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FSE-RBFNNs-Based Robust Adaptive DSC Design for a Larger Class of Nonlinear Strict-Feedback Systems

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ABSTRACT A novel set-invariance adaptive neural dynamic surface (DSC) control scheme is presented for an extended class of the periodically disturbed nonlinear MIMO strict-feedback systems whose control gain functions are possibly unbounded. The most advanced is that the restrictive bounds assumption is removed after introducing appropriate compact sets, which are constructed in such a way that all the closed-loop trajectories stay in those sets all the time. To tackle the tracking control problem in the presence of more general periodic disturbances, a novel function approximator is well constructed by combining the radial basis function neural networks (RBFNNs) with the Fourier series expansion (FSE). In addition, the DSC technique is employed to overcome the problem of ''explosion of complexity''. Furthermore, the Lyapunov theory and invariant set theorem are utilized to prove the closed-loop systems semi-globally uniformly ultimately bounded (SGUUB) stability, and the tracking errors can converge to an arbitrarily small residual set after appropriately choosing design parameters. Finally, the simulation results verify the effectiveness of the proposed method.

INDEX TERMS Nonlinear MIMO systems, adaptive neural control, DSC, periodic disturbances, invariant set theorem.

I. INTRODUCTION

In practical control engineering, approximation-based adaptive control methodologies have attracted much attention, emerging as promising approaches for controlling highly uncertain and nonlinear dynamical systems [1]–[4]. Based on the universal approximation theorem, the fuzzy logic systems (FLSs) and neural networks (NNs) have been successfully employed to approximate the unknown nonlinear functions with little knowledge of system plant [5]–[10]. When combined with the backstepping approach, approximation-based adaptive control has been extensively shown to achieve global stability for many classes of nonlinear systems [11]–[15]. However, an obvious drawback with the backstepping technique is the problem of ''explosion of complexity'', which is caused by repeated differentiations of certain nonlinear functions, such as virtual controls. Thus, the dynamic surface control (DSC) technique has been creatively proposed to avoid this problem effectively by introducing a first-order low-pass filter at each step in the conventional backstepping design procedure. Approximation-based adaptive controllers stemming from this technique have been successfully constructed for many nonlinear SISO and MIMO strict-feedback systems, see [3], [16]–[22] and references therein. To just name a few, an adaptive neural DSC scheme is proposed for a class of uncertain nonlinear strict-feedback systems subject to input constraint in [16]. In [20], an adaptive neural state-feedback control scheme is presented for a class of stochastic nonlinear switched systems. Recently, in [22], a neural adaptive hierarchical sliding-mode control scheme has been proposed for a class of MIMO strict-feedback nonlinear time-vary systems with dead zone. Furthermore, an adaptive finite-time tracking control scheme is developed for a class of MIMO strict-feedback nonlinear continuous-time systems in the presence of full state constraints and dead-zone in [3]. Nevertheless, it should be pointed out that, for the DSC

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technique to work, there is a common assumption that bounded control gain functions is required, which is very restrictive because the bounds of control gain functions may be difficult to acquire in practical applications, or even nonexistent [23]. This problem is first investigated in [24] by assuming continuous (possibly unbounded) control gain functions, which are bounded on compact set. However, the aforementioned research is only limited to non-strict-feedback systems but not for more complex periodically disturbed strict-feedback systems.

Actually, periodically time-varying disturbances frequently exist in a wide range of practical applications, such as industrial robots [25], numerical control machines [26] and van der Pol oscillator [27]. Therefore, the adaptive tracking control scheme of disturbed systems has the applicable important and received increasing attention. As for periodically disturbed nonlinear uncertain systems, one of the main difficulties is that the unmeasured disturbances appear nonlinearly in unknown functions, and it would be extremely difficult to find an appropriate feasible scheme to solve the tracking problem of strict-feedback systems with more general time-varying nonlinearly parameterized disturbances [27]. In order to overcome this problem, some efforts have been made: most notably, in [28], an adaptive backstepping control scheme is presented for a class of nonlinear systems with periodic disturbances utilizing Fourier series expansion (FSE) and fuzzy logic systems (FLSs). Recently, two robust adaptive tracking control methods are proposed for a class of strict-feedback nonlinear systems by combining FSE with radial basis function neural networks (RBFNNs) and multilayer neural networks (MNNs) in [30], [31], respectively. However, it should be pointed out that, for all above existing researches [25]–[31], upper and lower bounds of the control gain functions always assumed to exist, which is a restrictive assumption. Moreover, the aforementioned researches are only limited to SISO periodically disturbed strict-feedback systems. To our best knowledge, to date, no results on a control design considering periodically time-varying disturbances have been reported for MIMO strict-feedback systems with possibly unbounded control gain functions [31].

Motivated by the aforementioned observations, in this paper, a novel set-invariance adaptive neural DSC scheme is proposed for a larger class of MIMO strict-feedback with periodic disturbances. The main contributions of this paper are highlighted as follows.

1. It seems that this is supposed to be the first work that concerns the problem of stabilization for MIMO nonlinear systems with periodic disturbance.

2. In this paper, we consider a huger class of MIMO strict-feedback systems with periodic disturbances. In this paper, RBFNNs and FSE are combined to approximate the unknown continuous system functions and DSC technique is constructively employed in the adaptive neural control design.

3. Semi-globally uniformly ultimately boundedness (SGUUB) of all signals in the closed loop systems is analytically proved using Lyapunov theory in combination with invariant set theorem, and the system output tracking errors are eventually proved to be within an arbitrarily small residual set by appropriately adjusting design parameters.

The rest of this paper is structured as follows. Section 2 presents the problem formulation and preliminaries. The adaptive neural controller design and stability analysis are given in Section 3 and Section 4. In Section 5, simulation results are presented to show the effectiveness of the proposed scheme. Finally, Section 6 concludes the work.

II. PROBLEM STATEMENT AND PRELIMINARIES

A. PROBLEM FORMULATION

Consider a class of uncertain MIMO strict-feedback nonlinear systems with periodic disturbances which is described by

$$\begin{cases} \dot{\xi}_{j,i_j} = \phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}(t)\right)\xi_{j,i_j+1} + \Delta_{j,i_j}(\xi, t) \\ \qquad + \varphi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}(t)\right), \quad 1 \le i_j \le \rho_j - 1 \\ \dot{\xi}_{j,\rho_j} = \phi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}(t)\right)u_j + \Delta_{j,\rho_j}(\xi, t) \\ \qquad + \varphi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}(t)\right) \\ y_j = \xi_{j,1}, \quad j = 1, \ldots, m \end{cases} \tag{1}$$

where $\xi_{j,i_j} \in R$ is the state of the j th subsystem, $\xi = \left[\bar{\xi}_{1,\rho_1}^T, \ldots, \bar{\xi}_{j,\rho_j}^T, \ldots, \bar{\xi}_{m,\rho_m}^T\right]^T \in R^{\sum_{k=1}^{m}\rho_k}$ is the state vector of the whole system, here $\bar{\xi}_{j,\rho_j} = \left[\xi_{j,1}, \ldots, \xi_{j,\rho_j}\right]^T \in R^{\rho_j}$ and ρ_j is the order of the jth subsystem. $\bar{\xi}_{j,i_j} = \left[\xi_{j,1}, \ldots, \xi_{j,i_j}\right]^T \in R^{i_j}$, u_j and $y_j \in R$ are the input and output of the j th subsystem respectively. $\varphi_{j,i_j}(\cdot,\cdot)$ are unknown continuous functions with $\varphi_{j,i_j}(0,\omega) = 0$, $\forall\omega$, $\phi_{j,i_j}(\cdot,\cdot)$ are unknown continuous control gain functions, and $\Delta_{j,i_j}(\xi, t)$ are the unknown external disturbances with $i_j = 1, \ldots, \rho_j, j = 1, \ldots, m$, $\tau_{j,i_j}(t) : [0, +\infty) \to R^{n_{j,i_j}}$ $(1 \le i_j \le \rho_j, j = 1, \ldots, m)$ are unknown and continuously time-varying disturbances with known periods T_{j,i_j}, namely, $\tau_{j,i_j}\left(t + T_{j,i_j}\right) = \tau_{j,i_j}(t)$. In what follows, denote $\tau_{j,i_j} = \tau_{j,i_j}(t)$ for brevity.

In this paper, the control aim is that a novel adaptive neural tracking controller u_j is constructed for system (1), such that the system output y_j converges to a small neighborhood of the reference signal $y_{j,d}$, as accurately as possible, and all trajectories of system (1) are SGUUB. The reference signal $y_{j,d}$ satisfies the following standard assumption.

Assumption 1: It is assumed that $y_{j,d}$ is a sufficient smooth function, and there exist constants $B_{j0} > 0$ such that

$$\Pi_{j0} = \left\{\left(y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right)^T | y_{j,d}^2 + \dot{y}_{j,d}^2 + \ddot{y}_{j,d}^2 \le B_{j0}^2\right\}.$$

To design appropriate controller for the system (1), the following related assumptions and lemmas are imposed.

Assumption 2: The unknown control gain functions satisfy

$$\phi_{j,i_j}(\bar{\xi}_{j,i_j}, \tau_{j,i_j}) > 0 \quad for\ i_j = 1, 2, \ldots, \rho_j, \quad j = 1, \ldots, m.$$

Remark 1: Note that the stability analysis in [25]–[31] rely on the fact that the control gain functions are bounded for any state (independently of stability), such a reasoning does not apply to our case, and a new stability analysis must be sought.

Assumption 3: For $\forall t > 0$, there exist constants $\Delta^*_{j,i_j} > 0$ such that $|\Delta_{j,i_j}(\xi, t)| \le \Delta^*_{j,i_j}$ $(i_j = 1, \ldots, \rho_j, j = 1, \ldots, m)$.

Lemma 1 [10]: Suppose the dynamic system as follows

$$\dot{\hbar}(t) = -a\hbar(t) + p\upsilon(t) \tag{2}$$

with $a > 0$ and $p > 0$ being constants and $\upsilon(t) > 0$ being a function. For $\hbar(t_0) \ge 0$, we obtain $\hbar(t) \ge 0, \forall t \ge 0$.

Lemma 2 [15]: The function $\tanh(\cdot)$ is uninterrupted and differentiable, and it accomplish that for $\forall \ell \in R$ and $\forall \varpi > 0$

$$0 \le |\ell| - \ell \tanh(\ell/\varpi) \le \kappa\varpi, \quad \kappa = 0.2785 \tag{3}$$

Lemma 3 [24]: (Young's inequality) For $\forall(\xi, y) \in R^2$, we can obtain the following inequality

$$\xi y \le \frac{\mu^2}{\iota}|\xi|^2 + \frac{1}{\varsigma\mu^2}|y|^2 \tag{4}$$

where $\iota > 1$, $\varsigma > 1$, $\mu > 0$, $(\iota - 1)(\varsigma - 1) = 1$.

