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Two-Sided Matching for Triangular Intuitionistic Fuzzy Numbers in Smart Environmental Protection

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ABSTRACT Triangular intuitionistic fuzzy numbers (TIFNs) are effective and flexible to characterize the fuzziness and uncertainty in real-world problems. The theories of TIFNs have been used in multi-attribute decision making but are rarely applied in a two-sided matching decision. Therefore, it is important and necessary to investigate the two-sided matching problem with TIFNs. This paper develops a decision method for two-sided matching with triangular intuitionistic fuzzy numbers and applies it to smart environmental protection. First, a similarity measure between generalized triangular fuzzy numbers (TFNs) is presented. Then, a novel similarity measure between TIFNs is extended, where the maximum membership degrees and minimum non-membership degrees, areas, and perimeters are considered. With respect to the two-sided matching problem with TIFNs, the two-sided matching model with TIFNs is established. Using similarity measures between TIFNs, the similarity matrices of triangular intuitionistic fuzzy preference matrices are constructed by using the positive idea vectors. Then, the two-sided matching model with similarity measures is obtained. Using the arithmetic mean, normalization formulas and linear weighting, the two-sided matching model with similarity measures is transformed into a mono-objective model. The optimum matching scheme is obtained by solving the model. Thus, a similarity measure-based two-sided matching decision method for TIFNs is proposed. Finally, a matching example in smart environmental protection is provided to illustrate the advantages of the proposed method.

INDEX TERMS Two-sided matching, smart environmental protection, triangular intuitionistic fuzzy number, similarity measure, model.

I. INTRODUCTION

Two-sided matching has garnered much attention and has been used for a wide range of applications, such as marriage [1], advertising [2], college enrollment [3], service matching [4], teachers-schools matching [5], and so on. Now, two-sided matching has been extended to a different environment, such as ordinal numbers, order relations, and linguistic terms [6]–[12]. However, considering the fuzziness, the complexity intangibility of practical decision problems and the cognitive limitation of agents, the agents' preferences could be in the format of triangular intuitionistic fuzzy number (TIFN). As an extension of the intuitionistic fuzzy number (IFS) [13], it is effective and flexible to use TIFNs to characterize the fuzziness and uncertainty in real-world problems [14]–[17]. The difference between TIFNs and IFNs is that the degrees of membership and non-membership are triangular fuzzy numbers (TFNs), not real numbers. The related

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theories of TIFNs have been used in multi-attribute decision making [18]–[20]. However, TIFNs are rarely applied in a two-sided matching decision. Therefore, it is important and necessary to investigate the two-sided matching problem with TIFNs. On the other hand, smart environmental protection could be regarded as a typical two-sided matching problem. The monetary input of environmental protection monitoring equipment in China has surpassed ten billion RMB. But the effect is very poor. Hence, with respect to the smart environmental protection, the input is not matched with the output. In conclusion, the paper investigates the smart environmental protection problem from the perspective of two-sided matching in a TIFN environment.

The contribution of this paper is summarized as follows:

(1) A similarity measure between generalized triangular fuzzy numbers (TFNs) is presented. Then, a novel similarity measure between TIFNs is extended. The novel similarity measure between TIFNs considers the maximum membership degrees, the minimum non-membership degrees, areas and the perimeters of TIFNs, which are comprehensive.

(2) Using the similarity measure between TIFNs, the similarity matrices of triangular intuitionistic fuzzy preference matrices are constructed. Then, the two-sided matching model with similarity measures is obtained using the two-sided matching model with TIFNs.

(3) Using the arithmetic mean, normalization formulas and linear weighting, the two-sided matching model with similarity measures is transformed into a mono-objective model. Thus, a similarity measure based two-sided matching decision method for TIFNs is proposed.

The remainder of this paper is explained below. In Section 2, the concepts of TIFN and two-sided matching are reviewed. A novel similarity measure between TIFNs is presented in Section 3. In Section 4, a similarity measure based two-sided matching decision method is proposed for addressing two-sided matching problems with TIFNs. In Section 5, an example is presented to explain the proposed method. The conclusions are discussed in Section 6.

II. PRELIMINARIES

In this section, the definitions of TIFNs are first introduced. Then, the concept of two-sided matching is presented.

A. CONCEPT OF TIFNs

Definition 1 [13], [14]: Let $\tilde{a} =< (\underline{a}, a', \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} >$ be a special intuitionistic fuzzy number on real set R, where $\underline{a} \leq a' \leq \bar{a}$. If membership degree $\omega_{\tilde{a}}(x)$ and non-membership degree $u_{\tilde{a}}(x)$ are respectively defined by

$$\omega_{\tilde{a}}(x) = \begin{cases} \dfrac{x-\underline{a}}{a'-\underline{a}}\omega_{\tilde{a}}, & \underline{a} \leq x < a', \\ \omega_{\tilde{a}}, & x = a', \\ \dfrac{\bar{a}-x}{\bar{a}-a'}\omega_{\tilde{a}}, & a' < x \leq \bar{a}, \\ 0, & x < \underline{a}, x > \bar{a}, \end{cases} \tag{1}$$

$$u_{\tilde{a}}(x) = \begin{cases} \dfrac{a'-x+(x-\underline{a})u_{\tilde{a}}}{a'-\underline{a}}, & \underline{a} \leq x < a', \\ u_{\tilde{a}}, & x = a', \\ \dfrac{x-a'+(\bar{a}-x)u_{\tilde{a}}}{\bar{a}-a'}, & a' < x \leq \bar{a}, \\ 1, & x < \underline{a}, x > \bar{a}, \end{cases} \tag{2}$$

where $\omega_{\tilde{a}}$ and $u_{\tilde{a}}$ are the maximum membership degree and minimum non-membership degree, respectively, and satisfy $0 \leq \omega_{\tilde{a}} \leq 1$, $0 \leq u_{\tilde{a}} \leq 1$ and $0 \leq \omega_{\tilde{a}} + u_{\tilde{a}} \leq 1$; then, $\tilde{a}$ is called a TIFN.

Remark 1: In the real world, TIFNs can be obtained through online reviews [21], [22].

B. TWO-SIDED MATCHING

Let $\Gamma = \{\tau_1, \tau_2, \cdots, \tau_m\}$ and $\mathrm{E} = \{\varepsilon_1, \varepsilon_2, \cdots \varepsilon_n\}$ be the set of agents on two separated sides, where $\tau_i(\varepsilon_j)$ is the i th(j th) agent on side $\Gamma(\mathrm{E})$, and $n \geq m \geq 2$. Let $M = \{1, 2, \cdots, m\}$, $N = \{1, 2, \cdots, n\}$. Inspired by references [23]–[25], the definitions of two-sided matching can be described below.

Definition 2: Let $\ell : \Gamma \cup \mathrm{E} \rightarrow \Gamma \cup \mathrm{E}$ be a one-to-one mapping, then ℓ is called two-sided matching if it meets the following three properties:

1) $\ell(\tau_i) \in \mathrm{E}$;
2) $\ell(\varepsilon_j) \in \Gamma \cup \{\varepsilon_j\}$; and
3) $\ell(\tau_i) = \varepsilon_j$ if $\ell(\varepsilon_j) = \tau_i$, for all τ_i and ε_j.

Thereafter, $\ell(\tau_i) = \varepsilon_j$ (or matching pair (τ_i, ε_j)) represents that τ_i matches with ε_j in ℓ; $\ell(\varepsilon_j) = \varepsilon_j$ (or single pair $(\varepsilon_j, \varepsilon_j)$) represents that ε_j does not match (or single).

Definition 3: Let $\ell : \Gamma \cup \mathrm{E} \rightarrow \Gamma \cup \mathrm{E}$ be the two-sided matching, then it can be expressed as $\ell = \ell_{Ma} \cup \ell_{Si}$, where ℓ_{Ma} is the set of matching pairs, and ℓ_{Si} is the set of single pairs.

III. NOVEL SIMILARITY MEASURE BETWEEN TIFNs

A. NOVEL SIMILARITY MEASURE BETWEEN GENERALIZED TFNs

Inspired by references [26], [27], a novel similarity measure between generalized TFNs can be expressed below.

