Systematic Literature Review of Sampling Process in Rapidly-Exploring Random Trees

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ABSTRACT Path planning is one of the most important process on applications such as navigating autonomous vehicles, computer graphics, game development, robotics, and protein folding. It ensures that a path is planned between an initial and final position on the collision-free region of a search space if one exists. One of the most wide algorithms used for this purpose is the rapidly-exploring random tree (RRT), in which each node of a tree data structure is generated from a search space by a random sampling process, which originally follows a uniform spatial distribution. However, some authors claim that the addition of a non-uniform/informed approach into the sampling process of the RRT could accelerate the planning time of the algorithm. Actually, many works on literature propose different strategies to include non-uniform/informed behavior on RRT-based algorithms. However, the large number of studies on path planning subject impose difficulties on the identification of new solutions on a review process. The aim of this paper is to structure a review process to deal with the massive volume of works on this subject, by presenting the planning, development, and results of a systematic literature review (SLR), to investigate non-uniform/informed sampling solutions applied to RRT-based algorithms on path planning literature. A review protocol with two scientific questions was developed to guide the investigation. As a result, 1136 studies were selected in the path planning literature, of which 53 were identified as claiming to contain a solution with non-uniform/informed sampling on RRT-based algorithms. As a specific work is considered a scientific contribution only when it has not yet been explored in scientific circles, the results of the SLR can be used as a tool to search for what has not yet been proposed, helping to identify opportunities to contribute with new sampling processes of RRT-based algorithms. To the best knowledge of the authors, this paper presents the first development of an SLR of a topic related to the RRT algorithm.

INDEX TERMS Non-uniform sampling, informed sampling, path planning, RRT, systematic literature review.

I. INTRODUCTION

Path planning consists in describing a sequence of states through a scenario to move objects from an initial state to an end state, avoiding the non-navigable regions (obstacles, danger zones, etc) of a search space. The object can be a wheel robot, a robotic arm, a virtual agent, an unmanned aerial vehicle, among others. Each state is an element of the search space, which is the set of possible transformations that can be applied an object, at each moment of its trajectory [1].

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In a simplified way, the state of a path can be defined as a set of information related to the object, such as position, inclination angles, speed, etc. The state of an object can still be composed by other elements, depending on the type, the level of detail of its mathematical modeling and context of application.

The planning occurs by the exploration of the search space regions. In the context of path planning by sampling-based algorithms, it delimits where the new samples are collected to construct new paths. The representation of the search space depends on the application scope of the planning. The samples can represent an element from the configuration

space (robotics view), state space (control view) or spatial position on an environment (artificial intelligence view) [1]. On robotics, the sample of a configuration space usually gives the relative position of the robot to a referential frame, represented as its spatial position and angular direction. On control and dynamic systems applications, it is defined as state space, where the motion constraints are verified during the planning, incorporating kinodynamics constraints on the sample (velocity, acceleration, curvature, etc). On artificial intelligence, the sampling process usually imposes only geometric constraints, where the sample is a position on the free collision region of the search space. In practice, many views are merged together to create the planning solutions.

Rapidly-Exploring Random Tree (RRT) algorithm [2] is one of several techniques that can be applied to path planning on all cited application scopes by the search of feasible path steps. The RRT algorithm allows the planning of paths for holonomic and non-holonomic systems, through efficient exploration of multidimensional search spaces, with convex and non-convex obstacles. Its operation consists on expanding a tree-shaped data structure from a initial position (root node) of the path to be planned, until one of its tree branches reaches the final/goal position (last leaf node). Each branch corresponds to a connection between two nodes, and its length is an input parameter of the algorithm. A node can be any collision free position in a search space and is obtained through a random sampling process.

Many applications have been benefited by the properties of RRT-based algorithms. The RRT is used on virtual reality planning as a high-degree virtual humans [3] and a physics-based virtual reality [4]. In artificial intelligence on game development, the RRT is a tool to quickly plan random paths [5] and exploring levels [6]. In robotics, a path can be designed for systems such as mobile robots, manipulators, space robots, underwater robots, helicopters and humanoids [7]. The work presented in [8] relates a RRT-based planning algorithm to the estimation of the movements of a dog-like robot. On [9], is presented a arm robot with movements planned by a RRT-based algorithm. A human-system interaction on industries processes is presented in [10]. Path planning applications on space robotics can be also found on [11]. On [12] and [13], applications on analog circuits and protein manifolds are presented, respectively. Path planners applied to many types of autonomous vehicles were also built over RRT structure, as Aerial vehicles (Micro Vehicle Aircraft (AVM) [14], Hypersonic Air Vehicles (HAV) [15], Blastless Unmanned Aerial Vehicles (BUAVS) [16], Unmanned Aerial Vehicles (RUAV) [17] and Combat Unmanned Aerial Vehicle (UCAV) [18]), Unmanned Ground Vehicles (Unmanned Terrestrial Vehicles (UGV) [19]) and Unmanned Surface Vehicles (Autonomous Submarine Vehicles (AUV) [20], Unmanned Maritime Surface Vehicles (USSV) [21]).

In the RRT algorithm, originally in [2], the random sampling follows a uniform spatial distribution to collect new samples. Thus, any position in the search space has the same probability to be collected. However, some authors argue that introducing non-uniform/informed behavior into the sampling process would increase the convergence of RRT to obtain a viable solution. In a non-uniform/informed sampling, some positions of the search space have higher probability to be collected than others. In [22], it is stated that some authors believe that introducing the use of non-uniformly/informed distributed sampling may satisfy parameters, such as dispersion, which have better performance than randomized sampling with uniform distribution. The main difference between these two types of sampling is that the non-uniform distribution can direct the expansion of the RRT to more promising regions of the navigation environment, e.g., regions where a path has higher chance to be found, bias towards the goal position, etc.

Although some authors have drawn attention to the theme, as stated in [22], the study of effects of the non-uniform/informed sampling in the RRT is a recent topic, as pointed on [23], and new studies are needed to understand its effects compared to the uniform approach. However, the scientific community has already developed a considerable quantity of methods and applied to the sampling process of RRT-based algorithms. The search for related academic works, mainly to the RRT algorithm, is a very complex task due the huge quantity of works proposed on literature. Some bibliographic review methodologies can standardize the method to research a particular topic in the literature to deal with the huge volume of works that can be encountered in a epoch like today of quick and large dissemination of scientific information. One increasingly popular technique that deal with these matters in several areas of research is the *Systematic Literature Review (SLR)*. A guide to perform this type of review is described in [24].

SLR is a popular review methodology in the areas of Software Engineering [24], [25] and Medicine [26]. This type of bibliographic review is organized in a structured way, where the search engines (websites were indexed academic works can be queried), strings for the search, criteria used to include and exclude a study, and all the information about the review process are specified in the *Review Protocol*. The protocol is developed together with a team of researchers with experience on the subject, which defines what type of scientific evidence should be researched. The review is then carried out respecting to the research protocol. Thus, in addition to allowing an evaluation of the review process by a third party, the interpretation bias caused by the personal vision of the researchers involved in the review is diminished [24].

The objective of this work is to report the results of the SLR methodology to address the study of different non-uniform/informed sampling processes in the RRT algorithm. This review prioritizes works in which non-uniform/informed sampling strategies are used to reduce the planning time of a path or optimize it in some specific aspect. This work only consider strategies where the sampling process is biased in some way. Another optimization strategies, as like sample rejection [27], [28] and parallel approaches [29], [30], are not

considered, since the sampling process is not modified, but other processes of the RRT. Usually, the modified algorithms are the RRT and RRT* [31]. It is notable the increase of academic interest on the last one, because of its optimal asymptotic cost property. On [23], a review of general optimizations of RRT*-based algorithm is presented.

The main contribution of this work is the use of an SLR approach to the study of different processes of non-uniform/informed sampling in RRT-based algorithms. Thus, it must be clear that it is not our intended to provide an exhaustive survey on sampling methods applied to RRT, but report the SLR results, showing that is possible to extract academic information from the vast literature available about path planning. The solutions identified on SLR can then later be applied to robotics, autonomous vehicles, game development, virtual reality or another application which search space can be explored by RRT in a biased way.

To the best of our knowledge, this is the first work that uses an SLR procedure to review a path planning technique. No evidence was found that SLR has ever been applied to the topic addressed in this study, or that such review was performed for the RRT sampling process. The search engines for this evidence are the same as those specified in the research protocol considered in this review. However, the closest theme to this work addressed by an SLR is presented in [32], where the results of a *Systematic Mapping Study* (A type of SLR with a broader theme) on modeling techniques of systems for control of mobile robots is shown. RRT-based algorithms surveys are available on [23] and [33]. However, neither of these works has focused on non-uniform/informed sampling processes of RRT-based algorithms.

This work is organized as follows: a review of path planning algorithms is presented in section II; in section III, the RRT algorithm is described; the influence of uniform and non-uniform distributions on the RRT is discussed in section IV; section V presents the modeling of the SLR protocol based on the use of non-uniform sampling processes in RRTs; section VI describes the development of the SLR through the protocol modeled and a validation of review process; in section VII, the selected papers as the result of the execution of the SLR are indicated and discussed; section VIII presents the conclusion of this work.

