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An Algorithm for Mapping a Traffic Domain Into a Complex Network: A Social Internet of Things Approach

SIFATUL MOSTAFI¹, FARZANA KHAN¹, AMITABHA CHAKRABARTY¹, DOUG YOUNG SUH^{©2}, (Member, IEEE), AND MD. JALIL PIRAN^{©3}, (Member, IEEE)

¹Department of Computer Science and Engineering, BRAC University, Dhaka 1212, Bangladesh

²Department of Electronic Engineering, College of Electronics and Information, Kyung Hee University, Yongin 17104, South Korea ³Department of Computer Science and Engineering, Sejong University, Seoul 05006, South Korea

Corresponding authors: Doug Young Suh (suh@khu.ac.kr) and Md. Jalil Piran (piran@sejong.ac.kr)

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ABSTRACT Traffic congestion is expected to be an inescapable problem in the near future, as traffic is growing exponentially and becoming congested with thousands of millions of vehicles across the world. All existing approaches to examine this traffic domain are centralized and not feasible in terms of cost, time, and scalability issue. To investigate the probable reasons for traffic congestion with efficiency, this paper proposes a novel algorithm to reform any traffic domain into a complex network using the principles of decentralized Social Internet of Things (SIoT). By integrating social networking concepts into the Internet of Things (IoT), the concept of SIoT has been proposed. The idea presented below is that every vehicle acts as a smart Thing, communicate with nearby vehicles within a particular distance in a decentralized manner and together form a complex network. We have built a generalized mathematical model to measure the least amount of distance between two vehicles within the complex network. By extracting key network properties from the complex network of vehicles built from our proposed algorithm, probable reasons for traffic congestion can be investigated efficiently.

INDEX TERMS Traffic domain, complex network, social IoT, decentralization.

I. INTRODUCTION

Over the past decade, a complex network is being studied as a theme in diverse research fields [1], [2]. As the Internet and world wide web (WWW) are growing with time and especially in the near future by introducing the fifth generation of cellular networks (5G) [3], [4], social scientists tend to help users navigate vast information by managing the complexity of complex networks. In recent times, several types of research are being conducted to explore the possibilities of incorporating social networking concepts into IoT and find solutions of many uprising problems [5]–[8]. IoT aims to transform any object in the real-world into a computing device that has sensing, communication and control capabilities [9]–[11]. SIoT, a new paradigm that has been procreated by the integration of social networking concepts into IoT, potentially supports novel applications and networking services for the IoT in more effective and efficient ways [12]–[19]. When a large number of individuals are tied in a social network, it can provide far more accurate answers to complex problems than a single individual can. The IoT integrates a large number of technologies and envisions various smart objects [20]. Smart objects can interact with each other and cooperate with their neighbors to accomplish common goals [21], [22]. The underlying notion of SIoT is that every object will communicate with each other for searching their desired service by using its relationships, querying its friends and the friends of its friends in a distributed manner for an efficient and scalable discovery of objects and services by the characteristic principles of social networks for humans. From Miligram's experiment, Kleinberg concluded that there are structural clues that help people to find a short path efficiently without a global knowledge of a network [23], [24]. Traffic congestion is a global problem

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and it is increasing tremendously around the world [25]. It could be mitigated by identifying the probable reasons for congestion. To incorporate the ideology and implications of decentralized SIoT into the traffic domain, the traffic domain should be considered as a complex network of vehicles. Based on this ideology, we have proposed an algorithm that maps any traffic domain to a complex network of vehicles. All the vehicles within the complex network are considered as a social agent and are socially connected based on their distance and position within the network. Based on our proposed algorithm, a traffic domain of a particular location could easily be converted into a complex network in which each vehicle is considered as a node and the distance between each two vehicles is considered as an edge. We have conducted our research on two distinct datasets of a particular traffic domain and proposed a mathematical model to figure out the minimum distance between two vehicles to form social relationship between them. Using our algorithm, we have formed two complex networks of vehicles from two distinct datasets and extracted some key network properties from the complex networks. We have analyzed those extracted network properties and stated predictions regarding traffic situation.

II. RELATED WORKS

Both developed and developing countries are facing disruption caused by traffic congestion [26], [27]. In recent times, the number of vehicles has increased tremendously [28]–[30]. Almost all major cities around the world suffer from inevitable heavy traffic in peak hours. Huge traffic jams and unfortunate accidents can be caused even by small road maintenance. Traffic jam forecasts can reduce driver fatigue and save time on roads.

Alam *et al.* [31] and Guinard *et al.* [32] discussed how Web of Things (WoT) could share their functionality interfaces using a human social network infrastructure, such as Facebook, Linkedin, Twitter etc. In their system, every object wants to share its functionality on the web either has a built-in embedded web server or proxy smart gateways, such as radio frequency identification tag based devices. The mart Things of an individual person share their web crawlable public interfaces with the owner's groups and friends through a social network. Smart-Its Friends [33] looked into how qualitative wireless connections can be established between smart artifacts. Their system introduces context proximity based match making and respective connections. Their proposed system introduces context proximity based match making and respective connections [34].

Rezende *et al.* [35] talked about mobile Ad Hoc networks (MANETs), where nodes can exchange messages without depending on any previously established infrastructure. Establishment of a wide ubiquitous network has become possible with portable network devices capable of wireless communication. With support of these type of networks, users can facilitate services anywhere anytime. The authors analyzed the next generation of MANets formed by moving vehicles named vehicular Ad Hoc networks (VANETs) from the perspective of complex networks [35]–[37]. This is an uprising technology that enables vehicles to become self-organized without the need for any permanent infrastructure [38], [39]. It has been utilized in recent applications to increase traffic fluency over roads [40], [41]. In these types of networks, vehicles are able to communicate with each other. Real time position of vehicles assumed as nodes can be assumed by protocols, algorithms and applications since the global positioning system (GPS) can be installed easily in vehicles [36].

State-of-the-art vehicles are embellished with advanced technologies [42], [43] that enable them to communicate with nearby vehicles by forming VANETs [43]. In a vehicular social network (VSN) [44]–[46], passengers can engage in entertainment, utility and emergency related data exchanges [47]–[50]. This type of social network belongs to the mobile social network (MSN) category where mobile users share user-centric information with each other using mobile devices [43].

According to [22], IoT is a worldwide network of inter-connected objects uniquely addressable, based on standard communication protocols. IoT incorporates a large number of heterogeneous extensive objects which continuously generate information about the physical world [51]. Social applications of the IoT have been transforming into a new novel paradigm of social network of intelligent objects, known as SIoT [52]. Atzori et al. [20] and [53] have introduced SIoT terminology that focuses on establishing and exploiting social relation-ships among things rather than their owners. They have identified different types of things relationships are based on location, such as co-work, ownerships etc. These things can crawl in their social network to discover other Things and services which can be exploited to build various IoT applications. Based on the idea of social relationships among objects, the first definition of SIoT has been provided [53]. SIoT is a network where every node is an object capable of establishing social relationships with other objects in an autonomous way with respect to the rules set by the owner. These relationships can provide useful traffic information to guide the drivers in less congested routes.

The target of the IoT is to upgrade real-life physical objects with computing and communication power so that they can interact among themselves. Social Internet of Vehicles (SIoV) is a vehicular instance of the SIoT. According to the concept of SIoV, vehicles are the key entities for the social network. The rapidly increasing interconnected objects would provide secure and convenient transportation environment through increased interconnection and cooperation [34]. Just like the objectives of IoT, vehicles play an important role for safe and convenient travel and that is why the idea of the Internet of Vehicles (IoV) has also been inaugurated [54].

Contemporary research conducted on vehicular socializing consider humans as a social entity but in our research, we have analyzed vehicular social network SIoT theory where vehicles are central social entities. Masudul *et al.* [55] focused on some rudimentary concepts of the vehicular social network from the SIoT perspective. Nitti *et al.* [52] also explained some key aspects of SIoV and also talked about the integration of SIoV middleware in the ITS station architecture.