Definition 4: Suppose $a^g = (\underline{a}, a', \bar{a}; \mu_{a^g})$ and $b^g = (\underline{b}, b', \bar{b}; \mu_{b^g})$ are generalized TFNs, in which $\underline{a} \leq a' \leq \bar{a}$, $\underline{b} \leq b' \leq \bar{b}$. Thereafter, a similarity measure between a^g and b^g is defined in (3) as shown at the top of the next page, where $Ar(a^g)$ and $Ar(b^g)$ are the areas of a^g and b^g, respectively; $Pe(a^g)$ and $Pe(b^g)$ are the perimeters of a^g and b^g, respectively, which are calculated by:

$$Ar(a^g) = \frac{1}{2} \cdot (\bar{a} - \underline{a}) \cdot \mu_{a^g}, \tag{4}$$

$$Ar(b^g) = \frac{1}{2} \cdot (\bar{b} - \underline{b}) \cdot \mu_{b^g}, \tag{5}$$

$$Pe(a^g) = \sqrt{(a'-\underline{a})^2 + (\mu_{a^g})^2} + \sqrt{(\bar{a}-a')^2 + (\mu_{a^g})^2} + (\bar{a} - \underline{a}), \tag{6}$$

$$Pe(b^g) = \sqrt{(b'-\underline{b})^2 + (\mu_{b^g})^2} + \sqrt{(\bar{b}-b')^2 + (\mu_{b^g})^2} + (\bar{b} - \underline{b}), \tag{7}$$

$$S(a^g, b^g) = 2^{-\frac{|\underline{a}-\underline{b}+2a'-2b'+\bar{a}-\bar{b}|}{4}} \times \frac{\min(Ar(a^g), Ar(b^g)) + \min(Pe(a^g), Pe(b^g)) + \min(\mu_{a^g}, \mu_{b^g})}{\max(Ar(a^g), Ar(b^g)) + \max(Pe(a^g), Pe(b^g)) + \max(\mu_{a^g}, \mu_{b^g})} \tag{3}$$

$$S(a_1^g, b_1^g) = 2^{-\frac{|\underline{a}-\underline{b}+2a'-2b'+\bar{a}-\bar{b}|}{4}} \times \frac{\min(Ar(a_1^g), Ar(b_1^g)) + \min(Pe(a_1^g), Pe(b_1^g)) + \min(\omega_{\tilde{a}}, \omega_{\tilde{b}})}{\max(Ar(a_1^g), Ar(b_1^g)) + \max(Pe(a_1^g), Pe(b_1^g)) + \max(\omega_{\tilde{a}}, \omega_{\tilde{b}})}, \tag{8}$$

$$S(a_2^g, b_2^g) = 2^{-\frac{|\underline{a}-\underline{b}+2a'-2b'+\bar{a}-\bar{b}|}{4}} \times \frac{\min(Ar(a_2^g), Ar(b_2^g)) + \min(Pe(a_2^g), Pe(b_2^g)) + \min(1-u_{\tilde{a}}, 1-u_{\tilde{b}})}{\max(Ar(a_2^g), Ar(b_2^g)) + \max(Pe(a_2^g), Pe(b_2^g)) + \max(1-u_{\tilde{a}}, 1-u_{\tilde{b}})}, \tag{9}$$

Theorem 1: Let $a^g = (\underline{a}, a', \bar{a}; \mu_{a^g})$ and $b^g = (\underline{b}, b', \bar{b}; \mu_{b^g})$ be generalized TFNs, then $S(a^g, b^g)$ meets the characteristics:

1) $0 \leq S(a^g, b^g) \leq 1$;
2) $S(a^g, b^g) = 1$ if $a^g = b^g$;
3) $S(a^g, b^g) = S(b^g, a^g)$.

Theorem 1 can be easily proved by Definition 4.

B. NOVEL SIMILARITY MEASURE BETWEEN TIFNs

According to Definition 4, some similarity measures are expressed below.

Definition 5: Let $\tilde{a} =< (\underline{a}, a', \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} >$ and $\tilde{b} = (\underline{b}, b', \bar{b}; \omega_{\tilde{b}}, u_{\tilde{b}})$ be TIFNs. The similarity measure $S(a_1^g, b_1^g)$ between generalized TFNs $a_1^g = (\underline{a}, a', \bar{a}; \omega_{\tilde{a}})$ and $b_1^g = (\underline{b}, b', \bar{b}; \omega_{\tilde{b}})$ is defined in (8) as shown at the top of this page, where the areas $Ar(a_1^g)$ and $Ar(b_1^g)$ and the perimeters $Pe(a_1^g)$ and $Pe(b_1^g)$ can be calculated by using Eqs. (4)-(7).

Similarly, the similarity measure $S(a_2^g, b_2^g)$ between generalized TFNs $a_2^g = (\underline{a}, a', \bar{a}; u_{\tilde{a}})$ and $b_2^g = (\underline{b}, b', \bar{b}; u_{\tilde{b}})$ is defined in (9) as shown at the top of this page, where the areas $Ar(a_2^g)$ and $Ar(b_2^g)$ and the perimeters $Pe(a_2^g)$ and $Pe(b_2^g)$ are respectively calculated by:

$$Ar(a_2^g) = \frac{1}{2} \cdot (\bar{a} - \underline{a}) \cdot (1 - u_{\tilde{a}}), \tag{10}$$

$$Ar(b_2^g) = \frac{1}{2} \cdot (\bar{b} - \underline{b}) \cdot (1 - u_{\tilde{b}}), \tag{11}$$

$$Pe(a_2^g) = \sqrt{(a' - \underline{a})^2 + (1 - u_{\tilde{a}})^2} + \sqrt{(\bar{a} - a')^2 + (1 - u_{\tilde{a}})^2} + (\bar{a} - \underline{a}), \tag{12}$$

$$Pe(b_2^g) = \sqrt{(b' - \underline{b})^2 + (1 - u_{\tilde{b}})^2} + \sqrt{(\bar{b} - b')^2 + (1 - u_{\tilde{b}})^2} + (\bar{b} - \underline{b}), \tag{13}$$

Definition 6: Let $\tilde{a} =< (\underline{a}, a', \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} >$ and $\tilde{b} = (\underline{b}, b', \bar{b}; \omega_{\tilde{b}}, u_{\tilde{b}})$ be two TIFNs. Then, the similarity measure $S(\tilde{a}, \tilde{b})$ between $\tilde{a} =< (\underline{a}, a', \bar{a}); \omega_{\tilde{a}}, u_{\tilde{a}} >$ and $\tilde{b} = (\underline{b}, b', \bar{b}; \omega_{\tilde{b}}, u_{\tilde{b}})$ is defined as:

$$S(\tilde{a}, \tilde{b}) = \frac{S(a_1^g, b_1^g) + S(a_2^g, b_2^g)}{2}. \tag{14}$$

where $S(a_1^g, b_1^g)$ and $S(a_2^g, b_2^g)$ are calculated by Eqs. (8) and (9), respectively.

IV. SIMILARITY MEASURE BASED TWO-SIDED MATCHING DECISION METHOD FOR TIFNs

The section describes the two-sided matching problem with TIFNs. Then, a similarity measure based two-sided matching decision method is proposed.

A. DESCRIPTION OF TWO-SIDED MATCHING PROBLEM WITH TIFNs

With respect to the two-sided matching problem with TIFNs, assume $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ is the triangular intuitionistic fuzzy preference matrix from Γ to E, in which TIFN $\tilde{a}_{ij}$ is expressed as $\tilde{a}_{ij} =< (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} >$. Therein, $\omega_{\tilde{a}_{ij}}$ represents the maximum membership degree of τ_i to ε_j, and $u_{\tilde{a}_{ij}}$ represents the of minimum membership degree of agent τ_i to ε_j. Assume $\tilde{B} = [\tilde{b}_{ij}]_{m\times n}$ is the triangular intuitionistic fuzzy preference matrix from E to Γ, in which TIFN $\tilde{b}_{ij}$ is expressed as $\tilde{b}_{ij} =< (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}} >$. Therein, $\omega_{\tilde{b}_{ij}}$ represents the maximum membership degree of ε_j to τ_i, and $u_{\tilde{b}_{ij}}$ represents the minimum membership degree dissatisfaction of ε_j to τ_i. Assume ℓ^* is the "optimum" alternative of two-sided matching.