II. PATH PLANNING ALGORITHMS

The complexity of a path planning problem depends directly on the computational model adopted to represent the search space [34]. Initially, methods with explicit representation of the search space were used in path planning problems. Algorithms such as cell decomposition [35], potential fields [36], visibility graphs [37], Voronoi diagrams [38] and those presented in [39]–[41], use the explicit representation of the search space. However, these algorithms can generate excessive computational effort, especially when there are a lot of obstacles in the search space [42]. A class of algorithms, known as Discretization methods, can be applied to fragment the search space into small pieces represented by cells,

reducing the complexity of a planning problem [43]. The complexity of the planning problem is reduced because it can be resolved in each cell at a time until a complete path between two positions can be planned. These cells have an intrinsic relationship to each other that form a graph structure. Search algorithms such as A* [44] and Dijkstra [45] can be used to find the path with smallest cost associated to the edges of these structures (cost can be represented by edge length, for example), as can be seen in [46]. Some search methods in dynamic graphs are also used as D* [47], AD* [48] and Life-Long A* (LPA*) [49]. However, the efficiency and effectiveness of the search methods in graphs depends on the resolution adopted in the search space discretization. In addition, representations of the search space at high discretization resolutions generate a high computational cost for planning [33]. Artificial intelligence techniques were applied as an alternative in path planning, many of them being biologically inspired. Examples of these techniques are Artificial Fish School Algorithm, [18], Fuzzy Logic [50]–[52], Ant Colony Optimization [53]–[55], Machine Learning [1], Neural Networks [56] and Genetic Algorithms [57]. Currently, the most commonly used methods are sampling based algorithms, due to their efficiency and ease in coupling the planning with the kinodynamic constraints of a system. The main sampling-based path planning algorithms are Probabilistic Roadmaps (PRM) [58], Rapidly-exploring Random Tree (RRT) [2], Expansive Space Trees (EST) [59], Ariadne's Clew [57] and Fast Marching Tree (FMT) [60]. Many of the planned paths by the mentioned methods may not be viable for navigation due to the dynamics and kinematics restrictions of the considered system for which it was generated. The methods usually employed smooth these paths by generating curves that allow this system to run its trajectory along the planned path. Some techniques used for path smoothing are clothoids [61], β-spline [62], Bézier curves [63], [64], Pythagorean-Hodograph (PH) curves [65]–[68], logistic curves [69] and Support Vector Machine (SVM) [70].

III. RAPIDLY-EXPLORING RANDOM TREE (RRT)

The RRT algorithm for path planning is described in Alg. 1. RRT plans a navigation path in a Q configuration space, which is the set of points/positions of a navigation environment. Q is divided in two subsets, Q_{free}, representing the navigable regions of the navigation environment, i. e., the regions without obstacles and collision risks, and Q_{obs}, the spatial representation of the obstacles.

The root node of the tree $q_{init} \in Q_{free}$ is the starting point of the path to be planned. The algorithm works by expanding a G tree randomly from the root node until one of its branches reaches the final point $q_{goal} \in Q_{free}$ of the path to be planned, or until a maximum number of iterations n is reached. Since each node is a sample/point of the navigation environment and has the information about its predecessor node, the path is planned from the q_{init} point to the q_{goal} origin point, adding each node that connects them to a path

Algorithm 1 RRT Algorithm Pseudocode

1: **procedure** RRT($Q, q_{init}, q_{goal}, \Delta q, l_d, n$)
2: $\quad G \leftarrow \{\}$
3: $\quad R \leftarrow \{\}$
4: $\quad s \leftarrow 0$
5: $\quad i \leftarrow 0$
6: $\quad$ **while** ($s = 0$) and ($i \leq n$) **do**
7: $\quad\quad i \leftarrow i + 1$
8: $\quad\quad q_{rand} \leftarrow RAND\_CONFIG(Q)$
9: $\quad\quad q_{near} \leftarrow NEAREST\_VERTEX(q_{rand}, G)$
10: $\quad\quad q_{new} \leftarrow NEW\_CONFIG(q_{near}, q_{rand}, \Delta q)$
11: $\quad\quad$ **if** $\overline{q_{near}q_{new}}$ do not intercept Q_{obs} **then**
12: $\quad\quad\quad EXTEND(G, q_{near}, q_{new})$
13: $\quad\quad\quad$ **if** $d(q_{new}, q_{goal}) \leq l_d$ and ($\overline{q_{new}q_{goal}}$) do not intercept Q_{obs}) **then**
14: $\quad\quad\quad\quad EXTEND(G, q_{new}, q_{goal})$
15: $\quad\quad\quad\quad s \leftarrow 1$
16: $\quad\quad\quad\quad R \leftarrow ROUTE(q_{init}, q_{goal})$

R. A leaf node q_{new} is a point of the straight line segment $\overline{q_{near}q_{rand}}$, such that: $q_{goal} \in Q_{free}$ is a point generated as a random sampling of the navigation environment; q_{near} is the closest node to q_{rand}; the distance between q_{near} and q_{new} has the constant value Δq; and the straight line segment $\overline{q_{near}q_{new}}$ is collision-free, that means it does not intercept any obstacle of Q_{obs} of the navigation environment.

Several extensions of the RRT algorithm have been published over the years: RRT-Connect [71], Resolution Complete Rapidly-Exploring Random Tree (RC-RRT) [72], Real-time Closed-loop Rapidly Exploring Random Trees (CL-RRT) [73] and its derivation Closed-loop Random Belief Trees (CL-RBT) [14], Execution extended Rapidly Exploring Random Tree (ERRT) [74], Chance-Constraint Rapidly Exploring Random Tree (CC-RRT) [75], RRT Star (RRT*) [31], CC-RRT*-D [76], Adaptive RRT Based on Dynamic Step (DRRT) [77], Fast RRT [78], among others. Therefore, the variability of studies focused on improving or adapting some aspect of the RRT algorithm is remarkable.

IV. ROLE OF SAMPLING IN RRT

In the literature, the following two types of sampling are considered: uniform and non-uniform. In the first type, the search space samples are randomly collected, considering a uniform spatial distribution. In the second type, the sample collection process can be biased by determining the most promising regions of the search space. In this way, a path can be planned more efficiently within the context of the problem in which the sampling methods are applied. Further discussion of sampling issues in path planning can be found in [79].

The best performance of non-uniform sampling, regarding uniform sampling, can be observed by the best convergence for a solution in a search space [79]. Usually, two approaches are used to implement the non-uniform sampling

in RRT. In the first one, the expansion of RRT can be regionally biased, collecting samples next to the major regions of interest on the configuration space. This approach is called importance sampling. In the second, the sampling is changed during the planning, depending on the restrictions that may be encountered in the navigation space. This last approach is called adaptive sampling. By adopting one of these approaches, the sampling process is biased by a non-uniform distribution, which reduces the amount of samples needed to find a feasible path. Both approaches can be used together or individually. However, implicitly, non-uniform sampling is usually a fusion with uniform sampling. Therefore, the application of a non-uniform distribution in concomitance with the uniform distribution is an indispensable requirement to guarantee a successful performance of methods based on non-uniform sampling [79].

In the standard RRT [2], sample collection is governed by a uniform spatial distribution. As the number of samples collected in configuration space increase, the possibility of finding a collision-free path will increase [79]. As described in this literature review, some researches study the role of non-uniform distribution in RRT. We investigated the improvements, as pointed out in the literature, in path planning using RRT sampling techniques based on informed/non-uniform distribution.

V. SYSTEMATIC LITERATURE REVIEW PROTOCOL

This section describes the protocol that guides the SLR process of collecting scientific evidences in the literature, about the solutions of non-uniform/informed sampling processes applied to RRT-based path planning algorithms.

To find works that help to answer the scientific questions developed to this review, an SLR was developed to understand how the subject is currently explored. Within the context of this kind of review, some concepts are followed to define the types of works that should be returned and which of them should be included into the review results. These concepts follow the guidelines stated in [24]. The works are then classified as following: those that present innovative methods, results, procedures, technology, or schema related to the review subject are called primary studies; reviews and comparative studies are secondary studies; the reviews of secondary studies are considered tertiary studies. The SLR that is elaborated in this paper considers only the primary studies as an evidence of the solution.

A. DEFINITION OF THE SCOPE OF THE RESEARCH REVIEW QUESTIONS

The main objective of this review is to report the set of evidences that show what is being investigated about RRT-based algorithms with informed/non-uniform sampling processes through a systematic review of the literature. In this paper, only proposed informed/non-uniform sampling strategies are considered. During the execution of the SLR, a sampling strategy will be considered informed/non-uniform if it generates random samples that do not follow

TABLE 1. Research questions developed in systematic literature review.

Label	Research Question	Description and intent
Question 1	Which are the proposed solutions to include non-uniform/informed behavior into the RRT sampling process found in the path planning literature?	Identify the methods proposed in path planning literature that implement non-uniform/informed strategies on sampling process of the RRT algorithm or any of their variants (for example, RRT*).
Question 2	Which is the RRT-based algorithm to which a proposed non-uniform/informed sampling solution is applied to improve its planning performance?	This question helps to identify the base techniques (RRT or some of its variants) extended to generate a new non-uniform RRT-based algorithm.

a uniform distribution, i.e., they are biased. It is important to emphasize that the random samples determine the growth direction of the tree of the RRT. In this way, if the samples are biased by an informed/non-uniform sampling, the tree of the RRT will be biased as well. To be selected in the SLR, a primary study should introduce a new non-uniform/informed sampling strategy with one of the following objectives: to accelerate the convergence of RRT planning for a path, reduce its planning time, or optimize some other characteristic of RRT-based algorithms. To compare and identify if a selected study introduces some modification that adds biased behavior in some RRT-based algorithms, the *Random_state* procedure of the RRT algorithm in [2] and *Rand_config* in Alg. 1 are considered as the canonical sampling.

