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Maximum Likelihood Recursive Identification for the Multivariate Equation-Error Autoregressive Moving Average Systems Using the Data Filtering

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ABSTRACT The maximum likelihood principle has wide applications in system identification. This paper studies the maximum likelihood identification problems of the multivariate equation-error systems with colored noise. The system is broken down into several subsystems based on the number of the outputs. The key is to transform the subsystem into a controlled autoregressive moving average model and a noise model. Based on the maximum likelihood principle and the data filtering technique, a filtering-based maximum likelihood recursive generalized extended least squares algorithm is presented for estimating the parameters of these two models. For comparison, a maximum likelihood recursive generalized extended least squares algorithm is presented. Finally, the simulation example results confirm the effectiveness of the two algorithms.

INDEX TERMS Parameter estimation, maximum likelihood, data filtering, multivariate system.

I. INTRODUCTION

For the actual control systems, system modeling and model identification are the basis of all control problems. Parameter estimation methods can be applied to many areas [1]–[4]. Parameter estimation is the eternal theme of the identification field and has wide applications in one-dimensional and multidimensional signal processing and filtering. The research on model uncertainty has become a hot topic. Recently, a family of robust filtering approaches under model uncertainty have been introduced [5] and the robust estimation problem of a signal given noisy observations is dealt with [6]. Multivariable systems are widely used in practical control processes because of their complex structures and uncertain interference. Consequently, the parameter estimation problem of different systems has attracted a lot of attention [7]–[10]. Some examples of applications include fermentation processes [11] and distillation columns [12]. Wu [13] presented a model predictive control based proportional-integral-derivative controller for the multivariable process in the distillation column. Li *et al.* [14] provided an adaptive control method to deal with the tracking control problems for the nonlinear MIMO time-varying delay systems.

The maximum likelihood principle is widely used in the field of system identification because of good statistical properties [15]. In recent years, many maximum likelihood identification algorithms have been proposed [16], [17]. For instance, Wang *et al.* [18] focused on the robust Chinese remainder theorem problem for real numbers and derived

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a maximum likelihood estimation based robust remainder theorem algorithm. Chen and Ding [19] considered the parameter estimation problems of the multiple-input single-output systems and delivered a filtering based maximum likelihood recursive least squares algorithm by using the data filtering. Söderström and Soverini [20] dealt with the identification problems of the errors-in-variables models and developed a frequency domain maximum likelihood estimator. Chang and Chang [21] extended the maximum likelihood principle to the multiple-input multiple-output systems and presented a novel maximum likelihood detection algorithm which does not need QR decomposition and matrix inversion.

The data filtering technique is an effective way to solve the parameter estimation problem of systems with colored noise [22]–[24]. The main idea is to transform the system into a filtered system and a filtered noise system. The data filtering method in system identification only changes the structure of the system and does not change the relationship between the inputs and outputs [25]–[27]. In this field, Li and Liu [28] extended the filtering technique to the bilinear systems with colored noises and proposed a maximum likelihood least squares based iterative algorithm . Ding *et al.* [29] derived a filtering and auxiliary model based recursive least squares algorithm for the dual-rate state space systems with time-delay based on the data filtering and the auxiliary identification idea. In order to improve the estimation accuracy, Pan *et al.* [30] presented a filtering based multi-innovation extended stochastic gradient algorithm for multivariate moving average systems by using the filtering technique and the multi-innovation identification theory.

For the multivariate equation-error systems with autoregressive noise, Liu *et al.* [31] proposed the maximum likelihood recursive extended least squares algorithm to obtain the parameter estimates . On the basis of the work in [31], this paper considers the parameter estimation problem for the multivariate equation-error systems with autoregressive moving average noise. The main contributions of this paper are as follows.

- A filtering based maximum likelihood recursive generalized extended least squares algorithm is derived for the multivariate equation-error system with autoregressive moving average noise by using the data filtering technique and the maximum likelihood principle.
- The system is broken down into several subsystems. The identification model of the subsystem is obtained by defining the parameter vectors and the information vectors of the subsystem.
- The simulation example proved the effectiveness of the filtering based maximum likelihood recursive generalized extended least squares algorithm.

Briefly, the rest of this paper is organized as follows. The multivariate equation-error autoregressive moving average system is broken down into several subsystems and the identification model of the subsystem is given in Section II. In Sections III and IV, a maximum likelihood recursive generalized extended least squares algorithm and a filtering based maximum likelihood recursive generalized extended least squares algorithm are presented for the multivariate equation-error autoregressive moving average systems. In addition, Section V offers the numerical simulation. Some concluding remarks are given in Section VI.

II. SYSTEM DESCRIPTION

Let us introduce some notation. The symbol $\boldsymbol{I}_m$ is an $m \times m$ identity matrix; 1_n is an n-dimensional column vector whose elements are 1; the superscript T denotes the matrix transpose; z represents unit forward shift operator: $z\boldsymbol{x}(r) = \boldsymbol{x}(r+1)$ and $z^{-1}\boldsymbol{x}(r) = \boldsymbol{x}(r-1)$.

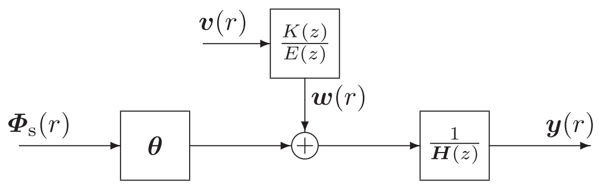

FIGURE 1. The multivariate equation-error autoregressive moving average system.

The multivariate equation-error autoregressive moving average system in Figure 1 can be expressed as follows,

$$\boldsymbol{H}(z)\boldsymbol{y}(r) = \boldsymbol{\Phi}_s(r)\boldsymbol{\theta} + \frac{K(z)}{E(z)}\boldsymbol{v}(r), \tag{1}$$

where $\boldsymbol{\Phi}_s(r) \in \mathbb{R}^{m\times n}$ is the measured information matrix which contains the input-output data, $\boldsymbol{y}(r) := [y_1(r), y_2(r), \cdots, y_m(r)]^{\mathrm{T}} \in \mathbb{R}^m$ is the output vector of the system, $\boldsymbol{v}(r) := [v_1(r), v_2(r), \cdots, v_m(r)]^{\mathrm{T}} \in \mathbb{R}^m$ is a white noise vector with zero mean and variance σ_j^2 for $v_j(r), j = 1, 2, \cdots, m$, $\boldsymbol{\theta} \in \mathbb{R}^n$ is the parameter vector to be identified, and $\boldsymbol{H}(z)$, $E(z)$ and $K(z)$ are polynomials (matrix) in the unit backward shift operator z^{-1} $(z^{-1}y(r) = y(r-1))$:

$$\begin{aligned}
\boldsymbol{H}(z) &:= \boldsymbol{I}_m + \boldsymbol{H}_1 z^{-1} + \boldsymbol{H}_2 z^{-2} + \cdots + \boldsymbol{H}_{n_h} z^{-n_h},\\
E(z) &:= 1 + e_1 z^{-1} + e_2 z^{-2} + \cdots + e_{n_e} z^{-n_e}, \quad e_i \in \mathbb{R},\\
K(z) &:= 1 + k_1 z^{-1} + k_2 z^{-2} + \cdots + k_{n_k} z^{-n_k}, \quad k_i \in \mathbb{R}.
\end{aligned}$$

