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Multiple Radios for Fast Rendezvous in Heterogeneous Cognitive Radio Networks

AOHAN LI[1], (Student Member, IEEE), GUANGJIE HAN[2], (Senior Member, IEEE), AND TOMOAKI OHTSUKI[1], (Senior Member, IEEE)

[1]Department of Information and Computer Science, Keio University, Yokohama 223-8522, Japan
[2]Department of Information and Communication Systems, Hohai University, Changzhou 213000, China

Corresponding author: Aohan Li (liaohan@ohtsuki.ics.keio.ac.jp)

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ABSTRACT In cognitive radio networks (CRNs), if two unlicensed secondary users (SUs) want to communicate with each other, they need to rendezvous with each other on the same channel at the same time. Rendezvous is the first key step for SUs to be able to communicate with each other. Channel hopping (CH) is a representative technique to solve the rendezvous problem in CRNs. SUs equipped with multiple radios can significantly reduce the time-to-rendezvous (TTR) for several existing CH algorithms while the additional cost is low. However, several drawbacks exist in the existing CH algorithms based on multiple radios. One of the main drawbacks is that they cannot be well applied in heterogeneous CRNs. The reason is that the number of radios for different SUs is implicitly assumed same or must be more than one in the existing CH algorithms based on multiple radios, which is unrealistic for heterogeneous CRNs. In heterogeneous CRNs, SUs may be equipped with different numbers of radios including one radio. To mainly address the above issue, hybrid radios rendezvous (HRR) algorithm is proposed in this paper. Moreover, the upper bounds of maximum TTR (MTTR) for the HRR algorithm are derived by a theoretical analysis. Furthermore, extensive simulations are performed to evaluate the expected TTR (ETTR), the MTTR, and the channel qualities of the rendezvous channels for the HRR algorithm. Simulation results show that rendezvous can be guaranteed by the HRR algorithm in heterogenous CRNs. Besides, the qualities of the rendezvous channels can be improved by the HRR algorithm. In addition, our algorithms can achieve rendezvous faster than several existing algorithms.

INDEX TERMS Cognitive radio networks, multiple radios, blind rendezvous, channel hopping.

I. INTRODUCTION

The usage of spectrum resources and regulation of radio emissions are coordinated by national regulatory bodies like the Federal Communications Commission (FCC). The FCC assigns spectrum to licensed holders, also known as Primary Users (PUs), on a long-term basis for large geographical regions [1]. However, a large portion of the assigned spectrum is used sporadically and geographical variations in the utilization of assigned spectrum ranges from 15% to 85% with a high variance in time [2]. Meanwhile, the unlicensed spectrum which is free to use for wireless devices, such as the Industrial Scientific and Medical (ISM) band is overcrowding with the increasing demands for wireless services [3]. To solve this issue, Cognitive Radio (CR) was introduced to improve the spectrum efficiency [4]. In Cognitive Radio Networks (CRNs), unlicensed Secondary Users (SUs) can opportunistically access the licensed spectrum without interfering PUs [5].

Prior to data transmission, SUs need first learn about the presences of their target SUs and establish communication links with them. This procedure, which is referred to as channel rendezvous, plays a critical role in configuring a CRN [6]. Existing solutions for the rendezvous problem can be classified into aided rendezvous schemes and unaided rendezvous schemes. Under the aided system, a centralized controller (e.g., Base Station (BS) in IEEE 802.22 Wireless Regional Area Networks (WRAN)) is necessary for accomplishing

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rendezvous [7]. A dedicated control channel is employed for exchanging information in this system. Most early works on this system to facilitate the rendezvous process owing to the fact that it is simple to implement. However, the dedicated control channel is not scalable, flexible, or robust [8]. The dedicated control channel may face problem of controlling channel saturation when the bottleneck of system becomes closely tied to the its capacity. This problem may lead to the result that no more users are allowed to achieve rendezvous in large scale networks. Moreover, it is vulnerable to jamming attacks [9].

The unaided rendezvous schemes that can solve the rendezvous problem without any centralized controller are more preferable. The unaided rendezvous schemes can be further classified into single control channel schemes, multiple control channels schemes and no control channel schemes. Single control channel schemes incur overhead and may act as a single point of failure while multiple control channels schemes have overhead in finding and identifying the control channels [10]. If no control channel is available, the SUs need figure out a way to find each other blindly, which is referred to as the blind rendezvous problem [8]. Blind rendezvous algorithms are applied for the rendezvous problem in distributed CRNs where there is no centralized controller, which is more flexible and scalable.

In blind rendezvous, a representative technique is Channel Hopping (CH), i.e., each SU hops among its available channels according to its CH sequence generated by the CH algorithm to attempt to rendezvous with its target SUs. However, SUs have no consensus about the channels that their target SUs access in distributed CRNs before rendezvous, which imposes great challenges to the blind rendezvous problem [11]. Besides, there also exist some other challenges for designing a CH-based rendezvous algorithm, they include:

(i) Asynchronous local clock. It is necessary to support the asynchronous scenario in distributed CRNs due to the difficulty and unrealistic to achieve clock synchronization between spatially dispersed SUs [12].

(ii) Heterogeneity. SUs in heterogeneous CRNs may have different spectrum sensing capabilities, different ranges of observable channels and different numbers of radios [13], [14].

(iii) Symmetric roles. Symmetric-role algorithms that do not need pre-assigned role (sender or receiver) are more applicable in practice. Because the prior knowledge of roles is unrealistic, and it is impossible to design different rules for different SUs according to their roles [15].

(iv) Anonymous information. Unique IDentifications (IDs) of SUs are utilized to generate CH sequences for the ID-based CH algorithms [24]. Because IEEE 802.22 uses a 48-bit universal MAC address to identify SUs, the IDs of SUs are usually generated by exploiting these MAC addresses in the existing ID-based CH algorithms [4], [16]. The unique ID can be expressed as a binary string with equal length terms as ID string [9]. In general, the Time To Rendezvous (TTR) for the ID-based CH algorithms is a multiplier of the length of ID string [29]. The TTR for the non-ID based CH algorithms is not related to the length of ID string. In general, the non-ID based algorithms have shorter TTR compared with the ID-based CH algorithms. Thus, the non-ID based CH algorithms are more favorable.

Among the extensive research literatures on CH algorithm, two performance metrics, namely Expected TTR (ETTR) and Maximum TTR (MTTR), are usually of the top concerns. The ETTR is the average (expected) latency before the first successfully rendezvous while the MTTR is the rendezvous latency in the worst case.

To shorten the rendezvous process under the challenges above, multiple radios technique is utilized when designing CH rendezvous algorithm in several latest researches [17]–[21]. In the multi-radio scenario, one SU can access multiple channels at the same time. Meanwhile, as the multi-radio wireless devices become realistic and popular, the cost of that is dropping sharply as well. Hence, the Time-To-Rendezvous (TTR) can be reduced by a large amount by multiple radios while the additional cost is low.

However, the existing multi-radio CH rendezvous algorithms present several disadvantages, which are listed as follows: (1) Rendezvous cannot be guaranteed within finite time and hence the MTTR is infinity for the random algorithm in [18] and [19]. (2) Different SUs are implicitly assumed to be equipped with the same number of radios for the Adaptive Rendezvous (AR) algorithm in [17] and the parallel sequence algorithm in [18] and [19], which is unpractical for heterogeneous CRNs. (3) The number of radios for each SU is implicitly assumed to be more than one for the Role-based Parallel Sequence (RPS) algorithm in [24] and [25] and the Adjustable Multi-Radio Rendezvous (AMRR) algorithm in [20]. Rendezvous can not be guaranteed when at least one SU is equipped with one radio for a pair of SUs. (4) Different radios of one SU may access the same channel at the same time for the AMRR algorithm in [20], the Multiple-radios Sunflower-Sets-based pairwise rendezvous (MSS) algorithm in [21], the independent sequence algorithm, the parallel sequence algorithm and the RPS algorithm in [18] and [19], which is a waste of radio resources. (5). The MTTR can not be shortened for the SUs with multiple radios compared with the SUs with one radio for the independent sequence algorithm in [18] and [19].

To address the above disadvantages, we develop a new CH rendezvous algorithm called Hybrid Radios Rendezvous (HRR) algorithm in this paper. Both symmetric model and asymmetric model are considered. In symmetric model, SUs have the same available channel sets. Symmetric model is suitable for SUs who are located in a relatively small area compared with their distance to PUs, in which scenario, the available channels for different SUs are influenced by the same PUs. In asymmetric model, different SUs have different available channel sets. Asymmetric model is applicable when the geographical locations of SUs are far apart from each other, in which scenario, the available channels for different SUs may be influenced by different PUs. Both symmetric model and asymmetric model have their applicable situation in practice. The contributions of this paper are summarized as follows.

- We present an Available Channel Distribution (ACD) algorithm, by which available channels are evenly divided among different radios of one SU before generating CH sequences. The proposed ACD algorithm can guarantee that different radios of one SU access different channels at the same time when the SU is equipped with multiple radios.
- We propose the HRR algorithm, which consists of Single Radio Rendezvous (SRR) algorithm and Multiple Radios Rendezvous (MRR) algorithm. When the SUs are equipped with one radio, their CH sequences are generated by the SRR algorithm. When the SUs are equipped with more than one radio, their CH sequences are generated by the MRR algorithm.
- We derive the upper bounds of MTTR for the HRR algorithm both under the symmetric model and the asymmetric model.
- We evaluate the proposed HRR algorithm and demonstrate its superiority compared with several state-of-the-art CH rendezvous algorithms through extensive simulations.

The rest of the paper is organized as follows. Section II reviews the state-of-the-art CH algorithms. Section III introduces the system model and problem formulation. Section IV presents the HRR algorithm. Section V derives the upper bounds of MTTR for the proposed HRR algorithm by theoretical analysis. Section VI demonstrates the simulation results. Finally, Section VII concludes the paper.

II. RELATED WORK

In this section, we review some representative CH rendezvous algorithms. According to the number of radios that SUs are equipped with, existing CH algorithms fall into two categories: single-radio-based CH algorithms and multi-radio-based CH algorithms. Since MTTR can be shortened for the SUs with multiple radios, we emphatically review the CH algorithms based on multiple radios.

A. SINGLE-RADIO-BASED CH ALGORITHMS

Most of the previous works have been focusing on the single-radio-based CH algorithm in which each SU is only equipped with one radio and can only access one channel at the same time slot [12], [22]–[26]. The Jump-Stay (JS) algorithm [22] is a typical single-radio-based CH algorithm, which generates CH sequences in rounds. Each round consists of one jump pattern and one stay pattern, SUs switch to access different available channels during jump pattern while stay at one specific available channel during stay pattern. The Enhanced Jump-Stay (EJS) algorithm [23] is based on the previous JS algorithm. The EJS algorithm lowers the upper bounds of MTTR compared with the JS algorithm. The T-Channel Hopping (T-CH) algorithm [24] generates CH sequences by concatenating rows in the default matrix sequentially. The default matrix contains two kinds of columns, which are jump column and stay column. The jump column is filled with distinct available channels while the stay column is filled with the same available channel. The Symmetric Asynchronous Rendezvous Channel Hopping (SARCH) algorithm [25] is proposed for symmetric asynchronous scenario. The pre-assignment is not required to the SARCH algorithm. The advanced Heterogeneous Channel Hopping (HCH) algorithm [26] based on a systematic approach utilizes group theory to design CH sequences. Two-step approach is used when constructing CH sequences. The first step is to create distinct bit strings whose lengths are same. The second step is to generate CH sequence for SU by using its selected bit string. The Modified Local Sequence (MLS) [12] algorithm generates sequences of varying lengths for different SUs. The MLS algorithm works significantly better than the JS algorithm in terms of MTTR when the number of the available channels is small.

B. MULTI-RADIO-BASED CH ALGORITHMS

Multiple radios are only utilized in a few existing CH rendezvous algorithms. Some of the multi-radio-based CH algorithms are designed based on the single-radio-based CH algorithms. One of the simplest way to implement the single-radio-based CH algorithms for the setting with multiple radios is presented as follows. Apply the CH sequence generated by the existing single-radio-based algorithm on all radios in parallel [18]. Up to now, there is almost no ID-based CH algorithm using multiple radios. In our opinion, the method mentioned above can be also used to implement the ID-based single-radio-based CH algorithms for the setting multiple radios. We present some state-of-the-art multi-radio-based CH algorithms as follows.