Modifications and optimizations in RRT algorithm processes other than sampling will not be considered in the SLR, for example, edge reconfiguration, parallelization, node reduction by rejections, collision detection optimizations, and kinodynamic optimization. Although these processes directly influence the planning performance of RRT-based algorithms, they are not related directly to sampling process, and are coupled independently of other processes on sampling-based algorithms. In addition, the subject of this review is limited only to methods applied to sampling processes of RRT-based algorithms. In the case of path planning with kinodynamic constraints in RRT-based algorithms, some proposed strategies can be selected in the SLR. Problems of path planning with kinodynamic constraints are quite complex, since many dimensions may be necessary to represent their search space, and not every state can be considered valid or reached from a previous one, as in the case of non-holonomic systems [80]. However, traditionally the planning process in these cases is accomplished by rejecting or adapting a random sample to the generation of a trajectory that meets the constraints of the movement of a given system or path smoothing [33], which will not be considered an informed sampling strategy/non-uniform in this review. On the other hand, if the state space of this system is used to model or influence the distribution of the random samples in the RRT, then the proposed strategy will be considered informed/non-uniform. The study presented in [81] corresponds to the last described situation.

Different search engines for scientific papers were consulted to find related studies to the theme. The data extracted from these studies were compared and analyzed to identify the main contributions of the use of non-uniform sampling as an alternative to the traditional uniform sampling of RRT.

The review protocol that guides the process of this review is described below.

B. REVIEW PROTOCOL

The review protocol that guides the process of collecting scientific evidence is composed by six elements: research questions; terms of research; queries for primary studies; search engines; selection criteria for primary studies; and data extraction.

1) RESEARCH QUESTION

A research question conditions the whole process by presenting the main discrepancies about the non-uniform sampling methods used in RRTs for path planning. To assist in the process of defining the research questions, the methodology proposed in [82], called PICOC (Population, Intervention, Comparator, Outcome, Context) was used. To the problem of the use of non-uniform sampling processes in RRTs, the following values were defined for each parameter of the PICOC methodology:

- **Population**: RRT-based path planning algorithms.
- **Intervention**: Addition of non-uniform/informed strategy into RRT sampling to obtain better path cost and/or convergence/time in path planning process.
- **Comparator**: Non-uniform/informed strategy used in the RRT sampling process.
- **Results**: Existing works in the literature on non-uniform/informed sampling applied to RRT and its characteristics improved by the approach used.
- **Context**: Academic research on any application context.

Thus, two research questions were developed to meet these parameters. These issues are presented in Tab. 1.

The first question is "Which are the proposed solutions to include non-uniform/informed behavior into the RRT sampling process found in the path planning literature?". This is the main problem of the review. To discover the answer, it is necessary to identify primary studies in which the problems that the authors propose to find solutions involve improving some property (usually planning time and/or path cost optimization [33]) of the RRT, using as the main strategy the change of its uniform sampling process. This is the expectation of the review; however, there may be studies with different approaches that fit the review criteria.

The second question is "Which is the RRT-based algorithm to which a proposed non-uniform/informed sampling solution is applied to improve its planning performance?". This question is related to the method from which the solution,

TABLE 2. Research queries structured for systematic literature review.

Label	Query Definition
Query 1	("Rapidly Exploring Random Tree" OR "Rapidly-Exploring Random Tree" OR RRT) AND Dispersion
Query 2	("Rapidly Exploring Random Tree" O "Rapidly-Exploring Random Tree" OR RRT) AND Sampling AND ("Path Planning" OR "Trajectory Planning" OR "Motion Planning")

proposed in a given primary study, was originated. The methods can be the RRT itself or some of its variants (RRT*, DRRT, RRT-Connect, etc). The answer to this question will help to identify which type of RRT is used as the basis of the studies to reduce its randomness, whether the new aspects that the variants include are considered, or whether versions that already have reduced planning time are taken into account in the literature of path planning.

The SLR results obtained from the above research questions will allow the analysis of which informed/non-uniform sampling solutions proposed to the RRT can yield the most promising results in path planning, which may help research teams in initial experiments of future research. For example, a non-uniform sampling strategy may have been proposed for RRT, but its convergence acceleration efficiency for a path may not have been verified along with some RRT*-based algorithm, which has asymptotic optimality. Analyzing the SLR results, a research team can verify what has already been proposed as an informed/non-uniform sampling solution for specific versions of the RRT algorithm and identify what has not yet been. Beyond that, a technological project team can take knowledge of which strategies exists through the proposed SLR and analyze which of them can fill some requirements of its project.

2) SEARCH STRINGS
Search strings should be related to the scientific problem of interest. In this way, the probability of retrieving a work related to the subject is greater. Each string is also a component of the search query applied to search engines. Considering variations between similar keywords and terms, the following strings were used in this work: "Rapidly-exploring Random Tree", "Rapidly exploring Random Tree", "RRT", "Sampling", "Dispersion", "Path Planning", "Trajectory Planning", "Motion Planning".

3) QUERIES FOR SEARCH PRIMARY STUDIES
Through strings and defined search questions, queries were formulated to be used in the search engines considered in this review. A query is the representation of a search question in a format suitable for the search engines consulted.

Queries are structured using search strings and logical operators, which are acceptable in all search engines considered in this review. Using the search strings previously defined, two queries related to the research questions were structured to search the primary studies for the SLR. In the Tab. 2 are summarized the research queries created for this SLR. All works/primary studies returned by the execution of these queries must be analyzed and extracted.

TABLE 3. List of search engines as source of primary studies for systematic literature review.

Search engine name	Web site
Scopus	www.scopus.com
IEEEXplore	www.ieeexplore.ieee.org
ScienceDirect	www.link.springer.com
ACM Digital Library	www.dl.acm.org
Engineering Village	www.engineeringvillage.com

4) ENGINES CONSIDERED IN THE PROCESS REVIEW
The process of researching primary studies involves the definition of the search engines, which are the tools that index the publication source of these works. The primary studies returned by these engines are filtered according to the inclusion and exclusion definitions considered for the review, and are described in the next section. On the Tab. 3 are listed the search engines used in the SLR elaborated in this work.

5) CRITERIA FOR SELECTING PRIMARY STUDIES
Through the query results in the search engines, a filtering of which studies are related to the search questions is performed. To guide this task, one inclusion criteria and a set of exclusion criteria was defined. These two types of criteria help to decide which work is relevant or not for the review. The following inclusion criteria was defined for the review in question:

- Academic works with real contribution to path planning by the RRT method. The solutions presented should propose improvements on RRT sampling process through non-uniform/informed strategies, focusing on reducing planning time or improving parameters and related characteristics.

In contrast to the inclusion criteria, the set of exclusion criteria was defined to decide what types of papers returned by search engines are not included in the review, even if they meet the inclusion criteria. For example, survey documents, white papers, gray literature, etc. can be returned, and this review does not include them in its results. The defined exclusion criteria are:

- paper does not present solution applied to path planning;
- paper does not include new solution to sampling process of the RRT algorithm;
- paper only presents comparison or tests between methods based on RRT;
- presentation documents (slides);
- secondary and tertiary studies;
- paper is an older version from another one, with same authors and problem;
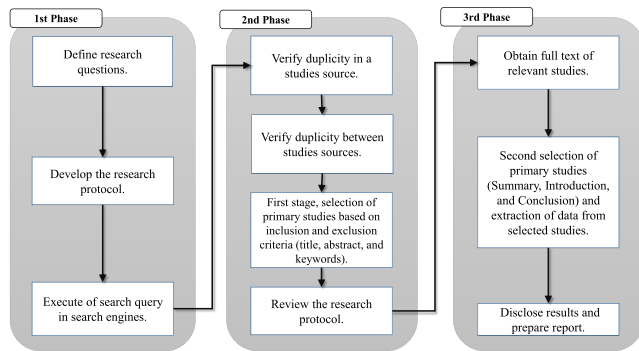- paper is not written in English;
- abstract only;

FIGURE 1. SLR development process: – in the 1st Phase, the scope of the review is outlined; – in the 2nd Phase, first filtering of the primary studies returned by the search engines is executed; – in the 3rd Phase, data from primary studies are extracted and analyzed to report results.

- duplicated outcomes. These works are counted just once;
- white paper, and gray literature.

6) DATA EXTRACTION

To analyze and compare the primary studies returned by the review, a list was made of data/characteristics that should be extracted from each of them. These characteristics of the primary studies are: title, publication year, journal, conference, congress, symposium, or workshop where the evidence was published; authors; RRT methods used as a base to develop the presented solution; sampling method used; other RRTs methods compared with the presented solution to test its effectiveness; completeness; optimality; acquired improvements resulting from proposed solution.

VI. DEVELOPMENT OF THE SYSTEMATIC LITERATURE REVIEW

This section describes the development of the Systematic Literature Review (SLR) based on the review protocol previously described. With the application of the review process, we expected to obtain only the studies related to the review scope to answer the research questions, thus reducing the bias caused by the reviewer's personal view on traditional review process. Fig. 1 summarizes the SLR process developed in this paper.

A. PHASES DESCRIPTION OF THE SYSTEMATIC LITERATURE REVIEW

The SLR is divided into three phases: in the first phase, the scope of the review is outlined by a research team; in the second phase, the first filtering of the primary studies is return by the search engines; in the final phase, the data of primary studies are extracted and analyzed. Then the results of the review are disseminated through a report document, which corresponds to this paper.

In the first phase, initially the scientific questions of interest are defined as described in Section V-B1. The second step was to develop the review protocol. This should be built together with a team of experts on the reviewed subject. Protocol reformulations before reaching a final version can be performed. These formulations are consequences of insights on how to improve the results of the returned studies, obtained in the later steps of the review development. The last step of this phase is to access the website of each search engine and insert search queries formed by search strings in each of them. A set of 1136 preliminary works were returned, considering the sum of the results of all search engines.