In our research, we have experimented in the complex network of the traffic domain where social relationships can be exploited. The combination of these relationships create a social network in traffic domain where vehicles can gather information in a decentralized manner by navigating the network from friend to friend like humans do in social network [56]. The main innovation of this paper is developing a novel algorithm for converting a traffic domain into a complex network of vehicles by considering that each vehicle in the traffic domain will act just like a human acts in social networking sites. Also we developed a mathematical model and analyzed how different properties of a complex network fluctuate with different distance values between vehicles assumed as nodes in our developed complex network of the traffic domain according to the algorithm.

III. DATA STRUCTURE AND THE PROPOSED ALGORITHM

For the research purposes, we considered a traffic domain on which we will do our experiments. We have collected traffic information from a busy junction called "BijoySarani" in the city of Dhaka, Bangladesh using the satellite view of Google maps and Bing maps. From these collected traffic information, we have prepared two different datasets each representing a complex network of vehicles.

A. DATA COLLECTION RULES

We have converted the traffic domain into a complex network by collecting data from satellite view of Google and Bing maps as mentioned before. While collecting traffic information from satellite view of the maps, we have assumed some rules as follows:

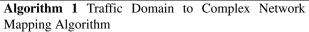
- 1) A vehicle builds relationship with all the vehicles that exist within one hop distance from that vehicle. The maximum range for one hop is 100m.
- 2) Edge is calculated in two ways. Those are:
 - (i) In the case of two vehicles existing side by side in parallel position, we did not calculated distance among those and we have assumed that the connecting edge between them, two vehicles is "1".
 - (ii) Except case (i), in all other cases we have measured distance between two particular vehicles from Google maps' satellite view and considered the distance between two vehicles as the value of their connecting edge.

We have developed an algorithm for converting the traffic domain into a complex network based on the above mentioned rules.

B. THE PROPOSED ALGORITHM

The algorithm that converts a traffic domain into a complex network using some heuristics of SIoT is as follows:

The conversion algorithm iterates over each and every pair of vehicles v and n in the traffic domain D, where v and n



Input: The Traffic Domain **Output:** A Complex Network 1 for $\forall v : v \in D$ do for $\forall n : n \in D$ && $n \notin v$ do 2 3 **if** *InTheSameRoute(v,n)== true* **then if** WithinOneHop(v,n) == true **then** 4 **if** $HopDistance(v,n) \le HD$ **then** 5 **if** StaySideBySide(v, n) = = true **then** 6 EdgeWeight = 1; 7 8 else EdgeWeight = HopDistance(v, 9 n): CreateFriendship(v, n, EdgeWeight); 10 else 11 **if** ConnectedRoute(v,n) = = true **then** 12 **if** *WithinOneHop*(*v*,*n*)==*true* **then** 13 **if** $HopDistance(v, n) \le HD$ **then** 14 15 EdgeWeight = HopDistance(v,n); CreateFriendship(v,n, 16 EdgeWeight);

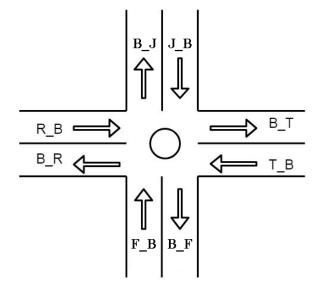


FIGURE 1. Identifying different routes.

are two distinguished vehicles $(n \neq v)$ as mentioned in [line 1] and [line 2] in the algorithm. In [line 3], the function InTheSameRoute(v, n) checks whether the two distinguished vehicles v and n, are on the same route or not. For instance, if both v and n are on the same route F_B (Figure 1), then the function InTheSameRoute(v, n) will return true. If the function returns true, then we check another condition which

is whether there is any other vehicle in between these two vehicles or not using the *WithinOneHop*(v, n) function as shown in [line 4] of the algorithm. If there is no vehicle between the two distinguished vehicles, *v* and *n*, then we say they are within one hop and move on checking the distance between v and n which we define as "Hop Distance". In the next line [line 5], we calculate the hop distance using the function HopDistance(v, n) and check whether the hop distance between v and n is within a certain range of hop distance HD. The value of HD plays an important role in converting the traffic domain into a complex network. We cannot create a relationship between any two vehicles who are within one hop. We need to calculate the distance in between them and create a relationship in between them only if they stay within a certain distance. The HD is that distance which is the limiting distance to create a relationship in between any two vehicles who meet all other conditions executed by the algorithm. In Section IV of this paper, we have discussed the ways to measure the proper limiting hop distance HD and proposed a generalized mathematical model to do so. In [line 6], the function *StaySideBySide*(*v*, *n*) checks whether the position of the two vehicles are side by side or not. If their position is side by side, then we assign the value 1 as the edge weight when creating an edge in between them while preparing a graph of the complex network [line 7]. However, if their position is not side by side, then we assign the distance in between them which is their hop distance as their edge weight as we can see in [line 9] of the algorithm. The function CreateFriendship(v, n, EdgeWeight) in [line 10] actually creates a friendship in between the two vehicles, v and n, who meets all the conditions executed by the algorithm. We can see *CreateFriendship*(v, n, *EdgeWeight*) function takes "EdgeWeight" as the last parameter which is nothing but the distance in between the vehicle v and n, which has been calculated based on their position, being whether their position is side by side or not.

From [line 11], the else portion of the condition InTheSameRoute(v, n) starts. That means obviously in this portion any pair of the vehicles will not be on the same route. Apart from creating a friendship in between vehicles who are on the same route, it is also vital to create a possible friendship in between vehicles who are not on the same route. There are routes which are connected with each other meaning a vehicle can move along one route to another. For instance, any vehicle in the route F_B can move along any one of the tree routes B R, B J or B T. We say that these three distinguished routes B_R, B_J and B_T are connected with the route F_B . Any two vehicles who are not on the same route but in any of these connected routes can also form a friendship based on the conditions discussed earlier. The function ConnectedRoute(v, n) in [line 12] checks whether the vehicle v and n are within the connected routes or not. In the next step, we check the similar conditions and logic as we have checked when assigning friendships in between two vehicles in the same route except for the condition StaySideBySide(v, n). Since it is not possible for any tow vehicle to share two separate routes and still stay side by side.

C. DATASET STRUCTURE

In our research, we assumed each vehicle as a node, and a node can build relationship with all nodes which are one hop away from that node. We made two different files for nodes and edges in comma separated values (CSV) format. In the Nodes file we have following fields:

- ID: We have assigned a unique ID for each vehicle. The ID of any two vehicles will never match.
- Category: It represents the type of vehicle. For example, if the vehicle is a private car then we put "Private Car" in this column.
- Label: Label will be the same as weight so that we can visualize the ID numbers of the nodes while simulating in Gephi [57].
- Route: It detects in which route the vehicle is currently. For instance, in Figure 1 we have identified a total of 8 routes connected to the junction. For our research purposes, we have denoted route names in short forms as shown in Figure 1.
- In the "Edges" file we have the following fields:
- 1) Source: The vehicle acting as a source in a connection.
- 2) Target: The vehicle with which the source vehicle builds the connection.
- Type: Represents if the connection is directed or undirected. In our research, we have assumed all connections "Undirected".
- 4) Weight: The distance between two connecting vehicles.
- 5) Calculated by distance: A one-digit binary value which determines if the edge value is calculated by distance or if it is measured according to our rules.
- 6) Label: Label will be the same as weight so that we can visualize values of the connecting edges while simulating in Gephi [57].
- 7) Route: It represents the short form of the route in which the vehicle is directed.

 TABLE 1. Edge dataset structure.