B. TRIANGULAR FUZZY TWO-SIDED MATCHING MODEL

To maximize the satisfaction degrees with TIFNs, a triangular fuzzy two-sided matching model (M-1) is constructed by using a multi-objective assignment model as follows:

$$(\mathrm{M}-1)\begin{cases} \max G_{\tau_i} = \sum_{j=1}^{n} < (\underline{a}_{ij},\ a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} > \rho_{ij},\ i \in M \\ \max G_{\varepsilon_j} = \sum_{i=1}^{m} < (\underline{b}_{ij},\ b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}} > \rho_{ij},\ j \in N \\ \text{s.t. } \sum_{j=1}^{n} \rho_{ij} = 1, \quad i \in M; \\ \quad \sum_{i=1}^{m} \rho_{ij} \leq 1, \quad j \in N; \\ \quad \rho_{ij} \in \{0,\ 1\}, \quad i \in M, j \in N \end{cases}$$

where ρ_{ij} is a 0-1 variable, $\rho_{ij} = 1$ signifies $\ell(\tau_i) = \varepsilon_j$, and $\rho_{ij} = 0$ signifies $\ell(\tau_i) \neq \varepsilon_j$; $\sum_{j=1}^{n} \rho_{ij} = 1$ implies each agent of side Γ has exactly a matching agent of side E; $\sum_{i=1}^{m} \rho_{ij} \leq 1$ implies each agent of side E has at most one matching agent of side Γ.

$$\tilde{A}=[\tilde{a}_{ij}]_{m\times n}=\begin{bmatrix} <(\underline{a}_{11},a'_{11},\bar{a}_{11});\omega_{\tilde{a}_{11}},\ u_{\tilde{a}_{11}}> & \ldots & <(\underline{a}_{1n},a'_{1n},\bar{a}_{1n});\omega_{\tilde{a}_{1n}},\ u_{\tilde{a}_{1n}}> \\ <(\underline{a}_{21},a'_{21},\bar{a}_{21});\omega_{\tilde{a}_{21}},\ u_{\tilde{a}_{21}}> & \ldots & <(\underline{a}_{2n},a'_{2n},\bar{a}_{2n});\omega_{\tilde{a}_{2n}},\ u_{\tilde{a}_{2n}}> \\ \ldots & \ldots & \ldots \\ <(\underline{a}_{m1},a'_{m1},\bar{a}_{m1});\omega_{\tilde{a}_{m1}},\ u_{\tilde{a}_{m1}}> & \ldots & <(\underline{a}_{mn},a'_{mn},\bar{a}_{mn});\omega_{\tilde{a}_{mn}},\ u_{\tilde{a}_{mn}}> \end{bmatrix},$$

$$\tilde{B}=[\tilde{b}_{ij}]_{m\times n}=\begin{bmatrix} <(\underline{b}_{11},b'_{11},\bar{b}_{11});\omega_{\tilde{b}_{11}},u_{\tilde{b}_{11}}> & \ldots & <(\underline{b}_{1n},b'_{1n},\bar{b}_{1n});\omega_{\tilde{b}_{1n}},u_{\tilde{b}_{1n}}> \\ <(\underline{b}_{21},b'_{21},\bar{b}_{21});\omega_{\tilde{b}_{21}},u_{\tilde{b}_{21}}> & \ldots & <(\underline{b}_{2n},b'_{2n},\bar{b}_{2n});\omega_{\tilde{b}_{2n}},u_{\tilde{b}_{2n}}> \\ \ldots & \ldots & \ldots \\ <(\underline{b}_{m1},b'_{m1},\bar{b}_{m1});\omega_{\tilde{b}_{m1}},u_{\tilde{b}_{m1}}> & \ldots & <(\underline{b}_{mn},b'_{mn},\bar{b}_{mn});\omega_{\tilde{b}_{mn}},u_{\tilde{b}_{mn}}> \end{bmatrix},$$

C. SIMILARITY MEASURE BASED TWO-SIDED MATCHING MODEL

Then, we use the similarity measures between TIFNs to solve the above triangular fuzzy two-sided matching model (M-1). First, with respect to preference matrix $\tilde{A}=[\tilde{a}_{ij}]_{m\times n}$, the positive ideal vector on side τ can be expressed in $\bar{A}^{PI}=(\tilde{a}_1^{PI},\tilde{a}_2^{PI},\ldots,\tilde{a}_m^{PI})^T$, where the i th positive ideal component $\tilde{a}_i^{PI}=<(\underline{a}_i^{PI},a'^{PI}_i,\bar{a}_i^{PI});\omega_{\tilde{a}_i^{PI}},\ u_{\tilde{a}_i^{PI}}>$ can be calculated by

$$\begin{cases} \underline{a}_i^{PI}=\max\{\underline{a}_{ij}\,|j\in N\} \\ a'^{PI}_i=\max\{a'_{ij}\,|j\in N\} \\ \bar{a}_i^{PI}=\max\{\bar{a}_{ij}\,|j\in N\} \\ \omega_{\tilde{a}_i^{PI}}=\max\{\omega_{\tilde{a}_{ij}}\,|j\in N\} \\ u_{\tilde{a}_i^{PI}}=\min\{u_{\tilde{a}_{ij}}\,|j\in N\} \end{cases} \tag{15}$$

With respect to preference matrix $\tilde{B}=[\tilde{b}_{ij}]_{m\times n}$, the positive ideal vector on side ε can be expressed in $\bar{B}^{PI}=(\tilde{b}_1^{PI},\tilde{b}_2^{PI},\ldots,\tilde{b}_n^{PI})$, where the j th positive ideal component $\tilde{b}_j^{PI}=<(\underline{b}_j^{PI},b'^{PI}_j,\bar{b}_j^{PI});\omega_{\tilde{b}_j^{PI}},\ u_{\tilde{b}_j^{PI}}>$ can be calculated by

$$\begin{cases} \underline{b}_j^{PI}=\max\{\underline{b}_{ij}\,|i\in M\} \\ b'^{PI}_j=\max\{b'_{ij}\,|i\in M\} \\ \bar{b}_j^{PI}=\max\{\bar{b}_{ij}\,|i\in M\} \\ \omega_{\tilde{b}_j^{PI}}=\max\{\omega_{\tilde{b}_{ij}}\,|i\in M\} \\ u_{\tilde{b}_j^{PI}}=\min\{u_{\tilde{b}_{ij}}\,|i\in M\} \end{cases} \tag{16}$$

According to preference matrix $\tilde{A}$ and $\tilde{B}$, as shown at the top of this page, the positive ideal vector of triangular intuitionistic fuzzy number $\bar{A}^{PI}=(<(\underline{a}_1^{PI},a'^{PI}_1,\bar{a}_1^{PI});\omega_{\tilde{a}_1^{PI}},\ u_{\tilde{a}_1^{PI}}>,\ldots,<(\underline{a}_m^{PI},a'^{PI}_m,\bar{a}_m^{PI});\omega_{\tilde{a}_m^{PI}},\ u_{\tilde{a}_m^{PI}}>)^T$ and Definitions 5 and 6, the similarity measure $S(\tilde{a}_{ij},\tilde{a}_i^{PI})=S(<(\underline{a}_{ij},a'_{ij},\bar{a}_{ij});\omega_{\tilde{a}_{ij}},\ u_{\tilde{a}_{ij}}>,<(\underline{a}_i^{PI},a'^{PI}_i,\bar{a}_i^{PI});\omega_{\tilde{a}_i^{PI}},u_{\tilde{a}_i^{PI}}>)$ can be calculated by

$$S(\tilde{a}_{ij},\tilde{a}_i^{PI})=\frac{S(a_{ij1}^g,a_{i1}^{PIg})+S(a_{ij2}^g,a_{i2}^{PIg})}{2}. \tag{17}$$

Therein, $a_{ij1}^g=(\underline{a}_{ij},a'_{ij},\bar{a}_{ij};\omega_{\tilde{a}_{ij}})$, $a_{ij2}^g=(\underline{a}_{ij},a'_{ij},\bar{a}_{ij};u_{\tilde{a}_{ij}})$, $a_{i1}^{PIg}=(\underline{a}_i^{PI},a'^{PI}_i,\bar{a}_i^{PI};\omega_{\tilde{a}_i^{PI}})$, $a_{i2}^{PIg}=(\underline{a}_i^{PI},a'^{PI}_i,\bar{a}_i^{PI};u_{\tilde{a}_i^{PI}})$, $S(a_{ij1}^g,a_{i1}^{PIg})$ and $S(a_{ij2}^g,a_{i2}^{PIg})$ can be calculated by using Eqs. (4)-(14). Similarly, according to preference matrix the positive ideal vector of triangular intuitionistic fuzzy number

$$\bar{B}^{PI}=(<(\underline{b}_1^{PI},b'^{PI}_1,\bar{b}_1^{PI});\omega_{\tilde{b}_1^{PI}},\ u_{\tilde{b}_1^{PI}}>,\ldots,<(\underline{b}_n^{PI},b'^{PI}_n,\bar{b}_n^{PI});\omega_{\tilde{b}_n^{PI}},\ u_{\tilde{b}_n^{PI}}>)$$

and Definitions 5 and 6, the similarity measure

$$S(\tilde{b}_{ij},\tilde{b}_j^{PI})=S(<(\underline{b}_{ij},b'_{ij},\bar{b}_{ij});\omega_{\tilde{b}_{ij}},u_{\tilde{b}_{ij}}>,<(\underline{b}_j^{PI},b'^{PI}_j,\bar{b}_j^{PI});\omega_{\tilde{b}_j^{PI}},\ u_{\tilde{b}_j^{PI}}>)$$

can be calculated by

$$S(\tilde{b}_{ij},\tilde{b}_j^{PI})=\frac{S(b_{ij1}^g,b_{j1}^{PIg})+S(b_{ij2}^g,b_{j2}^{PIg})}{2}. \tag{18}$$

Therein, $b_{ij1}^g=(\underline{b}_{ij},b'_{ij},\bar{b}_{ij};\omega_{\tilde{b}_{ij}})$, $b_{ij2}^g=(\underline{b}_{ij},b'_{ij},\bar{b}_{ij};u_{\tilde{b}_{ij}})$, $b_{j1}^{PIg}=(\underline{b}_j^{PI},b'^{PI}_j,\bar{b}_j^{PI};\omega_{\tilde{b}_j^{PI}})$, $b_{j2}^{PIg}=(\underline{b}_j^{PI},b'^{PI}_j,\bar{b}_j^{PI};u_{\tilde{b}_j^{PI}})$, $S(b_{ij1}^g,b_{j1}^{PIg})$ and $S(b_{ij2}^g,b_{j2}^{PIg})$ are calculated by using Eqs. (4)-(14).

