IEEE Access
Multidisciplinary | Rapid Review | Open Access Journal

Received January 30, 2019, accepted February 17, 2019, date of publication March 14, 2019, date of current version April 1, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2901966

Adaptive Integral-Type Terminal Sliding Mode Fault Tolerant Control for Spacecraft Attitude Tracking

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This work was supported in part by the National Nature Science Foundation under Grant 61573378, and in part by the Natural Science Foundation of Shandong Province under Grant ZR2011FM002.

ABSTRACT In this paper, the attitude tracking control for a spacecraft system based on a novel adaptive integral-type terminal sliding mode fault-tolerant control strategy is investigated. First, a basic fault-tolerant control scheme is designed to ensure the spacecraft attitude tracking performance in the presence of the actuator fault, external disturbance, and actuator saturation when the rotational inertia of a spacecraft system is known. Second, combined with adaptive laws, a modified fault-tolerant control scheme is proposed to compensate the actuator fault and system uncertainty which contains external disturbance and the inertia uncertainty, separately. The actuator fault is compensated more directly and accurately. The benefits of this approach are that the proposed control scheme can enable the merits of the integral-type terminal sliding mode control, such as nonsingularity, fast convergence speed, and small steady-state errors. Finally, the simulation results are given to demonstrate the effectiveness of the proposed fault-tolerant control schemes.

INDEX TERMS Actuator fault, adaptive integral-type terminal sliding mode control, attitude tracking control, fault tolerant control.

I. INTRODUCTION

Attitude tracking control problem has been an active research topic for recent years. Many advanced control methods have been developed for this kind of space vehicle [1]–[4]. However, the afore-mentioned studies did not consider the actuator fault, which is possible and even likely for practical spacecraft. How to improve the tracking performance and transient response for spacecraft system, particularly in the presence of external disturbance and unexpected actuator fault is still a challenge. Therefore, the fault tolerant control (FTC) emerges as an effective approach to deal with the actuator fault and increase system reliability.

As is well known, the FTC schemes can be classified into two categories: passive approach and active approach. However, the active FTC relies on the availability of the fault detection and diagnosis (FDD) [5]–[8]. Therefore, obviously that a huge computational burden and decision steps are required in active FTC. According to that, the long computation time may generate the time delay, and consequently, the control performance may be decreased, even out of control. Contrary to the active FTC, passive FTC is simpler and requires less computation time and power to compensate the unexpected fault without explicit FDD procedure [9], [10]. Considerable effective passive FTC methods have been proposed, such as linear matrix inequalities (LMIs) [11], pole assignment [12], and sliding mode control (SMC) [13]–[15], [15].

Among these proposed methods, SMC approach has been widely applied for spacecraft system due to the robustness against uncertainty and disturbance. However, the traditional SMC can only ensure asymptotic stability. With a view to achieving the finite time convergence, terminal SMC (TSMC) has been proposed by [16]. The application of TSMC to design the spacecraft attitude control first emerged in [17]. In [18], the TSMC-based FTC law has been designed for the spacecraft attitude control under the inertia uncertainty, external disturbance and actuator fault. However, TSMC suffers with two weaknesses: one is the singular problem, and the other is the slower convergence rate. Hence, the nonsingular

The associate editor coordinating the review of this manuscript and approving it for publication was Fanbiao Li.

TSMC (NTSMC) and fast TSMC (FTSMC) have been proposed in [19] and [20] to solve the problem of singularity and convergence speed, respectively. Further, by using NTSMC and FTSMC, the attitude controllers for spacecraft have been designed in [21]–[23], respectively. The NTSMC-based and FTSMC-based FTC laws for spacecraft attitude control have been proposed in [24] and [25], respectively. Unfortunately, the presented controller based on NTSMC or FTSMC can only solve one aspect and ignore the other problem. To tackle both the singular problem and faster convergence rate together, the nonsingular FTSMC (NFTSMC) has been proposed in [26]. Inspired by [26], the NFTSMC-based attitude control laws for spacecraft have been designed in [27] and [28]. Then, in [29], the NFTSMC-based FTC scheme for the spacecraft attitude tracking has been presented.