The random algorithm, the independent sequence algorithm, the parallel sequence algorithm and the RPS algorithm in [18] and [19] specifically exploit multiple radios for faster rendezvous. In the random algorithm, each radio randomly and independently selects an available channel in each time slot. Unfortunately, the MTTR for the random algorithm is equal to infinity. In the independent sequence algorithm, an existing algorithm in each radio which generates CH sequence independently. Unfortunately, the MTTR can not be shortened by the independent sequence even though the SUs are equipped with multiple radios. In the parallel algorithm, an existing algorithm is applied to generate a CH sequence. The CH sequence is in parallel performed in all radios. However, SUs need equip with the same number of radios. In the RPS algorithm, radios are divided into two groups, one dedicated radio and some general radios. The dedicated radio stays at one specific channel for a while, while the general radios keep on switching to access different available channels in the round-robin manner. If the generated channel is unavailable for SUs, it will be replaced by randomly selecting an available channel. However, although the RPS algorithm supposes that the number of radios for SUs can

TABLE 1. Comparison of CH algorithms.

Algorithms	Upper bounds of MTTR		Assumptions				
	Symmetric Model	Asymmetric Model	Asynchronous local clock	Symmetric roles	Anonymous Information	Common labelling	Multiple radios
JS [22]	$3P$	$3MP(P-G)+3P$	√	√	√	√	×
EJS [23]	$4P$	$4P(P+1-G)$	√	√	√	√	×
T-CH [24]	–	$2M^2+\left\lfloor \frac{M}{2} \right\rfloor M^*$	√	√	√		×
SARCH [25]	–	$8M^2+8M$	√	√	√	√	×
HCH [26]	–	$O(N_A N_B)$	√	√	×	×	×
MLS [12]	$O(l_A N_A)$	$O(l_B N_B^2)$ if $P_B \geq 2P_A$ or $P_B = P_A$ $O(l_B N_B^2 N_A)$ if $P_A < P_B < 2P_A$	√	√	×	√	×
RPS [18], [19]	$O(\frac{\lvert M\rvert}{\max\{m_A,m_B\}})$	$O(\frac{M^2}{\min\{m_A,m_B\}})$	√	√	√	√	√
AMRR [20]	$O(\frac{\lvert C_A\rvert}{\max\{m_A,m_B\}})$	$O(\frac{\lvert C_A\rvert\lvert C_B\rvert}{m_A m_B})$	√	√	√	√	√
MSS [21]	$\frac{MTTR(SSS)}{m_A m_B}$, where $MTTR(SSS)=6P_A$	$\frac{MTTR(SSS)}{m_A m_B}$, where $MTTR(SSS)= 6P_A^2, if\ P_A=P_B\ or\ P_A \geq 2P_B$ $MTTR(SSS)= 6P_A^2 P_B,\quad if P_B < P_A < 2P_B$	√	√	√	√	√
AR [17]	–	$\frac{3P\times M}{G\times I}$	√	√	√	√	√

Remarks: 1) M is the number of the licensed channels; $*$: M is a prime number; P is the smallest prime number greater than M; N_A and N_B are the numbers of sensible channels for SU_A and SU_B, respectively; $|C_A|$ and $|C_B|$ are the numbers of the available channels for SU_A and SU_B, respectively; P_A and P_B are the smallest prime numbers not smaller than $|C_A|$ and $|C_B|$, respectively; G is the number of commonly available channels between SU_A and SU_B; $l_i = \left\lfloor log_{P_{i-1}} M^c \right\rfloor + 1$, where P_i is the smallest prime number not smaller than $\max\{N_i, 3\}$, and c is a constant; m_A and m_B are the numbers of radios for SU_A and SU_B, respectively; I is the number of radios for SUs when all SUs are assumed to be equipped with the same number of radios; k_A and k_B are the numbers of stay radios for SU_A and SU_B, respectively. 2) $MTTR(SSS)$ is the upper bound of MTTR for the SSS algorithm. 3) "–" means that the upper bound of MTTR is not given in the corresponding reference.

be equal to 1, the upper bounds of MTTR will be infinity when the number of radios for SUs is equal to 1. Besides, different radios of one SU may access the same channel at the same time, which is a waste of radio resources. Because the random algorithm and the independent algorithm can not improve the MTTR. Besides, the parallel algorithm cannot be well used in heterogeneous CRNs, we are not consider the performance of these three algorithms in this paper. The AMRR algorithm [20] only uses the available channel set to generate CH sequences for SUs. In the AMRR algorithm, radios of one SU are divided into two groups, which are stay radios and jump radios. Jump radios parallelly access the available channels while stay radios stay at one specific channel for a while and then switch to stay at another channel during next duration. However, the AMRR algorithm has the same disadvantages with the RPS algorithm. The MSS algorithm [21] is based on the Single-radio Sunflower-Sets-based pairwise rendezvous (SSS) algorithm. Mathematical construction of sunflower sets is exploited to develop the SSS rendezvous algorithm. The SSS algorithm is used to generate periodic CH sequences for the first radio while the MSS algorithm cyclically rotates the sequence of the previous radio for the remaining radios. For instance, the CH sequence for the second radio of SU_A is generated by cyclically rotating the CH sequence of its previous radio (i.e., the first radio) by $2P_A$ time slots, where P_A is the smallest prime number not smaller than the number of local available channels for SU_A. However, P_A needs to be not smaller than 3 in the MSS algorithm, which is inapplicable to the condition when the number of the available channels for SU_A is equal to 1. Besides, different radios of one SU may access the same channel at the same time. In the AR algorithm [17], the possibility of rendezvous increases when multiple radios are available for SUs. However, the number of the radios for different SUs is assumed to be same, which is unpractical for heterogeneous CRNs.

The comparison of the state-of-the-art CH algorithms in terms of the upper bound of MTTR under the symmetric model and the asymmetric model, and assumptions (including asynchronism, symmetric roles, anonymity, common labeling and multiple radios) are shown in TABLE 1.

III. SYSTEM MODEL AND PROBLEM FORMULATION

In this section, we present the system model and the formulation of the CH rendezvous problem.

A. SYSTEM MODEL

We consider a CRN with $|N|$ non-overlapping licensed channels denoted as $N = \{c_1, c_2, \cdots c_i, \cdots, c_{|N|}\}$, where c_i is the i^{th} licensed channel in CR. P is the smallest prime number greater than $|N|$. The available channel set of SU_A is referred to as C_A. The number of available channels for SU_A is $|C_A|$. These available channels are indicated as $C_A = \{C_A(1),\ C_A(2), \cdots, C_A(i), \cdots, C_A(|C_A|)\}$, where $C_A(i)$ represents the i^{th} available licensed channel for SU_A. Without loss of generality, we consider the rendezvous between a pair of SUs, such as SU_A and SU_B. Besides, We assume that there exists at least one commonly available channel between SU_A and SU_B, i.e., $C_A \cap C_B \neq \emptyset$. The rendezvous between a pair of SUs can be extended to multiple pairs of SUs. Typically, the TTR is usually in the order of tens of milliseconds, which is very small compared with the PU dynamic [18]. Therefore, the status of channels (available or unavailable) is assumed to be static during the rendezvous process. The network time is divided into time slots. The length of each time slot is equal to $2t_e$. t_e is the sufficient time for SUs to successfully complete the processes of beaconing, handshaking, and establishing a link if they access the same available channel at the same time slot. In general, $t_e = 10$ms according to the IEEE 802.22 [27]. The local clock of SU_A is t_A. The clock drift between SUs is denoted as δ.

SU_A is equipped with m_A ($m_A \geq 1$) radios while SU_B is equipped with m_B ($m_B \geq 1$) radios. Note that m_A may be not equal to m_B in heterogeneous CRNs. SU_A can access m_A channels at each time slot to attempt rendezvous with other SUs. When $m_A > 1$, the radios of SU_A are generally divided into jump radios and stay radios. Assume that the number of jump radios for SU_A is equal to k_A. Hence, the number of stay radios for SU_A will be $m_A - k_A$. The available channel sets for the stay radios and the jump radios of SU_A is denoted as C_A^S and C_A^J, respectively. Note that $C_A^J = C_A \backslash C_A^S$. For guaranteeing that the radios of one SU access different channels at the same time slot, the available channels for the jump radios of SU are first allocated to each jump radio before generating CH sequences. The set consisting of the available channel sets for the jump radios of SU_A is denoted as C_A^{J*}. The available channel set for the j^{th} radio of SU_A is $C_A^{J*}(j)$. The length of half period of the CH sequence generated by the MRR algorithm for SU_A is w_A, where $w_A = \left\lceil \frac{|C_A|-(m_A-k_A)}{k_A} \right\rceil$. In order to explicitly present the proposed algorithm, we define $C[i]$ and $|C|$ as the i^{th} channel and the number of channels in the channel set C, respectively.

B. PROBLEM FORMULATION

The CH rendezvous problem is how to devise a fully distributed CH algorithm whereby each SU autonomously generates its CH sequence such that the SU can achieve small ETTR and bounded MTTR, in spite of random clock drift between them.

The CH sequence for SU_A can be denoted as $S_A = \{S_A^1, S_A^2, \cdots, S_A^{x_A}, \cdots, S_A^{m_A}\}$, where $S_A^{x_A}$ is the CH sequence for the x^{th} radio of SU_A. The $S_A^{x_A}$ during T time slots can be denoted as $\{S_A^{x_A}(1), S_A^{x_A}(2), \cdots, S_A^{x_A}(t_A), \cdots, S_A^{x_A}(T)\}$, where $S_A^{x_A}(t_A)$ is the channel that the x^{th} radio of SU_A accesses at its t^{th} local time slot. The channel set consisting of the channels that SU_A accesses at its t^{th} local time slot is denoted as $S_A(t_A)$, which can be expressed as:

$$S_A(t_A) = \left\{S_A^1(t_A), S_A^2(t_A), \cdots, S_A^{x_A}(t_A), \cdots, S_A^{m_A}(t_A)\right\}.$$

The CH rendezvous problem for multiple radios scenario is different with that for single radio scenario. In the multiple radios scenario, rendezvous can be achieved when any radio of one SU and any radio of its target SU access the same channel simultaneously. Hence, a new formulation for the multi-radio-based CH rendezvous problem needs to be presented, which can be formulated as:

If $\forall\delta, \forall C_A, C_B, \exists C \in C_A \cap C_B, x_A, x_B,\ s.t.\ S_A^{x_A}(t_A) = S_B^{x_B}(t_A + \delta) = C$, and C is available for SU_A and SU_B at time slot t_A, then the rendezvous is achieved.

Let $\Gamma(A, B, \delta)$ denote the TTR between SU_A and SU_B given that the local clock of SU_A is δ time slots behind that of SU_B. The TTR is indexed in accordance with the local clock left behind. This idea is natural because the zeroth slot of the clock left behind denotes when both of SUs start CH [28].

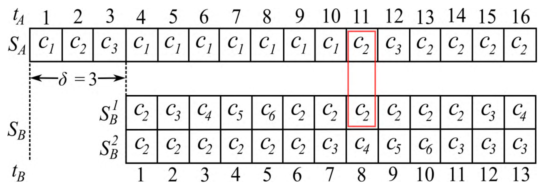

FIGURE 1. An example of CH rendezvous.

Fig. 1 illustrates an example of CH rendezvous process, where $C_A = \{c_1, c_2, c_3\}$, $C_B = \{c_2, c_3, c_4, c_5, c_6\}$, SU_B starts CH process behind SU_A for $\delta = 3$ time slots, $m_A = 1$, and $m_B = 2$. In this case, the TTR is indexed in accordance with the local clock of SU_B. From Fig. 1, we can see that SU_A and SU_B rendezvous with each other on channel c_2 at the 8^{th} time slot of SU_B. Hence, the $\Gamma(A, B, \delta) = t_B = 8$ while the rendezvous channel is c_2.

In this paper, we analyze and evaluate the performance of the HRR algorithm in terms of ETTR and MTTR. The ETTR can be expressed as

$$ETTR(A, B) = E[\min \Gamma(A, B, \delta)].$$

where $E[\cdot]$ denotes the expection operation. The MTTR can be expressed as

$$MTTR(A, B) = \max_{\forall\delta} \min \Gamma(A, B, \delta).$$

Some important notations being used in this paper are listed in Table 2.

