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Soft Set Based Parameter Value Reduction for Decision Making Application

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ABSTRACT The soft set theory is a completely new mathematical tool for modeling vagueness and uncertainty, which can be applied to decision making. However, in the process of making decision, there are some unnecessary and superfluous information which should be reduced. Normal parameter reduction is a good way to reduce superfluous information, which keeps the entire decision ability. However, the algorithm has a low redundant degree, which involves a great amount of computation. It is not certain that normal parameter reduction has the solution, that is, it has a low success rate of finding reduction. Parameterization value reduction is another reduction method, which improves redundant degree, amount of computation, and success rate of finding reduction. However, this method only considers the best choice, but it does not concern the suboptimal choice, the sequence of choice, and added parameter, that is, it loses some part of decision ability. In order to settle these problems, in this paper, we introduce the parameter value reduction which keeps the entire decision ability while having a very high redundant degree and success rate of finding reduction and low amount of computation. Maximal parameter value reduction is defined as the special cases of parameter value reduction and the related heuristic algorithms are presented, which reaches an extreme degree to reduce the redundant information. The comparison result among maximal parameter value reduction, parameterization value reduction, and normal parameter reduction on 30 datasets shows that the proposed algorithm outperforms parameterization value reduction and normal parameter reduction.

INDEX TERMS Soft sets; Reduction; Parameter reduction; Parameter value reduction; maximal parameter value reduction.

I. INTRODUCTION

Soft set theory as a mathematical tool for dealing with uncertainties becomes more and more popular, when we have to face difficulties which involve uncertain and fuzzy information in the real and complicated application circumstances. In resent twenty years, Soft set theory [1] initiated by a Russian mathematician D. Molodtsov has gained all-round and rapid development.

Presently, theoretical research on the soft set theory is one of main research branches. Some related definitions and operations on soft sets were added in the documents of [2] and [3]. Many researchers discussed the relationships and differences between soft set theory and other mathematical tools and then extended soft set into the new theory such as fuzzy soft set [4], [34], [38], intuitionistic fuzzy soft sets [5], [6], intuitionistic fuzzy parameterized soft set [7], interval-valued fuzzy soft set [8], [35], [36], interval-valued intuitionistic fuzzy soft set theory [9], bijective soft set [11], [12], trapezoidal interval type-2 fuzzy soft sets [13], [14], soft rough set [15], hesitant N-soft set [10], Z-soft fuzzy rough set [16], [17], confidence soft sets [19], Belief interval-valued soft set [20] and rough soft sets [18] so on. Except for the development of theoretical research, soft set also fully express in the real life applications such as the smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability theory, and measurement theory [1], data analysis, screening alternative problem [21] and data mining [22], [23] so on. It is remarkable that we apply soft set into the field of decision making [24]–[26], [37]. However, in the process of making decision, there are some unnecessary and superfluous information which should be reduced. Hence, parameter reduction is the important research branch when we apply soft set theory into the field of decision making. Excellent research results have been published about the issues of reduction of soft sets [31], [40].

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Firstly, Maji *et al.* [24] made use of the reduction solution of rough set to solve the reduction of soft set. Chen *et al.* [27] indicated that above method does not apply to reduction of soft set, and then give a new idea of parameterization reduction when we use soft set theory to solve the decision making, which only involves the optimal choice. The idea of normal parameter reduction was introduced in [28], which involves suboptimal choice and added parameter set of soft sets. This algorithm was greatly improved in [28], [29], [30], [32], and [33] from the computational complexity. However, the normal parameter reduction has a low redundant degree, involves a great amount of computation and it is not certain that normal parameter reduction has the solution. Parameterization value reduction [39] improves redundant degree, amount of computation and success rate of finding reduction. But this method only considers the best choice while does not concern the suboptimal choice, sequence of choice and added parameter, that is, it loses some part of decision ability. In order to solve these above problems, in this paper, we propose the parameter value reduction of soft sets. In detail, parameter value reduction (PVR) which keeps the classification ability and rank of choice objects invariant for decision making is discussed. That is, PVR keeps the entire decision ability. More specifically, maximal parameter value reduction (MPVR) of soft sets is defined as the special cases of PVR and the related heuristic algorithms are presented. In order to clarify that the method of NPR does not reduce all of redundant information, we introduce the algorithm of maximal parameter value reduction based on normal parameter reduction (MPVR-NPR), which is another way to obtain the maximal parameter value reduction. Furthermore, we make comparison among maximal parameter value reduction, maximal parameter value reduction based on normal parameter reduction parameterization value reduction and normal parameter reduction from the four aspects of redundant degree, computation complexity, decision ability and success rate of finding reduction. The experiment results show that MPVR outperform NPR, MPVR-NPR and parameterization value reduction.

The remainder of this paper is organized as follows. Section II reviews the basic notions of soft set theory and the related reduction algorithms. Section III depicts the definition of parameter value reduction (PVR) and maximal parameter value reduction (MPVR) maximal parameter value reduction based on normal parameter reduction, respectively. Furthermore, the related heuristic algorithms are presented. Section IV makes comparison among maximal parameter value reduction, maximal parameter value reduction based on normal parameter reduction parameterization value reduction and normal parameter reduction. Finally Section V presents the conclusion from our study.

II. BASIC CONCEPTS AND RELATED WORK

In this section, we review the basic concept with regard to soft sets and the related normal parameter reduction.