The aforementioned methods, although robust against the system uncertainty and actuator fault, however, the dynamics may be damaged during the reaching phase in which the system states stay at a distance from sliding surface. In order to tackle this problem, the concept of PID-SMC or integral SMC (ISMC) has been developed in [30]–[34]. Furthermore, the integral TSMC (ITSMC), which combined the advantages of ISMC and TSMC, has been proposed in [35]–[37]. However, none of them has been applied to FTC. Then, the FTC schemes based on ITSMC have been proposed in [38]–[40]. Although the ITSMC to FTC for spacecraft has been studied extensively, there are some limitations to existing researches. More specifically, in [39], the control scheme has been proposed based on modified Rodrigues parameters (MRPs), so the FTC method was non-global due to the intrinsic singularity of MRPs. The method of [40] can only ensure that the tracking errors converge to a small neighborhood about zero. Meanwhile, in most previous work [41]–[43], which focused on known specific fault scenarios. In order to tackle these limitations, an appropriate design procedure is necessary to guarantee the performance in the presence of the unknown actuator fault and system uncertainty.

Purposed by the above analysis, the endeavor here is to propose a novel integral-type TSMC fault tolerant control algorithm for spacecraft system with actuator fault and system uncertainty. Starting from the selection of an integral-type TSM surface, the second-order dynamics model of the spacecraft system is transferred into another standard second-order dynamics model based on the selected sliding surface. Following this, a basic fault tolerant controller based on integral-type TSMC is designed to ensure the attitude tracking performance in the presence of the actuator fault, external disturbance and actuator saturation when the spacecraft inertia is known. Then, a modified FTC scheme is proposed to compensate the actuator fault and the system total uncertainty including inertia uncertainty, separately. Such actuator fault is unknown in pattern, time instant, value. The main contributions of this paper are given as follows:

1) The novel integral-type TSM fault tolerant control schemes are proposed for spacecraft attitude tracking control.

2) A modified adaptive FTC technique for spacecraft system with redundant actuators is proposed, the actuator fault and the system total uncertainty are estimated by adaptive laws, separately. As a result, the actuator fault can be compensated more accurately and directly without exact fault diagnosis procedure.

3) Compared with other existing FTC schemes, such as NTSMC, FTSMC, and PID-SMC, the proposed fault tolerant scheme will provide fast convergence speed and small steady state error without singular problem.

The rest of this paper is organized as follows. In Section 2, the spacecraft attitude model and actuator faults are introduced. In Section 3, basic integral-type TSM FTC scheme and modified adaptive integral-type TSM FTC scheme are presented, respectively. Simulation results are demonstrated to show the effectiveness of the proposed control scheme in Section 4. Finally, conclusion is given in Section 5.

Remark 1: Beside the references cited and discussed above, the integral sliding mode control is proposed in the literature for difference applications. For instance, in [44], a novel integral sliding mode controller has been designed for a general type of underwater robots based on multiple-input and multiple-output extended-state-observer (MIMO-ESO). In [45], a dynamic integral sliding sliding mode control has been investigated for Markovian jump singular system (MJSS) with model-dependent derivative-term coefficient. However, the existing results are mainly confined to linear MJSS, which imposes a considerable limitation in real applications since most practical dynamics are highly nonlinear. In [46], a novel integral-type fuzzy sliding surface has been put forward for T-S fuzzy-model-based nonlinear MJSS subject to matched/unmatched uncertainties, and a new projection term with projection matrices is introduced, which distinguishes the existing surfaces for linear MJSS. In practical applications, the unexpected faults may occur during operation and, if they do occur, can degrade the designed attitude control performance or even result in mission faults. Therefore, the focus of this paper is to propose adaptive passive FTC schemes using an integral-type terminal sliding mode surface.

