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Event-Triggered Control for Strict-Feedback Nonlinear Systems With External Disturbances

JIANPING CAI[1], RUI YU[2], QIUZHEN YAN[3], CONGLI MEI[1], AND LUJUAN SHEN[4]

[1]School of Electrical Engineering, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China
[2]College of Mechanical Electrical Engineering, China Jiliang University, Hangzhou 310018, China
[3]College of Information Engineering and Art Design, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China
[4]School of Marxism, Zhejiang University of Water Resources and Electric Power, Hangzhou 310018, China

Corresponding author: Jianping Cai (caijianping2001@163.com)

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ABSTRACT In this paper, the event-trigger adaptive control problem is investigated for a class of nonlinear systems with unknown external disturbance. An estimator is designed to estimate the unknown upper bound of the disturbance-like term, which presents the uncertainties caused by external disturbance, a measurement error of input, and an unknown gain parameter. Unlike the existing results, the bound of tracking error is not only independent of the size of unknown system parameters but also can converge to zero. Moreover, new proof of Zeno-free under the discontinuous input is presented. It is shown that the designed adaptive controller ensures the stability of the closed-loop system and achieves desired tracking performance.

INDEX TERMS Event-trigger, backstepping, adaptive control, nonlinear system.

I. INTRODUCTION

As we all know, the efficiency of networked control systems [1]–[7] is usually constrained by limited network resources in practice. Distributed complex network systems provide a realistic example for us. In such systems computation abilities and channel bandwidth have always played important roles on execution efficiency of complex control schemes. The constraint on efficiency caused by limited network resources is becoming more and more obvious along with the widely using of nonlinear and adaptive control technologies. Therefore networked control has gained more and more attention on improving control efficiency [8]–[10]. On the other hand, input signal applied on the system is allowed any value at any time instant in the classical control framework. Limited network resources are overwhelmed and system performance including stability, tracking and transient performance are greatly reduced, especially when unknown time delays [11]–[13] exist in signal transmission. Event-triggered control strategy is just such a technology which has been proposed to overcome the aforementioned drawbacks.

In the recent twenty years, many studies have focused on event-triggered control [14]–[21]. Specially for decentralized networked control systems which has the higher requirement on channel bandwidth, event-triggered control has been deeply studied [18]–[20] and some basic results have been proposed. In [21] a decentralized event-triggered consensus controller is proposed without requiring continuous communication among agents. For nonlinear systems, a static event-triggered rule with state is proposed in [14], while a dynamic controller trigger event is designed based on an additional internal dynamic variable in [22]. For a class of uncertain nonlinear systems with unknown actuator failures which and missing data have been widely studied [23]–[25], fixed threshold, relative threshold and switching threshold strategy have been proposed in [26] and [27] respectively. The output event-trigger based control scheme has been proposed in [28]. Based above analysis, in the context of event-trigger based control, the results for nonlinear systems are still every limited. Especially for nonlinear systems with parameter uncertainties and external disturbance, there is still short of available results. It's precisely because external disturbance is inevitable for practical systems that we must consider its uncertainty influence in event-trigger based adaptive control scheme design. In this paper, we address such a problem by considering controlling a class of unknown nonlinear systems with external disturbance. Due to the existence of external disturbance, the requirement on avoiding the Zeno

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behavior can not be guaranteed by traditional event triggered controller. To solve it, $\bar{T}$ as a positive design parameter is introduced in event triggering controller (18). In addition, we introduce an estimator to estimate the unknown upper bound of disturbance-like term in the design of the proposed event-trigger based control scheme. Unlike the existing results, the bound of tracking error is not only independent of the size of unknown system parameters but also can converges to zero. Moreover, a new proof of Zeno-free under the discontinuous input are proved.

The main contributions of this paper are summarized as follows: (1) The event-trigger based adaptive control problem is investigated for a class of non-linear systems with unknown parameters and unknown external disturbances. (2) A new event-triggering mechanism is proposed and an adaptive event-trigger based control scheme is developed based on such mechanism. (3) In addition, a new proof of Zeno-free under the discontinuous input are provided based on the proposed event-triggering mechanism. (4) Moreover, unlike some existing event-trigger based control schemes for nonlinear systems, the uncertainties caused by external disturbance can be compensated accurately by estimating its unknown upper bound under the control scheme developed in this paper, thus a better system performance can be achieved.