According to similarity measures $S(\tilde{a}_{ij},\tilde{a}_i^{PI})$ and $S(\tilde{b}_{ij},\tilde{b}_j^{PI})$, the triangular fuzzy two-sided matching model (M-1) is transformed into the similarity measure based two-sided matching model (M-2) as follows:

$$(\text{M-2})\begin{cases} \max O_{\tau_i}=\sum_{j=1}^{n}S(\tilde{a}_{ij},\tilde{a}_i^{PI})\rho_{ij}, & i\in M \\ \max O_{\varepsilon_j}=\sum_{i=1}^{m}S(\tilde{b}_{ij},\tilde{b}_j^{PI})\rho_{ij}, & j\in N \\ \text{s.t.}\ \sum_{j=1}^{n}\rho_{ij}=1,\quad i\in M;\ \sum_{i=1}^{m}\rho_{ij}\le 1,\ j\in N; \\ \rho_{ij}\in\{0,\ 1\},\quad i\in M,\ j\in N. \end{cases}$$

D. SOLUTION OF THE SIMILARITY MEASURE BASED TWO-SIDED MATCHING MODEL

If we consider that the statuses of τ_i $(i=1,2,\ldots,m)$ are the same in most actual situations, as are ε_j $(j=1,2,\ldots,n)$, using the arithmetic mean, model (M-2) is converted into a two-goals model (M-3):

$$\text{(M-3)}\begin{cases}\max O_\tau = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{m}S(\tilde{a}_{ij},\tilde{a}_i^{PI})\rho_{ij}\\ \max O_\varepsilon = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{1}{n}S(\tilde{b}_{ij},\tilde{b}_j^{PI})\rho_{ij}\\ \text{s.t.}\ \sum_{j=1}^{n}\rho_{ij}=1,\quad i\in M;\ \ \sum_{i=1}^{m}\rho_{ij}\le 1,\ j\in N;\\ \rho_{ij}\in\{0,\ 1\},\quad i\in M,\ j\in N.\end{cases}$$

Remark 2: In the real problems, if the statuses of τ_i ($i = 1, 2, \ldots, m$) are different, then the weights of τ_i ($i = 1, 2, \ldots, m$) can be obtained through AHP approach, as are ε_j ($j = 1, 2, \ldots, n$).

To solve model (M-3), considering that the dimensions of $\frac{1}{m}S(\tilde{a}_{ij},\tilde{a}_i^{PI})$ and $\frac{1}{n}S(\tilde{b}_{ij},\tilde{b}_j^{PI})$ may not be consistent, they should be normalized. The normalization formulas can be displayed as:

$$S^{Nor}(\tilde{a}_{ij},\tilde{a}_i^{PI}) = \frac{\frac{1}{m}S(\tilde{a}_{ij},\tilde{a}_i^{PI})}{\max\left\{\frac{1}{m}S(\tilde{a}_{ij},\tilde{a}_i^{PI})\,|i\in M, j\in N\right\}}, \tag{19}$$

$$S^{Nor}(\tilde{b}_{ij},\tilde{b}_j^{PI}) = \frac{\frac{1}{n}S(\tilde{b}_{ij},\tilde{b}_j^{PI})}{\max\left\{\frac{1}{n}S(\tilde{b}_{ij},\tilde{b}_j^{PI})\,|i\in M, j\in N\right\}}. \tag{20}$$

By using Eqs. (19) and (20), model (M-3) can be transformed into model (M-4):

$$\text{(M-4)}\begin{cases}\max O_\tau^{Nor} = \sum_{i=1}^{m}\sum_{j=1}^{n}S^{Nor}(\tilde{a}_{ij},\tilde{a}_i^{PI})\rho_{ij}\\ \max O_\varepsilon^{Nor} = \sum_{i=1}^{m}\sum_{j=1}^{n}S^{Nor}(\tilde{b}_{ij},\tilde{b}_j^{PI})\rho_{ij}\\ \text{s.t.}\ \sum_{j=1}^{n}\rho_{ij}=1,\quad i\in M;\ \ \sum_{i=1}^{m}\rho_{ij}\le 1,\ j\in N;\\ \rho_{ij}\in\{0,\ 1\},\quad i\in M,\ j\in N.\end{cases}$$

Let w_τ and w_ε be the weights of objectives O_τ^{Nor} and O_ε^{Nor}, respectively. Then, using the linear weighting, model (M-4) is transformed into model (M-5):

$$\text{(M-5)}\begin{cases}\max O = \sum_{i=1}^{m}\sum_{j=1}^{n}S_{ij}\rho_{ij}\\ \text{s.t.}\ \sum_{j=1}^{n}\rho_{ij}=1,\quad i\in M;\sum_{i=1}^{m}\rho_{ij}\le 1,\ j\in N;\\ \rho_{ij}\in\{0,\ 1\},\quad i\in M,\ j\in N.\end{cases}$$

where $S_{ij} = w_\tau S^{Nor}(\tilde{a}_{ij},\tilde{a}_i^{Id}) + w_\varepsilon S^{Nor}(\tilde{b}_{ij},\tilde{b}_j^{PI})$.

Remark 3: In actual problems, weights w_τ and w_ε could be regard as the important degrees of agents of two sides respectively, which can be determined on the basis of the statuses of agents.

Solving model (M-5) yields the optimal solutions ρ_{ij}^*. Furthermore, the "optimum" matching alternative $\ell*$ can be obtained.

E. SIMILARITY MEASURE BASED TWO-SIDED MATCHING DECISION METHOD FOR TIFNs

According to the aforementioned analysis, a similarity measure based two-sided matching decision method for TIFNs is summarized as follows.

Step 1: Establish model (M-1) based on the triangular intuitionistic fuzzy preference matrices $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}$ and $\tilde{B} = [\tilde{b}_{ij}]_{m\times n}$ and the two-sided matching matrix $\Theta = [\rho_{ij}]_{m\times n}$.

Step 2: Calculate the positive ideal vectors $\tilde{A}^{Id} = (\tilde{a}_1^{Id}, \tilde{a}_2^{Id}, \ldots, \tilde{a}_m^{Id})^T$ and $\tilde{B}^{Id} = (\tilde{b}_1^{Id}, \tilde{b}_2^{Id}, \ldots, \tilde{b}_n^{Id})$ using Eqs. (15) and (16), respectively.

Step 3: Calculate the similarity measure $S(\tilde{a}_{ij},\tilde{a}_i^{PI})$ according to preference matrix $\tilde{A} = [\tilde{a}_{ij}]_{m\times n} = [< (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}},\ u_{\tilde{a}_{ij}}\ >]_{m\times n}$ and the positive ideal vector $\tilde{A}^{PI} = (\tilde{a}_1^{PI}, \tilde{a}_2^{PI}, \ldots, \tilde{a}_m^{PI})^T = (< (\underline{a}_i^{PI}, a'^{PI}_i, \bar{a}_i^{PI}); \omega_{\tilde{a}_i^{PI}},\ u_{\tilde{a}_i^{PI}} > \Big| i = 1, 2, \ldots, m)^T$ by using Eqs. (4)-(14) and (17).

Step 4: Calculate the similarity measure $S(\tilde{b}_{ij},\tilde{b}_j^{PI})$ according to preference matrix $\tilde{B} = [\tilde{b}_{ij}]_{m\times n} = [< (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}}\ >]_{m\times n}$ and the positive ideal vector $\tilde{B}^{PI} = (\tilde{b}_1^{PI}, \tilde{b}_2^{PI}, \ldots, \tilde{b}_n^{PI}) = (< (\underline{b}_j^{PI}, b'^{PI}_j, \bar{b}_j^{PI}); \omega_{\tilde{b}_j^{PI}}, u_{\tilde{b}_j^{PI}} > \Big| j = 1, 2, \ldots, n)$ by using Eqs. (4)-(14) and (18).

