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Intuitionistic Fuzzy Multiple Criteria Group Decision Making: A Consolidated Model With Application to Emergency Plan Selection

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ABSTRACT With complexity and uncertainty having an increasing impact on the decision-making environment, much attention is being paid to the development and application of multiple criteria group decision-making (MCGDM) models owing to the potential for fully exploiting the diverse strengths and expertise of various members. In general, inevitable interactions among decision makers (DMs), when a number of DMs share similar knowledge and experiences, can have a significant impact on the management of decision information directly or indirectly related to DMs, and can easily lead to distorted and unconvincing decision outcomes. In order to model the MCGDM problem in which DMs share a similar background, a consolidated MCGDM model in the context of intuitionistic fuzzy sets (IFSs) is developed. First, we refine the constructive principles for intuitionistic fuzzy entropy (IFE) and use them as a basis to produce a novel IFE measure simultaneously factoring in the intuitionism and fuzziness of IFSs. With the aim of dealing with the impact on the specifications of the weights of DMs and criteria, an integrated method is then proposed based on the novel IFE measure, 2-additive fuzzy measure, and Choquet integral. Due to their capability of modeling effectively the interrelationships among arguments, the weighted intuitionistic fuzzy Bonferroni mean (WIFBM) and the weighted intuitionistic fuzzy geometric Bonferroni mean (WIFGBM) are introduced to fuse the individual evaluation values of alternatives on criteria. In addition, simple additive weighting based on the WIFBM or WIFGBM is applied to rank alternatives and select the best one. Finally, the feasibility and effectiveness of the proposed model are explored with a case study of an emergency plan decision-making problem accompanied with sensitivity and comparison analysis.

INDEX TERMS Intuitionistic fuzzy sets, Bonferroni mean, intuitionistic fuzzy entropy, 2-additive fuzzy measure, choquet integral, multiple criteria group decision making.

I. INTRODUCTION

Decision-making environments faced by enterprises and organizations today have become increasingly complex

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and uncertain, necessitating scientific decision-making techniques that can help provide decision makers (DMs) or experts with clear qualitative thinking or with tools for quantitative analysis. Multiple criteria decision making (MCDM) is an important decision-making technique which can provide DMs with a systematic framework to facilitate decision

analysis and conduct effective decision making by evaluating the multiple and conflicting criteria that stakeholders factor into [33], [35], [36], [40], [52], [61], [85]. Groups and teams are reported to be collectively wiser than their individual members and usually play a significant role in societies and organizations [2], [4], [22], [49], [62]. Therefore, a number of MCDM researchers have been attempting to extend the theories and methods of the classic MCDM model to develop multiple criteria group decision making (MCGDM) models and to apply the MCGDM methods to various socioeconomic areas, including management science, social science, economics, public administration, military research, and emergency management evaluation [7], [10], [11], [26], [27], [76].

The MCGDM process involves multi-faceted problems that need to be addressed when conducting efficient and feasible decision analysis. Several attempts have been made to enhance the ability of an MCGDM model to deal with the increasing complexity of the socioeconomic environment by introducing theoretical and methodological concepts for studying MCGDM processes from different perspectives. The scope of these efforts includes the following concerns: the articulation and representation of decision information; the determination of weights of DMs and criteria; information aggregation; and the ranking of alternatives.

In real-life group decision-making scenarios, if a number of DMs share similar knowledge and experiences, the inevitable interactions among them can have a significant impact on the management of decision information directly or indirectly related to DMs, including the specifications of the weights of DMs and criteria, and the fusion of individual arguments. When building the structure of an MCGDM model, if we fail to tackle explicitly the interactions among DMs, the decision information obtained during the MCGDM process may be overestimated or underestimated and can easily lead to distorted and unconvincing decision outcomes. However, the literature on the construction of MCGDM models has not delved into the interactions among DMs and their effects on decision information [4], [7], [18], [23], [24], [26], [27], [50], [56], [70], [75], [76], [81]. Bridging this gap has provided the motivation to develop a novel MCGDM model in the current study.

This paper develops a consolidated model to address the MCGDM problem in which there exist the interactions among DMs arisen from their similar knowledge and experiences. In the proposed model, intuitionistic fuzzy numbers (IFNs) [2], [3] are introduced to characterize the uncertain evaluation information on alternatives. To obtain the weights of DMs and criteria, an integrated method capable of dealing with the impacts of the interactions among DMs is proposed based on a novel intuitionistic fuzzy entropy measure, 2-additive fuzzy measure, and Choquet integral. Because of their ability to model the interrelationships among arguments, the weighted intuitionistic fuzzy Bonferroni mean (WIFBM) [82] and the weighted intuitionistic

fuzzy geometric Bonferroni mean (WIFGBM) [84] are employed to fuse individual arguments into collective evaluation information. Subsequently, the traditional simple additive weighting (SAW) method based on the WIFBM or the WIFGBM is used to rank the alternatives. Finally, the paper presents a practical application to an emergency plan decision-making problem combined with sensitivity analysis and comparison analysis to validate the effectiveness and practicability of the proposed MCGDM model. The contributions of the proposed model can be summarized as follows:

- (i) The model introduces the intuitionistic fuzzy set (IFS), which is coined by Atanassov [2], [3] as an extension of the fuzzy set [88] and has drawn much attention from notable scholars owing to its capacity to deal with uncertainty and imprecision, to characterize uncertain evaluation values in the proposed MCGDM model. Additionally, we refine the constructive principles for intuitionistic fuzzy entropy (IFE) and establish a novel IFE measure, which performs better than the existing IFE measures [34], [60], [72], [73], [78] in measuring the uncertainty of IFS. The measure can support the construction of an entropy-based method to determine the individual criterion weights.
- (ii) To cope with the impact resulting from the interactions among DMs on the specifications of the weights of DMs and criteria, the paper presents an integrated method of determining simultaneously the criterion weights and the DM weights, building on the proposed IFE measure, 2-additive fuzzy measure, and Choquet integral [31].
- (iii) The WIFBM and the WIFGBM, which can both model effectively the interrelationships among decision information [82], [84], are introduced in building the structure of the MCGDM model, not only to fuse individual evaluation values into a collective value but also to serve as a decision-making method for deriving the final ranking order of alternatives. In addition, the two aggregation means can provide DMs with many choices of parameters depending on their levels of optimism or pessimism.

The rest of this paper is organized as follows. Relevant basic concepts are introduced in Section II. An improved axiomatic definition for IFE and a novel IFE measure are proposed in Section III. This section also presents a comparison with existing IFE measures to validate the effectiveness of the proposed IFE measure. Section IV develops a consolidated MCGDM model in which the IFNs are embedded, based on the novel IFE measure, 2-additive fuzzy measure, Choquet integral, the WIFBM, and the WIFGBM. Section V presents a practical case of an emergency planning decision making problem accompanied by sensitivity analysis and comparison analysis to illustrate the merits of the new consolidated MCGDM model. Section VI presents the conclusions.

II. LITERATURE REVIEW

A. MULTI-CRITERIA GROUP DECISION MAKING (MCGDM)

MCGDM problems have traditionally been addressed by applying a selection process to choose the best alternative or subset of alternatives based on multiple or conflicting criteria. These problems are generally categorized as multiple criteria discrete alternative problems. Different models have been proposed in the literature to solve diverse MCGDM problems. With respect to a common MCGDM problem in which the criterion values are set in different forms by DMs or experts and the weight information on criteria and that on DMs are both unknown, the process of addressing the MCGDM problem mainly includes, but is not limited to, the following steps [24], [56]:

Step1: Establishing individual decision matrices in groups and normalizing these matrices.

Step2: Determining the weights of criteria and DMs.

Step3: Fusing individual normalized decision matrices into a collective decision matrix with the utilization of an appropriate information aggregation operator in which the criterion weights are embedded.

Step4: Using an appropriate decision-making method to rank alternatives and select the best one.

The MCGDM models in the literature have concentrated on the following research topics:

1) THE ARTICULATION AND REPRESENTATION OF INDIVIDUAL EVALUATION INFORMATION

During an MCGDM process, each DM is required to provide individual evaluation information, which would constitute a decision matrix or a preference matrix. The elements of a decision matrix comprise the performance evaluation information of alternatives on each criterion, while the elements of a preference matrix comprise the preference evaluation information obtained by pairwise comparisons of each two alternatives with regard to a criterion [17], [18], [68]. The evaluation information in these two types of matrix takes different formats, such as real numbers (crisp), 2-tuple models, fuzzy numbers, or linguistic information [13], [15], [25], [53], [62]. Owing to the increasing complexity and uncertainty of the decision-making environment, DMs prefer to articulate evaluation information in the form of fuzzy sets. Since its creation by Zadeh [88], the fuzzy set and its extended forms (such as intuitionistic fuzzy sets [24], [33], [35], [36], [39], [40]; hesitant fuzzy sets [10], [53]; type-2 fuzzy sets [51]; neutrosophic sets; and Pythagorean fuzzy sets) have occupied a prominent place in the theoretical development and practical application of MCDM and MCGDM [52], [54]. In the proposed MCGDM model, the IFNs are utilized by DMs to characterize uncertain evaluation information owing to their strong application potential.

2) THE DETERMINATION OF WEIGHTS OF DMS AND CRITERIA

Assigning weights to criteria and DMs is a basic step in building an MCGDM model, but the literature related to

MCGDM places more focus on deriving the weights of criteria than on determining the weights of DMs. The existing methods for deriving weights of DMs and criteria can be classified into the following three categories [21], [23]: subjective methods; objective methods; and integrated methods.

- (i) In subjective methods, the weights of criteria are obtained depending on the DMs' preference information on criteria, while the weights of DMs are assigned by a supervisor or by a mutual evaluation of the DMs depending on expert knowledge and experiences. The subjective methods include analytic hierarchy process (AHP) [51],[53], analytic network process (ANP), and Delphi method.
- (ii) In objective methods, the weights of DMs and criteria are acquired depending on the objective decision information. The objective methods for DM weights encompass similarity-based approaches, consensus-based approaches, and consistency-based approaches [23]. The objective methods for criteria weights include the entropy-based method, the standard deviation (SD) method, the criteria importance through inter-criteria correlation (CRITIC) method, the correlation coefficient and standard deviation (CCSD) method, the ideal point (IP) method, and the maximizing deviation (MD) method [48]
- (iii) In integrated methods, the weights of DMs or criteria are determined by integrating several methods, especially the combination of a subjective method and an objective method.

In this paper, with the aim of handling the impacts of the interactions among DMs on the weights of DMs and criteria, we design an integrated method to simultaneously specify the weights of DMs and criteria, based on a novel IFE measure, 2-additive fuzzy measure, and Choquet integral.

3) THE AGGREGATION OF INDIVIDUAL EVALUATION INFORMATION

Aggregating several numerical values to output a single representative value is an instrumental step in building an MCGDM model [14]. This necessitates the construction or selection of aggregation functions in accordance with the specific requirements of diverse information aggregation applications. Various families of aggregation functions have been put forward in the literature, such as the family of traditional averaging aggregation functions [33], the family of OWA-type aggregation functions [53], and the family of Bonferroni mean (BM) type aggregation functions [11], [37], [82], [84]. Among the numerous families of aggregation functions, the BM and its extensions has been identified as the one useful aggregation function owing to its ability to model the homogeneous interrelationships among the given arguments in decision-making contexts. Xu [82] extended the BM to an IFNs context and developed the intuitionistic fuzzy Bonferroni mean (IFBM) and the weighted IFBM (WIFBM). Xia *et al.* [84] proposed the concept of the geometric Bonferroni mean (GBM) to generalize BM [12] and the geometric

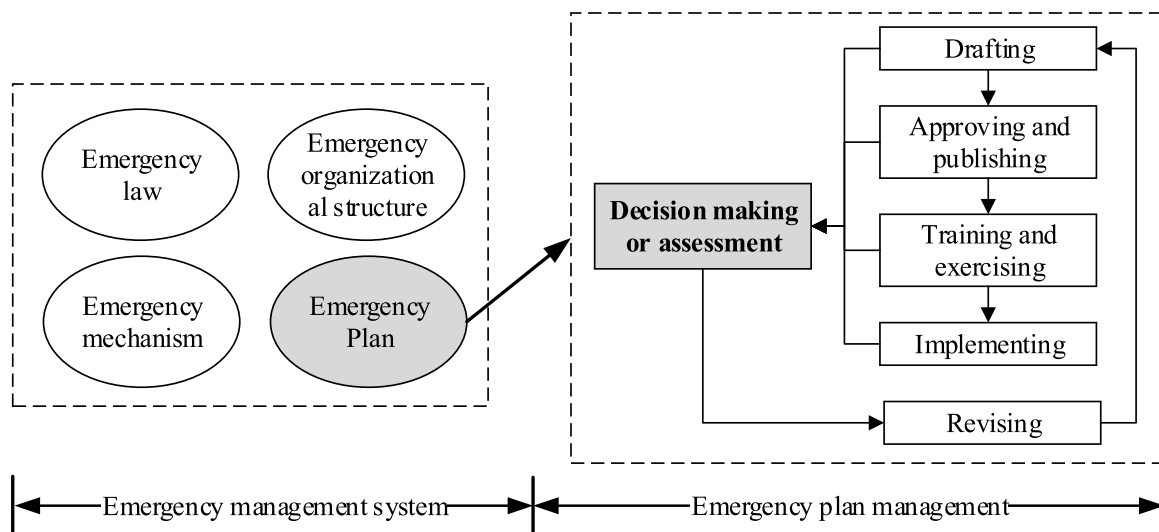


FIGURE 1. Emergency management system and emergency plan management.

mean [40], and further extended the BM to the intuitionistic fuzzy GBM (IFGBM) and the weighted version (WIFGBM). Thus, the proposed MCGDM model introduces the WIFBM and the WIFGBM to synthesize the individual evaluation information.

4) THE RANKING AND SELECTION OF ALTERNATIVES

The literature describes many MCDM methods for ranking or selecting alternatives and each method has its own characteristics. These methods can be categorized into four main groups [50]: non-compensatory methods; value-based methods; analytic hierarchical process (AHP) methods; and outranking methods. Compared with the other categories of MCDM methods, value-based methods such as simple additive weighting (SAW) [50], Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [24], [28], [31], [35], and Višekriterijumsko Kompromisno Rangiranje (VIKOR) [75], are the most popular methods in the academic and application fields. Furthermore, SAW, in which the overall performance evaluations of each alternative obtained by aggregation functions serve as the basis for ranking alternatives, is the simplest and most widely used MCDM method among value-based methods [50]. Given the interrelationships among the evaluation information of each alternative on criteria, the SAW method based on WIFBM and WIFGBM is applied in this paper to rank alternatives.