The rest of the paper is organized as follows. In section 2, we formulate the strict feedback systems with unknown external disturbance and constant parameters. In section 3, the event-triggered based adaptive control scheme is proposed and stability analyses of the closed-loop system is presented. In section 4, simulation results are shown in detail. Finally, the paper is concluded in section 5.

II. MODELS AND PROBLEM STATEMENT

To illustrate our design scenario, the following strict-feedback nonlinear systems similar to [23] and [26] is considered:

$$\begin{aligned}\dot{x}_1 &= x_2 + g_1^T(x_1)\theta\\ \dot{x}_2 &= x_3 + g_2^T(x_1, x_2)\theta\\ &\vdots\\ \dot{x}_n &= g_0(x) + g_n^T(x)\theta + bu + \bar{d}(t)\\ y &= x_1\end{aligned} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$ is system state and $y \in \mathbb{R}$ is the output. $u \in \mathbb{R}$ is input. $g_j(j = 0, 1, 2, \ldots, n) \in \mathbb{R}^p$ are known and sufficiently smooth functions. $\theta \in \mathbb{R}^p, b \in \mathbb{R}$ are unknown constant parameters. $\bar{d}(t)$ denotes bounded disturbance.

In what follows, the event-trigger based adaptive controller design strategies for systems (1) will be proposed. To obtain control laws and update laws of unknown parameters, several assumptions are needed.

Assumption 1: $b \neq 0$ and the sign of b is known.

Assumption 2: Reference signal $y_r(t) \in \mathbb{R}$ and its ith-order $(i = 1, 2, \ldots, n-1)$ derivatives are known and bounded.

III. DESIGN AND ANALYSIS OF ADAPTIVE CONTROLLERS

Our objective is to guarantee all the signals of the closed loop system are bounded under the proposed event-trigger based adaptive controller. To carry out the design of control law and adaptive update laws based on backstepping technique, the following change of coordinates is required.

$$\begin{aligned}z_1 &= x_1 - y_r;\\ z_i &= x_i - \alpha_{i-1} - y_r^{(i-1)}, \quad (i = 2, \ldots, n)\end{aligned} \tag{2}$$

where $\alpha_{i-1}(i = 2, \ldots, n)$, is a virtual control at step $n-1$. z_1 denotes the tracking error.

Step 1: From (1) and (2), the derivative of z_1 can be rewritten as

$$\dot{z}_1 = z_2 + \alpha_1 + g_1^T\theta \tag{3}$$

where α_1 is the virtual control. We consider the following Lyapunov function

$$V_1 = \frac{1}{2}z_1^2 + \frac{1}{2}\tilde{\theta}^T\Gamma_\theta^{-1}\tilde{\theta} \tag{4}$$

where $\tilde{\theta} = \theta - \hat{\theta}$ is the estimation error. $\hat{\theta}$ is an estimate of unknown parameter θ. $\Gamma_\theta \in \mathbb{R}^{p\times p}$ is a positive definite matrix. We chose virtual control α_1 as

$$\alpha_1 = -L_1 z_1 - g_1^T\hat{\theta} \tag{5}$$

where $L_1 \in \mathbb{R} > 0$ is a design parameter. With (3) and (5), the derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= z_1(z_2 - L_1 z_1 - g_1^T\hat{\theta} + g_1^T\theta) - \tilde{\theta}^T\Gamma_\theta^{-1}\dot{\hat{\theta}}\\ &= -L_1 z_1^2 + z_1 z_2 - \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \Gamma_\theta g_1 z_1)\end{aligned} \tag{6}$$

We choose

$$\tau_1 = \Gamma_\theta g_1 z_1 \tag{7}$$

Then we can get

$$\dot{V}_1 = -L_1 z_1^2 + z_1 z_2 - \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \tau_1) \tag{8}$$

where τ_1 is a tuning function.

Step i($i = 2, 3, \cdots, n-1$): Easily, we can get

$$\begin{aligned}\dot{z}_i &= \dot{x}_i - \dot{\alpha}_{i-1} - y_r^{(i)}\\ &= x_{i+1} + g_i^T\theta - \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}\dot{x}_k + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right)\\ &\quad - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - y_r^{(i)}\\ &= z_{i+1} + \alpha_i + \left(g_i^T - \sum_{k=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_k}g_k^T\right)\theta\\ &\quad - \sum_{k=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_k}x_{k+1} + \frac{\partial\alpha_{i-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\right) - \frac{\partial\alpha_{i-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\end{aligned} \tag{9}$$

