IEEE Access
Multidisciplinary : Rapid Review : Open Access Journal

Received March 10, 2019, accepted March 22, 2019, date of publication March 27, 2019, date of current version April 11, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2907778

Optimal Control Approach to Advertising Strategies of a Supply Chain Under Consignment Contract

ZHIHUI WU
[1]Department of Mathematics, Harbin University of Science and Technology, Harbin 150080, China
[2]Heilongjiang Provincial Key Laboratory of Optimization Control and Intelligent Analysis for Complex Systems, Harbin University of Science and Technology, Harbin 150080, China
e-mail: wuzhihui@hrbust.edu.cn

This work was supported by the Fundamental Research Foundation for Universities of Heilongjiang Province of China under Grant LGYC2018JC007.

ABSTRACT The retailer's platform goodwill and the manufacturer's brand goodwill are important factors that influence the consumer's purchasing behavior under a consignment contract. In order to examine these effects on firm's decisions under different channel structures, a supply chain, including a single manufacturer and a single retailer, is considered, where the retailer provides the sales platform and the manufacturer sells the product through the retailer's sales platform under the consignment contract. By constructing a price-dependent and goodwill-dependent demand and using the differential game theory, the optimal equilibrium strategies are obtained under the decentralized and centralized structures. Our results demonstrate that the relationship of retail prices under two channel structures depends on the share of the revenue, and the centralized scenario could not always lead to a higher advertising effort. Subsequently, the decentralized supply chain is coordinated by designing the linear goodwill-dependent contract. Finally, the effects of the share of revenue and the effectiveness of advertising efforts on the equilibrium strategies as well as profits are illustrated by some simulations.

INDEX TERMS Optimal control, consignment contract, brand's goodwill, platform's goodwill, advertising, supply chain coordination.

I. INTRODUCTION

During the past two decades, a great deal of effort has been devoted on the supply chain management, see e.g., inventory, advertising, pricing, contract choice [1]–[4]. In particular, the consignment contract has been widely adopted in many online sale platforms, such as Amazon.com, JD.com, the mobile platforms with "Apps", and so on [5], [6]. Accordingly, these platforms themselves offer the consumers with goods available. Meanwhile, the platform owner usually invites various third-party suppliers to sell their products through the platforms, where the platform owner would deduct a certain percentage from the sales price when the products are sold. For instance, Amazomn.com charges 6–20% of the sales price only when the third-party supplier's products are sold [7], and iTunes App Store keeps 30% of the revenues from the related Apps [8]. Moreover, most enterprises indeed face the threat of severe competition, shorter product lifetime and dynamic yet complex market. In this case, the platform owner and the third-party suppliers usually invest in the advertising in order to promote their business goodwill (platform goodwill and brand goodwill) and attract the customers. Hence, we aim to investigate the pricing and advertising equilibrium strategies for supply chain under consignment contract, where the platform goodwill can affect the brand goodwill.

In recent years, the consignment contract has attracted increasing research interests [9]–[13]. In [14], the definition of relative strength of preference between consignment and wholesale contracts during the contract choice has been given, and the effect from many factors (e.g. cost share, price elasticity, and price markup) onto the strength of contract preferences has been discussed. In [15], the inventory risk has been compared under the wholesale/consignment contracts,

The associate editor coordinating the review of this manuscript and approving it for publication was Ton Do.

and the impact of a financial constraint on efficiency of supply chain has been discussed. In [16], the contract preferences of the two players have been discussed by considering the dynamic effect of the advertising under the consignment and wholesale contracts. On the other hand, both the vendor-managed consignment inventory (VMCI) and the retailer-managed consignment inventory (RMCI) have been widely compared in the literature. It should be pointed out that the channel performance under VMCI is more efficient as mentioned in [11]. Subsequently, the customer return behavior has been taken into account in [17], and it has been revealed that the VMCI is still advantageous for the channel performance even though the full-refund policy is allowed. In [18], the consignment contracts with bonus or side payment have been examined, the results showed that they can better coordinate the addressed supply chain. When the supply chain subjects to uncertainty and stochastic demand, both the advance-purchase discount (APD) contract and the revenue sharing contract can achieve the coordination [19]. In [20], the pricing and investment of shelf-space strategies in a two-echelon supply chain have been studied and the supply chain has been coordinated by providing the two-part tariff contract. Furthermore, how to enhance the channel efficiency under a dynamic setting has been discussed in [21] by employing a discrete-time model. However, it should be noted that although the managerial implications have been analyzed in the above mentioned references, it is additionally necessary to discuss the framework for supply chain under consignment contract with the dynamics of platform goodwill. The major reason is that the platform goodwill is important factor influencing the consumer purchasing behavior in the environment of consignment sale.

Another research stream of related literature studies the advertising's effect on the goodwill, and increasing research attention has been devoted concerning on the advertising strategy problems [22]–[26]. For example, the brand goodwill of channel members has been viewed as an important influential factor on product demand in [27], in which the channel members have invested their own advertising effort to establish the brand goodwill. In addition, in order to increase the demands and brand goodwill, both long term and short term advertising efforts have been considered by players in [28]. Motivated by [28], subsequently, the advertising strategies with competition between the national brand and private brand have been studied in [29]. Moreover, the advertising strategies through the mobile platform have been given in [30] by building the differential game model, where the results showed that the owner can obtain more profits under certain conditions if it participates the apps' advertising. When the competition becomes a concern, the frameworks have been proposed in [31], [32] to discuss the "multiple manufacturer vs single retailer" and "single manufacturer vs multiple retailer". For example, the negative effects from the competitor's advertising onto the brand goodwill have been considered in [33], where a differential game incorporating goodwill dynamics of manufacturer brands has been established. Furthermore, a non-cooperative differential game has been considered and the Feedback Stackelberg equilibrium strategies have been given in [34], where a retailer sells manufacturer's brand as well as own private label and each brand's goodwill obeys the modified Nerlove-Arrow model. Regarding the contract coordination issue considering the dynamics of goodwill, the two-part tariff contract and the committed dynamic transfer price contract have been discussed in [35], [36]. Nevertheless, it is worthwhile to notice that the above results have focused mainly on the traditional supply chain without taking the consignment contract into account. In addition, most of the existing results have considered the case that the advertising plays an important role on increasing the goodwill level, but ignored the fact that the platform's goodwill can positively affect the brand's goodwill, which constitutes one of the current motivations.

Based on the above discussions, we aim to discuss the advertising strategies for a supply chain under consignment contract with the retailer's platform goodwill and the manufacturer's brand goodwill. The major contributions of the paper can be listed: 1) by adopting a differential game framework, our model captures the effect from retailer's platform goodwill onto manufacturer's the brand goodwill clearly; 2) the equilibrium strategies of supply chain under consignment contract within decentralized and centralized scenarios are given by taking the dynamic of platform retailer's goodwill and manufacturer's brand goodwill into account; and 3) a contract with goodwill-dependent fees is designed, which could coordinate the decentralized supply chain under consignment contract within the dynamic environment. At last, we use a simulation example to depict the effects of the share of the revenue and the effectiveness of advertising efforts on the equilibrium strategies as well as profits.