B. FSE-RBFNNs-BASED APPROXIMATOR

Consider the unknown function $h_{j,i_j}(\bar{\lambda}_{j,i_j}, \tau_{j,i_j})$, let $\bar{\lambda}_{j,i_j} \in \Pi_{j,i_j} \times \Pi_{j0}$ being a measured signal with $\Pi_{j,i_j} \times \Pi_{j0}$ a compact set, and $\tau_{j,i_j} = [\tau_{j,i_j1}, \tau_{j,i_j2}, \ldots, \tau_{j,i_jm}]^T$ is an unknown continuous disturbance vector of known period T defined on a compact set $\Pi_{j\tau} = \left\{(\tau_{1,1}, \ldots, \tau_{m,\rho_m}) \mid \sum_{j=1}^{m}\sum_{i_j=1}^{\rho_j} \tau^T_{j,i_j}\tau_{j,i_j} \le D^2_{j\tau}\right\}$ with $D_{j\omega} > 0$ being a constant. The vector τ_{j,i_j} can also be expressed by a linearly parameterized FSE as follows

$$\tau_{j,i_j} = \vartheta^T_{j,i_j}\wp_{j,i_j} + \delta_{\tau_{j,i_j}} \tag{5}$$

where $\vartheta_{j,i_j} = [\vartheta_{j,i_j1}, \ldots, \vartheta_{j,i_jm}] \in R^{q\times m}$ is a constant matrix with $\vartheta_{j,i_jj} \in R^q$ being a vector consisting of the first q coefficients of the FSE of τ_{j,i_jj} (q is an odd integer), $\delta_{\tau_{j,i_j}}$ is the truncation error and $\wp_{j,i_j}(t) = [\wp_{j,i_j1}(t), \ldots, \wp_{j,i_jq}(t)]^T$ with $\wp_{j,i_j1}(t) = 1$, $\wp_{j,i_j2k}(t) = \sqrt{2}\sin(2\pi kt/T)$ and $\wp_{j,i_j2k+1}(t) = \sqrt{2}\cos(2\pi kt/T)$, $(k = 1, \ldots, (q-1)/2)$, whose derivatives up to n-order are smooth and bounded.

The RBFNNs will be employed to approximate the unknown continuous function as

$$h_{j,i_j}(\bar{\lambda}_{j,i_j}, \tau_{j,i_j}) = \theta^T_{j,i_j}\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \vartheta^T_{j,i_j}\wp_{j,i_j}) + \varepsilon_{j,i_j}(\bar{\lambda}_{j,i_j}) \tag{6}$$

where θ_{j,i_j} is a vector of adjustable parameters; $\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \tau_{j,i_j})$ is a known smooth vector-valued function; $\varepsilon_{j,i_j}(\bar{\lambda}_{j,i_j})$ is the approximation error with $|\varepsilon_{j,i_j}(\bar{\lambda}_{j,i_j})| \le \varepsilon^*_{j,i_j}$ $(\varepsilon^*_{j,i_j} > 0)$.

For (6), the estimation errors can be given by

$$\begin{aligned}&\theta^T_{j,i_j}\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \vartheta^T_{j,i_j}\wp_{j,i_j}) - \hat{\theta}^T_{j,i_j}\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \hat{\vartheta}^T_{j,i_j}\wp_{j,i_j})\\ &= \tilde{\vartheta}^T_{j,i_j}(\hat{\psi}_{j,i_j} - \hat{\psi}'_{j,i_j}\hat{\vartheta}^T_{j,i_j}\wp_{j,i_j}) + \hat{\theta}^T_{j,i_j}\hat{\psi}'_{j,i_j}\tilde{\vartheta}^T_{j,i_j}\wp_{j,i_j} + d_{j,i_j}\end{aligned} \tag{7}$$

where $\hat{\psi}_{j,i_j} = \psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \hat{\vartheta}^T_{j,i_j}\wp_{j,i_j})$, $\hat{\psi}'_{j,i_j} = [\hat{\psi}'_{j,i_j1}, \hat{\psi}'_{j,i_j2}, \ldots, \hat{\psi}'_{j,i_jq}]^T \in R^{q\times m}$ with $\hat{\psi}'_{j,i_jk} = (\partial\psi_{j,i_jk}(\bar{\lambda}_{j,i_j}, \tau_{j,i_j}))/\partial\omega_{j,i_j}|_{\tau_{j,i_j}=\hat{\vartheta}^T_{j,i_j}\wp_{j,i_j}}$, and the residual terms d_{j,i_j} are bounded by

$$\begin{aligned}|d_{j,i_j}| \le d^*_{j,i_j} = \|\vartheta_{j,i_j}\|_F \left\|\wp_{j,i_j}\hat{\theta}^T_{j,i_j}\hat{\psi}'_{j,i_j}\right\|_F + |\theta_{j,i_j}|_1\\ + \|\theta_{j,i_j}\| \left\|\hat{\psi}'_{j,i_j}\hat{\vartheta}^T_{j,i_j}\wp_{j,i_j}\right\|\end{aligned} \tag{8}$$

For the sake of clarity, let $\|\cdot\|$ denotes the Euclidean norm of a vector, $\|\cdot\|_F$ denotes the Frobenius norm of a matrix, $\lambda_{\max}(T)$ and $\lambda_{\min}(T)$ denote the largest and smallest eigenvalues of a square matrix T, respectively.

III. ADAPTIVE NEURAL CONTROLLER DESIGN

This section will propose an adaptive backstepping-based neural controller for the MIMO strict-feedback nonlinear systems (1) with DSC technique. Firstly, consider the following change of coordinates:

$$\begin{cases} z_{j,1} = \xi_{j,1} - y_{j,d} \\ z_{j,i_j} = \xi_{j,i_j} - \nu_{j,i_j}, \quad i_j = 2, \ldots, \rho_j \end{cases} \tag{9}$$

where ν_{j,i_j} are the outputs of the following first-order filters

$$\omega_{j,i_j+1}\dot{\nu}_{j,i_j+1} + \nu_{j,i_j+1} = s_{j,i_j}, \quad i_j = 1, \ldots, \rho_j - 1 \tag{10}$$

where $\omega_{j,i_j+1} > 0$, $\nu_{j,i_j+1}(0) = s_{j,i_j}(0)$ and s_{j,i_j} are the virtual control laws to be designed later.

The virtual control laws and the actual control law will be designed as follows

$$\begin{aligned} s_{j,i_j} &= -c_{j,i_j}z_{j,i_j} - \hat{\theta}^T_{j,i_j}\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \hat{\vartheta}^T_{j,i_j}\wp_{j,i_j})\alpha_{j,i_j} \\ \alpha_{j,i_j} &= \tanh\left(\frac{z_{j,i_j}\hat{\theta}^T_{j,i_j}\psi_{j,i_j}(\bar{\lambda}_{j,i_j}, \hat{\vartheta}^T_{j,i_j}\wp_{j,i_j})}{\upsilon_{j,i_j}}\right)\end{aligned} \tag{11}$$

$$\begin{aligned} u_j &= -c_{j,\rho_j}z_{j,\rho_j} - \hat{\theta}^T_{j,\rho_j}\psi_{j,\rho_j}(\bar{\lambda}_{j,\rho_j}, \hat{\vartheta}^T_{j,\rho_j}\wp_{j,\rho_j})\alpha_{j,\rho_j} \\ \alpha_{j,\rho_j} &= \tanh\left(\frac{z_{j,\rho_j}\hat{\theta}^T_{j,\rho_j}\psi_{j,\rho_j}(\bar{\lambda}_{j,\rho_j}, \hat{\vartheta}^T_{j,\rho_j}\wp_{j,\rho_j})}{\upsilon_{j,\rho_j}}\right)\end{aligned} \tag{12}$$

Further, the corresponding adaptive laws are provided as follows:

$$\dot{\hat{\vartheta}}_{j,i_j} = \Gamma_{\vartheta_{j,i_j}}\left[z_{j,i_j}\wp_{j,i_j}\hat{\theta}^T_{j,i_j}\hat{\psi}'_{j,i_j} - \sigma_{j,i_j}\hat{\vartheta}_{j,i_j}\right] \tag{13}$$

$$\dot{\hat{\theta}}_{j,i_j} = \Gamma_{\theta_{j,i_j}}\left[z_{j,i_j}(\hat{\psi}_{j,i_j} - \hat{\psi}'_{j,i_j}\hat{\vartheta}^T_{j,i_j}\wp_{j,i_j}) - \sigma_{j,i_j}\hat{\theta}_{j,i_j}\right] \tag{14}$$

where $\Gamma_{\vartheta_{j,i_j}} = \Gamma^T_{\vartheta_{j,i_j}} > 0$ and $\Gamma_{\theta_{j,i_j}} = \Gamma^T_{\theta_{j,i_j}} > 0$ are adaptive gain matrices, and $c_{j,i_j} > 0$, $\sigma_{j,i_j} > 0$, $\upsilon_{j,i_j} > 0$ are design parameters.

It should be noticed that

$$\xi_{j,i_j+1} = z_{j,i_j+1} + s_{j,i_j} + \beta_{j,i_j+1} \tag{15}$$

where $\beta_{j,i_j+1} = \nu_{j,i_j+1} - s_{j,i_j}$ are the output errors of filters (10).

Invoking (10) and (11), we obtain $\dot{\upsilon}_{j,i_j+1} = -\left(\beta_{j,i_j+1}/\omega_{j,i_j+1}\right)$ and

$$\dot{\beta}_{j,i_j+1} = -\frac{\beta_{j,i_j+1}}{\omega_{j,i_j+1}} + B_{j,i_j+1}\left(\bar{z}_{j,i_j+1}, \bar{\beta}_{j,i_j+1}, \bar{\hat{\vartheta}}_{j,i_j}, \bar{\hat{\theta}}_{j,i_j}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right) \quad (16)$$

where $\bar{z}_{j,i_j+1} = \left[z_{j,1}, \ldots, z_{j,i_j+1}\right]^T$, $\bar{\beta}_{j,i_j+1} = \left[\beta_{j,2}, \ldots, \beta_{j,i_j+1}\right]^T$, $\bar{\hat{\vartheta}}_{j,i_j} = \left[\hat{\vartheta}_{j,1}, \ldots, \hat{\vartheta}_{j,i_j}\right]^T$, $\bar{\hat{\theta}}_{j,i_j} = \left[\hat{\theta}_{j,1}, \ldots, \hat{\theta}_{j,i_j}\right]^T$ and

$$B_{j,i_j+1}\left(\bar{z}_{j,i_j+1}, \bar{\beta}_{j,i_j+1}, \bar{\hat{\vartheta}}_{j,i_j}, \bar{\hat{\theta}}_{j,i_j}, y_{j,d}, \dot{y}_{j,d}, \ddot{y}_{j,d}\right) = -\frac{\partial s_{j,i_j}}{\partial z_{j,i_j}}\dot{z}_{j,i_j} - \frac{\partial s_{j,i_j}}{\partial \beta_{j,i_j}}\dot{\beta}_{j,i_j} - \frac{\partial s_{j,i_j}}{\partial \vartheta_{j,i_j}}\dot{\vartheta}_{j,i_j} - \frac{\partial s_{j,i_j}}{\partial \xi_{j,i_j}}\dot{\xi}_{j,i_j} - \frac{\partial s_{j,i_j}}{\partial \theta_{j,i_j}}\dot{\theta}_{j,i_j} \quad (17)$$

where $B_{j,i_j+1}(\cdot)$ are the continuous functions. Based on Lemma 1, for any given bounded initial condition $\hat{\vartheta}_{j,i_j}(0) \geq 0$ and $\hat{\theta}_{j,i_j}(0) \geq 0$, we have $\hat{\vartheta}_{j,i_j}(t) \geq 0$ and $\hat{\theta}_{j,i_j}(t) \geq 0$ for $\forall t \geq 0$, respectively.