We choose Lyapunov function V_i as

$$V_i = V_{i-1} + \frac{1}{2}z_i^2 \tag{10}$$

and let virtual control α_i be

$$\alpha_i = -L_i z_i - z_{i-1} - (g_i^T - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} g_k^T)\hat{\theta} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}} \tau_i + \sum_{k=2}^{i-1} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} \Gamma_\theta (g_i - \sum_{l=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_l} g_l) z_k + \sum_{k=1}^{i-1} (\frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)}) \tag{11}$$

and tuning function τ_i be

$$\tau_i = \tau_{i-1} + \Gamma_\theta (g_i - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} g_k) z_i \tag{12}$$

where $L_i \in \mathbb{R}$ is a positive design parameter. Then we can get

$$\dot{V}_i = -\sum_{k=1}^{i} L_k z_k^2 + z_i z_{i+1} - \tilde{\theta}^T \Gamma_\theta^{-1} (\dot{\hat{\theta}} - \tau_i) - \sum_{k=2}^{i} \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} (\dot{\hat{\theta}} - \tau_i) z_k \tag{13}$$

Step n: From (1) and (2), the derivative of z_n is

$$\dot{z}_n = \dot{x}_n - \dot{\alpha}_{n-1} - y_r^{(n)} = g_0(x) + g_n^T(x)\theta + bu + \bar{d}(t) - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} (x_{k+1} + g_k^T \theta) - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_r^{(k-1)}} y_r^{(k)} - y_r^{(n)} \tag{14}$$

We take

$$\alpha = -L_n z_n - z_{n-1} - (g_n^T - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} g_k^T)\hat{\theta} + \Gamma_\theta \sum_{k=2}^{n-1} z_k \frac{\partial \alpha_{k-1}}{\partial \hat{\theta}} (g_n - \sum_{l=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_l} g_l) + \sum_{k=1}^{n-1} (\frac{\partial \alpha_{n-1}}{\partial x_k} x_{k+1} + \frac{\partial \alpha_{n-1}}{\partial y_r^{(k-1)}} y_r^{(k)}) + \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}} \tau_n - sign(z_n)\hat{D} - g_0(x) + y_r^{(n)} \tag{15}$$

where $L_n \in \mathbb{R}$ is a positive design parameter. $\hat{D}$ is an estimation of D. D is the unknown upper bound of $d(t)$.

$$d(t) = \bar{d}(t) - b\varepsilon(t) \tag{16}$$

The boundedness of $\varepsilon(t)$ will be given in the following event-trigger based controller. The tuning function τ_n can be chosen as

$$\tau_n = \tau_{n-1} + \Gamma_\theta (g_n - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} g_k) z_n \tag{17}$$

Now the proposed event-trigger based controller can be designed as follows:

$$\begin{aligned} w &= \hat{\rho}\alpha; \\ u(t) &= w(t_k), t \in [t_k, t_{k+1}) \\ t_{k+1} &= min\{t_{k+1}^0, t_k + \bar{T}\}; \\ t_{k+1}^0 &= inf\{t \in R, ||\varepsilon(t)| \geq m\}; \end{aligned} \tag{18}$$

where $\hat{\rho}$ is an estimation of unknown parameter $\rho = \frac{1}{b}$. $\varepsilon(t) = w(t) - u(t)$ denotes the measurement error. $\bar{T} \in \mathbb{R}$ and $m \in \mathbb{R}$ are positive constants. t_k $(k = 1, 2, \cdots)$ is the controller update time and during the time interval $[t_k, t_{k+1})$ the value of input signal holds a constant $w(t_k)$. Compared with the existing results shown in [26]–[28], a constraint function $t_{k+1} = min\{t_{k+1}^0, t_k + \bar{T}\}$ is imposed on update time t_{k+1}. Such constraint function can guarantee the length of time interval $[t_k, t_{k+1})$ being smaller than a prescribed bound. Such a bound is important to the proof of Zeno-free. Update laws are chosen as

$$\begin{aligned} \dot{\hat{D}} &= \eta_d |z_n|; \\ \dot{\hat{\theta}} &= \tau_n; \\ \dot{\hat{\rho}} &= -sign(b)\eta_\rho \alpha z_n \end{aligned} \tag{19}$$

where $\eta_d \in \mathbb{R}$ and $\eta_\rho \in \mathbb{R}$ are positive design parameters.

Remark 1: Different from [26], a designed parameter $\bar{T}$ is introduced in the proposed event-trigger based controller (18). It can ensure the length of time interval $[t_k, t_{k+1})$ is limited. It is important for the establishment and proof of following Theorem 1.