IV. HYBRID RADIOS RENDEZVOUS ALGORITHM

In this section, we first present the ACD algorithm, by which the available channels are divided among different jump radios of one SU. Then, we propose the MRR algorithm for the SUs who are equipped with multiple radios to generate their CH sequences. The ACD algorithm is invoked by the MRR algorithm for guaranteeing that radios of one SU access different channels at the same time slot. Next, we introduce the SRR algorithm. The SRR algorithm is utilized to generate CH sequences for the SUs who are equipped with one radio. Finally, we present the HRR algorithm. The HRR algorithm consists of the MRR algorithm and the SRR algorithm. SUs can generate their CH sequences using the HRR algorithm

TABLE 2. Notations.

Variables	Definitions
N	The licensed channel set
$\|N\|$	The number of licensed channels
P	The smallest prime number greater than $\|N\|$
C_A	The available channel set of SU_A
$\|C_A\|$	The number of available channels for SU_A
$C_A(i)$	The i^{th} available licensed channel for SU_A
t_A	The local clock of SU_A
δ	The clock drift between two SUs
m_A	The number of radios for SU_A
k_A	The number of jump radios for SU_A
C_A^S	The available channel set for the stay radios of SU_A
C_A^J	The available channel set for the jump radios of SU_A, $C_A^J = C_A \backslash C_A^S$
C_A^{J*}	The set consisting of the available channel sets for the jump radios of SU_A
$C_A^*(j)$	The available channel set for the j^{th} radio of SU_A
w_A	The length of half period of the CH sequence generated by the MRR algorithm for SU_A
$C[i]$	The i^{th} channel in channel set C
$\|C\|$	The number of channels in channel set C
S_A	The CH sequence for SU_A
S_A^{xA}	The CH sequences for the x^{th} radio of SU_A
$S_A^{xA}(t_A)$	The channel that the x^{th} radio of SU_A accesses at its t^{th} local time slot
$S_A(t_A)$	The channel set consisting of the channels that SU_A accesses at its t^{th} local time slot
$\Gamma(A, B, \delta)$	The TTR between SU_A and SU_B

regardless of the number of radios. The MRR algorithm is invoked by the HRR algorithm when the SU is equipped with multiple radios while the SRR algorithm is invoked when the SU is equipped with one radio. The HRR algorithm can guarantee rendezvous between any pair of SUs in heterogeneous CRNs.

A. ACD ALGORITHM

For guaranteeing that the radios of one SU access different channels at the same time slot, we propose the ACD algorithm to divide the available channels among the jump radios for each SU. After dividing, each jump radio of one SU possesses an individual available channel set. The intersection of any two individual available channel sets for two different jump radios is empty. Besides, the jump radios only access the channels in their individual available channel sets. Hence, the ACD algorithm can guarantee that the jump radios of one SU access different channels at the same time slot. The ACD algorithm is formally presented in Algorithm 1. Note that $|C_A^J|$ denotes the number of channels allocated to the jump radios of SU_A in the ACD algorithm. Besides, from the first radio to the $(m_A - k_A)^{th}$ radio are stay radios while from the $(m_A - k_A + 1)^{th}$ radio to the m_A^{th} radio are jump radios for SU_A.

An example for the ACD algorithm is depicted in Fig. 2, where 15 sorted available channels ($|C_A^J| = 15$) for the jump radios of SU_A are evenly divided among its four jump radios ($k_A = 4$). The available channels are sorted in descending order according to the channel quality in the HRR algorithm before carrying out the ACD algorithm. The channel quality can be measured by noise and/or interference. The reason

Algorithm 1 ACD Algorithm

Require: $C_A^J,\ w_A,\ m_A,\ k_A$ *for* SU_A
Ensure: C_A^{J*}
$C_A^{J*} = \emptyset$
for $j = m_A - k_A + 1$ to m_A **do**
  for $q = 0$ to $w_A - 1$ **do**
    if $qk_A + j - (m_A - k_A) \le |C_A^J|$ **then**
      $C_A^{J*}(j) = C_A^{J*}(j) \cup C_A^J[qk_A + j - (m_A - k_A)]$
    end if
  end for
end for

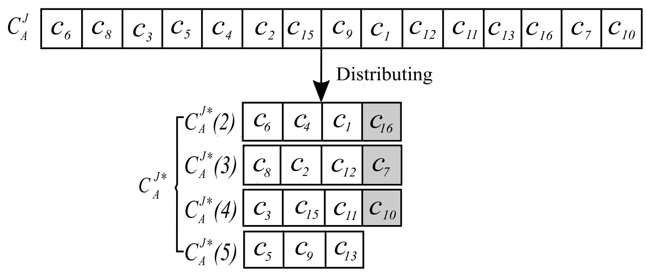

FIGURE 2. An example for the ACD algorithm.

for sorting available channels is to increase the probability of rendezvousing on the channels whose qualities are better. $m_A = 5$, and $w_A = 4$. The jump radios of SU_A start from its 2^{nd} radio to 5^{th} radio. After allocation, each jump radio of SU_A possesses an individual available channel set. For instance, the 1^{st} jump radio of SU_A possesses an individual available channel set $C_A^{J*}(2)$.

B. MRR ALGORITHM

The MRR algorithm is proposed to generate CH sequences for SUs who are equipped with more than one radio. For guaranteeing rendezvous between the SUs in spite of the clock drift, the radios of one SU generally consist of stay radios and jump radios. Each stay radio stays at one specific available channel during one period, and changes the stay channel every period. Each jump radio sequentially accesses the available channels in its individual available channel set generated by invoking the ACD algorithm at different time slots. All available channels except the channels that the stay radios stay at are visited at least once by the jump radios of SU_A during any continuous w_A time slots within one period. Hence, rendezvous must be achieved on the stay channel that the stay radio of the SU with a longer CH sequence period stays at regardless of the clock drift. The situation where the number of available channels is smaller than that of radios for one SU is also considered in the MRR algorithm. In such situation, all radios are stay radios, and each radio stays at the same channel all the time. Hence, rendezvous must be achieved on any commonly available channel for a pair of SUs in such situation.

The length of one period for the CH sequences generated by the MRR algorithm is equal to $2w_A$, where

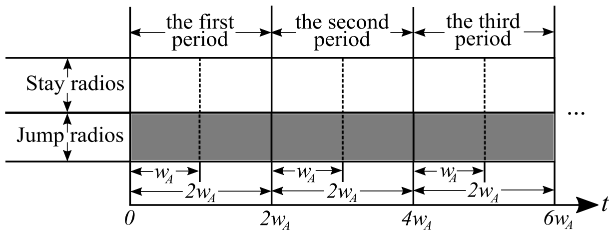

FIGURE 3. Structure of the CH sequences generated by the MRR algorithm.

$w_A = \left\lceil \frac{|C_A| - (m_A - k_A)}{k_A} \right\rceil$. The structure of the CH sequences generated by the MRR algorithm is shown in Fig. 3 while the pseudo code for the MRR algorithm is presented in Algorithm 2.

Algorithm 2 MRR Algorithm

Require: $C_A,\ m_A,\ k_A,\ t_A$ \\for SU_A
Ensure: $S_A\,(t_A)$

1: $S_A\,(t_A) = \emptyset$; $C_A^S = \emptyset$;
2: **if** $|C_A| > m_A$ **then**
3: $\quad w_A = \left\lceil \frac{|C_A| - (m_A - k_A)}{k_A} \right\rceil$;
4: $\quad$**for** $i = 1$ to $m_A - k_A$ **do**
5: $\quad\quad h = \left(\left(\left(\left\lfloor \frac{(t_A - 1)}{2w_A} \right\rfloor (m_A - k_A) + i - 1\right) \bmod |C_A|\right) + 1\right)$
6: $\quad\quad S_A^i\,(t_A) = C_A\,[h]$
7: $\quad\quad C_A^S = C_A^S \cup C_A\,[h]$
8: $\quad$**end for**
9: $\quad$**if** $(t_A - 1) \bmod 2w_A = 0$ **then**
10: $\quad\quad C_A^J = C_A \backslash C_A^S$
11: $\quad\quad$Invoke ACD Algorithm to divide the available channels among k_A jump radios
12: $\quad$**end if**
13: $\quad$**for** $i = m_A - k_A + 1$ to m_A **do**
14: $\quad\quad h = (t_A - 1) \bmod |C_A^{J*}(i)| + 1$
15: $\quad\quad S_A^i\,(t_A) = C_A^{J*}\,(i)\,[h]$
16: $\quad$**end for**
17: $\quad$**for** $x = 1$ to m_A **do**
18: $\quad\quad S_A\,(t_A) = S_A\,(t_A) \cup S_A^x\,(t_A)$
19: $\quad$**end for**
20: **else**
21: $\quad$**for** $q = 1$ to m_A **do**
22: $\quad\quad h = ((q - 1) \bmod |C_A|) + 1$
23: $\quad\quad S_A^q\,(t_A) = C_A\,[h]$; $S_A\,(t_A) = S_A\,(t_A) \cup S_A^q\,(t_A)$;
24: $\quad$**end for**
25: **end if**

Two simple examples of the CH sequences generated by the MRR algorithm are depicted in Fig. 4. In Fig. 4(a), $C_A = \{c_3,\ c_2,\ c_4,\ c_1\}$ after being sorted in descending order according to the channel quality, $m_A = 5$. Since $m_A > |C_A| = 4$, all radios of SU_A are stay radios and then the CH sequences for the radios of SU_A are generated by lines 21∼24 in Algorithm 2. For instance, the first radio of SU_A stays at channel c_3 derived by $C_A\,[h] = C_A\,[(1-1) \bmod 4 + 1] = C_A\,[1] = c_3$ while the second radio of SU_A stays at channel c_2 derived by $C_A\,[h] = C_A\,[(2-1) \bmod 4 + 1] = C_A\,[2] = c_2$. In Fig. 4(b), $C_B = \{c_6, c_3, c_2, c_5, c_4, c_1, c_7\}$ after being sorted in descending order according to the channel quality. m_B=4 while k_B=2. Since $m_B < |C_B| = 7$, the CH sequence is generated by lines 2∼19 in Algorithm 2. Fist, the length of half period for the CH sequence of SU_B is calculated by $w_B = \left\lceil \frac{|C_B| - (m_B - k_B)}{k_B} \right\rceil = \left\lceil \frac{7-(4-2)}{2} \right\rceil = 3$. In the first period (i.e., form the 1^{th} time slot to the 6^{th} time slot), the index of the channel that the first stay radio of SU_B stays at is calculated by line 5 (i.e., $h = \left(\left(\left(\left\lfloor \frac{(t_B-1)}{2w_B} \right\rfloor (m_B - k_B) + i - 1\right) \bmod |C_B|\right) + 1\right) = \left(\left(\left(\left\lfloor \frac{(1-1)}{6} \right\rfloor (4-2) + 1 - 1\right) \bmod 7\right) + 1\right) = 1$). Hence, the first stay radio of SU_B stays at channel $C_B[h] = C_B[1] = c_6$ during the first period. The second stay radio of SU_B gets its stay channel c_3 during the first period as the same way as the first radio. Therefore, the available channel set for the stay radios of SU_B during the first period is $C_B^S = \{c_6, c_3\}$ while the available channel set for the jump radios of SU_A during the first period is $C_B^J = C_B \backslash C_B^S = \{c_2, c_5, c_4, c_1, c_7\}$. Then, the ACD algorithm is invoked to divide C_B^J among the 2 jump radios of SU_B. After dividing, each jump radio of SU_B possesses its individual available channel set for the first period, which are $C_B^{J*}\,(3) = \{c_2, c_4, c_7\}$ and $C_B^{J*}\,(4) = \{c_5, c_1\}$, respectively. After that, the channels that the jump radios access are generated by lines 14 and 15 based on $C_B^{J*}\,(3)$ and $C_B^{J*}\,(4)$. For instance, the first jump radio gets its access channel c_2 at the 1^{st} time slot by $h = (1-1) \bmod |2| + 1 = 1$, and $C_B^{J*}(3)[h] = c_2$.

