{2m,g}^T \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1.g}^T & w_{n2_g}^T & \cdots & w_{nm,g}^T \end{bmatrix}$$

$$S_g = \begin{bmatrix} Z_f & 0 & \cdots & 0 \\ 0 & Z_f & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Z_f \end{bmatrix}$$

$f_1(x|w_{f_1}) = w_{f_1}^T Z_f$, $g_{ij} = w_{ij,g}^T Z_f$.

Due to the approximation capabilities of the dynamic neural networks, we can assume without loss of generality, that the unknown system (1) can be completely described by a linear in the weights neural network structures, plus a modeling error term $\xi(x, u)$. In other words, there exist constant but unknown optimal weight values W_f^* and W_g^* such that the system (1) can be written as

$$\begin{aligned} E(\varepsilon)\dot{x} &= W_f^{*T} Z_f(x) + W_g^{*T} S_g(x)u \\ &\quad + [f(x) - W_f^{*T} Z_f(x)] + [g(x) - W_g^{*T} S_g(x)]u \end{aligned} \tag{7}$$

where

$$W_f^* = arg \min_{W_f\in\Omega_f} \left[\sup_{x\in\Omega_x} \left|f(x) - W_f^T Z_f(x)\right|\right]$$

$$W_g^* = arg \min_{W_g\in\Omega_g} \left[\sup_{x\in\Omega_x} \left|g(x) - W_g^T S_g(x)\right|\right]$$

Let

$$\xi(x,u) = [f(x) - W_f^{*T} Z_f(x)] + [g(x) - W_g^{*T} S_g(x)]u$$

Then (7) can be written as

$$E(\varepsilon)\dot{x} = W_f^{*T} Z_f(x) + W_g^{*T} S_g(x)u + \xi(x,u) \tag{8}$$

Remark 3: Because of approximation properties of neural networks and the boundedness of u, $|\xi(x,u)|$ can be arbitrarily small if the neuron number p is chosen large enough [26]. Therefore, without loss of generality we assume that

$$\sup_{0\le t} |\xi(x,u)| \le \bar{\xi}_0$$

where $\bar{\xi}_0$ is an unknown bound.

III. CONTROLLER DESIGN AND THE STABILITY ANALYSIS

In this section, the adaptive neural network controller design procedure is given. The controllers guarantees a uniform ultimate boundedness property for the tracking error, as well as for all other signals in the closed-loop systems with unknown functions.

Define the tracking error e as

$$e = x - x_d \tag{9}$$

Differentiating e and substituting (1) and (7), we can get

$$\begin{aligned} E(\varepsilon)\dot{e} &= E(\varepsilon)(\dot{x} - \dot{x}_d) \\ &= E(\varepsilon)\dot{x} - E(\varepsilon)\dot{x}_d \\ &= W_f^{*T} S(x) + W_g^{*T} S_g(x)u - E(\varepsilon)\dot{x}_d \\ &\quad + f(x) - W_f^{*T} Z_f(x) + \left[g(x) - W_g^{*T} S_g(x)\right]u \\ &= W_f^{*T} Z_f(x) + W_g^{*T} S_g(x)u - E(\varepsilon)\dot{x}_d + \xi(x) \end{aligned} \tag{10}$$

Adding and subtracting the terms $W_f^T Z_f(x)$, $W_g^T S_g(x)$, where W_f, W_g are estimates of the unknown weight values W_f^*, W_g^*, respectively, we can rearrange (10) as

$$\begin{aligned} E(\varepsilon)\dot{e} &= E(\varepsilon)\dot{x} - E(\varepsilon)\dot{x}_d \\ &= W_f^{*T} Z_f(x) + W_g^{*T} S_g(x)u - E(\varepsilon)\dot{x}_d + \xi(x) \\ &= \left(W_f - \tilde{W}_f\right) S(x) + \left(W_g - \tilde{W}_g\right) S_1(x)u \\ &\quad - E(\varepsilon)\dot{x}_d + \omega(x,u) \\ &= -\tilde{W}_f Z_f(x) - \tilde{W}_g S_g(x)u - E(\varepsilon)\dot{x}_d \\ &\quad + W_f Z_f(x) + W_g S_g(x)u + \xi(x,u) \end{aligned} \tag{11}$$

where the parameter errors $\tilde{W}_f$, $\tilde{W}_g$ are defined as

$$\begin{aligned} W_f^* &= W_f - \tilde{W}_f \\ W_g^* &= W_g - \tilde{W}_g \end{aligned}$$

Thus the tracking control problem is to find the appropriate control and update laws to drive the solution of (11) to a small neighborhood of the origin.

