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Design of Parametric Controller for Two-Dimensional Polynomial Systems Described by the Fornasini-Marchesini Second Model

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ABSTRACT In this paper, a general parametric controller design method is presented for two-dimensional (2D) linear discrete systems described by the Fornasini–Marchesini second model. This method is based on the discriminant systems of polynomial and Hurwitz theorem. By the fractional linear transformation, the problem of stability analysis for 2D systems can be turned to a new problem whether the polynomials are Hurwitz stable. Thus, the process of stability analysis is changed into a problem whether some polynomials are positive definite, which can be easily checked by the discriminant system of the polynomial. It simplifies some existing methods of analyzing stability for 2D systems. Then, we apply the process proposed stability analysis method to consider the stabilization problem. A parametric controller is derived. The form of the parametric controller designed by the proposed method is simple, the parameters of the controller law can be fully solved. Finally, we give two numerical examples and a practical example of a chemical reactor thermal process to show the validity of the proposed methods.

INDEX TERMS Parametric controller, 2D system, the Fornasini-Marchesini second model, the discriminant system of polynomial.

I. INTRODUCTION

Two dimensional systems are a class of dynamic systems which propagate information in two independent directions. This propagation feature makes 2D system's dynamics depend upon two independent variables [1]. That make the analysis of 2D systems much more complicated and difficult than common one-dimensional (1D) systems, especially for uncertain 2D system [1], [2]. Recently, the heightened research interest in the study of 2D systems can be attributed to its widespread applications in processing 2D sampled data, such as seismic data, photogrammetric data [1], [3]–[6]. And 2D systems under different scenarios have been widely studied in [7]–[9]. Various types of 2D state space models are proposed [3], [4], [10]. The Fornasini-Marchesini second model studied in this paper is first introduced in the research of 2D image processing [4], [10].

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The Fornasini-Marchesini second model for 2D system is as follow:

$$\mathbf{x}(k+1, l+1) = \mathbf{A}_1\mathbf{x}(k, l+1) + \mathbf{A}_2\mathbf{x}(k+1, l), \quad (1)$$

where $\mathbf{A}_1, \mathbf{A}_2$ are $n \times n$ real constant matrices, $x(k, l)$ is the state vector at (k, l).

The stability properties of 2D systems described by the Fornasini-Marchesini second model have been investigated extensively [11]–[16]. Precise definition of the asymptotic stability of the 2D system was first made in [4]. The Fornasini-Marchesini second model (1) is asymptotically stable if and only if

$$det(\mathbf{I}_n - z_1\mathbf{A}_1 - z_2\mathbf{A}_2) \neq 0, \quad (z_1, z_2) \in \bar{U}, \quad (2)$$

where $\bar{U} = \{(z_1, z_2) : z_1, z_2 \in C \mid |z_1| \leq 1, |z_2| \leq 1\}$, $\mathbf{I}_n$ denotes the $n \times n$ identity matrix and 'det' stands for determinant [4], [5].

Unfortunately, condition (2) is not numerically tractable since they should be checked at infinitely many points over the range of (z_1, z_2) [17], [18]. It is difficult to determine the stability of system (1) by directly analyzing condition (2). To overcome the difficulty, Galkowski and Ito et al. provide brief linear matrix inequality (LMI) conditions for condition (2) to hold [19]–[21]. These methods in terms of LMI made the effort to convert condition (2) into feasibility tests of LMIs via Lyapunov's stability theory [18], [21], [22].

Note that most LMI conditions in [11], [12], [21], [22] are only sufficient and far from necessity. These methods use Lyapunov functions of restricted form [21] and thus satisfactory results have not been obtained on the exactness of the resulting LMI-based conditions [18]. And the analysis conditions are computationally demanding since the size and the number of scalar variables of the proposed LMIs grow rapidly with respect to the problem size [18], [21], [23], [24]. Although Ebihara et al. report the necessary and sufficient LMIs conditions for the 2D state space model in [18]. However, the proposed conditions in [18] are computationally more demanding than the only sufficient conditions in [20], [22]. In particular, the LMI conditions in [20], [22] is promising for state-feedback controller design, whereas the necessary and sufficient conditions in [18] are not suitable for controller synthesis [18].

As for the issue of controller design, many fruitful theoretical methods have been extensively studied in many fields [9], [20], [22], [25]–[28]. Among these methods, the significant results of various practical 1D systems on the control design have been reported in [25]–[28]. And the stabilization issue of 2D systems have been derived in [9], [20], [22]. Note that the methods based on LMIs [9], [20], [22] are derived for 2D systems that can only derive a control law of the specified system. In practical engineering, the designed controller is complex and difficult to achieve. In this paper, by the fractional linear transformation and applying the Hurwitz Theorem, some polynomial inequality conditions are obtained. Afterwards, the process of stability analysis is changed into a problem whether some polynomials are positive definite, which can be easily checked by the discriminant system method of polynomial [29]. Then, we apply the process proposed stability analysis method to consider the stabilization problem. Thus, the interval values of the parameters can be obtained and the controller meeting the requirements can be achieved.

Due to the method of the controller designed in this paper gives a complete range of parameters of control law, as long as the controller law is selected in this range when the controller is designed in practical engineering, the system would be stabilized. It is more applicable than the existing methods based on LMI. This kind of method can be widely used in practice. On the other hand, multiple control laws can be selected in a region to optimize the performances of the system. Putting it in another way, different control laws can be selected according to the requirements of the controller and the application scenarios.

Many achievements in [30]–[32] on the parametric controller design have been made, they both focus on 1D systems. However, to our best knowledge, no work considering the design of parametric controller for 2D system has been done up to now. In this paper, a parametric controller design method is proposed for 2D linear discrete systems described by the Fornasini-Marchesini second model. The form of the parametric controller designed by the proposed method is simple. The parameters of the controller law can be fully solved and the controller does not change the position of the equilibrium point of the system.

It should be noted that the existing methods based on LMI are conservative. They are only sufficient and need external parameters to complete the stability test. And the LMI methods that only give a control law, which does not satisfy the actual demand of production process. Thus, for the problem of stability analysis and controller design of 2D systems, there is still room for further improvement. Motivated by the previous aspects, we present an improved method such that the proposed conservatism can be reduced. We change the problem of the analysis stability into a new problem whether some polynomials are positive definite which is easily to test. And we expand the process proposed stability analysis method to solve the stabilization problem. The conditions in this note are necessary and sufficient. In stark contrast with the existing results [18], [20], [22], [33], the conditions of analysis stability and stabilization in this note are non-conservative and have less computational burden. Compared to the LMI methods that only give a control law, this method has a further field of practical application and a better result to solve the stability and stabilization problems of the systems.