The remainder of the paper is listed as follows. In Section II, we develop a differential game model by considering the dynamics of platform goodwill and brand goodwill for the supply chain under the consignment contract. In Sections III and IV, we derive the equilibrium strategies under the decentralized and centralized scenarios. In Section V, we design a linear goodwill-dependent contract, which is used to coordinate the decentralized supply chain. In Section VI, some numerical analyses are carried out to gain more managerial insights. In Section VII, we conclude the paper.

II. THE PROBLEM DESCRIPTION AND THE MODEL DEVELOPMENT

In this paper, we consider a two-echelon supply chain consisting of a retailer and a manufacturer, where the retailer owns a sale platform with higher goodwill and the manufacturer sells own brand product through the retailer's platform. The retailer offers a consignment contract with revenue sharing to the manufacturer, which specifies that the retailer keeps ϕ share of the retail price for per unit of the product sold. In particular, let c_M be the manufacturer' constant unit production cost and c_R be the retailer' unit handling cost for selling the product to the consumer. Hence, we can use $c = c_M + c_R$ to represent the total unit cost for the channel and $\alpha = \frac{c_R}{c}$ to depict the share of the channel cost for a retailer.

We consider the dynamic characteristics of the retailer's platform goodwill $E(t)$, which is given by:

$$\dot{E}(t) = \varepsilon I(t) - \delta E(t), \quad E(0) = E_0, \tag{1}$$

where $I(t)$ depicts the advertising effort at time t, which is made by the retailer in order to improve the platform goodwill, ε is the effectiveness of retailer's advertising effort, δ stands for the decay rate, and E_0 denotes the initial retailer's platform goodwill.

The retailer's platform goodwill could enlarge the manufacturer's product demand, the reason is that the loyal users of the platform are potential consumers for manufacturer. That is to say, the retailer's platform goodwill always plays an important role in accumulating the manufacturer's brand goodwill. As such, we assume that the retailer's platform goodwill is a manufacturer's brand goodwill-building factor. Introducing this effect of the retailer's platform goodwill $E(t)$ on manufacturer's brand goodwill $G(t)$, the Nerlove and Arrow model is modified as

$$\dot{G}(t) = \beta A(t) + \theta E(t) - \tau G(t), \quad G(0) = G_0, \tag{2}$$

where $A(t)$ is the advertising effort at time t introduced by the manufacturer with hope to improve the product goodwill, the parameter β depicts the effectiveness of manufacturer's advertising effort, θ denotes the effectiveness of the retailer's platform goodwill on the brand goodwill, τ represents the decay rate, and G_0 stands for the initial product goodwill. As in [30], the quadratic advertising cost functions with respect to the advertising effort are utilized, i.e., $C(A(t)) = \frac{1}{2}k_M A^2(t)$ and $C(I(t)) = \frac{1}{2}k_R I^2(t)$.

According to [37], the product demand is dependent on the retailer's platform goodwill $E(t)$, the manufacturer's brand goodwill $G(t)$, and the retail price of product $p(t)$ in a separable multiplicative way, i.e.,

$$D(t) = (a - bp(t))(\eta E(t) + \gamma G(t)), \tag{3}$$

where a depicts the market capacity, b denotes the price sensitivity of demand. In addition, η and γ represent the effectiveness of the retailer's platform goodwill and the product goodwill on demand.

For an infinite time horizon and a common discount rate ρ, the following objective functions of the retailer and the manufacturer are given:

$$J_R = \int_0^{+\infty} e^{-\rho t}[(\phi p(t) - c_R)D(t) - C(I(t))]dt, \tag{4}$$

$$J_M = \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t) - c_M)D(t) - C(A(t))]dt. \tag{5}$$

Then, the objective functional of the whole supply chain is the sum of the objective functions of the retailer and the manufacturer, i.e.,

$$J_C = \int_0^{+\infty} e^{-\rho t}[(p(t) - c_M - c_R)D(t) - C(I(t)) - C(A(t))]dt. \tag{6}$$

III. THE OPTIMAL STRATEGIES UNDER THE CENTRALIZED SCENARIO

Under the centralized scenario, the retailer and the manufacturer can be seen as a whole system. Therefore, we are in a position to maximize the objective functional of whole supply chain by properly determining the retail price p and advertising efforts A as well as I.

Theorem 1: Under the centralized channel structure, the optimal retail price is given by:

$$p^{C*} = \frac{a + bc}{2b}, \tag{7}$$

the optimal advertising efforts of the manufacturer and retailer can be described as:

$$A^{C*} = \frac{\beta\gamma(a - bc)^2}{4bk_M(\rho + \tau)}, \tag{8}$$

$$I^{C*} = \frac{\varepsilon(a - bc)^2}{4bk_R(\rho + \delta)}\left(\eta + \frac{\gamma\theta}{\rho + \tau}\right). \tag{9}$$

Furthermore, the optimal trajectories of the retailer's platform goodwill and manufacturer's brand goodwill are

$$E^C(t) = (E_0 - E_{SS}^C)e^{-\delta t} + E_{SS}^C, \tag{10}$$

$$G^C(t) = (G_0 - G_{SS}^C)e^{-\tau t} + G_{SS}^C, \tag{11}$$

where

$$E_{SS}^C = \frac{\varepsilon^2(a - bc)^2}{4b\delta k_R(\rho + \delta)}\left(\eta + \frac{\gamma\theta}{\rho + \tau}\right),$$

$$G_{SS}^C = \frac{(a - bc)^2}{4b\tau}\left(\frac{\gamma\beta^2}{k_M(\rho + \tau)} + \frac{\theta\eta\varepsilon^2}{\delta k_R(\rho + \delta)} + \frac{\gamma\theta^2\varepsilon^2}{\delta k_R(\rho + \delta)(\rho + \tau)}\right),$$

here E_{SS}^C and G_{SS}^C corresponds to the steady state of the platform goodwill and brand goodwill, respectively.

Proof: The optimization problem for the centralized channel is an optimal control problem, which can be described by:

$$\max_{p>0, I>0, A>0} J_C = \int_0^{+\infty} e^{-\rho t}[(p(t) - c_M - c_R)D(t) - C(I(t)) - C(A(t))]dt$$

$$\text{s.t. } \dot{E}(t) = \varepsilon I(t) - \delta E(t), \quad E(0) = E_0,$$
$$\dot{G}(t) = \beta A(t) + \theta E(t) - \tau G(t), \quad G(0) = G_0.$$

The current-value Hamiltonian is:

$$H_C = (p(t) - c)(a - bp(t))(\eta E(t) + \gamma G(t)) - \frac{1}{2}k_R I^2(t) - \frac{1}{2}k_M A^2(t) + \lambda_1^C(\varepsilon I(t) - \delta E(t)) + \lambda_2^C(\beta A(t) + \theta E(t) - \tau G(t)), \tag{12}$$

where λ_i^C $(i = 1, 2)$ are costate variables. Then, it follows from the necessary conditions of maximum principle that

$$\frac{\partial H_C}{\partial p} = 0, \tag{13}$$
$$\frac{\partial H_C}{\partial I} = 0, \tag{14}$$
$$\frac{\partial H_C}{\partial A} = 0, \tag{15}$$
$$\frac{\partial H_C}{\partial E} = \rho\lambda_1^C - \dot{\lambda}_1^C, \tag{16}$$
$$\frac{\partial H_C}{\partial G} = \rho\lambda_2^C - \dot{\lambda}_2^C. \tag{17}$$