Let $\boldsymbol{w}(r) := \frac{K(z)}{E(z)}\boldsymbol{v}(r) \in \mathbb{R}^m$ be the intermediate vector. Let $\boldsymbol{\Phi}_s(r) := [\boldsymbol{\phi}_1(r), \boldsymbol{\phi}_2(r), \cdots, \boldsymbol{\phi}_m(r)]^{\mathrm{T}} \in \mathbb{R}^{m\times n}$, where $\boldsymbol{\phi}_j(r) \in \mathbb{R}^n$. Let $\boldsymbol{H}_i := [\boldsymbol{h}_{i1}^{\mathrm{T}}, \boldsymbol{h}_{i2}^{\mathrm{T}}, \cdots, \boldsymbol{h}_{im}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{m\times m}$, where $\boldsymbol{h}_{ij} \in \mathbb{R}^{1\times m}$ is the jth row values of $\boldsymbol{H}_i$. From (1), we have

$$\begin{bmatrix} y_1(r)\\ y_2(r)\\ \vdots\\ y_m(r)\end{bmatrix} + \begin{bmatrix} \boldsymbol{h}_{11}\\ \boldsymbol{h}_{12}\\ \vdots\\ \boldsymbol{h}_{1m}\end{bmatrix} z^{-1}\boldsymbol{y}(r) + \begin{bmatrix} \boldsymbol{h}_{21}\\ \boldsymbol{h}_{22}\\ \vdots\\ \boldsymbol{h}_{2m}\end{bmatrix} z^{-2}\boldsymbol{y}(r) + \cdots + \begin{bmatrix} \boldsymbol{h}_{n_h1}\\ \boldsymbol{h}_{n_h2}\\ \vdots\\ \boldsymbol{h}_{n_hm}\end{bmatrix} z^{-n_h}\boldsymbol{y}(r) = \begin{bmatrix} \boldsymbol{\phi}_1^{\mathrm{T}}(r)\\ \boldsymbol{\phi}_2^{\mathrm{T}}(r)\\ \vdots\\ \boldsymbol{\phi}_m^{\mathrm{T}}(r)\end{bmatrix}\boldsymbol{\theta} + \frac{K(z)}{E(z)}\begin{bmatrix} v_1(r)\\ v_2(r)\\ \vdots\\ v_m(r)\end{bmatrix}.$$

Afterwards, System (1) can be broken down into m subsystems. Subsystem j can be expressed as

$$\boldsymbol{H}_j(z)\boldsymbol{y}(r) = \boldsymbol{\phi}_j^{\mathrm{T}}(r)\boldsymbol{\theta} + \frac{K(z)}{E(z)}v_j(r), \quad j = 1, 2, \cdots, m, \tag{2}$$

where $\boldsymbol{H}_j(z) := \boldsymbol{h}_{0j} + \boldsymbol{h}_{1j}z^{-1} + \cdots + \boldsymbol{h}_{n_hj}z^{-n_h} \in \mathbb{R}^{1\times m}$ is the jth row of $\boldsymbol{H}(z)$.

Then we have

$$v_j(r) = \frac{E(z)}{K(z)}[\boldsymbol{H}_j(z)\boldsymbol{y}(r) - \boldsymbol{\phi}_j^{\mathrm{T}}(r)\boldsymbol{\theta}], \quad j = 1, 2, \cdots, m.$$

Define the subsystem parameter vectors $\boldsymbol{h}_j \in \mathbb{R}^{mn_h}$, $\boldsymbol{e} \in \mathbb{R}^{n_e}$ and $\boldsymbol{k} \in \mathbb{R}^{n_k}$ as

$$\boldsymbol{h}_j := \begin{bmatrix} \boldsymbol{h}_{1j} \\ \boldsymbol{h}_{2j} \\ \vdots \\ \boldsymbol{h}_{n_hj} \end{bmatrix}, \quad \boldsymbol{e} := \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_{n_e} \end{bmatrix}, \quad \boldsymbol{k} := \begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{n_k} \end{bmatrix}.$$

Define the subsystem parameter vectors and the information vectors as

$$\begin{aligned}
\boldsymbol{\eta}_j &:= [\boldsymbol{\theta}^{\mathrm{T}}, \boldsymbol{h}_j^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n+mn_h}, \\
\boldsymbol{\vartheta}_j &:= [\boldsymbol{\eta}_j^{\mathrm{T}}, \boldsymbol{e}^{\mathrm{T}}, \boldsymbol{k}^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{n_0}, \quad n_0 := n + mn_h + n_e + n_k, \\
\boldsymbol{\psi}_j(r) &:= [\boldsymbol{\phi}_j^{\mathrm{T}}(r), -\boldsymbol{y}^{\mathrm{T}}(r-1), -\boldsymbol{y}^{\mathrm{T}}(r-2), \cdots, \\
&\qquad -\boldsymbol{y}^{\mathrm{T}}(r-n_h)]^{\mathrm{T}} \in \mathbb{R}^{n+mn_h}, \\
\boldsymbol{\varphi}_j(r) &:= [\boldsymbol{\psi}_j^{\mathrm{T}}(r), -w_j(r-1), -w_j(r-2), \cdots, -w_j(r-n_e), \\
&\qquad v_j(r-1), v_j(r-2), \cdots, v_j(r-n_k)]^{\mathrm{T}} \in \mathbb{R}^{n_0}.
\end{aligned}$$

From (2), we can get

$$y_j(r) = \boldsymbol{\varphi}_j^{\mathrm{T}}(r)\boldsymbol{\vartheta}_j + v_j(r), \tag{3}$$

$$w_j(r) = y_j(r) - \boldsymbol{\psi}_j^{\mathrm{T}}(r)\boldsymbol{\eta}_j, \quad j = 1, 2, \cdots, m. \tag{4}$$

Remark 1: The system is broken down into m subsystems based on the number of the system outputs. The identification model of Subsystem j is shown in Equations (3) and (4).

Remark 2: The objective of this paper is to present maximum likelihood recursive identification algorithms to estimate the unknown parameter vectors $\boldsymbol{\theta}$, $\boldsymbol{e}$ and $\boldsymbol{k}$ and the unknown parameter matrix $\boldsymbol{H}_i$.

III. THE MAXIMUM LIKELIHOOD RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

According to the maximum likelihood principle, the maximum likelihood criterion function is obtained as follows [32],

$$J_j(\boldsymbol{\vartheta}_j)\Big|_{\hat{\boldsymbol{\vartheta}}_{jML}} = \frac{1}{2}\sum_{r=1}^{N} v_j^2(r)\Big|_{\hat{\boldsymbol{\vartheta}}_{jML}} = \min, \tag{5}$$

subject to

$$v_j(r) = \frac{E(z)}{K(z)}[\boldsymbol{H}_j(z)\boldsymbol{y}(r) - \boldsymbol{\phi}_j^{\mathrm{T}}(r)\boldsymbol{\theta}].$$

The research [31] derived the maximum likelihood recursive extended least squares algorithm for the multivariate equation-error systems with autoregressive noise. Referring to the method in [31], minimizing the cost function $J_j(\boldsymbol{\vartheta}_j)$, we can obtain the maximum likelihood recursive generalized extended least squares (ML-RGELS) algorithm:

$$\hat{\boldsymbol{\vartheta}}_j(r) = \hat{\boldsymbol{\vartheta}}_j(r-1) + \boldsymbol{L}_j(r)[y_j(r) - \hat{\boldsymbol{\varphi}}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\vartheta}}_j(r-1)], \tag{6}$$

$$\boldsymbol{L}_j(r) = \boldsymbol{P}_j(r-1)\hat{\boldsymbol{\varphi}}_{jf}(r)[1 + \hat{\boldsymbol{\varphi}}_{jf}^{\mathrm{T}}(r)\boldsymbol{P}_j(r-1)\hat{\boldsymbol{\varphi}}_{jf}(r)]^{-1}, \tag{7}$$

$$\boldsymbol{P}_j(r) = [\boldsymbol{I} - \boldsymbol{L}_j(r)\hat{\boldsymbol{\varphi}}_{jf}^{\mathrm{T}}(r)]\boldsymbol{P}_j(r-1), \tag{8}$$

$$\begin{aligned}
\hat{\boldsymbol{\varphi}}_{jf}(r) = [&\hat{\boldsymbol{\phi}}_{jf}^{\mathrm{T}}(r), -\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-1), -\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-2), \cdots, \\
&-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-n_h), -\hat{w}_{jf}(r-1), \\
&-\hat{w}_{jf}(r-2), \cdots, -\hat{w}_{jf}(r-n_e), \\
&\hat{v}_{jf}(r-1), \hat{v}_{jf}(r-2), \cdots, \hat{v}_{jf}(r-n_k)]^{\mathrm{T}},
\end{aligned} \tag{9}$$

$$\begin{aligned}
\boldsymbol{\psi}_j(r) = [&\boldsymbol{\phi}_j^{\mathrm{T}}(r), -\boldsymbol{y}^{\mathrm{T}}(r-1), -\boldsymbol{y}^{\mathrm{T}}(r-2), \cdots, \\
&-\boldsymbol{y}^{\mathrm{T}}(r-n_h)]^{\mathrm{T}},
\end{aligned} \tag{10}$$