Remark 2: It is clear that $d(t)$ is bound because $|\varepsilon(t)|$ is bounded by m shown in (18). To compensate such uncertainty we can establish the estimation of the upper bound of $d(t)$ which includes external disturbance and unknown measurement error. Then the estimation $\hat{D}$ will be used in the design of event-trigger based controller shown in (15) and (18).

We now establish the boundedness of all signals in the closed loop system, as stated in the following theorem.

Theorem 1: Consider the uncertain nonlinear systems (1) and the event-trigger based adaptive controller given in (18) and (19). Under Assumptions 1 and 2, the following results hold

- All signals in the closed-loop system are globally bounded.
- There exists a time $T^* > 0$ such that $t_{k+1} - t_k \geq T^*$.
- The tracking error z_1 satisfies

$$lim_{t\to\infty} |x_1(t) - y_r(t)| = 0$$

Proof: We consider the following Lyapunov function

$$V = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2\eta_d} \tilde{D}^2 + \frac{|b|}{2\eta_\rho} \tilde{\rho}^2 \tag{20}$$

From (13) and (14) the derivative of V is

$$\begin{aligned}\dot{V} &= \dot{V}_{n-1} + \frac{1}{2}z_n\dot{z}_n - \frac{1}{\eta_d}\tilde{D}\dot{\hat{D}} - \frac{|b|}{\eta_\rho}\tilde{\rho}\dot{\hat{\rho}} \\ &= -\sum_{k=1}^{n-1} L_k z_k^2 + z_n z_{n-1} - \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \tau_{n-1}) \\ &\quad -\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{n-1}) + \frac{1}{2}z_n\Big(g_0 + g_n^T\theta \\ &\quad + bu + \bar{d}(t) - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}(x_{k+1} + g_n^T\theta) - y_r^{(n)} \\ &\quad -\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_r^{(k-1)}}y_r^{(k)}\Big) - \frac{1}{\eta_d}\tilde{D}\dot{\hat{D}} \\ &\quad -\frac{|b|}{\eta_\rho}\tilde{\rho}\dot{\hat{\rho}} \end{aligned} \tag{21}$$

With (16) we can get

$$\begin{aligned} bu(t) + \bar{d}(t) &= bw(t) - b\varepsilon(t) + \bar{d}(t) \\ &= b\hat{\rho}\alpha + d(t) \\ &= b(\rho - \tilde{\rho})\alpha + d(t) \\ &= \alpha - b\tilde{\rho}\alpha + d(t) \end{aligned} \tag{22}$$

Then we can get

$$\begin{aligned}\dot{V} &= -\sum_{k=1}^{n} L_k z_k^2 - \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \tau_{n-1}) \\ &\quad + (g_n^T - \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}g_k^T)\tilde{\theta} - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}z_n(\dot{\hat{\theta}} - \tau_n) \\ &\quad - sign(z_n)\hat{D}z_n + z_n d(t) - \frac{1}{\eta_d}\tilde{D}\dot{\hat{D}} - b\tilde{\rho}\alpha z_n \\ &\quad -\frac{|b|}{\eta_\rho}\tilde{\rho}\dot{\hat{\rho}} - \sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{n-1}) \\ &\quad + z_n\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma_\theta(g_n - \sum_{l=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_l}g_l) \end{aligned} \tag{23}$$

Note that

$$\begin{aligned} -sign(z_n)\hat{D}z_n + z_n d(t) &\le |z_n|D - |z_n|\hat{D} = |z_n|\tilde{D} \\ -b\tilde{\rho}\alpha z_n - \frac{|b|}{\eta_\rho}\tilde{\rho}\dot{\hat{\rho}} &= -\frac{|b|}{\eta_\rho}\tilde{\rho}(\dot{\hat{\rho}} + sign(b)\eta_\rho z_n\alpha) \end{aligned} \tag{24}$$

and with (17), we have

$$\begin{aligned} -\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n) &= -\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_{n-1}) \\ &\quad + z_n\sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}\Gamma_\theta(g_n - \sum_{l=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_l}g_l) \end{aligned} \tag{25}$$

and

$$\begin{aligned} \sum_{k=2}^{n} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n) &= \sum_{k=2}^{n-1} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n) \\ &\quad + \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}z_n(\dot{\hat{\theta}} - \tau_n) \end{aligned} \tag{26}$$