The main contributions of this paper are as follows. Firstly, we propose necessary and sufficient conditions for the asymptotic stability of 2D systems. And we give an exact analysis method with simple and efficient algebraic computations. We transform the problem of stability analysis for 2D systems to a new problem of checking whether some polynomials are Hurwitz stable employing linear fraction transformation. And a concise algorithm to show the process of examining the stability of the considered system is given. Secondly, via the analysis of the stability problem, the stabilization problem is tackled and the desired conditions and a concise algorithm to show the processing of working out the stability regions of the parametric control law are presented. A parametric controller is derived. Finally, we compare the main results in this note with the results stated in [18], [33] to show the effectiveness of proposed approach in examples. And the proposed stabilization criterion in this paper can be applied to a practical example, the thermal processes in chemical reactors, to demonstrate the effectiveness.

The organization of the rest of this paper is as follows. Section II presents the main results of stability analysis, state feedback parametric stabilization of 2D model system. Besides, two algorithms are presented to show the processes of analyzing stability and getting the stability parameter regions of the control law respectively. In Section III, the corresponding obtained results are used to thermal processes in chemical reactors and two numerical examples to show the validity of the proposed method. Conclusions are drawn in Section IV.

Notations: R denotes the real number set. C denotes the complex number set. C^+ denotes the set of complex numbers with non-negative real part. I_n denotes identity matrix of $n \times n$ dimension. j denotes imaginary unit. And 'det' stands for determinant. $P \equiv \{z \in C \mid |z| = 1\}$, $D \equiv \{z \in C \mid |z| \leq 1\}$.

II. MAIN RESULTS

A. STABILITY ANALYSIS

The aim of this section is to change condition (2) into new condition that is necessary and sufficient by linear fraction transformation and present an algorithm for analyzing stability of the system under consideration.

It is known [17] that condition (2) is equivalent to the following criteria.

$$M(z_1, 0) \neq 0, \quad z_1 \in D, \tag{3}$$

and

$$M(z_1, z_2) \neq 0, \quad z_1 \in P, \ z_2 \in D, \tag{4}$$

where $M(z_1, z_2) = det(\mathbf{I}_n - z_1\mathbf{A}_1 - z_2\mathbf{A}_2)$.

The above-mentioned stability criteria (Conditions (3) and (4)) are not numerically tractable since they should be checked at infinitely many points over the range of (z_1, z_2) [18]. It is difficult to check the stability of the considered 2D system by directly analyzing criteria. In the following section, conditions (3) and (4) are changed into some new conditions.

Making the change of variable in $M(z_1, z_2)$,

$$z_1 = \frac{1 + js}{1 - js},$$

where $s \in R$. By the transformation, the variable z_1, $z_1 \in P$ is changed into s, $s \in R$.

And we denote $F(s, z_2)$ as follows:

$$F(s, z_2) = (1 - js)^m M(\frac{1 + js}{1 - js}, z_2), \tag{5}$$

where m and n are the degrees of $F(s, z_2)$ in s and z_2, respectively.

Remark 1: $F(s, z_2)$ can be regarded as a complex coefficient polynomial in $z_2 \in D$ with a real parameter s. Thus, it's convenient for further processing to consider a variable z_2 of $F(s, z_2)$ instead of studying two variables z_1, z_2 of $M(z_1, z_2)$.

From (5), it's easy to know that condition (4) is equivalent to

$$M(-1, z_2) \neq 0, \quad z_2 \in D, \tag{6}$$

and

$$F(s, z_2) \neq 0, \quad s \in R, \ z_2 \in D. \tag{7}$$

Remark 2: Under the linear fraction transformation $z_1 = \frac{1+js}{1-js}$, $z_1 = -1$ map into $s = \infty$. Condition (6) is required for the equivalence.

From the above discussion, condition (2) is equivalent to the conditions (3), (6) and (7). If the polynomial $M(z_1, z_2)$ satisfies these conditions, then the system (1) is stable.

Now we only need to consider conditions (3), (6) and (7). We continue to analyze and simplify them. By the fractional linear transformations, conditions (3) and (6) can be turned to a problem whether the real polynomial is Hurwitz stable, which can be easily checked by the discriminant systems of polynomial [29]. Some details are as follows.

Making the change of variables z_1 in (3) and z_2 in (6),

$$z_1 = \frac{1 - x}{1 + x},$$
$$z_2 = \frac{1 - y}{1 + y}.$$

We can obtain, respectively

$$Y_1(x, 0) = (1 + x)^m M(\frac{1 - x}{1 + x}, 0), \tag{8}$$

and

$$Y_2(-1, y) = (1 + y)^m M(-1, \frac{1 - y}{1 + y}). \tag{9}$$

Let $C^+ \equiv \{z \in C \mid Re(z) \geq 0\}$.

By the transformation (8) and (9), the unit disk D in the z_1 plane is mapped into C^+ in the x plane, and the unit disk D in the plane z_2 is mapped into C^+ in the y plane. Therefore, condition (3) is equivalent to

$$M(-1, 0) \neq 0, \quad Y_1(x, 0) \neq 0, \ x \in C^+, \tag{10}$$

and condition (6) is equivalent to

$$M(-1, -1) \neq 0, \quad Y_2(-1, y) \neq 0, \ y \in C^+. \tag{11}$$

Remark 3: Under the linear fraction transformation $z_1 = \frac{1-x}{1+x}$ and $z_2 = \frac{1-y}{1+y}$,$z_1 = -1$ and $z_2 = -1$ map into $x = \infty$ and $y = \infty$, respectively. $M(-1, 0) \neq 0$ in condition (10) and $M(-1, -1) \neq 0$ in condition (11) are required.

$Y_1(x, 0)$, $Y_2(-1, y)$ are real coefficient polynomial in x and y, respectively. So $Y_1(x, 0) \neq 0, x \in C^+$ in condition (10) and $Y_2(-1, y) \neq 0, y \in C^+$ in condition (11) can be regard as the following Hurwitz criterion of real coefficient polynomials.