$$\begin{aligned}
\hat{\boldsymbol{\varphi}}_j(r) = [&\boldsymbol{\psi}_j^{\mathrm{T}}(r), -\hat{w}_j(r-1), -\hat{w}_j(r-2), \cdots, \\
&-\hat{w}_j(r-n_e), \hat{v}_j(r-1), \\
&\hat{v}_j(r-2), \cdots, \hat{v}_j(r-n_k)]^{\mathrm{T}},
\end{aligned} \tag{11}$$

$$\hat{\boldsymbol{\phi}}_{jf}(r) = \boldsymbol{\phi}_j(r) + \sum_{i=1}^{n_e}\hat{e}_i(r-1)\boldsymbol{\phi}_j(r-i) - \sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{\boldsymbol{\phi}}_{jf}(r-i), \tag{12}$$

$$\hat{\boldsymbol{y}}_f(r) = \boldsymbol{y}(r) + \sum_{i=1}^{n_e}\hat{e}_i(r)\boldsymbol{y}(r-i) - \sum_{i=1}^{n_k}\hat{k}_i(r)\hat{\boldsymbol{y}}_f(r-i), \tag{13}$$

$$\hat{w}_{jf}(r) = y_j(r) + \sum_{i=1}^{n_h}\hat{\boldsymbol{h}}_{ij}(r)\boldsymbol{y}(r-i) - \boldsymbol{\phi}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\theta}}(r) - \sum_{i=1}^{n_k}\hat{k}_i(r)\hat{w}_{jf}(r-i), \tag{14}$$

$$\hat{v}_{jf}(r) = v_j(r) - \sum_{i=1}^{n_k}\hat{k}_i(r)\hat{v}_{jf}(r-i), \tag{15}$$

$$\hat{\boldsymbol{\vartheta}}_j(r) = [\hat{\boldsymbol{\eta}}_j^{\mathrm{T}}(r), \hat{\boldsymbol{e}}_j^{\mathrm{T}}(r), \hat{\boldsymbol{k}}_j^{\mathrm{T}}(r)]^{\mathrm{T}}, \tag{16}$$

$$\hat{w}_j(r) = y_j(r) - \boldsymbol{\psi}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\eta}}_j(r), \tag{17}$$

$$\hat{v}_j(r) = y_j(r) - \hat{\boldsymbol{\varphi}}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\vartheta}}_j(r), \tag{18}$$

$$\hat{\boldsymbol{\theta}}(r) = \frac{\hat{\boldsymbol{\theta}}_1(r) + \hat{\boldsymbol{\theta}}_2(r) + \cdots + \hat{\boldsymbol{\theta}}_m(r)}{m}, \tag{19}$$

$$\hat{\boldsymbol{e}}(r) = \frac{\hat{\boldsymbol{e}}_1(r) + \hat{\boldsymbol{e}}_2(r) + \cdots + \hat{\boldsymbol{e}}_m(r)}{m}, \tag{20}$$

$$\hat{\boldsymbol{k}}(r) = \frac{\hat{\boldsymbol{k}}_1(r) + \hat{\boldsymbol{k}}_2(r) + \cdots + \hat{\boldsymbol{k}}_m(r)}{m}, \tag{21}$$

$$\hat{\boldsymbol{\vartheta}}(r) = [\hat{\boldsymbol{\theta}}^{\mathrm{T}}(r), \hat{\boldsymbol{h}}_1^{\mathrm{T}}(r), \hat{\boldsymbol{h}}_2^{\mathrm{T}}(r), \cdots, \hat{\boldsymbol{h}}_m^{\mathrm{T}}(r), \hat{\boldsymbol{e}}^{\mathrm{T}}(r), \hat{\boldsymbol{k}}^{\mathrm{T}}(r)]^{\mathrm{T}}. \tag{22}$$

The flowchart of the ML-RGELS algorithm for computing $\hat{\boldsymbol{\vartheta}}(r)$ is shown in Figure 2.

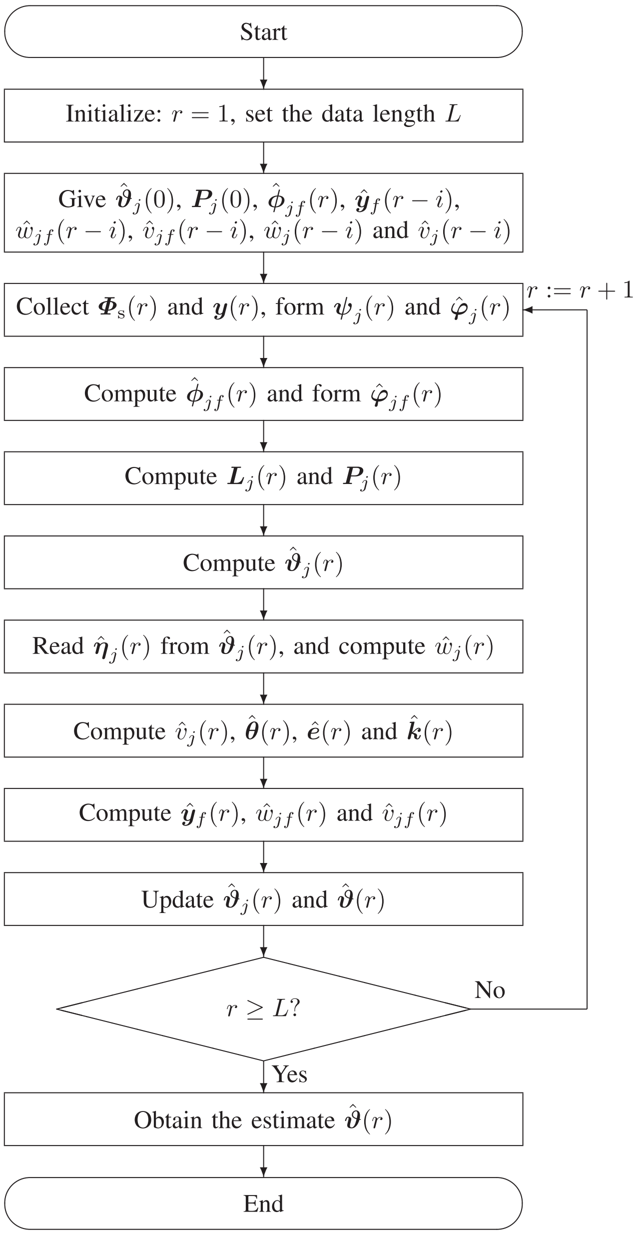

FIGURE 2. The flowchart of the ML-RGELS algorithm for computing $\hat{\boldsymbol{\vartheta}}(r)$.

IV. THE FILTERING BASED MAXIMUM LIKELIHOOD RECURSIVE GENERALIZED EXTENDED LEAST SQUARES ALGORITHM

The data filtering technology is an effective method to solve the parameter estimation problems of the colored noise systems. A filtering based maximum likelihood recursive generalized extended least squares algorithm is derived in this section. Through the data filtering, the subsystem can be transformed into a controlled autoregressive moving average model and a noise model.