So we can get

$$\begin{aligned}\dot{V} &\le -\sum_{k=1}^{n} L_k z_k^2 - \tilde{\theta}^T\Gamma_\theta^{-1}(\dot{\hat{\theta}} - \tau_n) \\ &\quad -\sum_{k=2}^{n} z_k\frac{\partial\alpha_{k-1}}{\partial\hat{\theta}}(\dot{\hat{\theta}} - \tau_n) - \frac{1}{\eta_d}\tilde{D}(\dot{\hat{D}} - \eta_d|z_n|) \\ &\quad -\frac{|b|}{\eta_\rho}\tilde{\rho}(\dot{\hat{\rho}} + sign(b)\eta_\rho z_n\alpha) \end{aligned} \tag{27}$$

With update laws (19), we have

$$\dot{V} \le -\sum_{k=1}^{n} L_k z_k^2 \tag{28}$$

Clearly, we can get that V is non-increasing with (28). Therefore, all signals in closed-loop $z_i(i = 1, \cdots, n)$, $\hat{\theta}$, $\hat{\rho}$, $\hat{D}$ are bounded. Then all virtual control $\alpha_i(i = 1, \cdots, n-1)$, α and states $x_i(i = 1, \cdots, n)$ are bounded. From the control law (18), $u(t)$, $w(t)$ are ensured bounded.

In the following we will show that there exists a constant T^* such that $t_{k+1} - t_k \ge T^*$ $(\forall k \in Z^+)$. It is clear that $w(t)$ is a discontinuous function due to $sign(z_n)$ existing in α shown in (15). Discontinuity points caused by $sign(\cdot)$ are jump discontinuity, rather than infinite discontinuity points. That is to say the one-sided derivatives of such discontinuity points is constant. Now we use $t_k^i, (i = 1, \cdots, l-1)$ denotes such discontinuity points in time interval $[t_k, t_{k+1})$. Then we have

$$t_k = t_k^0 < t_k^1 < \cdots < t_k^{l-1} < t_k^l = t_{k+1} \tag{29}$$

Firstly, we want to reassurance that the length of time interval (t_k^i, t_k^{i+1}) divided by above discontinuity points of $sign(z_n)$ must be greater than 0. Namely, (t_k^i, t_k^{i+1}) is not empty. It is not hard to understand. Because it can be guaranteed by the continuity of z_n. Note that $t_{k+1} - t_k \le \bar{T}$ has been given in (18). So the above discontinuity points are finite. Namely, l is a finite number. Considering the time interval $[t_k^i, t_k^{i+1})$, we have

$$|\dot{\varepsilon}(t)| = sign(\varepsilon(t))\dot{\varepsilon}(t) \le |\dot{w}(t)| \tag{30}$$

Clearly, $|\dot{w}(t)|$ is continuous with $x_i, \hat{\alpha}, \hat{D}, y_r$. As all signals $x_i, \hat{\alpha}, \hat{D}, y_r$ in time interval $[t_k^i, t_k^{i+1})$ are globally bounded, there must exist a positive constant κ_k^i such that $|\dot{w}(t)| \le \kappa_k^i$. Based on event-trigger controller (18), we consider the following two cases:

- $t_{k+1} = t_k + \bar{T}$,
  It indicates the length of time interval $[t_k, t_{k+1})$ is $\bar{T}$.
- $t_{k+1} = t_{k+1}^0$,
  It indicates $\varepsilon(t_{k+1}) = m$. Note that

$$\begin{aligned} m &= \varepsilon(t_{k+1}) - \varepsilon(t_k) \\ &= \sum_{q=0}^{l-1}(\varepsilon(t_k^{q+1}) - \varepsilon(t_k^q)) \\ &\le \sum_{q=0}^{l-1}|\dot{\varepsilon}(\zeta_k^q)||t_k^{q+1} - t_k^q| \\ &\le \kappa_k\sum_{q=0}^{l-1}|t_k^{q+1} - t_k^q| \\ &= \kappa_k(t_{k+1} - t_k) \end{aligned} \tag{31}$$

where $\exists\ \zeta_k^q \in (t_k^q, t_k^{q+1})$ and $\kappa_k = max_{q=1,\cdots,l-1}\{\kappa_k^q\}$ ensure that above inequality (31) is established. It is easy to get

$$t_{k+1} - t_k \geq \frac{m}{\kappa_k} \tag{32}$$

So we have

$$t_{k+1} - t_k \geq min\{\frac{m}{\kappa_k}, \bar{T}\} \triangleq T^* \tag{33}$$

From (28), we can easily get V is non-increasing. Similar in [29] and [30], by applying the Lasalle-Yoshizawa theorem to (28), we have

$$lim_{t\to\infty} z_i(t) = 0 \ \ (i = 1, \cdots, n) \tag{34}$$

This implies that

$$lim_{t\to\infty}|x_i(t) - y_r(t)| = 0 \tag{35}$$

■

Remark 3: Compared with [26], the tracking performance has been greatly improved in this paper under the consideration of external disturbance. the tracking error can convergence to zero by introducing an estimator in control scheme to realize the online estimation of the unknown upper bound of external disturbance.