Source	Target	Туре	Weight	Distance Calcu- lated	Label	Route
16	25	Undirected	7	1	7	B_J
412	89	Undirected	1	0	1	R_B
109	354	Undirected	15	1	15	F_B
51	18	Undirected	1	0	1	B_T

In Table 1, the dataset structure of the edge file is shown where the first row indicates the fields which are source, target, vehicle type, vehicle weight, label and the route. The following rows are example of relationships between two vehicles. For instance, if we look at the second row we can see that two vehicles with the IDs 16 and 25 are moving towards B_J route and they form relationship between them. Their distance from each other is 7 meters.

IV. MEASURING THE HOP DISTANCE

The most important parameter of our algorithm is the hop distance- the maximum distance between two vehicles who form a social relation in between them. Depending on the hop distance considered by our algorithm, the resulting network might differ in many ways. To see how the difference in hop distance can affect the overall network we have considered three key network properties. These are average degree, average weighted degree and connected components of the network. We have conducted our algorithm with a hop distance from 1 meter to 100 meter and formed their corresponding complex networks. From those complex networks we have extracted the average degree, average weighted degree and connected components. To avoid any unintentional biasness, we have conducted the whole procedure two times on two different datasets of the very same traffic junction to do our analysis.

A. AVERAGE DEGREE VS. HOP DISTANCE

We have plotted Average Degree vs. Hop Distance for two datasets, dataset 01 and dataset 02. In Figure 2, the average degree vs. hop distance for Dataset 01 is indicated by blue curve and the average degree vs. hop distance for dataset 02 is indicated by red curve.

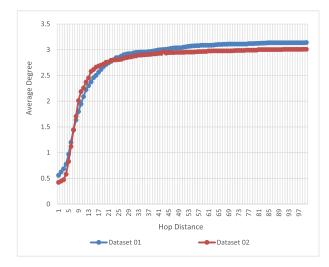


FIGURE 2. Average Degree vs. Hop Distance.

In each dataset, there are 100 observations of hop distance and the corresponding average degree extracted from the associated complex network of that particular hop distance. We can see the value of average degree distribution of both the datasets moves together. The initial response of the average degree with an increase of hop distance is much higher as we can see from the graph. When the hop distance is in between 1 to 25 approximately, the marginal average degree is much higher than the latter portion of the graph. This outcome is interesting. This implies that the average degree fluctuates a lot when the hop distance is within a certain low range. After that certain range no matter how much we increase the hop distance the average degree will not response much. However, looking at the graph and identifying the range is not a mathematical way to find the appropriate hop distance where the curve differs the most. We need to build a mathematical model to generalize the approach to identify the proper range of hop distance within which the curve shows a different characteristic from the rest of the part.

Lets assume that the set of hop distances $H = \{h_x : x = 1, ..., n\}$ and the set of corresponding values of average degree $Y = \{y_x : x = 1, ..., n\}$. That means for each hop distance h_x we will have an average degree y_x . The function, $f : H \rightarrow Y$, shows the one to one relationship between hop distance and the network property average degree.

Now if d_k is an observation of hop distance where $1 < d_k < n$ then $1 \le k \le n-2$. Lets divide the set of hop distances, H into two parts $H_i = \{h_i : i = 1, ..., d_k\}$ and $H_j = \{h_j : j = d_k, ..., n\}$. Because of the function $f : H \to Y$ we will have two different sets of the average degree for the set of hop distances H_i and H_j which are $Y_i = \{y_i : i = 1, ..., d_k\}$ and $Y_j = \{y_j : j = d_k, ..., n\}$. If we run a regression of the set of average degree Y_i on the set of hop distances H_i , then we will have an SLR (Simple Linear Regression) model as shown in (1). Similarly, if we run a regression of the set of average degree Y_j on the set of hop distances H_j then we will have another SLR model as shown in (2).

$$y_i(h_i) = \alpha_0 + \alpha_1 h_i + \epsilon_i, \quad i = 1, \dots, d_k, \tag{1}$$

$$y_j(h_j) = \beta_0 + \beta_1 h_j + \epsilon_j, \quad j = d_k, \dots, n,$$
(2)

where α_0 denotes the constant term. In our model, the average degree of the network will remain α_0 even if there are no changes in hope distance. α_1 is the coefficient of h_i that says how explanatory variable h_i explains y_i . That means for each unit change in hop distance h_i , the average degree y_i changes by α_1 units. ϵ_i refers to the overall error term. Similarly, in (2), β_0 is the constant term, β_1 is the coefficient of h_j that says how explanatory variable h_j explains y_j . That means for each unit change in hop distance h_j , the average degree y_j changes by α_1 units. ϵ_j is the error term of the equation.

We would like to see how the value of α_1 and β_1 differ for all possible values of $d_k(1 < d_k < n$ therefore, $1 \le k \le n-2$). To see that, we would like to plot the absolute difference between regression coefficients, $|\alpha_1(d_k) - \beta_1(d_k)|$ for different values of hop distance d_k . Our main objective is to find the value of d_k where the value of regression coefficients α_1 and β_1 differs the most.

Figure 3 shows the absolute difference between regression coefficients graph, $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. observation of hop distance, d_k . As we can see from the graph, when the value of d_k is 11, the difference between regression coefficients α_1 and β_1 becomes the highest, 0.1858. That means when the hop distance is in between 1 and 11, the change of the average degree relative to the hop distance is higher than the latter portion of the graph. Although we have estimated from Figure 2 that when the hop distance is in between 1 to 25 approximately, the marginal average degree is much

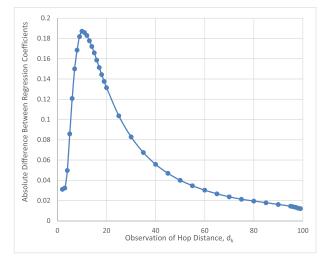


FIGURE 3. The absolute difference between regression coefficients, $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. observation of hop distance, d_k .

higher than the latter portion of the graph, the outcome that we have found using simple linear regression is very robust and has a mathematical base.

However, we need to consider some important statistical properties of our regression analysis to see whether our outcome is acceptable or not. In other words, is there any way we can reject our outcome based on statistical analysis or how much explanatory capability our model has to explain about our outcome. To discuss further, we consider two important statistical properties, P-value and the R-squared value. In the next portion of the paper we will go further with these two constraints and see how we can fit our model to meet these constraints.

1) P-value Measurement: The P-value measurement is associated with the probability of rejecting the null hypothesis. If we can reject our null hypothesis then the coefficient estimates of our simple linear regressions will be equal to 0 at a different significance level. In (3), the null hypothesis states that hop distance h_i has no effect on the average degree y_i . The null hypothesis in (4) states that h_j has no effect on the average degree y_i ,

$$H_0^{\alpha_1}: \alpha_1 = 0, \tag{3}$$

$$H_0^{\beta_1}:\beta_1=0.$$
 (4)

To resolve whether the null hypothesis could be rejected or not in a certain level of significance and for that particular coefficient, we need to measure the P-values associated with the coefficients. If the P-value of the regression of average degree y_i on hop distance h_i is 0, the null hypothesis $H_0^{\alpha_1} : \alpha_1 = 0$ could be rejected at a different significance level, for instance at 10%, 5%, and 1% significance levels. That means the estimate of the model in (1) has 0% probability to be random choice alone and has meaningful implication towards the average degree y_i . Similarly we want

the P-value of the regression of average degree y_i on hop distance h_i to be 0 so that we can reject the null hypothesis $H_0^{\beta_1}$: $\beta_1 = 0$ at the lowest possible significance level. We would like to see how the P-value of the coefficients α_1 and β_1 changes for all possible values of hop distance d_k . To see that, we would like to plot the summation of P-values of the coefficients α_1 and β_1 , which is $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ where $p_{\alpha_1}(d_k)$ is referring to the P-value associating with the null hypothesis $H_0^{\alpha_1}$: $\alpha_1 = 0$ and $p_{\beta_1}(d_k)$ is referring to the P-value associating with the null hypothesis $H_0^{\beta_1}$: $\beta_1 = 0$. Our main objective is to find the value of hop distance d_k where the summation of P-values of α_1 and β_1 becomes 0. Since the *P* value of any coefficient cannot be negative, the only way a summation of two P-values can become zero is the case where both the P-values are zero, $p_{\alpha_1}(d_k) = 0$ and $p_{\beta_1}(d_k) = 0$.