B. EMERGENCY PLAN DECISION MAKING

In recent decades, natural, technological, or human-induced incidents and disasters have increased in frequency, scope, complexity, and severity, and have exerted severely negative impacts on social and economic development. This has necessitated the scientific management of various emergencies to eliminate or reduce the effects. Emergency management as

an interdisciplinary research area has attracted considerable interest from researchers and practitioners [6], [9], [29], [63], [86]. In China, an emergency management system, consisting of emergency law, emergency organizational structure, emergency mechanisms, and emergency plans, has been developed at each level of a public organization to cope with emergencies [30], [87]. In the system, emergency plan (EP) not only provides guidance for stakeholders by specifying the activities, the resources required, and the assignment of responsibility for managing emergencies, but can also help discover unrecognized risks or deficiencies and promote safety awareness among stakeholders. The EP management process is a dynamic and cyclical process consisting of drafting, approving and publishing, training and exercising, implementing, and revising, in which decision making is reliant on evaluation or assessment and is deemed to be one of the cornerstones of EP management [41]. Conducting EP decision making or assessment can help identify strengths, weaknesses, and opportunities for the improvement of EP [5]. Fig. 1 depicts the structures of the emergency management system and EP management. In this section, we concentrate on the application of the proposed hybrid MCGDM model to EP decision making.

To date, the practitioners and researchers who focus on EP decision making or assessment mainly prefer to structure guidelines or standards for EP development, but seldom provide a consolidated model for the assessment of EPs or decision making. The Federal Emergency Management Agency (FEMA) [29] provided a series of criteria for successful planning which included acceptability, adequacy, completeness, consistency and standardization of products, feasibility, flexibility, interoperability, and collaboration. Alexander [1] put forward 18 principles for the judgment of EP quality. Perry and Lindell [65] presented 10 guidelines for emergency planning. Lindell and Perry [57] identified a set of general

TABLE 1. The criteria for emergency plan decision making.

Criteria	Content	References
Completeness	An EP should cover all the phases of emergency management consisting of mitigation, preparedness, response, and recovery. A complete EP should also incorporate the personnel and resources required and sound concepts for how those will be deployed, employed, sustained, and demobilized.	[1],[5],[16],[29]
Appropriateness	The activities involved must be established not only depending on the accurate knowledge of the hazard derived from risk assessment and vulnerability assessment, but also in accordance with practical experience and relevant laws and regulations, in order to guarantee the appropriateness of these activities.	[1],[5],[16],[29],[65]
Responsibility and coordination	The involvement of emergency managers, law enforcement, hospitals, public health departments, the military, and a host of other organizations necessitates the clear assignment of responsibilities to each group and the mechanisms for communication and coordination.	[1],[5],[16],[29],[65]
Flexibility	The plan should focus on the principles of response rather than trying to elaborate the process to include specific details. It should accommodate all hazards ranging from small-scale incidents to wider national contingencies.	[1],[5],[16],[29],[65]
Feasibility	The activities in an EP should be developed according to the availability of diverse resources, including human, materials, and finance, to guarantee the feasibility of these activities.	[1],[5],[16],[29]
Acceptability	An EP should meet the requirements of anticipated scenarios and should be implemented within the costs and timeframes that the stakeholders or the public can support.	[1],[5],[29]

criteria for the evaluation of emergency response plans which fall into two categories: substantive criteria and supporting criteria. Lumbroso et al. [58] developed a series of metrics for assessment of flood EPs, and further put forward a systematic method named as the FIM FRAME method for the assessment and improvement of EPs [59]. The method includes the following three phases: a quick assessment phase to evaluate the content and quality of a plan; a more detailed analytical phase for the specification of specific issues; and an implementation phase for the improvement of a plan. Ren et al. [66] provided 30 indicators for assessing the effectiveness of an earthquake EP and further developed a model based on the hesitant analytic hierarchy process for EP assessment. Girard et al. [42] introduced the Function-Interactions-Structure (FIS) to conduct the assessment of an EP under multi-state degradation. Cheng and Qin [16] introduced a fuzzy comprehensive assessment model to conduct EP decision making using the criteria of completeness, operability, effectiveness, flexibility, rapidity, and rationality of responsibility assignment.

This review of the literature shows that there are only a few studies that concentrate on the development of an EP decision-making model. It also shows that those studies have acknowledged the necessity of including fuzzy set theory in the construction of an EP decision-making model. This necessity provides further motivation for developing a consolidated MCGDM model which could then be applied to the EP decision-making problem. Before developing the model, we specify a series of criteria for EP decision making by reviewing the existing studies and regulations related to EP. These are depicted in Table 1.

III. PRELIMINARIES

This section introduces the basic notions, definitions, and properties of IFS. It also presents the axiomatic definition of intuitionistic fuzzy entropy, the WIFBM, the WIFGBM, 2-additive Fuzzy Measure, and the Choquet integral.

A. INTUITIONISTIC FUZZY SETS

Definition (1) [2], [3]: Let X be a non-empty set called the universe of discourse. An IFS A defined on X according to Atanassov's definition is

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \},$$

where $\mu_A(x) : X \rightarrow [0, 1]$ and $\nu_A(x) : X \rightarrow [0, 1]$ are called respectively the membership degree and non-membership degree of x to A with the condition $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for all $x \in X$. Now define $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all $x \in X$. This value is called the hesitancy degree of x to A , which is also a mapping from X to $[0, 1]$ and satisfies $\pi_A(x) \in [0, 1]$.

In particular, if $\pi_A(x) = 0$ for all $x \in X$, then an IFS is degenerated to a fuzzy set in the sense that every fuzzy set can be viewed as a special case of IFS. For convenience, the set $IFS(X)$ is used to stand for all IFSs in X . To facilitate further analysis, each pair of $(\mu_A(x), \nu_A(x))$ in A is called an IFN [82]. We denote an IFN by $\alpha = (\mu_\alpha, \nu_\alpha)$, where

$$\mu_\alpha, \nu_\alpha \geq 0, \mu_\alpha + \nu_\alpha \leq 1, \pi_\alpha = 1 - \mu_\alpha - \nu_\alpha,$$

and $S_\alpha = \mu_\alpha - \nu_\alpha$ and $H_\alpha = \mu_\alpha + \nu_\alpha$ are called the score and accuracy degree, respectively, of the IFN α . To rank any two IFNs $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ ($i = 1, 2$), Xu [81] and Xu and Yager [82] proposed the following order relation between any two IFNs.

Definition 2: Let $S_{\alpha_i} = \mu_{\alpha_i} - \nu_{\alpha_i}$ be the score of α_i ($i = 1, 2$) and $H_{\alpha_i} = \mu_{\alpha_i} + \nu_{\alpha_i}$ be the accuracy degree of α_i ($i = 1, 2$).

- 1) If $S_{\alpha_1} > S_{\alpha_2}$, then $\alpha_1 > \alpha_2$.
- 2) If $S_{\alpha_1} = S_{\alpha_2}$, then
 - 2a) If $H_{\alpha_1} = H_{\alpha_2}$, then $\alpha_1 = \alpha_2$;
 - 2b) If $H_{\alpha_1} > H_{\alpha_2}$, then $\alpha_1 > \alpha_2$.

Xu [81] and Xu and Yager [82] developed the following basic operations of the IFNs $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i})$ ($i = 1, 2$) and $\alpha = (\mu_\alpha, \nu_\alpha)$, which are employed in the next section.

- 1) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1}\nu_{\alpha_2})$.
- 2) $\alpha_1 \otimes \alpha_2 = (\mu_{\alpha_1}\mu_{\alpha_2}, \nu_{\alpha_1} + \nu_{\alpha_2} - \nu_{\alpha_1}\nu_{\alpha_2})$.

- 3) $\lambda\alpha = (1 - (1 - \mu_\alpha)^\lambda, \nu_\alpha)^\lambda, \lambda > 0.$
- 4) $\alpha^\lambda = (\mu_\alpha^\lambda, 1 - (1 - \nu_\alpha)^\lambda), \lambda > 0.$

All the results of the aforementioned operations are also IFNs. No loss of information occurs for operations 1) to 4) when aggregating the arguments. These operations are also capable of dealing with situations where the importance of the arguments must be considered. The classic axiomatic definition of intuitionistic fuzzy entropy is also introduced to facilitate further analysis.

Definition (3) [8]: Let $A, B \in IFS(X)$. The intuitionistic fuzzy entropy measure is a real-valued function $E(A) : IFS(X) \rightarrow [0, 1]$ that satisfies the following axiomatic principles:

- 3.1) $E(A) = 0$ iff A is a crisp set, that is, $\{\mu_A(x_i) = 1 \text{ and } \nu_A(x_i) = 0\}$ or $\{\mu_A(x_i) = 0 \text{ and } \nu_A(x_i) = 1\}$ for all $x_i \in X$.
- 3.2) $E(A) = 1$ iff $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$.
- 3.3) $E(A) = E(A^c)$ for all $x_i \in X$.
- 3.4) $E(A) \leq E(B)$ if A is less fuzzy than B ; that is, for all $x_i \in X$:
 - $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$; and
 - $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$.

Xu and Yager [82] proposed the WIFBM based on the BM. Xia et al. [84] further extended the BM by proposing the WIFGBM, based on the usual geometric mean and the BM. Some related concepts are introduced below.

Definition (4) [82]: Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}) (i = 1, 2, \dots, n)$ be a collection of IFNs and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of $\alpha_i (i = 1, 2, \dots, n)$, where ω_i indicates the importance degree of α_i that satisfies $\omega_i > 0 (i = 1, 2, \dots, n)$ and $\sum_{i=1}^n \omega_i = 1$. If

$$IFB_{\omega}^{p,q}(\alpha_1, \alpha_2, \dots, \alpha_n) = \left(\frac{1}{n(n+1)} \left(\bigoplus_{i,j=1; i \neq j}^n ((\omega_i \alpha_i)^p \otimes (\omega_j \alpha_j)^q) \right) \right)^{\frac{1}{p+q}} \quad (1)$$

for any $p, q > 0$, then $IFB_{\omega}^{p,q}$ is called the WIFBM.

Definition (5) [84]: Let $\alpha_i = (\mu_{\alpha_i}, \nu_{\alpha_i}) (i = 1, 2, \dots, n)$ be a collection of IFNs and $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ be the weight vector of $\alpha_i (i = 1, 2, \dots, n)$, where ω_i indicates the importance degree of α_i that satisfies $\omega_i > 0 (i = 1, 2, \dots, n)$ and $\sum_{i=1}^n \omega_i = 1$. If

$$IFGB_{\omega}^{p,q}(\alpha_1, \alpha_2, \dots, \alpha_n) = \frac{1}{p+q} \left(\bigotimes_{i,j=1; i \neq j}^n (p\alpha_i^{\omega_i} \oplus q\alpha_j^{\omega_j})^{\frac{1}{n(n-1)}} \right) \quad (2)$$

for any $p, q > 0$, then $IFGB_{\omega}^{p,q}$ is called the WIFGBM.

B. 2-ADDITIVE FUZZY MEASURE AND CHOQUET INTEGRAL

We consider a finite referential set $Y = \{1, 2, \dots, n\}$ of n elements. This set is usually regarded as criteria in the MCGDM problem. Let us denote the set of subsets of Y by $P(Y)$. Subsets of Y are denoted by A, B, \dots

Definition (6) [45]: A fuzzy measure, non-additive measure, or capacity over Y , is a set function $\mu : P(Y) \rightarrow [0, 1]$ that satisfies the following:

- (i) $\mu(\emptyset) = 0, \mu(Y) = 1$ (boundary conditions).
- (ii) $\forall A, B \in P(Y)$, if $A \subseteq B$, then $\mu(A) \leq \mu(B)$ (monotonicity).

The set of all fuzzy measures on Y is denoted by \mathcal{Y} . As usual, algebras and σ -algebras are required in definitions on general spaces but not in the discrete case.

In the framework of MCGDM, the number $\mu(A)$ can be interpreted as the importance of the subset $A \subseteq X$, and the monotonicity (condition (ii) in Definition 6) of the fuzzy measure means that the importance of a subset of criteria (or DMs) cannot decrease when new criteria (or DMs) are added to it. A fuzzy measure is mainly characterized by its non-additivity, which enables it to represent flexibly the heterogeneous interactions among decision criteria (or DMs), ranging from redundancy (negative interaction) to synergy (positive interaction) [43]–[45].

The pseudo-Boolean function is an important concept that is often used to represent a set function. Moreover, this function can represent a fuzzy measure, which is characterized by exponential complexity.

Definition (7) [47]: A pseudo-Boolean function is a real-valued function $f : \{0, 1\}^n \rightarrow \mathfrak{R}$.

Any pseudo-Boolean function can be expressed in the form of a multilinear polynomial in n variables given by:

$$f(y) = \sum_{T \subset Y} \left[a(T) \prod_{i \in T} y_i \right], \quad (3)$$

with $a(T) \in \mathfrak{R}$, and $y = (y_1, \dots, y_n) \in \{0, 1\}^n$.

A fuzzy measure can be seen as a particular case of the pseudo-Boolean function that is defined for any $A \subset Y$ such that A is equivalent to a point $y = (y_1, \dots, y_n)$ in $\{0, 1\}^n$, where $y_i = 1$ iff $i \in A$.

The set of coefficients $a(T), T \subset Y$ can be viewed as a set function, which in fact corresponds to the Möbius transform. If we denote any set function $\mu : P(Y) \rightarrow \mathfrak{R}$ by μ , then the Möbius transform of μ is a set function a on Y defined by [45] as follows:

$$a(T) = \sum_{K \subset T} (-1)^{|T \setminus K|} \mu(K), \forall T \subset Y. \quad (4)$$

This transformation is invertible. When a is given, we can recover the original μ using the so-called Zeta-transform, which is given by:

$$a(T) = \sum_{S \subset T} \mu(S), \forall T \subset Y. \quad (5)$$

Let us consider the case of additive measures. Following Eq. (1), we know that additive measures have a linear representation given by $f(y) = \sum_{i=1}^n a_i y_i$, where $\mu_i \equiv a_i$ and the notations $\mu_i = \mu(\{i\})$ and $a_i = a(\{i\})$ are used. By extension, fuzzy measures with a polynomial representation of the degree of any fixed integer k can be defined.

The k -order additive fuzzy measure (or simply k -additive measure) is defined below.

Definition (8) [45]: A fuzzy measure μ defined on Y is said to be a k -order additive if its corresponding pseudo-Boolean function is a multilinear polynomial of degree k ; that is, its Möbius transform is $a(T) = 0$ for all T such that $|T| > k$, and \exists at least one subset T of Y with exactly k elements such that $a(T) \neq 0$.