The second phase starts by eliminating duplicated results. The works that were returned more than once by the search engines were removed. A total of 403 duplicate papers were identified and removed, resulting in 733 unique items. In the next step, the inclusion and exclusion criteria were applied by reading the title, abstract, and keywords of each work. The works resulting from the application of such criteria are considered to be works with potential solutions related to the theme of the review. After this phase, 143 potential works from 733 unique studies were selected.

In the third and last phase, the data was extracted and the review is finished. The first step of this phase was to gain access to the full text of the selected studies. The second step consisted in the complete reading of each study to identify and extract the data required for the review, as described in subsection V-B6. After the data extraction, those works that violated the inclusion criteria or that met the set of exclusion criteria are removed from the review. At the end of the SLR process, 53 primary studies were selected. Finally, the last step was to disclose the results of the review through some report, which in the case of this review, is this article.

B. VALIDATION OF THE SYSTEMATIC LITERATURE REVIEW

All the process of leading the SLR was executed by just one of the authors in this work. The other authors contributed on the development review protocol, working mainly as supervisors of the review. Thus, a bias on the studies' selection can be strongly presented on the performed process. As a mean to identify possible misunderstanding of which studies attend the scientific questions, a validation process was performed, as suggested on [24]. This SLR validation consists of a calibration of the query search processes by identifying if the most cited/relevant works of the literature were returned on the review process.

The query calibration process is important to ensure that search engines return relevant articles to the research question considered in this review. Initial queries were performed and the returned results were analyzed based on the scientific topic addressed in the review. The queries were modified until relevant works could be returned.

It was identified that studies like [22] and [83] were returned. These are considered important for the RRT sampling theme, which according to their authors, bring the first attempts to further discuss the importance of non-uniform/informed sampling on RRT, giving a historical insight of the discussions about biased sampling on the based-sampling algorithms. In addition, one of the most

TABLE 4. Evidence collected in systematic literature review process that fall on goal-biased sampling class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[87]	ERRT, DRRT	Goas bial sampling and forest bias	MP-RRT	Probabilistic complete	non-complete	RRT, ERRT, DRRT
[88]	Multi-RRT	Goal-bias sampling	Multi-RRT-GoalBias	Probabilistic complete	non-optimal	Multi-RRT
[89]	RRT	Goal oriented sampling	ERRT	Probabilistic complete	non-optimal	RRT
[12]	RRT	Learning approach	Goal-oriented test generation	Probabilistic Complete	non-optimal	RRT
[90]	RRT	Scent pervasion based sampling	Two stage RRT	Probabilistic complete	non-optimal	RRT, RRT-Connect
[91]	RRT*	Artificial Potential Field	P-RRT*	Probabilistic complete	optimal	RRT*
[92]	RRRT	Goal-biased sampling	Goal oriented RRT (GO RRT)	Probabilistic complete	non-optimal	RRT, RRT-Connect and RRT*
[93]	RRT	Goal-bias sampling	GRRT	Probabilistic complete	non-optimal	RRT
[94]	RRT	Potential fields based sampling	RRT with potential fields-based sampling	Probabilistic complete	non-optimal	RRT
[95]	B-RRT*	Artificial Potential Fields	PB-RRT*, PIB-RRT*	Probabilistic complete	optimal	IB-RRT*, P-RRT*, RRT*

recent and successful algorithms proposed with bias sampling strategy, the Informed-RRT* algorithm [84], was returned on the SLR process. Even some extensions of it were identified through the SLR. Also, as stated on [23], one of the most recent review works of RRT-based algorithms, the RRT* algorithm has been actively researched on path planning community on the last few years. Many works listed [23] that are identified as proposing novel non-uniform/informed sampling on RRT* are actually returned by the SLR as well. Even more RRT*-based algorithms with biased sampling extensions were identified on SLR compared to [23]. Therefore, we believe that were assembled queries that considerably covered the possibilities of works about non-uniform/informed sampling on RRT-based algorithms.

VII. RESULTS OF THE SYSTEMATIC LITERATURE REVIEW
This section presents the results of the review process guided by the review protocol developed. As described earlier, the subject of interest in this review is the use of non-uniform/informed samplings strategies that results in a biased behavior of the sampling process of RRT-based algorithms. The data about each strategy were extracted from the selected papers in the review process. These data were analyzed and summarized to show the solutions that were returned by the SLR execution.

During the data extraction of the selected primary studies, was possible to identify common characteristics between various proposed biased sampling strategies. On [85], a classification of the kinds of non-uniform/informed sampling is given. Some examples of classes of sampling bias are given on [33], [84], and [86] too. Inspired by these works, the results were clustered on classes, considering the two approaches of sampling introduced on [79]: importance sampling and adaptive sampling. Thus, each class can be considered a subset of a approach defined on [79]. The primary studies solutions

were classified by sampling strategies that shares similar mechanisms. The importance sampling classes identified on the SLR were: goal-biased sampling, obstacle-biased sampling, region-biased sampling, path-biased sampling, narrow passage-biased sampling. The adaptive sampling classes identified on the SLR were: sampling by reduction of the search space, bias by sampling distribution.

A. IMPORTANCE SAMPLING
1) GOAL-BIASED SAMPLING
In this class of bias sampling the RRT grow is drove towards the goal region of the search space. Usually the strategies proposed use the goal position of planning explicitly to extend the tree to a given frequency during the planning. In contrast, another strategies apply methods that captured the relations between the obstacles and the goal position, which makes the collection of new samples to this region implicit, as the case of the Potential Fields Method (APF). The primary studies selected that fall into this class are listed next and in Tab. 4.

On [87], the algorithm Multipartite RRT (MP-RRT) is proposed over the foundation of the ERRT and DRRT algorithms to planning on a search space with static and dynamic obstacles. The algorithm maintains a forest that is disconnected from the tree that contains the planning initial position (principal tree). As the tree grows, the new samples are collected between the roots of the forest trees to a given probability value. If the obstacles on the search space has their position changed, the principal tree is reinitialized and its nodes are stored to the forest, that will be used to on future iteration of MP-RRT. Also, a probability chance of goal bias sampling is given. If random generated probability value p is lesser than a predefined threshold, so the sampling is biased to the goal region. Otherwise, if p is lesser than the probability of the goal bias threshold plus the forest bias threshold, than the new

sample is collected from the forest sampling. Finally, if all checks fail, a random sample is collected.

The Multi-RRT-GoalBias is proposed on [88], which is a goal biased version of the Multi-RRT algorithm. Its sampling strategy utilizes a goal probability threshold, a value that defines when the goal bias sampling is applied on the path planning. When used, the tree is expanded on the direction of the goal position. The authors offer also a neural network based approach to define the optimal threshold value to a given search space.

The proposed algorithm ERRT [89] inserts a probability value to decide if the sampling process is defined by the goal position or by uniform distribution. If a value p is higher than a certain parameter, so the goal position is defined as the sample to bias the RRT grow.

The Goal-oriented test generation method is proposed in [12]. The method plans a path in a two-dimensional search space representing the states of an analog non-linear electrical circuit. In this method, RRT tends to grow towards regions where most samples of states of the circuit occur in its search space. To this, two phases are added to RRT: (a) a learning phase, where the state samples are collected to identify the regions of interest on the circuit search space; (b) exploration phase, where RRT has its expansion oriented to these regions of the greatest interest. In the learning phase k samples of the search space of the analog circuit are collected. So the search space is partitioned in a network with equally-sized grids. These grids are then clustered by the quantity of samples that relies on each one of them. Grids with the same number of samples are related to a same cluster. The cluster with the higher number of grids is defined as the goal to where the expansion of the RRT must be biased/oriented. In the exploration phase the samples of the cluster are used as bias to grow the RRT. At each iteration, a sample is collected from one of the selected clusters. Thus, the RRT converges faster to these regions, which could be associated with fail states of the circuit. Thus, paths between the initial and final states of the circuit can be generated in a faster way. The paths planned by RRT are used to execute circuit tests, which manually could be exhausting.

The RRT-based method introduced on [90], called Two Stages RRT, apply an initial decomposition of the search space, and then, based on decomposition result, the RRT is constructed. In the first stage, the proposed method utilizes a scent pervasion algorithm to construct a discrete representation of the search space. A scent information is diffused through the neighborhood regions of the search space starting from the goal region, which is the scent source. This way, tags are given to each cell of the discretized search space, that represents the intensity scent, an information of proximity between a cell and the goal region. After the discretization stage, the RRT is constructed based on the scent information. The cells that are less selected and has lesser value of tags, has more preference to be selected to expand the tree. Its supposed by the authors that this process grow the RRT more quickly into direction of the goal region.

A solution to reduce the effect of randomness in RRT using the Artificial Potential Fields (APF) technique is proposed in [91] to create the Potential Guided Directional-RRT* (P-RRT*) method. In the new algorithm, goal, samples and obstacles emit forces of opposite polarities to drive the expansion of the tree. Each node of the tree has the same polarity of the obstacles, which creates a repulsion effect between the samples on the extension of the tree, existing tree nodes and the obstacles. In contrast, the iteration between new samples and the goal point results in a force of attraction that converges the RRT to solution. Thus, the P-RRT* maintains the balance between an exploration of randomly collected samples and the use of non-uniform/informed exploration. According to the authors, the new method has better convergence in the search for an optimized path planning solution than RRT*, an RRT-based method that optimize the planning path cost.

The Goal Oriented RRT (GO RRT) is proposed on [92]. The sampling process is biased by the goal region of the search space by a strategy called Pre-GO sampling. This sampling processing defines the size of the search space based on the distance between the goal and its farthest node on the tree. This distance is used as radius of a ball region centered on goal position. New samples are collected on the ball search space by a constant value that determines the concentration/probability in the collect of a sample from the goal trough the limit ball defined by the radius, with the higher probabilities attached to the goal position.