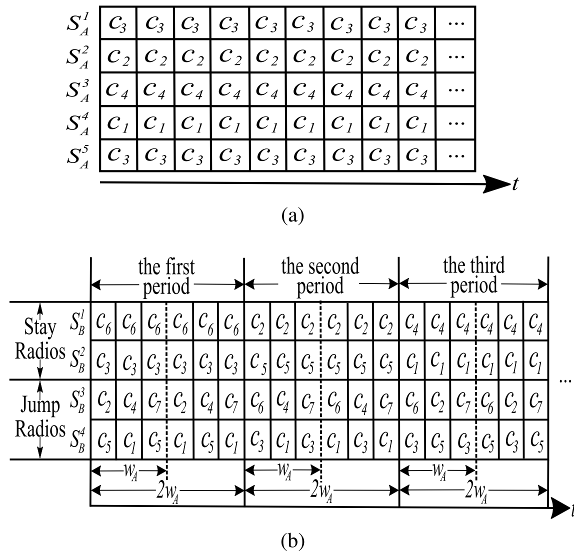

FIGURE 4. CH sequences generated by the MRR algorithm. (a) $m_A > |C_A|$. (b) $m_B < |C_B|$.

From Fig. 4, we can see that the MRR algorithm can guarantee that different radios of one SU access different channels at the same time slot when the number of radios is not larger than that of the available channels for the SU.

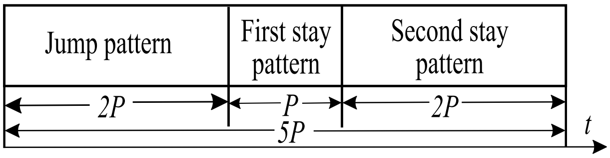

FIGURE 5. Structure of the CH sequences generated by the SRR algorithm.

C. SRR ALGORITHM

The SRR algorithm is proposed to generate CH sequences for the SUs who are equipped with only one radio. For guaranteeing rendezvous between the SUs in heterogeneous CRNs regardless of the clock drift as soon as possible, the length of one period for the CH sequences generated by the SRR algorithm lasts for $5P$ time slots in our setting. Three patterns exist during one period, which are jump pattern, first stay pattern and second stay pattern. The jump pattern lasts for $2P$ time slots while the first stay pattern lasts for P time slots, and the second stay pattern lasts for $2P$ time slots during one period. The structure of the CH sequences generated by the SRR algorithm is shown in Fig. 5. During the jump pattern, SU_A incessantly switches to access different available channels according to the step length s_A and the initial channel index i_A. The value of the s_A is generated by the HRR algorithm while the value of i_A is calculated from the initial channel index i_{A0} generated by the HRR algorithm. In the HRR algorithm, s_A is randomly selected from the subscripts of channels in C_A while i_{A0} is randomly selected from $[1, |C_A|]$. It is worth mentioning that the unavailable channels generated for the jump pattern are replaced by the available channels in the sorted available channel set sequentially in the SRR algorithm. The main intention of the method mentioned above is to increase the frequency of accessing the channels whose channel qualities are better. During the first stay pattern, SU_A always stays at the channel c_{s_A}. During the second stay pattern, SU_A stays at one specific available channel during one period and then changes to stay at another specific channel during next period. Rendezvous between SUs in heterogeneous CRNs can be guaranteed by the SRR algorithm and the MRR algorithm. The detailed proofs for guaranteeing rendezvous will be presented in Section V. The steps of the SRR algorithm are presented in Algorithm 3.

An example of the CH sequence generated by the SRR algorithm is depicted in Fig. 6, where $s_A = 4$, $i_{A0} = 2$, $N = \{c_4, c_2, c_3, c_1\}$, and $C_A = \{c_4, c_3, c_1\}$ after being sorted in descending order according to the channel quality. When $t = 1$, the channel index $j = 2$ is derived by line 4. Because $N[2] = c_2 \notin C_A$, c_2 is replaced by lines 17∼19, where $k = k + 1 = 1$ and $S_A(1) = C_A[1] = c_4$. When $t = 2$, the channel index $j = 5$ is also derived by line 4. Because $j = 5 > |N| = 4$, j is replaced by lines 14∼16. $j = (j - 1) \bmod |N| + 1 = 1$ after replacing. Because $N[1] = c_4 \in C_A$, $S_A(2) = N[1] = c_4$. Other channels during the jump pattern are also generated by this way. When $t = 11$, the CH sequence of SU_A enters its first stay pattern. The stay channel during the first stay pattern is generated by line 6.

Algorithm 3 SRR Algorithm

Require: $N, P, C_A, t_A, i_{A0}, s_A$ *for* SU_A
Ensure: $S_A(t_A)$
1: $t^* = (t_A - 1) \bmod 5P$;
2: $n = \lfloor (t_A - 1)/5P \rfloor$; $i_A = (i_{A0} + n) \bmod P$;
3: **if** $t^* < 2P$ **then**
4: $\quad j = ((i_A + t^* \cdot s_A - 1) \bmod P) + 1$
5: **else if** $2P \leq t^* < 3P$ **then**
6: $\quad S_A(t_A) = c_{s_A}$
7: **else**
8: $\quad j = (n \bmod |C_A|) + 1$; $S_A(t_A) = C_A[j]$;
9: **end if**
10: **if** $t^* = 0$ **then**
11: $\quad k = 0$
12: **end if**
13: **if** $t^* < 2P$ **then**
14: $\quad$ **if** $j > |N|$ **then**
15: $\quad\quad j = (j - 1) \bmod |N| + 1$
16: $\quad$ **end if**
17: $\quad$ **if** $N[j] \notin C_A$ **then**
18: $\quad\quad k = k + 1$
19: $\quad\quad S_A(t_A) = C_A[(k - 1) \bmod |C_A| + 1]$
20: $\quad$ **else**
21: $\quad\quad S_A(t_A) = N[j]$
22: $\quad$ **end if**
23: **end if**

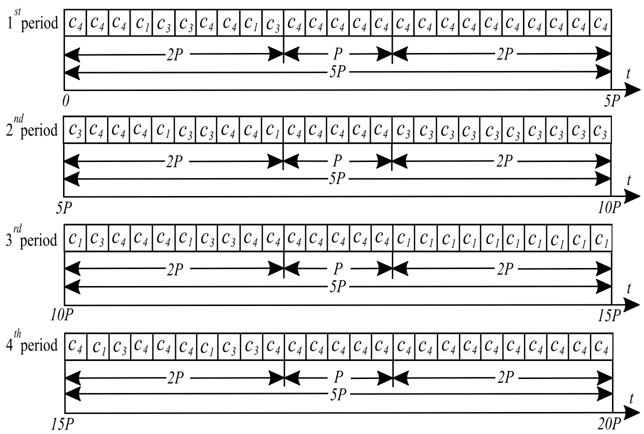

FIGURE 6. CH sequence generated by the SRR algorithm.

Because $s_A = 4$, $S_A(11) = c_{s_A} = c_4$. When $t = 16$, the CH sequence of SU_A enters its second stay pattern. The stay channel is generated by line 8, i.e. $S_A(16) = C_A[1] = c_4$. After $5P$ time slots, the CH sequence of SU_A enters its second period. At the beginning of the second period, the initial channel index is changed to 3 derived by $i_A = i_{A0} + n = 2 + 1 = 3$ while the counter k is reset to zero.

D. HRR ALGORITHM

The HRR algorithm consists of the MRR algorithm and the SRR algorithm. The HRR algorithm is utilized to generate

CH sequences for all SUs in heterogeneous CRNs. Rendezvous between SUs can be guaranteed by the HRR algorithm. In the HRR algorithm, SU keeps attempting rendezvous on the channel generated by invoking the SRR algorithm or the MRR algorithm until achieving rendezvous with its target SUs. The SRR algorithm is invoked to generate CH sequences for the SUs with one radio while the MRR algorithm is invoked to generate CH sequences for the SUs with multiple radios. The details of the HRR algorithm are shown in Algorithm 4.

Algorithm 4 HRR Algorithm

Require: t_A, m_A, k_A, C_A, N *for* SU_A

t_A=1
P = the smallest prime number greater than $|N|$
s_A = the step length randomly selected from the subscripts of channels in C_A
i_{A0} = the initial channel index randomly selected form $[1, |C_A|]$
Sort C_A and N in descending order according to channel quality
while not rendezvous **do**
  if $m_A = 1$ **then**
    Invoke SRR algorithm to generate CH sequence
  end if
  if $m_A > 1$ **then**
    Invoke MRR algorithm to generate CH sequence
  end if
  Attempt rendezvous on $S_A(t_A)$
  $t_A = t_A + 1$
end while

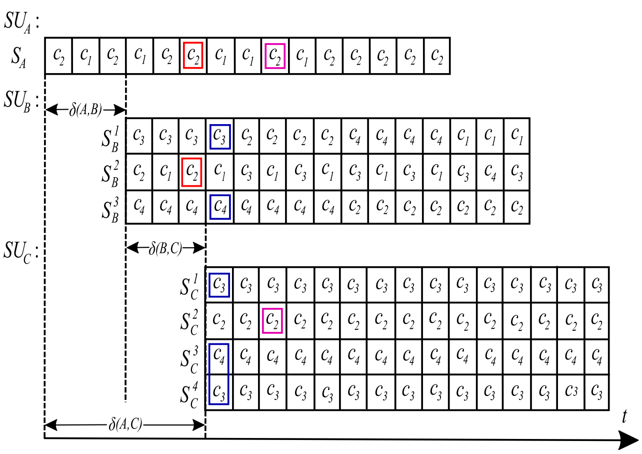

FIGURE 7. An example of rendezvous by using the HRR algorithm.

An example of rendezvous by using the HRR algorithm is shown in Fig. 7, where $N = \{c_1, c_2, c_3, c_4\}$, $C_A = \{c_1, c_2\}$, $C_B = \{c_1, c_2, c_3, c_4\}$, $C_C = \{c_2, c_3, c_4\}$, $m_A = 1$, $m_B = 3$, $k_B = 2$, $m_C = 4$, s_A=2, and $i_{A0} = 2$. After sorting the channels in descending order according to their qualities, $N = \{c_3, c_2, c_4, c_1\}$, $C_A = \{c_2, c_1\}$, $C_B = \{c_3, c_2, c_4, c_1\}$, and $C_C = \{c_3, c_2, c_4\}$. Let $\delta(A, B)$ denote the clock drift between SU_A and SU_B. $\delta(B, C)$ denotes the clock drift between SU_B and SU_C. $\delta(A, C)$ denotes the clock drift between SU_A and SU_C. In this example, $\delta(A, B) = 3$, $\delta(B, C) = 3$, and $\delta(A, C) = 6$. From Fig. 7, we can see that SU_B can achieve rendezvous with SU_A at its third local time slot on channel c_2, while SU_C can achieve rendezvous with SU_B at its first local time slot on channels c_3 and c_4. Besides, SU_C can achieve rendezvous with SU_A at its third local time slot on channel c_2. Hence, $\Gamma(A, B, \delta(A, B)) = 3$, $\Gamma(B, C, \delta(B, C)) = 1$, and $\Gamma(A, C, \delta(A, C)) = 3$.

V. PERFORMANCE ANALYSIS

The proposed HRR algorithm is applicable for realistic CRNs with the following characteristics: asynchronous local clock, heterogeneity, symmetric roles and anonymous information. For the characteristics of symmetric roles and anonymous information, because pre-assigned roles and IDs are not required when generating CH sequences by the HRR algorithm. Hence, the HRR algorithm is applicable for the CRNs with the characteristics of symmetric roles and anonymous information. Specifically, for proving that the HRR algorithm is also applicable for the CRNs with the characteristic of asynchronous local clock and heterogeneity, we derive the upper bounds of MTTR for the HRR algorithm under both symmetric model and asymmetric model in the following scenarios.

- Both of two SUs are equipped with only one radio ($m_A = m_B = 1$).
- One SU is equipped with one radio while the other SU is equipped with multiple radios ($m_A > 1, m_B = 1$ or $m_A = 1, m_B > 1$).
- Both of two SUs are equipped with multiple radios ($m_A > 1, m_B > 1$).

To formally derive the upper bounds of MTTR for the HRR algorithm, we first give the following Lemmas.

Lemma 1: Given the number of global licensed channels $|N|$, prime number P, step length $s_A \in (0, P)$, arbitrary initial channel index i_{A0}, all $|N|$ channels are visited in any consecutive P time slots during the jump pattern of the CH sequence generated by the SRR algorithm.