A pair (F, A) is defined as a soft set over non-empty initial universe of objects U, where F is a mapping given by $F : A \to P(U)$, $P(U)$ is the power set of U, and A is the subset of E (E is termed as a set of parameters in relation to objects in U). Example 1 tells us what is the soft set.

Example 1: Mary wants a truly memorable and fantastic wedding. The wedding market is big business nowadays. There are six companies providing six candidate wedding design schemes. Mary needs to determine one scheme which is the most satisfactory. We make use of the mathematical tool of soft set (F, E) to describe the six candidate wedding design schemes. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$,where there are six wedding design schemes in the universe U and E is a set of parameters, $e_i (i = 1, 2, 3, 4, 5)$ standing for the parameters "romantic", "affordable value", "exquisite", "grand", and "modern" respectively. In detail, Table 1 depicts six candidate wedding designs from five respects by the style of soft set. From such case, we may see the structure of a soft set can classify the objects into two classes (yes/1 or no/0). "1" and "0" stand for "yes" and "no", respectively. For example, from Table 1, we can find that Many think the first wedding design scheme h_1 has affordable value, is grand and modern, but not romantic and exquisite. Mary is planning to choose the best one from candidates. According to the decision approach, find k, for which $c_k = \max c_i$, where $c_i = \sum_j h_{ij}$. Hence, h_1 and h_3 are the best wedding design schemes.

The notion of a reduct plays an essential role in analyzing an information table. In the process of making decision, there are some unnecessary and superfluous information which should be reduced. A reduct is a minimum subset of attributes that provides the same descriptive ability as the entire set of attributes.

Maji and Roy [24] made use of the reduction solution of rough set to solve the reduction of soft set. However, in rough set theory the attributes reduction is designed to keep the classification ability of conditional attributes relative to the decision attributes. There is not straightforward connection between the conditional attributes and the decision attributes. But for the soft set, the connection between the decision values and the conditional parameters are straightforward, i.e., the decision values are computed by the conditional parameters, and the reduction of parameters is designed to offer minimal subset of the conditional parameters set to keep the optimal choice objects. The problems tackled by attributes reduction in rough set theory and parameters reduction in soft set theory are different and their methods should be also different. Thus the reduction of parameter sets in soft set theory and the reduction of attributes in rough set theory are different tools for different purposes. We can not apply the reduction method of rough set for dealing with the reduction of soft set.

Therefore, the parameterization reduction of soft sets [27] was introduced to deal with the decision problems based on soft sets as below.

Suppose that $U = \{h_1, h_2, \ldots, h_n\}$, $E = \{e_1, e_2, \ldots, e_m\}$, (F, E) is a soft set with tabular representation.

TABLE 1. Tabular representation of a soft set in example 1.

U	e_1	e_2	e_3	e_4	e_5	c_i
h_1	0	1	0	1	1	3
h_2	1	0	0	0	0	1
h_3	0	1	1	1	0	3
h_4	1	0	1	0	0	2
h_5	0	0	1	1	0	2
h_6	0	0	0	0	0	0

Define $f_E(h_i) = \sum_j h_{ij}$ where h_{ij} are the entries in the table of (F, E).

Definition 2: Denote M_E as the collection of objects in U which takes the max value of f_E. For every $A \subset E$, if $M_{E-A} = M_E$, then A is called a dispensable set in E, otherwise A is called an indispensable set in E. Roughly speaking, $A \subset E$ is dispensable means that the difference among all objects according to the parameters in A does not influence the final decision. The parameter set E is called independent if every $A \subset E$ is indispensable in E, otherwise E is dependent. $B \subseteq E$ is called a reduction of E if B is independent and $M_B = M_E$, i.e., B is the minimal subset of E that keeps the optimal choice objects invariant.

Clearly, after the reduction of the parameter set E, less parameters can be obtained and the optimal choice objects have not been changed. However this idea does not involve suboptimal choice and added parameter set of soft sets. In order to deal with these above problems, Kong *et al.* [28] presented the algorithm of normal parameter reduction as follows:

(1) Input the soft set (F, E) and the parameter set E;
(2) Compute parameter importance degree r_{e_i}, $1 \le i \le m$;
(3) Find maximum subset $A = \{e'_1, e'_2, \cdots, e'_p\} \subset E$ in which that sum of $r_{e'_i}$, for $1 \le i \le p$ is nonnegative integer, then put the A into a feasible parameter reduction set;
(4) Filter in the feasible parameter reduction set, if $f_A(h_1) = f_A(h_2) = \ldots = f_A(h_n)$, then $E - A$ is the normal parameter reduction, otherwise A is deleted.
(5) Get the maximum cardinality of A in feasible parameter reduction set.
(6) Compute $E - A$ as the optimal normal parameter reduction.

FIGURE 1. The algorithm of normal parameter reduction of [28].

This algorithm was improved in [28]–[30], [32], and [33] from the computational complexity. The main idea of these normal parameter reduction algorithms is to delete the entire redundant parameters, which leads to three weaknesses. The algorithm has a low redundant degree, involves a great amount of computation and it is not certain that normal parameter reduction has the solution. Ma *et al.* [39] proposed the definition of parameterization value reduction of soft set theory. However, this idea only considers the best choice while does not concern the suboptimal choice, sequence of choice and added parameter. In order to solve these above problems, in this paper, we propose the parameter value reduction of soft sets.

III. PARAMETER VALUE REDUCTION OF SOFT SETS AND ITS ALGORITHM

In this section, we introduce the definition of parameter value reduction (PVR), maximal parameter value reduction (MPVR) which is defined as the special cases of parameter value reduction, respectively. Furthermore, the related heuristic algorithms are presented.