A goal-bias based sampling is proposed on [93]. The sampling scheme is applied to RRT algorithm. The proposed solution collect two random samples simultaneously. The nearest sample between them from the goal position is selected to expand the tree. Second the authors, this strategy avoids local collisions caused by goal bias while improving the path length and computing time.

A strategy based on potential fields is proposed on [94]. Potential fields methods is explored in two distinct way by the proposed algorithm: on a stochastic manner and on a deterministic manner. In the stochastic sampling strategy a bunch of samples is collected per iteration, and that with the lowest potential value is selected to extend the tree. In the deterministic sampling, each node of the tree has a rank, based on its relative position to a target and the obstacle region. That node with the higher rank is selected and a sample is collected into direction of the potential difference with the goal position. Each strategy is selected given a probabilistic value as a way to bring an equilibrium to the use of the both. The proposed sampling strategy was developed to a manipulator application. According to authors, with the proposed method the manipulator show better ability to respond sudden situations during its movement process.

Huazhong *et al.* [95] extended the bidirectional RRT* (B-RRT*) and Intelligent Bi-directional RRT* (IB-RRT*) attaching to them the Artificial Potential Fields (APF) strategy, creating the PB-RRT* and PIB-RRT* algorithms, respectively. In these algorithms, a new random sample is

TABLE 5. Evidence collected in systematic literature review process that fall on obstacle-biased sampling class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[96]	RRT-ConCon	Random Triangle State	Dangerzone RRT	Probabilistic complete	non-optimal	DPRM
[97]	RRT	Danger zones sampling	Dangerzones RRT (DRRT)	Probabilistic complete	non-optimal	DPRM
[98]	RRT	Obstacle boundary information	GeneteRRT	Probabilistic complete	non-optimal	RRT
[99]	BIT*	Obstacle-guided Sampling	BIT*-H	Probabilistic complete	optimal	BIT*, BIT*-G (With Gaussian distribution)

potentially guided on the expansion of the B-RRT*. The strategy decides if the APF bias will be biased related to initial position or final position depending on if the iteration count of the expansion is even or not.

2) OBSTACLE-BIASED SAMPLING

Many primary studies usually introduces bias by information of the search space's obstacles to reduces the path length during the planning and to improve the sampling effectiveness on difficult areas, as narrow passages. Here are the primary studies that present solutions based on obstacle-biased sampling. Their data are listed too in Tab. 5.

In [96] a RRT-based algorithm is proposed where new samples are collected near of danger zones of the space search. This algorithm is called DRRT (Dangerzone RRT) and presents a modified sampling process on the RRT-ConCon algorithm. In a search space constituted from obstacles and danger zones (a region that is collision free but undesirable to navigation), the proposed sampling process use triangle surfaces obtained from the danger zones. These surfaces bias the tree grow after a first path is planned. A triangle is selected randomly and a random sample is generated over the surface of the triangle. This way, the tree grows in direction of the danger zones boundaries.

The algorithm proposed on [97], called Danger zones RRT (DRRT), extends RRT by existing techniques for danger zones sampling and local planning. Two samples strategies are applied on DRRT. In the first, a density measure is calculated to each node of the tree whenever a new node if generated. Thus, these nodes that has lower density is more preferable to be selected to expand the RRT. Finally, a new sample is generated on the neighborhood (defined as a circle region on the search space) of the node with the lowest density value. The second utilizes danger zones, regions of the search space that are preferable not to collect samples. To extends the tree, a random sample is collected from triangles that models the obstacles of the danger zones. A triangle of a danger zone is selected randomly, defining the direction to where the RRT must grow on the danger zone. Thus, the nearest node of the tree from the danger zone sample is selected to extend the RRT. Second the authors, the effect to apply these sampling strategies is a more effective planning (total nodes of the tree, total planning time) on scenarios with danger zones.

The RRT-based algorithm proposed on [98] use the information about the obstacles that are touched by the segment formed by the initial position and final position of planning.

In the proposed sampling process, each one of the touched obstacles are classified as valuable to the planning process. In the boundaries of these obstacles are generated samples that corresponding to its mid points. The obstacle that is nearest to the initial position are selected to extend the tree. Thus, one of its mid points are selected randomly to be added as a new sample of the RRT. This way, the tree is biased into the goal direction and a path with reduced length can be obtained quickly.

On [99], an algorithm with an obstacle bias sampling that generate denser sample distribution near obstacles with underlying uniform spreed, called BIT*-H, is proposed as extension of the BIT* algorithm. The sampling strategy is hybrid with two possibilities of processes to be executed. In the first, the ellipsoidal-based sampling of BIT* is used. However, when a sample is generated uniformly inside the ellipse, a neighborhood sampling is executed to explore local information around uniform samples. Finally, the mean of the neighborhood is collected as the sample to extends the tree. If this mean sample is no collision free, a new sample is generated around the uniform sample by a Gaussian distribution. These nodes collected by Gaussian distribution are defined as navigators by certain criteria. In the second process, the navigators are used to bias the tree grow. The use of each sampling process is defined by a dynamic ratio between the optimal cost and current cost of the planning solution. This ratio defines the proportion of use of each sampling process over the planning by the BIT*-H.

3) REGION-BIASED SAMPLING

On region-biased sampling more important regions on planning, as lesser cost regions, regions, has more probability to be samples collected. Usually, pre-information about the search space is given to define the classification of the most promising regions. Sampling learning are used to discovery these regions or hypotheses are made to infer what are the most promising regions to be explored on the search space. Next, the primary studies of this class returned by the SLR are listed and their extracted data are in Tab. 6.

Two planning algorihtms based on the concept of Voronoi bias is proposed by [83]. The first approach, called Volume-based RRT (VB-RRT), approximates the size of the Voronoi areas to decide where to expand the tree. To avoid explicitly Voronoi Diagram construction, which is computationally expensive, a strategy to collect uniformly k samples on search space is proposed. Thus, the tree expansion is

TABLE 6. Evidence collected in systematic literature review process that fall on region-biased sampling class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[83]	RRT	Voronoi diagram	VB-RRT and DB-RRT	Probabilistic complete	Non-optimal	RRT-Connect
[22]	RRT	Multiple Sampling	MS-RRT (has a and b versions)	Probabilistic Complete	Non-optimal	RRTConCon
[100]	bidirectional RRT	offline EA based sampling and online EA based sampling	RET	Probabilistic complete	non-optimal	Bidirectional RRT
[101]	RRT	Sampling around the tree node with the smallest densisty	DAS-RRT	Probabilistic complete	non-optimal	RRT, Adaptative-DDRRT
[102]	StRRT	Voronoi based bias	Voronoi Based StRRT	Probabilistic complete	non-optimal	StRRT
[103]	Bi-RRT	N-dimensional Cuboids	E-RRT	Probabilistic complete	non-optimal	RRT, Bi-RRT
[104]	RRT	Learn of the propagation of search space points of a dynamic system	LPCA-RRT	Probabilistic complete	non-optimal	RRT, GPCA-RRT
[105]	RRT*	Sphere-based sampling	Cloud RRT*	Probabilistic complete	optimal	RRT*-Smart, RRT*
[106]	RRT	decrease/increase the sampling area	EB-RRT	Probabilistic complete	non-optimal	RRT, biased-RRT
[107]	RRT	Moving-Window Unilateral Gaussian Sampling Strategy	MW-RRT	Probabilistic complete	non-optimal	Greedy-RRT, Connect-RRT
[108]	RRT	Any angle sampling	Theta*-RRT	Probabilistic complete	non-optimal	A*-RRT, RRT, RRT*, A*-RRT*
[109]	RRT	General Voronoi Diagram	VB-RT	Probabilistic complete	non-optimal	RRT
[110]	RRT	drivers' visual behavior sampling bias	DV-RRT	non-complete	non-optimal	RRT, Bi-RRT
[13]	RRT	Weighted Voronoi diagrams (WVD)	RRT-based search with Voronoi-based	Probabilistic complete	non-optimal	RRT
[111]	RRT	Goal-based bias sampling	RRT-DWA	Probabilistic complete	non-optimal	none
[112]	RRT	Sampling probabilities; fuzzy logic	Boundaries-bias and fuzzy bias RRT	Probabilistic complete	Non-optimal	RRT, Biased RRT to expand to goal waypoint, Biased RRT to expand to itself node, Biased RRT by cell decomposition

biased into direction of the mean point of the k samples. The second approach, called Dispersion-based RRT (DB-RRT), incorporates an expansion behavior that exhibit a degree of the Voronoi bias based on dispersion of k collected samples from the search space. The hypothesis of the approach is that growing the RRT toward the sample farthest from the tree between the k random samples, the algorithm will show an exact Voronoi-bias behavior. To implements this behavior, the algorithm sort the k samples in decreasing order of distance from their nearest node in the tree. The farthest random sample between the nearest nodes of the tree is selected to expand the RRT.

The Multi-Sample RRT (MS-RRT) method is proposed in [22]. In this approach, k samples are collected at the same time during the RRT expansion, in contrast to the traditional RRT sampling process in which only one sample is collected at a time. The tree is then biased to the mean point of the collected samples. Two algorithms are introduced based on the multi-sampling scheme. In the first, called MS-RRTa, the parameter k defines how many times random samples are collected per RRT iteration. For each one of these randomly

sampled points, its nearest neighbor node is computed. The node of the tree that was more frequently selected as the nearest neighbor, is defined as the best node for the tree expansion. Thus, the tree is extended by it. The second multi sample-based approach is the MS-RRTb. It uses a defined set of k samples that are used to estimate the Voronoi bias to the tree expansion too, but these samples are uniformly selected just when the planning starts. Thus, the set is reused on every iteration of the tree expansion, unlike MS-RRTa. The authors' hypothesis is that using a same set of samples that went uniformly selected has a similar bias effect than selecting them on each iteration.