Lemma 1 [34]: Let $f(\tau) = a_n\tau_n + a_{n-1}\tau_{n-1} + \ldots + a_0(a_0 > 0)$ be a real coefficient polynomial, where a_i, $i = 0, \ldots, n$ are real. The $n \times n$ Hurwitz matrix is constructed from $f(\tau)$, as follow:

$$H_f = \begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} & \cdots & 0 \\ a_n & a_{n-2} & a_{n-4} & \cdots & 0 \\ 0 & a_{n-1} & a_{n-3} & \cdots & 0 \\ 0 & a_n & a_{n-2} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_0 \end{bmatrix},$$

we denote the order principal minor determinant of H_f by $\triangle(f)_k$, $k = 1, 2 \ldots, n$, respectively. The necessary and sufficient condition for the roots' real part of $f(\tau)$ to be negative is $\triangle(f)_k > 0$, $k = 1, 2 \ldots, n$.

Similar to the process of condition (3) and (6), we process condition (7).

Making the change of variable in (7),

$$z_2 = \frac{1-z}{1+z},$$

we obtain

$$Y_3 = (1+z)^n F(s, \frac{1-z}{1+z}). \tag{12}$$

By the transformation (12), the unit disk D in the z_2 plane is mapped into C^+ in the z plane. Therefore, the condition (7) is equivalent to

$$F(s,-1) \neq 0, \quad s \in R,$$
$$Y_3(s,z) \neq 0, \quad s \in R, \ z \in C^+. \tag{13}$$

Remark 4: Under the linear fraction transformation $z_2 = \frac{1-z}{1+z}$, $z_2 = -1$ maps into $z = \infty$. $F(s,-1) \neq 0, s \in R$ in condition (13) is also required.

If $Y_3(s,z)$ can be considered as a complex coefficient polynomial in z with a real parameter s, that is as follow.

$$Y_3(s,z) = \sum_{i=0}^{n} a_i(s) z^i, \tag{14}$$

Then the condition (13) can be regard as the following Hurwitz criterion of complex coefficient polynomials.

Lemma 2 [34]: Let $f(\tau)$ be a complex coefficient polynomial and satisfy $f(j\tau) = b_n\tau_n + b_{n-1}\tau_{n-1} + \ldots + b_0 + j(a_n\tau_n + a_{n-1}\tau_{n-1} + \ldots + a_0)$, $a_n \neq 0$, where a_i and b_i, $i = 0, \ldots n$ are real. The $2n \times 2n$ Hurwitz matrix is constructed from $f(\tau)$, as follow:

$$H_f = \begin{bmatrix} a_n & a_{n-1} & a_{n-2} & \cdots & 0 \\ b_n & b_{n-1} & b_{n-2} & \cdots & 0 \\ 0 & a_n & a_{n-1} & \cdots & 0 \\ 0 & b_n & b_{n-1} & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \cdots & a_0 \\ 0 & 0 & 0 & \cdots & b_0 \end{bmatrix},$$

we denote the $2k_{th}$ order principal minor determinant of H_f by $\triangle_{2k}(f), k = 1, 2 \ldots, n$, respectively. If $\triangle(f)_{2k} \neq 0$, the necessary and sufficient condition for the roots' real part of $f(\tau)$ to be negative is $\triangle(f)_{2k} > 0, k = 1, 2 \ldots, n$.

From the above discussion, we change condition (2) to new conditions (10), (11) and (13). Thus, we can obtain the following theorem.

Theorem 1: The 2D system (1) is asymptotically stable if and only if the following conditions are satisfied,

1) $M(-1,0) \neq 0, \quad Y_1(x,0) \neq 0, \ x \in C^+$.
2) $M(-1,-1) \neq 0, \quad Y_2(-1,y) \neq 0, \ y \in C^+$.
3) $F(s,-1) \neq 0, \quad Y_3(s,z) \neq 0, s \in R, \ z \in C^+$.

Remark 5: It's necessary to state briefly that the considered 2D system described by state space model doesn't have any nonessential singularity of the second kind in this paper. So there are no the items as the use of bilinear applications which disadvantages for 2D systems described by transfer function are well known in [35]. Then we can obtain the above theorem.

Conditions 1) and 2) of Theorem 1 are the same as the problems whether the real polynomials are Hurwitz stable, and condition 3) of Theorem 1 is the same as Hurwitz criterion of complex coefficient polynomial. They can be easily checked by the discriminant systems of polynomial [29]. Then we can restate Theorem 1 as follows.

Theorem 2: System (1) is asymptotically stable if and only if the following conditions are satisfied,

1) $M(-1,0) \neq 0, \quad \triangle(Y_1)_k > 0, \ k = 1, 2 \ldots, n$.
2) $M(-1,-1) \neq 0, \quad \triangle(Y_2)_k > 0, \ k = 1, 2 \ldots, n$.
3) $F(s,-1) \neq 0, \quad \triangle(Y_3)_{2k} > 0, \ k = 1, 2 \ldots, n, s \in R$.

Proof: Conditions $Y_1(x,0) \neq 0, x \in C^+$ and $Y_2(-1,y) \neq 0, y \in C^+$ can be regarded as the real polynomials $Y_1(x,0), Y_2(-1,y)$ are Hurwitz stable. According to Lemma 1, the problems whether the real polynomials $Y_1(x,0), Y_2(-1,y)$ are Hurwitz stable can be solved by conditions $\triangle(Y_1)_k > 0, k = 1, 2 \ldots, n$ and $\triangle(Y_2)_k > 0, k = 1, 2 \ldots, n$, respectively. Similarly, condition $Y_3(s,z) \neq 0, s \in R, z \in C^+$ is equivalent to that the complex polynomial $Y_3(s,z)$ is Hurwitz stable. According to Lemma 2, $Y_3(s,z)$ is Hurwitz stable means the condition $\triangle(Y_3)_{2k} > 0, k = 1, 2 \ldots, n, s \in R$. Thus, we can restate Theorem 1 as Theorem 2. This proof is completed.

Based on the above results, by the fractional linear transformation, the problem of testing the stability is changed into a new problem whether some polynomials are positive definite, which can be easily checked by the discriminant systems of polynomial.

Then we can get the following algorithm for testing stability of 2D systems described by the Fornasini-Marchesini second model:

Algorithm 1 2DStabilityTest

Input $M(z_1,z_2) = det(\mathbf{I}_n - z_1\mathbf{A}_1 - z_2\mathbf{A}_2)$ of a 2D systems described by the Fornasini-Marchesin second model.

Output True if system is stable. False otherwise.