A. PARAMETER VALUE REDUCTION OF SOFT SETS (PVR)

Suppose $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, (F, E) is a soft set with tabular representation. Define $f_E(h_i) = \sum_{j=1}^{m} h_{ij}$ as decision value, where h_{ij} are the entries in the table of (F, E).

TABLE 2. A soft set(F, E).

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	*f(.)*
h_1	1	0	0	0	0	1	1	0	3
h_2	0	0	1	1	1	1	0	0	4
h_3	1	0	0	1	1	1	1	1	6
h_4	1	1	0	1	0	0	0	1	4
h_5	0	0	1	0	1	0	0	1	3
h_6	1	0	0	1	1	0	0	1	4

Definition 3: For soft sets (F, E), the decision partition based on decision value is referred to as

$$C_E = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1}, \{h_{i+1}, \ldots, h_j\}_{f_2}, \ldots, \{h_k, \ldots, h_n\}_{f_s}\right\},$$

where for subclass $\{h_v, h_{v+1}, \ldots, h_{v+w}\}_{f_i}$, $f_E(h_v) = f_E(h_{v+1}) = \cdots = f_E(h_{v+w}) = f_i$, and $f_1 \geq f_2 \geq \cdots \geq f_s$, s is the number of subclasses,f_s is the minimum decision choice value. In other words, objects in U are classified and ranked according to the decision value of $f_E(.)$.

Definition 4: The parameter value $h_{ij}$is defined as the dispensable parameter value, if we reduce these dispensable parameter values, the decision partition is

$$C'_E = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1-t}, \{h_{i+1}, \ldots, h_j\}_{f_2-t}, \ldots, \{h_k, \ldots, h_n\}_{f_s-t}\right\} (t \leq f_s).$$

From the definition of parameter value reduction, we know parameter value reduction of soft sets keeps the classification ability and rank invariant for decision making, that is, it keeps the entire decision ability.

Based on above definitions, we describe the heuristic algorithm of parameter value reduction of soft sets.

Algorithm 5: parameter value reduction of soft sets.

(1) Input the soft set (F, E) and the parameter set E;
(2) Obliterate the parameter values denoted by 0.
(3) Find f_s, here f_s is the minimum decision choice value.
(4) Given $t(t \leq f_s)$, and then reduce the t parameter values denoted by 1 for every $h_i(h_i \in U, 0 \leq i \leq n)$.
(5) Select remainder as the parameter value reduction which satisfies $C_E' = \{\{h_1, h_2, \ldots, h_i\}_{f_1-t}, \{h_{i+1}, \ldots, h_j\}_{f_2-t}, \ldots, \{h_k, \ldots, h_n\}_{f_s-t}\}$.

FIGURE 2. The algorithm of parameter value reduction.

Example 6: Let (F, E) be a soft set with the tabular representation displayed in Table 2. Suppose that $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$, and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$

From table 2, the decision partition is $C_E = \{\{h_3\}_6, \{h_2, h_4, h_6\}_4, \{h_1, h_5\}_3\}$.Clearly $f_E(h_3) = 6$ is the maximum choice value, thus h_3 is the optimal choice object. h_2, h_4, h_6 are the suboptimal choice object.

Given t = 1(namely, $f_{A_1}(h_1) = f_{A_2}(h_2) = \cdots = f_{A_n}(h_n) = t = 1$), it is evident that

$$A_1 = \{e_1, e_2, e_3, e_4, e_5, e_8\}, A_2 = \{e_1, e_2, e_3, e_7, e_8\},$$
$$A_3 = \{e_1, e_2, e_3\}, A_4 = \{e_1, e_3, e_5, e_6, e_7\},$$
$$A_5 = \{e_1, e_2, e_3, e_4, e_6, e_7\}, A_6 = \{e_1, e_2, e_3, e_6, e_7\}$$

in this example. The parameter value reduction of (F, E) is clearly shown in Table 3. And then we get that the decision partition is $C_E = \{\{h_3\}_5, \{h_2, h_4, h_6\}_3, \{h_1, h_5\}_2\}$.$h_3$ is still the optimal choice object. h_2, h_4, h_6 are still the suboptimal choice object, and so on. The parameter value reduction keeps the classification ability and rank invariant for decision making.

B. MAXIMAL PARAMETER VALUE REDUCTION OF SOFT SETS (MPVR)

If $t < f_s$, we can not reduce all of redundant information. In order to reduce all of redundant information, we introduce the algorithm of MVPR, which is a special case of PVR when $t = f_s$, heref_s is the minimum decision choice value.

Definition 7: The parameter value h_{ij} is defined as the dispensable parameter value, if we reduce these dispensable parameter values, the decision partition is

$$C'_E = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1-t}, \{h_{i+1}, \ldots, h_j\}_{f_2-t}, \ldots, \{h_k, \ldots, h_n\}_{f_s-t}\right\} (t = f_s).$$

(f_s*is the minimum decision choice value*)

TABLE 3. A parameter value reduction table of original table (Table 2).

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	*f(.)*
h_1	-	-	-	-	-	1	1	-	2
h_2	-	-	-	1	1	1	-	-	3
h_3	-	-	-	1	1	1	1	1	5
h_4	-	1	-	1	-	-	-	1	3
h_5	-	-	-	-	1	-	-	1	2
h_6	-	-	-	1	1	-	-	1	3

From the above definition it follows that MPVR does not differ essentially from PVR. Obviously, MPVR is a special case of PVR when $t = f_s$, heref_s is the minimum decision choice value.. Intuitively speaking, MPVR leads to the final decision partition

$$C'_E = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1-t}, \left\{h_{i+1}, \ldots, h_j\right\}_{f_2-t}, \ldots, \{h_k, \ldots, h_n\}_{f_s-t=0}\right\}.$$

*A*lgorithm 8: maximal parameter value reduction of soft sets.