Proof: According to the SRR algorithm, we denote the CH sequence during any consecutive P time slots of the jump pattern as $S = \{((i_A - 1) \bmod P) + 1,\ ((i_A + s_A - 1) \bmod P) + 1, \cdots, ((i_A + (P-1)s_A - 1) \bmod P) + 1\}$. Suppose that $((i_A + js_A - 1) \bmod P) + 1$ and $((i_A + ks_A - 1) \bmod P) + 1$ are identical, which implies that $((j - k) * s_A) \bmod P = 0$. Since P is a prime number and $s_A \in (0, P)$, P and s_A are co-prime. Hence, we must have $(j-k) \bmod P = 0$, which leads to $j = k$ due to $j-k < P$. Because j and k represent two different time slots, which contradicts that $j = k$. Hence, any two number in S must be different. i.e., Lemma 1 is proved. $\square$

Lemma 2: Given the number of channels $|C_A^J|$ and the length of half period w_A, all $|C_A^J|$ channels are visited by the jump radio of SU_A in any consecutive w_A time slots *during one period of the CH sequence generated by the MRR algorithm.*

Proof: According to the MRR algorithm, we denote the CH sequence during any consecutive w_A time slots of one period for the j^{th} radio of SU_A by $S_A^j = \{(t_A - 1) \bmod |C_A^{J*}(j)|+1,\ ((t_A+1)-1) \bmod |C_A^{J*}(j)|+1, \cdots, (t_A+(w_A-1)-1) \bmod |C_A^{J*}(j)|+1\}$, where $m_A - k_A + 1 \leq j \leq m_A$. Suppose that $(((t_A + i) - 1) \bmod |C_A^{J*}(j)| + 1)$ and $(((t_A + k) - 1) \bmod |C_A^{J*}(j)| + 1)$ are identical, which implies that $(i - k) \bmod |C_A^{J*}(j)| = 0$. According to the MRR algorithm, $|C_A^{J*}(j)| = w_A$ or $|C_A^{J*}(j)| = w_A - 1$. When $|C_A^{J*}(j)| = w_A$, since $i-k \leq w_A-1$, $(i-k) \bmod |C_A^{J*}(j)| = 0$ leads to $i = k$. Since i and k represent different time slots, which contradicts that $i = k$. Hence, any consecutive w_A channels in S_A^j must be different when $|C_A^{J*}(j)| = w_A$. When $|C_A^{J*}(j)| = w_A - 1$ and $i - k \leq w_A - 2$, $(i - k) \bmod |C_A^{J*}(j)| = 0$ leads to $i = k$, which means any consecutive $w_A - 1$ channels in S_A^j must be different. Thus, Lemma 2 is proved. □

Lemma 3: Given a prime number P, if s_A and s_B are two different numbers in $(0, P)$, then for any initial channel index $i_A \in [0, P)$ and $i_B \in [0, P)$, there must exist an integer $k \in [0, P)$ such that $(i_A+ks_A-1) \bmod P = (i_B+ks_B-1) \bmod P$.

Proof: The proof is given in [30]. □

Lemma 3 implies that rendezvous between two SUs can be guaranteed by the SRR algorithm when the CH sequences of the two SUs are in their jump patterns with different step lengths and the overlap between their jump patterns is not less than P time slots.

Based on the Lemma 1, Lemma 2 and Lemma 3, we derive the upper bounds of MTTR for the proposed HRR algorithm, which are presented in the following Theorems.

Theorem 1: The MTTR of the HRR algorithm is upper bounded by $3P$ time slots under the symmetric model when $m_A = m_B = 1$.

When $m_A = m_B = 1$, both of SU_A and SU_B generate their CH sequences by the SRR algorithm. The rendezvous between SUs can be divided into four scenarios according to the clock drift δ between them and their step lengths. The four rendezvous scenarios are: rendezvous between the jump patterns, rendezvous between the first stay patterns, rendezvous between the first stay pattern and the jump pattern, and rendezvous between the jump pattern and the second stay pattern. According to Lemma 3, rendezvous will be achieved between the jump patterns when the overlap between the jump patterns of two SUs is not less than P time slots and the step lengths of the two SUs are different. Owing to the fact that SU always stays at the same channel whose channel index is equal to its step length. Hence, rendezvous must be achieved between the stay patterns of two SUs when they have the same step length and at least one time slot overlap between their stay patterns. Besides, when the overlap between the jump pattern of one SU and the first stay pattern or the second stay pattern of the other SUs is not less than P, the rendezvous will be achieved between the jump pattern and the first stay pattern, or between the jump pattern and the second stay pattern. The reason is that all available channels are visited at least once for one SU while the other SU always stays at the same channel during these P time slots. Hence, rendezvous must be achieved on the stay channel that one of the SUs always stays at. Then, we give the specific proof as follows.

Proof: When $m_A = m_B = 1$, the CH sequences of SU_A and SU_B are generated by the SRR algorithm. Without loss of generality, we assume that SU_B starts rendezvous process later than SU_A for δ ($\delta \geq 0$) time slots. Let $\delta^* = \delta \bmod 5P$. The theoretical analysis also can be used for the case that SU_A starts rendezvous process later than SU_B.

Case1: $s_A = s_B = s$

Subcase 1.1: $0 \leq \delta^* < P$. As shown in Fig. 8(a), an overlap must exist between SU_A's and SU_B's first stay pattern. Since both SU_A and SU_B stay at the channel c_s during their first stay patterns, rendezvous must be achieved on channel c_s. Thus, we have $TTR \leq 2P + 1$.

Subcase 1.2: $P \leq \delta^* \leq 2P$. As shown in Fig. 8(b), there must exist P time slots overlap between SU_A's first stay pattern and SU_B's jump pattern. During these overlapping P time slots, SU_A stays at channel c_s while SU_B visits all $|N|$ channels according to Lemma 1. Thus, rendezvous must be achieved in one of these P time slots on channel c_s, we have $TTR \leq 2P$.

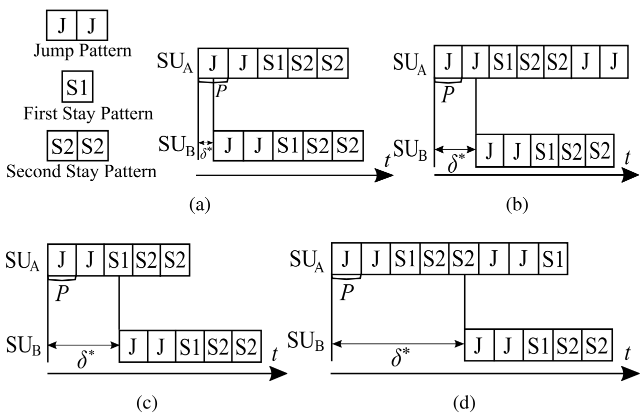

FIGURE 8. Guaranteed rendezvous when $m_A = m_B$=1. (a) $0 \leq \delta^* < P$. (b) $P \leq \delta^* \leq 2P$. (c) $2P < \delta^* \leq 4P$. (d) $4P < \delta^* < 5P$.

Subcase 1.3: $2P < \delta^* \leq 4P$. As shown in Fig. 8(c), there must exist P time slots overlap between SU_A's second stay pattern and SU_B's jump pattern. During these overlapping P time slots, SU_A stays at one specific channel while SU_B visits all $|N|$ channels. Thus, rendezvous must be achieved in one of these P time slots on the channel that SU_A stays at, we have $TTR \leq 2P$.

Subcase 1.4: $4P < \delta^* < 5P$. As shown in Fig. 8(d), an overlap must exist between SU_A's and SU_B's first stay pattern. Since both SU_A and SU_B stay at the same channel c_s during their first patterns, rendezvous must be achieved on channel c_s. Thus, we have $TTR \leq 3P$.

Case2: $s_A \neq s_B$

Subcase 1.1: $0 \leq \delta^* < P$. As shown in Fig. 8(a), there must exist P time slots overlap between SU_A's and SU_B's jump patterns. Since the step lengths of SU_A and SU_B are different in this Subcase, a rendezvous must happen in one of these overlapping P time slots according to Lemma 3. Thus, we have $TTR \leq P$.

Subcase 1.2: $P \leq \delta^* \leq 2P$. As shown in Fig. 8(b), there must exist P time slots overlap between SU_A's first stay pattern and SU_B's jump pattern. Hence, rendezvous must happen on channel c_{s_A}, we have $TTR \leq 2P$.

Subcase 1.3: $2P < \delta^* \leq 4P$. As shown in Fig. 8(c), there must exist P time slots overlap between SU_A's second stay pattern and SU_B's jump pattern. Rendezvous must happen in one of these P time slots on the channel that SU_A stays at, we have $TTR \leq 2P$.

Subcase 1.4: $4P < \delta^* < 5P$. As shown in Fig. 8(d), an overlap must exist between SU_A's and SU_B's jump patterns. Using the similar method described as Subcase 1.1, we have $TTR \leq 2P$.

To sum up, the MTTR of the HRR algorithm can be upper bounded by $3P$ time slots under the symmetric model when $m_A = m_B = 1$. $\square$

Theorem 2: The MTTR of the HRR algorithm is upper bounded by $(|N| - G + 1)5P$ time slots under the asymmetric model when $m_A = m_B = 1$, where G is the number of commonly available channels between two SUs (SU_A and SU_B).

The main idea of achieving rendezvous in this scenario is similar to that for Theorem 1. The difference between them is that the potential rendezvous channel may not be available for both of SUs simultaneously in this scenario owing to the heterogeneous available channels. Hence, several periods of CH sequences for two SUs may cost before rendezvous on the same commonly available channel. The specific proof is shown as follows.

Proof:

Case 1: $s_A = s_B = s$

Subcase 1.1: $0 \leq \delta^* < P$. As shown in Fig. 8(a), an overlap must exist between SU_A's and SU_B's first stay patterns. Thus, we have $TTR \leq 2P + 1$.

Subcase 1.2: $P \leq \delta^* \leq 2P$. As shown in Fig. 8(b), there must exist P time slots overlap between SU_A's first stay pattern and SU_B's jump pattern. Rendezvous must be achieved on channel c_s, we have $TTR \leq 2P$.

Subcase 1.3: $2P < \delta^* \leq 4P$. As shown in Fig. 8(c), there must exist P time slots overlap between SU_A's second stay pattern and SU_B's jump pattern. Since SU_A stays at one specific channel while SU_B visits all $|N|$ channels during these overlapping P time slots, the potential rendezvous on the channel that SU_A stays at must happen in one of these overlapping P time slots. However, the potential rendezvous channel may not be available for SU_B under the asymmetric model. Since the channel that SU_A stays at changes every period, the potential rendezvous must happen on different channels for different periods. Since the number of commonly available channels between SU_A and SU_B is G while each period lasts for $5P$ time slots. Thus, the TTR will not exceed $(|N| - G)5P + 2P$ time slots.

Subcase 1.4: $4P < \delta^* < 5P$. As shown in Fig. 8(d), an overlap must exist between SU_A's and SU_B's first stay patterns. We have $TTR \leq 3P$.

Case 2: $s_A \neq s_B$

Subcase 1.1: $0 \leq \delta^* < P$. As shown in Fig. 8(a), there must exist P time slots overlap between SU_A's and SU_B's jump patterns. A potential rendezvous must happen in one of these overlapping P time slots. However, the potential rendezvous channel may not be available for SU_A and SU_B simultaneously. According to the SRR algorithm, the initial channel index changes every period while the step length does not change for the SU. Hence, the potential rendezvous must happen on different channels during different periods. Thus, the TTR will not exceed $(|N| - G)5P + P$ time slots.

Subcase 1.2: $P \leq \delta^* \leq 2P$. As shown in Fig. 8(b), there must exist P time slots overlap between SU_A's jump pattern and SU_B's second stay pattern. Using the similar method described as before, we have $TTR \leq (|N| - G + 1)5P$.

Subcase 1.3: $2P < \delta^* \leq 4P$. As shown in Fig. 8(c), there must exist P time slots overlap between SU_A's second stay pattern and SU_B's jump pattern. Using the similar method described as before, we have $TTR \leq (|N| - G)5P + 2P$.

Subcase 1.4: $4P < \delta^* < 5P$. As shown in Fig. 8(d), there must exist P time slots overlap between SU_A's and SU_B's jump patterns. Using the similar method described as Subcase 1.1, we have $TTR \leq (|N| - G)5P + 2P$.

To sum up, the MTTR of the HRR algorithm can be upper bounded by $(|N| - G + 1)5P$ time slots under the asymmetric model when $m_A = m_B = 1$. $\square$

Theorem 3: Two SUs (SU_A and SU_B) performing the HRR algorithm achieve rendezvous in at most $5P + w_i$ time slots under the symmetric model when $m_A = 1, m_B > 1$ or $m_A > 1, m_B = 1$, where $i = B$ or A.