IV. STABILITY ANALYSIS

In this section, the main stability result for the whole system is summarized in the following Theorem 1. Choose the following Lyapunov function candidate as follows

$$V = \sum_{j=1}^{m} V_j = \sum_{j=1}^{m}\sum_{i_j=1}^{\rho_j} V_{j,i_j} \quad (18)$$

where V_j is the Lyapunov function for the j th subsystem

$$V_j = \frac{1}{2}\sum_{i_j=1}^{\rho_j}\left(z_{j,i_j}^2 + tr\left\{\frac{\underline{\phi}_{j,i_j}\tilde{\vartheta}_{j,i_j}^T\Gamma_{\vartheta_{j,i_j}}^{-1}\tilde{\vartheta}_{j,i_j}}{2}\right\} + \frac{\underline{\phi}_{j,i_j}\tilde{\theta}_{j,i_j}^T\Gamma_{\theta_{j,i_j}}^{-1}\tilde{\theta}_{j,i_j}}{2}\right) + \frac{1}{2}\sum_{i_j=1}^{\rho_j-1}\beta_{j,i_j+1}^2 \quad (19)$$

$$V_{j,i_j} = \frac{1}{2}z_{j,i_j}^2 + tr\left\{\frac{\underline{\phi}_{j,i_j}\tilde{\vartheta}_{j,i_j}^T\Gamma_{\vartheta_{j,i_j}}^{-1}\tilde{\vartheta}_{j,i_j}}{2}\right\} + \frac{\underline{\phi}_{j,i_j}\tilde{\theta}_{j,i_j}^T\Gamma_{\theta_{j,i_j}}^{-1}\tilde{\theta}_{j,i_j}}{2} + \frac{1}{2}\beta_{j,i_j+1}^2 \quad \left(i_j = 1, 2, \ldots, \rho_j - 1\right) \quad (20)$$

$$V_{j,\rho_j} = \frac{1}{2}z_{j,\rho_j}^2 + tr\left\{\frac{\underline{\phi}_{j,\rho_j}\tilde{\vartheta}_{j,\rho_j}^T\Gamma_{\vartheta_{j,\rho_j}}^{-1}\tilde{\vartheta}_{j,\rho_j}}{2}\right\} + \frac{\underline{\phi}_{j,\rho_j}\tilde{\theta}_{j,\rho_j}^T\Gamma_{\theta_{j,\rho_j}}^{-1}\tilde{\theta}_{j,\rho_j}}{2} \quad (21)$$

where $\tilde{\vartheta}_{j,i_j} = \vartheta_{j,i_j}x - \hat{\vartheta}_{j,i_j}$ and $\tilde{\theta}_{j,i_j} = \theta_{j,i_j} - \hat{\theta}_{j,i_j}\left(i_j = 1, \ldots, \rho_j\right)$ are the estimation errors of ϑ_{j,i_j} and θ_{j,i_j}, respectively.

Theorem 1: Consider Assumptions 1-3. The virtual control laws are chosen as (11), the actual control law is given by (12), and the adaptive laws are provided by (13) and (14). For bounded initial conditions, there exist design parameters c_{j,i_j}, σ_{j,i_j}, υ_{j,i_j} and ω_{j,i_j} such that.

i) The compact set $\Pi_{j,\rho_j} \times \Pi_{j0} \times \Pi_{j\tau}$ is an invariant set, namely, $V_j(t) \leq \chi$ for $\forall t > 0$, and hence all of the signals in the closed-loop systems are SGUUB;

ii) The system output tracking error $z_1 = \left[z_{1,1}, \ldots, z_{m,1}\right]^T$ satisfies $\lim_{t\to\infty}\|z_1(t)\| \leq \mathbb{Z}_1$ with $\mathbb{Z}_1$ being a positive constant that relies on the design parameters.

Proof: The proof for Theorem 1 contains ρ_j steps, At each recursive step $j, i_j\left(1 \leq i_j \leq \rho_j - 1, j = 1, \ldots, m\right)$, the time derivative of V_{j,i_j} will be studied. Finally, the two properties in Theorem 1 will be proven in Step $j, \rho_j\ (j = 1, \ldots, m)$.

Step j, 1: To begin with, considering the first equation in (1) as follows

$$\dot{\xi}_{j,1} = \varphi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right) + \phi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right)\xi_{j,2} + \Delta_{j,1}(\xi, t) \quad (22)$$

Let $z_{j,1} = \xi_{j,1} - y_{j,d}$, and the time derivative of $z_{j,1}^2/2$ is

$$z_{j,1}\dot{z}_{j,1} = z_{j,1}\left(\varphi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right) + \phi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right)\xi_{j,2}\right) + z_{j,1}\Delta_{j,1}(\xi, t) - z_{j,1}\dot{y}_{j,d} \quad (23)$$

Define the compact set $\Pi_{j,1} := \left\{z_{j,1}\left|z_{j,1}^2 \leq 2\chi\right.\right\}$, with $\chi > 0$ being any design constant. For $\Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau}$ and $\phi_{j,1}(\xi_{j,1}, \tau_{j,1})$, the following Lemma holds.

Lemma 4: The unknown continuous control-gain function $\phi_{j,1}(\xi_{j,1}, \tau_{j,1})$ has maximum and minimum over compact set $\Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau}$, namely, there exist constants $\underline{\phi}_{j,1} > 0$ and $\bar{\phi}_{j,1} > 0$ such that $\underline{\phi}_{j,1} = \min_{\Pi_{j,1}\times\Pi_{j0}\times\Pi_{j\tau}} \phi_{j,1}(\xi_{j,1}, \tau_{j,1})$ and $\bar{\phi}_{j,1} = \max_{\Pi_{j,1}\times\Pi_{j0}\times\Pi_{j\tau}} \phi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right)$.

Proof: Observing $z_{j,1} = \xi_{j,1} - y_{j,d}$ and $\xi_{j,1} = z_{j,1} + y_{j,d}$, the function $\phi_{j,1}(\xi_{j,1}, \tau_{j,1})$ can be given by

$$\phi_{j,1}(\xi_{j,1}, \tau_{j,1}) = \Phi_{j,1}(z_{j,1}, y_{j,d}, \tau_{j,1}) \quad (24)$$

where $\Phi_{j,1}(z_{j,1}, y_{j,d}, \tau_{j,1})$ is a continuous function of $z_{j,1}$, $y_{j,d}$ and $\tau_{j,1}$, and $\Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau}$ is a compact set since $\Pi_{j,1}$, Π_{j0} and $\Pi_{j\tau}$ are compact sets respectively. Furthermore, it is derive from (24) that all the variables of $\Phi_{j,1}(z_{j,1}, y_{j,d}, \tau_{j,1})$ lie in $\Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau}$. Thus, $\Phi_{j,1}(z_{j,1}, y_{j,d}, \tau_{j,1})$ has maximum $\bar{\phi}_{j,1}$ and minimum $\underline{\phi}_{j,1}$ over $\Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau}$. Consequently, we have

$$0 < \underline{\phi}_{j,1} \leq \phi_{j,1}(\xi_{j,1}, \tau_{j,1}) \leq \bar{\phi}_{j,1}, \quad \left(\xi_{j,1}, \tau_{j,1}\right) \in \Pi_{j,1} \times \Pi_{j0} \times \Pi_{j\tau} \quad (25)$$

Considering (25), we can rewrite (23) as

$$z_{j,1}\dot{z}_{j,1} = z_{j,1}\underline{\phi}_{j,1}\left(h_{j,1}\left(\bar{x}_{j,1}, \tau_{j,1}\right) + \xi_{j,2} + \phi_{j,1,0}\xi_{j,2}\right) + z_{j,1}\Delta_{j,1}(\xi, t) \quad (26)$$

where $h_{j,1}\left(\bar{x}_{j,1}, \tau_{j,1}\right) = \underline{\phi}_{j,1}^{-1}\left(\varphi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right) - \dot{y}_{j,d}\right)$ with $\bar{x}_{j,1} = \left[\xi_{j,1}, \dot{y}_{j,d}\right]^T$, $\phi_{j,1,0} = \underline{\phi}_{j,1}^{-1}\left(\phi_{j,1}\left(\xi_{j,1}, \tau_{j,1}\right)\right) - 1 > 0$.

In accordance with (8), (15) and Assumptions 3, (26) can be written as

$$\begin{aligned} z_{j,1}\dot{z}_{j,1} \leq\ & z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right)+\left|z_{j,1}\right|\Delta_{j,1}^{*} \\ & +z_{j,1}\phi_{j,1}\left(\theta_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\vartheta_{j,1}^{T}\wp_{j,1}\right)\right) \\ & +z_{j,1}\phi_{j,1}s_{j,1}+\left|z_{j,1}\right|\underline{\phi}_{j,1}\varepsilon_{j,1}^{*} \\ & +z_{j,1}\phi_{j,1}\phi_{j,1,0}s_{j,1} \end{aligned} \tag{27}$$

Substituting (11) into (27) yields

$$\begin{aligned} z_{j,1}\dot{z}_{j,1} \leq\ & \underline{\phi}_{j,1}\left(z_{j,1}\theta_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\vartheta_{j,1}^{T}\wp_{j,1}\right)\right)-c_{j,1}\underline{\phi}_{j,1}z_{j,1}^{2} \\ & +z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right)+\left|z_{j,1}\right|\Delta_{j,1}^{*} \\ & -\underline{\phi}_{j,1}\left(z_{j,1}\hat{\theta}_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\alpha_{j,1}\right) \\ & +\underline{\phi}_{j,1}\left(\left|z_{j,1}\hat{\theta}_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\right|\right) \\ & -\underline{\phi}_{j,1}\left(z_{j,1}\hat{\theta}_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\right) \\ & +\left|z_{j,1}\right|\underline{\phi}_{j,1}\varepsilon_{j,1}^{*}+z_{j,1}\underline{\phi}_{j,1}\phi_{j,1,0}s_{j,1} \end{aligned} \tag{28}$$

Invoking Lemma 2, it can be obtained that

$$\begin{aligned} z_{j,1}\dot{z}_{j,1} \leq\ & z_{j,1}\underline{\phi}_{j,1}\left(\theta_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\vartheta_{j,1}^{T}\wp_{j,1}\right)\right)+\left|z_{j,1}\right|\Delta_{j,1}^{*} \\ & -z_{j,1}\underline{\phi}_{j,1}\left(\hat{\theta}_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\right) \\ & +z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right) \\ & +z_{j,1}\underline{\phi}_{j,1}\phi_{j,1,0}s_{j,1}+\left|z_{j,1}\right|\underline{\phi}_{j,1}\varepsilon_{j,1}^{*} \\ & +0.2785\underline{\phi}_{j,1}\upsilon_{j,1}-c_{j,1}\phi_{j,1}z_{j,1}^{2} \end{aligned} \tag{29}$$

Considering (11), (25) and (26), the following inequality satisfies

$$\begin{aligned} & z_{j,1}\underline{\phi}_{j,1}\phi_{j,1,0}s_{j,1} \\ & =\underline{\phi}_{j,1}\phi_{j,1,0}\left(-c_{j,1}z_{j,1}^{2}-z_{j,1}\hat{\theta}_{j,1}^{T}\psi_{j,1}\left(\hbar_{j,1},\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\alpha_{j,1}\right)<0 \end{aligned} \tag{30}$$

Noting that (7) and (30), one reaches

$$\begin{aligned} z_{j,1}\dot{z}_{j,1} \leq\ & z_{j,1}\underline{\phi}_{j,1}\left(\tilde{\theta}_{j,1}^{T}\left(\hat{\psi}_{j,1}-\hat{\psi}_{j,1}'\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\right)+\left|z_{j,1}\right|\Delta_{j,1}^{*} \\ & +z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right)+z_{j,1}\underline{\phi}_{j,1}d_{j,1} \\ & +z_{j,1}\underline{\phi}_{j,1}\left(\hat{\theta}_{j,1}^{T}\hat{\psi}_{j,1}'\tilde{\vartheta}_{j,1}^{T}\wp_{j,1}\right)-c_{j,1}\underline{\phi}_{j,1}z_{j,1}^{2} \\ & +0.2785\underline{\phi}_{j,1}\upsilon_{j,1}+\left|z_{j,1}\right|\underline{\phi}_{j,1}\varepsilon_{j,1}^{*} \end{aligned} \tag{31}$$

with $d_{j,1}$ being bounded by

$$\begin{aligned} \left|d_{j,1}\right| \leq d_{j,1}^{*} =\ & \left\|\vartheta_{j,1}\right\|_{F}\left\|\wp_{j,1}\hat{\theta}_{j,1}^{T}\hat{\psi}'_{j,1}\right\|_{F} \\ & +\left\|\theta_{j,1}\right\|\left\|\hat{\psi}'_{j,1}\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right\|+\left|\theta_{j,1}\right|_{1} \end{aligned} \tag{32}$$

Substituting (32) into (31), we can further have

$$\begin{aligned} z_{j,1}\dot{z}_{j,1} \leq\ & z_{j,1}\underline{\phi}_{j,1}\left(\hat{\theta}_{j,1}^{T}\hat{\psi}_{j,1}'\tilde{\vartheta}_{j,1}^{T}\wp_{j,1}\right)+0.2785\underline{\phi}_{j,1}\upsilon_{j,1} \\ & +z_{j,1}\underline{\phi}_{j,1}\left(\tilde{\theta}_{j,1}^{T}\left(\hat{\psi}_{j,1}-\hat{\psi}_{j,1}'\hat{\vartheta}_{j,1}^{T}\wp_{j,1}\right)\right) \\ & +z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right) \\ & +\left|z_{j,1}\right|\underline{\phi}_{j,1}\gamma_{j,1}^{*}-c_{j,1}\underline{\phi}_{j,1}z_{j,1}^{2} \end{aligned} \tag{33}$$

where $\gamma_{j,1}^{*}=d_{j,1}^{*}+\varepsilon_{j,1}^{*}+\underline{\phi}_{j,1}^{-1}\Delta_{j,1}^{*}$.