The work on [100] proposed, called Rapidly Exploring Evolutionary Tree (RET), apply a evolutionary algorithm (EA) that generates individuals (points on the search space) and fitness values that determines how efficiently these individuals cause the RRT to grow. Thus, a bidirectional RRT is generated where the new samples are collected based on the EA individuals. A sample has 50% to be collected by EA individuals, otherwise it will be collected by uniform random sample.

A new sampling scheme is proposed on [101] to overcome situations that RRT is trapped by Voronoi bias on obstacles with maze shape. The proposed algorithm, called Density Avoided Sampling RRT (DDAS-RRT), where the tree is biased to escape from high density areas. A density (quantity of neighbors nodes of it) parameter is added to each node of the tree, that informs an estimated free space associated to it. The node of the tree with the lowest density value associated is selected to expand the tree. Thus, a neighborhood area around these nodes will be the region where a new sample will be collected. A random procedure collect a new sample inside this region.

The proposed Voronoi StRRT on [102], a modified version of Spatiotemporal-RRT (StRRT), has a sampling process based on generalized Voronoi diagram. The new collected samples are biased just toward the boundaries of the local Voronoi diagram, to reduces computational time in presence of static and dynamic obstacles on the search space. To reach this results, the Voronoi StRRT apply two steps. First, a collected sample is moved on the opposite direction of its closest point on an obstacle boundary. Second, the amount of movement is estimated based on the distribution neighboring obstacle and its distance of its nearest boundary point of obstacles. As result, a new sample is placed near of the Voronoi boundaries and safer paths are planned.

The algorithm proposed on [103] called extended bidirectional RRT (E-RRT) is an extension of the Bi-RRT. The sampling process of the algorithm is biased by a goal-oriented bias and an N-dimensional cuboid. On each iteration, the algorithm attempt extend the tree in direction of the goal position. Then, if the tree is not yet connect with the goal position, its is expanded by N-dimensional cuboids. Over the expansion of the RRT, each new node added has a cuboid added to it. This cuboid is generates to a given offset from the latest added node on the tree. Thus, on the sampling step of the RRT, a new random sample is collected then inside of the cuboids from the last extended node of the tree.

The proposed algorithm on [104] is called Local Principal Component Analysis RRT (LPCA-RRT). Models a cost function based on a dynamic system state space to improves local planning by a bias on its discretized search space sampling. The strategy is a two-step approach. The first step learns the direction of the propagation points on the discretized search space when the system are simulated on these points. In the second, the sampling process of RRT is biased to the regions where the points on the first step propagated. This way, RRT converge to the regions of the search space of the system that are more probably to occurs.

In the Cloud RRT* algorithm proposed on [105], the search space is first decomposed on a set of spheres with different radius sizes. Each one of the spheres has a unique selection probability value, which give the weight to be selected on the sampling process. The spheres position on the search space and its probability values are initially defined by a Generalized Voronoi Graph (GVG) based on the distance visibility of the initial planning position, Voronoi vertices and edges and overlap optimization between the spheres. In the sphere selection by the sampling process, a new sample is uniformly collected inside on it. During the planning by Cloud RRT*, new spheres are created based on previous paths. Thus, the path length is optimized by the samples obtained by spheres based on previous nodes.

To overcome the randomness of RRT and improves its search efficiency, [106] propose the Exponential Back-off sampling Rapid-exploring Random Tree (EB-RRT) algorithm. It biases the sampling trough collision-free regions of the search space denominated middle zones. They are Voronoi regions that are not poorly explored and not overwhelmed explored, they are median explored regions. To induces the RRT grow to them, a back-off sampling operation is executed when a collision on extend operation occurs. The back-off sampling operation consists in increase e reduces the sampling area of the search, which is associated to the goal position of the exploration. The effect is that the tree grows around obstacles.

The algorithm proposed on [107], named moving-window rapidly exploring random tree (WM-RRT), use unilateral Gaussian sample generator to collect new samples near nodes that are more probable to be fresh nodes to the tree. The hypothesis of the authors is that exists a higher probability to the tree expands from late added nodes than old added ones. Also, they state that samples collected around the segment line formed by initial position and goal position optimizes the length of the planned paths.

The Theta*-RRT [108] is a algorithm with a sampling process biased by a pre-planed any-angle path generated by Theta* search grid algorithm. A strip of width W centered around the pre-planned path is used to generate new samples randomly collected inside of it Each new sample has an associated angle direction randomly defined given an angular interval. The tree grows around the any-angle path. This way a shorter and smoother path can be planned rapidly on the continuous search space.

The algorithm proposed on [109], creates a 2D GVD to extract the spatial distribution of obstacles on 3D search space. Thus, points are distributed locally around 2D Voronoi edge nodes and are selected according to a cost function to grow the RRT in each iteration. The Voronoi nodes with lesser costs are sampled to bias the RRT grow.

Inspired by the studies of drivers' visual behavior, a concept that humans utilizes two viewpoints to guide a vehicle in curves, a far one and a near one, the work on [110] proposes a biased sampling algorithm called Drivers' Visual Behavior-guided RRT (DV-RRT). The algorithm is applied on the context of a car-like vehicle. The far and near points of drivers' visual behavior are utilized to bias the sampling process of RRT to collect samples that are more suitable to the vehicle's motions. Thus, one of the points are selected given velocity criteria and new samples are generated given a Gaussian distribution around it. Second the authors, this strategy simplifies the path by avoiding the generation of useless samples to the vehicle motion.

TABLE 7. Evidence collected in systematic literature review process that fall on path-biased sampling class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[113]	bi-RRT*	Local bias by middle of two neighbors	Bi-RRT* with local sampling	Probabilistic complete	optimal	bi-RRT*
[114]	RRT*	Cross-entropy based sampling	SCE-RRT*, TCE-RRT*	Probabilistic complete	optimal	RRT*
[116]	RRT*	Intelligent sampling	RRT*-Smart	Probabilistic complete	Optimal	RRT*
[81]	RRT	Bias by the most selected nodes on a reference path	GRRT	Probabilistic complete	non-optimal	RRT
[117]	RRT*	Directed acyclic graph (DAG) based sampling	RRT* using Waypoint Graph	Probabilistic complete	optimal	RRT*
[118]	T-RRT*	Biased path areas sampling	BT-RRT*	Probabilistic complete	optimal	T-RRT*
[119]	RRT*	The Focused-Refinement sampling	Goal Tree RRT*	Probabilistic complete	non-optimal	RRT*, RRT*-Smart

A RRT-based algorithm to tunnel detection on protein structures is proposed on [13]. The sampling method of the algorithm maintain a Voronoi vertices regions list of the search space. The region size of the Voronoi vertices is defined by its distance from their nearest obstacle (on the problem domain is an atom). Thus, new samples are collected around the Voronoi vertices that are new on the tree. When the tree has already explored a Voronoi vertices region, it is deactivated from the sampling process.

The RRT algorithm with Dynamic Window Approach (DWA) is proposed on [111] to autonomous vehicles' path planning, introducing a algorithm called RRT-DWA. A sampling scheme based on goal-bias sampling selection based on probability value is used by it. The probability value is defined given the obstacles density on the search space. If a random generated value is higher than the probability value, the sampling is goal-biasing. Otherwise, the sample is collected randomly. DWA is used to local planning of velocity of the vehicle, which takes its dynamic into consideration.

The use of sampling probabilities and fuzzy logic to obtain collision information and guide the expansion of RRT is considered in [112]. In the method, the search space is cluttered by cell decomposition and each cell are associated with probabilities and, according to their classification, some regions of the search space becomes more important than others. Based on this, two planners, where probabilities information are used as RRT advisors on the expansion process, are proposed by [112]: a boundary bias planner; and a fuzzy bias planner. In the boundary bias, regions classes are defined given the neighborhood relation of the regions/cells of the search space with the nodes of the RRT. Regions with lesser tree nodes are more important to sampling. In the fuzzy bias, weight values are calculated to each region/cell by fuzzy rules based on collision information. Second the authors, these strategies reduces the RRT grow to local minima situations (samples on the search space with no possibilities of exploration) and induces a better exploration of the search space.

4) PATH-BIASED SAMPLING
Previous planned paths can be used to generate promises candidates samples to accelerate the convergence to better

paths during planning. Usually, the nodes of the previous paths are used to bias the sampling of new node candidates. The strategies proposed by the primary studies in this class are discussed below and listed in Tab. 7.

On [113] a local sampling strategy is proposed to Bi-RRT*. The proposed algorithm use three sampling strategies, where which one is executed based on a probability value. They are: bias in direction of the goal position; the standard uniform random sampling; and the local bias that is presented as follows. When a path is planned, the local sampling is executed to bias the tree grow. First, a node of the path is selected randomly. Thus, its parent node and child node are obtained. These two node are represented as vectors. Based on this representation, the vector that represents the middle point between the parent and child nodes is calculated. Finally, a new sample is generated from the selected node of the tree in direction of the middle point calculated by a random generated distance between maximal and minimum defined values. The authors argue that this strategy generate straighten paths than uniform random sampling, which means that optimized paths are generated faster.