In order to find a control law, which would fulfill our goal and guarantee the stability, the following Lyapunov function is proposed:

$$V = \frac{1}{2} e^T E^T(\varepsilon) Pe + \frac{1}{2} tr\left\{\tilde{W}_f^T \tilde{W}_f\right\} + \frac{1}{2} tr\{\tilde{W}_g^T \tilde{W}_g\} \tag{12}$$

where matrix $P = \begin{bmatrix} P_{11} & \epsilon P_{12} \\ P_{12}^T & P_{22} \end{bmatrix}$ satisfies

$$E(\varepsilon)^T P = P^T E(\varepsilon) > 0 \tag{13}$$

Differentiating (12) with respect to time we have

$$\begin{aligned} \dot{V} &= e^T P^T E(\varepsilon)\dot{e} + tr\left\{\tilde{W}_f^T \dot{\tilde{W}}_f\right\} + tr\left\{\tilde{W}_g^T \dot{\tilde{W}}_g\right\} \\ &= e^T P^T\{-\tilde{W}_f Z_f(x) - \tilde{W}_g S_g(x)u - E(\varepsilon)\dot{x}_d \\ &\quad + W_f Z_f(x) + W_g S_g(x)u + \xi(x,u)\} \\ &\quad + tr\{\tilde{W}_f^T \dot{\tilde{W}}_f\} + tr\left\{\tilde{W}_g^T \dot{\tilde{W}}_g\right\} \end{aligned} \tag{14}$$

If we choose the update laws

$$\dot{W}_f = ePZ_f^T(x) - \sigma_1 W_f \tag{15}$$

$$\dot{W}_g = ePS_g^T(x)u - \sigma_2 W_g \tag{16}$$

where σ_1 and σ_2 are design parameter. Substituting (15) and (16) into (14), it follows that

$$\begin{aligned} \dot{V} &= -e^T P^T \tilde{W}_f Z_f(x) - e^T P^T \tilde{W}_g S_g(x) - e^T P^T E(\varepsilon)\dot{x}_d \\ &\quad + e^T P^T W_f Z_f(x) + e^T P^T W_g S_g(x)u + e^T P^T \xi(x,u) \\ &\quad + tr\left\{\tilde{W_f}^T\left[ePS^T(x) - \sigma_1 W_f\right]\right\} \\ &\quad + tr\left\{\tilde{W_g}^T\left[ePS^T(x)u - \sigma_2 W_f\right]\right\} \\ &= -e^T P^T E(\varepsilon)\dot{x}_d + e^T P^T W_f Z_f(x) + e^T P^T \xi(x,u) \\ &\quad + e^T P^T W_g S_g(x)u - \sigma_1 tr\left\{W_f^T \tilde{W}_f\right\} - \sigma_2 tr\left\{W_g^T \tilde{W}_g\right\} \end{aligned} \tag{17}$$

Choosing the control law

$$u = u_1 + u_2 + u_3 \tag{18}$$

with

$$\begin{aligned} u_1 &= \frac{S_g^T(x) W_g^T E(\varepsilon)\dot{x}_d}{1 + \|W_g\|^2 \|S_g(x)\|^2} \\ u_2 &= \frac{S_g^T(x) W_g^T E(\varepsilon)\dot{x}_d W_f Z_f(x)}{\lambda_2\left[1 + \|W_g\|^2 \|S_g(x)\|^2\right]} \\ u_3 &= [W_g S_g(x)]^\dagger\left\{-\alpha E(\varepsilon)e - \frac{1}{2}(1+k)P^T e\right\} \end{aligned}$$

where † denotes the Moore-Penrose pseudoinverse, then we can get

$$\begin{aligned} &e^T P W_g S_g(x)[u_1 + u_2 + u_3] \\ &\quad = e^T P W_g S_g(x)x \\ &\qquad \times \frac{S_g^T(x) W_g^T E(\varepsilon)\dot{x}_d}{1 + \|W_g\|^2 \|S_g(x)\|^2} \\ &\qquad + e^T P W_g S_g(x) \frac{S_g^T(x) W_g^T E(\varepsilon)\dot{x}_d W_f Z_f(x)}{\lambda_2\left[1 + \|W_g\|^2 \|S_g(x)\|^2\right]} \\ &\qquad + e^T P W_g S_g(x)[W_g S_g(x)]^\dagger\{-\alpha E(\varepsilon) \\ &\qquad - \frac{1}{2}(1+k)P^T e\} \end{aligned} \tag{19}$$