It follows from $\hat{\theta}_{j,1}^{T}\hat{\psi}_{j,1}'\tilde{\vartheta}_{j,1}^{T}\wp_{j,1} = tr\left\{\tilde{\vartheta}_{j,1}^{T}\wp_{j,1}\hat{\theta}_{j,1}^{T}\hat{\psi}_{j,1}'\right\}$, (13), (14), (16), (20) and (33) that the time derivative of $V_{j,1}$ is

$$\begin{aligned} \dot{V}_{j,1} \leq\ & z_{j,1}\phi_{j,1}\left(\xi_{j,1},\tau_{j,1}\right)\left(z_{j,2}+\beta_{j,2}\right)-c_{j,1}\underline{\phi}_{j,1}z_{j,1}^{2} \\ & +\left|z_{j,1}\right|\underline{\phi}_{j,1}\gamma_{j,1}^{*}+0.2785\underline{\phi}_{j,1}\upsilon_{j,1}-\beta_{j,2}^{2}\Big/\omega_{j,2} \\ & +\sigma_{j,1}tr\left\{\underline{\phi}_{j,1}\tilde{\vartheta}_{j,1}^{T}\hat{\vartheta}_{j,1}\right\}+\sigma_{j,1}\underline{\phi}_{j,1}\tilde{\theta}_{j,1}^{T}\hat{\theta}_{j,1} \\ & +\left|\beta_{j,2}B_{j,2}\left(\cdot\right)\right| \end{aligned} \tag{34}$$

Step j, $i_j(2 \leq i_j \leq \rho_j - 1, j = 1, \ldots, m)$: A similar procedure is recursively employed for each step j, i_j. Let $z_{j,i_j} = \xi_{j,i_j} - \nu_{j,i_j}$, the time derivative of $z_{j,i_j}^2\big/2$ is

$$\begin{aligned} z_{j,i_j}\dot{z}_{j,i_j} =\ & z_{j,i_j}\left(\varphi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\right)+z_{j,i_j}\Delta_{j,i_j}\left(\xi,t\right) \\ & +z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\xi_{j,i_j+1}-z_{j,i_j}\dot{\nu}_{j,i_j} \end{aligned} \tag{35}$$

Considering $\xi_{j,i_j} = z_{j,i_j} + \nu_{j,i_j}$ and $\beta_{j,i_j} = \nu_{j,i_j} - s_{j,i_j-1}$, we can obtain

$$\xi_{j,i_j} = z_{j,i_j} + \beta_{j,i_j} + s_{j,i_j-1} \tag{36}$$

In view of (11), we know that the virtual control s_{j,i_j-1} is a continuous function with respect to z_{j,i_j-1}, $\hat{\vartheta}_{j,i_j-1}$, β_{j,i_j-1} and $\hat{\theta}_{j,i_j-1}$. Therefore, ξ_{j,i_j} is a continuous function of z_{j,i_j}, β_{j,i_j}, $\hat{\vartheta}_{j,i_j-1}$ and $\hat{\theta}_{j,i_j-1}$. From $\xi_{j,1} = z_{j,1} + y_{j,d}$ and (36), the functions $\phi_{j,i_j}(\Xi_{j,i_j}, \tau_{j,i_j})$ can be rewritten as

$$\phi_{j,i_j}(\bar{\xi}_{j,i_j},\tau_{j,i_j}) = \Phi_{j,i_j}(\bar{z}_{j,i_j},\bar{\beta}_{j,i_j},\bar{\hat{\vartheta}}_{j,i_j-1},\bar{\hat{\theta}}_{j,i_j-1},y_{j,d},\tau_{j,i_j}) \tag{37}$$

with $\Phi_{j,i_j}(\cdot)$ being a continuous function.

Then, define the following sets Π_{j,i_j} $\left(i_j = 2, \ldots, \rho_j - 1\right)$ as

$$\begin{aligned} \Pi_{j,i_j} := \Bigg\{ & \left[\bar{z}_{j,i_j}^{T},\bar{\beta}_{j,i_j}^{T},\bar{\hat{\vartheta}}_{j,i_j-1}^{T},\bar{\hat{\theta}}_{j,i_j-1}^{T}\right]^{T} \mid \sum_{k=1}^{i_j} z_{j,k}^{2}+\sum_{k=2}^{i_j}\beta_{j,k}^{2} \\ & +\sum_{k=1}^{i_j-1}\left(\underline{\phi}_{j,k}\tilde{\theta}_{j,k}^{T}\Gamma_{\theta_{j,k}}^{-1}\tilde{\theta}_{j,k}+tr\left\{\underline{\phi}_{j,k}\tilde{\vartheta}_{j,k}^{T}\Gamma_{\vartheta_{j,k}}^{-1}\tilde{\vartheta}_{j,k}\right\}\right)\leq 2\chi\Bigg\} \end{aligned} \tag{38}$$

where $\chi > 0$ is an arbitrary constant. For Π_{j,i_j} and $\phi_{j,i_j}(\bar{\xi}_{j,i_j}, \tau_{j,i_j})$, in a similar fashion as Lemma 4 was derived, we have that the functions $\phi_{j,i_j}(\bar{\xi}_{j,i_j}, \tau_{j,i_j})$ have maximum and minimum in $\Pi_{j,i_j} \times \Pi_{j0} \times \Pi_{j\tau}$, namely, there exist constants $\bar{\phi}_{j,i_j} > 0$ and $\phi_{j,i_j} > 0$ satisfying

$$\begin{aligned} & 0 < \underline{\phi}_{j,i_j} \leq \phi_{j,i_j}(\bar{\xi}_{j,i_j},\tau_{j,i_j}) \leq \bar{\phi}_{j,i_j}, \\ & \left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right) \in \Pi_{j,i_j}\times\Pi_{j0}\times\Pi_{j\tau} \end{aligned} \tag{39}$$

By using (35) and (39), we have

$$\begin{aligned} z_{j,i_j}\dot{z}_{j,i_j} =\ & z_{j,i_j}\underline{\phi}_{j,i_j}h_{j,i_j}\left(\hbar_{j,i_j},\tau_{j,i_j}\right)+z_{j,i_j}\underline{\phi}_{j,i_j}\xi_{j,i_j+1} \\ & +z_{j,i_j}\underline{\phi}_{j,i_j}\phi_{j,i_j,0}\xi_{j,i_j+1}+z_{j,i_j}\Delta_{j,i_j}\left(\xi,t\right) \end{aligned} \tag{40}$$

where $\phi_{j,i_j,0} = \underline{\phi}_{j,i_j}^{-1}\left(\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)\right) - 1 > 0$, $h_{j,i_j}\left(\hbar_{j,i_j}, \tau_{j,i_j}\right) = \underline{\phi}_{j,i_j}^{-1}\left(\varphi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right) - \dot{\nu}_{j,i_j}\right)$ with $\hbar_{j,i_j} = \left[\bar{\xi}_{j,i_j}, \dot{\nu}_{j,i_j}\right]^T$.

From (8), (11), (15), (40) and Assumptions 3, we obtain

$$\begin{aligned} z_{j,i_j}\dot{z}_{j,i_j} \leq\ & \underline{\phi}_{j,i_j}\left(z_{j,i_j}\theta_{j,i_j}^T\psi_{j,i_j}\left(\hbar_{j,i_j}, \vartheta_{j,i_j}^T\wp_{j,i_j}\right)\right) + \left|z_{j,i_j}\right|\Delta_{j,i_j}^* \\ & - \underline{\phi}_{j,i_j}\left(z_{j,i_j}\hat{\theta}_{j,i_j}^T\psi_{j,i_j}\left(\hbar_{j,i_j}, \hat{\vartheta}_{j,i_j}^T\wp_{j,i_j}\right)\alpha_{j,i_j}\right) \\ & + z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)\left(z_{j,i_j+1} + \beta_{j,i_j+1}\right) \\ & - c_{j,i_j}\underline{\phi}_{j,i_j}z_{j,i_j}^2 + \left|z_{j,i_j}\right|\underline{\phi}_{j,i_j}\varepsilon_{j,i_j}^* \\ & + z_{j,i_j}\underline{\phi}_{j,i_j}\phi_{j,i_j,0}s_{j,i_j} \end{aligned} \tag{41}$$

By using Lemma 2, (41) can be rewritten as

$$\begin{aligned} z_{j,i_j}\dot{z}_{j,i_j} \leq\ & z_{j,i_j}\underline{\phi}_{j,i_j}\phi_{j,i_j,0}s_{j,i_j} - c_{j,i_j}\underline{\phi}_{j,i_j}z_{j,i_j}^2 + \left|z_{j,i_j}\right|\Delta_{j,i_j}^* \\ & + z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)\left(z_{j,i_j+1} + \beta_{j,i_j+1}\right) \\ & + z_{j,i_j}\underline{\phi}_{j,i_j}\left(\theta_{j,i_j}^T\psi_{j,i_j}\left(\hbar_{j,i_j}, \vartheta_{j,i_j}^T\wp_{j,i_j}\right)\right) \\ & - z_{j,i_j}\underline{\phi}_{j,i_j}\left(\hat{\theta}_{j,i_j}^T\psi_{j,i_j}\left(\hbar_{j,i_j}, \hat{\vartheta}_{j,i_j}^T\wp_{j,i_j}\right)\right) \\ & + 0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j} + \left|z_{j,i_j}\right|\underline{\phi}_{j,i_j}\varepsilon_{j,i_j}^* \end{aligned} \tag{42}$$

Considering (7), (11), (39) and (40), we can rewrite (42) as follows

$$\begin{aligned} z_{j,i_j}\dot{z}_{j,i_j} \leq\ & z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)z_{j,i_j+1} + \left|z_{j,i_j}\right|\underline{\phi}_{j,i_j}\gamma_{j,i_j}^* \\ & + z_{j,i_j}\underline{\phi}_{j,i_j}\left(\tilde{\theta}_{j,i_j}^T\left(\hat{\psi}_{j,i_j} - \hat{\psi}_{j,i_j}'\hat{\vartheta}_{j,i_j}^T\wp_{j,i_j}\right)\right) \\ & + 0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j} - c_{j,i_j}\phi_{j,i_j}z_{j,i_j}^2 \\ & + z_{j,i_j}\underline{\phi}_{j,i_j}\left(\hat{\theta}_{j,i_j}^T\hat{\psi}_{j,i_j}'\tilde{\vartheta}_{j,i_j}^T\wp_{j,i_j}\right) \\ & + z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)\beta_{j,i_j+1} \end{aligned} \tag{43}$$

where $\gamma_{j,i_j}^* = d_{j,i_j}^* + \varepsilon_{j,i_j}^* + \underline{\phi}_{j,i_j}^{-1}\Delta_{j,i_j}^*$.