Step 1. Calculate the polynomial $Y_1(x,0)$, and use Lemma 1 to check $M(-1,0) \neq 0, \triangle(Y_1)_k > 0, k = 1, 2 \ldots, n$. If not satisfied, output False.

Step 2. Calculate the polynomial $Y_2(-1,y)$, and use Lemma 1 to check $M(-1,-1) \neq 0, \triangle(Y_2)_k > 0, k = 1, 2 \ldots, n$. If not satisfied, output False.

Step 3. Calculate the complex polynomial $Y_3(s,z)$, and mark it as $Y_3(z) = q_nz_n + q_{n-1}z_{n-1} + \ldots + q_0 + j(p_nz_n + p_{n-1}z_{n-1} + \ldots + p_0)$ where q_i and p_i, $i = 0, \ldots n$ are real coefficient polynomials in s. Check whether $F(s,-1) \neq 0, \triangle(Y_3)_{2k} > 0, k = 1, 2 \ldots, n, s \in R$ is satisfied with Lemma 2. If not satisfied, output False.

Step 4. Output True.

This section shows that the process of the stability analysis for 2D systems. More specifically, we first present a transformation from the stability conditions (3) and (4) given in [17] to new conditions of Theorem 1. The conditions of Theorem 1 are equipment to the problems whether the polynomials are Hurwitz stable. Then Theorem 1 is rewritten as Theorem 2. Finally, an efficient algorithm for analyzing stability of 2D linear discrete system is given.

Because the designed controller is complex and difficult to achieve in practical control systems. In order to better apply controller to the practical engineering, next subsection we provide the method to design a parametric controller. This method of the controller designed in this paper gives a complete range of parameters of control law. As long as the controller law is selected in this range based on the actual situation when the controller is performed in practical engineering, the system would be stabilized.

B. PARAMETRIC CONTROLLER DESIGN

In this section, we shall deal with the problem of state feedback stabilization problem for the systems by Hurwitz theorem. More precisely, we are interested in finding a state feedback such that the closed-loop system with the control law is asymptotically stable. In this situation, the system is said to be stabilize via state feedback for the system as follow.

$$x(k+1,l+1) = \mathbf{A_1}x(k,l+1) + \mathbf{A_2}x(k+1,l) + \mathbf{B_1}u(k,l+1) + \mathbf{B_2}u(k+1,l) \quad (15)$$

The aim of solving the stabilization problem is to design a controller with form.

$$u(k,l) = \mathbf{K}x(k,l), \quad (16)$$

where $\mathbf{K}$ is the controller gain containing uncertain parameters with appropriate dimensions. Such that the corresponding closed-loop system with control law is given by

$$x(i+1,j+1) = (\mathbf{A_1}+\mathbf{B_1K})x(i,j+1) + (\mathbf{A_2}+\mathbf{B_2K})x(i+1,j) \quad (17)$$

For the stabilization problem we follow steps similar to the process of solving the stability and robust Stability problem.

Denote

$$\widetilde{M}(z_1,z_2) = det(\mathbf{I_n} - z_1(A_1+\mathbf{B_1K})) - z_2(A_2+\mathbf{B_2K})), \quad (z_1,z_2)\in\overline{U},$$
$$\widetilde{F}(s,z_2) = (1-js)^m\widetilde{M}(\frac{1+js}{1-js},z_2), \quad s\in R,\ z_2\in D,$$
$$\widetilde{Y}_1(z,0) = (1+z)^m\widetilde{M}(\frac{1-z}{1+z},0), \quad z\in C^+,$$
$$\widetilde{Y}_2(-1,z) = (1+z)^n\widetilde{M}(-1,\frac{1-z}{1+z}), \quad z\in C^+,$$
$$\widetilde{Y}_3(s,z) = (1+z)^n\widetilde{F}(s,\frac{1-z}{1+z}), \quad s\in R,\ z\in C^+.$$

where m and n are the degree of $\widetilde{M}(z_1,z_2)$ in z_1 and z_2, respectively.

The conditions of Theorem 2 can be established to test the stability of the system (17) and it is reported as the following Theorem.

Theorem 3: System (15) is stabilizable via state feedback if and only if there exists $\mathbf{K}$ such that the following conditions are satisfied,

1) $\widetilde{M}(-1,0)\neq 0, \quad \triangle(\widetilde{Y}_1)_k > 0,\ k=1,2\ldots,n.$
2) $\widetilde{M}(-1,-1)\neq 0, \quad \triangle(\widetilde{Y}_2)_k > 0,\ k=1,2\ldots,n.$
3) $\widetilde{F}(s,-1)\neq 0, \quad \triangle(\widetilde{Y}_3)_{2k} > 0,\ k=1,2\ldots,n.$

Proof: By replacing $\mathbf{A_1}$ and $\mathbf{A_2}$ in the system (1) with $\mathbf{A_1}+\mathbf{B_1K}$ and $\mathbf{A_2}+\mathbf{B_2K}$ in the system (17), respectively. Therefore, it follows from Theorem 2 that the system (17) is stable, which concludes the proof.

Then we can get the following algorithm for getting the regions of the parametric control law to finish the controller design of 2D systems described by the Fornasini-Marchesini second model:

Algorithm 2 Parametric Controller Design for Finding the Stability Parameter Regions in Parameter Space of the Designed Controller

Input: $\widetilde{M}(z_1,z_2) = det(\mathbf{I_n} - z_1(A_1+\mathbf{B_1K})) - z_2(A_2+\mathbf{B_2K}))$ of a 2D systems described by the Fornasini-Marchesini second model

Output: The stability parameter regions of the parametric control gain $\mathbf{K}$.

Step 1. Firstly, calculate the polynomial $\widetilde{Y}_1(x,0)$. Then solve the conditions $\widetilde{M}(-1,0)\neq 0, \triangle(\widetilde{Y}_1)_k > 0, k=1,2\ldots,n$ to get one of the controller parameter regions of $\mathbf{K}$.

Step 2. Firstly, calculate the polynomial $\widetilde{Y}_2(-1,y)$. Then solve the conditions $\widetilde{M}(-1,-1)\neq 0, \triangle(\widetilde{Y}_2)_k > 0, k=1,2\ldots,n$ to get one of the controller parameter regions of $\mathbf{K}$.