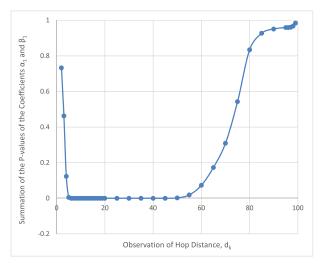


FIGURE 4. Summation of the P-values of the coefficients α_1 and β_1 vs. d_k .

In Figure 4, the graph summation of the P-values of the coefficients α_1 and β_1 , $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ vs. d_k have been shown. We can see that for certain values of d_k , the summation of the P-values are zero. On those values of d_k , we can reject both the null hypothesis $H_0^{\alpha_1}$: $\alpha_1 =$ 0 and $H_0^{\beta_1}$: $\beta_1 = 0$ at even a 1% significance level. That means there is 100% probability that both the coefficients α_1 and β_1 have meaningful implications towards their associated model. As we can see from the graph, when the value of d_k is in between 6 and 45, the summation of the P-values of the coefficients α_1 and β_1 remains at zero. That means we can take the value $d_k = 11$, as at that range, the difference between coefficients α_1 and β_1 becomes the highest and the summation of the P-values of the coefficients α_1 and β_1 remain zero.

 R-Squared Value Measurement: The R-squared value is also known as the coefficient of determination. It determines how much of the variations in the explained

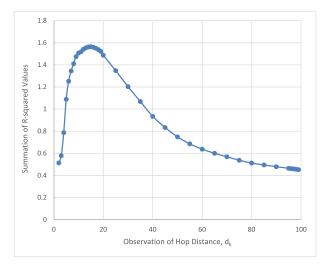


FIGURE 5. Summation of R-Squared values vs. hop distance, d_k .

variable (in our case it is the average degree of the network) are being explained by variations of explanatory variable (in our case which is the hop distance). The value of R-squared remains in between 0 to 1 or in between 0% to 100% while being represented as a percentage. Low R-squared value means very low amount of variations in the explained variable are being explained by the explanatory variable of the model and vice versa. We have calculated the R-squared values for equations (1) and (2) for all possible values of d_k and plotted the summation of R-squared values, $R_i^2(d_k) + R_i^2(d_k)$ vs. the hop distance d_k , where $R_i^2(d_k)$ means to the R-squared value for (1) and $R_i^2(d_k)$ means the R-squared value for (2) for a particular value of d_k . Our main objective is to find the d_k for which the summation of both the R-squared value of equations (1) and (2) becomes the highest. Therefore, we can claim that, at that particular value of d_k our model fits the best. In Figure 5, the graph $R_i^2(d_k) + R_i^2(d_k)$ vs. d_k is shown. We can see that there are certain values of d_k for which the summation of R-squared value is very high. In our previous case, we have considered $d_k = 11$ as in that range the coefficients α_1 and β_1 differ the most, the summation of their associated P-values are 0, and finally we can see that when $d_k = 11$, the summation of R-squared is also high (which is 1.52 and pretty close to the highest possible value 1.56. Based on our analysis and outcome now we can build our generalized model to measure the proper range of the hop distance.

However, before generalizing our mathematical model, we would like to experiment and see whether other network properties also show similar characteristics like the average degree or not.

B. AVERAGE WEIGHTED DEGREE VS. HOP DISTANCE

Similarly, we have plotted Average Weighted Degree vs. Hop Distance for two datasets, dataset 01 and dataset 02. In the Figure 6.(a), the average weighted degree vs. hop distance for

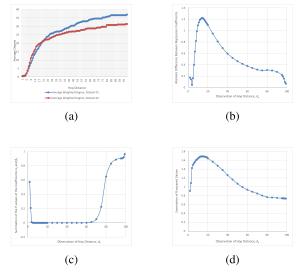


FIGURE 6. Average Weighted Degree vs. Hop Distance.

dataset 01 is indicated by blue curve and the average weighted degree vs hop distance for dataset 02 is indicated by red curve. In each dataset, there are 100 observations of hop distance and corresponding average weighted degree extracted from the associated complex network of that particular hop distance. Although the value of average weighted degree distribution of both the datasets moves together, they are not as close as how average degree vs hop distance moves together in Figure 1. However, the initial response of average weighted degree with an increase of hop distance is still much higher up to a certain value of hop distance and up to that certain range the marginal average weighted degree is much higher than the latter portion of the graph. That means the average weighted degree vs. hop distance curve behaves like the average degree vs. hop distance curve and rises fast when the hop distance is within a certain low range. The graph $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. d_k , $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ vs. d_k and $R_i^2(d_k) + R_i^2(d_k)$ vs. d_k for average weighted degree are shown in Figures 6.b, 6.c, and 6.d respectively. All these graphs show similar characteristics, like the network property average degree as shown in Figures 3, 4, and 5, respectively.

C. CONNECTED COMPONENTS VS. HOP DISTANCE

We have found the similar outcome when we have plotted the amount of connected components vs. hop distance for two datasets, dataset 01 and dataset 02. In the Figure 7.(a), the amount of connected components vs. hop distance for dataset 01 is indicated by the blue curve and the connected components vs. hop distance for dataset 02 is indicated by the red curve. In each dataset there are 100 observations of the hop distance and corresponding connected components extracted from the associated complex network of that particular hop distance. The difference between the number of connected components of both the dataset is highest when the hop distance is close to zero. Then the difference becomes smaller and smaller gradually. After a certain value of hop

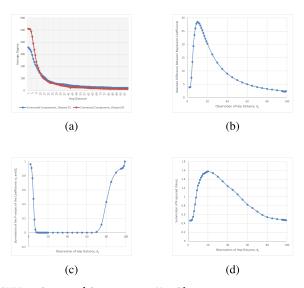


FIGURE 7. Connected Component vs. Hop Distance.

distance, both the curves move together. The initial response of the number of connected components with an increase of hop distance is higher as we can see from the graph. This outcome is similar with the previous two cases where both the average degree and average weighted degree fluctuate a lot when the hop distance is within a certain and low range. After that certain range the average degree and average weighted degree do not respond much like connected components in this example. The graph $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. d_k , $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ vs. d_k and $R_i^2(d_k) + R_j^2(d_k)$ vs. d_k for connected components are shown in Figures 7.b, 7.c, and 7.d, respectively. Figure 7.b shows reflects the similar type of characteristics as Figures 6.b and 3. Likewise Figure 7.c reflects the similar type of characteristics as Figures 6.c and 4. Figure 7.d shows similar characteristics as Figures 6.d and 5.

V. MATHEMATICAL MODEL

Lets assume the set of hop distances $H = \{h_x : x = 1, \dots, n\}$ and the set of corresponding values of a network property $Y = \{y_x : x = 1, \dots, n\}$. That means for each hop distance h_x we will have a particular value of that network property which is y_x . The function $f : H \to Y$ shows the one to one relationship between hop distance and the network property average degree.