Step 5: Transform model (M-1) into model (M-2) based on similarity measures $S(\tilde{a}_{ij},\tilde{a}_i^{PI})$ and $S(\tilde{b}_{ij},\tilde{b}_j^{PI})$.

Step 6: Convert model (M-2) into a two-objective model (M-3) by using the arithmetic mean.

Step 7: Tranform model (M-3) into a two-objective model (M-4) by using Eqs. (19) and (20).

Step 8: Convert model (M-4) into a mono-objective model (M-5) by using the linear weighting.

Step 9: Acquire the "optimum" two-sided matching ℓ^* by solving model (M-5).

V. SMART ENVIRONMENTAL PROTECTION AND ANALYSIS

This section provides an example to illustrate the advantages of similarity measure based two-sided matching decision method for TIFNs.

Suppose that the environmental monitoring station of Nanchang plans to hire environmental protection personnel in three positions, i.e., an environmental monitoring staff τ_1, a quality inspector τ_2, and a laboratory analyst τ_3. After preliminary screening, five employees ($\varepsilon_1, \varepsilon_2, \ldots,$ and ε_5) apply for the three vacant positions. The department heads evaluate the five employees from three perspectives: technical skills, reaction capabilities, and human relationship skills; they then provide the preference matrix $\tilde{A} = [\tilde{a}_{ij}]_{3\times 5}$, which is demonstrated in Table 1. Five employees estimate the three positions from three aspects: technical requirements, salary and welfare, and work environment; they then provide the preference matrix $\tilde{B} = [\tilde{b}_{ij}]_{3\times 5}$, which is demonstrated in Table 2. Therein, the score "1" denotes the lowest satisfaction degree, and the score "10" denotes the highest satisfaction degree.

TABLE 1. Triangular intuitionistic fuzzy preference matrix $\tilde{A} = [< (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} >]_{3\times 5}$.

position	employee				
	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	$<(3,5,7);0.5,0.3>$	$<(4,6,7);0.6,0.3>$	$<(4,5,7);0.5,0.4>$	$<(3,4,6);0.6,0.2>$	$<(3,4,8);0.7,0.1>$
τ_2	$<(3,5,8);0.6,0.2>$	$<(3,6,9);0.5,0.4>$	$<(3,4,9);0.7,0.2>$	$<(5,6,7);0.7,0.1>$	$<(4,7,8);0.6,0.2>$
τ_3	$<(3,6,8);0.6,0.3>$	$<(3,6,8);0.7,0.2>$	$<(4,5,8);0.7,0.2>$	$<(4,5,7);0.8,0.1>$	$<(3,5,9);0.6,0.3>$

TABLE 2. Triangular intuitionistic fuzzy preference matrix $\tilde{B} = [< (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}} >]_{3\times 5}$.

position	employee				
	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	$<(3,4,5);0.6,0.1>$	$<(3,4,7);0.5,0.1>$	$<(4,5,8);0.6,0.1>$	$<(5,6,7);0.6,0.2>$	$<(3,5,8);0.7,0.3>$
τ_2	$<(4,5,6);0.5,0.2>$	$<(3,4,6);0.7,0.2>$	$<(5,6,8);0.6,0.3>$	$<(4,5,7);0.6,0.3>$	$<(3,5,7);0.6,0.3>$
τ_3	$<(3,4,6);0.5,0.3>$	$<(5,6,6);0.6,0.2>$	$<(5,6,7);0.7,0.2>$	$<(3,5,6);0.7,0.2>$	$<(3,6,8);0.5,0.2>$

TABLE 3. Similarity measures $S(a^g_{ij1}, a^{PIg}_{i1})$ and $S(a^g_{ij2}, a^{PIg}_{i2})$.

$\begin{pmatrix} S(a^g_{ij1}, a^{PIg}_{i1}) \\ S(a^g_{ij2}, a^{PIg}_{i2}) \end{pmatrix}$	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	$\begin{pmatrix} 2^{-1}\times\frac{1+8.1231+0.5}{1.4+8.2379+0.7}=0.4654 \\ 2^{-1}\times\frac{1.4+8.2379+0.7}{1.8+8.3863+0.9}=0.4662 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{4}}\times\frac{0.9+6.2543+0.6}{1.4+8.2379+0.7}=0.6307 \\ 2^{-\frac{1}{4}}\times\frac{1.05+6.3396+0.7}{1.8+8.3863+0.9}=0.6136 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{0.75+6.1796+0.5}{1.4+8.2379+0.7}=0.4273 \\ 2^{-\frac{3}{4}}\times\frac{0.9+6.2543+0.6}{1.8+8.3863+0.9}=0.4159 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{7}{4}}\times\frac{0.9+6.2543+0.6}{1.4+8.2379+0.7}=0.223 \\ 2^{-\frac{7}{4}}\times\frac{1.2+6.4347+0.8}{1.8+8.3863+0.9}=0.2262 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{5}{4}}\times\frac{1.4+8.2379+0.7}{1.75+10.2814+0.7}=0.3282 \\ 2^{-\frac{5}{4}}\times\frac{1.8+8.3863+0.9}{2.25+10.4454+0.9}=0.3429 \end{pmatrix}$
τ_2	$\begin{pmatrix} 2^{-\frac{7}{4}}\times\frac{1.4+8.2379+0.6}{1.5+10.1475+0.7}=0.2465 \\ 2^{-\frac{7}{4}}\times\frac{1.8+8.3863+0.8}{2+10.2589+0.9}=0.2482 \end{pmatrix}$	$\begin{pmatrix} 2^{-1}\times\frac{1.4+8.2379+0.5}{1.5+9.0828+0.7}=0.4493 \\ 2^{-1}\times\frac{1.8+8.3863+0.6}{1.8+9.1188+0.9}=0.4563 \end{pmatrix}$	$\begin{pmatrix} 2^{-2}\times\frac{1.4+8.2379+0.7}{1.5+10.1475+0.7}=0.2093 \\ 2^{-2}\times\frac{2.1+12.2694+0.8}{2.4+12.3442+0.9}=0.2424 \end{pmatrix}$	$\begin{pmatrix} 2^{-1}\times\frac{0.7+6.4413+0.7}{1.4+8.2379+0.7}=0.3793 \\ 2^{-1}\times\frac{0.9+6.6907+0.9}{1.8+8.3863+0.9}=0.3829 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{2}}\times\frac{1.2+8.2256+0.6}{1.4+8.2379+0.7}=0.6857 \\ 2^{-\frac{1}{2}}\times\frac{1.6+8.3855+0.7}{1.8+8.3863+0.9}=0.6815 \end{pmatrix}$
τ_3	$\begin{pmatrix} 2^{-\frac{1}{2}}\times\frac{1.5+10.1475+0.6}{2+10.2589+0.8}=0.6632 \\ 2^{-\frac{1}{2}}\times\frac{1.75+10.1995+0.7}{2.25+10.3253+0.9}=0.6638 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{2}}\times\frac{1.75+10.1995+0.7}{2+10.2589+0.8}=0.6849 \\ 2^{-\frac{1}{2}}\times\frac{2+10.2589+0.8}{2.25+10.3253+0.9}=0.6853 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{1.4+8.3012+0.7}{2+10.2589+0.8}=0.4736 \\ 2^{-\frac{3}{4}}\times\frac{1.6+8.3855+0.8}{2.25+10.3253+0.9}=0.4759 \end{pmatrix}$	$\begin{pmatrix} 2^{-1}\times\frac{1.2+6.4347+0.8}{2+10.2589+0.8}=0.3229 \\ 2^{-1}\times\frac{1.35+6.5385+0.9}{2.25+10.3253+0.9}=0.3261 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{1.8+10.2589+0.6}{2+11.1328+0.8}=0.5402 \\ 2^{-\frac{3}{4}}\times\frac{2.1+10.3253+0.7}{2.25+11.1798+0.9}=0.5446 \end{pmatrix}$

Lastly, the board of the environmental monitoring station will determine the "optimum" matching alternative $\ell*$ based on the preference matrices of TIFNs.

To solve the aforesaid problem with TIFNs, a simple description of the proposed similarity measure based two-sided matching decision method is provided below.

Step 1: Based on the triangular intuitionistic fuzzy preference matrices $\tilde{A} = [\tilde{a}_{ij}]_{3\times 5}$ and $\tilde{B} = [\tilde{b}_{ij}]_{3\times 5}$, and the two-sided matching matrix $\Theta = [\rho_{ij}]_{3\times 5}$, the two-sided matching model (M-1) can be established, i.e.,

$$\text{(M-1)}\begin{cases} \max O_{\tau_i} = \sum_{j=1}^{5} < (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} > \rho_{ij}, \ i \in M \\ \max O_{\varepsilon_j} = \sum_{i=1}^{3} < (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}} > \rho_{ij}, \ j \in N \\ \text{s.t.} \sum_{j=1}^{5} \rho_{ij} = 1, \quad i \in M; \sum_{i=1}^{3} \rho_{ij} \le 1, \quad j \in N; \\ \rho_{ij} \in \{0, 1\}, \quad i \in M, \ j \in N. \end{cases}$$

where $i \in M = \{1, 2, 3\}, j \in N = \{1, 2, \ldots, 5\}$.