Step 3. Firstly, calculate the complex polynomial $\widetilde{Y}_3(s,z)$, then we can get $Y_3(jz) = q_nz_n + q_{n-1}z_{n-1} + \ldots + q_0 + j(p_nz_n + p_{n-1}z_{n-1} + \ldots + p_0)$
where q_i and $p_i, i = 0,\ldots n$ are real coefficient polynomials in s. Finally solve the conditions the conditions $F(s,-1)\neq 0, and \triangle(Y_3)_{2k} > 0, k=1,2\ldots,n, s\in R$ to get one of the controller parameter regions of $\mathbf{K}$.

Step 4. For satisfying three conditions in step 2-3, we obtain the final results of the stability parameter regions of the control gain $\mathbf{K}$.

Step 5. Output the parameters region of control gain $\mathbf{K}$. Finish the parametric controller design.

III. NUMERICAL EXAMPLES AND APPLICATION

In this section, we shall illustrate the results via the following examples. All computations were performed with Maple.

Example 1: Let us consider the 2D system described by (1) where

$$\mathbf{A_1} = \begin{bmatrix} 0.5 & 0.5 & 0.4 & 1.1 \\ 0.1 & -0.1 & 0.6 & 0.1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\mathbf{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.1 & -0.1 & -0.2 & -0.5 \\ -0.2 & 0.6 & -0.1 & -0.7 \end{bmatrix}.$$

Now we follow the Algorithm 1 to analyze the stability of this system.

Based on this system, we firstly get the polynomial $M(z_1,z_2) = det(\mathbf{I_n} - z_1\mathbf{A_1} - z_2\mathbf{A_2}) = 1 + 0.9z_2 + 0.09z_2^2 - 0.4z_1 - 0.1z_1z_2 + 0.194z_1z_2^2 - 0.1z_1^2 - 0.086z_2z_1^2 - 0.688z_1^2z_2^2$.

Step 1: Calculate the polynomial $Y_1(x, 0)$, as follow

$$Y_1 = (1+x)^2M(\frac{1-x}{1+x}, 0) = 1.3x^2 + 2.2x + 0.5.$$

And use Lemma 1 to check whether the conditions $M(-1, 0) \neq 0, \triangle(Y_1)_k > 0, k = 1, 2$ are satisfied, as follow

$$M(-1, 0) = 1.3 \neq 0,$$

$$H_{Y_1} = \begin{bmatrix} 2.2 & 0 \\ 1.3 & 0.5 \end{bmatrix}.$$

We can get the principal minor determinant $\triangle(Y_1)_k > 0, k = 1, 2$. From Hurwitz criterion, we know that the polynomial Y_1 is Hurwitz stable, hence condition 1) of Theorem 2 is satisfied.

Step 2: Calculate the polynomial $Y_2(-1, y)$, as follow

$$Y_2=(1+y)^2M(-1, \frac{1-y}{1+y})=0.2132y^2+2.9456y+2.0412.$$

And use Lemma 1 to check whether the conditions $M(-1, -1) \neq 0, \triangle(Y_2)_k > 0, k = 1, 2$ are satisfied, as follow

$$M(-1, -1) = 0.2131 \neq 0,$$

$$H_{Y_2} = \begin{bmatrix} 2.9456 & 0 \\ 0.2132 & 2.0412 \end{bmatrix}.$$

We can get the principal minor determinant $\triangle(Y_2)_k > 0, k = 1, 2$. From Hurwitz criterion, we know that the polynomial Y_2 is Hurwitz stable, hence condition 2) of Theorem 2 is satisfied.

Step 3: Calculate the complex polynomial $Y_3(s, z)$

$$\begin{aligned} Y_3(z) &= (1-js)^2(1+z)^2F(\frac{1+js}{1-js}, \frac{1-z}{1+z}) \\ &= 1.4292 - 2.0412s^2 - 0.2132s^2z^2 - 2.9456s^2z \\ &\quad +0.0012z^2 + 0.5696z + j(-4.4896s - 0.5456sz^2 \\ &\quad -3.7648sz) \end{aligned}$$

Then, we can know

$$\begin{aligned} Y_3(jz) &= (0.2132s^2 - 0.0012)z^2 + 3.7648sz + 1.4292 \\ &\quad -2.0412s^2 + j(0.5456sz^2 + 0.5696)z - 4.4896s), \end{aligned}$$

Check whether $F(s, -1) \neq 0, \triangle(Y_3)_{2k} > 0, k = 1, 2, s \in R$, are satisfied, as follow

$$F(s, -1) = -0.2132s^2 - 0.5456js + 0.0012 \neq 0, \quad s \in R$$

According to Lemma 2, we can draw H_{Y_3} as follow:

$$H_{Y_3} = \begin{bmatrix} 0.5456s & -2.9456s^2 + 0.5696 & 4.4896s & 0 \\ 0.2132s^2 - 0.0012 & 3.7648s & 1.4292 - 2.0412s^2 & 0 \\ 0 & 0.5456s & -2.9456s^2 + 0.5696 & 4.4896 \\ 0 & 0.2132s^2 - 0.0012 & 3.7648s & 1.4292 - 2.0412s^2 \end{bmatrix}$$

According to H_{Y_3}, we obtain the $2k_{th}$ order principal minor determinant as follows:

$$\begin{aligned} \triangle(Y_3)_2 =& (0.62800192s^2+1.92887890043452) \\ & (s_2+0.000354361282009991), \\ \triangle(Y_3)_4 =& (3.775898421s^2+11.1139321489633) \\ & (s^2+1.99546879286784)(s^2+0.435300446686488) \\ & (s^2+0.000576384669062484) \end{aligned}$$

We can get $\triangle(Y_3)_{2k} > 0, k = 1, 2, s \in R$. From Lemma 2, the polynomial Y_3 is Hurwitz stable. Condition 3) of Theorem 2 is satisfied. Thus, it can be concluded that the system is stable by Theorem 2.

Remark 6: The example given above has shown the proposed approach for stability analysis for 2D systems described by the Fornasini-Marchesini second model. We have shown all the steps to test stability of 2D system in order to understand the testing process more clearly. All the results in the process can be easily calculated by Maple. The conditions in this note are necessary and sufficient. Because they are equivalent to condition (2). Note that the proposed method is different from the methods based on LMI that need external parameters to complete the stability test and most are only sufficient. We change the problem of the analysis stability into a new problem whether some polynomials are positive definite that is easily to test. For the same system in example 1, the testing stability process of the method in [18] is to find an exact value γ and check whether it satisfies condition. It needs 32.66s to complete computation. In [20], [22], the paper also show that it takes 2.26s get a conservative result in term of the LMI condition stated in [18]. While in this example it only takes 0.12s using the new necessary and sufficient condition. Obviously, the method in this note is computationally less demanding than the methods in [18], [20], [22]. This method is more efficient than some existing methods.