Here below, we provide an algorithm to illustrate how to achieve the maximal parameter value reduction of soft sets.

(1) Input the soft set (F, E) and the parameter set E;

(2) Delete the parameter values denoted by 0.

(3) Find f_s, here f_s is the minimum decision choice value.

(4) If $f_s \neq 0$, reduce the f_s parameter values denoted by 1 for every $h_i (h_i \in U, 0 \le i \le n)$ until $f_s = 0$.

(5) Put the left values as the maximal parameter value reduction which satisfies $C_E{}' = \left\{\{h_1, h_2, \ldots, h_i\}_{f_1-t}, \{h_{i+1}, \ldots, h_j\}_{f_2-t}, \ldots, \{h_k, \ldots, h_n\}_{f_s-t=0}\right\}$.

FIGURE 3. The algorithm of maximal parameter value reduction.

Example 9: Suppose we have a soft set (F, E) with the tabular representation displayed in Table 2. The maximal parameter value reduction is generated and shown in Table 4.

Clearly $f_s = 3$ is the minimum decision choice value in original table (Table 2). Consequently, let $f_{A_1}(h_1) = f_{A_2}(h_2) = \cdots = f_{A_n}(h_n) = f_s = t = 3$. And then we can obtain

$$A_1 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\},$$
$$A_2 = \{e_1, e_2, e_3, e_4, e_5, e_7, e_8\},$$
$$A_3 = \{e_1, e_2, e_3, e_4, e_5\}, A_4 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\},$$
$$A_5 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\},$$
$$A_6 = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}.$$

The final decision partition is $C_E = \{\{h_3\}_3, \{h_2, h_4, h_6\}_1, \{h_1, h_5\}_0\}$. The results from Table 3 and Table 4 indicate that MPVR can delete more parameter values in comparison with PVR, in the case of keeping the classification ability and rank invariant for decision making. Thus MPVR can be generally interpreted as the maximal degree of PVR.

C. MAXIMAL PARAMETER VALUE REDUCTION OF SOFT SETS BASED ON NORMAL PARAMETER REDUCTION (MPVR-NPR)

In order to illustrate the relation between MPVR and NPR, we give the algorithm of MPVR-NPR. This is another way to get the maximal parameter value reduction. We will find that NPR can not reduce all of redundant information. That is, NPR only reduces part of redundant information. After carrying out NPR, we can go on reducing all of redundant information by the idea of Algorithm 8 until the result satisfies the definition 7,

*A*lgorithm 10: maximal parameter value reduction of soft sets based on normal parameter reduction (MPVR-NPR) Firstly constructing a feasible normal parameter reduction and then excluding redundant parameter values, the algorithm of MPVR-NPR can be proposed as follows:

Example 11: Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the set of objects and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ be the

TABLE 4. A maximal parameter value reduction table of Table 2.

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	$f(.)$
h_1	-	-	-	-	-	-	-	-	0
h_2	-	-	-	-	-	1	-	-	1
h_3	-	-	-	-	-	1	1	1	3
h_4	-	-	-	-	-	-	-	1	1
h_5	-	-	-	-	-	-	-	-	0
h_6	-	-	-	-	-	-	-	1	1

TABLE 5. A soft set (G, E).

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	$f(.)$
h_1	0	1	0	0	0	1	0	0	0	1	3
h_2	1	0	1	0	1	0	1	1	0	0	5
h_3	0	1	1	1	1	1	1	0	1	0	7
h_4	0	1	0	1	0	1	1	1	0	0	5
h_5	0	0	0	0	1	1	0	0	1	0	3
h_6	1	0	1	0	1	0	1	1	0	0	5

parameter set. Suppose (G, E) be a soft set with the tabular representation displayed in Table 5.

It can be derived from the above example that $\{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ is the normal parameter reductions with respect to the soft set (G, E), which is shown in Table 6. It is worthwhile to notice that the minimum decision choice value after normal parameter reduction $f_s' = 2 \neq 0$ in Table 6. Hence, the method of normal parameter reduction does not reduce all of redundant information. In other words, there still exists redundant information. In order to reduce remainder redundant information, motivated by the desire to construct the maximal parameter value reduction, we execute algorithm 10 to achieve it which is represented in Table 7.

This method is another way to get the maximal parameter value reduction. Obviously, MPVR-NPR is more complicated and time consuming than MPVR. MPVR-NPR is not a good way to get the maximal parameter value reduction. Here, we introduce MPVR-NPR in order to verify NPR can not reduce all of redundant information and NPR is only the sub-process of MPVR.

IV. COMPARISON RESULT

In order to explain and clarify them, we elaborate the comparison result between normal parameter reduction (NPR), parameterization value reduction(PZVR), maximal parameter value reduction (MPVR), maximal parameter value reduction based on normal parameter reduction (MPVR-NPR) of soft sets through the thirty synthetic generated Boolean data sets. We discussMPVR, MPVR-NPR, PZVR and NPR from

TABLE 6. A normal parameter reduction table of Table 5.