Since

$$\frac{\|W_f\|^2\|S_f(x)\|^2}{1+\|W_f\|^2\|S_f(x)\|^2}\leq 1$$

we have

$$\begin{aligned} e^T W_g S_g(x)[u_1+u_2+u_3] &\leq e^T PE(\varepsilon)\dot{x}_d+\frac{1}{\lambda_2}e^T PW_f Z_f(x) \\ &\quad -\alpha e^T PE(\varepsilon)e-\frac{1}{2}(1+k)\,|e|^2\,\|P\|^2 \end{aligned} \tag{20}$$

Using the facts that

$$tr\{W_f^T\tilde{W}_f\}=\frac{1}{2}\|W_f\|+\frac{1}{2}\|\tilde{W}_f\|-\frac{1}{2}\|W_f^*\|^2]$$

$$tr\{W_g^T\tilde{W}_g\}=\frac{1}{2}\|W_g\|+\frac{1}{2}\|\tilde{W}_g\|-\frac{1}{2}\|W_g^*\|^2]$$

equation (17) can be rewritten

$$\begin{aligned} \dot{V} &= -e^T P^T E(\varepsilon)\dot{x}_d+e^T P^T W_f Z_f(x)+e^T P^T W_g S_g(x)u \\ &\quad +e^T P^T\xi(x,u)-\sigma tr\left\{W_f^T\tilde{W}_f\right\}-\sigma_1 tr\left\{W_g^T\tilde{W}_g\right\} \\ &\leq -e^T P^T E(\varepsilon)\dot{x}_d+e^T P^T W_f Z_f(x)+\frac{1}{2}\,|e|^2\,\|P\|^2 \\ &\quad +\frac{1}{2}\,|\xi(x,u)|^2+e^T PE(\varepsilon)\dot{x}_d+\frac{1}{\lambda_2}e^T PW_f Z_f(x) \\ &\quad -\alpha e^T PE(\varepsilon)e-\frac{1}{2}(1+k)\,|e|^2\,\|P\|^2 \\ &\quad -\sigma_1 tr\left\{W_f^T\tilde{W}_f\right\}-\sigma_2 tr\left\{W_g^T\tilde{W}_g\right\} \\ &\leq -\alpha e^T PE(\varepsilon)e+\left(1+\frac{1}{\lambda_2}\right)e^T P^T W_f Z_f(x) \\ &\quad -\frac{1}{2}k\,|e|^2\,\|P\|^2-\frac{\sigma_1}{2}\left\|W_f\right\|^2-\frac{\sigma_2}{2}\left\|W_g\right\|^2 \\ &\quad -\frac{\sigma_1}{2}\left\|\tilde{W}_f\right\|^2-\frac{\sigma_2}{2}\left\|\tilde{W}_g\right\|^2 \\ &\quad +\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2+\frac{\sigma_2}{2}\left\|W_f^*\right\|^2+\frac{1}{2}|\xi\,(x,u)|^2 \end{aligned} \tag{21}$$

According to properties of neural network universal approximation, we have

$$\left|Z_f(x)\right|\leq l \tag{22}$$

where $l>0$ known constants number of nodes neural network. Choose the design constant