An extremely important case of additive fuzzy measures is when k takes the value of 2. The 2-additive fuzzy measure is defined by:

$$\mu(K) = \sum_{i=1}^n a_i y_i + \sum_{\{i,j\} \subseteq Y} a_{ij} y_i y_j. \quad (6)$$

For any $K \subseteq Y$, $|K| \geq 2$ with $y_i = 1$ if $i \in K$; otherwise, $y_i = 0$. With $\mu_i = a_i$ for all i , the following expression is obtained:

$$\mu_{ij} = a_i + a_j + a_{ij} = \mu_i + \mu_j + a_{ij}. \quad (7)$$

The general formula of the 2-additive fuzzy measure is:

$$\mu(K) = \sum_{i \in K} a_i + \sum_{\{i,j\} \subseteq K} a_{ij} = \sum_{\{i,j\} \subseteq K} \mu_{ij} - (|K| - 2) \sum_{i \in K} \mu_i \quad (8)$$

for any $K \subseteq Y$ such that $|K| \geq 2$. The 2-additive fuzzy measure is clearly determined by the coefficients μ_i and μ_{ij} .

When using the 2-additive fuzzy measure to model the importance of the subsets of criteria (or DMs), a suitable aggregation function is the Choquet integral [79].

Definition (9) [79]: Let $\mu : F \rightarrow [0, 1]$ be a fuzzy measure on a measurable space (X, F) and $f : X \rightarrow [0, \infty)$ be a measurable function. The Choquet integral of f with respect to μ is defined by:

$$C_\mu(f) = (C) \int f \circ \mu = \int_0^\infty \mu(H_\alpha) d\alpha, \quad (9)$$

where $H_\alpha = \{x \in Y | f(x) \geq \alpha\}$, $\forall \alpha \in [0, 1]$.

Suppose that the function $f(x)$ is discrete on $X = [x_1, x_2, \dots, x_n]$, and let the value of a function at a point $x_i \in Y$ be denoted by f_i . Consider a permutation of the function values in increasing order denoted by $f(x_{(1)}), \dots, f(x_{(n)})$, with $f(x_{(0)}) = 0$, $X_{(i)} = \{x_{(i)}, x_{(i+1)}, \dots, x_{(n)}\}$, and $x_{(n+1)} = \emptyset$. The Choquet integral can then be written as follows [79]:

$$C_\mu(f) = \sum_{i=1}^n (f(x_{(i)}) - f(x_{(i-1)})) \mu(X_{(i)}) \quad (10)$$

or

$$C_\mu(f) = \sum_{i=1}^n (\mu(X_{(i)}) - \mu(X_{(i+1)})) f(x_{(i)}). \quad (11)$$

The Choquet integral has some well-known properties that suit information aggregation; these properties include idempotence, compensativeness, and comonotonic additivity [79].

IV. A NOVEL IFE MEASURE AND COMPARISON ANALYSIS

Since its introduction by Zadeh [90], the concept of fuzzy entropy, a measure of uncertainty of FSSs, has captured a great deal of attention from large numbers of researchers who have proposed axiomatic structures for entropy and quantified the entropy measure for FS as well as IFS [32], [38]. Firstly, De Luca and Termini [20] axiomatized the structure of fuzzy entropy based on the concept of Shannon's entropy [69]. Subsequently, Burillo and Bustince [8] presented an axiomatic definition of an IFE measure and provided a group of IFE measures. Garg [34] proposed a generalized entropy measure for interval-valued IFSs (IVIFSs) and put forward a family of fuzzy entropies for IVIFSs which can be extended to IFSs. However, the entropy measures for IFSs proposed by Burillo and Bustince [8] and Garg [34] factor in only one aspect of uncertainty which amounts to the hesitancy degree and is known as the intuitionism of IFS [8]. Szmidi and Kacprzk [72] formulated a new axiomatic definition of entropy for IFSs and proposed another IFE based on a geometric interpretation of IFSs. As an extension of the definitions in [72], Szmidi and Kacprzk [73] proposed a distance-based entropy measure for IFSs. However, the IFE proposed by [73] focuses only on the other aspect of uncertainty induced by the deviation from the membership degree to the non-membership degree, which is known as the fuzziness of IFS. A number of researchers have reached a consensus that the IFE measure should simultaneously take into account the intuitionism and fuzziness of IFS [60], [64], [78]. Wei et al. [78] and Mao et al. [60] proposed new IFEs respectively, simultaneously taking into account these two aspects. Mao et al. [60] further pointed out that the entropy value increases along with enhanced fuzziness under the same intuitionism, and decreases along with weakened intuitionism under the same fuzziness. In this section, we propose a novel axiomatic definition of IFE and present a novel IFE based on this axiom. We then conduct a comparison with existing IFE measures to validate the advantages of the proposed IFE measure.

A. A NOVEL IFE MEASURE

Firstly, suppose the universe X is a finite set; that is, $X = \{x_1, x_2, \dots, x_n\}$, where $n \geq 1$ and $n \in N^+$. Let $\phi_A(x_i) = |\mu_A(x_i) - \nu_A(x_i)|$ for an IFS $A (\in IFS(X))$, which can be interpreted as the balance of power between support and opposition in a voting model [60].

Definition 10: Let $A \in IFS(X)$. We then define $\phi_A(x_i)$ as the deviation of A from the membership degree $\mu_A(x_i)$ to the non-membership degree $\nu_A(x_i)$ for all $x_i \in X$, and we define $\mu_A(x_i) \nu_A(x_i)$ as the linear product of $\mu_A(x_i)$ and $\nu_A(x_i)$ in terms of A for all $x_i \in X$.

Using Definition 3 as a basis, we propose a new axiomatic definition of the IFE measure.

Definition 11: Let $A, B \in IFS(X)$. The intuitionistic fuzzy entropy measure is a real-valued function $E(A)$ of $\pi_A(x_i)$ and $\phi_A(x_i)$; that is,

$E(A) = f(\pi_A(x_i), \phi_A(x_i)) : IFS(X) \rightarrow [0, 1]$, where $E(A)$ satisfies the following axiomatic principles:

(E1) $E(A) = 0$ iff A is a crisp set; that is, $\{\mu_A(x_i) = 1 \text{ and } \nu_A(x_i) = 0\}$ or $\{\mu_A(x_i) = 0 \text{ and } \nu_A(x_i) = 1\}$ for all $x_i \in X$.

(E2) $E(A) = 1$ iff $\pi_A(x_i) = 1$ or $\phi_A(x_i) = 0$ for all $x_i \in X$.

(E3) $E(A) = E(A^c)$ for all $x_i \in X$.

(E4) $E(A) \leq E(B)$ holds in the following two situations:

a) When A is less fuzzy than B ; that is, for all $x_i \in X$:

$\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$; and

$\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$.

b) When $Y \subset X$ such that, for $x_i \in Y$,

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu'_A(x_i), \nu'_A(x_i) \rangle,$$

where $\phi'_A(x_i) = \phi_B(x_i)$ and $\pi'_A(x_i) < \pi_B(x_i)$, and

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \text{ for } x_i \notin Y.$$

Situation a) in Definition 11 shows that $f(\pi_A(x_i), \phi_A(x_i))$ is a real-valued continuous function that decreases with respect to $\phi_A(x_i)$ for all $x_i \in X$ when the value of $\pi_A(x_i)$ is fixed. The entropy value thus decreases along with enhanced fuzziness under the same intuitionism. Meanwhile, situation b) shows that $f(\pi_A(x_i), \phi_A(x_i))$ increases with respect to $\pi_A(x_i)$ for all $x_i \in X$ when the value of $\phi_A(x_i)$ is fixed, thus indicating that the entropy value increases along with enhanced intuitionism under the same fuzziness.

On the basis of the concepts defined in Definition 11, we can discuss some properties of intuitionistic fuzzy entropy.

Proposition 1: Let $A, B \in IFS(X)$ and let $Y \subset X$ such that, for $x_i \in Y$,

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu'_A(x_i), \nu'_A(x_i) \rangle,$$

where $\phi'_A(x_i) \geq \phi_B(x_i)$ and $\pi'_A(x_i) = \pi_B(x_i)$; and, for $x_i \notin Y$,

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle.$$

We then obtain $E(A) \leq E(B)$.

Proof: See Appendix A.

Proposition 2: Let $A, B \in IFS(X)$ and let $Y \subset X$ such that, for $x_i \in Y$,

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu'_A(x_i), \nu'_A(x_i) \rangle,$$

where $\pi'_A(x_i) = \pi_B(x_i)$ and $\mu'_A(x_i) \nu'_A(x_i) < \mu_B(x_i) \nu_B(x_i)$; and, for $x_i \notin Y$,

$$\langle x_i, \mu_B(x_i), \nu_B(x_i) \rangle = \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle.$$

We then obtain $E(A) \leq E(B)$.

Proof: See Appendix B.

Theorem 1: Proposition 2 is equivalent to situation a) of principle (E4) in Definition 11.

Following the proofs of Proposition 1 and Proposition 2, we can easily obtain this theorem. We know from Theorem 1 that the deviation between the membership degree $\mu_A(x_i)$ and the non-membership degree $\nu_A(x_i)$ or the linear

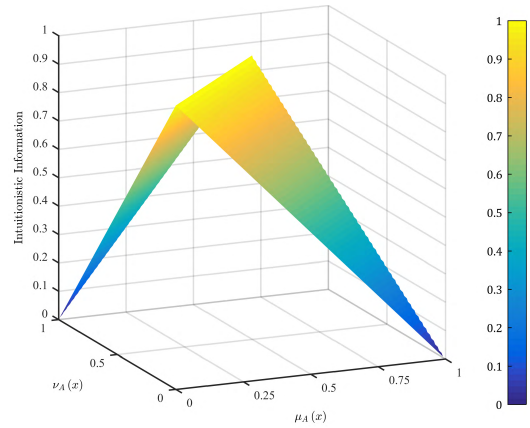


FIGURE 2. Variations in the intuitionism of A . with respect to $\mu_A(x)$ and $\nu_A(x)$.

product of $\mu_A(x_i)$ and $\nu_A(x_i)$ can both reflect the fuzziness of an IFS. We define a novel intuitionistic fuzzy entropy measure based on the above analysis.

Theorem 2: For an IFS $A \in IFS(X)$, then the equation Eq. (12), as shown at the bottom of the next page, is an intuitionistic fuzzy entropy measure.

Proof: See Appendix C.

The uncertain information of IFSs generally consists of intuitionistic information and fuzzy information. Based on the intuitionistic fuzzy entropy $E(A)$, the intuitionistic information and the fuzzy information of A are given by

$$f(x_i) = 1 - |\mu_A(x_i) - \nu_A(x_i)| = 1 - \phi_A(x_i) \quad (13)$$

and

$$\begin{aligned} g(x_i) &= 1 + \pi_A(x_i) (\mu_A(x_i) - \nu_A(x_i))^2 \\ &= 1 + \pi_A(x_i) \phi_A^2(x_i) \end{aligned} \quad (14)$$

respectively.

Here, we treat $g(x_i)$ as a form that describes the intuitionism of A and $f(x_i)$ as a form that depicts the fuzziness of A . Suppose that the universe X has only one object; that is, $X = \{x\}$. We then determine the variations in the intuitionism and fuzziness of A with respect to $\mu_A(x)$ and $\nu_A(x)$. These variations are illustrated in Figs. 2 and 3.

As shown in Figs. 2 and 3, the real-valued function $g(x_i)$ measures the uncertain information caused by the intuitionism in an IFS, whereas the real-valued function $f(x_i)$ depicts the uncertain information caused by fuzziness. The two real-valued functions are reviewed as effective tools for interpreting properly how IFE measures the uncertain information of IFSs.

B. COMPARISON ANALYSIS

This subsection presents a comparison analysis to demonstrate some prominent characteristics of the proposed measure and its advantages.

The family of fuzzy entropies provided by Garg [34] is analogous to those proposed by Burillo and Bustince [8] but focuses only on the intuitionism of an IFS. We conduct an

TABLE 2. The comparison of different intuitionistic fuzzy entropy measures.

IFS	$\phi_A(x_i)$	$\pi_A(x_i)$	Intuitionistic fuzzy entropy measures					
			$E(A)$	$E_G(A)$	$E_{SK1}(A)$	$E_{SK2}(A)$	$E_W(A)$	$E_M(A)$
$A_1 = \{ \langle x, 0.4, 0.4 \rangle \}$	0	0.2	1	0.023	1	1	1	0.6
$A_2 = \{ \langle x, 0.5, 0.3 \rangle \}$	0.2	0.2	0.806	0.023	0.714	0.9	0.926	0.31
$A_3 = \{ \langle x, 0.6, 0.2 \rangle \}$	0.4	0.2	0.619	0.023	0.5	0.8	0.733	0.196
$A_4 = \{ \langle x, 0.4, 0.2 \rangle \}$	0.2	0.4	0.813	0.105	0.75	0.9	0.933	0.41
$A_5 = \{ \langle x, 0.3, 0.1 \rangle \}$	0.2	0.6	0.819	0.271	0.778	0.9	0.943	0.51
$A_6 = \{ \langle x, 0.3, 0.3 \rangle \}$	0	0.4	1	0.105	1	1	1	0.7
$A_7 = \{ \langle x, 0.5, 0.5 \rangle \}$	0	0	1	0	1	1	1	0.5
$A_8 = \{ \langle x, 0.6, 0.4 \rangle \}$	0.2	0	0.8	0	0.667	0.9	0.923	0.21
$A_9 = \{ \langle x, 0.8, 0.2 \rangle \}$	0.6	0	0.4	0	0.25	0.7	0.471	0.036

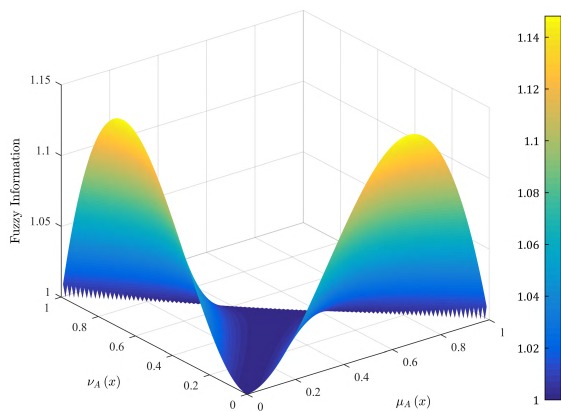


FIGURE 3. Variations in the fuzziness of A with respect to $\mu_A(x)$ and $\nu_A(x)$.

equivalence conversion from one of the fuzzy entropy measures for IVIFSs proposed by Garg [34] to an IFE measure presented in Eq. (15), as shown at the bottom of the next page.

Szmidt and Kacprzk provided an IFE measure in [72], which results from the geometric interpretation of IFSs and is exhibited in Eq. (16). In Eq. (16), as shown at the bottom of the next page, for each i , A_i denotes the single-element in an IFS corresponding to the element x_i in X , and is described as $A_i = \{ \langle x_i, \mu_A(x_i), \nu_A(x_i) \rangle \}$. They then came up with another novel IFE measure depicted in Eq. (17), as shown at the bottom of the next page, which factors in only the fuzziness of IFS [73].