Now if d_k is an observation of hop distance where $1 < d_k < n$ then $1 \le k \le n-2$. Let's divide the set of hop distances, H into two parts $H_i = \{h_i : i = 1, \dots, d_k\}$ and $H_j = \{h_j : j = d_k, \dots, n\}$. Because of the function $f : H \to Y$, we will have two different sets of that network property for the set of hop distances H_i and H_j which are $Y_i = \{y_i : i = 1, \dots, d_k\}$ and $Y_j = \{y_j : j = d_k, \dots, n\}$. If we run a regression of the set of network property Y_i on the set of hop distances H_i then we will have a SLR (Simple Linear Regression) model as shown (5). Similarly, if we run a regression of the set of that network property Y_i on the set of that network property Y_i on the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the set of that network property Y_i on the set of the

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of hop distances H_j then we will have another SLR model.

$$f(h) = y(h) = \begin{cases} y_i(h_i) = \alpha_0 + \alpha_1 h_i + \epsilon_i, & i = 1, ..., d_k \\ y_j(h_j) = \beta_0 + \beta_1 h_j + \epsilon_j, & j = d_k, ..., n \end{cases}$$
(5)

We want to find the value of hop distance d_k for which we can maximize the absolute difference of regression coefficients $|\alpha_1(d_k) - \beta_1(d_k)|$. However, we have two constraints to satisfy. One of the constraints is that we have to make sure the summation of the P-values of the coefficients α_1 and β_1 are zero. That means the value of $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ must be zero where $p_{\alpha_1}(d_k)$ refers to the P-value of the null hypothesis $H_0^{\alpha_1} : \alpha_1 = 0$ and $p_{\beta_1}(d_k)$ refers to the P-value of the null hypothesis $H_0^{\beta_1} : \beta_1 = 0$. Another constraint is that the summation of R-Square values of coefficients α_1 and β_1 need to be maximized. Therefore, we need to find a point where the first derivative of the function $r(d_k) = R_i^2(d_k) + R_j^2(d_k)$ becomes zero. In a nutshell, we need to maximize the following function,

$$\underset{d}{\text{maximize}} f(d_k) = |\alpha_1(d_k) - \beta_1(d_k)|, \tag{6}$$

subject to:
$$p(d_k) = [p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)] = 0,$$
 (7)

$$\dot{r}(d_k) = \frac{\partial r(d_k)}{\partial d_k} = 0, \tag{8}$$

where,

$$p_{\alpha_1}(d_k) = p - value(H_0^{\alpha_1} : \alpha_1 = 0),$$
 (9)

$$p_{\beta_1}(d_k) = p - value(H_0^{\beta_1} : \beta_1 = 0), \tag{10}$$

$$r(d_k) = R_i^2(d_k) + R_j^2(d_k).$$
(11)

We can maximize our objective function by constrain optimization using Lagrange Multipliers λ_1 and λ_2 as following:

$$L(d_k, \lambda_1, \lambda_2) = f(d_k) + \lambda_1 p(d_k) + \lambda_2 \acute{r}(d_k).$$
(12)

Putting the values from (6), (7), and (8) in (12), we can obtain:

$$L(d_k, \lambda_1, \lambda_2) = |\alpha_1(d_k) - \beta_1(d_k)| + \lambda_1[p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)] + \lambda_2 \frac{\partial}{\partial d_k} [R_i^2(d_k) + R_j^2(d_k)], \quad (13)$$

where $L(d_k, \lambda_1, \lambda_2)$ is the Lagrange function where $f(d_k)$ is our objective function that we want to maximize based on two constraints $p(d_k)$ and $\dot{r}(d_k)$. Now we can solve for d_k ,

$$\frac{\partial L(d_k, \lambda_1, \lambda_2)}{\partial d_k} = 0.$$
(14)

This is our desire value of hop distance d_k for which our objective function $f(d_k)$ is maximized by optimizing the constraints. That means in that particular value of hop distance the difference between coefficients becomes the highest, satisfying all the constrains. If we decrease the value of d_k , we can see that the value of that particular network property will change more than compared to the change of network property if we could increase the value of d_k . Therefore, we say that the hop distance in the observation of d_k is the limiting hop distance because even if we move a little further to the left network property will change a lot. To create any complex network we need to fix our hop distance at a point that is higher than the value of hop distance in the observation d_k . Therefore, the set of hop distance where the network property will be relatively stable is,

$$H_s = \{h_s : d_k \le s \le n\}. \tag{15}$$

VI. REGRESSION ANALYSIS

Using our mathematical model we have calculated the hop distance of observation d_k , which is 15. We have divided the regression analysis in two parts. The first part consists of the Simple Linear Regression on the dataset consists of observations of the hop distance from 1 to d_k and their corresponding average degree (a total of 30 observations including dataset 01 and dataset 02). The second part describes the regression from hop distance of observations d_k to n (a total 170 observations including dataset 01 and dataset 02). The linear relationship between hop distance (HD) and average degree (AD) of the network for hop distances 1 to d_k can be represented as follows:

$$AD_{i(HD_i)} = \alpha_0 + \alpha_1 HD_i + \epsilon_i, \quad i = 1, \dots, d_k, \quad (16)$$

and the linear relationship between hop distance (HD) and average degree (AD) of the network for observation of hop distances d_k to n is represented as,

$$AD_{j(HD_i)} = \beta_0 + \beta_1 HD_j + \epsilon_j, \quad j = d_k, \dots, n.$$
(17)

In Table 2, the mean, standard deviation, and the number of observations are listed for the average degree for both the range of 1 to d_k and d_k to n.

TABLE 2. Descriptive statistics.

Analysis of Variation	Average Degree		
Analysis of variation	1 to d_k	d_k to n	
No. of observations	30	170	
Mean	1.5536	2.9807	
Standard Deviation	0.7615	0.1230	

TABLE 3. Statistical Properties.

Analysis of Variation	Average Degree		
Analysis of variation	1 to d_k	d_k to n	
Coefficient Value	$\alpha_1 = 0.1697$	$\beta_1 = 0.0038$	
F-Value	655.48	257.17	
P-Value	0.0000	0.0000	
R-squared	0.9590	0.6049	
Adjusted R-squared	0.9576	0.6025	
Root MSE	0.1568	0.0775	

Now let's first look at the statistical properties of Table 3. The R-squared value of average degree is 0.9590 for observations 1 to d_k . That means approximately 95.90% of the variations in the overall average degree of the network are being explained by variations of our only explanatory variable

hop distance when the observations are from 1 to d_k . The R-squared value of average degree is 0.6049 for observations d_k to n that represents 60.49% variations in the overall average degree are being explained by the hop distance when the observations are from d_k to *n*. The adjusted R-squared value for observations of hop distances from 1 to d_k and d_k to n are 95.76% and 60.25% respectively. We have calculated the R-Squared value to get off the excessive effect of the degree of freedom on the R-squared value in the model. The R-squared values of our model are very near to the adjusted R-squared values of the model. That means the explanatory variables of the model are capable to explain the dependent variable. Moreover, there is no chance that the R-squared value of our model is over rated. We can measure the mean of squares and residual components for our model using the sum of squares and degrees of freedom. The F-value of the model is calculated using these mean of squares' values. The F-value for the average degree are 655.48 and 257.17 for observations 1 to d_k and d_k to *n*, respectively.

From Table 2, we can see that the P-value of the regression of the average degree on the hop distance for observations of 1 to d_k is 0. As a result, the null hypothesis $H_0^{(\alpha_1)}$: $\alpha_1 = 0$ could be rejected at a 10%, 5% and 1% significance level. That means the estimate of the model in (16) has 0% probability to be a random choice alone and has meaningful implications towards the overall average degree of the network. Here, the null hypothesis $H_0^{(\beta_1)}$: $\beta_1 = 0$ can also be rejected at a 1% significance level. As a result, there is a 0% probability for the average degree of the network not to be effected by the hop distance for observations of d_k to n. In Table 3, the Root MSE is referring to the standard error of the regression (SER), which is the root of mean squared error. On average, how much each observation of explanatory variable is missing from the prediction of the regression model is denoted by the Root MSE value. In our case, the value of Root MSE is 0.15% and 0.07% for the regression model in (16) and (17), respectively.

From the estimates α_1 and β_1 represented in Table 3, we can rewrite (16) and (17) as such in (18) and (19).