II. PROBLEM FORMULATION

A. SPACECRAFT ATTITUDE KINEMATICS AND DYNAMICS

Consider the kinematics of spacecraft attitude control system described as [47]

$$\dot{q}_0 = -\frac{1}{2}q^T w, \tag{1}$$

$$\dot{q} = Q(q)w, \tag{2}$$

where $q_s = [q_0 \;\; q^T]^T \in \mathbb{R}^4$ is regarded as the quaternion of the body frame with respect to the inertial frame and satisfies the constraint $q_0^2 + q^T q = 1$, q_0 is the scalar component and $q = [q_1 \;\; q_2 \;\; q_3]^T$ is the vector part; $w = [w_1 \;\; w_2 \;\; w_3]^T \in \mathbb{R}^3$ represents the angular velocity of the spacecraft with respect to the inertial frame; $Q(q) \in \mathbb{R}^{3\times 3}$ is defined as $Q(q) = \frac{1}{2}(q_0\mathbf{I}_{3\times 3} + q^\times)$. $q^\times$ represents a skew-symmetric matrix which can be described as

$$q^{\times} = \begin{pmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{pmatrix}. \quad (3)$$

The dynamics of spacecraft attitude control system can be given by [47]

$$J\dot{w} = -w^{\times}Jw + u(t) + d(t), \quad (4)$$

where $J \in \mathbb{R}^{3\times 3}$ is the positive-definite inertia matrix, and $J = J^T$. $u(t) = [u_1 \quad u_2 \quad u_3]^T \in \mathbb{R}^3$ and $d(t) \in \mathbb{R}^3$ are the control torque and external disturbance acting on the spacecraft, respectively.

Let $q_{ev} = [q_{e0} \quad q_e^T]^T$ represents the relative attitude quaternion error between the body frame and the desired frame. Then, one can get that

$$q_{ev} = q_s \otimes q_{ds}^*, \quad (5)$$

where q_{ds} denotes the attitude motion of a desired frame with respect to the inertial frame. q_{ds}^* is defined by

$$q_{ds}^* = [q_{d0} \quad -q_d^T]^T. \quad (6)$$

Given that two unit quaternion $q_{as} = [q_{a0} \quad q_a^T]^T$ and $q_{bs} = [q_{b0} \quad q_b^T]^T$, the operator $\otimes$ is for quaternion multiplication that can be defined as [48]

$$q_{as} \otimes q_{bs} = \begin{pmatrix} q_{a0}q_b + q_{b0}q_a - q_a^{\times}q_b \\ q_{a0}q_{b0} - q_a^T q_b \end{pmatrix}. \quad (7)$$

The angular velocity error is defined as $w_e = w - Cw_d \in \mathbb{R}^3$, $w_d \in \mathbb{R}^3$ is the angular velocity of the desired frame. It is noted that $\|C\| = 1$ and $C \in \mathbb{R}^{3\times 3}$ is the corresponding attitude rotation matrix:

$$\begin{cases} C = (q_0^2 - q^T q)I + 2qq^T - 2q_0 q^{\times} \\ \dot{C} = -w_e^{\times} C \end{cases}. \quad (8)$$

As a result, the spacecraft attitude tracking problem is transformed into the stabilization problem of the attitude tracking errors, and then the mathematic model of the attitude tracking error system can be obtained that

$$\dot{q}_{e0} = -\frac{1}{2}q_e^T w_e, \quad (9)$$

$$\dot{q}_e = Q(q_e)w_e, \quad (10)$$

$$J\dot{w}_e = -(w_e + Cw_d)^{\times}J(w_e + Cw_d) + J(w_e^{\times}Cw_d - C\dot{w}_d) + u(t) + d(t), \quad (11)$$

where $Q(q_e) = \frac{1}{2}(q_{e0}\mathbf{I}_{3\times 3} + q_e^{\times})$ and $\|Q(q_e)\| = \frac{1}{2}$.