Define

$$\boldsymbol{\phi}_{1j}(r) := E(z)\boldsymbol{\phi}_j^{\rm T}(r) \in \mathbb{R}^n,$$
$$\boldsymbol{y}_1(r) := E(z)\boldsymbol{y}(r) \in \mathbb{R}^m.$$

Multiplying both sides of Equation (2) by $E(z)$ yields

$$\boldsymbol{H}_j(z)E(z)\boldsymbol{y}(r) = E(z)\boldsymbol{\phi}_j^{\rm T}(r)\boldsymbol{\theta} + K(z)v_j(r). \tag{23}$$

Then Equation (23) can be written as

$$\boldsymbol{H}_j(z)\boldsymbol{y}_1(r) = \boldsymbol{\phi}_{1j}(r)\boldsymbol{\theta} + K(z)v_j(r). \tag{24}$$

This is the controlled autoregressive moving average model. Define the parameter vector $\boldsymbol{\vartheta}_{1j}$ and the information vector $\boldsymbol{\varphi}_{1j}(r)$:

$$\begin{aligned}
\boldsymbol{\vartheta}_{1j} &:= [\boldsymbol{\theta}^{\rm T}, \boldsymbol{h}_{1j}, \boldsymbol{h}_{2j}, \cdots, \boldsymbol{h}_{n_hj}, k_1, k_2, \cdots, k_{n_k}]^{\rm T} \in \mathbb{R}^{n_1},\\
n_1 &:= n + mn_h + n_k,\\
\boldsymbol{\varphi}_{1j}(r) &:= [\boldsymbol{\phi}_{1j}^{\rm T}(r), -\boldsymbol{y}_1^{\rm T}(r-1), -\boldsymbol{y}_1^{\rm T}(r-2), \cdots,\\
&\quad -\boldsymbol{y}_1^{\rm T}(r-n_h), v_j(r-1),\\
&\quad v_j(r-2), \cdots, v_j(r-n_k)]^{\rm T} \in \mathbb{R}^{n_1}.
\end{aligned}$$

Then we can get the identification model of this model:

$$y_{1j}(r) = \boldsymbol{\varphi}_{1j}^{\rm T}(r)\boldsymbol{\vartheta}_{1j} + v_j(r). \tag{25}$$

Define the intermediate variable

$$w_j(r) := \frac{K(z)}{E(z)}v_j(r). \tag{26}$$

Define the information vectors $\boldsymbol{\varphi}_{ej}(r)$ and $\boldsymbol{\varphi}_{kj}(r)$:

$$\begin{aligned}
\boldsymbol{\varphi}_{ej}(r) &:= [-w_j(r-1), -w_j(r-2), \cdots,\\
&\quad -w_j(r-n_e)]^{\rm T} \in \mathbb{R}^{n_e},\\
\boldsymbol{\varphi}_{kj}(r) &:= [v_j(r-1), v_j(r-2), \cdots, v_j(r-n_k)]^{\rm T} \in \mathbb{R}^{n_k}.
\end{aligned}$$

Then Equation (26) can be written as

$$\begin{aligned}
w_j(r) &= [1-E(z)]w_j(r) + K(z)v_j(r)\\
&= \boldsymbol{\varphi}_{ej}^{\rm T}(r)\boldsymbol{e} + \boldsymbol{\varphi}_{kj}^{\rm T}(r)\boldsymbol{k} + v_j(r).
\end{aligned} \tag{27}$$

Define and minimize the criterion function

$$J_1(\boldsymbol{\vartheta}_{1j}) := \frac{1}{2}\sum_{r=1}^{N} v_j^2(r)\Big|_{\hat{\boldsymbol{\vartheta}}_{1jML}} = \min,$$

subject to

$$v_j(r) = \frac{1}{K(z)}[\boldsymbol{H}_j(z)\boldsymbol{y}_1(r) - \boldsymbol{\phi}_{1j}(r)\boldsymbol{\theta}]. \tag{28}$$

Let $\hat{\boldsymbol{h}}_j(r)$, $\hat{\boldsymbol{e}}(r)$, $\hat{\boldsymbol{k}}(r)$ and $\hat{\boldsymbol{\vartheta}}_{1j}(r)$ be the estimates of $\boldsymbol{h}_j$, $\boldsymbol{e}$, $\boldsymbol{k}$ and $\boldsymbol{\vartheta}_{1j}$ at time r, respectively,

$$\begin{aligned}
\hat{\boldsymbol{h}}_j(r) &:= [\hat{\boldsymbol{h}}_{1j}(r), \hat{\boldsymbol{h}}_{2j}(r), \cdots, \hat{\boldsymbol{h}}_{n_hj}(r)]^{\rm T},\\
\hat{\boldsymbol{e}}(r) &:= [\hat{e}_1(r), \hat{e}_2(r), \cdots, \hat{e}_{n_e}(r)]^{\rm T},\\
\hat{\boldsymbol{k}}(r) &:= [\hat{k}_1(r), \hat{k}_2(r), \cdots, \hat{k}_{n_k}(r)]^{\rm T},\\
\hat{\boldsymbol{\vartheta}}_{1j}(r) &:= [\hat{\boldsymbol{\theta}}^{\rm T}(r), \hat{\boldsymbol{h}}_j^{\rm T}(r), \hat{\boldsymbol{k}}^{\rm T}(r)]^{\rm T}.
\end{aligned}$$

Then the estimates of $E(z)$ and $F(z)$ at time r can be written as

$$\begin{aligned}
\hat{E}(r,z) &:= 1 + \hat{e}_1(r)z^{-1} + \hat{e}_2(r)z^{-2} + \cdots + \hat{e}_{n_e}(r)z^{-n_e},\\
\hat{K}(r,z) &:= 1 + \hat{k}_1(r)z^{-1} + \hat{k}_2(r)z^{-2} + \cdots + \hat{k}_{n_k}(r)z^{-n_k}.
\end{aligned}$$

Furthermore, the estimates $\hat{\boldsymbol{\phi}}_{1j}(r)$ and $\hat{\boldsymbol{y}}_1(r)$ of $\boldsymbol{\phi}_{1j}(r)$ and $\boldsymbol{y}_1(r)$ can be obtained by

$$\begin{aligned}\hat{\boldsymbol{\phi}}_{1j}(r) &= \hat{E}(r-1,z)\boldsymbol{\phi}_j^{\mathrm{T}}(r)\\ &= \boldsymbol{\phi}_j^{\mathrm{T}}(r)+\hat{e}_1(r-1)\boldsymbol{\phi}_j^{\mathrm{T}}(r-1)+\cdots\\ &\quad+\hat{e}_{n_e}(r-1)\boldsymbol{\phi}_j^{\mathrm{T}}(r-n_e),\\ \hat{\boldsymbol{y}}_1(r) &= \hat{E}(r-1,z)\boldsymbol{y}(r)\\ &= \boldsymbol{y}(r)+\hat{e}_1(r-1)\boldsymbol{y}(r-1)+\cdots\\ &\quad+\hat{e}_{n_e}(r-1)\boldsymbol{y}(r-n_e).\end{aligned}$$

Define the filtered information vector

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_{1jf}(r) &:= -\left.\frac{\partial v_j(r)}{\partial \boldsymbol{\vartheta}_{1j}}\right|_{\hat{\boldsymbol{\vartheta}}_{1j}(r-1)}\\ &= -\Big[\frac{\partial v_j(r)}{\partial \boldsymbol{\theta}},\frac{\partial v_j(r)}{\partial \boldsymbol{h}_{1j}},\frac{\partial v_j(r)}{\partial \boldsymbol{h}_{2j}},\cdots,\frac{\partial v_j(r)}{\partial \boldsymbol{h}_{n_hj}},\\ &\qquad \frac{\partial v_j(r)}{\partial k_1},\frac{\partial v_j(r)}{\partial k_2},\cdots,\frac{\partial v_j(r)}{\partial k_{n_k}}\Big]^{\mathrm{T}}_{\hat{\boldsymbol{\vartheta}}_{1j}(r-1)}. \qquad (29)\end{aligned}$$

From (28), the filtering values are calculated as follows:

$$\begin{aligned}\left.\frac{\partial v_j(r)}{\partial \boldsymbol{\theta}}\right|_{\hat{\boldsymbol{\vartheta}}_j(r-1)} &= -[\hat{K}(r-1,z)]^{-1}\hat{\boldsymbol{\phi}}_{1j}(r) = -\hat{\boldsymbol{\phi}}_{jf}(r),\\ \left.\frac{\partial v_j(r)}{\partial \boldsymbol{h}_{ij}}\right|_{\hat{\boldsymbol{\vartheta}}_j(r-1)} &= [\hat{K}(r-1,z)]^{-1}\hat{\boldsymbol{y}}_1(r-i) = z^{-i}\hat{\boldsymbol{y}}_f(r),\\ \left.\frac{\partial v_j(r)}{\partial k_i}\right|_{\hat{\boldsymbol{\vartheta}}_j(r-1)} &= -[\hat{K}(r-1,z)]^{-1}\hat{v}_j(r-i) = -z^{-i}\hat{v}_{jf}(r),\end{aligned}$$

where $\hat{\boldsymbol{\phi}}_{jf}(r)$, $\hat{\boldsymbol{y}}_f(r)$ and $\hat{v}_{jf}(r)$ can be computed by

$$\begin{aligned}\hat{\boldsymbol{\phi}}_{jf}(r) &:= [\hat{K}(r-1,z)]^{-1}\hat{\boldsymbol{\phi}}_{1j}(r)\\ &= \hat{\boldsymbol{\phi}}_{1j}(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{\boldsymbol{\phi}}_{jf}(r-i),\\ \hat{\boldsymbol{y}}_f(r) &:= [\hat{K}(r-1,z)]^{-1}\hat{\boldsymbol{y}}_1(r)\\ &= \hat{\boldsymbol{y}}_1(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{\boldsymbol{y}}_f(r-i),\\ \hat{v}_{jf}(r) &:= [\hat{K}(r-1,z)]^{-1}\hat{v}_j(r)\\ &= \hat{v}_j(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{v}_{jf}(r-i).\end{aligned}$$