On [114], an adaptive sampling scheme based on cross-entropy (CE) method [115], a global stochastic optimization, is proposed to the RRT* algorithm. In the proposed strategy, initially the optimal path is obtained without considerations about constraints. Thus, the optimal path is used to feed the probabilistic model, based on Gaussian mixture model, to indicate the regions with the lowest cost on the search space that generates constrained optimal paths to an autonomous system. Two versions of the RRT* algorithm with CE sampling are proposed. The first is the trajectory-cross-entropy RRT* (TCE), where a distribution is update with the bests parameters of trajectories with the lowest costs and better trajectories (instead of samples) are generated. The second is the state-cross-entropy (SCE) RRT*, where samples are collected from the search space directly by the distributions modified by cross-entropy method.

An approach based on intelligent sampling and path optimization called RRT*-Smart is proposed in [116]. This algorithm is applied to scenarios where path planning is executed until the optimal path (the shortest path) to a search space can

be encountered. The method works similar to the cost-based algorithm RRT*. However, since a path is found between the initial and final planning nodes, it is optimized by connecting directly visible nodes in the navigation environment. To optimize the length of the planned paths, new samples are collected near points named beacons, which are generated close to nodes previous planned paths. After the first path is encountered, the sampling process becomes based on these beacons. The new samples are spawned within balls centered on the beacons. According to the authors, the method accelerates the convergence time to find optimal solution and the path maintains lower length compared to a path generated by RRT*, considering the same pair of initial and final nodes.

On [81] are presented a combination of geometric planning and kinodynamic planning called Guided RRT. The algorithm has to layers: one were the RRT plan the tree representing geometric constraints and another that is sampled based on the nodes of the first one. The nodes of the kinodaynamic tree is sampled from the geometric tree nodes given a probability value p, otherwise its use a random sample. Each time a node on the geometric node is selected by the kinodaynamic sample, its selection probability is reduced. The Guided RRT algorithm sampling of the kinodynamic tree is biased by the nodes of the geometric tree that were less selected previously.

The sampling process proposed on [117] is based on a directed acyclic graph (DAG) of waypoints constructed by cell decomposition techniques and General Diagram Voronoi (GDV). The waypoints of the DAG are utilized to expand the RRT*. Two kinds of waypoints are maintained on the DAG: traversed and frontier. When one of the waypoints are added to the RRT, its ancestors are classified as traversed. The descendants of the waypoint is classified as frontier. Thus, the tree is expanded from the frontier with probability value than the traversed waypoints. Authors argues that expanding the tree by the frontier nodes optimize the cost of the planned path.

The algorithm proposed on [118], introduces a sampling strategy called biased path area sampling to the Transition-based RRT* (T-RRT*) algorithm, creating the BT-RRT* algorithm. The biased sampling starts after the first path is planned or when a path with lesser cost than the previous path is founded. The nodes of a planned path is used on the sampling bias. To each one of the path nodes, a sample is randomly generated around it given a radius distance value. If each sample optimizes the local cost of the path considering through possible connection with its neighbors, them it is added to the tree. The process optimizing the global cost of the path each time a path with lesser cost is founded by the tree.

The algorithm presented on [119] introduces the focused-refinement algorithm to bias the RRT* grow after a first path is planned. To a given planned path, the algorithm uses the maximal and minimum k-component value of the nodes of the path. These values define the interval from which a random uniform k-component sample are collected (for example, the x component of the sample). Next, after the first

component definition, another k-component (for example, y component of the sample) is defined based on the nearest k-component node from the first previously selected. Then, inside a range defined by the last k-component plus a shift value, the last component is randomly generated and finally a new sample is collected, defined by its all k-components. This sampling scheme is executed based on a constant value that defines its ratio of use with the random uniform sampling on all search space. The work yet presents two additional improvements to RRT*: The Goal tree and Grandparent-Connection Modifications. The first is a tree rooted on goal position to replan on scenarios with dynamical obstacle and the second is an optimization path process that try connects a node to it grandparent to reduces the quantity of nodes into a path.

5) NARROW PASSAGE-BIASED SAMPLING

This kind of sampling bias is used to overcome the RRT gap in explores difficult areas as narrow passages. Accelerating the exploration of RRT into these regions locally, has a positive effect on global planning time. The bias sampling strategies proposed by the works into this class are listed below and in Tab. 8.

The Interactive RRT in Contact (I-RRT-C) proposed on [120], is an extended version of the human-autonomous planning algorithm Iteration RRT (I-RRT) [121] to path planning on 3D search spaces. A new sample schema is added to the tree based on the contact with the obstacles. When the tree is near of an obstacle, the algorithm enter on the called contact mode. On this mode, a local tangent plane to the nearest obstacle is used to collect new samples. The samples are collected randomly over the tangent, allowing the RRT slide on the obstacles.

On [122], an algorithm based on RRT, called Retraction-based RRT, is proposed with optimization sampling on narrow passages and another in-contact situations. The algorithm utilizes a retraction based strategy when in a collision position, translating the in-collision samples to the boundaries of the obstacles, improving then the exploration on narrow passages. Initially, for a randomly generated in-colliding sample, its nearest node on the tree is selected. This node will be the initial guess for where the in-collision sample must be repositioned. In the narrow passage, the nearest selected node has a high probability to be near the boundary of an obstacle. So a new sample is generated by the optimization-based retraction also near the boundary of the obstacle. Finally, sample is added to the tree as a new node. This new node has probably the highest density of Voronoi, so its more probable to it be selected on the next iterations. This way, a sample created by the retraction operation is almost ever selected on the subsequent iterations, which to narrow passages accelerates the exploration of the tree through these regions of the search space.

In [123] the application of an optimized retraction-based technique to bias RRT grow on narrow passages was studied. The aim of the retraction-based approach is retract samples

TABLE 8. Evidence collected in systematic literature review process that fall on narrow passage-biased sampling class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[120]	IRRT, RRT-Connect	Contact algorithm sampling	I-RRT-C	Probabilistic complete	non-optimal	RRT-Connect, I-RRT
[122]	RRT	Retraction-based sampling	RRRT	Probabilistic complete	non-otpimal	RRT
[123]	RRT	Retraction-based techniques	SR-RRT	Probabilistic complete	Non-optimal	RRT, RRRT

TABLE 9. Evidence collected in systematic literature review process that fall on sampling by reduction of the search space class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[84]	RRT*	Hyperellipsoid-based sampling	Informed-RRT*	Probabilistic complete	non-optimal	RRT*
[124]	Informed-RRT*	Hyperellipsoid-based sampling	Wrapping-based Informed RRT* (WIRRT*)	Probabilistic complete	optimal	RRT*, Informed RRT*
[125]	Informed-RRT*	RRG batch sampling	BIT*	Probabilistic complete	optimal	RRT*, FMT*, Informed-RRT*, RRT-Connect.
[85]	RRT*	self-learning process	self-learning RRT*	Probabilistic complete	optimal	RRT, DD-RRT

that occur in the obstacle region of the search space to a free collision sample near or over the boundary of obstacles or the medial axis, which is useful on situations of narrow passages. The proposed algorithm based on this concept is called Selective Retraction-based RRT (SR-RRT). Two tests are proposed to identify narrow passages by SR-RRT: bridge line-test and non-colliding line-test. A bridge line-test consists in identify the existence of a narrow passage through a straight line when random samples are collected over the obstacle region of the search space. Thus, a line generated from the nearest-node of the samples is used to identify the narrow passage. If the corridor is detected, so the retraction operation is performed to collect a new sample over the boundaries of the obstacles. The non-colliding line-test records wide-open free spaces in the navigation environment and discard samples generated within them and to generate more samples outside those areas, which potentially towards RRT to narrow passages. Second the authors, this strategy bias new sample collection into direction of boundaries of the obstacles.

B. ADAPTATIVE SAMPLING
1) SAMPLING BY REDUCTION OF THE SEARCH SPACE
Many problems have a necessary restriction of maintain the exploration capabilities of the RRT while improving performance of planning. To allow this, many works proposed the reduction of search space region. Then, new samples could be collected just into this reduced space, disregarding samples that probably could not improve the quality of the path. Each strategy based on the reduction of the search space are discussed below and listed in Tab. 9.

The Informed-RRT* [84] algorithm reduces the search space based on the cost of the planned path. This cost defines the size of a hyperellipsoidal subspace, from where new samples are collected. The authors affirm that this ellipsoidal

region has a higher probability to contain the samples that generates the best paths. The size of the ellipsoidal region is reduced if the reduction of the path cost.

On [124] is proposed a modified version of the Informed-RRT* algorithm called Wrapping-based Informed RRT* (WIRRT*). A wrapping process is applied to accelerate the reduction of the ellipsoidal region that limits the sampling space. The wrapping process optimize a planned path replacing its intermediate nodes by ones that is wrapped onto the obstacles. Consequently, the cost of the path is optimized and the size of the ellipse, that is dependent of this cost, is also reduced.

Based on generation of the hyperellipsoide sub region on Informed-RRT*, the Batch Informed Tree (BIT*) is proposed on [125]. Initially, a Random Geometric Graph (RGG) is defined by uniformly distributed random samples on the non-collision region of the search space. A tree that connects the initial position and the final position of planning is heuristically searched over the RGG. The resulted tree is used as a batch to generate an initial ellipsoidal region that is a subregion of the search space that theoretically contains the bests samples to optimize the path. Thus, the process is restarted with a lesser subregion formed by the current tree cost. Each time a tree with better cost is planned the new batch is formed. The process continues as planning time allows. This way, optimized paths are generated faster than others planning algorithms based on RRT.

The algorithm presented on [85] introduces a self-learning of the search space to adapt the sampling depending on if the region being explored it is a difficult one or an open one where goal bias is more promising. The collision check process give information about the distribution of the obstacles regions into the search space. To achieve this, each node of the tree has a disc containing several sectors with information based on visibility of the search space generated by a virtual sensors

TABLE 10. Evidence collected in systematic literature review process that fall on bias by sampling distribution class.