$AD_{i(HD_i)} = 0.195 + 0.169HD_i + \epsilon_i,$	$i=1,\ldots,d_k,$	(18)
$AD_{j(HD_j)} = 2.755 + 0.0038HD_j + \epsilon_j,$	$j=d_k,\ldots,n.$	(19)

If we compare (16) and (17) with (18) and (19) respectively, we can see that $\alpha_0 = 0.195$, $\alpha_1 = 0.169$, $\beta_0 = 2.755$ and $\beta_1 = 0.0038$. The values of the constant terms α_0 and β_0 do not have much implication in our analysis. However, the values of the coefficients α_1 and β_1 , which are associated with the explanatory variable HD for different observations of hop distances take us to meaningful outcome.

Looking at the estimates we can see that the hop distance HD is strongly related with the average degree AD for the observations from 1 to d_k , but not for the observations from d_k to n.

For a 1-meter increment in the hop distance, the amount of the average degree will increase by 0.1697 when the

TABLE 4. Ranges of coefficient with 95% confidence interval.

Range	95% Confidence Interval		
Kange	Lower Bound	Upper Bound	
1 to d_k	0.1561	0.1832	
d_k to n	0.00341	0.00436	

observations are from 1 to d_k keep all other variables constant. However, for a 1-meter increment in the hop distance, the amount of the average degree will increase by a negligible amount of 0.0038 when the observations are from 1 to d_k , keeping all other variables constant. The lower bounds and upper bounds for estimates are mentioned for a 95% confidence interval in Table 4. For instance, 95% of the time the value of α_1 will be in between 0.1561 and 0.1832. That means, 95% of the time every 1-meter increment in the hop distance will be associated with an increment in the average degree in between 0.1561 and 0.1832 units when the observations of the hop distance are in between 1 to d_k . The lower bound and upper bound of estimate β_1 is also mentioned.

VII. SIMULATION AND ANALYSIS

In this part we are going to create a complex network with a hop distance of 100 meters as this hop distance agrees with the suggestions of our mathematical model because 100 is higher than the value $d_k = 15$ and also does not exceed the value of the total observations of the hop distances in our dataset 01 and 02. However, we can take any value of hop distance that is higher or equal than the hop distance $d_k = 15$ and within the total of no observation of hop distance, which is 100. The reason for taking the hop distance as 100 is that we want to cover as much distance as possible as a hop distance. Because when the hop distance is large, any two vehicles within one hop can be connected and can create a social relationship between each other. But if we do not take a hop distance that satisfies our mathematical model and also covers a really large distance then our network will surely miss some edges between numerous vehicles. To make the complex network from dataset, we have used the Forced Atlas 2.0 algorithm where the scaling amount was 500, and strong gravity was imposed on the network view in order to keep the network view centered and round shaped. Now that we can have a deeper look into the network and do necessary analysis in order to extract valuable information from the network like amounts of a particular node, giant component, amount of a particular type of vehicle in the giant component, their combination in the whole network, clustering coefficient, etc.

A. NETWORK ANALYSIS BASED ON VEHICLE CATEGORIES

In the Figure 8, the network shows different vehicle categories. Each node in the network denotes a vehicle. Each color in the network refers to a particular vehicle category.

There are 489 vehicles in the network, and there are 751 social connections in the network. From Table 5, we can see the participation of each category of vehicle that forms the network together.

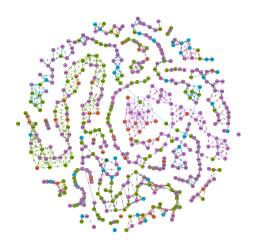


FIGURE 8. Network showing vehicle categories built from Dataset 01.

 TABLE 5. Percentage of vehicles form different categories in the network built from dataset 01.

Color	Percentage
	54.4%
	29.65%
	7.98%
	7.77%
	0.2%
	Color

In Figure 8, any node refereeing to a private car is represented by the color purple, any node refereeing to a CNG auto rickshaw is represented by the color light green, any node referring to a pickup car is represented by the color orange, any node referring to a bus is represented by the color sky blue and any node referring to an ambulance is represented by the color cyan. In the total network built from the information of dataset 01, there are 54.4% private cars, 29.65% CNGs, 7.98% pickup cars, 7.77% buses and 0.2% ambulances. From the achieved ratio of vehicles, we can come to the conclusion that among all types of vehicles in that network, most of the vehicles are private cars followed by the CNG auto rickshaws. That means mostly private cars and CNG auto rickshaws are responsible for network congestion.

B. NETWORK ANALYSIS BASED ON DIFFERENT ROUTES

From Figure 9, we can see vehicles in different routes in the network. Each color in the network represents a particular route in the network.

For instance, the highest number of vehicles are in the T_B route wich is 22.09% as mentioned in Table 6. Also smallest number of vehicles are in the B_F route which is 3.07%.

C. NETWORK ANALYSIS BASED ON CLUSTERING COEFFICIENTS

From our previous analysis we have observed that the highest number of vehicles category that form the network is private cars. Now we want to see how these private cars tend to cluster together and create congestion in the network. To see that we want a new measurement of the network that is clustering coefficient.

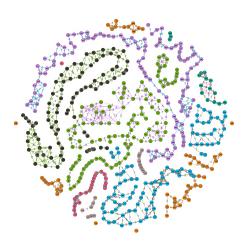


FIGURE 9. Network showing different vehicle routes built from Dataset 01.

 TABLE 6. Amount of vehicles in a particular route of the network built from dataset 01.

Route	Color	Percentage
T_B		22.09%
J_B		21.47%
R_B		18.4%
F_B		16.56%
B_R		10.43%
B_J		4.7%
B_T		3.27%
B_F		3.07%



FIGURE 10. Network of private cars with lowest clustering coefficient built from Dataset 01.

Figure 10 shows the network of private cars that have lowest clustering coefficients. There are only 123 private cars have lowest number of clustering coefficient and these private cars form total of 67 relationships in between them. In percentage, these private cars with the lowest clustering coefficients covers only 25.15% of the total network and the number of relationship they form among them is only 8.92% of the total number of relationships in the network. As these numbers suggest, very small amount of private cars do not tend to cluster. That means most of the private cars in the network tend to create clusters among them. Among all the vehicles within the network a total of 123 (25.15% of the total vehicles) private cars have lowest clustering coefficient in the network built from dataset 01.

D. NETWORK ANALYSIS BASED ON GIANT COMPONENT

In this portion we are going to extract the giant component of the network, which is how many vehicles within this network are connected together and can contact with each other, either directly or via other vehicles. Figure 11 shows the giant component of the network.

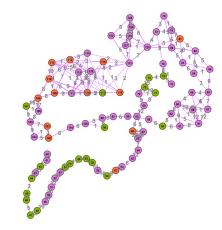


FIGURE 11. Giant component showing vehicles of differnet categories built from Dataset 01.

Among all the vehicles, about 88 vehicles (18% of the total vehicles) are responsible for forming the giant component in the network. That means all these vehicles within this giant component are connected together and are responsible for creating traffic congestion. Now lets have a look at how many private cars are there in the giant component of the network.

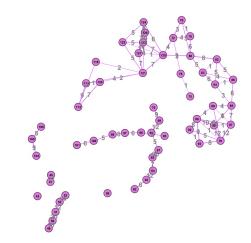


FIGURE 12. Only private cars within the Giant Component of the network built from Dataset 01.

From Figure 12, we can see that among all the vehicles, about 55 private cars (11.25% of the total vehicles) are responsible for forming the giant component in the network.

They form about 70 social connections among them which is 10.52% of the total no of relationships in the network. Even in this analysis we can see that private cars are responsible for forming the giant component of the network.

Different filtering	Dataset 01		Dataset 02	
Different intering	Nodes	Edges	Nodes	Edges
Network	489	751	649	977
Private cars	266	245	502	612
	(54.4%)	(32.62%)	(77.35%)	(62.64%)
Giant component	88	169	103	126
	(18%)	(22.5%)	(15.87%)	(12.9%)
Private cars in	55	79	73	66
giant component	(11.25%)	(10.52%)	(11.25%)	(6.76%)
Lowest clustering	219	186	302	258
coefficient	(44.9%)	(24.77%)	(46.53%)	(26.41%)
Private cars with	123	67	226	158
the lowest	(25.15%)	(8.92%)	(34.82%)	(16.17%)
clustering				
coefficient				

 TABLE 7. Statistics of nodes and edges for different datasets.