Step 2: By Eq. (15), the triangular intuitionistic fuzzy number positive ideal vector $\bar{A}^{PI} = (\tilde{a}^{PI}_1, \tilde{a}^{PI}_2, \tilde{a}^{PI}_3)^T$ can be calculated as: $\bar{A}^{Id} = \begin{bmatrix} < (4, 6, 8); 0.7, 0.1 > \\ < (5, 7, 9); 0.7, 0.1 > \\ < (4, 6, 9); 0.8, 0.1 > \end{bmatrix}$.

By Eq. (16), the triangular intuitionistic fuzzy number positive ideal vector $\bar{B}^{PI} = (\tilde{b}^{PI}_1, \tilde{b}^{PI}_2, \ldots, \tilde{b}^{PI}_5)$ can be calculated as: $\bar{B}^{PI} = (<(4, 5, 6); 0.6, 0.1>, <(5, 6, 7); 0.7, 0.1>, <(5, 6, 8); 0.7, 0.1>, <(5, 6, 7); 0.7, 0.2>, <(3, 6, 8); 0.7, 0.2>)$.

Step 3-4: According to preference matrix $\tilde{A} = [\tilde{a}_{ij}]_{3\times 5} = \left[< (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} >\right]_{3\times 5}$ and the triangular intuitionistic fuzzy number positive ideal vector $\bar{A}^{PI} = \begin{bmatrix} < (4, 6, 8); 0.7, 0.1 > \\ < (5, 7, 9); 0.7, 0.1 > \\ < (4, 6, 9); 0.8, 0.1 > \end{bmatrix}$, the similarity measure $S(a^g_{ij1}, a^{PIg}_{i1})$ between generalized TFNs $a^g_{ij1} = (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}; \omega_{\tilde{a}_{ij}})$ and $a^{PIg}_{i1} = (\underline{a}^{PI}_i, a'^{PI}_i, \bar{a}^{PI}_i; \omega_{\tilde{a}^{PI}_i})$ can be calculated by using Eqs. (4)-(8). The similarity measure $S(a^g_{ij2}, a^{PIg}_{i2})$ between generalized TFNs $a^g_{ij2} = (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}; u_{\tilde{a}_{ij}})$ and $a^{PIg}_{i2} = (\underline{a}^{PI}_i, a'^{PI}_i, \bar{a}^{PI}_i; u_{\tilde{a}^{PI}_i})$ can be calculated by using Eqs. (9)-(13). Therein, $\begin{pmatrix} S(a^g_{ij1}, a^{PIg}_{i1}) \\ S(a^g_{ij2}, a^{PIg}_{i2}) \end{pmatrix}$ is displayed in Table 3. According to prefer-

TABLE 4. Similarity measures $S(b_{ij1}^{g}, b_{j1}^{PIg})$ and $S(b_{ij2}^{g}, b_{j2}^{PIg})$.

$\begin{pmatrix} S(b_{ij1}^{g}, b_{j1}^{PIg}) \\ S(b_{ij2}^{g}, b_{j2}^{PIg}) \end{pmatrix}$	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	$\begin{pmatrix} 2^{-1}=0.5 \\ 2^{-1}=0.5 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{2}}\times\frac{0.7+4.4413+0.5}{1+8.1594+0.6}=0.2044 \\ 2^{-\frac{3}{2}}\times\frac{0.9+4.6907+0.9}{1.8+8.4775+0.9}=0.2053 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{1.05+6.3396+0.6}{1.2+8.2256+0.7}=0.4692 \\ 2^{-\frac{3}{4}}\times\frac{1.35+6.5385+0.9}{1.8+8.4775+0.9}=0.4675 \end{pmatrix}$	$\begin{pmatrix} \frac{0.6+4.3324+0.6}{0.8+4.5612+0.7}=0.9128 \\ \frac{0.8+4.5612+0.8}{0.9+4.6907+0.8}=0.9641 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{2}}\times\frac{1.75+10.1995+0.7}{1.75+10.1995+0.7}=0.7071 \\ 2^{-\frac{1}{2}}\times\frac{1.75+10.1995+0.7}{2+10.2589+0.8}=0.6849 \end{pmatrix}$
τ_2	$\begin{pmatrix} \frac{0.5+4.2361+0.5}{0.6+4.3324+0.6}=0.9464 \\ \frac{0.8+4.5612+0.8}{0.9+4.6907+0.9}=0.9492 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{7}{4}}\times\frac{0.7+4.4413+0.7}{1.05+6.3396+0.7}=0.2147 \\ 2^{-\frac{7}{4}}\times\frac{0.9+4.6907+0.8}{1.2+6.4347+0.9}=0.2226 \end{pmatrix}$	$\begin{pmatrix} \frac{0.9+6.2543+0.6}{1.05+6.3396+0.7}=0.9586 \\ \frac{1.05+6.3396+0.7}{1.35+6.5385+0.9}=0.9205 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{0.8+4.5612+0.6}{0.9+6.2543+0.7}=0.4513 \\ 2^{-\frac{3}{4}}\times\frac{0.9+4.6907+0.7}{1.05+6.3396+0.8}=0.4567 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{1.2+8.1761+0.6}{1.75+10.1995+0.7}=0.4689 \\ 2^{-\frac{3}{4}}\times\frac{1.4+8.2379+0.7}{2+10.2589+0.8}=0.4707 \end{pmatrix}$
τ_3	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{0.6+4.3324+0.5}{0.75+6.1796+0.6}=0.4290 \\ 2^{-\frac{3}{4}}\times\frac{0.9+4.6907+0.7}{1.05+6.3396+0.9}=0.4512 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{4}}\times\frac{0.3+2.7662+0.6}{0.7+4.4413+0.7}=0.5278 \\ 2^{-\frac{1}{4}}\times\frac{0.4+2.9316+0.8}{0.9+4.6907+0.9}=0.5353 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{1}{4}}\times\frac{0.8+4.5612+0.7}{1.05+6.3396+0.7}=0.6300 \\ 2^{-\frac{1}{4}}\times\frac{0.9+4.6907+0.8}{1.35+6.5385+0.9}=0.6115 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{5}{4}}\times\frac{0.8+4.5612+0.7}{1.05+6.3396+0.7}=0.3150 \\ 2^{-\frac{5}{4}}\times\frac{0.9+4.6907+0.8}{1.2+6.4347+0.8}=0.3190 \end{pmatrix}$	$\begin{pmatrix} 2^{-\frac{3}{4}}\times\frac{1.25+10.1029+0.5}{1.75+10.1995+0.7}=0.5572 \\ 2^{-\frac{3}{4}}\times\frac{2+10.2589+0.8}{2+10.2589+0.8}=0.5946 \end{pmatrix}$

TABLE 5. Similarity measures $S(\tilde{a}_{ij}, \tilde{a}_i^{PI})$ and $S(\tilde{b}_{ij}, \tilde{b}_j^{PI})$.

$\left(S(\tilde{a}_{ij},\tilde{a}_i^{PI}), S(\tilde{b}_{ij},\tilde{b}_j^{PI})\right)$	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	(0.4658, 0.5000)	(0.6222, 0.2049)	(0.4216, 0.4684)	(0.2246, 0.9358)	(0.3356, 0.6960)
τ_2	(0.2474, 0.9478)	(0.4528, 0.2187)	(0.2259, 0.9396)	(0.3811, 0.4540)	(0.6836, 0.4698)
τ_3	(0.6635, 0.4401)	(0.6851, 0.5316)	(0.4748, 0.6208)	(0.3245, 0.3170)	(0.5424, 0.5759)

ence matrix $\tilde{B} = [\tilde{b}_{ij}]_{3\times5} = \left[<(\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}}>\right]_{3\times5}$ and the triangular intuitionistic fuzzy number positive ideal vector $\tilde{B}^{PI} = (<(4, 5, 6); 0.6, 0.1>, <(5, 6, 7); 0.7, 0.1>, <(5, 6, 8); 0.7, 0.1>, <(5, 6, 7); 0.7, 0.2>, <(3, 6, 8); 0.7, 0.2>)$, the similarity measure $S(b_{ij1}^{g}, b_{j1}^{PIg})$ between generalized TFNs $b_{ij1}^{g} = (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}; \omega_{\tilde{b}_{ij}})$ and $b_{j1}^{PIg} = (\underline{b}_j^{PI}, b'^{PI}_j, \bar{b}_j^{PI}; \omega_{\tilde{b}_j^{PI}})$ can be calculated by using Eqs. (4)-(8). The similarity measure $S(b_{ij2}^{g}, b_{j2}^{PIg})$ between generalized TFNs $b_{ij2}^{g} = (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}; u_{\tilde{b}_{ij}})$ and $b_{j2}^{PIg} = (\underline{b}_j^{PI}, b'^{PI}_j, \bar{b}_j^{PI}; u_{\tilde{b}_j^{PI}})$ can be calculated by using Eqs. (9)-(13). Therein, $\begin{pmatrix} S(b_{ij1}^{g}, b_{j1}^{PIg}) \\ S(b_{ij2}^{g}, b_{j2}^{PIg}) \end{pmatrix}$ is shown in Table 4.