Next, it follows from (13)–(15) that

$$p = \frac{a + bc}{2b}, \tag{18}$$
$$I = \frac{\varepsilon}{k_R}\lambda_1^C, \tag{19}$$
$$A = \frac{\beta}{k_M}\lambda_2^C. \tag{20}$$

Substituting (18) into (16)–(17), we have

$$\dot{\lambda}_1^C = (\rho + \delta)\lambda_1^C - \theta\lambda_2^C - \frac{\eta(a - bc)^2}{4b}, \tag{21}$$
$$\dot{\lambda}_2^C = (\rho + \tau)\lambda_2^C - \frac{\gamma(a - bc)^2}{4b}. \tag{22}$$

Solving the above differential equations and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_i^C(t) = 0$ $(i = 1, 2)$, one has

$$\lambda_1^C = \frac{\gamma(a - bc)^2}{4b(\rho + \tau)}, \tag{23}$$
$$\lambda_2^C = \frac{(a - bc)^2}{4b(\rho + \delta)}\Big(\eta + \frac{\gamma\theta}{\rho + \tau}\Big). \tag{24}$$

Substituting (23)–(24) into equations (19)–(20), the optimal advertising strategies of retailer and manufacturer under the centralized channel structure can be given. Furthermore, by using the models (1)–(2), the optimal trajectories of the retailer's platform goodwill and the manufacturer's brand goodwill are obtained, which ends the proof of this theorem.

From Theorem 1, it is not difficult to obtain the optimal profit of supply chain described by:

$$\begin{aligned} J_C^* = {} & \frac{\eta(a - bc)^2}{4b(\rho + \delta)}E_0 + \frac{\eta\delta(a - bc)^2}{4\rho b(\rho + \delta)}E_{SS}^C \\ & + \frac{\gamma(a - bc)^2}{4b(\rho + \tau)}G_0 + \frac{\gamma\tau(a - bc)^2}{4\rho b(\rho + \tau)}G_{SS}^C \\ & - \frac{k_R(I^{C*})^2}{2\rho} - \frac{k_M(A^{C*})^2}{2\rho}. \end{aligned} \tag{25}$$

Moreover, we can obtain the following result.

Proposition 1: Under the centralized channel structure, the impacts of key parameters on the optimal strategies can be given by:

(1) $\frac{\partial p^{C*}}{\partial b} < 0, \quad \frac{\partial A^{C*}}{\partial b} < 0, \quad \frac{\partial I^{C*}}{\partial b} < 0,$

(2) $\frac{\partial A^{C*}}{\partial \beta} > 0, \quad \frac{\partial A^{C*}}{\partial \gamma} > 0, \quad \frac{\partial A^{C*}}{\partial k_M} < 0,$

(3) $\frac{\partial I^{C*}}{\partial \varepsilon} > 0, \quad \frac{\partial I^{C*}}{\partial \eta} > 0, \quad \frac{\partial I^{C*}}{\partial \gamma} > 0,$
$\frac{\partial I^{C*}}{\partial \theta} > 0, \quad \frac{\partial I^{C*}}{\partial k_R} < 0.$

It follows from Proposition 1 that some managerial implications can be obtained. Firstly, when the price impact becomes higher (i.e., a higher b), the manufacturer will decrease the retail price and invest less in the advertising effort. Similarly, the retailer should invest less in advertising effort when b becomes higher. Secondly, it should be noted that the advertising efforts of the manufacturer/retailer are increasing functions with respect to the effectiveness of the manufacturer's brand goodwill (i.e., γ), which mean that both the manufacturer and retailer need to invest more advertising efforts when the effectiveness of the retailer's platform goodwill onto the market demand becomes higher (i.e., a higher γ). Thirdly, the retailer will invest more advertising effort as the effectiveness of the platform's goodwill becomes higher (i.e., a higher η). Moreover, a higher effectiveness of the retailer's platform goodwill on the manufacturer's brand goodwill will result in the fact that the retailer should invest more advertising efforts (i.e., a higher θ).

IV. THE OPTIMAL STRATEGIES UNDER THE DECENTRALIZED SCENARIO

In the decentralized scenario, the retailer and the manufacturer should make their own decisions in order to maximize their profits, where the retailer is the Stackelberg leader. Moreover, the sequence of events is described as: the retailer declares the advertising effort $I(t)$ firstly, and then the manufacturer decides the retail price of product $p(t)$ and the advertising effort $A(t)$ based on the retailer's decision.

Theorem 2: According to the decentralized channel structure, the optimal retail price is given by:

$$p^{D*} = \frac{(1 - \phi)a + (1 - \alpha)bc}{2b(1 - \phi)}, \tag{26}$$

the optimal advertising efforts of the manufacturer and retailer are:

$$A^{D*} = \frac{\beta\gamma((1 - \phi)a - \alpha bc)^2}{4bk_M(1 - \phi)(\rho + \tau)}, \tag{27}$$
$$\begin{aligned} I^{D*} = {} & \frac{\varepsilon\big(\eta + \frac{\gamma\theta}{\rho+\tau}\big)}{4bk_R(\rho + \delta)(1 - \phi)^2}((1 - \phi)a - (1 - \alpha)bc) \\ & \times(\phi(1 - \phi)a + ((1 + \alpha)\phi - 2\alpha)bc). \end{aligned} \tag{28}$$

Furthermore, the optimal trajectories of the retailer's platform goodwill and manufacturer's brand goodwill are:

$$E^D(t) = (E_0 - E_{SS}^D)e^{-\delta t} + E_{SS}^D, \tag{29}$$
$$G^D(t) = (G_0 - G_{SS}^D)e^{-\tau t} + G_{SS}^D, \tag{30}$$

where

$$E_{SS}^{D} = \frac{\varepsilon^2((1-\phi)a-(1-\alpha)bc)(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc)}{4b\delta k_R(\rho+\delta)(1-\phi)^2}(\eta+\frac{\gamma\theta}{\rho+\tau}),$$

$$G_{SS}^{D} = \frac{1}{4b\tau}\times\left(\frac{\theta\varepsilon^2((1-\phi)a-(1-\alpha)bc)(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc)}{\delta k_R(\rho+\delta)(1-\phi)^2}\times(\eta+\frac{\gamma\theta}{\rho+\tau})+\frac{\gamma\beta^2((1-\phi)a-\alpha bc)^2}{k_M(1-\phi)(\rho+\tau)}\right),$$

here, E_{SS}^{D} and G_{SS}^{D} corresponds to the steady state of the retailer's platform goodwill and the manufacturer's brand goodwill.