Table 7 shows a comparison among statistics of nodes and edges in different filtering of the networks built from dataset 01 and dataset 02.

In the complex network built from dataset 01, there are a total of 489 nodes and 751 edges. A total of 266 nodes are private cars, and they form about 245 edges among them. That means almost 54.4% of all the nodes and 32.62% of all the edges are due to the presence of a private car. About 88 nodes together form the giant component of the complex network which is 18% nodes among all the nodes in the complex network. The nodes within the giant component have 169 edges in between them, which is about 22.5% of the total edges of the network. Within these 88 vehicles that form the giant component, about 55 of the vehicles are private cars. That means 11.25% of the private car nodes of the whole network are responsible for forming the giant component. These private cars within the giant component form 79 edges that is about 10.52% of all the edges of the network. Among all the nodes, 219 nodes that have the lowest clustering coefficients, which is about 44.9% of all the nodes. These nodes with lowest clustering coefficient form 186 edges, which is about 24.77% of all edges of the network. Among these 219 nodes with lowest clustering coefficients, 123 nodes are private cars which is about 25.15% of all the nodes. These private cars with the lowest clustering coefficients form 67 edges or about 8.92% edges in the complex network.

In the complex network built from dataset 02, there are a total of 649 nodes and 977 edges. A total of 502 nodes are private cars and they form about 612 edges among them. That means almost 77.35% of all the nodes and 62.64% of all the edges are due to the presence of a private car. About 103 nodes or 15.87% nodes, together form the giant component of the complex network. The nodes within the giant component have 126 edges in between them which is about 12.9% of the total edges of the network. Within these 103 vehicles that form the giant component, about 79 of the vehicles are private cars. That means 11.25% of the private car nodes of the whole network are responsible for forming the giant component. These private cars within the giant component form 66 edges, which is about 6.76% of all the edges of the network. Among all the nodes, 302 nodes have the lowest clustering coefficients which is about 46.53% of all the nodes. These nodes with the lowest clustering coefficient form 258 edges which is about 26.41% of all edges of the network. Among these 302 nodes with the lowest clustering coefficients, 226 nodes are private cars which is about 34.82% of all the nodes. These private cars with the lowest clustering coefficients form 158 edges or about 16.17% of the edges in the complex network.

VIII. GENERALITY OF THE MATHEMATICAL MODEL FOR DIFFERENT ROAD CONNECTIONS

In this study we have conducted our research on datasets that have been collected from cross road traffic sections and built a mathematical model to measure the proper hop distance that has led us to create a complex traffic network. However, there are different types of typical road connections such as a three-way T, a three-way fork, etc. To establish the generality of our mathematical model, it is really vital to see whether some other road connection types, if not all, lead us to our mathematical model.

The cross-road connection as shown in Figure 1, has a total of 8 routes. A road connection is a combination of different routes with different directions. Let us represent our cross-road connection as C, which is a set of vectors where each vector represents a different route along with their direction as following:

$$C = \{\vec{BJ}, \vec{BR}, \vec{BF}, \vec{BT}, \vec{JB}, \vec{RB}, \vec{TB}, \vec{FB}\}.$$
 (20)

If we can define C as a finite set of n elements, the number of subsets of the set C will be as follows:

$$|P(C)| = 2^n. (21)$$

The number of elements in the set *C* is 8 as shown in (20). Therefore, the value of n in (21) is 8. The number of elements in power set of C, |P(C)| is 256. That means the set *C* in (20) has a total of 256 subsets. In other words, a total of 256 types of different road connections can be derived from our cross-road connection. Two of these road connections are three-way T road connection, we will make an insight on these two different road sections and verify whether these typical road connections, other than the cross-road connection, can also be explained by our mathematical model or not.

A. THREE-WAY T ROAD CONNECTION

There are four pairs of adjacent routes in our cross-road connection set C. The set of these pairs of adjacent routes are shown as a set P as follows.

$$P = \{\{\vec{BJ}, \vec{JB}\}, \{\vec{BR}, \vec{RB}\}, \{\vec{BF}, \vec{FB}\}, \{\vec{BT}, \vec{TB}\}\}$$
(22)

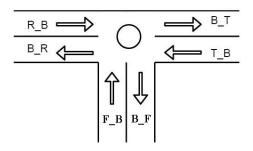


FIGURE 13. Routes in a three-way T road connection.

To form a three-way T road connection set, we have to exclude any pair of two adjacent routes from the cross-road connection *C*. Therefore, we have excluded two adjacent routes \vec{BJ} and \vec{JB} from the set *C* because the pair $\{\vec{BJ}, \vec{JB}\}$ is an element of the set *P* as shown in (23) and formed a three-way T road connection set *T* as shown in (24).

$$\{\vec{BJ}, \vec{JB}\} \in P,\tag{23}$$

$$T = \{\vec{BR}, \vec{BF}, \vec{BT}, \vec{RB}, \vec{TB}, \vec{FB}\},$$
(24)

where the set T is a subset of the powerset of the cross-road connection set C.

$$T \subseteq P(C), \tag{25}$$

where the routes of the set T together forms a three-way road connection as shown in Figure 13. We have prepared two datasets, dataset 01 and dataset 02 for the three-way T road connection and extracted the network property average degree up to a certain range of the hop distance like before.

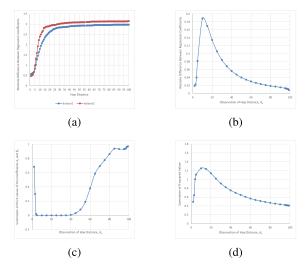


FIGURE 14. Average Degree vs. Hop Distance for three-way T road connection.

In Figure 14.(a), the average degree vs. hop distance for dataset 01 is indicated by the blue curve, and the average weighted degree vs hop distance for dataset 02 is indicated by the red curve. In each dataset, there are 100 observations of hop distance and corresponding average degree extracted

from the associated complex network of that particular hop distance. The value of average degree distribution of both the datasets moves very closely and together. Like our previous experiments in cross-road connection, the initial response of the average degree with an increase in hop distance is much higher up to a certain value of hop distance, and up to that certain range, the marginal average degree is much higher than the latter portion of the graph. That means the average degree vs. hop distance curve in the three-way T road connection behaves like the average degree vs. hop distance curve in the cross-road connection and rises fast when the hop distance is within a certain and low range. The graph $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. d_k , $p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ vs. d_k and $R_i^2(d_k) + R_i^2(d_k)$ vs. d_k for the average degree are shown in Figure 14.(b), 14c and 14.(d) respectively. All these graphs for the network property average degree show similar characteristics like the cross-road connection as shown in Figure 3, 4 and 5, respectively.

B. THREE-WAY FORK ROAD CONNECTION

As stated before, if we want to form a set F that contains all the routes of a three-way fork road connection, then F will be a subset of the set C that contains all the routes for a cross-road connection as shown in (26).

$$F \subseteq P(C). \tag{26}$$

However, there is a particular rule to follow. First of all, the set F containing routes of the three-way fork road connection must contain one route directed towards the junction. All other routes of the set F need to start from the junction and directed towards different destinations. We can see that there are only four routes in the cross-road connection set Cthat are directed towards the junction. All these four routes are sorted together in a set J as follows.

$$J = \{JB, RB, TB, FB\}.$$
 (27)

As we can see from the (27), all the routes in the set J directed towards the junction. We can include any one of the four elements from set J in the set F containing all the routes for a three-way fork road connection. We have included \vec{FB} as such a route in the set F. Along with the route \vec{FB} , we have included another three routes in the set F to form a three-way fork road connection as stated in (28).

$$F = \{\vec{BR}, \vec{BJ}, \vec{BT}, \vec{FB}\}.$$
(28)

Figure 15 represents the three-way fork road connection that is built from the routes contained in set F.