Wei et al. [78] and Mao et al. [60] introduced novel IFEs respectively which focus simultaneously on the fuzziness

and intuitionism of IFS and are represented by Eq. (18) and Eq. (19), as shown at the bottom of the next page.

We present an example below to illustrate the difference between the proposed IFE measure $E(A)$ and the existing measures $E_G(A)$, $E_{SK1}(A)$, $E_{SK2}(A)$, $E_M(A)$ and $E_W(A)$.

Example 1: Suppose the universe $X = \{x\}$ has the 9 IFSs $\{A_1, A_2, \dots, A_9\}$ that are presented in Table 2. We can make use of the intuitionistic fuzzy entropy measures provided above to calculate the entropy values of the nine IFSs. The entropy values are shown in Table 2.

1) The comparison of $E(A)$ with $E_G(A)$.

From Table 2, we see that $E_G(A)$ cannot distinguish A_7 , A_8 , and A_9 , owing to the fact that $E_G(A)$ focuses only on the intuitionism of IFS and the three IFSs have the same degree of hesitancy or intuitionism. It is apparent that the deviation degree of the supportability and opposability of A_9 is larger than that of A_7 and A_8 , indicating that the uncertainty degree of A_9 is larger than that of A_7 and A_8 . In contrast, the proposed IFE measure $E(A)$ produces the result $E(A_9) > E(A_8) > E(A_7)$, indicating that $E(A)$ is inconsistent with human intuition and that $E(A)$ performs better than $E_G(A)$.

2) The comparison of $E(A)$ with $E_{SK2}(A)$.

Table 2 indicates that $E_{SK2}(A)$ cannot tell the difference between A_2 , A_4 , and A_5 . Although A_2 , A_4 , and A_5 take the same value in the deviation degree of supportability and opposability, it is apparent that A_5 is more uncertain than A_4 and A_2 because of the larger hesitancy degree embedded in A_5 . Applying $E(A)$ produces the result $E(A_5) > E(A_4) > E(A_2)$, which accords with human intuition.

$$\begin{aligned}
 E(A) &= \frac{1}{n} \sum_{i=1}^n (1 - |\mu_A(x_i) - \nu_A(x_i)|) \left[1 + (1 - \mu_A(x_i) - \nu_A(x_i)) (\mu_A(x_i) - \nu_A(x_i))^2 \right] \\
 &= \frac{1}{n} \sum_{i=1}^n (1 - \phi_A(x_i)) (1 + \pi_A(x_i) \phi_A^2(x_i))
 \end{aligned} \tag{12}$$

3) The comparison of $E(A)$ with $E_M(A)$.

From Table 2, we see that $A_1, A_6,$ and A_7 take the same value 1 when $E(A)$ is applied, whereas $E_M(A)$ produces different results on $A_1, A_6,$ and A_7 . The main reason is that $E(A)$ is constructed according to the principle (E2) in Definition 11, which points out that $E(A) = 1$ iff $\pi_A(x_i) = 1$ OR $\phi_A(x_i) = 0$ for all $x_i \in X$. However, the corresponding constructive principle for $E_M(A)$ is that $E(A) = 1$ iff $\pi_A(x_i) = 1$ AND $\phi_A(x_i) = 0$ for all $x_i \in X$. We hold that the principle (E2) in Definition 11 is essential for the construction of an IFE measure owing to the fact that entropy is introduced to measure the uncertainty degree, whereas the counterpart held by $E_M(A)$ is appropriate to measure the amount of information embedded in an IFS which has been called the knowledge measure of an IFS [19], [46], [74]. Essentially, the entropy measure and the knowledge measure differ significantly from a practical point of view [19], [46], [74].

4) The comparison of $E(A)$ with $E_{SK1}(A)$ and $E_W(A)$.

Firstly, we convert $E_{SK1}(A)$ to the formula $E'_{SK1}(A)$ depicted in Eq. (20), as shown at the bottom of this page, which has been proved to be equivalent to $E_{SK1}(A)$ by Wei et al. [77]. From Eq. (20), we find that $E'_{SK1}(A)$ and $E_W(A)$ take a similar form. Table 2 indicates that it is hard to tell the difference between $E(A), E_{SK1}(A),$ and $E_W(A)$. However, if we fix the intuitionism of the three IFE measures, we find that $E(A)$ is a linear function and is more uniform than $E_{SK1}(A)$ and $E_W(A)$. Thus, we hold that the proposed IFE measure is more consistent with human intuition. To further facilitate the discussion, consider the universe $X = \{x\}$ for an IFS $A = \{(x, \mu_A(x), \nu_A(x))\}$. The variations in the entropy values of A with respect to $\mu_A(x)$ and $\nu_A(x)$ within the range of $[0, 1]$ using the IFE measures $E(A), E'_{SK1}(A),$ and $E_W(A)$ are presented in Figs. 4 to 6.

As shown in the figures, the entropy values obtained by the proposed IFE measure (Fig. 4) are more uniform than those obtained by its counterparts (Figs. 5 and 6). To conclude

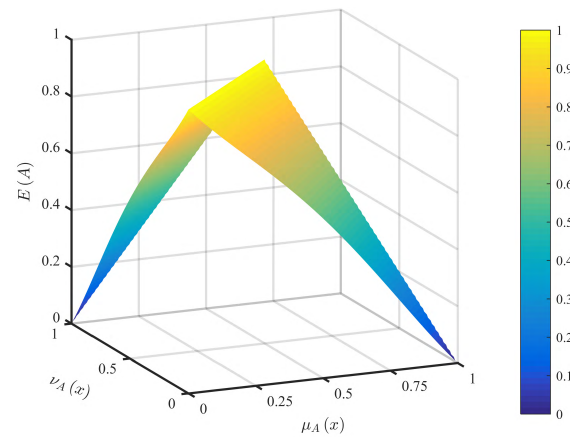


FIGURE 4. Variations in the entropy values of A with respect to $\mu_A(x)$ and $\nu_A(x)$ using $E(A)$.

this analysis, the comparisons confirm the rationality and efficiency of the proposed intuitionistic fuzzy entropy measure. We can sum up the characteristics of the proposed measure as follows:

- (i) The constructive function for the IFE measure is symmetric;
- (ii) The IFE measure takes the value of 0 if $\pi_A(x_i) = 1$ or $\mu_A(x) = \nu_A(x)$;
- (iii) The IFE value increases with the hesitancy degree $\pi_A(x_i)$ when the deviation between the membership degree $\mu_A(x)$ and the non-membership degree $\nu_A(x)$ is fixed, and decreases with the deviation from $\mu_A(x)$ to $\nu_A(x)$ when $\pi_A(x_i)$ is fixed;
- (iv) The IFE value decreases with respect to the linear product $\mu_A(x) \nu_A(x)$ with a fixed hesitancy degree $\pi_A(x_i)$.

V. A CONSOLIDATED MODEL FOR MCGDM

In this section, we structure a consolidated model based on IFNs for an MCGDM problem in which interactions among DMs, arising from their similar backgrounds, lead to

$$E_G(A) = \frac{1}{n} \sum_{i=1}^n \left(1 - (\mu_A(x_i) + \nu_A(x_i)) e^{1 - (\mu_A(x_i) + \nu_A(x_i))} \right) = \frac{1}{n} \sum_{i=1}^n \left(1 - (1 - \pi_A(x_i)) e^{\pi_A(x_i)} \right) \tag{15}$$

$$E_{SK1}(A) = \frac{1}{n} \sum_{i=1}^n \frac{\max \text{Count}(A_i \cap A_i^C)}{\max \text{Count}(A_i \cup A_i^C)} \tag{16}$$

$$E_{SK2}(A) = 1 - \frac{1}{2n} \sum_{i=1}^n |\mu_A(x_i) - \nu_A(x_i)| = 1 - \frac{1}{2n} \sum_{i=1}^n \phi_A(x_i) \tag{17}$$

$$E_W(A) = \frac{1}{n} \sum_{i=1}^n \frac{2\mu_A(x_i) \nu_A(x_i) + \pi_A(x_i)}{\mu_A^2(x_i) + \nu_A^2(x_i) + \pi_A(x_i)} = \frac{1}{n} \sum_{i=1}^n \frac{1 - \phi_A^2(x_i) + \pi_A^2(x_i)}{1 + \phi_A^2(x_i) + \pi_A^2(x_i)} \tag{18}$$

$$E_M(A) = \frac{1}{2n \ln 2} \sum_{i=1}^n \left(\pi_A(x_i) \ln 2 + \phi_A(x_i) \ln \phi_A(x_i) + (\phi_A(x_i) + 1) \ln \frac{2}{\phi_A(x_i) + 1} \right) \tag{19}$$

$$E'_{SK1}(A) = \frac{1}{n} \sum_{i=1}^n \frac{1 - |\mu_A(x_i) - \nu_A(x_i)| + \pi_A(x_i)}{1 + |\mu_A(x_i) - \nu_A(x_i)| + \pi_A(x_i)} = \frac{1}{n} \sum_{i=1}^n \frac{1 - \phi_A(x_i) + \pi_A(x_i)}{1 + \phi_A(x_i) + \pi_A(x_i)} \tag{20}$$

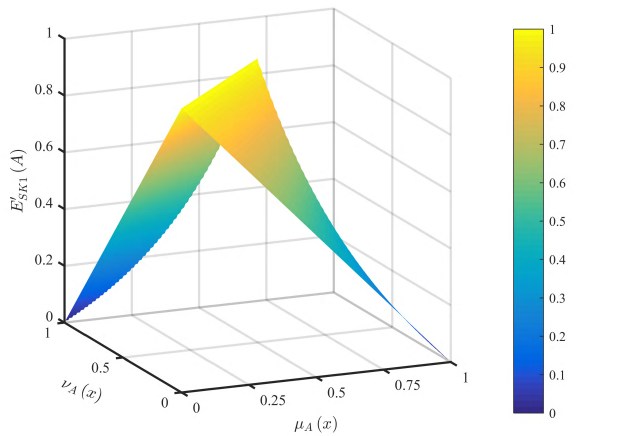


FIGURE 5. Variations in the entropy values of A with respect to $\mu_A(x)$ and $\nu_A(x)$ using $E_{SK1}^*(A)$.

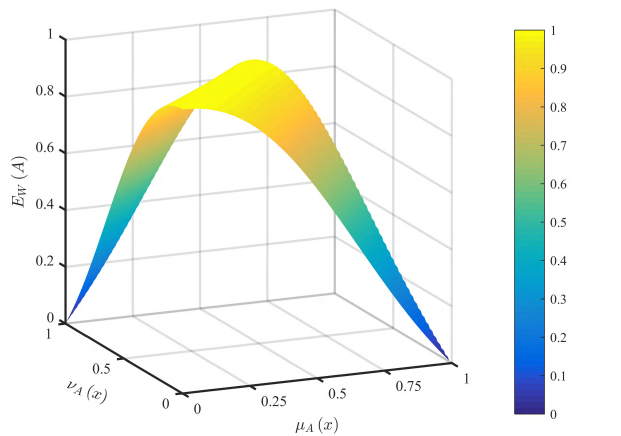


FIGURE 6. Variations in the entropy values of A with respect to $\mu_A(x)$ and $\nu_A(x)$ using $E_W(A)$.

interrelationships among the DM-related decision information, which includes the individual criteria weights, the DM weights, and the evaluation information on alternatives. In the model, an integrated method capable of tackling the interrelationships among the decision information associated with the individual criteria weights and the DM weights is put forward based on the proposed IFE measure, 2-additive fuzzy measure and Choquet integral to determine simultaneously the DM weights and the criterion weights. In addition, because of its ability to model the interrelationships among the evaluation information on alternatives, the WIFBM (or WIFGBM) is introduced to fuse the individual evaluation information. Based on the WIFBM or the WIFGBM, the SAW method is then applied to determine the best alternative.

The following notations are used to denote the indices, sets, and variables in an MCGDM problem.

- m : Total number of alternatives;
- n : Total number of criteria;
- t : Total number of DMs involved in the decision process;

- $i \in M = \{1, 2, \dots, m\}$: Index of alternatives;
- $j \in N = \{1, 2, \dots, n\}$: Index of criteria;
- $k \in T = \{1, 2, \dots, t\}$: Index of DMs;
- A_i : The i th alternative;
- $A = \{A_1, A_2, \dots, A_m\}$: A set of m alternatives;
- C_j : The j th criterion;
- $C = \{C_1, C_2, \dots, C_n\}$: A set of n criteria, which are considered to be independent;
- D_k : The k th expert;
- $D = \{D_1, D_2, \dots, D_t\}$: A set of t DMs;
- δ_k : Weight of the k th DM;
- $\delta = (\delta_1, \delta_2, \dots, \delta_t)^T$: Vector of the weights of the DMs, where $\sum_{k=1}^t \delta_k = 1$, $0 \leq \delta_k \leq 1$, and $k = 1, 2, \dots, t$;
- ω_j^k : Weight of the j th criterion with respect to DM D_k ;
- $\omega^k = (\omega_1^k, \omega_2^k, \dots, \omega_n^k)^T$: Vector of criterion weights with respect to DM D_k , where $\sum_{j=1}^n \omega_j^k = 1$, $0 \leq \omega_j^k \leq 1$, and $j = 1, 2, \dots, n$;
- $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$: Overall weight vector of the criteria, where $\sum_{j=1}^n \omega_j = 1$, $0 \leq \omega_j \leq 1$, and $j = 1, 2, \dots, n$;
- N_b : A collection of benefit criteria (i.e., larger criterion value indicates greater preference);
- N_c : A collection of cost criteria (i.e., smaller criterion value indicates greater preference) such that $N_b \cup N_c = N$ and $N_b \cap N_c = \emptyset$;
- r_{ij}^k : Evaluation of alternative A_i concerning criterion C_j that is given by DM D_k and is an IFN;
- $R = (R^1, R^2, \dots, R^k)^T$: Vector of intuitionistic fuzzy decision matrices with respect to all DMs.

In this work, we focus on addressing an MCGDM problem aimed at ranking alternatives and selecting the most desirable alternative(s) from an alternative set A based on the criteria set C . The basic process for solving this MCGDM problem is illustrated in Fig. 7.

Step 1: Establish and normalize decision matrices.

Each DM provides his or her intuitionistic fuzzy decision information on alternatives, with respect to each criterion, which can constitute a matrix depicted as follows:

$$R^k = \begin{pmatrix} r_{ij}^k \end{pmatrix}_{m \times n} = \begin{bmatrix} (\mu_{11}^k, \nu_{11}^k) & (\mu_{12}^k, \nu_{12}^k) & \dots & (\mu_{1n}^k, \nu_{1n}^k) \\ (\mu_{21}^k, \nu_{21}^k) & (\mu_{22}^k, \nu_{22}^k) & \dots & (\mu_{2n}^k, \nu_{2n}^k) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{m1}^k, \nu_{m1}^k) & (\mu_{m2}^k, \nu_{m2}^k) & \dots & (\mu_{mn}^k, \nu_{mn}^k) \end{bmatrix}$$

where r_{ij}^k ($i \in M, j \in N, k \in T$) represents the intuitionistic fuzzy value of criterion C_j ($\in C$) of the alternative A_i ($\in A$), which is provided by D_k ($\in D$).