Then the filtered information vector $\hat{\boldsymbol{\varphi}}_{jf}(r)$ can be rewritten as

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_{jf}(r) = [&\hat{\boldsymbol{\phi}}_{jf}^{\mathrm{T}}(r),-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-1),-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-2),\cdots,-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-n_h),\\ &\hat{v}_{jf}(r-1),\hat{v}_{jf}(r-2),\cdots,\hat{v}_{jf}(r-n_k)]^{\mathrm{T}}.\end{aligned}$$

From (25), we have $v_j(r) = y_{1j}(r)-\boldsymbol{\varphi}_{1j}^{\mathrm{T}}(r)\boldsymbol{\vartheta}_{1j}$. Replacing $y_{1j}(r)$, $\boldsymbol{\varphi}_{1j}(r)$ and $\boldsymbol{\vartheta}_{1j}$ with $\hat{y}_{1j}(r)$, $\hat{\boldsymbol{\varphi}}_{1j}(r)$ and $\hat{\boldsymbol{\vartheta}}_{1j}(r)$, the estimate $\hat{v}_j(r)$ of $v_j(r)$ can be calculated as follows:

$$\begin{aligned}\hat{v}_j(r) &= \hat{y}_{1j}(r)-\hat{\boldsymbol{\varphi}}_{1j}^{\mathrm{T}}(r)\hat{\boldsymbol{\vartheta}}_{1j}(r),\\ \hat{\boldsymbol{\varphi}}_{1j}(r) &= [\hat{\boldsymbol{\phi}}_{1j}^{\mathrm{T}}(r),-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-1),-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-2),\cdots,-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-n_h),\\ &\quad \hat{v}_j(r-1),\hat{v}_j(r-2),\cdots,\hat{v}_j(r-n_k)]^{\mathrm{T}}.\end{aligned}$$

Minimizing the cost function $J_1(\boldsymbol{\vartheta}_{1j})$, then the filtering based maximum likelihood recursive generalized extended least squares algorithm for estimating $\boldsymbol{\vartheta}_{1j}$ can be summarized as follows:

$$\hat{\boldsymbol{\vartheta}}_{1j}(r) = \hat{\boldsymbol{\vartheta}}_{1j}(r-1)+\boldsymbol{L}_{1j}(r)[\hat{y}_{1j}(r)-\hat{\boldsymbol{\varphi}}_{1j}^{\mathrm{T}}(r)\hat{\boldsymbol{\vartheta}}_{1j}(r-1)], \qquad (30)$$

$$\boldsymbol{L}_{1j}(r) = \frac{\boldsymbol{P}_{1j}(r-1)\hat{\boldsymbol{\varphi}}_{1jf}(r)}{1+\hat{\boldsymbol{\varphi}}_{1jf}^{\mathrm{T}}(r)\boldsymbol{P}_{1j}(r-1)\hat{\boldsymbol{\varphi}}_{1jf}(r)}, \qquad (31)$$

$$\boldsymbol{P}_{1j}(r) = [\boldsymbol{I}-\boldsymbol{L}_{1j}(r)\hat{\boldsymbol{\varphi}}_{1jf}^{\mathrm{T}}(r)]\boldsymbol{P}_{1j}(r-1), \qquad (32)$$

$$\hat{\boldsymbol{\theta}}(r) = \frac{1}{m}[\hat{\boldsymbol{\theta}}_1(r)+\hat{\boldsymbol{\theta}}_2(r)+\cdots+\hat{\boldsymbol{\theta}}_m(r)], \qquad (33)$$

$$\hat{\boldsymbol{k}}(r) = \frac{1}{m}[\hat{\boldsymbol{k}}_1(r)+\hat{\boldsymbol{k}}_2(r)+\cdots+\hat{\boldsymbol{k}}_m(r)], \qquad (34)$$

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_{1jf}(r) = [&\hat{\boldsymbol{\phi}}_{jf}^{\mathrm{T}}(r),-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-1),-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-2),\cdots,\\ &-\hat{\boldsymbol{y}}_f^{\mathrm{T}}(r-n_h),\hat{v}_{jf}(r-1),\\ &\hat{v}_{jf}(r-2),\cdots,\hat{v}_{jf}(r-n_k)]^{\mathrm{T}}, \qquad (35)\end{aligned}$$

$$\begin{aligned}\hat{\boldsymbol{\varphi}}_{1j}(r) = [&\hat{\boldsymbol{\phi}}_{1j}^{\mathrm{T}}(r),-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-1),-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-2),\cdots,\\ &-\hat{\boldsymbol{y}}_1^{\mathrm{T}}(r-n_h),\hat{v}_j(r-1),\\ &\hat{v}_j(r-2),\cdots,\hat{v}_j(r-n_k)], \qquad (36)\end{aligned}$$

$$\hat{\boldsymbol{\phi}}_{jf}(r) = \hat{\boldsymbol{\phi}}_{1j}(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{\boldsymbol{\phi}}_{jf}(r-i), \qquad (37)$$

$$\hat{\boldsymbol{y}}_f(r) = \hat{\boldsymbol{y}}_1(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{\boldsymbol{y}}_f(r-i), \qquad (38)$$

$$\hat{v}_{jf}(r) = \hat{v}_j(r)-\sum_{i=1}^{n_k}\hat{k}_i(r-1)\hat{v}_{jf}(r-i), \qquad (39)$$

$$\hat{\boldsymbol{\phi}}_{1j}(r) = \boldsymbol{\phi}_j^{\mathrm{T}}(r)+\sum_{i=1}^{n_e}\hat{e}_i(r-1)\boldsymbol{\phi}_j^{\mathrm{T}}(r-i), \qquad (40)$$

$$\hat{\boldsymbol{y}}_1(r) = \boldsymbol{y}(r)+\sum_{i=1}^{n_e}\hat{e}_i(r-1)\boldsymbol{y}(r-i), \qquad (41)$$

$$\hat{v}_j(r) = \hat{y}_{1j}(r)-\hat{\boldsymbol{\varphi}}_{1j}^{\mathrm{T}}(r)\hat{\boldsymbol{\vartheta}}_{1j}(r), \qquad (42)$$

$$\hat{\boldsymbol{\vartheta}}_{1j}(r) = [\hat{\boldsymbol{\eta}}_j(r),\hat{\boldsymbol{k}}^{\mathrm{T}}(r)]^{\mathrm{T}}. \qquad (43)$$