Furthermore, according to the similarity measures $S(a_{ij1}^{g}, a_{i1}^{PIg})$ and $S(a_{ij2}^{g}, a_{i2}^{PIg})$, the similarity measure $S(\tilde{a}_{ij}, \tilde{a}_i^{PI})$ between $\tilde{a}_{ij} =< (\underline{a}_{ij}, a'_{ij}, \bar{a}_{ij}); \omega_{\tilde{a}_{ij}}, u_{\tilde{a}_{ij}} >$ and $\tilde{a}_i^{PI} =< (\underline{a}_i^{PI}, a'^{PI}_i, \bar{a}_i^{PI}); \omega_{\tilde{a}_i^{PI}}, u_{\tilde{a}_i^{PI}} >$ can be calculated by using Eq. (17). According to the similarity measures $S(b_{ij1}^{g}, b_{j1}^{PIg})$ and $S(b_{ij2}^{g}, b_{j2}^{PIg})$, the similarity measure $S(\tilde{b}_{ij}, \tilde{b}_j^{PI})$ between $\tilde{b}_{ij} =< (\underline{b}_{ij}, b'_{ij}, \bar{b}_{ij}); \omega_{\tilde{b}_{ij}}, u_{\tilde{b}_{ij}} >$ and $\tilde{b}_j^{PI} =< (\underline{b}_j^{PI}, b'^{PI}_j, \bar{b}_j^{PI}); \omega_{\tilde{b}_j^{PI}}, u_{\tilde{b}_j^{PI}} >$ can be calculated by using Eq. (18). Therein, $\left(S(\tilde{a}_{ij}, \tilde{a}_i^{PI}), S(\tilde{b}_{ij}, \tilde{b}_j^{PI})\right)$ is depicted in Table 5.

Step 5-7: Based on similarity measures $S(\tilde{a}_{ij}, \tilde{a}_i^{PI})$ and $S(\tilde{b}_{ij}, \tilde{b}_j^{PI})$, model (M-1) with TIFNs can be converted into model (M-2). Then, by using the arithmetic mean approach, model (M-2) can be converted into a two-objective model (M-3). Furthermore, model (M-3) can also be converted into the two-objective model (M-4), i.e.,

$$\text{(M-4)} \begin{cases} \max O_{\tau}^{Nor} = \sum_{i=1}^{3}\sum_{j=1}^{5} S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI})\rho_{ij} \\ \max O_{\varepsilon}^{Nor} = \sum_{i=1}^{3}\sum_{j=1}^{5} S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})\rho_{ij} \\ \text{s.t. } \sum_{j=1}^{5} \rho_{ij} = 1, \quad i \in M; \\ \sum_{i=1}^{3} \rho_{ij} \le 1, \quad j \in N; \\ \rho_{ij} \in \{0, 1\}, \quad i \in M, j \in N. \end{cases}$$

where the normalized similarity measures $S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI})$ and $S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})$ can be calculated by using Eqs. (19) and (20). Therein, $(S^{Nor}(\tilde{a}_{ij},\tilde{a}_i^{PI}), S^{Nor}(\tilde{b}_{ij},\tilde{b}_j^{PI}))$ are depicted in Table 6.

Step 8: If we consider the fairness of agents on both sides, we have $w_\tau = 0.5$ and $w_\varepsilon = 0.5$. Hence, model (M-4) is changed to a mono-objective model (M-5) through using the linear weighting approach, i.e.,

$$\text{(M-5)} \begin{cases} \max O = \sum_{i=1}^{3}\sum_{j=1}^{5} S_{ij}\rho_{ij} \\ \text{s.t. } \sum_{j=1}^{5} \rho_{ij} = 1, \quad i \in M; \\ \sum_{i=1}^{3} \rho_{ij} \le 1, \quad j \in N; \\ \rho_{ij} \in \{0, 1\}, \quad i \in M, j \in N. \end{cases}$$

TABLE 6. Normalized similarity measure $\left(S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI}), S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})\right)$.

$\left(S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI}), S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})\right)$	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	(0.6799, .5275)	(0.9082, 0.2162)	(0.6154, 0.4942)	(0.3278, 0.9873)	(0.4899, 0.7343)
τ_2	(0.3611, 1)	(0.6609, 0.2307)	(0.3297, 0.9913)	(0.5563, 0.479)	(0.9978, 0.4957)
τ_3	(0.9685, 0.4643)	(1, 0.5609)	(0.693, 0.655)	(0.4737, 0.3345)	(0.7917, 0.6076)

TABLE 7. Coefficient $S_{ij} = 0.5S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI}) + 0.5S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})$.

S_{ij}	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	0.6037	0.5622	0.5548	0.6576	0.6121
τ_2	0.6806	0.4458	0.6605	0.5177	0.7468
τ_3	0.7164	0.7805	0.674	0.4041	0.6997

TABLE 8. Solution matrix $\Theta^* = [\rho_{ij}^*]_{3\times 5}$.

ρ_{ij}^*	ε_1	ε_2	ε_3	ε_4	ε_5
τ_1	0	0	0	1	0
τ_2	0	0	0	0	1
τ_3	0	1	0	0	0

TABLE 9. Eleven cases for weights w_τ and w_ε.

cases	values of weights
case I	$w_\tau = 1, w_\varepsilon = 0$
case II	$w_\tau = 0.9, w_\varepsilon = 0.1$
case III	$w_\tau = 0.8, w_\varepsilon = 0.2$
case IV	$w_\tau = 0.7, w_\varepsilon = 0.3$
case V	$w_\tau = 0.6, w_\varepsilon = 0.4$
case VI	$w_\tau = 0.5, w_\varepsilon = 0.5$
case VII	$w_\tau = 0.4, w_\varepsilon = 0.6$
case VIII	$w_\tau = 0.3, w_\varepsilon = 0.7$
case IX	$w_\tau = 0.2, w_\varepsilon = 0.8$
case X	$w_\tau = 0.1, w_\varepsilon = 0.9$
case XI	$w_\tau = 0, w_\varepsilon = 1$

where $S_{ij} = 0.5S^{Nor}(\tilde{a}_{ij}, \tilde{a}_i^{PI}) + 0.5S^{Nor}(\tilde{b}_{ij}, \tilde{b}_j^{PI})$ is displayed in Table 7.

Step 9: By solving model (M-5), the "optimum" solution matrix $\Theta^* = [\rho_{ij}^*]_{3\times 5}$ can be obtained, which is shown in Table 8.

Therefore, the "optimum" two-sided matching ℓ^* is $\ell^* = \ell_{Ma}^* \cup \ell_{Si}^*$, where $\ell_{Ma}^* = \{(\tau_1, \varepsilon_4), (\tau_2, \varepsilon_5), (\tau_3, \varepsilon_2)\}$ and $\ell_{Si}^* = \{(\varepsilon_1, \varepsilon_1), (\varepsilon_3, \varepsilon_3)\}$.

In the following, we analyze how the weights w_τ and w_ε influence objective function value O, comprehensive similarity measure S_{ij} and two-sided matching coefficient ρ_{ij}, and we study the relation among O, S_{ij} and ρ_{ij}. For convenience, eleven cases corresponding to weights w_τ and w_ε are listed in Table 9.

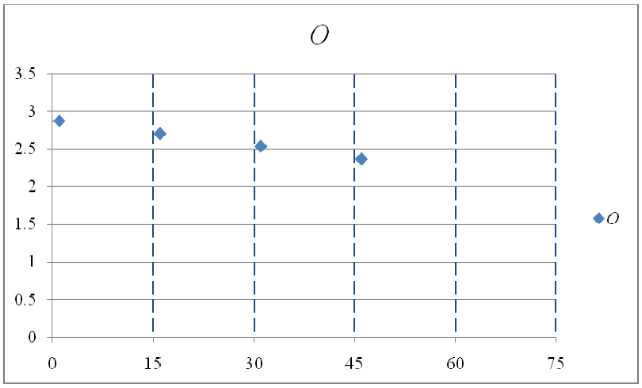

FIGURE 1. The trend graph of O from Case I to Case IV.