The evaluations do not need normalization if all the criteria C_j ($1, 2, \dots, n$) are of the same type. However, MCGDM commonly comprises benefit and cost criteria. In such cases, we may transform the evaluations of the cost type into evaluations of the benefit type. According to Xu [83], $R^k = \begin{pmatrix} r_{ij}^k \end{pmatrix}_{m \times n}$ can be transformed into the normalized

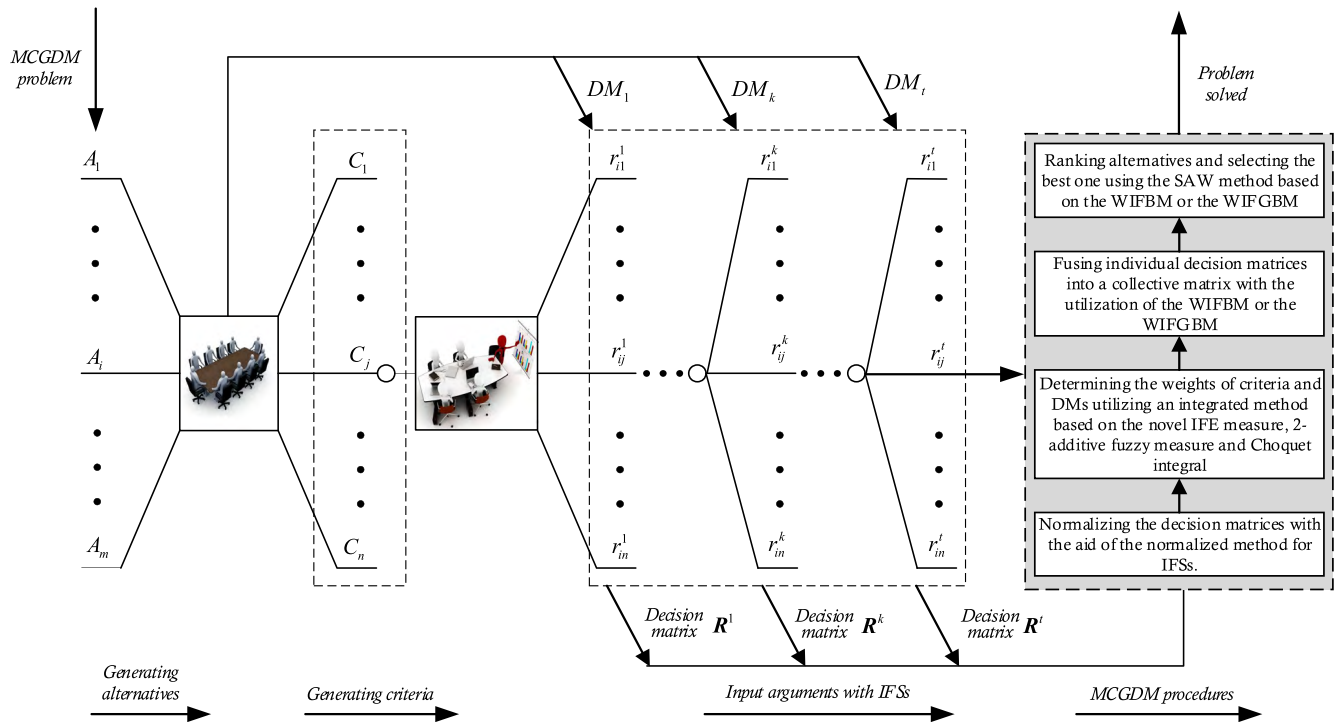


FIGURE 7. The proposed MCGDM process.

intuitionistic fuzzy decision matrix $Q^k = (q_{ij}^k)_{m \times n}$, where

$$q_{ij}^k = (r_{ij}^k, \bar{r}_{ij}^k) = \begin{cases} r_{ij}^k, & \text{for benefit criterion } C_j \in N_b \\ \bar{r}_{ij}^k, & \text{for cost criterion } C_j \in N_c \end{cases}$$

and \bar{r}_{ij}^k is the complement of r_{ij}^k such that $\bar{r}_{ij}^k = (v_{ij}^k, \mu_{ij}^k)$.

Step 2: Determine the weights of DMs and criteria.

A key element in the MCGDM process is specifying the weights of DMs and criteria. If we overlook a key criterion or a key DM (e.g., we assign a higher weight than necessary to a criterion or DM), then information that could be used to distinguish the options on this dimension may be ignored, thereby increasing the likelihood of an inappropriate decision. Hence, the method for specifying the weights of DMs and criteria should factor in the impact resulting from the interactions among the DMs who share a similar knowledge. Inspired by Shi et al. [70] who put forward a method for determining the weights of DMs considering the interactions among them, we propose an integrated method to simultaneously determine the weights of DMs and criteria.

Step 2.1: Determine the individual criterion weights.

The entropy-based method based on the proposed IFE measure is introduced to obtain the individual weights of the criteria $\omega^k = (\omega_1^k, \omega_2^k, \dots, \omega_n^k)^T$ with respect to each DM $D_k (\in D)$ which are specified by the following formula:

$$\omega_j^k = \frac{1 - \sum_{i=1}^m E(r_{ij}^k)}{n - \sum_{j=1}^n \sum_{i=1}^m E(r_{ij}^k)} \quad (21)$$

Step 2.2: Determine the initial weights of DMs and measure the degrees of resemblance between DM knowledge levels.

A supervisor who is supposed to be familiar with all of the DMs is required to assign initial weights to each of the DMs, which are denoted by $\zeta = (\zeta_1, \zeta_2, \dots, \zeta_t)^T$, and to further determine the degree of resemblance between the knowledge levels of any two DMs. If any two of the DMs (e.g., D_k and D_l) share similar knowledge levels to a large degree, then the resemblance degree can be denoted by d_{kl} , which takes the value of 1 in this case. Conversely, if DMs D_k and D_l lack resemblance, then d_{kl} takes the value of -1.

Step 2.3: Measure the degrees of resemblance between DM preferences.

According to Shi et al. [70], the DM preferences are reflected in the weight vectors of the criteria corresponding to each DM $D_k (\in D)$. Thus, the degree of resemblance between the preferences of D_k and D_l is defined as:

$$s_{kl} = \cos^{kl} \theta = \frac{\omega^k \cdot \omega^l}{|\omega^k| |\omega^l|} \quad (22)$$

where $s_{kl} = s_{lk}$ and $s_{kl} \in [0, 1]$.

Obviously, a large s_{kl} indicates a large degree of resemblance between the preferences of D_k and D_l .

Step 2.4: Calculate the fuzzy measure μ_k with respect to DMD_k .

The fuzzy measure μ_k with respect to D_k is assumed to be linearly correlated with the initial weight of D_k . Thus,

$$\mu_k = \lambda \zeta_k \quad (k \in T) \quad (23)$$

where λ represents the linear coefficient of Eq. (23) and $\lambda \geq 0$. If the information associated with the interactions among DMs is redundant, we then have $\lambda > 1$. In this case, a great degree of information redundancy results in a large λ . If the information associated with the interactions among DMs is complementary, we then have $\lambda < 1$. In this case, a great degree of complementary information results in a small λ . In particular, if the information redundancy and the information complementation are of the same degree, then $\lambda = 1$.

By combining Steps 2.2 and 2.3, we can define the measure of the degree of information redundancy or information complementation associated with the interactions among DMs; that is,

$$\chi = \sum_{k,l \in T} \zeta_k \zeta_l d_{kl} s_{kl}, \tag{24}$$

where $\chi > 0$ represents information redundancy, $\chi < 0$ represents information complementation, and $\chi = 0$ shows the homogeneity of the degree of information redundancy and that of information complementation. The relationship between λ and χ can be defined by:

$$\lambda = \frac{1}{1 - \chi}. \tag{25}$$

Hence, the fuzzy measure μ_k with respect to DM D_k is determined by $\mu_k = \lambda \zeta_k$.

Step 2.5: Calculate the fuzzy measure μ_{kl} corresponding to any two DMs.

Let us consider DMs D_k and D_l . If the information provided by D_k is considered as redundant with respect to D_l (that is, D_k would provide the same information that D_l may have provided because of their similarity in knowledge levels and preferences), then D_k is supposed to be assigned a low degree of importance. However, if the information provided by any one of the two DMs is considered as complementary with respect to the other (that is, DM D_k can provide information that complements DM D_l), then D_k is supposed to be assigned a large degree of importance. Considering the interactions among DMs, we can define the importance degree of D_k as:

$$\delta'_k = \zeta_k - \sum_{l=1, l \neq k}^t \frac{1}{2} \zeta_k \zeta_l d_{kl} s_{kl}. \tag{26}$$

Thus the weight of D_k can be obtained by normalizing δ'_k , which leads to:

$$\delta_k = \delta'_k / \sum_{l=1}^t \delta'_l. \tag{27}$$

Furthermore, the fuzzy measure μ_{kl} can be determined by:

$$\mu_{kl} = \mu_k + \mu_l - \lambda \zeta_k \zeta_l d_{kl} s_{kl} \tag{28}$$

The 2-additive fuzzy measure is determined by μ_k and μ_{kl} as:

$$\mu(S) = \sum_{\{k,l\} \subset S} \mu_{kl} - (|S| - 2) \sum_{l \subset S} \mu_l \tag{29}$$

for any $S \subseteq T$ such that $|S| \geq 2$.

Step 2.6: Determine the collective weight vector of the criteria $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$.

The collective weight of criterion C_j can be written using the Choquet integral as:

$$\omega_j = \frac{\omega_j^*}{\sum_{j=1}^n \omega_j^*} = \frac{\sum_{k=1}^t \omega_j^{(k)} [\mu(D_{(k)}) - \mu(D_{(k+1)})]}{\sum_{j=1}^n \sum_{k=1}^t \omega_j^{(k)} [\mu(D_{(k)}) - \mu(D_{(k+1)})]}, \tag{30}$$

where (\cdot) indicates a permutation on N such that $\omega_j^{(1)} \leq \omega_j^{(2)} \leq \dots \leq \omega_j^{(n)}$, and $D_{(i)} = \{D_{(i)}, D_{(i+1)}, \dots, D_{(t)}\}$ with $D_{(t+1)} = \emptyset$.

Both the collective weight vector of the criteria and the weight vector of DMs can be determined by following Steps 2.1 to 2.6.

Step 3: Fuse individual normalized decision matrices.

To capture the interrelationships among the individual evaluations of each alternative with respect to each criterion, we use the WIFBM or the WIFGBM for fusing the individual evaluation values $\{q_{ij}^1, q_{ij}^2, \dots, q_{ij}^t | i \in M, j \in N\}$ to derive the collective evaluation value $z_{ij} = (\phi_{ij}, \varphi_{ij})$ of each alternative on each criterion. The collective preference value is obtained in the following cases.

Case 1: If we use the WIFBM, we then have

$$z_{ij} = (\phi_{ij}, \varphi_{ij}) = IFB_{\delta}^{p,q} (q_{ij}^1, q_{ij}^2, \dots, q_{ij}^t).$$

Case 2: If we use the WIFGBM, we then have

$$z_{ij} = (\phi_{ij}, \varphi_{ij}) = IFGB_{\delta}^{p,q} (q_{ij}^1, q_{ij}^2, \dots, q_{ij}^t).$$

Step 4: Rank alternatives and select the best one.

It is worth noting that there are also interrelationships among the collective evaluation values $\{z_{i1}, z_{i2}, \dots, z_{in} | i \in M\}$ of each alternative. Thus, the SAW method based on the WIFBM or the WIFGBM is introduced to generate a final ranking of alternatives. With the aid of the WIFBM or the WIFGBM, we can derive the overall evaluation value $c_i = (\kappa_i, \eta_i)$ of alternative A_i by aggregating the collective preference values of alternative A_i on all the criteria.

Case 1: If we use the WIFBM, we then have

$$c_i = (\kappa_i, \eta_i) = IFB_{\omega}^{p,q} (z_{i1}, z_{i2}, \dots, z_{in}).$$

Case 2: If we use the WIFGBM, we then have

$$c_i = (\kappa_i, \eta_i) = IFGB_{\omega}^{p,q} (z_{i1}, z_{i2}, \dots, z_{in}).$$

We can then rank the alternatives depending on the score and accuracy values of $c_i = (\kappa_i, \eta_i)$ ($i = 1, 2, \dots, 6$) and find the most desirable alternative.

VI. CASE STUDY

A. APPLICATION TO EP DECISION-MAKING

A security department of a railway administrative body in China intends to draw up a new EP to respond to train derailment, which is one of the most frequent accidents in the railway sector. An EP can be decomposed into a sequence of actions required for dealing with specific scenarios. As a new derailment accident shares features and requirements with similar cases experienced previously, the department can adopt a case-based reasoning method to generate the EP for derailment, in which the actions required are specified by retrieving and revising the operations for the most similar scenarios which have been stored in a case database in a structured form. After the retrieval procedure has generated a few candidates, a selection procedure for the most appropriate EP can then be activated with the aid of the proposed MCGDM model. Given the alternatives $\{A_1, A_2, A_3, A_4, A_5, A_6\}$ and the criteria $\{C_1, C_2, C_3, C_4, C_5, C_6\}$ described in Table 1, the department establishes a temporary decision-making team $\{D_1, D_2, D_3, D_4\}$ composed of the relevant experts, managers, and staff involved in train derailments to conduct the selection and revision procedures. Additionally, a supervisor in the department assigns weights $\zeta = (0.4, 0.25, 0.15, 0.2)^T$ to DMs based on their knowledge, experience, status, and so on.

Step 1: Establish and normalize decision matrices.

Each DM D_k ($k = 1, 2, 3, 4$) is asked to provide the evaluation information for alternatives on each criterion in the form of IFNs, which can be denoted by $R^k = (r_{ij}^k)_{6 \times 6}$ in which r_{ij}^k is an IFN and represents the performance of EP A_i under criterion C_j provided by the DM D_k ($i = 1, 2, \dots, 6, j = 1, 2, \dots, 6, k = 1, 2, 3, 4$). The results are shown in Table 3. As all the criteria for EP evaluation are the benefit criteria, there is no need to normalize the decision matrices.

Step 2: Determine the weights of DMs and criteria.

Step 2.1: Determine the individual criterion weights.