For the identification model in (27), minimizing the cost function

$$\boldsymbol{J}_1(\boldsymbol{e}) := \sum_{r=1}^{N}[w_j(r)-\boldsymbol{\varphi}_{ej}^{\mathrm{T}}(r)\boldsymbol{e}-\boldsymbol{\varphi}_{kj}^{\mathrm{T}}(r)\boldsymbol{k}]^2,$$

we can obtain the following recursive least squares relation to estimate the parameter vector $\boldsymbol{e}$:

$$\begin{aligned}\hat{\boldsymbol{e}}_j(r) &= \hat{\boldsymbol{e}}_j(r-1)\\ &\quad+\boldsymbol{L}_{ej}(r)[\hat{w}_j(r)-\boldsymbol{\varphi}_{ej}^{\mathrm{T}}(r)\hat{\boldsymbol{e}}(r-1)-\hat{\boldsymbol{\varphi}}_{kj}^{\mathrm{T}}(r)\hat{\boldsymbol{k}}(r)],\\ \boldsymbol{L}_{ej}(r) &= \boldsymbol{P}_{ej}(r-1)\boldsymbol{\varphi}_{ej}(r)[1+\boldsymbol{\varphi}_{ej}^{\mathrm{T}}(r)\boldsymbol{P}_{ej}(r-1)\boldsymbol{\varphi}_{ej}(r)]^{-1},\\ \boldsymbol{P}_{ej}(r) &= [\boldsymbol{I}-\boldsymbol{L}_{ej}(r)\boldsymbol{\varphi}_{ej}^{\mathrm{T}}(r)]\boldsymbol{P}_{ej}(r-1), \quad \boldsymbol{P}_{ej}(0)=p_0\boldsymbol{I}_{n_e}.\end{aligned}$$

Let the estimate of $\boldsymbol{\varphi}_{ej}(r)$ be:

$$\hat{\boldsymbol{\varphi}}_{ej}(r) := [-\hat{w}_j(r-1), -\hat{w}_j(r-2), \cdots, -\hat{w}_j(r-n_e)]^{\mathrm{T}} \in \mathbb{R}^{n_e}.$$

From (4), we have $w_j(r) = y_j(r) - \boldsymbol{\psi}_j^{\mathrm{T}}(r)\boldsymbol{\eta}_j$, replacing $\boldsymbol{\eta}_j$ with its estimate $\hat{\boldsymbol{\eta}}_j(r)$, the estimate $\hat{w}_j(r)$ of $w_j(r)$ can be calculated as follows:

$$\hat{w}_j(r) = y_j(r) - \boldsymbol{\psi}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\eta}}_j(r). \tag{44}$$

Therefore, the recursive generalized extended least squares algorithm for estimating the parameter vector $\boldsymbol{e}$ is summarized as follows:

$$\hat{\boldsymbol{e}}_j(r) = \hat{\boldsymbol{e}}_j(r-1) + \boldsymbol{L}_{ej}(r) \times [\hat{w}_j(r) - \hat{\boldsymbol{\varphi}}_{ej}^{\mathrm{T}}(r)\hat{\boldsymbol{e}}(r-1) - \hat{\boldsymbol{\varphi}}_{kj}^{\mathrm{T}}(r)\hat{\boldsymbol{k}}(r)], \tag{45}$$

$$\boldsymbol{L}_{ej}(r) = \boldsymbol{P}_{ej}(r-1)\hat{\boldsymbol{\varphi}}_{ej}(r)[1 + \hat{\boldsymbol{\varphi}}_{ej}^{\mathrm{T}}(r)\boldsymbol{P}_{ej}(r-1)\hat{\boldsymbol{\varphi}}_{ej}(r)]^{-1}, \tag{46}$$

$$\boldsymbol{P}_{ej}(r) = [\boldsymbol{I} - \boldsymbol{L}_{ej}(r)\hat{\boldsymbol{\varphi}}_{ej}^{\mathrm{T}}(r)]\boldsymbol{P}_{ej}(r-1), \tag{47}$$

$$\hat{\boldsymbol{e}}(r) = \frac{1}{m}[\hat{\boldsymbol{e}}_1(r) + \hat{\boldsymbol{e}}_2(r) + \cdots + \hat{\boldsymbol{e}}_m(r)], \tag{48}$$

$$\hat{\boldsymbol{\varphi}}_{ej}(r) = [-\hat{w}_j(r-1), -\hat{w}_j(r-2), \cdots, -\hat{w}_j(r-n_e)]^{\mathrm{T}}, \tag{49}$$

$$\hat{\boldsymbol{\varphi}}_{kj}(r) = [\hat{v}_j(r-1), \hat{v}_j(r-2), \cdots, \hat{v}_j(r-n_k)]^{\mathrm{T}}, \tag{50}$$

$$\hat{w}_j(r) = y_j(r) - \boldsymbol{\psi}_j^{\mathrm{T}}(r)\hat{\boldsymbol{\eta}}_j(r), \tag{51}$$

$$\boldsymbol{\psi}_j(r) = [\boldsymbol{\phi}_j^{\mathrm{T}}(r), -\boldsymbol{y}^{\mathrm{T}}(r-1), -\boldsymbol{y}^{\mathrm{T}}(r-2), \cdots, -\boldsymbol{y}^{\mathrm{T}}(r-n_h)]^{\mathrm{T}}, \tag{52}$$

$$\hat{\boldsymbol{\vartheta}}(r) = [\hat{\boldsymbol{\theta}}^{\mathrm{T}}(r), \hat{\boldsymbol{h}}_1^{\mathrm{T}}(r), \hat{\boldsymbol{h}}_2^{\mathrm{T}}(r), \cdots, \hat{\boldsymbol{h}}_m^{\mathrm{T}}(r), \hat{\boldsymbol{e}}^{\mathrm{T}}(r), \hat{\boldsymbol{k}}^{\mathrm{T}}(r)]^{\mathrm{T}}. \tag{53}$$

Equations (30)–(43) and (45)–(53) make up the filtering based maximum likelihood recursive generalized extended least squares (F-ML-RGELS) algorithm. The computation process of the F-ML-RGELS algorithm is summarized as follows:

1) To initialize. Let $r = 1$, and give the initial values $\hat{\boldsymbol{\vartheta}}_{1j}(0) = \boldsymbol{1}_{n_1}/p_0$, $\hat{\boldsymbol{e}}_j(0) = \boldsymbol{1}_{n_e}/p_0$, $\hat{\boldsymbol{\phi}}_{jf}(r) = \boldsymbol{1}_n/p_0$, $\hat{\boldsymbol{y}}_f(r-i) = \boldsymbol{1}_m/p_0$, $\hat{v}_{jf}(r-i) = 1/p_0$, $\hat{v}_j(r-i) = 1/p_0$, $\hat{w}_j(r-i) = 1/p_0$, $\boldsymbol{P}_{1j}(0) = p_0\boldsymbol{I}_{n_1}$, $\boldsymbol{P}_{ej}(0) = p_0\boldsymbol{I}_{n_e}$, $i = 1, 2, \cdots, \max[n_h, n_e, n_k]$, $p_0 = 10^6$, $j = 1, 2, \cdots, m$. Set the data length L.
2) Gather the observation data $\boldsymbol{y}(r)$ and $\boldsymbol{\Phi}_s(r)$, compute the filtered information vector $\hat{\boldsymbol{\phi}}_{1j}(r)$ and the filtered output vector $\hat{\boldsymbol{y}}_1(r)$ by (40) and (41), and construct the information vectors $\hat{\boldsymbol{\varphi}}_{1j}(r)$ and $\boldsymbol{\psi}_j(r)$ by (36) and (52).
3) Calculate the filtered vector $\hat{\boldsymbol{\phi}}_{jf}(r)$ using (37) and construct the filtered information vector $\hat{\boldsymbol{\varphi}}_{1jf}(r)$ using(35).
4) Calculate the gain vector $\boldsymbol{L}_{1j}(r)$ and the covariance matrix $\boldsymbol{P}_{1j}(r)$ using (31) and (32).
5) Calculate the parameter estimate $\hat{\boldsymbol{\vartheta}}_{1j}(r)$ using (30), $(j = 1, 2, \cdots, m)$.
6) Read $\hat{\boldsymbol{\eta}}_j(r)$ from $\hat{\boldsymbol{\vartheta}}_j(r)$ in (43) and calculate $\hat{w}_j(r)$ and $\hat{v}_j(r)$ using (51) and (42).
7) Compute the information vectors $\hat{\boldsymbol{\varphi}}_{ej}(r)$ and $\hat{\boldsymbol{\varphi}}_{kj}(r)$ by (49) and (50).
8) Compute the gain vector $\boldsymbol{L}_{ej}(r)$ by (46) and the covariance matrix $\boldsymbol{P}_{ej}(r)$ by (47).
9) Update the parameter estimate $\hat{\boldsymbol{e}}_j(r)$ by (45).
10) Compute the filtered output vector $\hat{\boldsymbol{y}}_f(r)$ and $\hat{v}_{jf}(r)$ by (38) and (39).
11) Compute the parameter estimates $\hat{\boldsymbol{\theta}}(r)$, $\hat{\boldsymbol{k}}(r)$ and $\hat{\boldsymbol{e}}(r)$ by (33), (34) and (48).
12) Update the parameter estimate $\hat{\boldsymbol{\vartheta}}(r)$ by (53).
13) Increase r by 1 and go to Step 2.