Let $p_1 = Cw_d$, $p_2 = C\dot{w}_d$, $M_e = Q(q_e)^{-1}$ and $M = Q(q_e)J^{-1}$. p_e is considered as the first derivative of the vector part of quaternion error, i.e., $p_e = Q(q_e)w_e$. Then (10) and (11) can be rewritten as

$$\dot{q}_e = p_e, \quad (12)$$

$$\dot{p}_e = f(q_e, p_e) + Mu(t) + Md(t), \quad (13)$$

where $f(q_e, p_e) = \dot{Q}_e M_e p_e - M[(M_e p_e + p_1)^{\times} J(M_e p_e + p_1) + J((M_e p_e)^{\times} p_1 - p_2)]$. To simplify the form of writing, the $Q(q_e)$ is written into Q_e.

Control Objective: The objective of this paper is to design the FTC scheme such that the spacecraft system can achieve attitude tracking performance in spite of actuator fault and the system uncertainty. That is to say, the tracking error (q_e, w_e) converges to zero asymptotically, which can be expressed as follows:

$$\lim_{t\mapsto\infty} q_e = 0, \quad \lim_{t\mapsto\infty} w_e = 0. \quad (14)$$

Remark 2: Here, the matrix Q_e should be invertible to ensure the existence of M_e, that is to say, $det(Q_e) = \frac{1}{2}q_{e0}(t) \neq 0$ for $t \geq 0$. Therefore, the initial state and control scheme should be chosen to guarantee $q_{e0}(t) \neq 0$ for all time. Actually, the initial states can be always selected as $q_{e0}(t) \neq 0$ for $t > 0$.

B. ACTUATOR FAULT

The actuators are vulnerable to various faults in practical operation. In this paper, the actuator multiplicative fault is considered as

$$u(t) = Ev(t), \quad (15)$$

where $E = diag(E_1, E_2, \cdots, E_m)$ is the actuator effectiveness matrix, and $0 < E_j \leq 1, j = 1, \cdots, m$. Note that, $E_j = 0$ implies that the j^{th} actuator completely fail, $E_j = 1$ shows that the j^{th} actuator is working fault-free, $0 < E_j < 1$ represents the efficiency factor of j^{th} actuator. $v(t)$ is the control input to be designed.

III. FAULT TOLERANT CONTROL DESIGN

A. BASIC FAULT TOLERANT CONTROL DESIGN

In this subsection, under control input saturation, the dynamics of spacecraft with the actuator fault (15) is given as

$$\begin{aligned} \dot{p}_e &= f(q_e, p_e) + MEsat(v(t)) + Md(t) \\ &= f(q_e, p_e) + Msat(v(t)) + Md(t) \\ &\quad + M(E - I_3)sat(v(t)) \\ &= f(q_e, p_e) + Mv(t) + H, \end{aligned} \quad (16)$$

where $H = M(E - I_3)sat(v(t)) + Md(t) + M\theta(t)$ is the so-called synthetic uncertainty. $sat(v_i(t)), i = 1, 2, 3$ denotes the nonlinear saturation characteristic of the actuators, and has the form as $sat(v_i(t)) = \theta_i(t) + v_i(t)$. The part of excess saturation $\theta_i(t)$ is defined as

$$\theta_i(t) = \begin{cases} 0 & |v_i(t)| < v_{imax}(t) \\ sgn(v_i(t))v_{imax}(t) - v_i(t) & |v_i(t)| \geq v_{imax}(t) \end{cases}, \quad (17)$$

where $v_i(t)$ and $v_{imax}(t)$ are the control torque and the maximum control torque of i^{th} actuator, respectively.

Assumption 1: The external disturbance d is assumed to be bounded by an unknown positive scalar δ, i.e., $\|d\| \leq \delta$.