Reference	RRT Type	Sampling	Solution	Completeness	Optimality	Comparison
[126]	RRT	Gaussian-based sampling distribution bounded by obstacles visibility.	Localized random sampling based RRT	Probabilistic complete	non-optimal	RRT, Distribution-based RRTs
[127]	RRT	Adaptive sampling based on Gaussian distribution	RRT with Adaptive Bias	Probabilistic complete	non-optimal	RRT
[128]	RRT	Adaptive sampling by probabilistic density function changing	enhanced-RRT	Probabilistic complete	non-optimal	Uniform RRT, RRT with Medial Bias, RRT with Heavy Bias
[129]	RRT	Poisson-disk sampling	Poisson-RRT	Probabilistic complete	Non-optimal	RRT-Connect, Lazy-RRT, pRRT (parallel processing)
[130]	RRT#	Adaptive Sampling Strategy	No name	Probabilistic complete	optimal	none
[131]	RRT	Gaussian Mixture Model (GMM)	GMM-based RRT	complete	non-optimal	RRT, Bi-directional RRT
[86]	RRT*	Ant colony optimization	ACO-RRT*	Probabilistic complete	non-optimal	ACO-RRT, RRT*, RRT
[132]	RRT*	Gaussian distribution sampling	RRT* with Gaussian distribution sampling	Probabilistic complete	optimal	RRT*
[133]	Informed-RRT*	Markov Chain Monte Carlo-based Informed Sampling (MCMC)	Informed-RRT* based on MCMC	Probabilistic complete	optimal	none

over the node. Each sector is classified based if is identified by its associated sensor an obstacle inside its range. The sectors that has visibility of an obstacle have its range of visibility reduced. Also, the sector in direction of the goal position is more important than others on the sample process. The sampling by the proposed sampling scheme starts by collecting a random sample and finding its nearest node on the tree. Thus, the disc of this node is used to collect a biased sample. Each sector has a probability chance of be selected. Once a sector is selected, a node is randomly selected inside of its area. Second the authors, is expected that with the self-learning sampling the tree can explore the search space more widely at the beginning of the planning and then improve the branches more finely later.

2) BIAS BY SAMPLING DISTRIBUTION

Traditionally, the RRT algorithm randomly selects a new sample from the search space following a uniform distribution. Thus, each position of the search space has equal probability to be collected. The strategies in this class modify the sampling distribution to one that allows the collection of more promising samples to include in the path. Usually, a modified distribution can be defined since the planning starts or adaptive distribution can be changed based on previous collected samples, increasing the efficiency of the sampling process as the RRT grows. The strategies observed to fall into this sampling class are listed below and their data are in Tab. 10.

In [126] is proposed a visibility-based RRT to evaluate open spaces in a cluttered search space. A Gaussian-based random distribution is considered to generate random samples around the surfaces of the obstacles. The distribution is affected by the quantity of obstacle visible by the nodes of the tree. With the expansion of RRT, eventually more obstacle

are "visible" by its nodes, changing the sampling distribution accordingly. The algorithm also use sampling biased by the goal position.

An adaptive scheme is proposed by [127] to change the uniform sampling of the RRT algorithm. The adaptive sampling algorithm begins with a biased search scheme and then, if the tree is growing rapidly, maintain or increase the bias. If the growth rate slows, a more uniform sampling strategy is used. If the tree growth rate is fast, so the samples are collected toward the unsafe regions. The level of uniformity is adjusted by changing the standard deviation of a Gaussian distribution. Then, the modified distribution is used to generate random samples to the tree. With the adaptive sampling strategy, paths are generated faster than using strictly uniform sampling.

A enhanced version of RRT algorithm is proposed on [128]. Its informed sampling generate a new adaptive biasing algorithm controlled by a factor that depends on how often a collected sample is successful in extends the tree by less costly edges. The quantity of iterations performed on the planning defines when a Gaussian-like density function is modified by this factor. Thus, subsequent samples are collected given the recently modified distribution. Second the authors, by this probability density function changing scheme the adaptive biasing algorithm improves the efficiency of RRT method compared to other fixed bias strategies rather dramatically.

An approach based on sampling Poisson disks is proposed in [129]. In this approach a pre-computed step is considered, where random samples (discs centers) are collected given a Poisson-disk distribution. The discs are sampled in a nested and tightly way, separated by a minimum distance r between the disk centers. The centers of the Poisson disks are used to

expand the RRT. This methodology has the objective to get maximum sampling process, which means cover navigation environment completely by Poisson disks. The authors state that this new approach reduces the number of nodes in the tree and can be implemented in parallel processing with multiple threads. Also, was proposed on the work an on-construction RRT sampling scheme to increase the ratio of samples nears challenging areas of the navigation space, like narrow passages.

On [130], a discovery approach of the search space is proposed based on machine learning. The learned information about the search space is used on an adaptive sampling strategy and applied to RRT# [134]. The strategy works as follows. A new sample is collected from the search space based on uniform distribution. Thus, if the sample is collision free or not, class density functions are approximated by a classifier based on these collected samples. Thus, the density functions (updated on each iteration) are used to determine the position of the new samples added to the tree. As result, the probability of generating free collision samples are higher than on uniform random distribution.

A Gaussian Mixture Model (GMM) is proposed as sampling process to RRT on [131]. GMM learns about collision region of the search space by a clustering approach based on the previous collected samples from which a distribution can be estimated. Two GMMs are used: one to represents the on collision samples; another to represents free collision samples. The difference between then is used to estimate collision positions. This way, the sampling by GMM has a higher probability to generate samples on free region of the search space, which can reduce the number of collision checks and reduces the planning time. The GMM is updated on each iteration of the RRT.

The algorithm presented on [86], called Ant Colony Optimization RRT (ACO-RRT), determines the optimal sampling distribution through ant colony optimization. The presented strategy works as follows. Initially, virtual ants are distributed according to its relevance. Based on the current exploration of the RRT*, these ants are updated, which modify the sampling distribution of the algorithm. So based on this estimated distribution, new nodes are sampled to expand the tree. Next, their utility to improves the path quality is evaluated based on the optimization of the current solution and exploration of the search space to find new, better solution. With these new information the ants are updated and a new distribution are estimated. This process follows until optimizing the capacity of exploration of the RRT* and reduces the cost path faster than others RRT-based algorithms.

On [132], the uniform random sampling process of RRT is replaced by a Gaussian sampling process of the search space. The mean value of the Gaussian distribution is set almost close to the target position, making the sampling process collect new samples on the goal region. The standard deviation of the Gaussian distribution determines the effectiveness of the proposed algorithm.

The algorithm proposed on [133] is based on Informed RRT*. In the initial steps of the planning, initially a first sample is collected uniformly over the informed region of the space search (defined by an n-dimensional hiperellipsoid). From there, sampling procedure follows over the same informed region based on a Markov Chain Monte Carlo (MCMC) process. The MCMC generate samples given a target distribution that is estimated based on previous collected samples and the cost that defines the informed region (a characteristic of Informed-RRT*). The authors propose that using MCMC reduces much more quickly the costs of the planned paths on higher dimensions of the search space.

C. CONSIDERATION ABOUT SLR RESULTS

The description of the selected studies responds the Question 1 of the SLR. Question 2 is answered using data from primary studies. The collected data show that the RRT and its RRT* variant are the principal bases considered in the primary studies on the use of non-uniform/informed strategies in the sampling process. However, more recent RRT-based are considered too, as Informed-RRT*, BIT*, RRT-ConCon, etc. Possibly, there would be a variant method of an already altered RRT-based algorithm, but with modification not in it sampling process but in another of its processes, as in the case with the RRT*, which introduces costs based reduction on length of planned path for the insertion of a new node. However, they were removed from the results, primarily because of scope of the scientific question of this review.

It is feasible to state that there are research opportunities within the topic addressed. A considerable quantity of works with RRT-based with non-uniform/informed samplings were selected on the SLR, showing the evolution of the topic on the last 14 years (considering the oldest works identified on the SLR, [126] and [83], that are from the 2004 year). However, 53 works is not yet a quantity that could define a topic as widely studied. Thus, could be inferred that the question of sampling on RRT-based algorithms is yet on preliminary discussion. Thus, this review reinforced the existence of research opportunities about this topic.

VIII. CONCLUSION

In this paper, a review of non-uniform/informed sampling strategies for RRT-based algorithms was presented using a review technical called Systematic Literature Review (SLR). Quite popular in the areas of Medicine and Software Engineering, this review methodology uses a research protocol to guide the review process. Its purpose is to reduce the revision bias inherent to the traditional revision approach, where the literature works in a given area are selected by the researcher's experience and can lead to subjective and non-objective selection.

First, two scientific questions were developed, outlining the scope of the review. The research protocol was developed together with a team of expert researchers. Based on it, the primary studies were consulted in several academic search engines. The works returned by each search engine

went through the selection process according to the inclusion and exclusion criteria defined in the research protocol.

A total of 1136 studies were selected, of which 53 were identified claiming to contain solution within the topic addressed. The extracted data from them may help in new research projects on the development of new RRT-based algorithms applied to path planning.

In future work, a study of the methods found in the review can be done for applications on the path planning for Unmanned Aerial Vehicles (UAVs). This type of autonomous vehicle is usually controlled by systems that produce responses in real time, which requires a fast enough path planning method to do so. As found in the review, the non-uniform/informed strategies on sampling process shows the promising increase in planning efficiency by RRT, which could initiate research about better path planning time for UAVs, as others applications. Also, a comparative study of the solutions proposed on the selected works on the SLR could give insights over the efficiency of the different non-uniform/informed sampling strategies on RRT-based algorithms.

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