Remark 7: Note that example 1 is given to show the proposed method is more efficient than the existing methods. However, the matrixes A1 and A2 have no generality as the zero rows in example 1. A general example is given to show the proposed method has generality as follows.

Example 2: Consider the 2D system with

$$\mathbf{A_1} = \begin{bmatrix} \frac{47}{100} & \frac{3}{25} \\ -\frac{1}{5} & \frac{9}{20} \end{bmatrix}, \quad \mathbf{A_2} = \begin{bmatrix} \frac{47}{100} & -\frac{3}{400} \\ \frac{33}{1000} & \frac{477}{1000} \end{bmatrix}.$$

We give the same steps as example 1 to analyze the stability of the general 2D system as follow:

$$\begin{aligned} M(z_1, z_2) &= det(\mathbf{I_n} - z_1\mathbf{A_1} - z_2\mathbf{A_2}) \\ &= 1 - \frac{23}{25}z_1 - \frac{947}{1000}z_2 + \frac{471}{2000}z_1^2 + \frac{43023}{100000}z_1z_2 \\ &\quad + \frac{3591}{16000}z_2^2 \end{aligned}$$

Step 1: Calculate the polynomial $Y_1(x, 0)$, as follow

$$Y_1(x, 0) = \frac{4311}{2000}x^2 + \frac{1529}{1000}x + \frac{631}{2000} \neq 0, \quad x \in C^+,$$

And use Lemma 1 to check whether the conditions $M(-1, 0) \neq 0, \triangle(Y_1)_k > 0, k = 1, 2$ are satisfied, as follow

$$M(-1, 0) = \frac{4311}{2000} \neq 0,$$
$$\mathbf{H_{Y_1}} = \begin{bmatrix} \frac{1529}{1000} & 0 \\ \frac{4311}{2000} & \frac{631}{2000} \end{bmatrix}.$$

We can get the principal minor determinant $\triangle(Y_2)_k > 0, k = 1, 2$. From Hurwitz criterion, we know that the polynomial Y_2 is Hurwitz stable, hence condition 2) of Theorem 2 is satisfied.

Step 2: Calculate the polynomial $Y_2(-1, y)$, as follow

$$Y_2(-1, y) = \frac{1502867}{400000}y^2 + \frac{30897}{8000}y + \frac{401083}{400000} \neq 0, \quad y \in C^+,$$

Use Lemma 1 to check whether the conditions $M(-1, -1) \neq 0, \triangle(Y_2)_k > 0, k = 1, 2$ are satisfied, as follow

$$M(-1, -1) = \frac{1502867}{400000} \neq 0,$$
$$\mathbf{H_{Y_2}} = \begin{bmatrix} \frac{30897}{8000} & 0 \\ \frac{1502867}{400000} & \frac{401083}{400000} \end{bmatrix}.$$

We can get the principal minor determinant $\triangle(Y_2)_k > 0, k = 1, 2$. From Hurwitz criterion, we know that the polynomial Y_2 is Hurwitz stable, hence condition 2) of Theorem 2 is satisfied.

Step 3: Calculate the complex polynomial $Y_3(s, z)$

$$\begin{aligned} Y_3(s, z) &= (-\frac{1502867}{400000}s^2 + \frac{422683}{400000})z^2 + (\frac{1457}{8000} - \frac{30897}{8000}s^2)z \\ &\quad + \frac{9267}{400000} - \frac{401083}{400000}s^2 \\ &\quad + j(\frac{1239}{320}sz^2 - \frac{8641}{4000}sz - \frac{671}{8000}s) \\ &\neq 0, \quad s \in R, \ z \in C^+, \end{aligned}$$

Then, we can get $Y_3(s, jz)$, For easier further processing, we simplify it as follows:

$$\begin{aligned} &Y_3(s, jz) \\ &= (1502867s^2 - 422683)z^2 + 864100sz - 401083s^2 \\ &\quad + 9267 + j(1548750sz^2 - 1544850s^2 + 72850z - 33550s) \end{aligned}$$

Check whether $F(s, -1) \neq 0, \triangle(Y_3)_{2k} > 0, \quad k = 1, 2, \ s \in R$, are satisfied, as follow

$$F(s, -1) = -\tfrac{1502867}{400000}s^2 - \tfrac{1239}{320}js + \tfrac{422683}{400000} \neq 0, \quad s \in R,$$

According to Lemma 2, we can draw H_{Y_3} as follow:

$$\mathbf{H_{Y_3}} = \begin{bmatrix} 1548750s & -1544850s^2 + 72850 & -33550s & 0 \\ 1502867s^2 - 422683 & 864100s & -401083s^2 + 9267 & 0 \\ 1548750s & & -1544850s^2 + 72850 & -33550s \\ 0 & 1502867s^2 - 422683 & 864100s & -401083s^2 + 9267 \end{bmatrix}$$

According to H_{Y_3}, we obtain the $2k_{th}$ order principal minor determinant as follows:

$$\begin{aligned} &\triangle(Y_3)_2 \\ &= 46434081699s^4 + 11516183630s^2 + 615849131 \\ &\triangle(Y_3)_4 \\ &= 5754232806511022852 49s^8 - 1100634564266144764s^6 \\ &\quad + 4986976676201550214s^4 - 2366394176212428 4s^2 \\ &\quad + 8315206667895489. \end{aligned}$$

We can get $\triangle(Y_3)_{2k} > 0, k = 1, 2, s \in R$. From Lemma 2, the polynomial Y_3 is Hurwitz stable. Condition 3) of Theorem 2 is satisfied. Thus, it can be concluded that the system is stable by Theorem 2. The obtained result is inline with that in [33]. The proposed method are general to test the 2D system stability.