$$\lambda_2\geq\frac{l}{\sqrt{2k_1\sigma_1}-l} \tag{23}$$

where

$$\sigma_1>\frac{l^2}{2k_1} \tag{24}$$

$$l(1+\frac{1}{\lambda_2})\leq 2\sqrt{\frac{k_1\sigma_1}{2}} \tag{25}$$

Then, we obtain

$$\begin{aligned} \dot{V} &\leq -\alpha e^T PE(\varepsilon)e+l(1+\frac{1}{\lambda_2})\,|e|\,\|P\|\,|W_f\| \\ &\quad -\frac{1}{2}k\,|e|^2\,\|P\|^2-\frac{\sigma_1}{2}\|W_f\|^2-\frac{\sigma_1}{2}\|\tilde{W}_f^2\|^2 \\ &\quad -\frac{\sigma_2}{2}\|W_g\|^2-\frac{\sigma_2}{2}\|\tilde{W}_g\|^2+\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2 \\ &\quad +\frac{\sigma_2}{2}\left\|W_g^*\right\|^2+\frac{1}{2}|\omega\,(x,u)|^2 \\ &\leq -\alpha e^T PE(\varepsilon)e+2\sqrt{\frac{k_1\sigma_1}{2}}\,|e|\,\|P\|\,|W_f\| \\ &\quad -\frac{1}{2}k\,|e|^2\,\|P\|^2-\frac{\sigma_1}{2}\|W_f\|^2-\frac{\sigma_1}{2}\|\tilde{W}_f^2\|^2 \\ &\quad -\frac{\sigma_2}{2}\|W_g\|^2-\frac{\sigma_2}{2}\|\tilde{W}_g\|^2+\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2 \\ &\quad +\frac{\sigma_2}{2}\left\|W_g^*\right\|^2+\frac{1}{2}|\omega\,(x,u)|^2 \end{aligned} \tag{26}$$

$$\begin{aligned} \dot{V} &\leq -\alpha e^T PE(\varepsilon)e-\frac{1}{2}k\,|e|^2\,\|P\|^2+k_1\,|e|^2\,\|p\|^2 \\ &\quad -\{k_1\,|e|^2\,\|p\|^2-2\sqrt{\frac{k_1\sigma_1}{2}}\,|e|\,\|P\|\,\|W_f\|+\frac{\sigma_1}{2}\|W_f\|^2\} \\ &\quad -\frac{\sigma_1}{2}\|\tilde{W}_f^2\|^2-\frac{\sigma_2}{2}||\tilde{W}_g\|^2 \\ &\quad +\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2+\frac{\sigma_2}{2}\left\|W_g^*\right\|^2+\frac{1}{2}|\omega\,(x,u)|^2 \end{aligned} \tag{27}$$

If we choose

$$\frac{k}{2}-k_1>0 \tag{28}$$

$\dot{V}$ can be rewritten as

$$\begin{aligned} \dot{V} &\leq -\alpha e^T PE(\varepsilon)e-\frac{\sigma_1}{2}\|\tilde{W}_f^2\|^2-\frac{\sigma_2}{2}||\tilde{W}_g\|^2 \\ &\quad +\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2+\frac{\sigma_2}{2}\left\|W_g^*\right\|^2+\frac{1}{2}|\omega\,(x,u)|^2 \end{aligned} \tag{29}$$

Let

$$c=min\{\alpha,\frac{\sigma_1}{2},\frac{\sigma_2}{2}\} \tag{30}$$

$$d=\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2+\frac{\sigma_2}{2}\left\|W_g^*\right\|^2+\frac{1}{2}|\omega\,(x,u)|^2 \tag{31}$$

Then (30) can be written as

$$\dot{V}\leq -cV+d \tag{32}$$

Remark 3: By selecting suitable design parameters $P,\sigma_1,\lambda_2,\ \alpha,k$, the performance of the proposed architecture is controlled. However, more research is required to understand in which way the system performance is affected. Some insight on this interesting and important subject is provided in the simulation section.

The main results are summarized by the following theorem.

Theorem 1 : Consider the closed-loop uncertain singularly perturbed plant (1), controller (18), adaptive updating laws (15) and (16). Under the conditions that Assumptions 1-4 are satisfied and the design parameters satisfy the conditions (13), (22), (23), (26) and (27), all signals in the closed-loop singularly perturbed system remain bounded, and the tracking error can be made arbitrarily small by choose parameters appropriately and increasing the approximation accuracy of the neural networks.