Remark 1: The system is broken down into several subsystems based on the number of the system outputs. In this way, the maximum likelihood principle can be used more easily in parameter identification.

Remark 2: The data filtering method in system identification only changes the structure of the system and does not change the relationship between the inputs and outputs. The subsystem to be identified is transformed into a controlled autoregressive moving average model and a noise model.

Remark 3: Compared with the ML-RGELS algorithm, the introduction of the data filtering technique improves the parameter estimation accuracy of the F-ML-RGELS algorithm.

V. EXAMPLE

Consider the following multivariate equation-error autoregressive moving average model:

$$\begin{aligned}
\boldsymbol{H}(z)\boldsymbol{y}(r) &= \boldsymbol{\Phi}_s(r)\boldsymbol{\theta} + \frac{K(z)}{E(z)}\boldsymbol{v}(r),\\
\boldsymbol{\theta} &= [\theta_1, \theta_2]^{\mathrm{T}} = [-0.85, 0.61]^{\mathrm{T}},\\
E(z) &= 1 + e_1 z^{-1} = 1 + 0.01 z^{-1},\\
K(z) &= 1 + k_1 z^{-1} = 1 - 0.26 z^{-1},\\
\boldsymbol{H}(z) &= \boldsymbol{I} + \boldsymbol{H}_1 z^{-1}\\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} z^{-1}\\
&= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.23 & 0.50 \\ -0.70 & 0.53 \end{bmatrix} z^{-1},\\
\boldsymbol{\vartheta}_1 &= [\theta_1, \theta_2, h_{11}, h_{12}, e_1, k_1]^{\mathrm{T}}\\
&= [-0.85, 0.61, 0.23, 0.50, 0.01, -0.26]^{\mathrm{T}},\\
\boldsymbol{\vartheta}_2 &= [\theta_1, \theta_2, h_{21}, h_{22}, e_1, k_1]^{\mathrm{T}}\\
&= [-0.85, 0.61, -0.70, 0.53, 0.01, -0.26]^{\mathrm{T}},\\
\boldsymbol{\vartheta} &= [\theta_1, \theta_2, h_{11}, h_{12}, h_{21}, h_{22}, e_1, k_1]^{\mathrm{T}}\\
&= [-0.85, 0.61, 0.23, 0.50, -0.70, 0.53, 0.01, -0.26]^{\mathrm{T}}.
\end{aligned}$$

In simulation, the data length $L = 3000$. $\boldsymbol{\Phi}_s(r)$ is a 2×2 matrix sequence which contains the input-output data, $\boldsymbol{y}(r) \in \mathbb{R}^2$ is the output vector of this model, and $\boldsymbol{v}(r) \in \mathbb{R}^2$ as the white noise vector with zero mean and the noise variances $\sigma_1^2 = \sigma_2^2 = \sigma^2 = 0.40^2$. Applying the ML-RGELS algorithm and the F-ML-RGELS algorithm to estimate the parameters of this model, the estimation errors

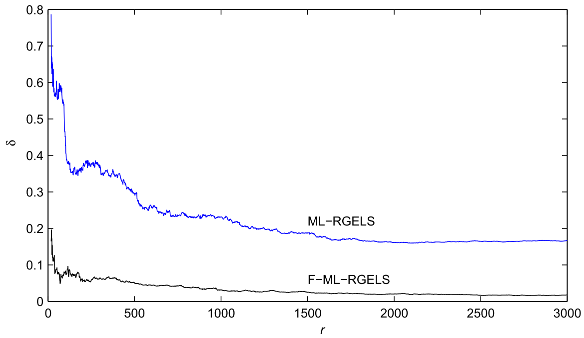

FIGURE 3. The F-ML-RGELS and ML-RGELS estimation errors versus r ($\sigma^2 = 0.40^2$).

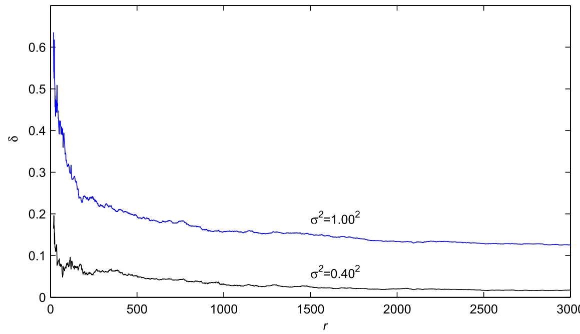

FIGURE 4. The F-ML-RGELS estimation errors versus r with different σ^2.

$\delta := \|\hat{\boldsymbol{\vartheta}}(r) - \boldsymbol{\vartheta}\|/\|\boldsymbol{\vartheta}\|$ versus r are shown in Figure 3. The F-ML-RGELS parameter estimates are plotted in Figures 5.

Applying the F-ML-RGELS algorithm to estimate the parameters of this model with different variances $\sigma^2 = 0.40^2$ and $\sigma^2 = 1.00^2$, the parameter estimates and their errors are shown in Table 1.

To show the effectiveness of the estimated model obtained by the F-ML-RGELS algorithm, we choose the parameter estimates of the seventh row in Table 1 as the final estimated model, that is

$$\begin{bmatrix} 1+0.22024z^{-1} & 0.49714z^{-1} \\ -0.70579z^{-1} & 1+0.51896z^{-1} \end{bmatrix}\boldsymbol{y}(r) = \boldsymbol{\Phi}_s(r)\begin{bmatrix} -0.85481 \\ 0.61899 \end{bmatrix} + \frac{1-0.26985z^{-1}}{1-0.00524z^{-1}}\boldsymbol{v}(r).$$

Then, the model predicted outputs can be represented as

$$\begin{aligned}
\hat{y}_1(r) &= y_1(r) - \hat{y}_{1f}(r) - \hat{h}_{11}z^{-1}\hat{y}_{1f}(r) - \hat{h}_{12}z^{-1}\hat{y}_{2f}(r) \\
&\quad + \hat{\Phi}_{1f}\theta_1 + \hat{\Phi}_{2f}\theta_2 \\
&= y_1(r) - \hat{y}_{1f}(r) - 0.22024\hat{y}_{1f}(r-1) \\
&\quad - 0.49714\hat{y}_{2f}(r-1) - 0.85481\hat{\Phi}_{1f} + 0.61899\hat{\Phi}_{2f}, \\
\hat{y}_2(r) &= y_2(r) - \hat{y}_{2f}(r) - \hat{h}_{21}z^{-1}\hat{y}_{2f}(r) - \hat{h}_{22}z^{-1}\hat{y}_{2f}(r) \\
&\quad + \hat{\Phi}_{3f}\theta_1 + \hat{\Phi}_{4f}\theta_2 \\
&= y_2(r) - \hat{y}_{2f}(r) + 0.70579\hat{y}_{1f}(r-1) \\
&\quad - 0.51896\hat{y}_{2f}(r-1) - 0.85481\hat{\Phi}_{3f} + 0.61899\hat{\Phi}_{4f}, \\
\hat{y}_{1f}(r) &= \hat{E}(z)y_1(r) + [1-\hat{K}(z)]y_{1f}(r) \\
&= y_1(r) - 0.00524y_1(r-1) + 0.26985\hat{y}_{1f}(r-1), \\
\hat{y}_{2f}(r) &= \hat{E}(z)y_2(r) + [1-\hat{K}(z)]y_{2f}(r) \\
&= y_2(r) - 0.00524y_2(r-1) + 0.26985\hat{y}_{2f}(r-1), \\
\hat{\Phi}_{1f}(r) &= \hat{E}(z)\Phi_{s1}(r) + [1-\hat{K}(z)]\hat{\Phi}_{1f}(r) \\
&= \Phi_{s1}(r) - 0.00524\Phi_{s1}(r-1) + 0.26985\hat{\Phi}_{1f}(r-1), \\
\hat{\Phi}_{2f}(r) &= \hat{E}(z)\Phi_{s2}(r) + [1-\hat{K}(z)]\hat{\Phi}_{2f}(r) \\
&= \Phi_{s2}(r) - 0.00524\Phi_{s2}(r-1) + 0.26985\hat{\Phi}_{2f}(r-1), \\
\hat{\Phi}_{3f}(r) &= \hat{E}(z)\Phi_{s3}(r) + [1-\hat{K}(z)]\hat{\Phi}_{3f}(r) \\
&= \Phi_{s3}(r) - 0.00524\Phi_{s3}(r-1) + 0.26985\hat{\Phi}_{3f}(r-1), \\
\hat{\Phi}_{4f}(r) &= \hat{E}(z)\Phi_{s4}(r) + [1-\hat{K}(z)]\hat{\Phi}_{4f}(r) \\
&= \Phi_{s4}(r) - 0.00524\Phi_{s4}(r-1) + 0.26985\hat{\Phi}_{4f}(r-1).
\end{aligned}$$