IV. SIMULATION STUDIES

We now apply the proposed control scheme to the following 2nd order system described as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \phi_1(x_1, x_2)\theta + \phi_2(x_1, x_2) + u + d(t) \end{aligned} \tag{36}$$

where x_1, x_2 are system states and u is input. θ is an unknown parameter. $\phi_1(x_1, x_2) = sin(x_1)$ and $\phi_2(x_1, x_2) = cos(x_1 + 2x_2)$ are known functions. $d(t)$ is external disturbance.

In simulation, the true value of unknown parameter θ is taken as 2 and disturbance is $0.2sin(t)$. The design parameters can be chosen as: $L_1 = L_2 = 5$, $\Gamma_\theta = \eta_d = 0.01$. The parameters in trigger are $m = 1$ and $T = 0.01$. The initial values are taken as: $x_1(0) = 2.5$, $x_2(0) = 0$, $\hat{\theta}(0) = 1$ and $\hat{D}(0) = 0$.

Fig.1 represents tracking error and the state x_2 is shown in Fig.2. Fig.3 shows the signal w. After the change of event-trigger (18), the signal $u(t)$ is given in Fig.4. Clearly, we can get that all signals of the systems (1) are all bounded under the controlling of the proposed control law.

Then in order to investigate the influence of design parameter m on system performance, the simulation results are shown in Fig.5 and Fig.6 where we take $m = 0.5$ and $m = 1$. By comparing these two cases, we can get that the better tracking performance can be realized when the design parameter is bigger. But the amplitude of control signal will also become bigger.

Finally, we take the external disturbance $d(t) = 0.4cos(t)$. The simulation results are shown in Fig.7 and Fig.8. It is clear that all signals of the systems (1) are bounded under such a different disturbance.

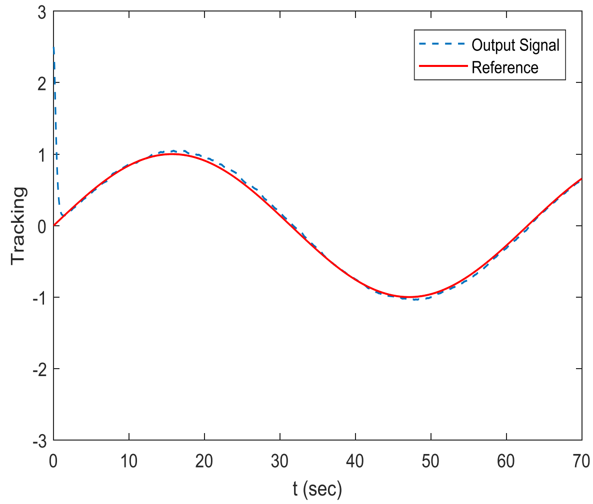

FIGURE 1. Tracking errors($m = 1, d(t) = 0.2sint$).

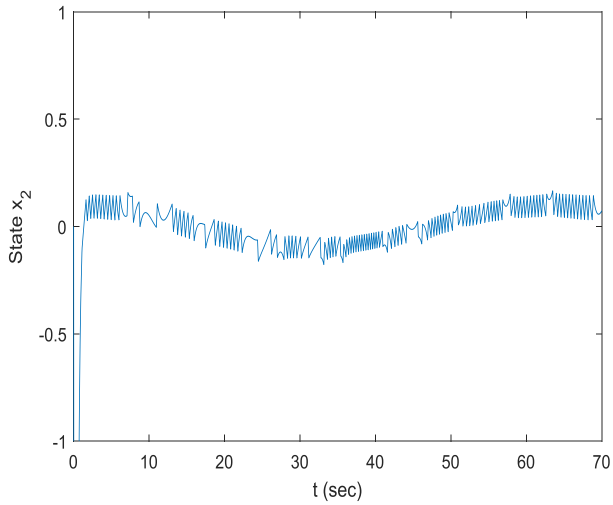

FIGURE 2. State x_2($m = 1, d(t) = 0.2sint$).