Assumption 2: The inertia matrix J is the known symmetric positive-definite matrix and satisfies that

$$\|J\| \leq J^*, \quad (18)$$

where $J^* > 0$ is the known upper bound of the norm of the inertia matrix.

Assumption 3: The control input satisfies the limit of maximum value, $\|u(t)\| \le u_{max}$, u_{max} is the maximum control input torque.

Assumption 4: In this paper, the so-called synthetic uncertainty is assumed to be bounded but unknown, that is to say $\|H\| \le c_1$ as [24], c_1 is a positive constant but unknown.

Remark 3: The actual control input torques are bounded due to practical physical limitations of the actuators as [49], thus the part of the excess saturation $\theta(t)$ is also bounded. In addition, due to the inertia matrix acting on the spacecraft system is bounded in practice [50], $M = Q_e J^{-1}$ is bounded. The external disturbance is also assumed to be bounded. Therefore, the above assumption is true.

In this paper, to achieve the attitude tracking problem for spacecraft system, the integral-type TSM surface is designed as

$$s = p_e + \alpha q_e + \beta \int_0^t q_e^{\frac{q}{p}}\, d\tau, \tag{19}$$

where α and β are both positive scalars, $\alpha > 0$ and $\beta > 0$. p and q are both positive odd integers satisfying the relation $p > q$. During the reaching phase, to guarantee the performance, the reaching law is selected as the following form:

$$\dot{s} = -ks - \varepsilon sgn(s), \tag{20}$$

where k and ε are both given positive constants.

Theorem 1: Consider the attitude control system (12) and (16) in the presence of the actuator fault, external disturbance and actuator saturation, the integral-type TSM fault tolerant control scheme (21)-(22) can guarantee the system trajectory reach to sliding mode surface (19) in finite time.

$$\begin{aligned} v(t) &= M^{-1}(-ks - \varepsilon sgn(s) - \beta q_e^{\frac{q}{p}} - \alpha p_e - \hat{c}_1 sgn(s) \\ &\quad -f(q_e, p_e)), \end{aligned} \tag{21}$$

$$\dot{\hat{c}}_1 = k_1 s^T sgn(s), \tag{22}$$

where $\hat{c}_1$ is the estimated value of c_1, and k_1 is a positive scalar.

Proof: Define that $\tilde{c}_1 = c_1 - \hat{c}_1$. $\tilde{c}_1$ is the parameter error. Choose the Lyapunov function

$$V_1 = \frac{1}{2} s^T s + \frac{1}{2} k_1^{-1} \tilde{c}_1^2. \tag{23}$$

Taking the derivative with respect to time of V_1, it has

$$\begin{aligned} \dot{V}_1 &= s^T \dot{s} + k_1^{-1} \tilde{c}_1 \dot{\tilde{c}}_1 \\ &= s^T(\dot{p}_e + \alpha \dot{q}_e + \beta q_e^{\frac{q}{p}}) + k_1^{-1} \tilde{c}_1 \dot{\tilde{c}}_1 \\ &= s^T(f(q_e, p_e) + Mv(t) + H + \alpha p_e + \beta q_e^{\frac{q}{p}}) + k_1^{-1} \tilde{c}_1 \dot{\tilde{c}}_1. \end{aligned} \tag{24}$$

Substituting the control input (21) and adaptive law (22) into (24), the above equation can be transformed into following form:

$$\begin{aligned} \dot{V}_1 &\le s^T(-ks - \varepsilon sgn(s) - \hat{c}_1 sgn(s) + c_1 sgn(s)) + k_1^{-1} \tilde{c}_1 \\ &\quad (-k_1 s^T sgn(s)) = s^T(-ks - \varepsilon sgn(s)) \\ &\le -2kV_1 - \sqrt{2}\varepsilon V_1^{\frac{1}{2}}. \end{aligned} \tag{25}$$