Proof: Construct the Lyapunov candidate function

$$V = \frac{1}{2}e^T E^T(\varepsilon)Pe + \frac{1}{2}tr\widetilde{W}_f^T\widetilde{W}_f + \frac{1}{2}tr(\widetilde{W}_g^T\widetilde{W}_g^T) \quad (33)$$

Calculating the time derivative of V along (18), (15), and (16) yields

$$\begin{aligned}\dot{V} \leq& -\alpha e^T PE(\varepsilon)e + \left(1+\frac{1}{\lambda_2}\right)e^T P^T W_f Z_f(x)\\ &-\frac{1}{2}k\,|e|^2\,\|P\|^2 - \frac{\sigma_1}{2}\left\|W_f\right\|^2 - \frac{\sigma_2}{2}\left\|W_g\right\|^2 - \frac{\sigma_1}{2}\left\|\tilde{W_f}\right\|^2\\ &-\frac{\sigma_2}{2}\left\|\tilde{W}_g\right\|^2 + \frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2 + \frac{\sigma_2}{2}\left\|W_f^*\right\|^2 + \frac{1}{2}|\xi\,(x,u)|^2\end{aligned} \quad (34)$$

From (22)–(25), (28) and (34), the following inequality follows

$$\begin{aligned}\dot{V} \leq& -\alpha e^T PE(\varepsilon)e - \frac{\sigma_1}{2}\|\tilde{W}_f^2\|^2 - \frac{\sigma_2}{2}||\tilde{W}_g\|^2\\ &+\frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2 + \frac{\sigma_2}{2}\left\|W_g^*\right\|^2 + \frac{1}{2}|\omega\,(x,u)|^2\end{aligned} \quad (35)$$

Define new variables as follows

$$c = min\{\alpha, \frac{\sigma_1}{2}, \frac{\sigma_2}{2}\}$$

$$d = \frac{\sigma_1}{2}\left\|W_f{}^*\right\|^2 + \frac{\sigma_2}{2}\left\|W_g^*\right\|^2 + \frac{1}{2}\bar{\xi}_0$$

Then, (35)can be rewritten as

$$\dot{V} \leq -cV + d \quad (36)$$

Applying Lemma 1 and based on (36), we can get

$$V \leq e^{-ct}V(0) + \frac{d}{c}(1-e^{-ct})$$

Based on the procedure above, we can conclude that the tracking error e are bounded for all $t \geq 0$, and will asymptotically converge to a compact set. This implies that the close-loop system is semiglobally uniformly bounded.

Remark 4: The adaptive control laws for normal nonlinear systems have been proposed by many researchers [31]–[34], however, these do not consider the singular perturbation parameter ε, ε is usually very small, which usually leads to ill-conditioned numerical problem. In this paper, the adaptive neural network tracking controller is designed based on the Lyapunonv function method. The novel adaptive laws including singular perturbation parameter ε are designed to avoid the ill-conditioned numerical problems. Moreover, neural networks are utilized to approximate the unknown nonlinear functions, but each networks node is subject to both slow and fast dynamics in this paper.

Remark 5: Neural network-based adaptive identification and control for such kind of singularly perturbed nonlinear system with unknown functions has been studied in [36], [37], [39]. However, tracking control problem of singularly perturbed systems with unknown functions has been rarely considered [40], which prompts the research work of this paper. Compared with the results in [40], the proposed method in this paper is free of identification procedure and the number of parameters to be tuned is less, which will reduce the computational burden.

IV. SIMULATION STUDY

In this section, two simulation examples are given to demonstrate the effectiveness of the proposed control technique.

A. EXAMPLE1

To demonstrate the effectiveness of the proposed adaptive neural networks tracking control algorithm. We consider the following nonlinear system:

$$\begin{bmatrix}1 & 0\\ 0 & \varepsilon\end{bmatrix}\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-x_1^2 - x_1x_2\\ 4x_1x_2 - x_2\end{bmatrix} + \begin{bmatrix}0.25 & 0\\ 0 & 0.2\end{bmatrix}u \quad (37)$$

where $x = [x_1, x_2]^T \in R^2$, initial condition for the states are chose as $x(0) = [0, 0]^T$, singular perturbation parameter $\varepsilon = 0.001$, $u \in R^2$ is the control input. The objective is to design an adaptive controller for the system (1) such that: 1) the state x follow the desired reference signals are $x_d = [0.5sin(0.5t), 0.3cos(2t)]^T$; 2) the boundedness of all the signals in he closed-loop system is guaranteed. According to the above design method, the controller is defined as