Combining (13), (14), (16), (20) and (43), the time derivative of V_{j,i_j} can be given by

$$\begin{aligned} \dot{V}_{j,i_j} \leq\ & \sigma_{j,i_j}tr\left\{\underline{\phi}_{j,i_j}\tilde{\vartheta}_{j,i_j}^T\hat{\vartheta}_{j,i_j}\right\} + \sigma_{j,i_j}\underline{\phi}_{j,i_j}\tilde{\theta}_{j,i_j}^T\hat{\theta}_{j,i_j} \\ & + z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j}, \tau_{j,i_j}\right)\left(z_{j,i_j+1} + \beta_{j,i_j+1}\right) \\ & + 0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j} + \left|z_{j,i_j}\right|\underline{\phi}_{j,i_j}\gamma_{j,i_j}^* \\ & + \left|\beta_{j,i_j+1}B_{j,i_j+1}(\cdot)\right| - c_{j,i_j}\underline{\phi}_{j,i_j}z_{j,i_j}^2 \\ & - \beta_{j,i_j+1}^2\Big/\omega_{j,i_j+1} \end{aligned} \tag{44}$$

Step j, $\rho_j(j = 1, \ldots, m)$: In this step, the two properties in Theorem 1 will be proven. Similar to Step j, i_j, noting $z_{j,\rho_j} = \xi_{j,\rho_j} - \nu_{j,\rho_j}$, the time derivative of $z_{j,\rho_j}^2\big/2$ is

$$\begin{aligned} z_{j,\rho_j}\dot{z}_{j,\rho_j} =\ & z_{j,\rho_j}\varphi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}\right) + z_{j,\rho_j}\Delta_{j,\rho_j}(\xi, t) \\ & + z_{j,\rho_j}\phi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}\right)u_j - z_{j,\rho_j}\dot{\nu}_{j,\rho_j} \end{aligned} \tag{45}$$

Similar to the former steps, the function $\phi_{j,\rho_j}(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j})$ can be expressed in the following form

$$\phi_{j,\rho_j}(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}) = \Phi_{j,\rho_j}\left(\bar{z}_{j,\rho_j}, \bar{\beta}_{j,\rho_j}, \bar{\hat{\vartheta}}_{j,\rho_j-1}, \bar{\hat{\theta}}_{j,\rho_j-1}, y_{j,d}, \tau_{j,\rho_j}\right) \tag{46}$$

where $\Phi_{j,\rho_j}(\cdot)$ is a continuous function.

Define the following set

$$\begin{aligned} \Pi_{j,\rho_j} := \Bigg\{ & \left[\bar{z}_{j,\rho_j}^T, \bar{\beta}_{j,\rho_j}^T, \bar{\hat{\vartheta}}_{j,\rho_j-1}^T, \bar{\hat{\theta}}_{j,\rho_j-1}^T\right]^T \mid \sum_{k=1}^{\rho_j} z_{j,k}^2 + \sum_{k=2}^{\rho_j}\beta_{j,k}^2 \\ & + \sum_{k=1}^{\rho_j-1}\left(\underline{\phi}_{j,k}\tilde{\theta}_{j,k}^T\Gamma_{\theta_{j,k}}^{-1}\tilde{\theta}_{j,k} + tr\left\{\underline{\phi}_{j,k}\tilde{\vartheta}_{j,k}^T\Gamma_{\vartheta_{j,k}}^{-1}\tilde{\vartheta}_{j,k}\right\}\right) \leq 2\chi \Bigg\} \end{aligned} \tag{47}$$

with $\chi > 0$ being an arbitrary constant.

It should be noted that all the variables of $\Phi_{j,\rho_j}(\cdot)$ are included in the compact set $\Pi_{j,\rho_j} \times \Pi_{j0} \times \Pi_{j\tau}$, that is, the function $\Phi_{j,\rho_j}(\cdot)$ has maximum $\bar{\phi}_{j,\rho_j} = \max_{\Pi_{j,\rho_j}\times\Pi_{j0}\times\Pi_{j\tau}} \phi_{j,\rho_j}(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j})$ and minimum $\underline{\phi}_{j,\rho_j} = \min_{\Pi_{j,\rho_j}\times\Pi_{j0}\times\Pi_{j\tau}} \phi_{j,\rho_j}(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j})$ such that

$$0 < \underline{\phi}_{j,\rho_j} \leq \phi_{j,\rho_j}(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}) \leq \bar{\phi}_{j,\rho_j} \tag{48}$$

Considering (45) and (48), it can be obtained that

$$\begin{aligned} z_{j,\rho_j}\dot{z}_{j,\rho_j} =\ & z_{j,\rho_j}\underline{\phi}_{j,\rho_j}h_{j,\rho_j}\left(\hbar_{j,\rho_j}, \tau_{j,\rho_j}\right) + z_{j,\rho_j}\underline{\phi}_{j,\rho_j}u_j \\ & + z_{j,\rho_j}\Delta_{j,\rho_j}(\xi, t) + z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\phi_{j,\rho_j,0}u_j \end{aligned} \tag{49}$$

where $h_{j,\rho_j}\left(\hbar_{j,\rho_j}, \tau_{j,\rho_j}\right) = \underline{\phi}_{j,\rho_j}^{-1}\left(\varphi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}\right) - \dot{\nu}_{j,\rho_j}\right)$ with $\hbar_{j,\rho_j} = \left[\bar{\xi}_{j,\rho_j}, \dot{\nu}_{j,\rho_j}\right]^T$, $\phi_{j,\rho_j,0} = \underline{\phi}_{j,\rho_j}^{-1}\left(\phi_{j,\rho_j}\left(\bar{\xi}_{j,\rho_j}, \tau_{j,\rho_j}\right)\right) - 1 > 0$.

In a similar way, utilizing (8), (12) and Lemma 2, we have

$$\begin{aligned} z_{j,\rho_j}\dot{z}_{j,\rho_j} \leq\ & z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\left(\theta_{j,\rho_j}^T\psi_{j,\rho_j}\left(\hbar_{j,\rho_j}, \vartheta_{j,\rho_j}^T\wp_{j,\rho_j}\right)\right) \\ & - z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\left(\hat{\theta}_{j,\rho_j}^T\psi_{j,\rho_j}\left(\hbar_{j,\rho_j}, \hat{\vartheta}_{j,\rho_j}^T\wp_{j,\rho_j}\right)\right) \\ & + 0.2785\underline{\phi}_{j,\rho_j}\upsilon_{j,\rho_j} + \left|z_{j,\rho_j}\right|\underline{\phi}_{j,\rho_j}\varepsilon_{j,\rho_j}^* \\ & - c_{j,\rho_j}\underline{\phi}_{j,\rho_j}z_{j,\rho_j}^2 + \left|z_{j,\rho_j}\right|\Delta_{j,\rho_j}^* \\ & + z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\phi_{j,\rho_j,0}u_j \end{aligned} \tag{50}$$

In accordance with (7), (12), (48) and (49), we rewrite (50) as follows

$$\begin{aligned} z_{j,\rho_j}\dot{z}_{j,\rho_j} \leq\ & z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\left(\tilde{\theta}_{j,\rho_j}^T\left(\hat{\psi}_{j,\rho_j} - \hat{\psi}_{j,\rho_j}'\hat{\vartheta}_{j,\rho_j}^T\wp_{j,\rho_j}\right)\right) \\ & + 0.2785\underline{\phi}_{j,\rho_j}\upsilon_{j,\rho_j} - c_{j,\rho_j}\phi_{j,\rho_j}z_{j,\rho_j}^2 \\ & + z_{j,\rho_j}\underline{\phi}_{j,\rho_j}\left(\hat{\theta}_{j,\rho_j}^T\hat{\psi}_{j,\rho_j}'\tilde{\vartheta}_{j,\rho_j}^T\wp_{j,\rho}\right) \\ & + \left|z_{j,\rho_j}\right|\underline{\phi}_{j,\rho_j}\gamma_{j,\rho_j}^* \end{aligned} \tag{51}$$

where $\gamma_{j,\rho_j}^* = d_{j,\rho_j}^* + \varepsilon_{j,\rho_j}^* + \underline{\phi}_{j,\rho_j}^{-1}\Delta_{j,\rho_j}^*$.

Recalling (13), (14), (21) and using the similar design procedure, the time derivative of V_{j,ρ_j} is

$$\begin{aligned} \dot{V}_{j,\rho_j} \leq\ & \sigma_{j,\rho_j}\underline{\phi}_{j,\rho_j}\tilde{\theta}_{j,\rho_j}^T\hat{\theta}_{j,\rho_j} + \sigma_{j,\rho_j}tr\left\{\underline{\phi}_{j,\rho_j}\tilde{\vartheta}_{j,\rho_j}^T\hat{\vartheta}_{j,\rho_j}\right\} \\ & + 0.2785\phi_{j,\rho_j}\upsilon_{j,\rho_j} + \left|z_{j,\rho_j}\right|\underline{\phi}_{j,\rho_j}\gamma_{j,\rho_j}^* - c_{j,\rho_j}\underline{\phi}_{j,\rho_j}z_{j,\rho_j}^2 \end{aligned} \tag{52}$$

In the following part, the stabilization of whole system will be investigated. Considering the Lyapunov function (18) and invoking (34), (44) and (52), it can be known that the time derivative of V_j is

$$\begin{aligned}\dot{V}_j \le& \sum_{i_j=1}^{\rho_j}\left(0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j}+|z_{j,i_j}|\underline{\phi}_{j,i_j}\gamma^*_{j,i_j}-c_{j,i_j}\underline{\phi}_{j,i_j}z^2_{j,i_j}\right)\\&+\sum_{i_j=1}^{\rho_j}\left(\sigma_{j,i_j}tr\left\{\underline{\phi}_{j,i_j}\tilde{\vartheta}^T_{j,i_j}\hat{\vartheta}_{j,i_j}\right\}+\sigma_{j,i_j}\underline{\phi}_{j,i_j}\tilde{\theta}^T_{j,i_j}\hat{\theta}_{j,i_j}\right)\\&+\sum_{i_j=1}^{\rho_j-1}z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\left(z_{j,i_j+1}+\beta_{j,i_j+1}\right)\\&+\sum_{i_j=1}^{\rho_j-1}\left(\left|\beta_{j,i_j+1}B_{j,i_j+1}(\cdot)\right|-\beta^2_{j,i_j+1}\Big/\omega_{j,i_j+1}\right)\end{aligned}\tag{53}$$

Using the following inequalities

$$\tilde{\theta}^T_{j,i_j}\hat{\theta}_{j,i_j}\le\frac{\|\theta_{j,i_j}\|^2}{2}-\frac{\left\|\tilde{\theta}_{j,i_j}\right\|^2}{2}$$

$$tr\left\{\tilde{\vartheta}^T_{j,i_j}\hat{\vartheta}_{j,i_j}\right\}\le\frac{\|\vartheta_{j,i_j}\|_F^2}{2}-\frac{\left\|\tilde{\vartheta}_{j,i_j}\right\|_F^2}{2}$$

we have

$$\begin{aligned}\dot{V}_j \le& \sum_{i_j=1}^{\rho_j}\left(0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j}+|z_{j,i_j}|\underline{\phi}_{j,i_j}\gamma^*_{j,i_j}-c_{j,i_j}\underline{\phi}_{j,i_j}z^2_{j,i_j}\right)\\&+\sum_{i_j=1}^{\rho_j-1}z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\left(z_{j,i_j+1}+\beta_{j,i_j+1}\right)\\&+\sum_{i_j=1}^{\rho_j-1}\left(\left|\beta_{j,i_j+1}B_{j,i_j+1}(\cdot)\right|-\beta^2_{j,i_j+1}\Big/\omega_{j,i_j+1}\right)\\&+\sum_{i_j=1}^{\rho_j}\left(\frac{1}{2}g_{j,i_j}\sigma_{j,i_j}\left(\|\theta_{j,i_j}\|^2+\|\vartheta_{j,i_j}\|_F^2\right)\right)\\&-\sum_{i_j=1}^{\rho_j}\left(\frac{1}{2}g_{j,i_j}\sigma_{j,i_j}\left(\left\|\tilde{\theta}_{j,i_j}\right\|^2+\left\|\tilde{\vartheta}_{j,i_j}\right\|_F^2\right)\right)\end{aligned}\tag{54}$$