According to Lemma 3, one can clearly see that $\mu_1 = 2k$, $\mu_2 = \sqrt{2}\varepsilon$ and $\iota = \frac{1}{2}$, the trajectory of the system (12) and (16) will be driven onto the sliding mode surface (19) in a finite time T_1, which is given as

$$\begin{aligned} T_1 &\le \frac{1}{2k(1-\frac{1}{2})} ln \frac{2kV_1^{1-\frac{1}{2}}(0) + \sqrt{2}\varepsilon}{\sqrt{2}\varepsilon} \\ &\le \frac{1}{k} ln \frac{\sqrt{2}kV_1^{\frac{1}{2}}(0) + \varepsilon}{\varepsilon}, \end{aligned} \tag{26}$$

where $V_1(0)$ is the initial value of V_1. □

Theorem 2: For the attitude error system (12) and (16), once the attitude trajectory reach onto the sliding mode surface (19), that is to say, $s = 0$, the tracking error will converge to zero asymptotically.

Proof: Once reach the sliding mode surface $s = 0$, it has

$$p_e + \alpha q_e + \beta \int_0^t q_e^{\frac{q}{p}}\, d\tau = 0. \tag{27}$$

Meanwhile, it is obtained that $\dot{s} = 0$,

$$\dot{p}_e + \alpha \dot{q}_e + \beta q_e^{\frac{q}{p}} = 0. \tag{28}$$

Thus, it has that

$$\dot{p}_e = -\alpha \dot{q}_e - \beta q_e^{\frac{q}{p}}. \tag{29}$$

Choose the Lyapunov function

$$V_2 = \tfrac{1}{2} p_e^2 + \tfrac{\beta p}{p+q} q_e^{\frac{q}{p}+1}. \tag{30}$$

Taking the time derivative of V_2, and applying (29) leads to

$$\begin{aligned} \dot{V}_2 &= p_e \dot{p}_e + \beta q_e^{\frac{q}{p}} \dot{q}_e \\ &= p_e(-\alpha \dot{q}_e - \beta q_e^{\frac{q}{p}}) + \beta q_e^{\frac{q}{p}} \dot{q}_e \\ &= p_e(-\alpha p_e - \beta q_e^{\frac{q}{p}}) + \beta q_e^{\frac{q}{p}} p_e \\ &= -\alpha p_e^2 \le 0. \end{aligned} \tag{31}$$

Therefore, all states are uniformly bounded. In addition, because $V_2 > 0$ and $\dot{V}_2 \le 0$, it can be obtained that $\lim_{t\to\infty} V_2 = V_2(\infty)$ exists for $V_2(\infty) \in \mathbf{R}^+$. According to the boundedness of all states within $\dot{V}_2(t)$, it is obviously to obtain that the uniform continuity for $\dot{V}_2(t)$. Then, according to the application of Barbalat lemma, it can be concluded that $\lim_{t\to\infty} p_e = 0$. From the sliding mode surface (19), it follows that $\lim_{t\to\infty} q_e = 0$. Therefore, the tracking error converges to zero asymptotically. □

B. MODIFIED FAULT TOLERANT CONTROL DESIGN

In the above subsection, the actuator fault and external disturbance are addressed as the synthetic uncertainty. However, this is too conservative since that the very stringent conditions need to be fulfilled. In addition, the inertia uncertainty, which is usually occur in real operation, is not considered. For the sake of tackling these limitations, a modified fault tolerant controller is designed for spacecraft attitude control system with redundant actuators.

Consider the dynamics of spacecraft system (11) with redundant actuators can be given by

$$J\dot{w}_e = -(w_e + Cw_d)^{\times} J(w_e + Cw_d) + J(w_e^{\times} Cw_d - C\dot{w}_d) + d(t) + Du(t), \quad (32)$$

where $D \in \mathbb{R}^{3\times m}$ is the actuators distribution matrix, which is available by properly assembling actuators at a certain location and direction for a given spacecraft, m is the number of actuators.