By the proposed IFE measure in Eq. (12) and the entropy-based method for determining criterion weights in Eq. (21), we can derive the individual criterion weights ω^k for each DM D_k ($k = 1, 2, 3, 4$) as follows:

$$\omega^1 = (0.167, 0.195, 0.125, 0.182, 0.157, 0.174)$$

$$\omega^2 = (0.152, 0.185, 0.183, 0.165, 0.159, 0.156)$$

$$\omega^3 = (0.189, 0.197, 0.152, 0.197, 0.14, 0.124)$$

and

$$\omega^4 = (0.154, 0.155, 0.182, 0.165, 0.182, 0.162).$$

Step 2.2: Measure the degrees of resemblance between DM knowledge levels.

The degree of resemblance between the knowledge levels of any two DMs is determined by the group leader as $d_{12} = +1, d_{13} = +1, d_{14} = +1, d_{23} = -1, d_{24} = -1,$ and $d_{34} = -1$.

Step 2.3: Measure the degrees of resemblance between DM preferences.

TABLE 3. Intuitionistic fuzzy decision matrices $R^1, R^2, R^3,$ and R^4 .

	C_1	C_2	C_3	C_4	C_5	C_6	
D_1	A_1	(0.6,0.2)	(0.7,0.1)	(0.7,0.2)	(0.7,0.3)	(0.6,0.2)	(0.6,0.3)
	A_2	(0.7,0.1)	(0.7,0.2)	(0.6,0.3)	(0.6,0.2)	(0.6,0.3)	(0.6,0.2)
	A_3	(0.7,0.2)	(0.7,0.1)	(0.7,0.3)	(0.7,0.3)	(0.6,0.4)	(0.6,0.2)
	A_4	(0.6,0.3)	(0.6,0.3)	(0.4,0.3)	(0.8,0.2)	(0.6,0.3)	(0.6,0.2)
	A_5	(0.6,0.1)	(0.8,0.2)	(0.6,0.2)	(0.7,0.1)	(0.8,0.1)	(0.8,0.1)
	A_6	(0.5,0.2)	(0.6,0.2)	(0.5,0.3)	(0.5,0.1)	(0.7,0.2)	(0.6,0.1)
D_2	A_1	(0.6,0.2)	(0.8,0.2)	(0.6,0.3)	(0.6,0.1)	(0.6,0.2)	(0.5,0.3)
	A_2	(0.7,0.3)	(0.7,0.3)	(0.6,0.1)	(0.6,0.3)	(0.6,0.3)	(0.6,0.3)
	A_3	(0.7,0.2)	(0.7,0.3)	(0.7,0.3)	(0.7,0.3)	(0.8,0.2)	(0.6,0.2)
	A_4	(0.6,0.3)	(0.6,0.2)	(0.7,0.1)	(0.8,0.2)	(0.6,0.3)	(0.6,0.2)
	A_5	(0.5,0.3)	(0.6,0.2)	(0.6,0.2)	(0.6,0.4)	(0.7,0.3)	(0.7,0.2)
	A_6	(0.7,0.2)	(0.7,0.1)	(0.8,0.2)	(0.7,0.2)	(0.6,0.2)	(0.7,0.1)
D_3	A_1	(0.8,0.2)	(0.9,0.1)	(0.7,0.1)	(0.8,0.2)	(0.7,0.2)	(0.6,0.2)
	A_2	(0.9,0.1)	(0.8,0.2)	(0.8,0.2)	(0.9,0.1)	(0.7,0.1)	(0.7,0.2)
	A_3	(0.8,0.1)	(0.8,0.1)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)	(0.8,0.2)
	A_4	(0.8,0.2)	(0.9,0)	(0.7,0.3)	(0.9,0)	(0.7,0.2)	(0.7,0.2)
	A_5	(0.8,0.1)	(0.8,0.1)	(0.7,0.1)	(0.8,0)	(0.7,0.3)	(0.6,0.2)
	A_6	(0.9,0.1)	(0.8,0.1)	(0.8,0.2)	(0.8,0.1)	(0.6,0)	(0.7,0.3)
D_4	A_1	(0.7,0.1)	(0.8,0.2)	(0.7,0.2)	(0.8,0.1)	(0.8,0)	(0.8,0.2)
	A_2	(0.6,0.2)	(0.7,0.1)	(0.8,0.1)	(0.8,0)	(0.7,0.2)	(0.8,0.2)
	A_3	(0.7,0.1)	(0.7,0.1)	(0.8,0.1)	(0.8,0.2)	(0.8,0)	(0.8,0.2)
	A_4	(0.8,0.2)	(0.8,0.3)	(0.7,0)	(0.7,0.1)	(0.7,0.1)	(0.7,0.1)
	A_5	(0.7,0.2)	(0.7,0.2)	(0.8,0.2)	(0.7,0.2)	(0.8,0.2)	(0.7,0.2)
	A_6	(0.8,0.1)	(0.7,0.1)	(0.8,0)	(0.7,0.3)	(0.8,0.1)	(0.8,0.2)

Following Eq. (22), we can obtain the degree of resemblance between the preferences of any two DMs as follows: $s_{12} = 0.964; s_{13} = 0.978; s_{14} = 0.987; s_{23} = 0.993; s_{24} = 0.917;$ and $s_{34} = 0.934$.

Step 2.4: Calculate fuzzy measure μ_k with respect to DM D_k .

By using Eq. (23), we can calculate $\chi = 0.123$ and $\lambda = 1.14$. Furthermore, the fuzzy measures μ_k ($k = 1, 2, 3, 4$) are $\mu_1 = 0.456, \mu_2 = 0.285, \mu_3 = 0.171,$ and $\mu_4 = 0.228$.

Step 2.5: Determine the weights of individual DMs.

Following Eq. (26) and Eq. (27), we determine the weights of DMs $D_1, D_2, D_3,$ and D_4 as $\delta_1 = 0.323, \delta_2 = 0.277, \delta_3 = 0.175,$ and $\delta_4 = 0.225,$ respectively. The 2-additive fuzzy measures can be calculated following Eq. (28) and Eq. (29). The results are shown in Table 4. Fig. 8 illustrates the process of calculating the 2-additive fuzzy measures.

Step 2.6: Determine the overall weight vector of the criteria.

Using the Choquet integral, the overall weight vector of the criteria is determined as:

$$\omega = (0.163, 0.183, 0.162, 0.176, 0.159, 0.158).$$

Step 3: Fuse individual normalized decision matrices.

To obtain the collective evaluations of the EPs on each criterion, the WIFBM (or WIFGBM) is used to synthesize the individual evaluations $\{q_{ij}^1, q_{ij}^2, q_{ij}^3, q_{ij}^4 | i \in M, j \in N\}$. Xu [82] pointed out that the value derived by the IFBM depends on the choice of parameters p and q , which are not robust. In general, an extensive calculation is needed when parameters p and q are large. In the special case where at least one of these two parameters takes the value of 0, the IFBM cannot capture the interrelationships among individual preferences. As a result, the values of the two parameters are taken as $p = q = 1$ in practical applications. This approach, which

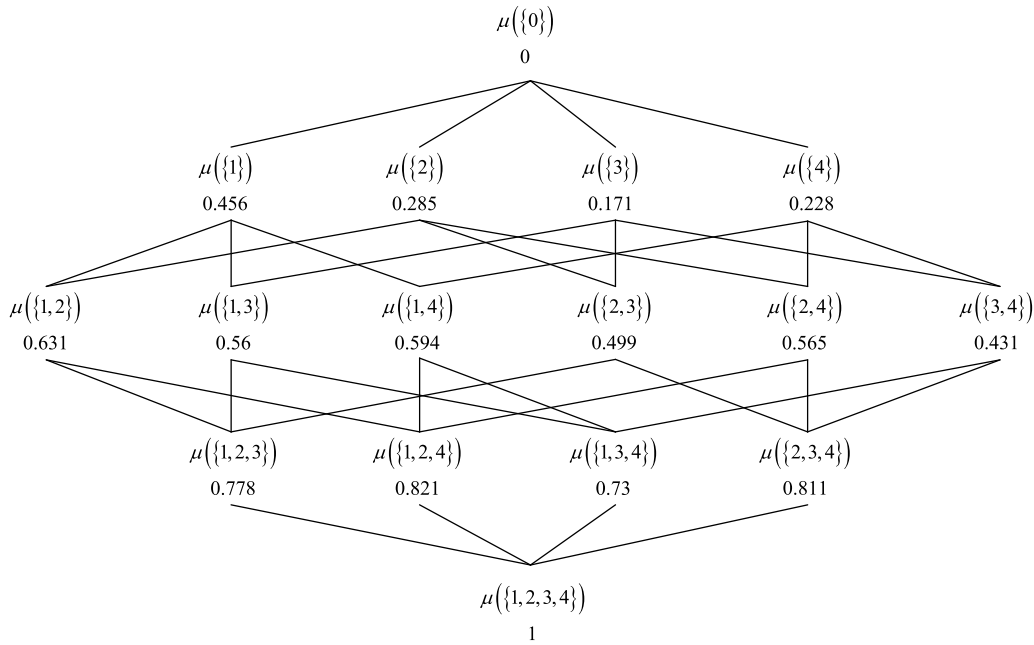


FIGURE 8. The process of calculating 2-additive fuzzy measures.

TABLE 4. The 2-additive fuzzy measures.

μ_1	μ_2	μ_3	μ_4	μ_{12}	μ_{13}	μ_{14}	μ_{23}
0.456	0.285	0.171	0.228	0.631	0.56	0.594	0.499
μ_{24}	μ_{34}	μ_{123}	μ_{124}	μ_{134}	μ_{234}	μ_{1234}	
0.565	0.431	0.778	0.821	0.73	0.811	1	

is intuitive and simple, considers fully the interrelationships among the individual arguments. The corresponding results obtained by the WIFBM and the WIFGBM with the overall weight vector of criteria ω are shown in Tables 5 and 6, respectively.

Step 4: Rank alternatives and select the best one.

The WIFBM or the WIFGBM (here, we take $p = q = 1$) and the DM weight vector δ are utilized to calculate the overall evaluation information of the EPs. The results are shown in Table 7. By Definition 2, we can obtain the ranking of alternatives $A_6 > A_4 > A_5 > A_3 > A_1 > A_2$ in Case 1 with the EP A_6 ranking first, and the ranking of alternatives $A_6 > A_3 > A_5 > A_1 > A_2 > A_4$ in Case 2 with EP A_2 ranking first. The ranking result using WIFBM in Case 1 is obviously different from the ranking of alternatives by WIFGBM in Case 2, but the alternative A_2 is obviously the best EP among the candidates. Table 7 shows that most of the scores obtained by the WIFBM are smaller than 0 and that most of those obtained by the WIFGBM are bigger than 0. This result indicates that the WIFGBM can obtain optimistic expectations, whereas the WIFBM can obtain pessimistic expectations.

With respect to the WIFBM and the WIFGBM, Xia et al. [84] pointed out that the values of parameters can be considered as representing either optimistic or

pessimistic levels. Therefore, the decision results may differ given changes to the parameters p and q . Fig. 9 shows the WIFBM scores for the alternatives when the parameters change in the following cases:

- i) $p = q \in [0, 10]$;
- ii) $p \in [0, 10], q = 1$;
- iii) $p \in [0, 10], q = 3$;
- iv) $p \in [0, 10], q = 5$;
- v) $p \in [0, 10], q = 7$;
- vi) $p \in [0, 10], q = 9$.

The scores for $q \in [0, 10], p = 1, 3, 5, 7, 9$ are exactly the same as $p \in [0, 10], q = 1, 3, 5, 7, 9$ because the WIFBM is a symmetric function. Fig. 10 shows the WIFGBM scores for the alternatives when the parameters change in the same cases. An intuitive feature reflected in these figures is that the ranking of the alternatives differs if the parameters change. Any intersection of these curves indicates the change in the ranking of any two alternatives. The rankings of the alternatives are slightly different in most cases. At some points, however, the scores for the alternatives change considerably. Therefore, the decision results often fluctuate under these situations. In fact, the parameters can be determined according to the subjective preferences of the DMs, and changes in the rankings of the alternatives can reflect the DMs' risk preferences.

B. SENSITIVITY ANALYSIS

In this subsection, we conduct a sensitivity analysis to explore how adjustments to the criterion weights influence the final ranking of alternatives. In Case 1, we firstly increase and decrease the weight of each criterion by 30% and 60%, respectively. The weights of other criteria are then adjusted

TABLE 5. The collective evaluation values of each EP on each criterion obtained by using the WIFBM.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.241,0.648)	(0.329,0.622)	(0.242,0.670)	(0.273,0.641)	(0.242,0.516)	(0.218,0.712)
A_2	(0.279,0.642)	(0.272,0.667)	(0.257,0.641)	(0.278,0.513)	(0.227,0.690)	(0.242,0.692)
A_3	(0.272,0.625)	(0.272,0.616)	(0.288,0.688)	(0.288,0.712)	(0.290,0.562)	(0.257,0.673)
A_4	(0.257,0.712)	(0.278,0.544)	(0.214,0.522)	(0.332,0.470)	(0.227,0.688)	(0.227,0.648)
A_5	(0.228,0.642)	(0.275,0.650)	(0.242,0.650)	(0.256,0.500)	(0.293,0.681)	(0.264,0.645)
A_6	(0.279,0.625)	(0.255,0.597)	(0.276,0.542)	(0.241,0.640)	(0.247,0.470)	(0.257,0.632)

TABLE 6. The collective evaluation values of each EP on each criterion obtained by using the WIFGBM.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.903,0.060)	(0.944,0.050)	(0.907,0.069)	(0.921,0.059)	(0.905,0.049)	(0.888,0.088)
A_2	(0.920,0.059)	(0.922,0.069)	(0.912,0.059)	(0.918,0.047)	(0.897,0.078)	(0.905,0.079)
A_3	(0.922,0.050)	(0.922,0.049)	(0.929,0.078)	(0.929,0.088)	(0.930,0.066)	(0.912,0.069)
A_4	(0.912,0.088)	(0.918,0.067)	(0.887,0.056)	(0.945,0.040)	(0.897,0.078)	(0.897,0.060)
A_5	(0.894,0.059)	(0.923,0.060)	(0.905,0.060)	(0.913,0.055)	(0.932,0.078)	(0.919,0.060)
A_6	(0.919,0.050)	(0.913,0.041)	(0.922,0.058)	(0.903,0.059)	(0.908,0.040)	(0.914,0.059)

TABLE 7. The overall preferences and the corresponding scores of each alternative for Case 1 and Case 2.