Proof: According to the backward induction method, we need to derive the manufacturer's retail price of product $p(t)$ and the advertising effort $A(t)$ when the retailer's advertising effort $I(t)$ is given. The optimal problem of the manufacturer can be given by:

$$\max_{p>0,A>0} J_M = \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t)-c_M)D(t)-C(A(t)]dt$$
$$\text{s.t. } \dot{E}(t)=\varepsilon I(t)-\delta E(t),\quad E(0)=E_0,$$
$$\dot{G}(t)=\beta A(t)+\theta E(t)-\tau G(t),\quad G(0)=G_0.$$

In view of the differential game theory, the following current-value Hamiltonian is introduced for the manufacturer:

$$H_M = ((1-\phi)p(t)-c_M)(a-bp(t))(\eta E(t)+\gamma G(t)) -\frac{1}{2}k_M A^2(t)+\lambda_1^M(\varepsilon I(t)-\delta E(t)) +\lambda_2^M(\beta A(t)+\theta E(t)-\tau G(t)), \quad (31)$$

where λ_i^M $(i=1,2)$ are costate variables.

From the necessary conditions of maximum principle, it follows that:

$$\frac{\partial H_M}{\partial p}=0, \quad (32)$$
$$\frac{\partial H_M}{\partial A}=0, \quad (33)$$
$$\frac{\partial H_M}{\partial E}=\rho\lambda_1^M-\dot{\lambda}_1^M. \quad (34)$$
$$\frac{\partial H_M}{\partial G}=\rho\lambda_2^M-\dot{\lambda}_2^M. \quad (35)$$

Equations (32)–(33) imply

$$p=\frac{(1-\phi)a+(1-\alpha)bc}{2b(1-\phi)}, \quad (36)$$
$$A=\frac{\beta}{k_M}\lambda_2^M. \quad (37)$$

Substituting (36) into (35), we have

$$\dot{\lambda}_2^M=(\rho+\tau)\lambda_2^M-\frac{\gamma((1-\phi)a-\alpha bc)^2}{4b(1-\phi)}. \quad (38)$$

Solving the above differential equation and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_2^M(t)=0$ yield

$$\lambda_2^M=\frac{\gamma((1-\phi)a-\alpha bc)^2}{4b(1-\phi)(\rho+\tau)}. \quad (39)$$

Therefore,

$$A^D=\frac{\beta\gamma((1-\phi)a-\alpha bc)^2}{4bk_M(1-\phi)(\rho+\tau)}. \quad (40)$$

Anticipating the manufacturer's response, the retailer's optimization problem is

$$\max_{I>0} J_R = \int_0^{+\infty} e^{-\rho t}[(\phi p(t)-c_R)D(t)-C(I(t)]dt$$
$$\text{s.t. } \dot{E}(t)=\varepsilon I(t)-\delta E(t),\quad E(0)=E_0,$$
$$\dot{G}(t)=\beta A(t)+\theta E(t)-\tau G(t),\quad G(0)=G_0.$$

Besides, the following current-value Hamiltonian is introduced for the retailer:

$$H_R = (\phi p(t)-c_R)(a-bp(t))(\eta E(t)+\gamma G(t)) -\frac{1}{2}k_R I^2(t)+\lambda_1^R(\varepsilon I(t)-\delta E(t)) +\lambda_2^R(\beta A(t)+\theta E(t)-\tau G(t)), \quad (41)$$

where λ_i^R $(i=1,2)$ are costate variables. Substituting (36) and (40) into (41), we have

$$H_R = \frac{(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc)}{4b(1-\phi)^2}\times((1-\phi)a-(1-\alpha)bc)(\eta E(t)+\gamma G(t)) -\frac{1}{2}k_R I^2(t)+\lambda_1^R(\varepsilon I(t)-\delta E(t))+\lambda_2^R\times\left(\frac{\beta^2\gamma((1-\phi)a-\alpha bc)^2}{4bk_M(1-\phi)(\rho+\tau)}+\theta E(t)-\tau G(t)\right), \quad (42)$$

Similarly, from the necessary conditions of maximum principle, it follows that

$$\frac{\partial H_R}{\partial I}=0, \quad (43)$$
$$\frac{\partial H_R}{\partial E}=\rho\lambda_1^R-\dot{\lambda}_1^R, \quad (44)$$
$$\frac{\partial H_R}{\partial G}=\rho\lambda_2^R-\dot{\lambda}_2^R. \quad (45)$$

Equation (43) implies

$$I=\frac{\varepsilon}{k_R}\lambda_1^R. \quad (46)$$

Solving the above differential equations (44)–(45) and according to transversality condition $\lim_{t\to\infty} e^{-\rho t}\lambda_i^R(t)=0$ $(i=1,2)$, one has

$$\lambda_1^R=\frac{((1-\phi)a-(1-\alpha)bc)}{4b(\rho+\delta)(1-\phi)^2}$$

$$\times(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc) \times\Big(\eta+\frac{\gamma\theta}{\rho+\tau}\Big), \quad (47)$$

Together with (36), (40) and (46)–(47), the optimal advertising effort of retailer under the decentralized decision can be given. Finally, by using the models (1)–(2), the optimal trajectories of the retailer's platform goodwill and the manufacturer's brand goodwill are obtained, which complete the proof of this theorem.

From Theorem 2, substituting p^{D*}, I^{D*} and A^{D*} into (4) and (5), the optimal profits of the retailer and the manufacturer can be obtained as:

$$\begin{aligned}J_R^{D^*} &= \frac{((1-\phi)a-(1-\alpha)bc)}{4b(1-\phi)^2} \\ &\quad\times(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc) \\ &\quad\times\Big(\frac{\eta}{\rho+\delta}E_0+\frac{\delta\eta}{\rho(\rho+\delta)}E_{SS}^D+\frac{\gamma}{\rho+\tau}G_0 \\ &\quad+\frac{\tau\gamma}{\rho(\rho+\tau)}G_{SS}^D\Big)-\frac{k_R(I^{D*})^2}{2\rho}, \quad (48)\\ J_M^{D^*} &= \frac{((1-\phi)a-(1-\alpha)bc)^2}{4b(1-\phi)^2}\Big(\frac{\eta}{\rho+\delta}E_0 \\ &\quad+\frac{\delta\eta}{\rho(\rho+\delta)}E_{SS}^D+\frac{\gamma}{\rho+\tau}G_0+\frac{\tau\gamma}{\rho(\rho+\tau)}G_{SS}^D\Big) \\ &\quad-\frac{k_M(A^{D*})^2}{2\rho}. \quad (49)\end{aligned}$$

Moreover, from Theorems 1-2, we can have the following proposition.