We have prepared two datasets, dataset 01 and dataset 02 for the three-way fork road connection and extracted the network property average degree up to a certain range of hop distance like before. We have found a similar outcome for the three-way fork road connection as well as when we have plotted the average degree vs. hop distance for two datasets, dataset 01 and dataset 02.

In Figure 16.(a), the average degree vs. hop distance for Dataset 01 is indicated by the blue curve, and the average

connection and a three-way fork road connection. In this research work, we have conducted our experiments initially

on a cross-road section and built a mathematical model to

authenticate our analysis. Later on, we have conducted the

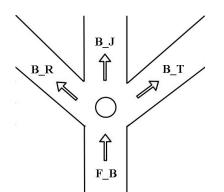


FIGURE 15. Routes in a three-way Fork road connection.

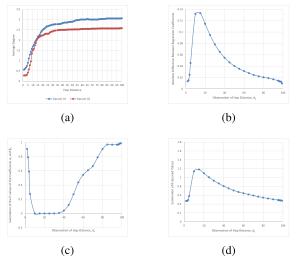


FIGURE 16. Average Degree vs. Hop Distance for three-way Fork road connection.

degree vs hop distance for dataset 02 are indicated by the red curve. In each dataset, there are 100 observations of hop distance and corresponding average degree extracted from the associated complex network of that particular hop distance. After a certain value of hop distance, both the curves move together. The initial response of average degree with an increase in hop distance is higher as we can see from the graph. This outcome is similar here for the three-way fork road connection with the previous two cases where we have experimented with a cross-road connection and a three-way T road connection. In every case, the average degree fluctuates a lot when the hop distance is within a certain and low range. After that certain range, the average degree does not respond much like the previous cases. The graph $|\alpha_1(d_k) - \beta_1(d_k)|$ vs. $d_k, p_{\alpha_1}(d_k) + p_{\beta_1}(d_k)$ vs. d_k and $R_i^2(d_k) + R_i^2(d_k)$ vs. d_k for the network property average degree for the three-way fork road connection are shown in Figure 16.(b), 16.(c) and 16.(d) respectively. Figure 16.(b) shows similar characteristics as Figure 14.(b) and Figure 3. Figure 16.(c) shows very similar kinds of characteristics as shown in Figure 14.(c) and Figure 4. Likewise, Figure 16.(d) reflects the similar type of characteristics as shown in Figure 14.(d) and Figure 5.

We have derived two different types of road connections from our cross-road connection which are a three-way T road

very same experiment in a three-way T road connection and in the three-way fork road connection. From our experiments in a three-way T road connection and in the three-way fork road connection, we have found very similar outcomes as we have found in the case of the cross-road connection. From our experiment, we can say that the algorithm and the mathematical model built to map a traffic domain into a complex network has generality of applications and has the authenticity to work for any traffic domain.
 IX. CONCLUSION The paramount contribution of this research is launching a new innovative algorithm to transform the traffic domain

new innovative algorithm to transform the traffic domain into a complex network, developing a mathematical model and analysis of that mathematical model to see different aspects and change of behavior of different properties of a complex network with different edge values. Our innovative algorithm can be implemented to transform a traffic domain into a complex network of vehicles where each vehicle of the traffic domain is considered a human being and acts just like a human acts on social networking sites. In the converted complex network of the traffic domain, each node represents a particular vehicle in the traffic domain. We also developed a mathematical model and analyzed the model with different distance values to see how different properties of complex network fluctuate with different distance values between vehicles. We have manually collected datasets from two different satellite views of the area of Bijay Sarani. Only two datasets are not enough for our further analysis. For that reason, in the future, we are thinking of using live traffic updates from Google maps. Using particular congestion indication from their maps, live information could be extracted. Then using our algorithm that has been developed upon characteristics and heuristics of Social Internet of Things, we will generate a pretty good amount of synthesized datasets that will reflect the traffic condition of "Bijay Sarani" at different times of day. Once we get the synthesized dataset we can find out the relationship between network properties using Multiple Regression Analysis. From that analysis we hope that we will be able to provide a generalized model that will describe the pattern and probability of traffic congestion in any traffic domain. Moreover, in order to visualize and analyze dynamic traffic data in real time, space and time complexity of our proposed algorithm need to be optimized since in any moment of time the whole traffic domain needs to be converted in a complex network.

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AMITABHA CHAKRABARTY received the M.Sc. degree from the Department of Computer Science and Engineering, University of Rajshahi, in 2004, and another M.Sc. degree in telecommunication engineering from Independent University, Bangladesh, and the Ph.D. degree from the Faculty of Engineering and Computing, Dublin City University, Dublin, Ireland, in 2012. He is currently an Associate Professor with the Department of Computer Science and Engineering, BRAC University,

Dhaka, Bangladesh. He leads the IoT and Embedded System Research Group at BRAC University. He is also involved in active research having number of graduate and undergraduate research groups in different research projects. He has published research papers in various national and international conferences, journals, and book chapters. His research interest includes the IoT, machine learning, deep learning, embedded systems, and switching theory. He is serving as a TCP member in various international journals and conferences. He is also serving as a Senior Judge in various national IT competition.



DOUG YOUNG SUH (S'89–M'90) received the B.Sc. degree from the Department of Nuclear Engineering, Seoul University, South Korea, in 1980, the M.Sc. and Ph.D. degrees from Department of Electrical Engineering, Georgia Institute of Technology, Atlanta, GA, USA, in 1986 and 1990, respectively. In 1990, he joined the Korea Academy of Industry and Technology and conducted research on HDTV, until 1992. Since 1992, he has been a Professor with the College of Elec-

tronics and Information Engineering, Kyung Hee University, South Korea. His research interests include networked video and video compression. He has been a Korean delegate for ISO/IEC MPEG, since 1996.



MD. JALIL PIRAN (S'10–M'16) received the Ph.D. degree in electronics and radio engineering from Kyung Hee University, South Korea, in 2016. Subsequently, he continued his work as a Postdoctoral Research Fellow in the field of resource management and quality of experience in 5G-Cellular Networks, and the Internet of Things with the Networking Laboratory, Kyung Hee University. He is currently a Professor with the Department of Computer Science and Engineering, Sejong

University, Seoul, South Korea. He has published substantial number of technical papers in well-known international journals and conferences in research fields of: resource allocation and management in; 5G mobile and wireless communication, HetNet, the Internet of Things (IoT), multimedia communication, streaming, adaptation, and QoE, cognitive radio networks, wireless sensor networks, machine learning, fuzzy logic, and neural networks. He was a recipient of IAAM Scientist Medal of the year 2017 for notable and outstanding research in the field of New Age Technology and Innovation, in Stockholm, Sweden. Moreover, He has been recognized as the Outstanding Emerging Researcher by the Iranian Ministry of Science, Technology, and Research, in 2017. In addition, his Ph.D. dissertation has been selected as the Dissertation of the Year 2016 by the Iranian Academic Center for Education, Culture, and Research in the field of electrical and communications engineering. In the worldwide communities, he has been an Active Member of the Institute of Electrical and Electronics Engineering (IEEE), since 2010, an Active Delegate from South Korea in the Moving Picture Experts Group (MPEG), since 2013, and an Active Member of the International Association of Advanced Materials (IAAM), since 2017.

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SIFATUL MOSTAFI received the bachelor's degree in computer science and engineering with a double major in Economics from BRAC University, Dhaka, Bangladesh, in 2018. He was a Teaching Assistant with the Department of Computer Science and Engineering, BRAC University. His research interest includes Internet of Things, wireless communication, vehicular network, social network, artificial intelligence, and statistics.



FARZANA KHAN received the bachelor's degree in computer science and engineering from BRAC University, Dhaka, Bangladesh, in 2018. Her research interest includes Internet of Things, social networking, and complex networks.