By completion of squares, we can have

$$\left|\beta_{j,i_j+1}B_{j,i_j+1}(\cdot)\right|\le\frac{\beta^2_{j,i_j+1}B^2_{j,i_j+1}(\cdot)}{2k_{j,1}}+\frac{k_{j,1}}{2}$$

$$|z_{j,i_j}|\underline{\phi}_{j,i_j}\gamma^*_{j,i_j}\le\frac{\underline{\phi}^2_{j,i_j}z^2_{j,i_j}}{2k_{j,3}}+\frac{k_{j,3}\gamma^{*2}_{j,i_j}}{2}$$

$$z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\beta_{j,i_j+1}\le\frac{k_{j,2}\phi^2_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\beta^2_{j,i_j+1}}{2}+\frac{z^2_{j,i_j}}{2k_{j,2}}$$

$$z_{j,i_j}\phi_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)z_{j,i_j+1}\le\frac{z^2_{j,i_j}}{2}+\frac{\phi^2_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)z^2_{j,i_j+1}}{2}$$

where $k_{j,1}>0$, $k_{j,2}>0$ and $k_{j,3}>0$ are unknown constants. Thus, (54) can be rewritten as

$$\begin{aligned}\dot{V}_j \le& \sum_{i_j=1}^{\rho_j-1}\left(\frac{\beta^2_{j,i_j+1}B^2_{j,i_j+1}(\cdot)}{2k_{j,1}}+\frac{k_{j,2}\phi^2_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)\beta^2_{j,i_j+1}}{2}\right)\\&+\sum_{i_j=1}^{\rho_j}\left(\frac{\underline{\phi}^2_{j,i_j}z^2_{j,i_j}}{2k_{j,3}}-c_{j,i_j}\underline{\phi}_{j,i_j}z^2_{j,i_j}\right)-\sum_{i_j=1}^{\rho_j-1}\left(\frac{\beta^2_{j,i_j+1}}{\omega_{j,i_j+1}}\right)\\&+\sum_{i_j=1}^{\rho_j-1}\left(\frac{z^2_{j,i_j}}{2k_{j,2}}+\frac{z^2_{j,i_j}}{2}+\frac{\phi^2_{j,i_j}\left(\bar{\xi}_{j,i_j},\tau_{j,i_j}\right)z^2_{j,i_j+1}}{2}\right)\\&-\sum_{i_j=1}^{\rho_j}\left(\frac{1}{2}\underline{\phi}_{j,i_j}\sigma_{j,i_j}\left(\left\|\tilde{\theta}_{j,i_j}\right\|^2+\left\|\tilde{\vartheta}_{j,i_j}\right\|_F^2\right)\right)+C_j\end{aligned}\tag{55}$$

where $C_j=(\rho_j-1)k_{j,1}/2+\sum_{i_j=1}^{\rho_j}\left(\frac{1}{2}\underline{\phi}_{j,i_j}\sigma_{j,i_j}\left(\|\theta_{j,i_j}\|^2+\|\vartheta_{j,i_j}\|_F^2\right)+0.2785\underline{\phi}_{j,i_j}\upsilon_{j,i_j}+k_{j,3}\gamma^{*2}_{j,i_j}/2\right)$.

Then, let us briefly investigate the characteristics of the continuous functions $B_{j,i_j+1}(\cdot)$ $(i_j=1,\ldots,\rho_j-1)$. It is apparent from (16) that all the variables of $B_{j,i_j+1}(\cdot)$ are included in the compact set $\Pi_{j,i_j}\times\Pi_{j0}\times\Pi_{j\tau}$, consequently, $B_{j,i_j+1}(\cdot)$ has a maximums D_{j,i_j+1} over $\Pi_{j,i_j}\times\Pi_{j0}\times\Pi_{j\tau}$, respectively. That is, based on $\Pi_{j,i_j}\times\Pi_{j0}\times\Pi_{j\tau}$, we have $\left|B_{j,i_j+1}(\cdot)\right|\le D_{j,i_j+1}$.

Noting that (25), (39) and (48), choose $c_{j,1}\ge\underline{\phi}^{-1}_{j,1}\left(1/2+1/2k_{j,2}+\underline{\phi}^2_{j,1}\Big/2k_{j,3}+\varsigma_j\right)$, $c_{j,i_j}\ge\underline{\phi}^{-1}_{j,i_j}\left(\varsigma_j+1/2+1/2k_{j,2}+\bar{\phi}^2_{j,i_j-1}\Big/2+\underline{\phi}^2_{j,i_j}\Big/2k_{j,3}\right)$, $(2\le i_j\le\rho_j-1)$, $c_{j,\rho_j}\ge\underline{\phi}^{-1}_{j,\rho_j}\times\left(\bar{\phi}^2_{j,\rho_j-1}\Big/2+\underline{\phi}^2_{j,\rho_j}\Big/2k_{j,3}+\varsigma_j\right)$ and $1/\omega_{j,i_j+1}\ge k_{j,2}\bar{\phi}^2_{j,i_j}\Big/2+\varsigma_j+D^2_{j,i_j+1}\Big/2k_{j,1}$, $(1\le i_j\le\rho_j-1)$. Therefore, we can obtain

$$\begin{aligned}\dot{V}_j \le& -\sum_{i_j=2}^{\rho_j-1}\left(c_{j,i_j}\underline{\phi}_{j,i_j}-\frac{1}{2}-\frac{1}{2k_{j,2}}-\frac{\bar{\phi}^2_{j,i_j-1}}{2}-\frac{\underline{\phi}^2_{j,i_j}}{2k_{j,3}}\right)z^2_{j,i_j}\\&-\sum_{i_j=1}^{\rho_j}\left(\frac{\sigma_{j,i_j}}{\lambda_{\max}\left(\Gamma^{-1}_{\vartheta_{j,i_j}}\right)}tr\left\{\frac{\underline{\phi}_{j,i_j}\tilde{\vartheta}^T_{j,i_j}\Gamma^{-1}_{\vartheta_{j,i_j}}\tilde{\vartheta}_{j,i_j}}{2}\right\}\right)\\&-\sum_{i_j=1}^{\rho_j}\left(\frac{\sigma_{j,i_j}}{\lambda_{\max}\left(\Gamma^{-1}_{\theta_{j,i_j}}\right)}\frac{\underline{\phi}_{j,i_j}\tilde{\theta}^T_{j,i_j}\Gamma^{-1}_{\theta_{j,i_j}}\tilde{\theta}_{j,i_j}}{2}\right)+C_j\\&-\sum_{i_j=1}^{\rho_j-1}\left(\frac{1}{\omega_{j,i_j+1}}-\frac{D^2_{j,i_j+1}}{2k_{j,1}}-\frac{k_{j,2}\bar{\phi}^2_{j,i_j}}{2}\right)y^2_{j,i_j+1}\\&-\left(c_{j,1}\underline{\phi}_{j,1}-\frac{1}{2}-\frac{1}{2k_{j,2}}-\frac{\underline{\phi}^2_{j,1}}{2k_{j,3}}\right)z^2_{j,1}\\&-\left(c_{j,\rho_j}\underline{\phi}_{j,\rho_j}-\frac{\bar{\phi}^2_{j,\rho_j-1}}{2}-\frac{\underline{\phi}^2_{j,\rho_j}}{2k_{j,3}}\right)z^2_{j,\rho_j}\end{aligned}\tag{56}$$

where $\varsigma_j=\min\left\{\sigma_{j,i_j}\Big/\lambda_{\max}\left(\Gamma^{-1}_{\vartheta_{j,i_j}}\right),\sigma_{j,i_j}\Big/\lambda_{\max}\left(\Gamma^{-1}_{\theta_{j,i_j}}\right)\right\}$.

From (56), one obtains

$$\dot{V}_j \leq -\varsigma_j V_j + C_j \tag{57}$$

It can be seen from ς_j that C_j/ς_j can be made arbitrarily small by increasing c_{j,i_j}, and meanwhile decreasing $\lambda_{\max}\left(\Gamma_{\theta_{j,i_j}}^{-1}\right)$, $\lambda_{\max}\left(\Gamma_{\vartheta_{j,i_j}}^{-1}\right)$, σ_{j,i_j}, υ_{j,i_j} and ω_{j,i_j}. It is always possible to make $C_j/\varsigma_j \leq \chi$ by choosing the design parameters appropriately. Then, in view of (57), we have that $\dot{V}_j \leq 0$ holds for $V_j = \chi$: consequently, the compact set $\Pi_{j,\rho_j} \times \Pi_{j0} \times \Pi_{j\tau}$ is an invariant set and all signals of closed-loop system are SGUUB. Therefore, property (i) of Theorem 1 is proved.

Multiplying (57) by $e^{\varsigma_j t}$ and integrating over $[0, t]$ yields

$$V_j(t) \leq \left[V_j(0) - \Lambda\right] e^{-\varsigma_j t} + \Lambda \tag{58}$$

with $\Lambda = C_j/\varsigma_j$ being a positive constant. Thus we have

$$\lim_{t\to\infty} \left|z_{j,1}\right| \leq \lim_{t\to\infty} \sqrt{2V_j(t)} \leq \sqrt{2\Lambda} = \mathbb{Z}_{j,1} \tag{59}$$

Now let us consider the Lyapunov function candidate for the whole systems as $V = \sum_{j=1}^{m} V_j$. From (58), it can be derived that

$$\dot{V} = \sum_{j=1}^{m} \dot{V}_j \leq \sum_{j=1}^{m} \left[-\varsigma_j V_j + C_j\right] \leq -\lambda V + \Sigma \tag{60}$$

where $\lambda = \min\{\varsigma_1, \ldots, \varsigma_m\}$ and $\Sigma = \sum_{j=1}^{m} C_j$. Then, we further have

$$V(t) \leq [V(0) - \Delta] e^{-\lambda t} + \Delta \tag{61}$$

where $\Delta = \frac{\Sigma}{\lambda}$ is a positive constant.

Similarly, we have $\lim_{t\to\infty} V(t) \leq \Delta$, which leads to

$$\lim_{t\to\infty} \|z_1(t)\| \leq \lim_{t\to\infty} \sqrt{2V(t)} \leq \sqrt{2\Delta} = \mathbb{Z}_1 \tag{62}$$

This completes the proof.

V. SIMULATION RESULTS

In this section, two simulation examples are given to illustrate the effectiveness of the proposed method in this paper.