Proposition 2: Comparing with optimal strategies under the decentralized and centralized scenarios, we have

(1) $I^{C^*} > I^{D^*}$,

(2) $p^{C^*} < p^{D^*}$ *if* $\alpha < \phi < 1$,

(3) $A^{C^*} > A^{D^*}$ *if*

$$1-\Big(\frac{a-bc+\sqrt{(a-bc)^2+4\alpha ab}}{2a}\Big)^2 < \phi < 1.$$

Proposition 2 shows that the retailer's advertising effort under the centralized scenario is always higher than the one under the decentralized scenario, which means that the centralized decision-making is conducive to the establishment of platform goodwill. The manufacturer's advertising effort under the centralized scenario is not always higher than one of the decentralized scenario. When the share of revenue higher than a certain value, the manufacturer invest more advertising effort. When the revenue of share is higher than the retailer's share of the channel cost, the retail price under the decentralized scenario is higher than the one under the centralized case.

V. THE COORDINATION CONTRACT

As mentioned in [38], the state-dependent incentive scheme is usually used to improve the performance of decentralized supply chain. Hence, we are ready to design a novel state-dependent contract (ϕ, m, n, l) in order to coordinate the supply chain under consignment contract. As in [20], [39], the main idea is to introduce the slotting fees $f(E, G)$ (transfer from the manufacturer to the retailer), which linearly depends on the retailer's platform goodwill $E(t)$ and manufacturer's brand goodwill $G(t)$. We construct the following contract provisions. Firstly, the retailer announces a constant revenue share for each unit sold ϕ and a linear state-dependent slotting fees $f(E, G)$, i.e., $f(E, G) = mE(t) + nG(t) + l$, where m, n and l are constants. Secondly, the manufacturer decides the retail price $p(t)$ and the advertising effort $A(t)$ by maximizing its own profit, while the retailer decides the advertising effort $I(t)$ by maximizing its own profit.

Theorem 3: Under the contract (ϕ, m, n, l), the optimal strategies of channel members are given by:

$$p^{E*} = \frac{(1-\phi)a+(1-\alpha)bc}{2b(1-\phi)}, \quad (50)$$

$$A^{E*} = \frac{\beta}{k_M(\rho+\tau)}\Big(\frac{\gamma((1-\phi)a-\alpha bc)^2}{4b(1-\phi)}-n\Big), \quad (51)$$

$$I^{E*} = \frac{\varepsilon}{k_R(\rho+\delta)}\Big(\eta\Delta+m+\frac{\theta}{\rho+\tau}(\gamma\Delta+n)\Big), \quad (52)$$

where $\Delta = \frac{((1-\phi)a-(1-\alpha)bc)(\phi(1-\phi)a+((1+\alpha)\phi-2\alpha)bc)}{4b(1-\phi)^2}$.

Proof: Under the state-dependent contract (ϕ, m, n, l), we introduce the following objective functions for the retailer and the manufacturer:

$$\begin{aligned}J_R^E &= \int_0^{+\infty} e^{-\rho t}[(\phi p(t)-c_R)D(t)+mE(t)+nG(t) \\ &\qquad +l-C(I(t))]dt, \quad (53)\\ J_M^E &= \int_0^{+\infty} e^{-\rho t}[((1-\phi)p(t)-c_M)D(t)-mE(t) \\ &\qquad -nG(t)-l-C(A(t)]dt. \quad (54)\end{aligned}$$

Along the same line in the proof of Theorem 2, we can obtain the optimal strategies under the state-dependent contract (ϕ, m, n, l).

Theorem 4: If the contract (ϕ, m, n, l) with $l_1 < l < l_2$ is utilized, where

$$\begin{aligned}\phi &= \alpha,\\ n &= \frac{\gamma(((1-\phi)a-\alpha bc)^2-(1-\alpha)(a-bc)^2)}{4b(1-\alpha)},\\ m &= \frac{(1-\alpha)(a-bc)^2}{4b(\rho+\delta)}\Big(\eta+\frac{\gamma\theta}{\rho+\tau}\Big)-\frac{\theta n}{\rho+\tau},\\ \Delta_1 &= m\Big(\frac{E_0}{\rho+\delta}+\frac{\delta E_{SS}^C}{\rho(\rho+\delta)}\Big)+n\Big(\frac{G_0}{\rho+\tau}+\frac{\tau G_{SS}^C}{\rho(\rho+\tau)}\Big)\\ &\qquad+\frac{\alpha k_M(A^{C*})^2}{2\rho}-\frac{(1-\alpha)k_R(I^{C*})^2}{2\rho},\\ l_1 &= \rho(J_R^{D*}-\alpha J_C^*-\Delta_1),\\ l_2 &= \rho((1-\alpha)J_C^*-J_M^{D*}-\Delta_1),\end{aligned}$$

then the supply chain can be coordinated.

Proof: In order to coordinate the supply chain, the following conditions are met:

$$p^{C*} = p^{E*}, \quad A^{C*} = A^{E*}, \quad I^{C*} = I^{E*}.$$

From (7), (8), (50) and (51), we have

$$\phi = \alpha, \quad n = \frac{\gamma(((1-\phi)a - \alpha bc)^2 - (1-\alpha)(a-bc)^2)}{4b(1-\alpha)}.$$

Subsequently, according to $I^{C*} = I^{E*}$, one has

$$m = \frac{(1-\alpha)(a-bc)^2}{4b(\rho+\delta)}\left(\eta + \frac{\gamma\theta}{\rho+\tau}\right) - \frac{\theta n}{\rho+\tau}.$$

Then, the profits of the manufacturer and the retailer are given as follows:

$$J_M^{S*} = (1-\alpha)J_C^* - \Delta_1 - \frac{l}{\rho},$$
$$J_R^{S*} = \alpha J_C^* + \Delta_1 + \frac{l}{\rho},$$

where $\Delta_1 = m\left(\frac{E_0}{\rho+\delta} + \frac{\delta E_{SS}^C}{\rho(\rho+\delta)}\right) + n\left(\frac{G_0}{\rho+\tau} + \frac{\tau G_{SS}^C}{\rho(\rho+\tau)}\right) + \frac{\alpha k_M (A^{C*})^2}{2\rho} - \frac{(1-\alpha)k_R(I^{C*})^2}{2\rho}$.

On the other hand, both the retailer and the manufacturer will participate the implementation of this contract, only when the contract can make them better off. Subsequently, according to $J_R^{S*} > J_R^{D*}$ and $J_M^{S*} > J_M^{D*}$, we can get that both the retailer and manufacturer can make more profits if $l_1 < l < l_2$.

Theorems 3-4 show that the retailer needs to set constant revenue share for each unit sold based on the share of the channel cost when the supply chain is coordinated by the contract (ϕ, m, n, l). Furthermore, the equilibrium strategies under this contract are independent of parameter l and the profits of the two players depend on the parameter l, therefore they may allocate the profits through this parameter.

VI. NUMERICAL ANALYSIS

In this section, we provide a numerical example to discuss the following three cases: (1) the effects from share of the revenue ϕ on the profits under decentralized scenario; (2) the effects from the share of the revenue ϕ on the equilibrium strategies under both decentralized and centralized scenarios; and (3) the effects from key system parameters $\beta, \varepsilon, \eta, \theta$ and γ onto the steady state, the advertising efforts as well as the profits of supply chain. In the sequel, assume that the gap of profits among the centralized and decentralized scenarios is $\Delta J = J_C^* - J_R^{D*} - J_M^{D*}$. To address the above questions, the following values are chose: $\phi = 0.08$, $a = 5$, $b = 2$, $\eta = 0.75$, $\gamma = 0.5$, $\delta = 0.2$, $\tau = 0.3$, $\beta = 1$, $\varepsilon = 0.8$, $\rho = 0.3$, $c = 1$, $\theta = 0.5$, $\alpha = 0.1$, $k_R = 1$, $k_M = 1$, $E_0 = 2$ and $G_0 = 2$. The above parameter values are selected from previous references as in [30], [37] in order to provide comprehensive illustrations.