Example 3: Consider the thermal processes in chemical reactors. Heat exchangers and pipe furnaces can be expressed as the following partial differential equation [36], [37].

$$\frac{\partial T(x, t)}{\partial x} = -\frac{\partial T(x, t)}{\partial t} - \partial T(x, t) + U(t), \tag{18}$$

where $\partial T(x, t)$ is usually the temperature at $x \in [0, x_f]$ and $t \in [0, \infty]$, and $U(t)$ is a given force function. Denote $x^T(i, j) = [T^T(i-1, j), T^T(i, j)]$, where $T(i, j) = T(i\triangle x, j\triangle t)$, then the system (18) can be converted into a 2D FMII model with

$$\mathbf{A_1} = \begin{bmatrix} 0 & 0 \\ \xi_1 & \xi_2 \end{bmatrix}; \ \mathbf{A_2} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; \ \mathbf{B_1} = \begin{bmatrix} 0 \\ \rho \end{bmatrix}; \ \mathbf{B_2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{19}$$

where $\xi_1 = 1 - \xi_2 - \rho, \xi_2 = \frac{\triangle t}{\triangle x}$ and $\rho = \triangle t$.

Let $\triangle t = 1$ and $\xi_2 = 1$.

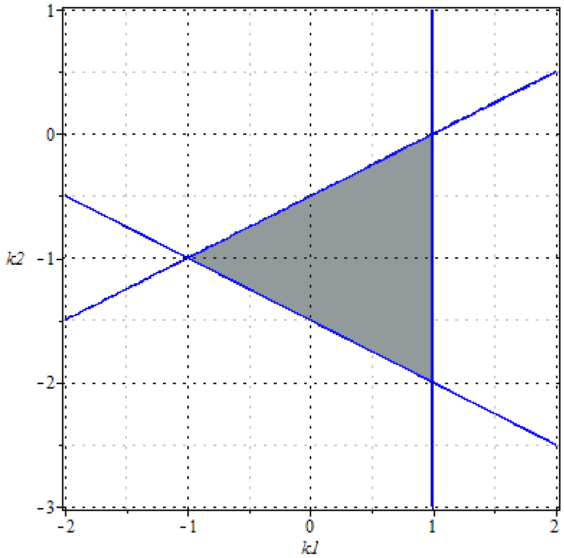

FIGURE 1. The stability parameter region of the parametric control gain K.

Now we follow Theorem 3 to analyze the stabilization of this system.

Let $K = [k_1 \quad k_2]$.

Based on the 2D model (19), we firstly get the polynomial

$$\begin{aligned}\widetilde{M}(z_1, z_2) &= det(\mathbf{I_n} - z_1(\mathbf{A_1} + \mathbf{B_1K})) - z_2(\mathbf{A_2} + \mathbf{B_2K})) \\ &= k_2z_1z_2 - k_1z_2 - k_2z_1 + z_1z_2 - z_1 + 1.\end{aligned}$$

Step 1: Calculate the polynomial $\widetilde{Y}_1(x, 0)$, as follow

$$\widetilde{Y}_1(x, 0) = 2z + k_2z - k_2 \neq 0, z \in C^+.$$

By considering the condition $\widetilde{M}(-1, 0) = 2 + k_2 \neq 0$ and solving the condition $\triangle(\widetilde{Y}_1)_k > 0, k = 1$ based on Lemma 1, we can get

$$\frac{k_2}{2 + k_2} < 0. \tag{20}$$

Step 2: Calculate the polynomial $\widetilde{Y}_2(-1, y)$, as follow

$$\widetilde{Y}_2(-1, y) = k_1z + 2k_2z - k_1 + 3z + 1 \neq 0, \quad z \in C^+.$$

Then, to consider the condition $\widetilde{M}(-1, -1) = 3 + 2k_2 + k_1 \neq 0$ and based on Lemma 1 to solve the condition $\triangle(\widetilde{Y}_2)_k > 0, k = 1$, we can get

$$\frac{k_1 - 1}{k_1 + 2k_2 + 3} < 0. \tag{21}$$

Step 3: Calculate the complex polynomial $\widetilde{Y}_3(s, z)$, as follow

$$\begin{aligned}\widetilde{Y}_3(s, z) = &-k_1z + 2k_2 + 2k_2z + k_1 + z - 1 \\ &+j(k_1sz + 2k_2sz - k_1s + 3sz + s \neq 0, z \in C^+.\end{aligned}$$

Then we can obtain

$$\begin{aligned}\widetilde{Y}_3(s, jz) = &-k_1sz - 2k_2sz - 3sz + k_1 - 1 \\ &+j(-k_1s - k_1z + 2k_2z + s + z).\end{aligned}$$

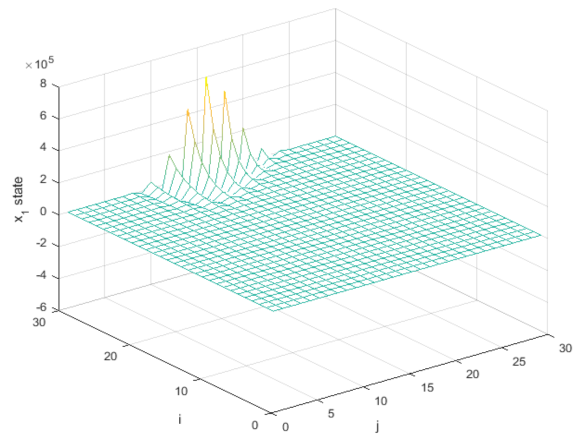

FIGURE 2. This is state space response of the system $x_1(i, j)$ before stabilization.

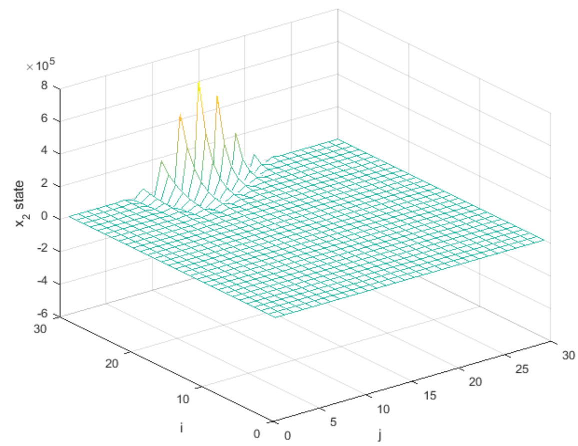

FIGURE 3. This is state space response of the system $x_2(i, j)$ before stabilization.

Finally, solve the conditions $\widetilde{F}(s, -1) = j(-k_1 - 2k_2 - 3)s + k_1 - 2k_2 - 1 \neq 0, s \in R$, and $\triangle(Y_3)_{2k} > 0, k = 1, 2, s \in R$.