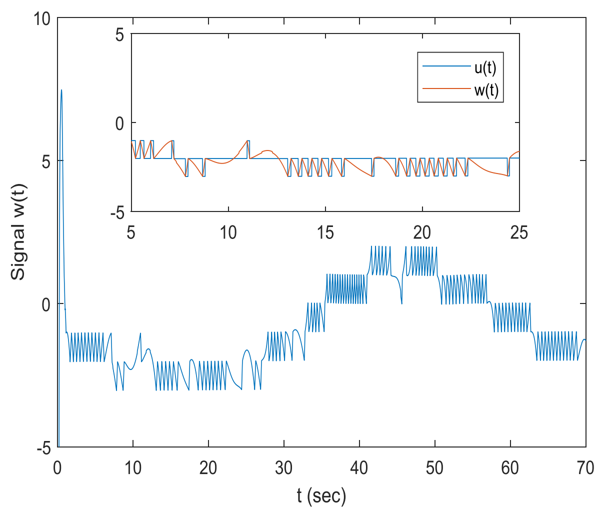

FIGURE 3. Signal w($m = 1, d(t) = 0.2sint$).

From the above simulation results, it is clear that the stability of the nonlinear system (36) can be ensured and the tracking performance can be realized under the controlling of

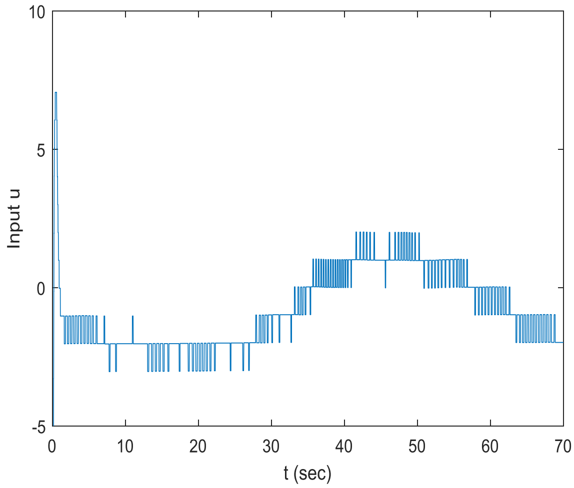

FIGURE 4. Input signal $u(m = 1, d(t) = 0.2sint)$.

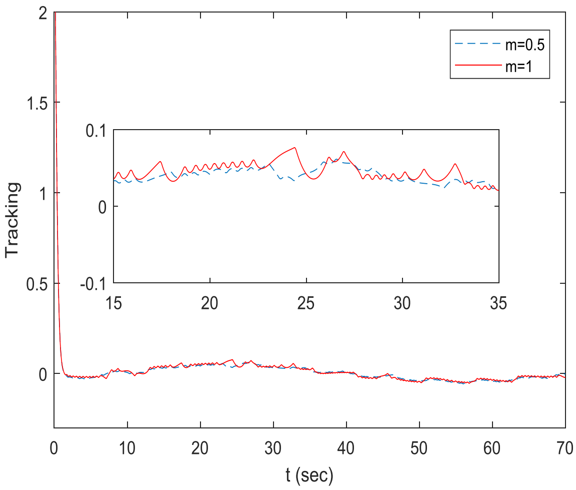

FIGURE 5. Comparison of tracking errors $m = 1$ and $m = 0.5$.

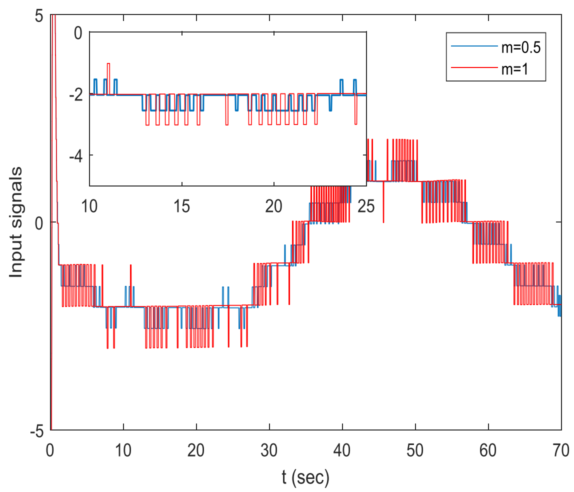

FIGURE 6. Comparison of input signals u $m = 1$ and $m = 0.5$.

the proposed event-trigger based adaptive controller when the unknown external disturbance exists. Compared with existing results in [26]–[28], the effect of external disturbance can be compensated by estimating its unknown upper bound directly rather than being ignore in controller design.