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	$f(.)$
h_1	0	1	0	0	0	1	0	-	-	-	2
h_2	1	0	1	0	1	0	1	-	-	-	4
h_3	0	1	1	1	1	1	1	-	-	-	6
h_4	0	1	0	1	0	1	1	-	-	-	4
h_5	0	0	0	0	1	1	0	-	-	-	2
h_6	1	0	1	0	1	0	1	-	-	-	4

TABLE 7. A maximal parameter value reduction based on normal parameter reduction table of Table 5.

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	$f(.)$
h_1	-	-	-	-	-	-	-	-	-	-	0
h_2	-	-	-	-	1	-	1	-	-	-	2
h_3	-	-	-	1	1	1	1	-	-	-	4
h_4	-	-	-	-	-	1	1	-	-	-	2
h_5	-	-	-	-	-	-	-	-	-	-	0
h_6	-	-	-	-	1	-	1	-	-	-	2

four perspectives: redundant degree, computation complexity decision ability and success rate of finding reduction.

A. REDUNDANT DEGREE

Definition 12: For soft set (F, E) with object set$U = \{h_1, h_2, \cdots, h_n\}$ and parameter set$E = \{e_1, e_2, \cdots, e_m\}$, the redundant degree of $(F, E)$is defined by

$$g = \frac{d}{|U| \times |E|} = \frac{d}{n \times m}$$

where $|.|$ denotes the cardinality of set and d expresses the number of reduced parameter values. Redundant degree g represents the ratio of the number of reduced parameter values to all of parameter values. Notice that the higher value of g means the higher efficiency of reduction and vice versa.

Property 13: For soft set (F, E) $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, $0 < g \leq 1$.

Proof:

$$\begin{aligned} g &= \frac{d}{|U| \times |E|} = \frac{d}{n \times m} \\ &\leq \frac{n \times m}{n \times m} = 1. \end{aligned}$$

Note that there is certainly some parameter values labeled by 0 which can be reduced, unless all of parameter values are equal to 1 which lead to $g = 1$. Therefore, $g \neq 0$.

It is easy to obtain $0 < g$. Therefore we have $0 \leq g \leq 1$. □

Property 14: $g = 1$ if and only if $f_E(h_1) = f_E(h_2) = \cdots = f_E(h_n)$.

TABLE 8. A maximal parameter value reduction table of Table 5.

h	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	$f(.)$
h_1	-	-	-	-	-	-	-	-	-	-	0
h_2	-	-	-	-	1	-	1	-	-	-	2
h_3	-	-	-	1	1	1	1	-	-	-	4
h_4	-	-	-	-	-	1	1	-	-	-	2
h_5	-	-	-	-	-	-	-	-	-	-	0
h_6	-	-	-	-	1	-	1	-	-	-	2

(1) Input the soft set (F, E) and the parameter set E;

(2) Carry out the algorithm of normal parameter reduction, and then obtain the normal parameter reduction (F', E').

(3) Reduce the parameter values denoted by 0 in the new soft set (F', E').

(4) For the new soft set (F', E'), find f'_s, here f'_s is the minimum decision choice value.

(5) Delete the parameter values denoted by 1 for every $h_i (h_i \in U, 0 \le i \le n)$ until $f'_s = 0$, If $f'_s \neq 0$.

(6) Find the left values as the maximal parameter value reduction based on normal parameter reduction.

FIGURE 4. The algorithm of maximal parameter value reduction based on normal parameter reduction.

Proof: Let $f_E(h_1) = f_E(h_2) = \cdots = f_E(h_n)$, it means that the parameter values $H_E(h_i)\,(i = 1, \ldots, n)$ are dispensable, namely, which can be deleted, according to definition 7. So we have $d = |U| \times |E|$. Thus $g = 1$. □

Theorem 15: For soft set $(F, E)\,U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, Denote$g_{MPVR}$as redundant degree of maximal parameter value reduction and$g_{MPVR-NPR}$ as redundant degree of maximal parameter value reduction based on normal parameter reduction, respectively. $g_{MPVR} = g_{MPVR-NPR}$.

Proof: Let $H_{B_i}(h_i)\,(i = 1, \ldots, n)$ the maximal parameter value reduction satisfying $f_{E-B_1}(h_1) = f_{E-B_2}(h_2) = \cdots = f_{E-B_n}(h_n) = t = f_s$, namely, $f_{A_1}(h_1) = f_{A_2}(h_2) = \cdots = f_{A_n}(h_n) = t = f_s$. It implies that nf_s parameter values labeled 1 can be reduced. Hence $g_{MPVR} = \frac{d}{n \times m} = \frac{nf_s + l}{n \times m}$, where l is defined as the number of parameter values labeled 0.

$E - A$ is the normal parameter reduction of E, so we get $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n) = t_1 (0 \le t_1 \le f_s)$ from which we can conclude that the number of reduced parameter values labeled 1 $d_{NPR} = nt_1$. From definition 12 we have $f_{A'_1}(h_1) = f_{A'_2}(h_2) = \cdots = f_{A'_n}(h_n) = t' = f'_s$, where $f'_s$is the minimum decision choice value after normal parameter reduction. Hence we also observe that nt' parameter values labeled 1 can be deleted in succession. To sum up,

$$g_{MPVR-NPR} = \frac{d}{|U| \times |E|} = \frac{d}{n \times m} = \frac{nt_1 + nt' + l}{n \times m} = \frac{n(t_1 + t') + l}{n \times m}.$$

Obviously, $t_1 + t' = t = f_s$ and then $g_{MPVR-NPR} = \frac{nf_s + l}{n \times m}$. *Therefore $g_{MPVR} = g_{MPVR-NPR}$. This completes the proof.* □

In order to elaborate this theorem, consider the following example.