Firstly, we discuss the effects from the share of the revenue ϕ on the profits under the decentralized scenario. Fig. 1 shows that the share of the revenue affects the retailer's profit in a proper manner. Initially, the retailer's profit increases with the share of the revenue, but the retailer's profit will decrease when the share of the revenue reaches ϕ_R. That is to say, the retailer's profit achieves its maximum value

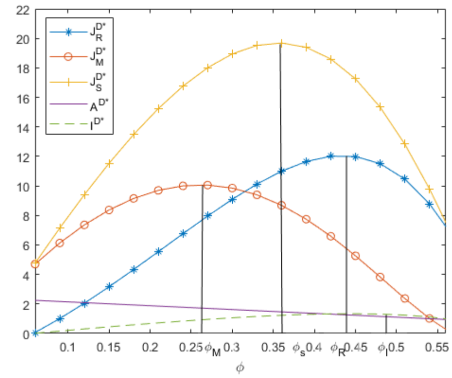


FIGURE 1. The impact of ϕ on the profits and decisions under the decentralized scenario.

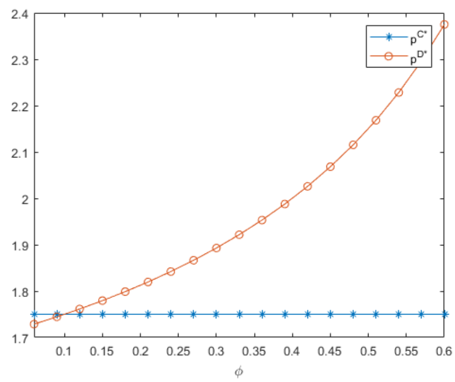


FIGURE 2. The impact of ϕ on the retail price p.

at ϕ_R, because a higher (lower) share of the revenue performs both positive and negative effects on the retailer's profit. To be more specific, a higher (lower) share of the revenue will lead to a higher (lower) retail price, which enhances (reduces) the profit margin and lower (increase) customer demand. In addition, the share of the revenue that makes the retailer invest maximum advertising effort (i.e., ϕ_I) does not bring the maximum profit to the retailer. Instead, a relatively low share of the revenue charged by the retailer (i.e., ϕ_R) can maximize the retailer's profit. It can be explained by the fact that a relatively lower share of the revenue charged by the retailer reduces the retailer's advertising effort, spurs the manufacturer to invest more in advertising effort, which enhance the profit of the retailer as well as the profits of the manufacturer and the whole supply chain. The manufacturer's profit reaches its maximum value at ϕ_M and the total profit of supply chain under decentralized scenario reaches its maximum value at ϕ_S, ($\phi_M < \phi_S < \phi_R$), which mean that the retailer charge a relatively lower share of the revenue in order to maximize the profits of the whole supply chain. Figs. 2–4 depict the changes of the equilibrium strategies with the share of the revenue. In particular, from Fig. 2, the price under the decentralized scenario is not always higher than the one under the centralized scenario. From Figs. 3–4, the advertising effort under the decentralized scenario is not

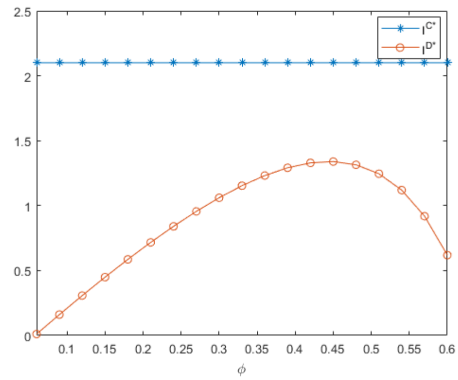


FIGURE 3. The impact of ϕ on the retailer's advertising effort I.

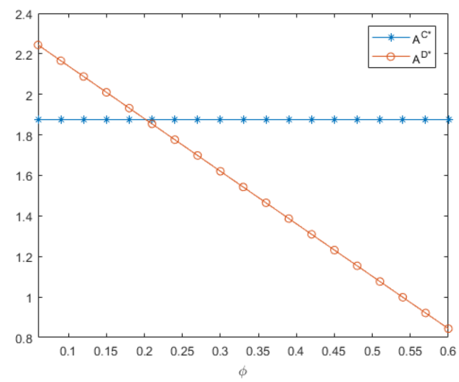


FIGURE 4. The impact of ϕ on the manufacturer's advertising effort A.

always lower than the one under the centralized scenario, which reaches the agreement with the assertion proposed in Proposition 2.

TABLE 1. Variations in the steady-state, advertising effort, and profit.

β	E_{SS}^C	E_{SS}^D	G_{SS}^C	G_{SS}^D	A^{C*}	A^{D*}	I^{C*}	I^{D*}	J_C^*	J_M^{D*}	J_R^{D*}	ΔJ
0.50	8.40	0.45	14.7	2.56	0.46	1.09	2.10	0.11	20.84	6.39	0.41	14.04
0.75	8.40	0.45	15.7	4.85	0.70	1.64	2.10	0.11	21.29	6.09	0.52	14.68
1.00	8.40	0.45	17.1	8.04	0.93	2.19	2.10	0.11	21.93	5.67	0.68	15.58
1.25	8.40	0.45	18.8	12.1	1.17	2.74	2.10	0.11	22.76	5.14	0.88	16.74
ε	E_{SS}^C	E_{SS}^D	G_{SS}^C	G_{SS}^D	A^{C*}	A^{D*}	I^{C*}	I^{D*}	J_C^*	J_M^{D*}	J_R^{D*}	ΔJ
0.30	1.18	0.06	5.09	7.41	0.93	2.19	0.78	0.04	8.85	4.61	0.64	3.60
0.55	3.97	0.21	9.74	7.65	0.93	2.19	1.44	0.07	13.91	5.02	0.66	8.23
0.80	8.40	0.45	17.1	8.04	0.93	2.19	2.10	0.11	21.93	5.67	0.68	15.58
1.05	14.4	0.76	27.2	8.58	0.93	2.19	2.75	0.14	32.94	6.56	0.71	25.67
η	E_{SS}^C	E_{SS}^D	G_{SS}^C	G_{SS}^D	A^{C*}	A^{D*}	I^{C*}	I^{D*}	J_C^*	J_M^{D*}	J_R^{D*}	ΔJ
0.10	3.72	0.19	9.32	7.63	0.93	2.19	0.93	0.04	8.71	1.78	0.49	6.44
0.35	5.52	0.29	12.3	7.79	0.93	2.19	1.38	0.07	13.26	3.22	0.56	9.48
0.60	7.32	0.38	15.3	7.95	0.93	2.19	1.83	0.09	18.48	4.73	0.63	13.12
0.85	9.12	0.48	18.3	8.11	0.93	2.19	2.28	0.12	24.37	6.31	0.71	17.35
θ	E_{SS}^C	E_{SS}^D	G_{SS}^C	G_{SS}^D	A^{C*}	A^{D*}	I^{C*}	I^{D*}	J_C^*	J_M^{D*}	J_R^{D*}	ΔJ
0.25	6.90	0.36	8.87	7.61	0.93	2.19	1.73	0.09	14.90	5.16	0.66	9.08
0.50	8.40	0.45	17.1	8.04	0.93	2.19	2.10	0.11	21.93	5.67	0.68	15.58
0.75	9.90	0.52	27.8	8.61	0.93	2.19	2.48	0.13	30.84	6.31	0.70	23.83
1.00	11.4	0.60	41.1	9.31	0.93	2.19	2.85	0.15	41.62	7.08	0.73	33.81
γ	E_{SS}^C	E_{SS}^D	G_{SS}^C	G_{SS}^D	A^{C*}	A^{D*}	I^{C*}	I^{D*}	J_C^*	J_M^{D*}	J_R^{D*}	ΔJ
0.25	6.90	0.36	13.0	4.26	0.46	1.09	1.72	0.09	12.87	4.91	0.34	7.62
0.50	8.40	0.45	17.1	8.04	0.93	2.19	2.10	0.11	21.93	5.67	0.68	15.58
0.75	9.90	0.52	21.1	11.8	1.40	3.28	2.47	0.13	33.61	6.08	1.21	26.32
1.00	11.4	0.60	25.2	15.6	1.87	4.38	2.85	0.15	47.89	6.13	1.92	39.84