The root mean square error is used to describe the error between the true outputs and the predicted outputs, which is defined as

$$Error1 = \Big[\frac{1}{200}\sum_{r=3001}^{3200}[\hat{y}_1(r) - y_1(r)]^2\Big]^{1/2} = 0.36164,$$

$$Error2 = \Big[\frac{1}{200}\sum_{r=3001}^{3200}[\hat{y}_2(r) - y_2(r)]^2\Big]^{1/2} = 0.41900.$$

The outputs $y_1(r)$ and $y_2(r)$, the predicted outputs $\hat{y}_1(r)$ and $\hat{y}_2(r)$, and the prediction errors $\hat{y}_1(r) - y_1(r)$ and $\hat{y}_2(r) - y_2(r)$ of the estimated model versus r are shown in Figure 6.

From Table 1 and Figures 3–6, we can draw the following conclusions.

TABLE 1. The F-ML-RGELS estimates and errors ($\sigma^2 = 0.40^2$, $\sigma^2 = 1.00^2$).

σ^2	r	θ_1	θ_2	h_{11}	h_{12}	h_{21}	h_{22}	e_1	k_1	δ (%)
0.40^2	100	-0.85854	0.59889	0.22024	0.50297	-0.66589	0.48864	0.01748	-0.36808	8.16610
	200	-0.85068	0.61407	0.21900	0.50950	-0.66597	0.52706	-0.01884	-0.32568	5.40846
	500	-0.83328	0.61413	0.18083	0.49930	-0.69214	0.53772	-0.03551	-0.29131	5.13112
	1000	-0.85197	0.61342	0.20620	0.49541	-0.70810	0.52730	0.00783	-0.29902	3.13939
	2000	-0.85804	0.62433	0.21766	0.49521	-0.70815	0.52600	-0.00140	-0.27563	2.00807
	3000	-0.85487	0.61899	0.22024	0.49714	-0.70579	0.51896	-0.00524	-0.26985	1.76011
1.00^2	100	-0.71768	0.35596	0.58101	0.50332	-0.61302	0.58499	0.04045	-0.32425	31.43129
	200	-0.73484	0.41986	0.46386	0.51492	-0.63071	0.58371	-0.08238	-0.36237	24.20665
	500	-0.73629	0.48025	0.25654	0.47016	-0.67163	0.57796	-0.16672	-0.39682	19.42952
	1000	-0.79842	0.52193	0.16576	0.47628	-0.70224	0.54589	-0.10725	-0.42308	15.78351
	2000	-0.83022	0.58493	0.12299	0.47718	-0.70724	0.53387	-0.08452	-0.39438	13.38358
	3000	-0.83520	0.58882	0.11410	0.48316	-0.70735	0.52012	-0.07125	-0.37942	12.58205
True values		-0.85000	0.61000	0.23000	0.50000	-0.70000	0.53000	0.01000	-0.26000	

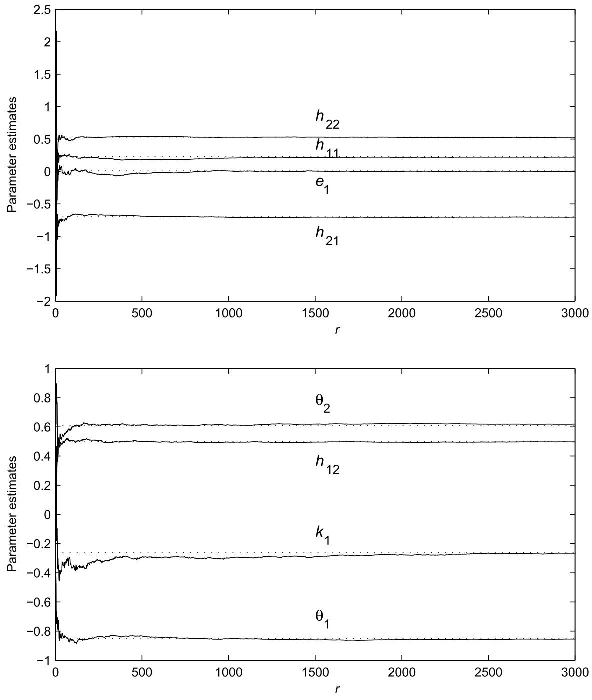

FIGURE 5. The F-ML-RGELS estimates versus r ($\sigma^2 = 0.40^2$).

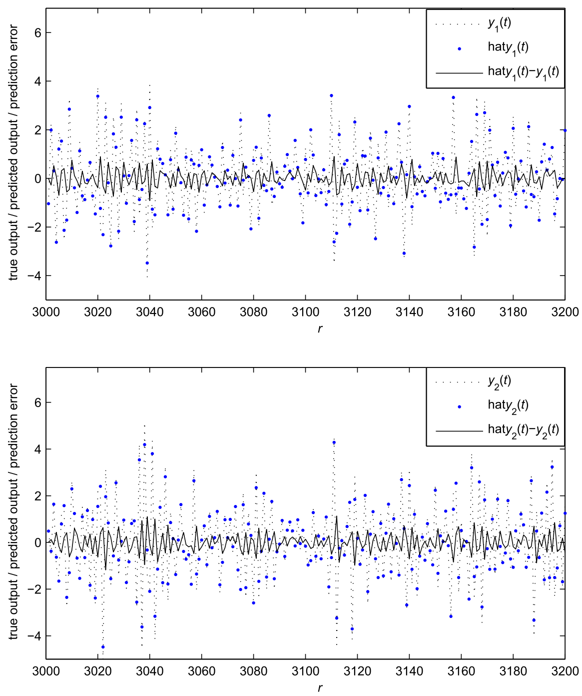

FIGURE 6. The system outputs, the predicted outputs and the prediction errors versus r for the F-ML-RGELS ($\sigma^2 = 0.40^2$).

- The F-ML-RGELS and ML-RGELS estimation errors are becoming small as the data length r increases. This shows that the two algorithms are effective – see Figure 3.
- The estimation errors are becoming smaller as the noise variance decreases under the same data length in the F-ML-RGELS algorithm – see Table 1 and Figure 4.
- The F-ML-RGELS estimates are very close to their true values – see Figures 5.
- The predicted outputs of the F-ML-RGELS algorithm are very close to the true outputs – see Figures 6.
- The F-ML-RGELS algorithm has smaller parameter estimation errors than the ML-RGELS algorithm – see Figure 3.

VI. CONCLUSIONS

This paper considers the parameter identification problems for the multivariate equation-error autoregressive moving average systems. An F-ML-RGELS algorithm is proposed for the multivariate equation-error systems by using the data filtering technique and the maximum likelihood principle. In addition, an ML-RGELS algorithm is presented as a comparison. The numerical example shows that the F-ML-RGELS algorithm is effective and has smaller parameter estimation errors than the ML-RGELS algorithm. The proposed algorithm can be extended to other multivariate systems [33], [34] and other fields [35]–[38]. The proposed algorithms in this paper can combine other identification methods [39]–[47], statistical strategies [48]–[55] and other methods [56]–[62] to study parameter identification of different systems and can be applied to other fields [63]–[71].

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