Example 16: Let (F, E) be a soft set with the tabular representation displayed in Table 5. We have obtained a maximal parameter value reduction based on normal parameter reduction shown in Table 7. Further we can also find a maximal parameter value reduction given in Table 8.

It is evident that $g_{MPVR} = g_{MPVR-NPR} = \frac{49}{60}$, in view of the above example. We also observe that two approaches keep the classification ability and rank invariant for decision making. They have the same final decision partition $C'_E = \{\{h_3\}_4, \{h_2, h_4, h_6\}_2, \{h_1, h_5\}_0\}$.

Theorem 17: For soft set $(F, E)\,U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, Denote$g_{MPVR}$as redundant degree of maximal parameter value reduction and $g_{MPVR-NPR}$as redundant degree of maximal parameter value reduction based on normal parameter reduction andg_{NPR}as redundant degree of normal parameter reduction, respectively. $g_{MPVR} = g_{MPVR-NPR} \ge g_{NPR}$.

Proof: Let $C'_{E(MPVR-NPR)}$ as the final decision partitions after carrying out maximal parameter value reduction and $C'_{E(NPR)}$ as the final decision partitions after carrying out normal parameter reduction respectively.

If $C'_{E(MPVR-NPR)} = C'_{E(NPR)}$,the maximal parameter value reduction only keep values denoted by 1, however, normal parameter reduction keep parameter columns consisting of values denoted by 0 and 1. In this case, obviously, $g_{MPVR} = g_{MPVR-NPR} \geq g_{NPR}$.

If $C'_{E(MPVR-NPR)} \neq C'_{E(NPR)}$, it means that $f'_s \neq 0$ ($f'_s$is the minimum decision choice value after normal parameter reduction).It is necessary to carry out maximal parameter value reduction based on normal parameter reduction algorithm to achieve it. It is certain that $g_{MPVR} = g_{MPVR-NPR} \geq g_{NPR}$.

To sum up, $g_{MPVR} = g_{MPVR-NPR} \geq g_{NPR}$. □

Example 18: From Table 6, Table 7 and Table 8, we can observe that $g_{MPVR} = g_{MPVR-NPR} = \frac{49}{60}$and $g_{NPR} = \frac{18}{60}$. It is easily obtained $g_{MPVR} = g_{MPVR-NPR} \geq g_{NPR}$.

Generally, PZVR has the a bit higher redundant degree than MPVR and MPVR-NPR. However, they have the different decision ability. PZVR only considers the best choice and then keeps some part of decision ability. MPVR and MPVR-NPR concern the suboptimal choice, sequence of choice and added parameter, which keep the entire decision ability. Hence, we can not compare PZVR with MPVR and MPVR-NPR in regard to redundant degree.

We perform the three algorithms on the thirty synthetic generated Boolean data sets, respectively. And then we compute the averages of redundant degree regarding the three algorithms. The average of redundant degree by MPVR and MPVR-NPR is up to 71.3% on thirty datasets; the mean of redundant degree by NPR is equal to 4.8%, which are shown in Figure 5.

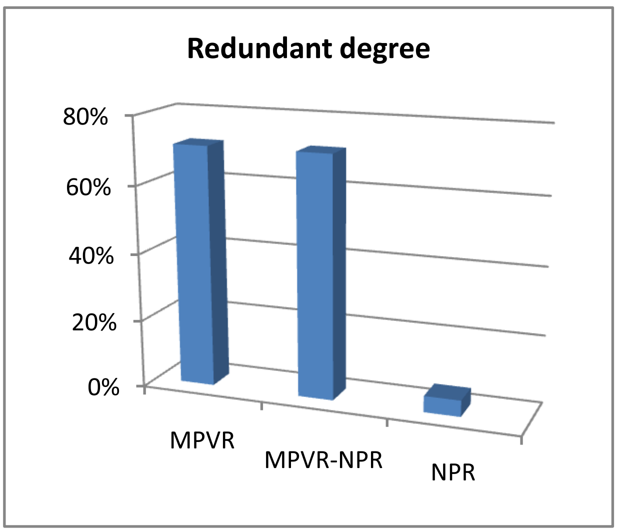

FIGURE 5. Average redundant degree of three algorithms.

B. COMPUTATION COMPLEXITY

In this section, computation complexity of NPR, PZVR, MPVR and MPVR-NPR will be discussed and compared. We estimate the computational complexity of the algorithm by counting the number of basic operation. Due to the basic operation perhaps vary with different implementation of the algorithm, we consider element access here as the basic operation.

a) Computation complexity of NPR [28]

For NPR, we need get the parameter importance degree. Firstly, compute $f_E(h_i) = \sum_j h_{ij}$, $1 \leq i \leq n$, $1 \leq j \leq m$. Every entry will be accessed one time, so the number of element access is $m \cdot n$. Secondly, Get $C_E = \{E_{f_1}, E_{f_2}, \cdots, E_{f_s}\}$, that is, classify objects according to $f_E(h_i)$.The column of $f_E(h_i)$ will be accessed one time, so the number of element access is n. Thirdly, Obtain decision partition deleted e_j. The numbers of element access are $n \cdot (m-1) + n$. Finally, Calculate parameter importance degree r_{e_j}. The number of element access is $2n$. From above steps, we can get the total number of element access for computing all of parameter importance degrees is $m^2n+2mn+mn+n = m^2n+3mn+n$. Taking big O notation, the complexity of computing all of parameter importance degree is $O(m^2n)$. Suppose $m = n$, the complexity will be $O(n^3)$. In order to find reduction results, the number of parameter importance degree access is $(C_m^1 + C_m^2 + \cdots + C_m^{\lfloor m/2 \rfloor}) \cdot m$. Hence, from above two points of view, NPR has a high computation complexity.