$$\begin{aligned}u =& \frac{GE(\varepsilon)\dot{x}_d}{1+\|G\|^2} + \frac{G^T E(\varepsilon)\dot{x}_d}{\lambda_2(1+G)}\\ &+G^{-1}(-\alpha E(\varepsilon)e - \frac{1}{2}(1+k)P^T e)\end{aligned} \quad (38)$$

where $e = x - x_d$ and $G = \begin{bmatrix}0.25 & 0\\ 0 & 0.2\end{bmatrix}$, the adaption law is given as follows:

$$\dot{W}_f = ePZ_f^T - \sigma_1 W_f \quad (39)$$

In the simulation studies, the centers and widths are selected on a regular lattice in the respective compact sets. The neural network contains 100 nodes with the centers ς_i evenly spaced in $[-1, 1] \times [-0.5, 0.5]$, and widths $\mu_i = 0.02$ where the input variable of the neural network is given as $x = [x_1, x_2]^T$. Initial value of the adaptation law is shown as $W_f(0) = 0$. The design parameters are selected as $P = \begin{bmatrix}1 & \varepsilon\\ 1 & 1\end{bmatrix}$, $\sigma_1 = 10$, $\lambda_2 = 0.5$, $\alpha = 0.1$, $k = 100$.

From Figs.1-4, it is clear that the simulation results which are achieve by employing the controller (38) to the systems (37). The tracking trajectories of x_i and $y_{di}, i = 1, 2$ are illustrated in Figs. 1-2 and the goof tracking performances can be observed. The controllers u_1 and u_2 are diagrammed in Fig.3. The trajectories of adaptation laws $\|W_{f1}\|, i = 1, 2$ are set in Fig.4. Form Figs.3-4, it can be seen that u_i and $W_{fi}, i = 1, 2$ are bounded. Form these simulation figures, we can see that all the signals in the closed-loop system are bounded.

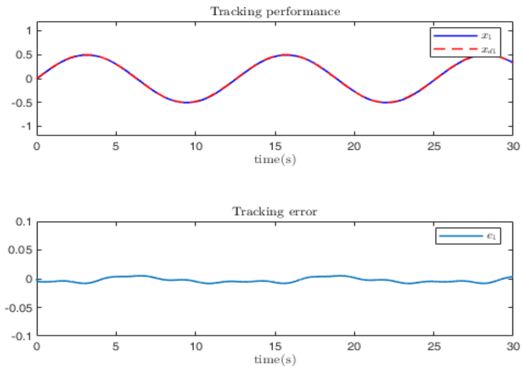

FIGURE 1. Trajectories of x_1, x_{d1}, and e_1.

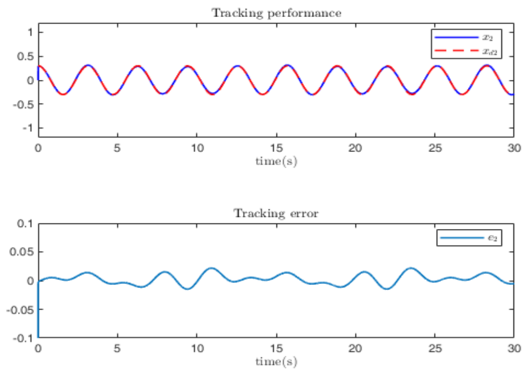

FIGURE 2. Trajectories of x_2, x_{d2}, and e_2.

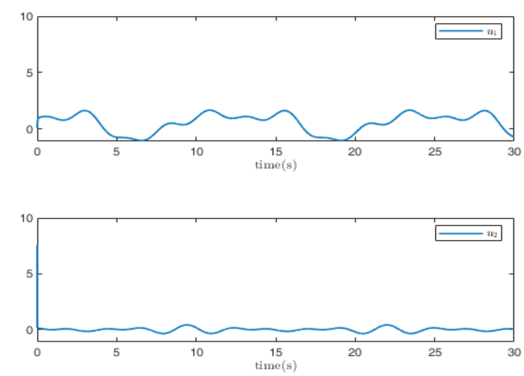

FIGURE 3. Trajectories of the control inputs u_1 and u_2.