In this scenario, two SUs generate their CH sequences by using the SRR algorithm and the MRR algorithm, respectively. The rendezvous must be achieved between the second stay pattern of the CH sequence for the SU with one radio and the other SU with multiple radios. Because the SU with one radio always stays at the same channel during its second stay pattern within one period while all available channels are visited at least once during any continuous w_i time slots within one period by the radios of the SU with multiple radios. Moreover, the overlap between the second stay pattern for the SU with one radio and one period of the CH sequence for the SU with multiple radios is not smaller than w_i. The specific proof is given as follows.

Proof: Without loss of generality, we assume that $m_A = 1$ and $m_B > 1$, the theoretical analysis also can be used for the case that $m_A > 1$ and $m_B = 1$. It can be easily seen that $w_B < P$ according to the HRR algorithm.

Case 1: SU_B starts hopping later than SU_A for δ^* time slots. As shown in Fig. 9(a), there must exist a w_B time slots overlap between SU_A's second stay pattern and SU_B's one period.

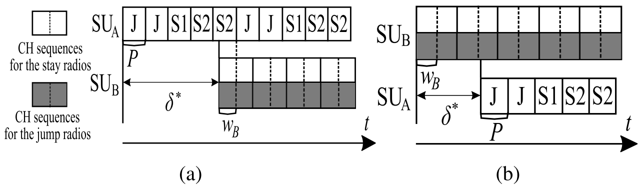

FIGURE 9. Guaranteed rendezvous when $m_A = 1$ and $m_B > 1$. (a) $t_A = t_B + \delta^*$. (b) $t_B = t_A + \delta^*$.

SU_A stays at one specific channel while SU_B visits all $|C_B|$ channels during these overlapping w_B time slots according to the MRR algorithm and Lemma 2. Hence, rendezvous must be achieved in one of these w_B time slots on the channel that SU_A stays at, we have $TTR < 5P + w_B$.

Case 2: SU_A starts hopping later than SU_B for δ^* time slots, as shown in Fig. 9(b), using similar method described as Case 1, we have $TTR < 5P$.

To sum up, the MTTR of the HRR algorithm can be upper bounded by $5P + w_i$ time slots under the symmetric model when $m_A = 1, m_B > 1$ or $m_A > 1, m_B = 1$, where $i = B$ or A. □

Theorem 4: Two SUs (SU_A and SU_B) performing the HRR algorithm achieve rendezvous in at most $(|N| - G + 1)5P$ time slots under the asymmetric model when $m_A = 1$, $m_B > 1$ or $m_A > 1, m_B = 1$.

The main idea of achieving rendezvous in this scenario is similar to that for Theorem 3. The difference between them is that the potential rendezvous channel may not be available for both of the SUs simultaneously in this scenario. Owing to the fact that the stay channel is changed every period for the second stay pattern of the CH sequences generated by the SRR algorithm. Rendezvous must be achieved when the SU with one radio stays at one commonly available channels during its second stay pattern. The specific proof is shown as follows.

Proof: Case 1: SU_B starts hopping later than SU_A for δ^* time slots. As shown in Fig. 9(a), there must exist w_B time slots overlap between SU_A's second stay pattern and SU_B's one period. The potential rendezvous must happen in one of these w_B time slots on the channel that SU_A stays at. Since the potential rendezvous channel may not be available for SU_B while the channel that SU_A stays at during its second stay pattern changes every period, the potential rendezvous must happen on different channels for different periods. Thus, we have $TTR < (|N| - G + 1)5P$.

Case 2: SU_A starts hopping later than SU_B for δ^* time slots. As shown in Fig. 9(b), using the similar method described as Case 1, we have $TTR < (|N| - G + 1)5P$.

To sum up, the MTTR of the HRR algorithm can be upper bounded by $(|N| - G + 1)5P$ time slots under the asymmetric model when $m_A = 1, m_B > 1$ or $m_A > 1, m_B = 1$. □

Theorem 5: Two SUs (SU_A and SU_B) performing the HRR rendezvous algorithm achieve rendezvous in at most $2\min(w_i)$ time slots under the symmetric model when $m_A > 1$ and $m_B > 1$, where $i = A, B$.

In this scenario, both of two SUs generate their CH sequences by the MRR algorithm. Rendezvous must be achieved between the stay radios of the SU with a larger CH sequence period and the radios of the other SU. The reason is that all available channels are visited at least once in any continuous w_i time slots during one period by the radios of the SU with a smaller CH sequence period. Meanwhile, the stay radios of the SU with a larger CH sequence period always stay at the same channels during one period. Moreover, the overlap between the CH sequences of one period for the SUs is long enough for achieving rendezvous. The specific proof is shown as follows.

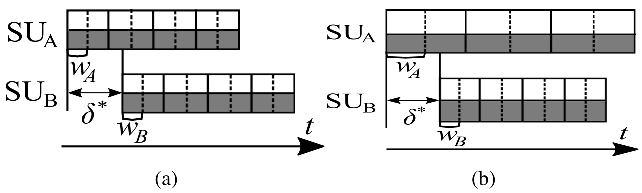

FIGURE 10. Guaranteed rendezvous when $m_A > 1$ and $m_B > 1$. (a) $w_A = w_B$. (b) $w_A \neq w_B$.

Proof: Case 1: $w_A = w_B$, as shown in Fig. 10(a). Without loss of generality, we assume that SU_B starts CH process later than SU_A, the theoretical analysis can also be used for the case that SU_B starts hopping earlier than SU_A. In this Case, there must exist w_B time slots overlap within one period of the CH sequences for SU_A and SU_B. Since the stay radios of SU_A and SU_B stay at specific channels while the radios of SU_A and SU_B visit all available channels during each period, rendezvous must be achieved in one of these w_B time slots between SU_A's stay radios and SU_B's radios, and between SU_B's stay radios and SU_A's radios. Thus, we have $TTR < 2w_B$.

Case 2: $w_A \neq w_B$. Without loss of generality, we assume that $w_A > w_B$. As shown in Fig. 10(b), there must exist w_B time slots overlap within one period of CH sequences for SU_A and SU_B. Thus, rendezvous must be achieved in one of these w_B time slots between SU_A's stay radios and SU_B's radios, we have $TTR < 2w_B$.

To sum up, the MTTR of the HRR algorithm can be upper bounded by $2\min(w_i)$ time slots under the symmetric model when $m_A > 1$ and $m_B > 1$, where $i = A, B$. □

Theorem 6: When $m_A > 1$ and $m_B > 1$, two SUs (SU_A and SU_B) performing the HRR algorithm achieve rendezvous in at most $\min\left(2\left\lfloor \frac{C_i - G}{m_i - k_i} \right\rfloor w_i\right) + 2w_B$ time slots when $w_A = w_B$ and at most $\left(2\left\lfloor \frac{C_A - G}{m_A - k_A} \right\rfloor w_A\right) + 2w_B$ time slots when $w_A > w_B$ under the asymmetric model, where $i = A, B$.

The reason for achieving rendezvous in this scenario is similar to that for Theorem 5. The difference between them is that the potential rendezvous channel may not be available for the two SUs simultaneously under the asymmetric model. Owing to the fact that the stay channels that the stay radios of the SU stay at are changed every period. The rendezvous must be achieved when any stay radio of the SU with a larger CH

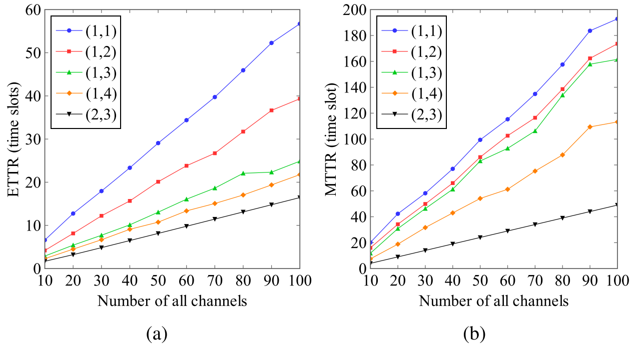

FIGURE 11. Comparison of different numbers of radios under the symmetric model. (a) ETTR VS. $|N|$. (b) MTTR VS. $|N|$.

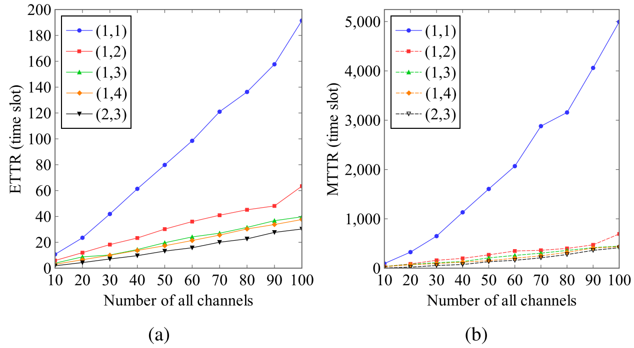

FIGURE 12. Comparison of different numbers of radios under the asymmetric model. (a) ETTR VS. $|N|$. (b) MTTR VS. $|N|$.

sequence period stays at one commonly available channel. The specific proof is shown as follows.

Proof: Case 1: $w_A = w_B$. As shown in Fig. 10(a), there must exist w_B time slots overlap within one period of the CH sequences for SU_A and SU_B. The potential rendezvous must happen in one of these w_B time slots. Since the potential rendezvous channel may not be available for SU_A and SU_B simultaneously, while the stay channels for the stay radios of SU_A and SU_B are changed every period. The potential rendezvous channels must be different for different periods. Thus, we have $TTR < min\left(2\left\lfloor \frac{C_i - G}{m_i - k_i} \right\rfloor w_i\right) + 2w_B,\ i = A, B$.

Case 2: $w_A \neq w_B$. As shown in Fig. 10(b), there must exist w_B time slots overlap within one period of CH sequences for SU_A and SU_B. The potential rendezvous must happen between SU_A's stay radios and SU_B's radios. Using the similar method described as Case 1, we have $TTR < \left(2\left\lfloor \frac{C_A - G}{m_A - k_A} \right\rfloor w_A\right) + 2w_B$. $\square$

From the above analysis, we can see that the HRR algorithm can guarantee rendezvous between SUs in spite of the clock drift and the number of radios under the asymmetric model. Hence, the HRR algorithm is also applicable for CRNs with the characteristics of asynchronous local clock and heterogeneity. In summary, the HRR algorithm is applicable for realistic CRNs with the characteristics of asynchronous local clock, heterogeneity, symmetric roles and anonymous information.

VI. SIMULATION

We conduct extensive simulations using MATLAB to evaluate the proposed HRR algorithm. First, we evaluate the performance of the HRR algorithm under different numbers of radios. Then, we evaluate the performance of the HRR algorithm under different parameter settings. Besides, the qualities of the rendezvous channel are evaluated. Moreover, we compare the performance of the HRR algorithm to several representative multi-radio-based CH rendezvous algorithms.

A. PERFORMANCE UNDER DIFFERENT NUMBERS OF RADIOS

In this subsection, we evaluate the ETTR and the MTTR of the HRR algorithm under different numbers of radios. Both symmetric model and asymmetric model are evaluated. We use the notation (m_A, m_B) to denote the case that SU_A and SU_B are equipped with m_A and m_B radios, respectively. The values of k_A and k_B are set to be equal to $\lceil \frac{m_A}{2} \rceil$ and $\lceil \frac{m_B}{2} \rceil$, respectively. The number of the global channels $|N|$ is varied from 10 to 100. All global channels are available to SUs under the symmetric model while different parts of global channels are available to different SUs under the asymmetric model. The numbers of available channels for SUs are set to be $0.8|N|$ while the number of commonly available channels between two SUs is set to be $0.6|N|$ under the asymmetric model.