From the above conditions, we can get

$$\begin{cases} -(k_1 - 1)(k_1 - 2k_2 - 1) > 0, \\ -(k_1 - 1)(k_1 + 2k_2 + 3) > 0 \end{cases} \tag{22}$$

From (20)(21) and (22), we have the solutions of the all the control laws satisfying the controller requirements as follows:

$$\begin{cases} k_1 < 1, \\ k_1 - 2k_2 - 1 > 0 \\ k_1 + 2k_2 + 3 > 0 \end{cases} \tag{23}$$

Thus, we finish the design of parametric controller. The trajectories of the two state variables for the system (19) with control input $u(i, j) = 0$ are shown in Figure 2 and Figure 3. The system (18) isn't stable before stabilization.

Next, let $u(i, j) \neq 0$ be imposed by controlled **K** as in (16), by solving conditions of Theorem 3 for getting the regions

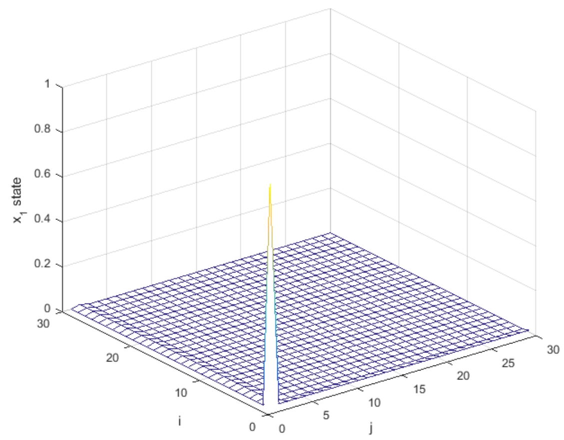

FIGURE 4. **This is state space response of the system $x_1(i,j)$ after stabilization, when K=[0 -1].**

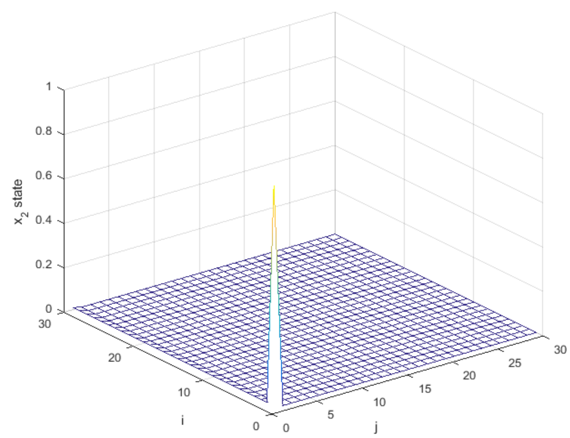

FIGURE 5. **This is state space response of the system $x_2(i,j)$ after stabilization, when K=[0 -1].**

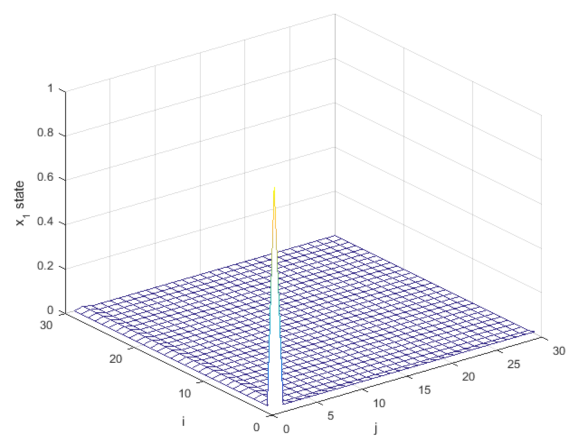

FIGURE 6. **This is state space response of the system $x_1(i,j)$ after stabilization, when K=[0.5 -1.5].**

of control gain **K**, we get the region (23) of the parametric control law shown in Figure 1.

According to the results shown in Figure 1, let **K**=[0 -1], and the trajectories of the two state variables of the stabilization system (19) are shown in Figure 4 and Figure 5. It's obvious that the system (18) is stable after stabilization when **K**=[0 -1]. What's more, let **K**=[0.5 -1.5] of the regions, and the trajectories of the two state variables of the stabilization system (19) are shown in Figure 6 and Figure 7. It is shown that the system (18) is also stable after stabilization when **K**=[0.5 -1.5] .

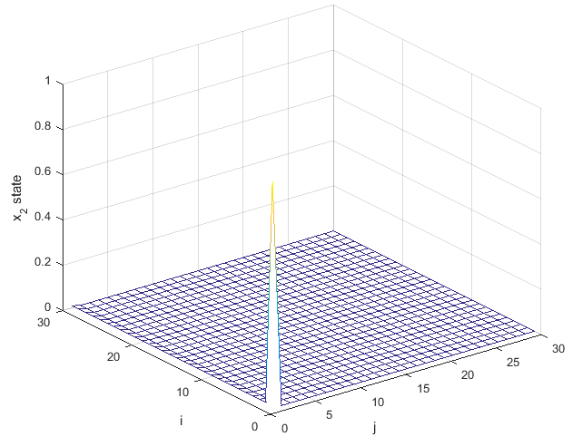

FIGURE 7. **This is state space response of the system $x_2(i,j)$ after stabilization, when K=[0.5 -1.5].**

Remark 8: This example illustrates some facts. First, before applying the controller to the system, it isn't stable. By designing a parametric controller, the system is stable. We can get controller parameter space satisfying system requests. Different control laws can be selected according to the requirements of the controller and the practical application scenarios.

IV. CONCLUSIONS

This paper has proposed a new stability analysis method of 2D discrete linear systems described by the Fornasini-Marchesini second model. By the fractional linear transformation, the problem of testing the stability is changed into a new problem whether some polynomials are Hurwitz stable, which can be easily checked by the discriminant systems of polynomial. More specifically, the problem of testing the stability is changed into a problem whether some polynomials are positive definite. Then this paper considers state feedback parametric controller design of 2D systems. To sum up, a necessary and sufficient condition which can guarantee the asymptotic stability of 2D systems, is derived in term of discriminant system method, and formulas can be given for the parametric controller design. Two concise algorithms are presented to show the processes of analyzing stability and getting the stability parameter regions of the control law to solve the 2D system stability and stabilization problems respectively. The effectiveness of the proposed stability has been illustrated by numerical examples. The computational burden is reduced. And we give a more practical example to consider the stabilization problem and show the advantages of parametric controller we derived. Finally, it should be point out that the results in this paper are also appropriate for iterative learning control. These are subjects of our further investigation in the future.

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