B. EXAMPLE2

To demonstrate the potential application of the control schemes for practical systems, we consider a nonlinear circuit [35] illustrated in Fig.5. The $v_c - I_R$ characteristics of the resistor is $I_R = \frac{1}{5}v_c^3 - \frac{1}{5}v_c$. Applying the Kirchoff law, we obtain the state equation

$$
\begin{aligned}
L\dot{I}_L &= -I_L R - v_c + u(t) \\
C\dot{v}_c(t) &= I_L - \frac{1}{5}(v_c^3 - v_c) + \alpha u(t)
\end{aligned}
\tag{40}
$$

where $\varepsilon = C$, $\alpha = 0.5$, $R = 2\Omega$ and $L = 0.1H$. Let $x(t) = L \cdot I_L$ and $z(t) = v_c$. The state equation (40) can be written by

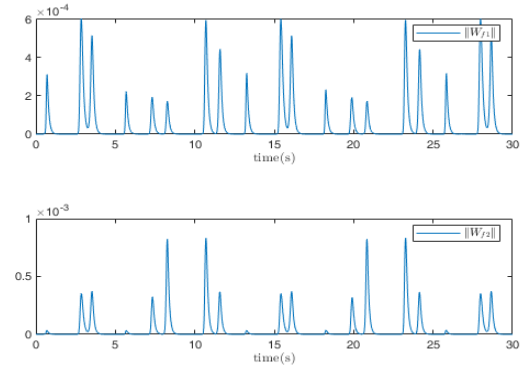

FIGURE 4. Trajectories of $\|W_{f1}\|$ and $\|W_{f2}\|$.

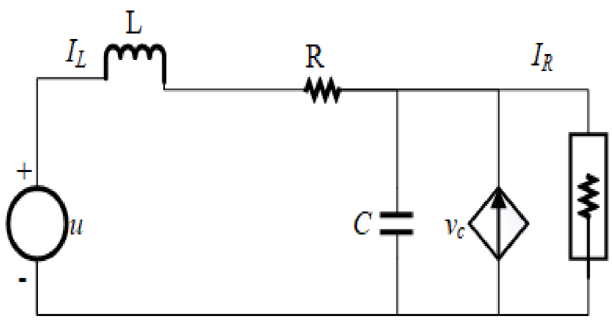

FIGURE 5. An electric circuit with parasitic capacitor and nonlinear resistor.

the following singularly perturbed nonlinear system

$$
\begin{aligned}
\dot{x}(t) &= -20x(t) - z(t) + u(t) \\
\varepsilon\dot{z}(t) &= 10x(t) - 0.2(z^2(t) - 1)z(t) + 0.5u(t)
\end{aligned}
$$

which can be rewritten as

$$
E(\epsilon)\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} -20x_1^2 - x_2 \\ 10x_1 - 0.2(x_2^2 - 1)x_2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \end{bmatrix} u \tag{41}
$$

where $E(\epsilon) = \begin{bmatrix} 1 & 0 \\ 0 & \varepsilon \end{bmatrix}$, $x = [x_1, x_2]^T \in R^2$, initial condition for the states are chose as $x(0) = [0, 0]^T$, singular perturbation parameter $\varepsilon = 0.001$, $u \in R^2$ is the control input. The control objective is to design an adaptive controller for the system (41) such that: 1) the tracking objective is to make the state x follow the desired reference signals are $x_d = [0.5sin(0.5t), 0.3cos(2t)]^T$; 2) the boundedness of all the signals in the closed-loop system is guaranteed.

In the simulation studies, the centers and widths are selected on a regular lattice in the respective compact sets. The neural network contain 100 nodes with the centers ς_i evenly spaced in $[-1, 1] \times [-0.5, 0.5]$, and widths $\mu_i = 0.02$ where the input variable of the neural network is given as $X = [x_1, x_2]^T$. The initial value of the adaptation law is shown as $W_f(0) = 0$. The design parameters are selected as $P = \begin{bmatrix} 1 & \varepsilon \\ 1 & 1 \end{bmatrix}$, $\sigma_1 = 8$, $\lambda_2 = 0.25$, $\alpha = 0.5$, $k = 100$.