Fig. 11 and Fig. 12 show the comparison of the ETTR and the MTTR among five combinations of (m_A, m_B) which are $(1, 1)$, $(1, 2)$, $(1, 3)$, $(1, 4)$, $(2, 3)$ under the symmetric model and the asymmetric model, respectively. We can see that SU_A and SU_B can achieve rendezvous with each other within the upper bounds of MTTR derived in Section V, which verifies the correctness of the theoretical analysis. For instance, when $|N| = 100$, i.e. $P = 101$, the upper bound of MTTR for (1,1) under the symmetric model derived by Theorem 1 is $3P$, which equals 303 time slots. While the MTTR under this situation obtained by simulation is about 190 time slots, which is less than 303 time slots. Besides, the ETTR and the MTTR increase with the increase of the total number of channels for SU_A and SU_B under both the symmetric model and the asymmetric model. The reason is that the ETTR and the MTTR are related to the length of one period for the CH sequences. When the numbers of all channels and available channels increase, the length of one period for the CH sequences increases, which directly leads to the rise of the ETTR and the MTTR. Moreover, when the total number of channels for SU_A and SU_B is fixed, the ETTR and the MTTR between them when both of them are equipped with multiple radios are shorter than that when one of them is equipped with one radio under both symmetric model and asymmetric model. The reason is listed as follows. First, we consider the scenario where a pair of SUs is equipped with a single radio and multiple radios, respectively. In this case, the rendezvous may be achieved between the stay patterns for the SU with a single radio and the jump radios for the

SU with multiple radios. Meanwhile, the rendezvous also can be achieved between the jump patterns for the SU with a single radio and the stay radios of the SU with multiple radios, simultaneously. Compared with the scenario where both SUs are equipped with a single radio, SUs have more chance to achieve rendezvous during each period in this scenario. Then, we consider the scenario where both SUs are equipped with multiple radios. In this scenario, the stay radios of one SU may rendezvous with the radios of the other SU. Meanwhile, the stay radios of the other may also rendezvous with the radios of that SU. With the increase in the number of radios, the rendezvous chance can be increased during each period.

B. PERFORMANCE UNDER DIFFERENT PARAMETER SETTINGS

In this subsection, we evaluate the performance of the proposed HRR algorithm under different parameter settings, including the allocation of radios, the number of commonly available channels, and the number of available channels.

1) COMPARISON OF DIFFERENT ALLOCATIONS OF RADIOS

The performance of the HRR algorithm influenced by different allocations of radios is evaluated. The total numbers of radios for SU_A and SU_B are fixed to 4. We use the notation (k_A, k_B) to denote the numbers of jump radios for SU_A and SU_B, which also can be used to denote the different allocations of radios. Six kinds of allocations exist when the total numbers of radios for SU_A and SU_B are equal to 4, including (1,1), (1,2), (1,3), (2,2), (2,3), and (3,3). The global channels, available channels, and commonly available channels are set as same as before.

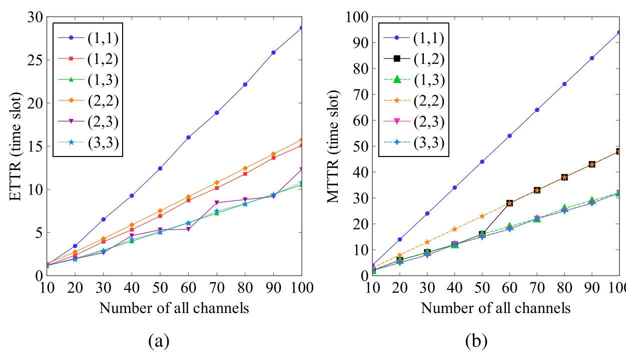

FIGURE 13. Comparison of different allocations of radios under the symmetric model. (a) ETTR VS. |*N*|. (b) MTTR VS. |*N*|.

Fig. 13 and Fig. 14 show the comparison of the ETTR and the MTTR among the six kinds of allocations under the symmetric model and the asymmetric model, respectively. From Fig. 13, we can see that the ETTR and the MTTR are shorter when one of SUs is equipped with three jump radios under the symmetric model. The reason is that the MTTR decrease with the increase of the number of jump radios for the SU with smaller CH sequence period when the numbers of its radios and available channels are fixed under the symmetric model,

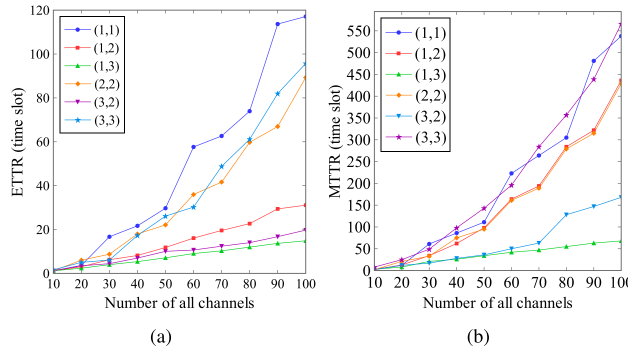

FIGURE 14. Comparison of different allocations of radios under the asymmetric model. (a) ETTR VS. |*N*|. (b) MTTR VS. |*N*|.

which can be easily seen from Theorem 5. Hence, it is better to set $(m_i - 1)$ jump radios for SU_i to obtain minimum TTR under the symmetric model. From Fig. 14, we can see that the allocation (1,3) can achieve the smallest ETTR and MTTR under the asymmetric model. The optimal radio allocation is the allocation that can minimize the MTTR derived by Theorem 6. Owing to the space limitations, we theoretically analyze how to allocate radios in [31].

2) COMPARISON OF DIFFERENT NUMBERS OF COMMONLY AVAILABLE CHANNELS

The performance of the HRR algorithm influenced by different numbers of commonly available channels is evaluated. The total numbers of radios for SU_A and SU_B are set to be 4. The numbers of jump radios for SU_A and SU_B are set to be 1 and 3, respectively. The numbers of available channels for SU_A and SU_B are set to be $0.6|N|$, while that of commonly available channels between SU_A and SU_B is varied from $0.2|N|$ to $0.6|N|$. The global channels are set as same as before.

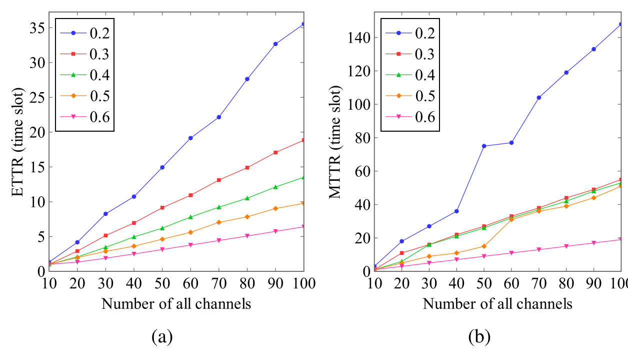

FIGURE 15. Comparison of different numbers of commonly available channels. (a) ETTR VS. |*N*|. (b) MTTR VS. |*N*|.

Fig. 15 shows the comparison of different numbers of commonly available channels between SU_A and SU_B. From Fig. 15, we can see that the ETTR and the MTTR decrease with the increase of the number of commonly available channels. When the number of commonly available channels between SU_A and SU_B is same as that of their available channels, i.e., under the symmetric model, the rendezvous between SU_A and SU_B can be achieved with the shortest ETTR and MTTR. The reason is that the commonly available channels may occur earlier with its number increases when executing the rendezvous process.

3) COMPARISON OF DIFFERENT NUMBERS OF AVAILABLE CHANNELS

The performance of the HRR algorithm influenced by the different numbers of available channels is evaluated. The total numbers of radios for two SU_A and SU_B are set to be 4 while that of jump radios for SU_A and SU_B are set to 1 and 3, respectively. The global channels $|N|$ varies from 10 to 100. The number of commonly available channels between SU_A and SU_B is fixed to $0.3|N|$. The numbers of available channels for SU_A and SU_B are varied from $0.3|N|$ to $0.6|N|$.

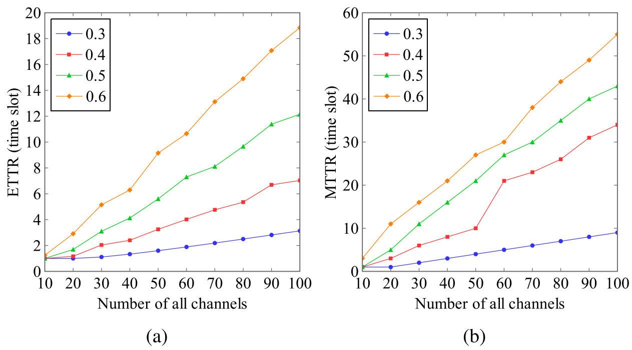

FIGURE 16. Comparison of different numbers of available channels. (a) ETTR VS. $|N|$. (b) MTTR VS. $|N|$.

Fig. 16 shows the comparison of different numbers of available channels. From Fig. 16, we can see that the ETTR and the MTTR decrease with the decrease of the number of available channels. When the numbers of available channels for SU_A and SU_B are equal to $0.3|N|$, i.e., under the symmetric model. The rendezvous between SU_A and SU_B can achieved with the shortest ETTR and MTTR. The reason is that the length of one period for the CH sequences generated by the MRR algorithm decrease with the decrease of the number of available channels when the numbers of total radios and that of the jump radios are fixed. Hence, the ETTR and MTTR decrease with the decrease of the number of available channels. Besides, we can see that the upper bound of MTTR for SUs under the symmetric model is less than that under the asymmetric model in any case form Theorem 5 and Theorem 6.

C. PERFORMANCE EVALUATION IN TERMS OF CHANNEL QUALITY

In this subsection, we evaluate the performance in terms of channel quality for our proposed HRR algorithm. Two places are related to the channel quality in our proposed HRR algorithm. We list them as follows. First, in the HRR algorithm, the licensed channels and the available channels of the SUs are sorted in descending order according to the channel quality. Second, generated unavailable channels are replaced by the available channels in the sorted available channel set sequentially for the SRR algorithm. To verify that SUs can rendezvous on the channels with higher channel qualities more frequent by using our proposed HRR algorithm, we compare the percentages of rendezvousing on different commonly available channels in our simulation.

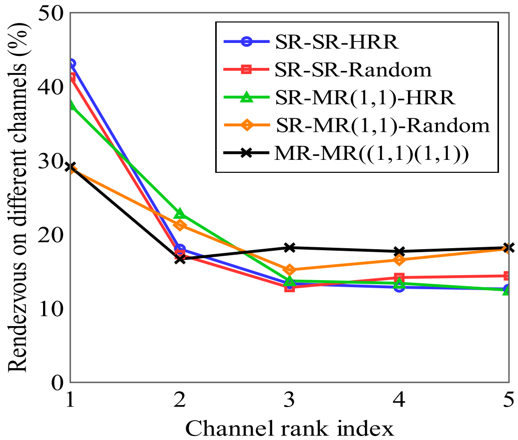

FIGURE 17. Performance evaluation in terms of channel quality.

In the simulation, we set the number of licensed channels as 20, the number of available channels for SUs as 10, and the number of commonly available channels between a pair of SUs as 5. Fig. 17 shows the simulation results. The abscissa of Fig. 17 is channel rank index, which means the position of the channel in the commonly available channel set between a pair of SUs. For instance, the channel with channel rank index 1 is the first channel in the commonly available channel set between a pair of SUs. We assume that the channel qualities for different SUs are same in our simulations. The vertical axis of Fig. 17 is the percentages of rendezvousing on different commonly available channels, which are measured by the average values of 1000 times simulations. In Fig. 17, SR-SR-HRR and SR-SR-Random denote that replacing unavailable channels by our proposed HRR algorithm and by randomly selecting available channels for the SUs with single radio, respectively. SR-MR(1,1)-HRR and SR-MR(1,1)-Random denote that replacing unavailable channels by our proposed HRR algorithm and by randomly selecting available channels for the scenario where one of the SUs is equipped with single radio and the other SU is equipped with multiple radios (one stay radio and one jump radio), respectively. MR((1,1)(1,1)) denotes the rendezvous between two SUs with multiple radios (one stay radio and one jump radio) using our proposed HRR algorithm. We only compare the unavailable channel replacement by using the proposed HRR algorithm and randomly replacement method when at least one of a pair of SUs is equipped with one radio. The reason is that the CH sequences generated based on the available channels when SUs are equipped with multiple radios. Hence, we do not need to replace channels when both SUs are equipped with multiple radios.

Form the performance results of the SR-SR-HRR and the SR-MR(1,1)-HRR in Fig. 17, we can see that the percentages of rendezvousing on the commonly available channels decrease with the ascending order of the channels in the commonly available channel set between SUs under the scenario where at least one SU is equipped with one radio. Hence, SUs can rendezvous on the channels with higher channel qualities more frequent after sorting the channels according to their channel qualities in this scenario. Besides, form the simulation results of the MR-MR((1,1)(1,1)) in Fig. 17, we can see that the percentage of rendezvousing on the first channel in the commonly available channel set between two SUs is higher than those of rendezvousing on the other channels. Moreover, the difference between the percentages of rendezvousing on the channels except the first channel is low. In summary, SUs can rendezvous on the commonly available channels with higher channel qualities more frequent after sorting the channels in descending order according to the channel quality. In addition, we evaluate the performance of replacing unavailable channels by randomly selecting available channels instead of replacing unavailable channels by the available channels with higher qualities under the scenario where at least one SU is equipped with single radio. From the simulation results, we can see that the percentages of rendezvousing on the front channels in the sorted commonly available channel set for the replacement method in our proposed algorithm are higher than random replacement. Hence, the frequent of rendezvousing on the channels with higher qualities can increase by the HRR algorithm.