b) **Computation complexity of MPVR**

Compute decision value $f_E(h_i) = \sum_j h_{ij}$, $1 \leq i \leq n$, $1 \leq j \leq m$. Every entry will be accessed once, so the number of element access is $m \cdot n$. Finding the minimum decision value, every decision value will be accessed once, so the number of element access is$n$. When we delete the parameter values in terms of the minimum decision value, every entry will be accessed once, so the number of element access is $m \cdot n$. Hence, the total number of element access is 2mn + n. Taking big O notation, the computation complexity of this proposed algorithm is O(mn).

c) Computation complexity ofMPVR-NPR

For MPVR-NPR, we should firstly perform NPR and then carry out MPVR. Hence, the computational complexity seems the sum of MPVR and NPR.

It is clear that the proposed algorithm of maximal parameter value reduction involves relatively much less computation.

d) **Computation complexity of**PZVR

Compute decision value $f_E(h_i) = \sum_j h_{ij}$, $1 \leq i \leq n$, $1 \leq j \leq m$. Every entry will be accessed once, so the number of element access is $m \cdot n$. Finding the maximum decision value, every decision value will be accessed once, so the number of element access is$n$. When we delete the parameter values except one parameter value corresponding to the maximum decision value, every entry will be accessed once, so the number of element access is $m \cdot n$. Hence, the total number of element access is 2mn + n. Taking big O notation, the computation complexity of this proposed algorithm is O(mn).

C. SUCCESS RATE OF FINDING REDUCTION

Theorem 19: For soft set (F, E) $U = \{h_1, h_2, \cdots, h_n\}$, $E = \{e_1, e_2, \cdots, e_m\}$, if $f_s \neq 0$ it is certain that we can get

PZVR, *MPVR and MPVR-NPR but it is not sure that we can obtain NPR.*

Proof: If $f_s \neq 0$, we have that nf_s parameter values are dispensable according to definition 3.3 and definition 12. Hence it is certain that we can get MPVR and MPVR-NPR. In a similar way, we surely find PZVR.

For NPR, there must exist a subset $A = \{e'_1, e'_2, \cdots, e'_p\} \subset E$ satisfying $f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$according to the related algorithm. However it is not sure that there certainly exists the subset. Thus it is not sure that we can obtain NPR.

Example 20: Assume that (F', E) is a soft set with the tabular representation displayed in Table 9. Let $U = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be a universal set and $E = \{e_1, e_2, e_3, e_4\}$ be the set of all parameters.

TABLE 9. A soft set (F', E).

h	e_1	e_2	e_3	e_4	f(.)
h_1	1	0	1	1	3
h_2	1	1	1	1	4
h_3	0	0	0	1	1
h_4	1	0	1	0	2
h_5	1	0	1	0	2

TABLE 10. A maximal parameter value reduction table of Table 9.

h	e_1	e_2	e_3	e_4	f(.)
h_1	-	-	1	1	2
h_2	-	1	1	1	3
h_3	-	-	-	-	0
h_4	-	-	1	-	1
h_5	-	-	1	-	1

In the view of the above example, the soft set in Table 9 has a maximal parameter value reduction illustrated in Table 10 and parameterization value reduction shown in Table 11, whereas we can not find the normal parameter reduction of this soft set.

Success rate of finding reduction refers to the radio of the number of whether finding reduction results on datasets to all of datasets. We perform the four algorithms on the thirty synthetic generated Boolean data sets, respectively. We find that we can find the reduction results on thirty datasets by PZVR, MPVR and MPVR-NPR, but we only obtain reduction on three datasets by NPR. The success rate of finding reduction by PZVR, MPVR and MPVR-NPR is up to 100% on thirty datasets; success rate of finding reduction by NPR is equal to 10%, which are shown in Figure 6.

TABLE 11. A parameterization value reduction table of Table 9.

h	e_1	e_2	e_3	e_4	f(.)
h_1	-	-	-	-	0
h_2	-	-	-	1	1
h_3	-	-	-	-	0
h_4	-	-	-	-	0
h_5	-	-	-	-	0

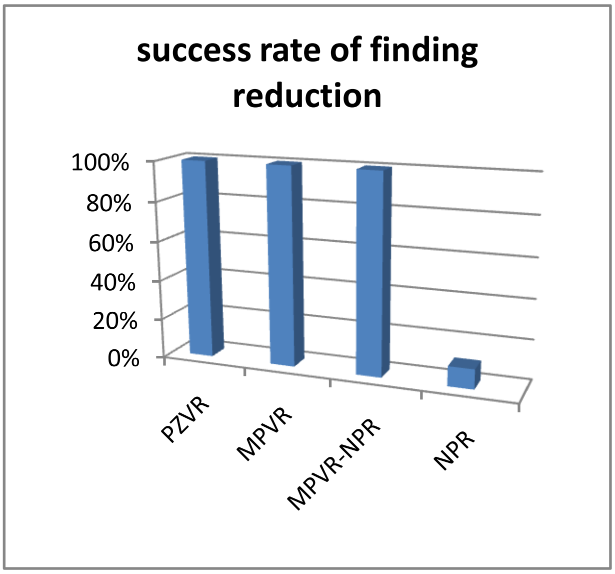

FIGURE 6. Success rate of finding reduction by four algorithms.