		A_1	A_2	A_3	A_4	A_5	A_6
Case 1	Overall preferences	(0.0487,0.9268)	(0.0489,0.9280)	(0.0528,0.9297)	(0.0483,0.9162)	(0.0489,0.9249)	(0.0487,0.9142)
	Scores	-0.878	-0.8792	-0.8769	-0.8679	-0.876	-0.8655
Case 2	Overall preferences	(0.9848,0.0106)	(0.9849,0.0111)	(0.9869,0.0114)	(0.9844,0.0110)	(0.9852,0.0105)	(0.9850,0.0087)
	Scores	0.9742	0.9738	0.9755	0.9734	0.9747	0.9763

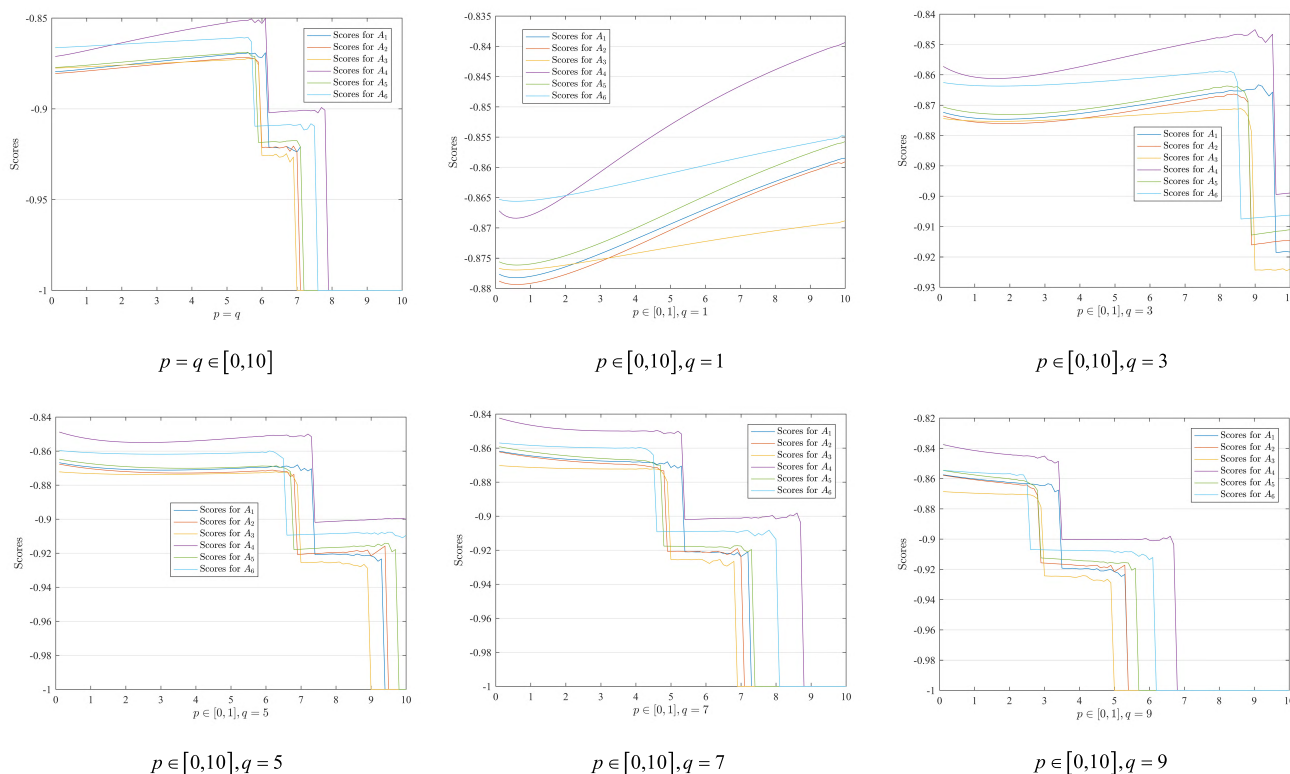


FIGURE 9. WIFBM scores for the EPs.

accordingly to ensure that the sum of all the criterion weights is equal to 1. Thus, we acquire four new groups of weights for each criterion, which would take 24 experiments to conduct

the sensitivity analysis. We fix other parameters including the weights of DMs and the criterion values, apply each new group of criterion weights to the proposed model, and arrive at

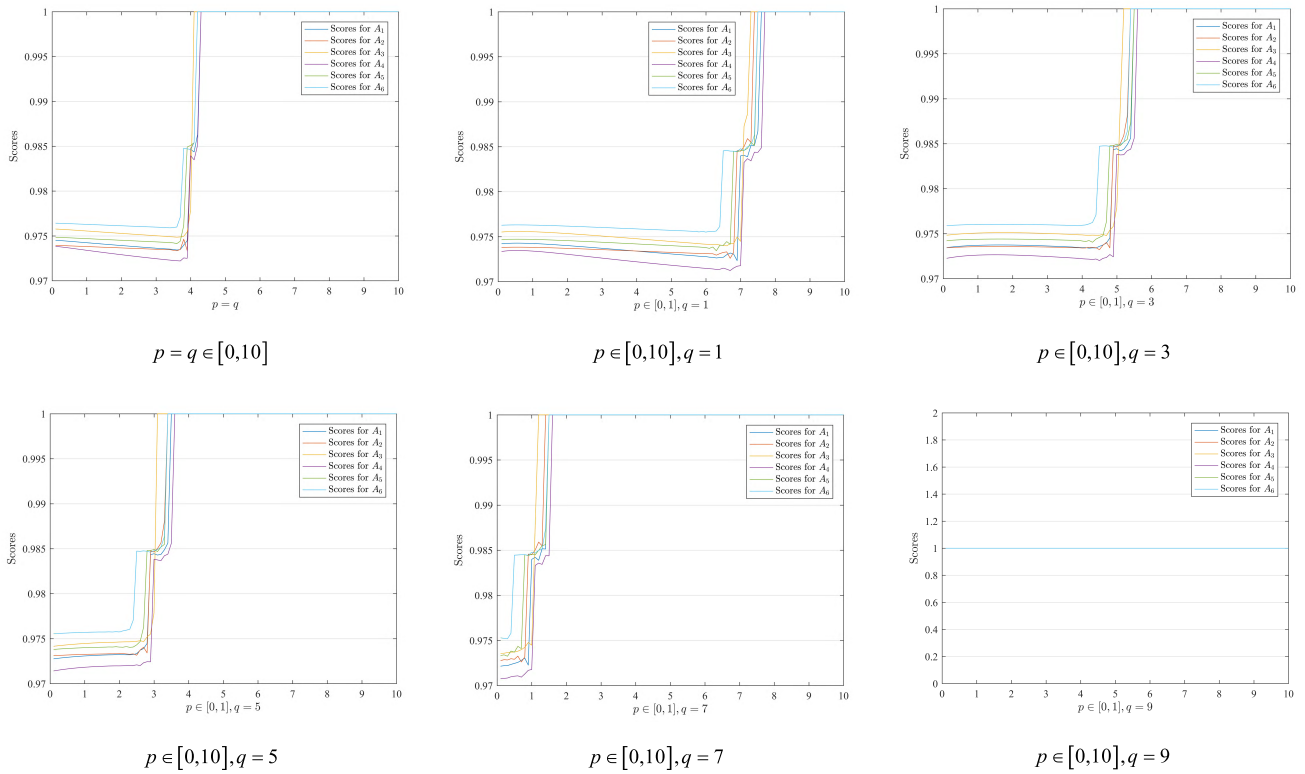


FIGURE 10. WIFGBM scores for the EPs.

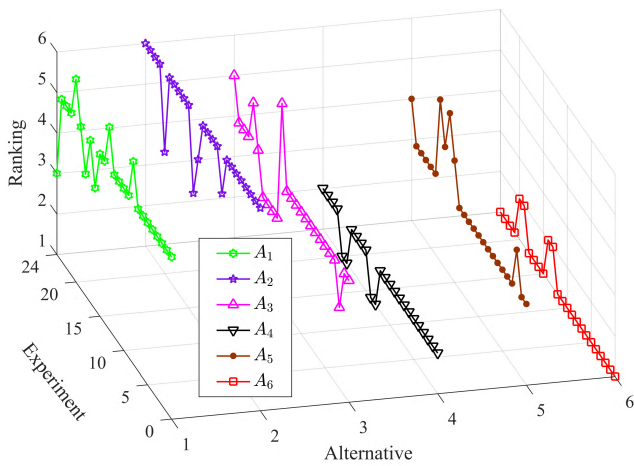


FIGURE 11. The results of sensitivity analysis in Case 1.

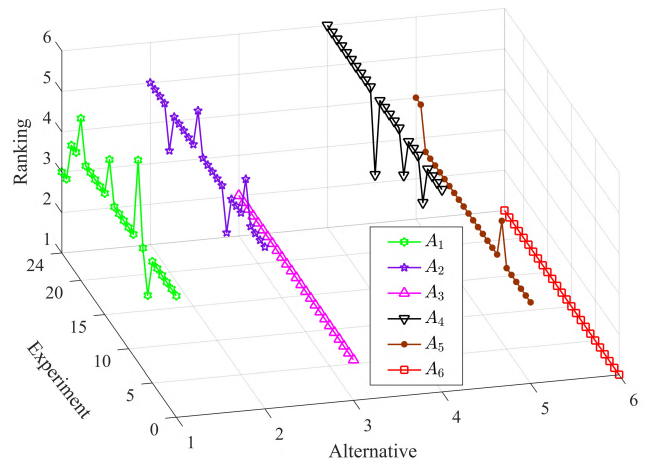


FIGURE 12. The results of sensitivity analysis in Case 2.

24 new rankings of alternatives, which are depicted in Fig. 11. We conduct the same experiments for Case 2 and the results are shown in Fig. 12. From Figs. 11 and 12, we arrive at the following two conclusions:

- (i) The change of criterion weights has a notable influence on the final ranking of alternatives, necessitating the input of precise criterion weights during the application of the proposed MCGDM model, and motivating an adjustment of the criterion weights to mitigate the influence of the interactions among DMs.

- (ii) The results obtained by SAW based on WIFGBM are more robust than the results from SAW based on WIFBM, indicating that WIFGBM performs better than WIFBM in modeling the interrelationships among the decision information.

C. COMPARISON ANALYSIS

1) Comparison analysis to demonstrate the necessity of factoring in the interactions among DMs.

TABLE 8. The collective evaluation values of each EP on each criterion obtained by using the WIFAM.

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	(0.660,0.174)	(0.788,0.137)	(0.678,0.199)	(0.720,0.172)	(0.666,0.000)	(0.632,0.260)
A_2	(0.731,0.151)	(0.718,0.193)	(0.686,0.172)	(0.717,0.000)	(0.638,0.235)	(0.666,0.221)
A_3	(0.718,0.157)	(0.718,0.132)	(0.740,0.227)	(0.740,0.260)	(0.736,0.000)	(0.686,0.200)
A_4	(0.686,0.260)	(0.717,0.000)	(0.604,0.000)	(0.805,0.000)	(0.638,0.227)	(0.638,0.174)
A_5	(0.640,0.151)	(0.742,0.180)	(0.666,0.180)	(0.697,0.000)	(0.765,0.178)	(0.734,0.152)
A_6	(0.712,0.157)	(0.683,0.132)	(0.711,0.000)	(0.654,0.148)	(0.690,0.000)	(0.690,0.135)

To achieve the goal of validating the necessity of factoring the interactions among DMs into the structure of the MCGDM model, we conduct a comparison of the proposed MCGDM model with the traditional MCGDM model which is constructed on the hypothesis that the DMs are independent.

Firstly, we structure a traditional MCGDM model within the domain of our research topic. This is achieved by the following procedure.

Step 1: Each DM is asked to provide the evaluation information $\left\{ R^k = \left(r_{ij}^k \right)_{m \times n} \mid k \in T \right\}$ in which $r_{ij}^k = \left(\mu_{ij}^k, \nu_{ij}^k \right)$ is an IFN. The decision matrices $\left\{ R^k = \left(r_{ij}^k \right)_{m \times n} \mid k \in T \right\}$ are then normalized to ensure that all the evaluation values are of the same type.

Step 2: We determine the weights of the criteria $\omega^k = \left(\omega_1^k, \omega_2^k, \dots, \omega_n^k \right)^T$ based on the entropy-based method in which the proposed IFE measure is also applied. The weights of the DMs are assigned directly by a supervisor familiar with all the DMs and represented as $\zeta' = \left(\zeta'_1, \zeta'_2, \dots, \zeta'_t \right)^T$. We can then acquire the collective weights of the criteria $\omega' = \left(\omega'_1, \omega'_2, \dots, \omega'_n \right)^T$ by the formula $\omega'_j = \sum_{k=1}^t \delta'_k \omega_j^k$.

Step 3: The weighted intuitionistic fuzzy arithmetic mean (WIFAM) [80] is introduced to fuse the individual decision matrices into a collective decision matrix denoted by $R' = \left(r'_{ij} \right)_{m \times n}$ in which the $r'_{ij} = \left(\mu'_{ij}, \nu'_{ij} \right)$ is also an IFN.

Step 4: The final ranking of alternatives is determined by the SAW method based on WIFAM.

Subsequently, we apply the traditional MCGDM model to the EP decision-making problem described above. The input decision information for this traditional MCGDM model $\left\{ R^k = \left(r_{ij}^k \right)_{6 \times 6} \mid k = 1, 2, 3, 4 \right\}$ remains the same and is depicted in Table 2. By the entropy-based method presented in Eq. (21), we obtain the individual criteria weights with the results $\left\{ \omega^k = \omega^k \mid k = 1, 2, 3, 4 \right\}$. Given the DM weights $\zeta' = (0.4, 0.25, 0.15, 0.2)^T$, we obtain the collective weights of the criteria as:

$$\omega = (0.1638, 0.1849, 0.1548, 0.1769, 0.1601, 0.1595).$$

Using the WIFAM, we can obtain the collective decision matrix depicted in Table 8. Subsequently, we use the SAW method based on WIFAM to obtain the final ranking of EPs $A_3 > A_5 > A_4 > A_6 > A_2 > A_1$ with the EP A_3 ranking first. We can easily find a considerable difference between

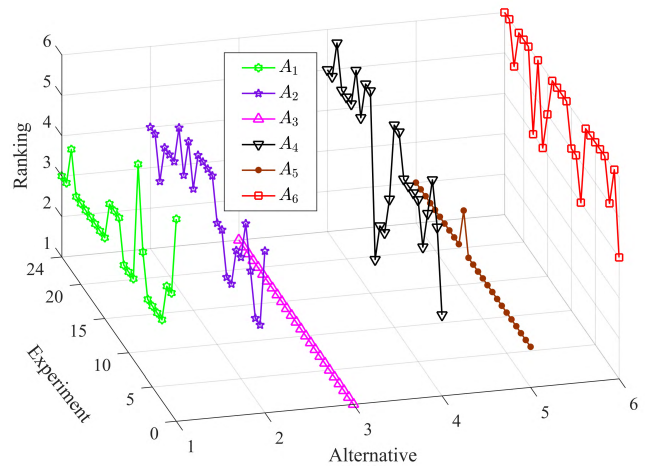


FIGURE 13. The results of sensitivity analysis using the traditional MCGDM model.

the result obtained by the traditional MCGDM model and that obtained by the proposed MCGDM model. The main reason for the difference is that the proposed model factors in the interactions among DMs whereas the traditional MCGDM model does not take into account these interactions. In order to further verify the superiority of the proposed MCGDM model over the traditional MCGDM model, we conduct a sensitivity analysis on the traditional MCGDM model based on the WIFAM by following the same procedure as that carried out on the proposed model. The results are shown in Fig. 13. Comparing Fig. 11 or Fig. 12 with Fig. 13, we can easily see that the proposed MCGDM model based on WIFBM or WIFGBM is more robust and performs better than the traditional MCGDM model based on WIFAM.