On the other hand, Table 1 indicates the variations of the steady states, advertising efforts and profits under the centralized/decentralized scenarios. For more details, the following specific observations can be obtained: (1) Accompanying with the increasing effectiveness of manufacturer's advertising effort on the brand goodwill (i.e., a higher β), the manufacturer's advertising effort increases. A higher manufacturer's advertising effort results in a higher brand goodwill, which in turn leads to higher profits of the supply chain. In contrast to the decentralized equilibria, the brand goodwill and the profit under the centralized case are relatively higher. In addition, the effects from the retailer's advertising effort ε onto the equilibria and profits are similar to the impacts from β onto the equilibria and profits; (2) The retailer's advertising effort increases if the effects from the retailer's platform goodwill onto both the customer demand and the brand goodwill (higher η and θ) increase. A higher advertising effort results in a higher retailer's platform goodwill and a higher brand goodwill, which in turn results in higher profits of the supply chain; and (3) An increasing effectiveness from the goodwill onto the customer demand (i.e., higher γ and η) can lead to higher profit of the supply chain. It can be interpreted by the fact that a higher γ or η can encourage the manufacturer and the retailer to invest more in the advertising with hope to obtain increasing goodwill. Meanwhile, a higher brand goodwill and a retailer's platform goodwill could promote the customer demand, which ultimately performs higher profits.

VII. CONCLUSION

By considering the advertising effort as an important influential factor on the manufacturer's brand goodwill and the retailer's platform goodwill, we have presented a differential game involving single manufacturer and single retailer in the supply chain under consignment contract. Based on the optimal control theory, we have provided the strategies of optimal pricing and advertising under the decentralized and centralized scenarios. Furthermore, we have investigated how to design a contract by introducing the slotting fees. The main novelties lie in that i) the effects from the retailer's platform goodwill and manufacture's brand goodwill on market demand have been considered; ii) both the pricing and advertising efforts have been given under the centralized and decentralized scenarios by constructing the differential game model, and then the decentralized supply chain has been coordinated by using a state-dependent contract. Finally, a numerical example has been utilized to demonstrate the effectiveness of advertising efforts, the effects from the retailer's platform goodwill and the brand goodwill onto the equilibria and profits. It is worthwhile to mention that the following assertions have been revealed: (1) the optimal retail price is not always low under the centralized setting, and the optimal advertising effort is not always high under the centralized setting; and (2) the linear state-dependent contract can effectively coordinate the decentralized supply chain under the consignment contract.

Motivated by the proposed method, some potential extensions can be listed. Firstly, the system considered is formed of a manufacturer and a retailer, a potential extension is to add competition between manufacturer and retailer or add competition between retailers as in [6]. Secondly, the demand function is described by a separable multiplicative way in this paper. Future research can also consider other function forms since different demand functions may lead to significantly different results.

REFERENCES

[1] X. Zhou, X. Zhong, H. Lin, Z. Qin, and X. Yang, "Lexicographic maximum solution of min-product fuzzy relation inequalities for modeling the optimal pricing with fixed priority grade in supply chain," *IEEE Access*, vol. 6, pp. 71306–71316, 2018.

[2] W. Zhao and D. Wang, "Simulation-based optimization on control strategies of three-echelon inventory in hybrid supply chain with order uncertainty," *IEEE Access*, vol. 6, pp. 54215–54223, 2018. doi: 10.1109/ACCESS.2018.2870856.

[3] Y. Jin, S. Wang, and Q. Hu, "Contract type and decision right of sales promotion in supply chain management with a capital constrained retailer," *Eur. J. Oper. Res.*, vol. 240, no. 2, pp. 415–424, Jan. 2015.

[4] R. Xu, M. Wang, and Y. Xie, "Optimally connected deep belief net for click through rate prediction in online advertising," *IEEE Access*, vol. 6, pp. 43009–43020, 2018.

[5] S. Li, Z. Zhu, and L. Huang, "Supply chain coordination and decision making under consignment contract with revenue sharing," *Int. J. Prod. Econ.*, vol. 120, no. 1, pp. 88–99, Jul. 2009.

[6] T. Avinadav, T. Chernonog, and Y. Perlman, "Consignment contract for mobile apps between a single retailer and competitive developers with different risk attitudes," *Eur. J. Oper. Res.*, vol. 246, no. 3, pp. 949–957, Nov. 2015.

[7] Y. Wang, L. Jiang, and Z.-J. Shen, "Channel performance under consignment contract with revenue sharing," *Manage. Sci.*, vol. 50, no. 1, pp. 1–131, Jan. 2004.

[8] T. Avinadav, T. Chernonog, and Y. Perlman, "The effect of risk sensitivity on a supply chain of mobile applications under a consignment contract with revenue sharing and quality investment," *Int. J. Prod. Econ.*, vol. 168, pp. 31–40, Oct. 2015.

[9] S. Kumar and S. P. Sethi, "Dynamic pricing and advertising for web content providers," *Eur. J. Oper. Res.*, vol. 197, no. 3, pp. 924–944, Sep. 2009.

[10] Y. Fang, Y.-Y. Wang, and Z. Hua, "Equilibrium contract selection strategy in chain-to-chain competition with demand uncertainty," *J. Oper. Res. Soc.*, vol. 67, no. 5, pp. 770–785, May 2016.