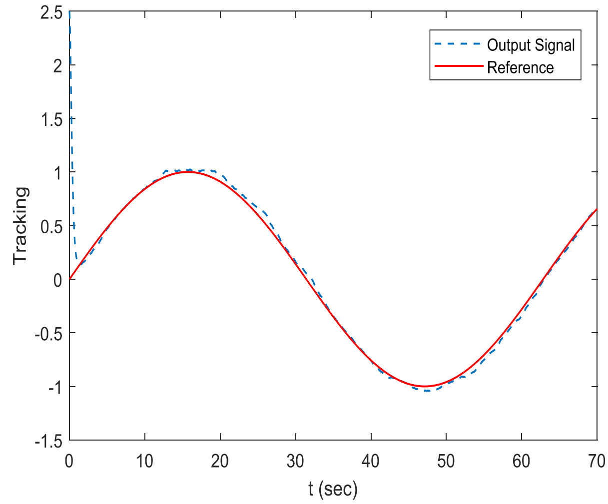

FIGURE 7. Tracking errors$(m = 1, d(t) = 0.4cost)$.

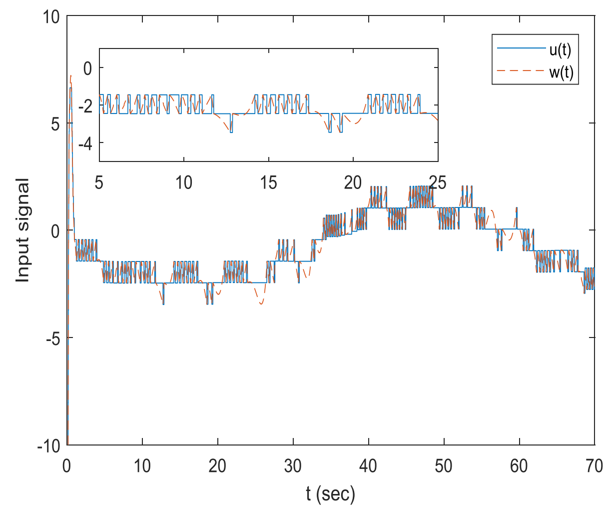

FIGURE 8. Input signal $u(m = 1, d(t) = 0.4cost)$.

V. CONCLUSION

An event-trigger based control scheme is proposed for strict feedback nonlinear systems with unknown constant parameters and external disturbance based on backstepping technique. It is shown that all signals of closed-loop system are bounded and the steady state tracking error is ensured to be zero. As we all know, design parameters including $L_1, \cdots, L_n$, η_d, η_ρ, Γ_θ, the event-triggering time $\bar{T}$ and the threshold m will affect the performance of control system. A possible direction for future work is to investigate how to choose the design parameters for the better performance.

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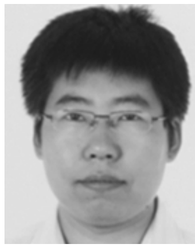

JIANPING CAI received the Ph.D. degree from Zhejiang University, in 2014. He is currently an Associate Professor with the Zhejiang University of Water Resources and Electric Power. His main research interests include nonlinear systems and adaptive control.

RUI YU is currently pursuing the master's degree with the Department of Control Engineering, China Jiliang University. Her research interests include adaptive control and nonlinear systems.

QIUZHEN YAN received the M.S. degree in computer science and the Ph.D. degree in control science and engineering from the Zhejiang University of Technology, Hangzhou, China, in 2005 and 2015, respectively. Since 2005, he has been with the College of Information Engineering, Zhejiang University of Water Resources and Electric Power, where he is currently a Lecturer. His present research interests include iterative learning control and repetitive control. He is a member of the Chinese Association of Automation.

CONGLI MEI received the Ph.D. degree from Zhejiang University, in 2007. He is currently an Associate Professor with the Zhejiang University of Water Resources and Electric Power. His main research interests include control engineering, pattern recognition, and information processing.

LUJUAN SHEN received the Ph.D. degree from Zhejiang University, in 2013. She is currently an Associate Professor with the Zhejiang University of Water Resources and Electric Power. Her main research interest includes application of mathematics.