Example 1. Consider the nonlinear MIMO strict-feedback uncertain systems with periodic disturbances as follows:

$$\begin{cases} \dot{\xi}_{1,1} = \dfrac{\xi_{1,1}^2\xi_{2,1}^2\tau_{1,1}^2 + \xi_{1,1}\xi_{2,1}\tau_{1,1}}{\xi_{1,1}^2\xi_{2,1}^2\tau_{1,1}^2 + 1} + \left(0.5 + e^{\xi_{1,1}^2\xi_{1,2}^2\tau_{1,1}^2}\right)\xi_{1,2} \\ \qquad + \Delta_{1,1}(\xi, t) \\ \dot{\xi}_{1,2} = \sin\left(\xi_{1,1}\xi_{1,2}\xi_{2,2}\tau_{1,2}\right) e^{\left(-\xi_{1,1}^2\xi_{1,2}^2\xi_{2,2}\tau_{1,2}^2\right)} + \Delta_{1,2}(\xi, t) \\ \qquad + \left(1 + e^{\xi_{1,1}^2\xi_{1,2}^3\xi_{2,1}^2\tau_{1,2}^2}\right) u_1 \\ \dot{\xi}_{2,1} = \dfrac{\xi_{1,1}\xi_{2,1}\tau_{2,1}}{\xi_{1,1}^2\xi_{2,1}^2\tau_{2,1}^2 + 0.5} + e^{\xi_{1,2}\xi_{2,1}\tau_{2,1}}\xi_{2,2} + \Delta_{2,1}(\xi, t) \\ \dot{\xi}_{2,2} = \left(\xi_{1,2}\xi_{2,1}\xi_{2,2}\tau_{2,2}\right)^3 e^{\left(-\xi_{1,2}^2\xi_{2,1}^2\xi_{2,2}^2\tau_{2,2}^2\right)} + \Delta_{2,2}(\xi, t) \\ \qquad + \left(1.5 + e^{\xi_{1,1}\xi_{2,1}\xi_{2,2}\tau_{2,2}}\right) u_2 \\ y_1 = \xi_{1,1}, \quad y_2 = \xi_{2,1} \end{cases} \tag{63}$$

where $\Delta_{1,1} = 0.5\cos\left(\xi_{1,1}^2\xi_{2,1}\xi_{2,2}\right)\sin(0.2t)$, $\Delta_{1,2} = 0.5 \times \cos\left(\xi_{1,2}^2 + \xi_{1,2}\xi_{2,1}\right)$, $\Delta_{2,1} = 2\sin\left(\xi_{1,1}\xi_{2,1}\xi_{1,2}^2\right)$, $\Delta_{2,2} = (\sin(t))^3 \times \sin\left(\xi_{2,2}^2 + \xi_{2,1}^2\right)$, and $\tau_{1,2}(t) = |\cos(0.25t)|$, $\tau_{1,1}(t) = |\cos(0.5t)|$, $\tau_{2,2}(t) = |\cos(t)|$ and $\tau_{2,1}(t) = |\sin(0.5t)|$ are the unknown time-varying disturbances with a known common period $T_{1,1} = 2\pi$, $T_{1,2} = 4\pi$, $T_{2,1} = 2\pi$ and $T_{2,2} = \pi$. We assume the reference signal $y_{1,d} = 0.5(\sin(t) + \sin(0.5t))$ and $y_{2,d} = \sin(t)$. Note that the control gain functions $\phi_{1,1} = \left(0.5 + e^{\xi_{1,1}^2\tau_{1,1}^2}\right)$, $\phi_{2,1} = e^{\xi_{2,1}\tau_{2,1}}$, $\phi_{1,2} = \left(1 + e^{\xi_{1,1}^2\xi_{1,2}^3\tau_{1,2}^2}\right)$ and $\phi_{2,2} = \left(1.5 + e^{\xi_{2,1}\xi_{2,2}\tau_{2,2}}\right)$ cannot be bounded a priori, but they apparently satisfy Assumption 2. Thus, where existing methods cannot be applied, our scheme can be used to the nonlinear system (63).

According to Theorem 1, the virtual control laws and actual control laws are constructed as

$$\begin{aligned} s_{1,1} &= -c_{1,1}z_{1,1} - \hat{\theta}_{1,1}^T\psi_{1,1}\left(\bar{x}_{1,1}, \hat{\vartheta}_{1,1}^T\phi_{1,1}\right) \\ &\quad \times \tanh\left(\frac{z_{1,1}\hat{\theta}_{1,1}^T\psi_{1,1}\left(\bar{x}_{1,1}, \hat{\vartheta}_{1,1}^T\phi_{1,1}\right)}{\upsilon_{1,1}}\right) \\ s_{2,1} &= -c_{2,1}z_{2,1} - \hat{\theta}_{2,1}^T\psi_{2,1}\left(\bar{x}_{2,1}, \hat{\vartheta}_{2,1}^T\phi_{2,1}\right) \\ &\quad \times \tanh\left(\frac{z_{2,1}\hat{\theta}_{2,1}^T\psi_{2,1}\left(\bar{x}_{2,1}, \hat{\vartheta}_{2,1}^T\phi_{2,1}\right)}{\upsilon_{2,1}}\right) \\ u_1 &= -c_{1,2}z_{1,2} - \hat{\theta}_{1,2}^T\psi_{1,2}\left(\bar{x}_{1,2}, \hat{\vartheta}_{1,2}^T\phi_{1,2}\right) \\ &\quad \times \tanh\left(\frac{z_{1,2}\hat{\theta}_{1,2}^T\psi_{1,2}\left(\bar{x}_{1,2}, \hat{\vartheta}_{1,2}^T\phi_{1,2}\right)}{\upsilon_{1,2}}\right) \\ u_2 &= -c_{2,2}z_{2,2} - \hat{\theta}_{2,2}^T\psi_{2,2}\left(\bar{x}_{2,2}, \hat{\vartheta}_{2,2}^T\phi_{2,2}\right) \\ &\quad \times \tanh\left(\frac{z_{2,2}\hat{\theta}_{2,2}^T\psi_{2,2}\left(\bar{x}_{2,2}, \hat{\vartheta}_{2,2}^T\phi_{2,2}\right)}{\upsilon_{2,2}}\right) \end{aligned}$$

where $z_{1,1} = \xi_{1,1} - y_{1,d}$, $z_{1,2} = \xi_{1,2} - \nu_{1,2}$, $z_{2,1} = \xi_{2,1} - y_{2,d}$ and $z_{2,2} = \xi_{2,2} - \nu_{2,2}$, and the adaption laws are given by (13) and (14) with design parameters $\sigma_{1,1} = \sigma_{1,2} = 0.2$, $\sigma_{2,1} = \sigma_{2,2} = 0.02$, $\Gamma_{\vartheta_{1,1}} = \Gamma_{\vartheta_{1,2}} = diag\{1\}$, $\Gamma_{\vartheta_{2,1}} = \Gamma_{\vartheta_{2,2}} = diag\{0.1\}$, $\Gamma_{\theta_{1,1}} = \Gamma_{\theta_{1,2}} = diag\{0.7\}$, $\Gamma_{\theta_{2,1}} = \Gamma_{\theta_{2,2}} = diag\{0.3\}$, $\upsilon_{1,1} = \upsilon_{1,2} = 0.25$, $\upsilon_{2,1} = \upsilon_{2,2} = 0.75$, $\omega_{1,2} = 10$, $\omega_{2,2} = 100$ and $c_{1,1} = 7$, $c_{1,2} = 6$, $c_{2,1} = 8$, $c_{2,2} = 9$. Let the initial conditions be $\left[\xi_{1,1}(0), \xi_{1,2}(0), \xi_{2,1}(0), \xi_{2,2}(0)\right]^T = [0.5, 0.3, 1.5, 2.0]^T$, $\hat{\vartheta}_{1,1}(0) = \hat{\vartheta}_{1,2}(0) = \hat{\vartheta}_{2,1}(0) = \hat{\vartheta}_{2,2}(0) = 0$ and $\hat{\theta}_{1,1}(0) = \hat{\theta}_{1,2}(0) = \hat{\theta}_{2,1}(0) = \hat{\theta}_{2,2}(0) = 0$. The simulation results of Example 1 are shown in Figs. 1-5.

It can be observed from Fig. 1 that the outputs y_1 and y_2 track the desired trajectories $y_{1,d}$ and $y_{2,d}$ with bounded error. Fig. 2 implies that the proposed scheme has a bounded control

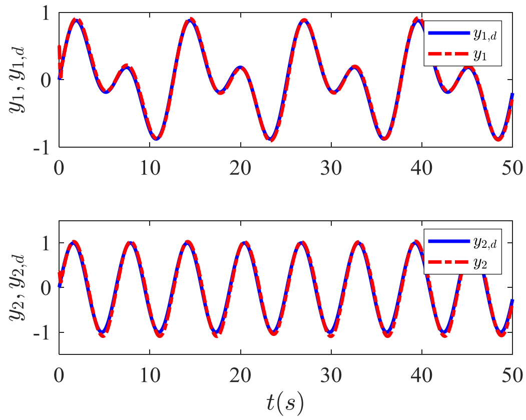

FIGURE 1. Outputs and desired trajectories.

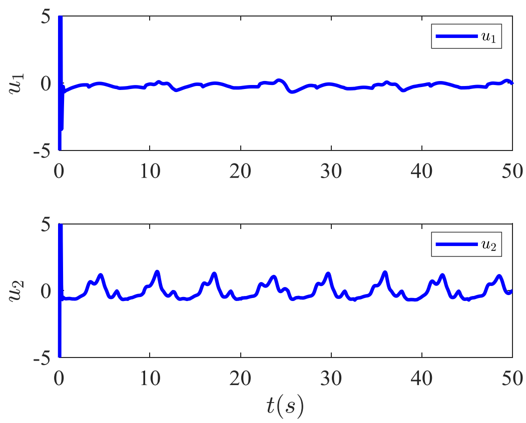

FIGURE 2. System inputs.

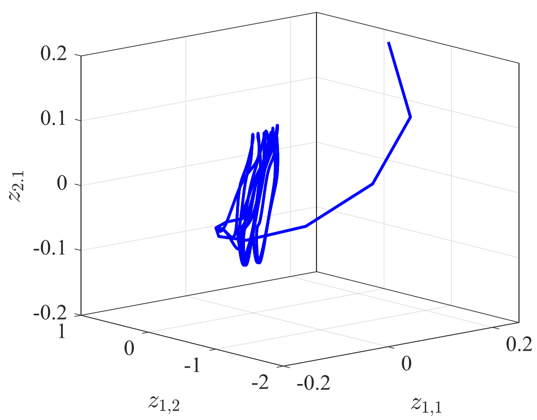

FIGURE 3. Phase portrait of $z_{1,1}, z_{1,2}$ and $z_{2,1}$.

input and Fig. 3 is given to explain phase portrait of $z_{1,1}$, $z_{1,2}$ and $z_{2,1}$. From this figure, it can be determined that the bounds for $z_{1,1}$, $z_{1,2}$ and $z_{2,1}$ are not overstepped. Additionally, the response curves of adaptive parameters $\hat{\vartheta}_{1,1}$, $\hat{\theta}_{1,1}$, $\hat{\vartheta}_{1,2}$, $\hat{\theta}_{1,2}$, $\hat{\vartheta}_{2,1}$, $\hat{\theta}_{2,1}$, $\hat{\vartheta}_{2,2}$ and $\hat{\theta}_{2,2}$ are depicted in Fig. 4, and Fig. 5 is presented to explain the boundedness of states $\xi_{1,1}$, $\xi_{1,2}$, $\xi_{2,1}$ and $\xi_{2,2}$.

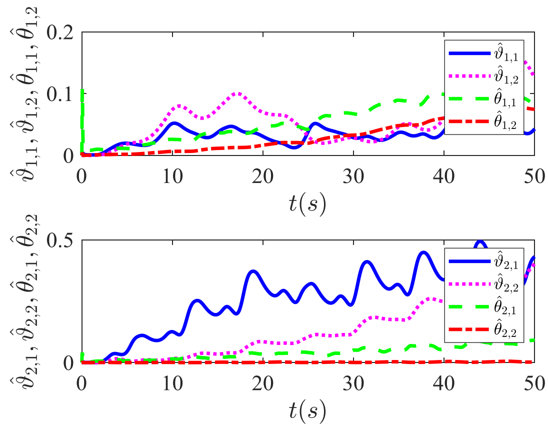

FIGURE 4. Curves of adaptation parameters.