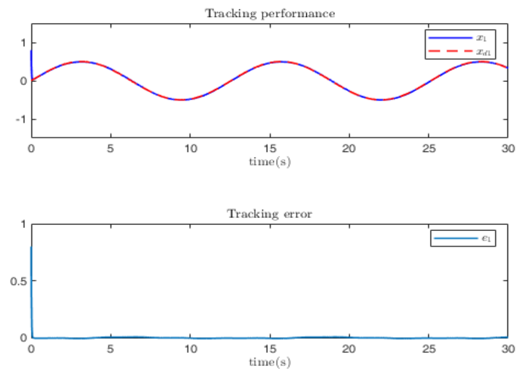

FIGURE 6. Trajectories of x_1, x_{d1}, and e_1.

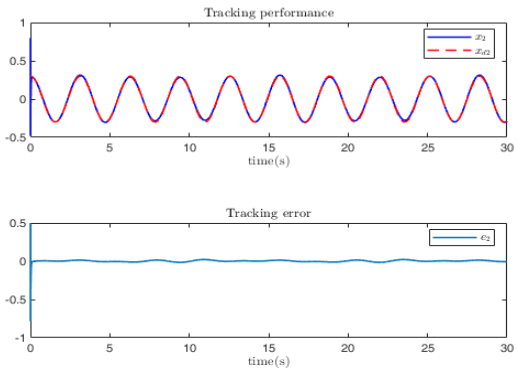

FIGURE 7. Trajectories of x_1, x_{d1}, and e_1.

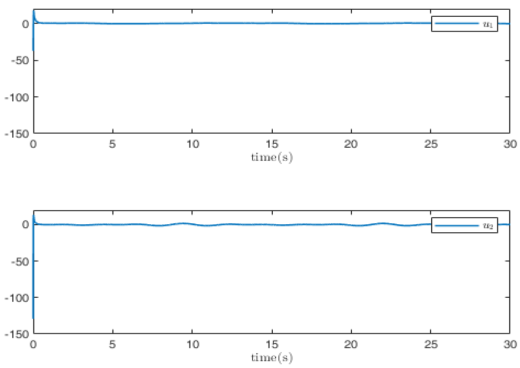

FIGURE 8. Trajectories of the control inputs u_1 and u_2.

Fig.8 show the simulation results which are obtained by applying the control controller u to the system (37). Figs.6-7 show the tracking trajectories of system and it can be seen that a good tracking performances is achieved as well as the tracking error of systems state converges to a small neighborhood of zero. The trajectories of the control signals and the adaptation laws are diagrammed in Figs.8-9. It is obvious that they are bounded. Therefore, it can be concluded from these figures that all the signals in the closed-loop system are bounded.

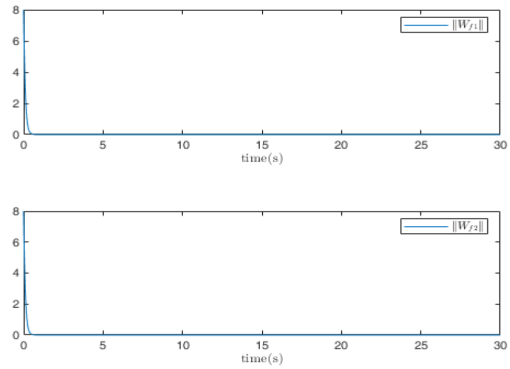

FIGURE 9. Trajectories of $\|W_{f1}\|$ and $\|W_{f2}\|$.

Remark 6: The specified references are $x_d = [0.5sin(0.5t), 0.3cos(2t)]^T$. Notice that the reference trajectories are chosen to maintain different time scale.

V. CONCLUSION

This paper has investigated the problem of adaptive neural network tracking control for a class of uncertain singularly perturbed nonlinear systems. Due to the presence of the singular perturbation parameter ε, which is a very 'small' parameter, the control design problem of singularly perturbed systems is a difficult task. By designing an adaptive neural network control and introducing a novel Lyapunonv function, the stability of the closed-loop systems has been proven. The approach can be used to cope with more general type of singularly perturbed systems compared with the existed works on singularly perturbed systems. It can be seen from simulation results that a good control performance is achieved.

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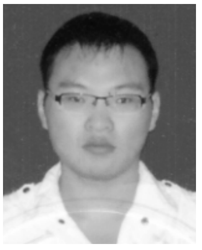

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