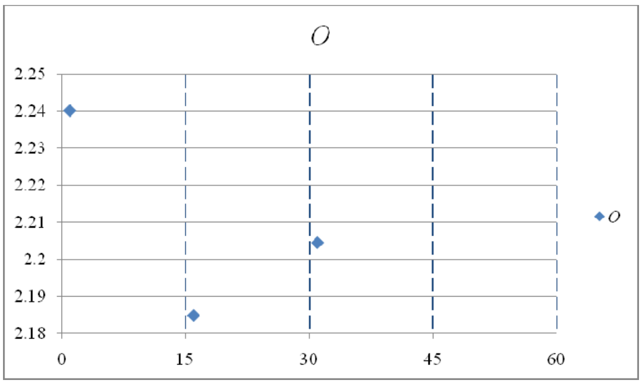

FIGURE 2. The trend graph of O from Case V to Case VII.

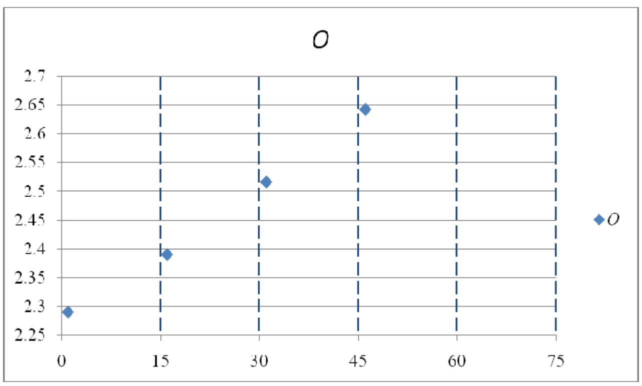

FIGURE 3. The trend graph of O from Case VIII to Case XI.

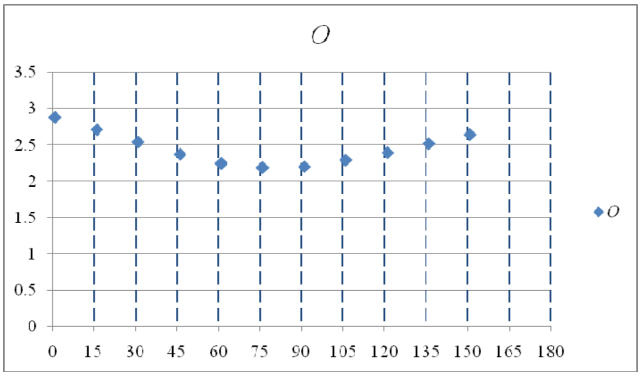

FIGURE 4. The trend graph of O from Case I to Case XI.

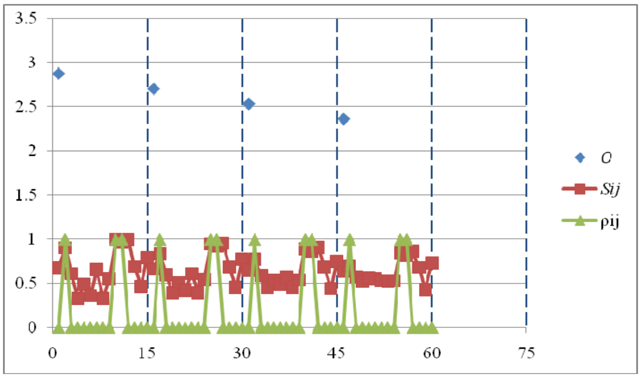

FIGURE 5. Relation among O, S_{ij} and ρ_{ij} from Case I to Case IV.

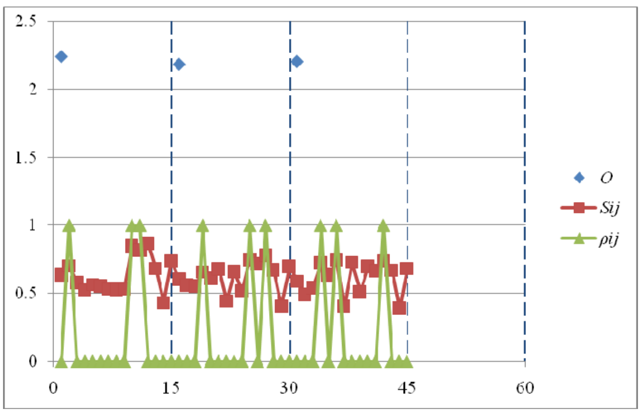

FIGURE 6. Relation among O, S_{ij} and ρ_{ij} from Case V to Case VII.

Figure 1 shows the trend on objective function value O from case I to case IV. Figure 2 shows the trend on objective function value O from case V to case VII. Figure 3 shows the trend on objective function value O from case VIII to case XI. From Figure 1 to Figure 3, the overall trend on O from case I to case XI can be obtained, as shown in Figure 4. We have known that objective function value O decreases first from case I to case VI and then increases from case VI to case XI. Figure 5 shows the relation between O, S_{ij} and ρ_{ij} from case I to case IV. Figure 6 shows the relation between O, S_{ij} and ρ_{ij} from case V to case VII. Figure 7 shows the relation between O, S_{ij} and ρ_{ij} from case VIII to case XI. From Figure 5 to Figure 7, the overall relation between O, S_{ij} and ρ_{ij} from case I to case XI can be obtained, as shown in Figure 8. We have known that objective function value O decreases first from case I to case VI and then increases from case VI to case XI; and comprehensive similarity measures S_{ij} $(i \in M = \{1, 2, 3\}, j \in N = \{1, 2, \ldots, 5\})$ are different for the eleven cases; But coefficient ρ_{ij} $(i \in M = \{1, 2, 3\}, j \in N = \{1, 2, \ldots, 5\})$ are not identical.

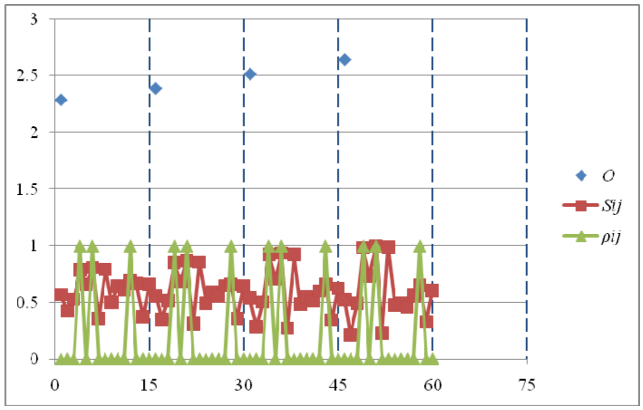

FIGURE 7. Relation among O, S_{ij} and ρ_{ij} from Case VIII to Case XI.

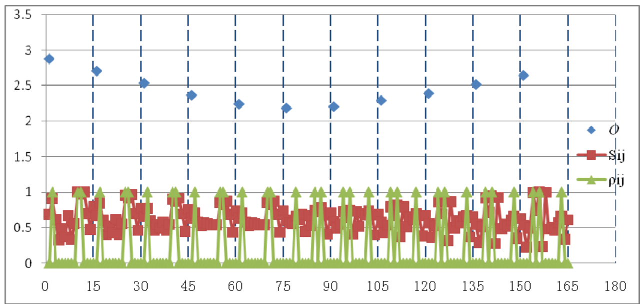

FIGURE 8. Relation among O, S_{ij} and ρ_{ij} from Case I to Case XI.

VI. CONCLUSION

This paper investigates a similarity measure based two-sided matching decision method for solving a two-sided matching problem with TIFNs. First, a similarity measure between TIFNs is defined by extending the similarity measure between generalized TFNs. In the two-sided matching problem with TIFNs, the two-sided matching model with TIFNs is constructed. Using the similarity measure between TIFNs, the similarity matrices of triangular intuitionistic fuzzy preference matrices are established. Then, the two-sided matching model with similarity measures is generated. Using the arithmetic mean, the normalization formulas and the linear weighting, the two-sided matching model with similarity measures is transformed into a mono-objective model to resolve. Thus, a similarity measure based two-sided matching decision method for TIFNs is proposed.

Comparing with other similar methods, the proposed method is based on the combination of two-sided matching and TIFNs, which are usually ignored in other similar methods. The contributions and achievements have been presented in introduction. However, two issues exist for further study.

One is how to address the decision-making problems where the preferences of agents are another type of fuzzy numbers. At the same time, the properties and characteristics of the novel similarity measure between TIFNs should be further explored.

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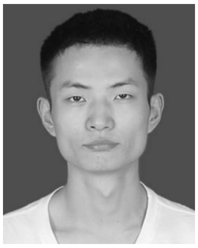

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