D. DECISION ABILITY

MPVR, MPVR-NPR and NPR have the same decision ability. They concern the suboptimal choice, sequence of choice and added parameter, which keep the entire decision ability. The three algorithms benefit the extension and combination of datasets. However, PZVR only considers the best choice and then keeps some part of decision ability. PZVR never think over the newly added parameters and parameter values, which goes against the extension of datasets.

TABLE 12. The comparison result.

Comparison	PZVR	NPR	MPVR	MPVR-NPR	Explanation
Decision ability	only considers the best choice	concern the suboptimal choice, sequence of choice and added parameter	concern the suboptimal choice, sequence of choice and added parameter	concern the suboptimal choice, sequence of choice and added parameter	NPR, MPVR and MPVR-NPR have the same decision ability and keep the entire decision ability. The three algorithms benefit the extension and combination of datasets. PZVR keeps some part of decision ability., which goes against the extension of datasets.
Final decision partition	-	VI. VII. $C_E{}' = \{\{h_1,h_2,...,h_i\}_{f_1-t}, \{h_{i+1},...,h_j\}_{f_2-t},..., \{h_k,...,h_n\}_{f_s-t}\}, t \le f_s$	$C_E{}' = \{\{h_1,h_2,...,h_i\}_{f_1-t}, \{h_{i+1},...,h_j\}_{f_2-t},..., \{h_k,...,h_n\}_{f_s-t}\}, t = f_s$	$C_E{}' = \{\{h_1,h_2,...,h_i\}_{f_1-t}, \{h_{i+1},...,h_j\}_{f_2-t},..., \{h_k,...,h_n\}_{f_s-t}\}, t = f_s$	NPR, MPVR and MPVR-NPR keep the classification ability and rank invariant for decision making.
redundant degree	$\frac{n \times m - 1}{n \times m}$	$\frac{nt_1}{n \times m}\ (t_1 < f_s)$	$\frac{nf_s + l}{n \times m}$	$\frac{nf_s + l}{n \times m}$	PZVR is the highest, MPVR and MPVR-NPR is much higher, NPR is much lower
computation complexity	O(mn)	$O(m^2 n)$	O(mn)	$O(m^2 n)$ + O(mn)	PZVR and MPVR involves relatively much less computation compared with NPR and MPVR-NPR
solution	Certainly has solution	Possibly has solution	Certainly has solution	Certainly has solution	Even if $f_s > 0$, NPR possibly has no solution, while PZVR, MPVR and MPVR-NPR surely have solution
The involved operation	Only deletion for parameter values	Addition, set operation, classification for parameter importance degree	Only deletion for parameter values	Addition, set operation, classification for parameter importance degree and deletion for parameter values	PZVR and MPVR are more simple and easily implemented, NPR and MPVR-NPR are much more complicated.

Table 12 illustrates the overall comparison results among MPVR, MNVP-NPR, PZVR and NPR. We find that PZVR only considers the best choice and then keeps some part of decision ability, which is based on the loss of some part decision ability. Compared with MPVR, MPVR-NPR and NPR, PZVR has the fatal weakness of loss of some part decision ability. Hence, MPVR, MPVR-NPR and NPR outperform PZVR. NPR has a very low success rate of finding reduction. In many real-life applications, it is not certain that normal parameter reduction has the solution, which is the fatal weakness for NPR. MPVR and MPVR-NPR outperform NPR. MPVR and MPVR-NPR are two different methods of obtaining the maximal parameter value reduction. They have the same redundant degree and success rate of finding reduction. MPVR-NPR is more complicated and time consuming than MPVR. That is, MPVR has the much lower computation complexity compared with MPVR-NPR. Therefore, MPVR is the best way among these algorithms.

V. CONCLUSION

Several algorithms exist to address the issues concerning reduction of soft sets, such as Normal Parameter Reduction (NPR) and Parameterization Value Reduction (PZVR). However, they have their respective fatal weaknesses. NPR has a very low success rate of finding reduction. In many real-life applications, normal parameter reduction has no solution. PVR can not keep the entire decision ability. In order to settle these problems, in this paper we introduce the parameter value reduction which not only keeps the entire decision ability, but also has very high success rate of finding reduction. Maximal parameter value reduction is (MPVR) defined as the special cases of parameter value reduction and the related heuristic algorithm is presented, which reaches an extreme degree to reduce the redundant information. In order to clarify that the method of NPR does not reduce all of redundant information, we introduce the algorithm of maximal parameter value reduction based on normal parameter reduction (MPVR-NPR), which is another way to obtain the maximal parameter value reduction. Furthermore, we make comparison among MPVR, MNVP-NPR, PZVR and NPR from the aspects of redundant degree, computation complexity, decision ability and success rate of finding reduction. The comparison result through the thirty Boolean data sets illustrates that MPVR, MPVR-NPR and NPR keep the entire decision ability while PZVR only keeps part of decision ability. By MPVR and MPVR-NPR, redundant degree and success rate of finding reduction are greatly improved to 93.3% and 90% as compared with NPR when these approaches have the same decision ability. MPVR has the much lower computation complexity compared with MPVR-NPR. Therefore, MPVR is the optimal reduction algorithm among these algorithms, which not only keeps the entire decision ability and but also greatly improves redundant degree, success rate and computation complexity.

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