2) Comparison analysis to demonstrate the feasibility of the proposed ranking method.

In order to verify the feasibility of the proposed ranking method (i.e., the SAW method based on WIFBM or WIFGBM), we conduct a comparison with other popular MCDM methods, namely SAW based on WIFAM, TOPSIS [7], and VIKOR [75]. Before carrying out the comparison process, we should ensure that the input information for these MCDM methods remains the same.

The input information is deemed to be the collective evaluation values of each EP on each criterion shown in Table 5 for Case 1 and in Table 6 for Case 2, and the collective criterion weights are as follows:

$$\omega = (0.163, 0.183, 0.162, 0.176, 0.159, 0.158).$$

TABLE 9. Comparison analysis to demonstrate the feasibility of the proposed ranking method.

Case 1	Ranking of alternatives	Case 2	Ranking of alternatives
The SAW based on the WIFBM	$A_6 \succ A_4 \succ A_5 \succ A_3 \succ A_1 \succ A_2$	The SAW based on the WIFGBM	$A_6 \succ A_3 \succ A_5 \succ A_1 \succ A_2 \succ A_4$
The SAW based on the WIFAM	$A_6 \succ A_4 \succ A_5 \succ A_2 \succ A_1 \succ A_3$	The SAW based on the WIFAM	$A_6 \succ A_3 \succ A_5 \succ A_1 \succ A_4 \succ A_2$
TOPSIS	$A_6 \succ A_4 \succ A_5 \succ A_2 \succ A_1 \succ A_3$	TOPSIS	$A_6 \succ A_1 \succ A_3 \succ A_3 \succ A_4 \succ A_2$
VIKOR	$A_6 \succ A_5 \succ A_4 \succ A_1 \succ A_2 \succ A_3$	VIKOR	$A_6 \succ A_1 \succ A_2 \succ A_3 \succ A_5 \succ A_4$

Additionally, the normalized Euclidean distance of an IFS, as coined by Szmidt and Kacprzyk [71], is introduced for the implementation of TOPSIS and VIKOR. The rankings of alternatives are then obtained for Case 1 using SAW method based on WIFBM, SAW based on WIFAM, TOPSIS, and VIKOR. The results are presented in Table 9. The ranking of alternatives for Case 2 using SAW method based on WIFGBM, SAW based on WIFAM, TOPSIS, and VIKOR are also presented in Table 9. From Table 9, we find that although the results from SAW based on WIFBM for Case 1 or on WIFGBM for Case 2 are slightly different from the results derived from SAW based on WIFAM, TOPSIS, and VIKOR, the alternative A_6 remains the optimal alternative when adopting different ranking methods. Thus, the SAW method based on WIFBM or WIFGBM is feasible, and the results obtained by this ranking method are reliable.

To conclude the above analysis, in order to address the problem of the interactions among DMs when a number of DMs share a similar background, we require an MCGDM model that is capable of modeling the various impacts resulting from these interactions. Analysis shows the proposed model to be feasible and applicable to this type of MCGDM problem, and the results of the proposed model to be reliable and robust. In addition, implementing the proposed MCGDM model only requires the input of decision information, including firstly the evaluation values of alternatives on criteria provided by DMs, and secondly the degree of resemblance between the knowledge levels of any two DMs provided by a supervisor.

VII. CONCLUSION

In this paper, with the aim of addressing the impact of the interactions among DMs when a number of DMs share a similar background, we have developed a consolidated MCGDM model within the IFNs context. Firstly, we put forward a new axiomatic definition for the construction of an IFE measure and produced a new IFE measure on the basis of this definition. Additionally, we validated its advantages over existing IFE measures by comparison analysis. Secondly, we presented an integrated method of obtaining the weights of DMs and criteria based on the new IFE measure, 2-additive fuzzy measure, and Choquet integral. Subsequently, we introduced WIFBM and WIFGBM for synthesizing the individual evaluation values, and the SAW method based on WIFBM or WIFGBM for ranking alternatives. Finally, the proposed MCGDM model was applied to an EP decision-making problem to exemplify its feasibility and effectiveness.

The main advantages of the proposed model can be summarized as follows:

- (i) The novel IFE measure for obtaining objective individual criterion weights factors in simultaneously the fuzziness and intuitionism of IFS and performs better than existing IFE measures in its consistency with human intuition and its precision in measuring the uncertainty of IFNs.
- (ii) The integrated method for determining the weights of DMs and criteria can tackle effectively the impact of the interactions among DMs on the specifications of the collective criterion weights and DM weights.
- (iii) The WIFBM and WIFGBM introduced for the aggregation of individual evaluation information and the ranking of alternatives can model effectively the interrelationships among arguments induced by the interactions among DMs.

Thus, the proposed model avoids the impacts resulting from the interactions among DMs on the specifications of collective criterion weights and DM weights, and on the fusion of individual evaluation values. The analysis has shown that it can model MCGDM problems effectively and can help groups of DMs to make specific decisions. In addition, the proposed MCGDM model can be extended to support situations where the information is in other forms, such as interval-valued IFNs, linguistic variables, or type-2 fuzzy numbers.

However, the proposed model does have some limitations. The model does not include the impact of interactions among DMs on the consensus-reaching problems in MCGDM, or the impact of interdependence among criteria. Those omissions indicate the main directions for future research.

**APPENDIX A
PROOF OF PROPOSITION 1**

Proof: Following Definition 11, we obtain the following for all $x_i \in X$:

a) $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i) \Leftrightarrow \phi_A(x_i) \geq \phi_B(x_i)$ when $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$; that is, $E(A) \leq E(B)$.

b) $\nu_A(x_i) \leq \nu_B(x_i) \leq \mu_B(x_i) \leq \mu_A(x_i) \Leftrightarrow \phi_A(x_i) \geq \phi_B(x_i)$ when $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$; that is, $E(A) \leq E(B)$.

As $\phi'_A(x_i) \geq \phi_B(x_i)$ for $x_i \in Y$, combining the analyses in a) and b) yields $E(A) \leq E(B)$.

**APPENDIX B
PROOF OF PROPOSITION 2**

Proof: For $x_i \in Y$, we have $\pi'_A(x_i) = \pi'_B(x_i)$. Then,

$$\begin{aligned} \mu'^2_A(x_i) + \nu'^2_A(x_i) + 2\mu'_A(x_i)\nu'_A(x_i) \\ = \mu^2_B(x_i) + \nu^2_B(x_i) + 2\mu_B(x_i)\nu_B(x_i) \end{aligned}$$

Following the condition that $\mu'_A(x_i)\nu'_A(x_i) < \mu_B(x_i)\nu_B(x_i)$, we have

$$\begin{aligned} \mu'^2_A(x_i) + \nu'^2_A(x_i) + 2\mu'_A(x_i)\nu'_A(x_i) - 4\mu'_A(x_i)\nu'_A(x_i) \\ > \mu^2_B(x_i) + \nu^2_B(x_i) + 2\mu_B(x_i)\nu_B(x_i) - 4\mu_B(x_i)\nu_B(x_i). \end{aligned}$$

Thus, we have

$$(\mu'_A(x_i) - \nu'_A(x_i))^2 > (\mu_B(x_i) - \nu_B(x_i))^2.$$

Therefore,

$$\phi'_A(x_i) > \phi_B(x_i).$$

Following Definition 11, we finally obtain $E(A) \leq E(B)$.

**APPENDIX C
PROOF OF THEOREM 2**

Proof: From $0 \leq \mu_A(x_i), \nu_A(x_i) \leq 1$ and $\mu_A(x_i) + \nu_A(x_i) \leq 1$, we obtain

$$(\mu_A(x_i) - \nu_A(x_i))^2 \leq |\mu_A(x_i) - \nu_A(x_i)|.$$

Then,

$$\begin{aligned} 0 &\leq (1 - |\mu_A(x_i) - \nu_A(x_i)|) \\ &\quad \times \left[1 + (1 - \mu_A(x_i) - \nu_A(x_i))(\mu_A(x_i) - \nu_A(x_i))^2 \right] \\ &\leq (1 - |\mu_A(x_i) - \nu_A(x_i)|) \\ &\quad \times [1 + (1 - \mu_A(x_i) - \nu_A(x_i))|\mu_A(x_i) - \nu_A(x_i)|] \\ &\leq (1 - |\mu_A(x_i) - \nu_A(x_i)|)(1 + |\mu_A(x_i) - \nu_A(x_i)|) \\ &= 1 - |\mu_A(x_i) - \nu_A(x_i)|^2 \leq 1. \end{aligned}$$

Thus, we have $0 \leq E(A) \leq 1$. Next, we prove that $E(A)$ satisfies the following axiomatic principles in Definition 11.

(E1) When A is a crisp set, $\mu_A(x_i) = 1$ and $\nu_A(x_i) = 0$ for all $x_i \in X$. We then have $\phi_A(x_i) = 1$ and $\pi_A(x_i) = 0$. $E(A) = 0$ can then be verified. Similarly, if $\mu_A(x_i) = 0$ and $\nu_A(x_i) = 1$ for all $x_i \in X$, we also obtain $E(A) = 0$. Conversely, suppose $E(A) = 0$. This value holds only when $1 - |\mu_A(x_i) - \nu_A(x_i)| = 0$ or when $1 + \pi_A(x_i)(\mu_A(x_i) - \nu_A(x_i))^2 = 0$. Obviously, $1 + \pi_A(x_i)(\mu_A(x_i) - \nu_A(x_i))^2 \geq 1$. Hence, $\phi_A(x_i) = 1$ holds; that is, $\{\mu_A(x_i) = 1 \text{ and } \nu_A(x_i) = 0\}$ or $\{\mu_A(x_i) = 1 \text{ and } \nu_A(x_i) = 0\}$. A is therefore a crisp set.

(E2) If $\pi_A(x_i) = 1$ or $\phi_A(x_i) = 0$ for all $x_i \in X$, then we have $E(A) = 1$ from Eq. (12). Conversely, if $E(A) = 1$ for all $x_i \in X$, then $1 - |\mu_A(x_i) - \nu_A(x_i)| = 1$ and $1 + \pi_A(x_i)(\mu_A(x_i) - \nu_A(x_i))^2 = 1$ should both hold. Thus, we have $\mu_A(x_i) = \nu_A(x_i)$, which means $\phi_A(x_i) = 0$ and $\pi_A(x_i) = 1$.

(E3) It is trivial.

(E4) First, situation a) is proven.

a.1) For all $x_i \in X$, if $\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$, then $\nu_A(x_i) \leq \nu_B(x_i) \leq \mu_B(x_i) \leq \mu_A(x_i)$. Thus, we have:

$$\mu_A(x_i) - \nu_A(x_i) \geq \mu_B(x_i) - \nu_B(x_i) \quad (31)$$

and

$$|\mu_A(x_i) - \nu_A(x_i)| \geq |\mu_B(x_i) - \nu_B(x_i)|. \quad (32)$$

Let:

$$f(\alpha, \beta) = (1 - \alpha) \left[1 + \beta\alpha^2 \right], \quad (33)$$

where $\alpha, \beta \in [0, 1]$. We then have:

$$f_\alpha(\alpha, \beta) = -3\beta \left(\alpha - \frac{1}{3} \right)^2 + \frac{\beta}{3} - 1. \quad (34)$$

Because $f_\alpha(\alpha, \beta)$ obtains the maximum value at $\alpha = 1/3$, then $f_\alpha(\alpha, \beta) \leq f_\alpha(1/3, \beta) = \beta/3 - 1 < 0$ for all $\alpha, \beta \in [0, 1]$. Thus, $f(\alpha, \beta)$ is decreasing with α . Following Eq. (32), we have:

$$\begin{aligned} (1 - |\mu_A(x_i) - \nu_A(x_i)|) \\ \times \left[1 + (1 - \mu_A(x_i) - \nu_A(x_i))(\mu_A(x_i) - \nu_A(x_i))^2 \right] \\ \leq (1 - |\mu_B(x_i) - \nu_B(x_i)|) \\ \times \left[1 + (1 - \mu_B(x_i) - \nu_B(x_i))(\mu_B(x_i) - \nu_B(x_i))^2 \right]. \end{aligned} \quad (35)$$

Therefore, $E(A) \leq E(B)$.

a.2) For all $x_i \in X$, if $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$, then $\mu_A(x_i) \leq \mu_B(x_i) \leq \nu_B(x_i) \leq \nu_A(x_i)$. Thus, we have:

$$\nu_A(x_i) - \mu_A(x_i) \geq \nu_B(x_i) - \mu_B(x_i) \quad (36)$$

and

$$|\mu_A(x_i) - \nu_A(x_i)| \geq |\mu_B(x_i) - \nu_B(x_i)|. \quad (37)$$

Similarly, we have:

$$\begin{aligned} (1 - |\mu_A(x_i) - \nu_A(x_i)|) \\ \times \left[1 + (1 - \mu_A(x_i) - \nu_A(x_i))(\mu_A(x_i) - \nu_A(x_i))^2 \right] \\ \leq (1 - |\mu_B(x_i) - \nu_B(x_i)|) \\ \times \left[1 + (1 - \mu_B(x_i) - \nu_B(x_i))(\mu_B(x_i) - \nu_B(x_i))^2 \right]. \end{aligned} \quad (38)$$

Therefore, combining a.1) and a.2) above yields $E(A) \leq E(B)$.

Second, we prove situation b).

b.1) If $\mu_A(x_i) = \nu_A(x_i)$ and $\mu_B(x_i) = \nu_B(x_i)$, then $E_1(A) = E_1(B) = 1$.

b.2) If $\phi_A(x_i) = \phi_B(x_i)$ and $\pi_A(x_i) < \pi_B(x_i)$, then:

$$(1 - \phi_A(x_i)) \left(1 + \pi_A(x_i) \phi_A^2(x_i)\right) < (1 - \phi_B(x_i)) \left(1 + \pi_B(x_i) \phi_B^2(x_i)\right) \quad (39)$$

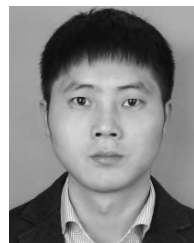
Thus, $E(A) \leq E(B)$.

Therefore, combining a) and b) yields $E(A) \leq E(B)$.

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