D. COMPARISON OF DIFFERENT CH ALGORITHMS

In this subsection, we compare the ETTR and the MTTR of the proposed HRR algorithm to several multi-radio-based representative CH rendezvous algorithms, including the AR algorithm [17], the RPS algorithm [18], the AMRR algorithm [20] and the MSS algorithm [21] under the symmetric model and the asymmetric model, respectively.

1) UNDER THE SYMMETRIC MODEL

The performance of the proposed HRR algorithm and several multi-radio-based CH rendezvous algorithms is compared under the symmetric model. The numbers of radios for SU_A and SU_B are set to be 4. The numbers of available channels for two SUs are set to be $0.5|N|$. The number of commonly available channels between SU_A and SU_B is set to be $0.5|N|$. The number of global channels varies from 10 to 100.

Fig. 18 shows the comparison of different algorithms under the symmetric model. From Fig. 18, we can see that the MTTR of the HRR algorithm outperforms all of the compared CH algorithms while its ETTR is close to that of other CH algorithms. Besides, although the ETTR of the AR algorithm, the RPS algorithm and the MSS algorithm outperforms the HRR algorithm under some situations, the gap between them is very small. Moreover, there exist several drawbacks in the

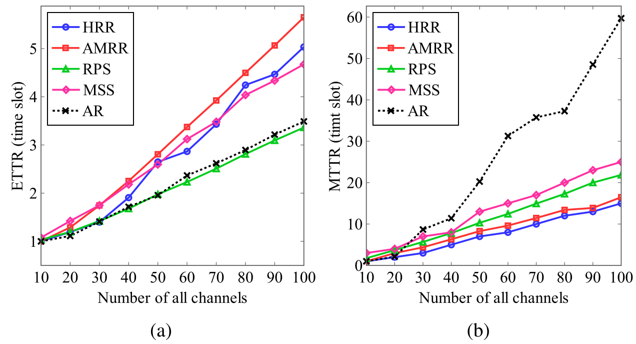

FIGURE 18. **Comparison of different algorithms under the symmetric model. (a) ETTR VS. $|N|$. (b) MTTR VS. $|N|$.**

compared CH algorithms, which are presented as follows. The numbers of radios for different SUs are assumed to be same by the AR algorithm while that for each SU cannot be equal to 1 for the RPS algorithm and the AMRR algorithm.

2) UNDER THE ASYMMETRIC MODEL

The performance of the proposed HRR algorithm and several representative multi-radio-based CH rendezvous algorithms is compared under the asymmetric model. The numbers of radios for SU_A and SU_B are set to be 4. The numbers of available channels for SU_A and SU_B are set to be $0.5|N|$. The number of commonly available channels between SU_A and SU_B is set to $0.2|N|$. The number of global channels varies form 10 to 100. The simulation results showed in Fig. 19 are measured by the average values of 10 times simulations.

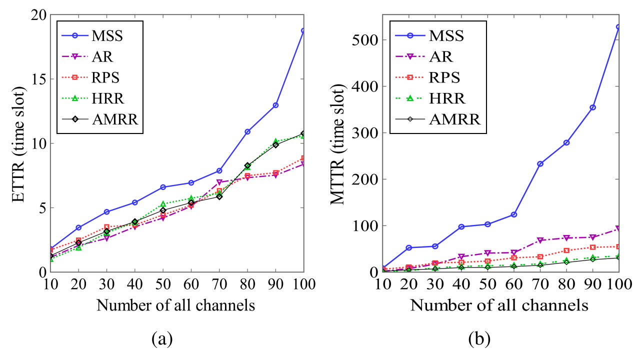

FIGURE 19. **Comparison of different algorithms under the asymmetric model. (a) ETTR VS. $|N|$. (b) MTTR VS. $|N|$.**

Fig.19 shows the comparison of different algorithms under the asymmetric model. From Fig. 19, we can see that the MTTR of the proposed HRR algorithm is smaller than that of the MSS algorithm, the AR algorithm, and the RPS algorithm. Besides, when the number of the licensed channels is small, the MTTR of the HRR algorithm is also shorter than the AMRR algorithm. When the number of the licensed channels is larger, the MTTR of the HRR algorithm is longer than that of the AMRR algorithm. However, the difference between them is tiny. Moreover, we can see that the ETTR of the proposed HRR algorithm is smaller than that of the MSS algorithm. Besides, when the number of licensed channels is smaller, the ETTR of the proposed HRR algorithm is also lower than the RPS algorithm, the AMRR algorithm, and the AR algorithm. When the number of licensed channel is larger, even though the ETTR of the proposed HRR algorithm is longer than that of the other algorithms, the difference between them is small.

In summary, although the ETTR of the AR algorithm, the AMRR algorithm, and the RPS algorithm outperform the HRR algorithm under several scenarios for the asymmetric model, the difference between them is very tiny. Besides, although the MTTR of the HRR algorithm is larger than that of the AMRR algorithm under several scenarios for the asymmetric model, the difference between them is very small. In addition, the AR algorithm assumed that the numbers of radios for different SUs are same, while the RPS algorithm and the ARMM algorithm are not applicable for the situation where one of SUs is equipped with one radio. Moreover, although the ETTR of the MSS algorithm is close to that of the HRR algorithm under the symmetric model, the MTTR of the MSS algorithm is much larger than that of the HRR algorithm under both symmetric model and asymmetric model, while the ETTR of the MSS algorithm is larger than that of the HRR algorithm under the asymmetric model. In a word, the proposed HRR algorithm is more applicable for the heterogeneous CRNs than any other compared representative multi-radio-based CH rendezvous algorithms.

VII. CONCLUSION

In this paper, a new Channel Hopping (CH) algorithm, Hybrid Radios Rendezvous (HRR) algorithm is proposed for heterogeneous Cognitive Radio Networks (CRNs). Theoretical analysis and simulation results show that the unlicensed Secondary Users (SUs) in heterogeneous CRNs can achieve rendezvous with their target SUs within upper bounded time by using the HRR algorithm. Moreover, simulation results showed that the Maximum Time-To-Rendezvous (MTTR) of the HRR algorithm outperforms the Multiple-radios Sunflower-Sets-based pairwise rendezvous (MSS) algorithm, the Adaptive Rendezvous (AR) algorithm, and the Role-based Parallel Sequence (RPS) algorithm both under the symmetric model and the asymmetric model. Although the MTTR of the HRR algorithm is larger than that of the Adjustable Multi-Radio Rendezvous (AMRR) algorithm under several scenarios, the difference between them is very tiny. In addition, the difference between the ETTRs of the HRR algorithm, the RPS algorithm, the AR algorithm, and the AMRR algorithm is very small both under the symmetric model and the asymmetric model. Meanwhile, their ETTRs are smaller than that of the MSS algorithm under the asymmetric model. Besides, we also showed that the proposed HRR algorithm could improve the qualities of the rendezvous channels by simulation. Up to now, most of the existing researches on CH problem aimed at shortening the Time-To-Rendezvous (TTR). However, there are still exist some other interesting aspects which were barely considered. For instance, dynamic available channels, competition between SUs and so on. The status of channels is commonly assumed to be static during the rendezvous process by most of the existing CH algorithms, which is inapplicable for realistic CRNs. The status of channels may be dynamic owing to the activities of PUs and the mobility of SUs. Besides, once multiple SUs intend to access the same channel at the same time, the completion among them may occur. The problems mentioned above will be deeply considered in our future work.

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AOHAN LI (S'17) received the B.S. and M.S. degrees from the College of Electronic Engineering, Heilongjiang University, Harbin, China, in 2012 and 2015, respectively. She is currently pursuing the Ph.D. degree with the Department of Information and Computer Science, Keio University, Yokohama, Japan. Her research interests include cognitive radio networks and machine learning. She is a Student Member of IEICE. She was a recipient of the 9th International Conference on Communications and Networking in China 2014 (CHINACOM 2014) Best Paper Award.

GUANGJIE HAN (S'03–M'05–SM'18) received the Ph.D. degree from Northeastern University, Shenyang, China, in 2004. From 2004 to 2005, he was a Product Manager of ZTE Company. From 2005 to 2006, he was the Key Account Manager of Huawei Company. Until 2008, he was a Postdoctoral Researcher with the Department of Computer Science, Chonnam National University, Gwangju, South Korea. From 2010 to 2011, he was a Visiting Research Scholar with Osaka University, Suita, Japan. In 2017, he was a Visiting Professor with the City University of Hong Kong, China. He is currently a Professor with the Department of Information and Communication Systems, Hohai University, Changzhou, China, and is also a Distinguished Professor with the Dalian University of Technology, Dalian, China. He has authored over 330 papers published in the related international conference proceedings and journals, including the IEEE COMST, the IEEE TII, the IEEE TMC, the IEEE TVT, the IEEE TIE, the IEEE TPDS, the IEEE TETC, the IEEE Internet of Things (IoT) Journal, the IEEE TETCI, the IEEE TCC, the IEEE Systems, the IEEE Sensors, the IEEE Wireless Communications, the IEEE Communications, and the IEEE Network. He holds 125 patents. His current *h*-index is 31 and i10-index is 88 in Google Citation (Google Scholar). Total citation of his papers by other people is more than 4354 times. His current research interests include the IoT, industrial Internet, machine learning and artificial intelligence, mobile computing, security, and privacy. He was a recipient of the ComManTel 2014, ComComAP 2014, Chinacom 2014, and Qshine 2016 best paper awards. He has served as a Co-Chair for more than 50 international conferences/workshops and as a Technical Program Committee Member for more than 150 conferences. He has served on the editorial boards of up to 16 international journals, including the IEEE JSAC, the IEEE Network, the IEEE Systems, the IEEE Access, the IEEE/CCA JAS, and *Telecommunication Systems*. He has guest edited a number of special issues in IEEE journals and magazines, including the IEEE Communications, the IEEE Wireless Communications, the IEEE Transactions on Industrial Informatics, and *Computer Networks*. He has served as a Reviewer for more than 60 journals.

TOMOAKI OHTSUKI (M'91–SM'01) received the B.E., M.E., and Ph.D. degrees in electrical engineering from Keio University, Yokohama, Japan, in 1990, 1992, and 1994, respectively. From 1994 to 1995, he was a Postdoctoral Fellow and a Visiting Researcher in electrical engineering with Keio University. From 1993 to 1995, he was a Special Researcher of Fellowships with the Japan Society for the Promotion of Science for Japanese Junior Scientists. From 1995 to 2005, he was with the Science University of Tokyo. From 1998 to 1999, he was with the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley, CA, USA. In 2005, he joined Keio University, where he is currently a Professor. He is engaged in research on wireless communications, optical communications, signal processing, and information theory. He has published over 140 journal papers and 340 international conference papers. He is a Fellow of the IEICE. He was a recipient of the 1997 Inoue Research Award for Young Scientist, the 1997 Hiroshi Ando Memorial Young Engineering Award, the Ericsson Young Scientist Award 2000, the 2002 Funai Information and Science Award for Young Scientist, the 1st Asia-Pacific Young Researcher Award 2001, the 5th International Communication Foundation (ICF) Research Award, the 2011 IEEE SPCE Outstanding Service Award, the 27th TELECOM System Technology Award, the ETRI Journal's 2012 Best Reviewer Award, and the CHINACOM 2014 Best Paper Award. He served as the Chair for the IEEE Communications Society, Signal Processing for Communications and Electronics Technical Committee. He served as a Technical Editor for *IEEE Wireless Communications Magazine*. He is currently serving as an Editor for the IEEE Communications Surveys and Tutorials and the *Elsevier Physical Communications*. He has served as the General Co-Chair and as the Symposium Co-Chair for many conferences, including the IEEE GLOBECOM 2008, SPC, the IEEE ICC 2011, CTS, and the IEEE GCOM 2012, SPC. He has given tutorials and keynote speeches in many international conferences, including the IEEE VTC and the IEEE PIMRC. He is currently serving as the Vice President of the Communications Society of the IEICE.