[11] J. Ru and Y. Wang, "Consignment contracting: Who should control inventory in the supply chain?" *Eur. J. Oper. Res.*, vol. 201, no. 3, pp. 760–769, Mar. 2010.

[12] Z. Wu, D. Chen, and H. Yu, "Coordination of a supply chain with consumer return under vendor-managed consignment inventory and stochastic demand," *Int. J. Gen. Syst.*, vol. 45, no. 5, pp. 502–516, 2016.

[13] J.-M. Chen, I.-C. Lin, and H.-L. Cheng, "Channel coordination under consignment and vendor-managed inventory in a distribution system," *Transp. Res. E, Logistics Transp. Rev.*, vol. 46, no. 6, pp. 831–843, Nov. 2010.

[14] R. E. de Matta, T. J. Lowe, and D. Zhang, "Consignment or wholesale: Retailer and supplier preferences and incentives for compromise," *Omega*, vol. 49, pp. 93–106, Dec. 2014.

[15] G. Lai, L. G. Debo, and K. Sycara, "Sharing inventory risk in supply chain: The implication of financial constraint," *Omega*, vol. 37, no. 4, pp. 811–825, Aug. 2009.

[16] F. Lu, J. Zhang, and W. Tang, "Wholesale price contract versus consignment contract in a supply chain considering dynamic advertising," *Int. Trans. Oper. Res.*, 2017. doi: 10.1111/itor.12388.

[17] W. Hu, Y. Li, and K. Govindan, "The impact of consumer returns policies on consignment contracts with inventory control," *Eur. J. Oper. Res.*, vol. 233, no. 2, pp. 398–407, Mar. 2014.

[18] D. Zhang, R. de Matta, and T. J. Lowe, "Channel coordination in a consignment contract," *Eur. J. Oper. Res.*, vol. 207, no. 2, pp. 897–905, Dec. 2010.

[19] Y. He and X. Zhao, "Contracts and coordination: Supply chains with uncertain demand and supply," *Nav. Res. Logistics*, vol. 63, no. 4, pp. 305–319, Jun. 2016.

[20] J.-M. Chen, H.-L. Cheng, and M.-C. Chien, "On channel coordination through revenue-sharing contracts with price and shelf-space dependent demand," *Appl. Math. Model.*, vol. 35, no. 10, pp. 4886–4901, Oct. 2011.

[21] L.-T. Chen, "Dynamic supply chain coordination under consignment and vendor-managed inventory in retailer-centric b2b electronic markets," *Ind. Marketing Manage.*, vol. 42, no. 4, pp. 518–531, May 2013.

[22] J. Xie and A. Neyret, "Co-op advertising and pricing models in manufacturer-retailer supply chains," *Comput. Ind. Eng.*, vol. 56, no. 4, pp. 1375–1385, May 2009.

[23] G. Aust and U. Buscher, "Vertical cooperative advertising and pricing decisions in a manufacturer-retailer supply chain: A game-theoretic approach," *Eur. J. Oper. Res.*, vol. 223, no. 2, pp. 473–482, Dec. 2012.

[24] G. Liu, J. Zhang, and W. Tang, "Strategic transfer pricing in a marketing-operations interface with quality level and advertising dependent goodwill," *Omega*, vol. 56, pp. 1–15, Oct. 2015.

[25] J. Zhang, J. Li, L. Lu, and R. Dai, "Supply chain performance for deteriorating items with cooperative advertising," *J. Syst. Sci. Syst. Eng.*, vol. 26, no. 1, pp. 23–49, Feb. 2017.

[26] J. Huang, M. Leng, and L. Liang, "Recent developments in dynamic advertising research," *Eur. J. Oper. Res.*, vol. 220, no. 3, pp. 591–609, Aug. 2012.

[27] P. K. Chintagunta and D. Jain, "A dynamic model of channel member strategies for marketing expenditures," *Marketing Sci.*, vol. 11, no. 2, pp. 117–206, 1992.

[28] S. Jørgensen, S. P. Sigué, and G. Zaccour, "Dynamic cooperative advertising in a channel," *J. Retailing*, vol. 76, no. 1, pp. 71–92, 2000.

[29] S. Karray and G. Zaccour, "A differential game of advertising for national and store brands," *Dyn. Games, Theory Appl.*, pp. 213–229, 2005.

[30] R. Wang, Q. Gou, T.-M. Choi, and L. Liang, "Advertising strategies for mobile platforms with 'Apps'," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 48, no. 5, pp. 767–778, May 2018.

[31] E. Adida and V. DeMiguel, "Supply chain competition with multiple manufacturers and retailers," *Oper. Res.*, vol. 59, no. 1, pp. iii–266, Feb. 2011.

[32] S. Karray and S. H. Amin, "Cooperative advertising in a supply chain with retail competition," *Int. J. Prod. Res.*, vol. 53, no. 1, pp. 88–105, Jan. 2015.

[33] A. Nair and R. Narasimhan, "Dynamics of competing with quality—And advertising-based goodwill," *Eur. J. Oper. Res.*, vol. 175, pp. 462–474, Nov. 2006.

[34] N. Amrouche, G. Martín-Herrán, and G. Zaccour, "Feedback Stackelberg equilibrium strategies when the private label competes with the national brand," *Ann. Oper. Res.*, vol. 164, no. 1, pp. 79–95, Nov. 2008.

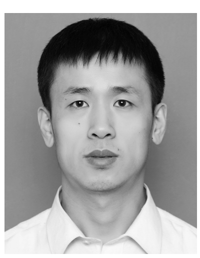
[35] L. Lambertini, "Coordinating static and dynamic supply chains with advertising through two-part tariffs," *Automatica*, vol. 50, no. 2, pp. 565–569, Feb. 2014.

[36] G. Liu, J. Zhang, and W. Tang, "Strategic transfer pricing in a marketing-operations interface with quality level and advertising dependent goodwill," *Omega*, vol. 56, pp. 1–15, Oct. 2015.

[37] Y. Zhou and X. Ye, "Differential game model of joint emission reduction strategies and contract design in a dual-channel supply chain," *J. Cleaner Prod.*, vol. 190, pp. 592–607, Jul. 2018.

[38] P. de Giovanni, "State—And control-dependent incentives in a closed-loop supply chain with dynamic returns," *Dyn. Games Appl.*, vol. 6, no. 1, pp. 20–54, Mar. 2016.

[39] A. Arya and T. Pfeiffer, "Manufacturer-retailer negotiations in the presence of an oligopolistic input market," *Prod. Oper. Manage.*, vol. 21, no. 3, pp. 534–546, May/Jun. 2012.



ZHIHUI WU received the B.Sc. degree in information and computation science and the M.Sc. degree in applied mathematics from the Hefei University of Technology, Hefei, China, in 2005 and 2008, respectively, and the Ph.D. degree in management science and engineering from the Harbin University of Science and Technology, Harbin, China, in 2017, where he is currently a Lecturer with the Department of Applied Mathematics. His current research interests include robust control, optimization approach, system optimization, and supply-chain management.