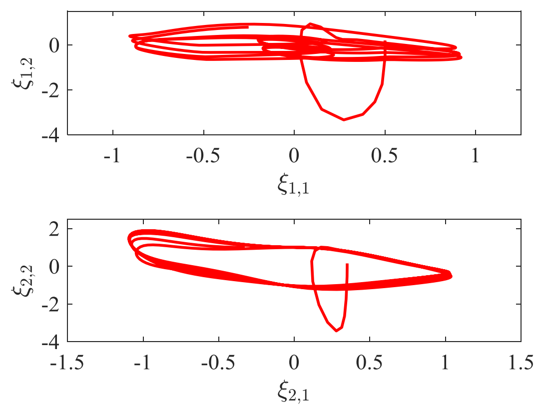

FIGURE 5. Phase portrait of states.

Example 2: Consider the following periodically disturbed nonlinear MIMO strict-feedback systems:

$$\begin{cases} \dot{\xi}_{1,1} = \left(1.1 - 0.1\cos\left(\xi_{1,1}\tau_{1,1}\right)\right)\xi_{1,2} + \Delta_{1,1}(\xi, t) \\ \qquad + \dfrac{1 - e^{-\xi_{1,1}\tau_{1,1}}}{1 + e^{-\xi_{1,1}\tau_{1,1}}} \\ \dot{\xi}_{1,2} = \left(0.9 - 0.1\tau_{1,2}^2 \sin^2\left(\xi_{1,1}\xi_{1,2}\right)\right)u_1 + \Delta_{1,2}(\xi, t) \\ \qquad + \dfrac{\xi_{1,1}^2\tau_{1,1}^2 + \xi_{1,2}^2\tau_{1,2}^2}{\xi_{1,1}^2\xi_{1,2}^2\tau_{1,2}^2 + 0.9} \\ \dot{\xi}_{2,1} = 0.2\cos\left(\xi_{2,1}\tau_{2,1}\right)\xi_{2,2} + \Delta_{2,1}(\xi, t) \\ \qquad + \dfrac{1 - e^{-1-\xi_{2,1}\tau_{2,1}}}{1 + e^{-1-\xi_{2,1}\tau_{2,1}}} \\ \dot{\xi}_{2,2} = \left(0.7 + 0.3\sin^2\left(\xi_{2,1}\xi_{2,2}\tau_{2,2}^2\right)\right)u_2 + \Delta_{2,2}(\xi, t) \\ \qquad + \dfrac{\xi_{2,1}\xi_{2,2}\tau_{2,2}}{\xi_{2,1}^2\xi_{2,2}^2\tau_{2,2}^2 + 0.3} \\ y_1 = \xi_{1,1} \\ y_2 = \xi_{2,1} \end{cases} \tag{64}$$

where $\Delta_{1,2} = 2\cos\left(\xi_{1,1} + \xi_{1,2} + \xi_{1,2}^2\xi_{2,1}^2\right)$, $\Delta_{1,1} = 2\cos(0.1t) \times \sin\left(\xi_{1,1}\xi_{1,2}^2\xi_{2,1}\right)$, $\Delta_{2,1} = 0.5\sin\left(\xi_{1,1}^2\xi_{1,2}^2\xi_{2,1}\right)$,

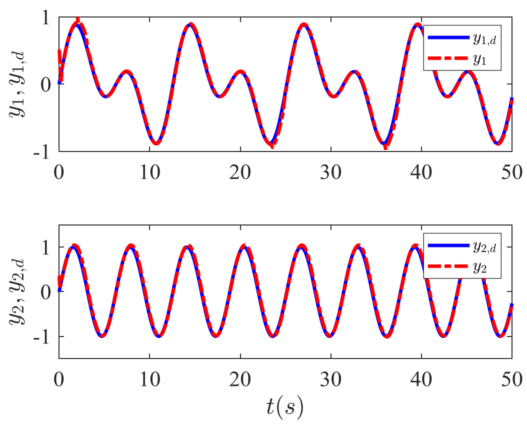

FIGURE 6. Outputs and desired trajectories.

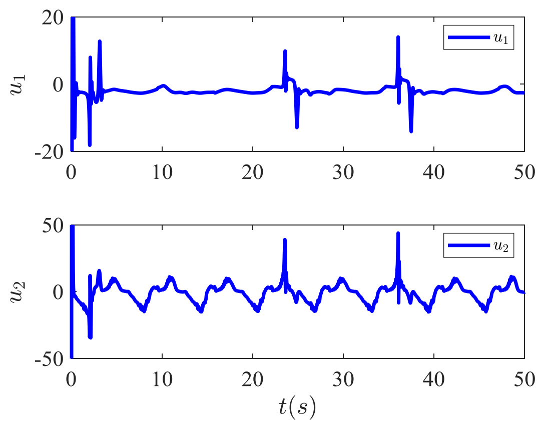

FIGURE 7. System inputs.

$\Delta_{2,2} = \sin^3(t) \times \sin\left(\xi_{1,1}^2 + \xi_{2,1}^2 + \xi_{2,2}^2\right)$, and $\tau_{1,1}(t) = |\sin(0.5t)|$, $\tau_{1,2}(t) = |\sin(0.25t)|$, $\tau_{2,1}(t) = |\cos(0.5t)|$ and $\tau_{2,2}(t) = |\cos(t)|$ are the unknown time-varying disturbances with a known common period $T_{1,1} = 2\pi$, $T_{1,2} = 4\pi$, $T_{2,1} = 2\pi$ and $T_{2,2} = \pi$. Define the desired tracking trajectories are $y_{1,d} = 0.5(\sin(t) + \sin(0.5t))$ and $y_{2,d} = \sin(t)$.

According to Theorem 1, the virtual control laws and actual control laws are constructed as

$$s_{1,1} = -c_{1,1}z_{1,1} - \hat{\theta}_{1,1}^T \psi_{1,1}\left(\bar{\lambda}_{1,1}, \hat{\vartheta}_{1,1}^T \phi_{1,1}\right) \times \tanh\left(\frac{z_{1,1}\hat{\theta}_{1,1}^T \psi_{1,1}\left(\bar{\lambda}_{1,1}, \hat{\vartheta}_{1,1}^T \phi_{1,1}\right)}{\upsilon_{1,1}}\right)$$

$$s_{2,1} = -c_{2,1}z_{2,1} - \hat{\theta}_{2,1}^T \psi_{2,1}\left(\bar{\lambda}_{2,1}, \hat{\vartheta}_{2,1}^T \phi_{2,1}\right) \times \tanh\left(\frac{z_{2,1}\hat{\theta}_{2,1}^T \psi_{2,1}\left(\bar{\lambda}_{2,1}, \hat{\vartheta}_{2,1}^T \phi_{2,1}\right)}{\upsilon_{2,1}}\right)$$

$$u_1 = -c_{1,2}z_{1,2} - \hat{\theta}_{1,2}^T \psi_{1,2}\left(\bar{\lambda}_{1,2}, \hat{\vartheta}_{1,2}^T \phi_{1,2}\right) \times \tanh\left(\frac{z_{1,2}\hat{\theta}_{1,2}^T \psi_{1,2}\left(\bar{\lambda}_{1,2}, \hat{\vartheta}_{1,2}^T \phi_{1,2}\right)}{\upsilon_{1,2}}\right)$$

$$u_2 = -c_{2,2}z_{2,2} - \hat{\theta}_{2,2}^T \psi_{2,2}\left(\bar{\lambda}_{2,2}, \hat{\vartheta}_{2,2}^T \phi_{2,2}\right) \times \tanh\left(\frac{z_{2,2}\hat{\theta}_{2,2}^T \psi_{2,2}\left(\bar{\lambda}_{2,2}, \hat{\vartheta}_{2,2}^T \phi_{2,2}\right)}{\upsilon_{2,2}}\right)$$

where $z_{1,1} = \xi_{1,1} - y_{1,d}$, $z_{1,2} = \xi_{1,2} - \nu_{1,2}$, $z_{2,1} = \xi_{2,1} - y_{2,d}$ and $z_{2,2} = \xi_{2,2} - \nu_{2,2}$, and the adaption laws are given by (13) and (14) with design parameter $\sigma_{1,1} = \sigma_{1,2} = 0.3$, $\sigma_{2,1} = \sigma_{2,2} = 0.05$, $\Gamma_{\vartheta_{1,1}} = \Gamma_{\vartheta_{1,2}} = diag\{2\}$, $\Gamma_{\vartheta_{2,1}} = \Gamma_{\vartheta_{2,2}} = diag\{0.2\}$, $\Gamma_{\theta_{1,1}} = \Gamma_{\theta_{1,2}} = diag\{0.8\}$, $\Gamma_{\theta_{2,1}} = \Gamma_{\theta_{2,2}} = diag\{0.5\}$, $\upsilon_{1,1} = \upsilon_{1,2} = 0.5$,

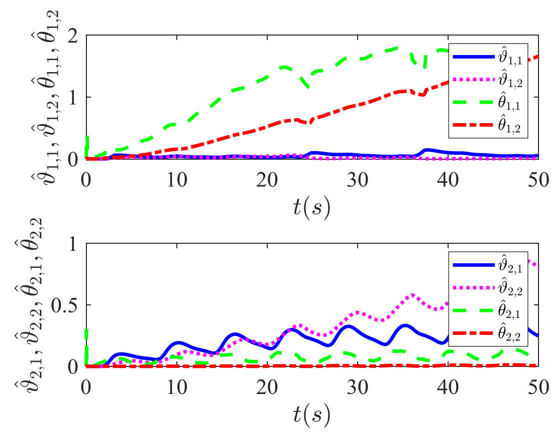

FIGURE 8. Curves of adaptation parameters.

$\upsilon_{2,1} = \upsilon_{2,2} = 0.35$, $\omega_{1,2} = 10$, $\omega_{2,2} = 100$ and $c_{1,1} = 10$, $c_{1,2} = 8$, $c_{2,1} = 6$, $c_{2,2} = 5$. Let the initial conditions be $\left[\xi_{1,1}(0), \xi_{1,2}(0), \xi_{2,1}(0), \xi_{2,2}(0)\right]^T = [0.25, 0.5, 1.2, 2.1]^T$, $\hat{\vartheta}_{1,1}(0) = \hat{\vartheta}_{1,2}(0) = 0$, $\hat{\vartheta}_{2,1}(0) = \hat{\vartheta}_{2,2}(0) = 0$ and $\hat{\theta}_{1,1}(0) = \hat{\theta}_{1,2}(0) = 0$, $\hat{\theta}_{2,1}(0) = \hat{\theta}_{2,2}(0) = 0$. The simulation results of Example 1 are shown in Figs. 6-8.

In these results, Fig. 6 shows that system outputs y_1 and y_2 track the desired trajectories $y_{1,d}$ and $y_{2,d}$ with small tracking error. From Fig. 7, we can see that the controller works very well and the signals of control inputs are bounded, and Fig. 8 illustrates the boundedness of adaptive parameters, respectively. From the above simulation results, the proposed scheme is able to guarantee the stability of the control systems and obtain fairly good control performance.

VI. CONCLUSION

An adaptive neural control design scheme has been presented for a class of nonlinear MIMO strict-feedback systems with periodic disturbances and possibly unbounded control gain control. In comparison with the existing research results, the restrictive assumption that the upper and lower bounds of control gain functions must be positive constants or coefficients has been removed by introducing appropriate compact sets where the maximums and minimums values of continuous control gain functions are well defined and used in the control design. Moreover, the novel FSE-RBFNNs-based approximation is used to model each suitable periodically disturbed function in systems, and the DSC technique is constructively employed to solve the problem of ''explosion of complexity''. Finally, the stability of the closed-loop system has been rigorously proved by Lyapunov analysis and invariant set theory, while the tracking error has been shown to converge to a residual set that can be made as small as desired by adjusting design parameters appropriately. The performance of the proposed approach has been verified through two simulation examples.

APPENDIX

The authors declare that there is no conflict of interests regarding the publication of this paper.

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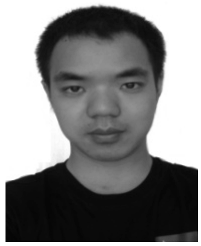

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