With the actuator fault (15), the dynamics of spacecraft system (32) becomes

$$J\dot{w}_e = -(w_e + Cw_d)^{\times} J(w_e + Cw_d) + J(w_e^{\times} Cw_d - C\dot{w}_d) + d(t) + DEv(t). \quad (33)$$

Actuation Redundancy: It is necessary to have a certain actuator redundancy to compensate actuator fault for spacecraft system. However, the existence of the redundant actuators may bring more challenges for controller design when the fault is unknown. Notice that fault-free actuators should not be less than the freedom of the dynamics system and the condition (34) needs to be satisfied for all fault cases that under consideration, otherwise the spacecraft attitude control system may become an under-actuated system [51]:

$$rank(DE) = 3. \quad (34)$$

Assumption 5: The inertia matrix J is in the form $J = J_0 + \Delta J$, where J_0 is the known nominal constant matrix, ΔJ represents the uncertainty and satisfies that $\|\Delta J\| \le J_\epsilon$, J_ϵ is a positive constant but unknown.

Assumption 6: The desired angular velocity and its derivative are both bounded, that is to say, there are unknown constants $r_1 \in \mathbb{R}^+$ and $r_2 \in \mathbb{R}^+$, such that $\|w_d\| \le r_1$, $\|\dot{w}_d\| \le r_2$.

Similarly, let $p_e = Q_e w_e$, $p_1 = Cw_d$, and $p_2 = C\dot{w}_d$. Then the system (33) with the inertia uncertainty has the following form

$$\dot{p}_e = f_1(q_e, p_e) + J_0^{-1}\bar{H} + zEv(t), \quad (35)$$

where

$$\begin{aligned} f_1(q_e, p_e) &= -M_0(Q_e^{-1}p_e + P_1)^{\times} J_0(Q_e^{-1}p_e + p_1) + Q_e \\ &\quad [(Q_e^{-1}p_e)^{\times} p_1 - p_2] + \dot{Q}_e Q_e^{-1} p_e, \quad (36) \\ \bar{H} &= -Q_e(Q_e^{-1}p_e + p_1)^{\times} \Delta J(Q_e^{-1}p_e + p_1) + \Delta J Q_e \\ &\quad [(Q_e^{-1}p_e)^{\times} p_1 - p_2] + Q_e d(t) - \Delta J \dot{p}_e \quad (37) \\ &\quad + \Delta J \dot{Q}_e Q_e^{-1} p_e. \end{aligned}$$

For the sake of brevity, let $Q_e J_0^{-1} = M_0$ and $z = M_0 D$.

In this subsection, the fault tolerant controller is designed as

$$v(t) = z^T v_0(t), \quad (38)$$

where $v_0(t) \in \mathbb{R}^3$ is the designed control input. With the condition (34), the following proposition is straightforward.

Proposition 1 [52]: If the condition (34) is satisfied, then $zEz^T = (zEz^T)^T > 0$.

Then, with the Proposition 1, we introduce that $F = [zEz^T]^{-1}$, and substitute it into (35) yields

$$\dot{p}_e = f_1(q_e, p_e) + J_0^{-1}\bar{H} + F^{-1}v_0(t). \quad (39)$$

Assumption 7: For the disposing of system uncertainty, it is assumed that $\bar{H}$ satisfies the bounded constraint similar to [53]:

$$\|FJ_0^{-1}\bar{H}\| \le c_2(1 + \|w\| + \|w\|^2) = c_2\varphi, \quad (40)$$

and $\varphi = 1 + \|w\| + \|w\|^2$.

The integral-type TSM surface is chosen as (19), and the reaching condition is selected as

$$F\dot{s} = -ks - \varepsilon sgn(s), \quad (41)$$

where k and ε are both positive integers.

As a result, the control input can be designed as

$$v_0(t) = -\hat{F}f_1(q_e, p_e) - \hat{c}_2 sgn(s)\varphi - \alpha\hat{F}p_e - \beta\hat{F}q_e^{\frac{q}{p}} - ks - \varepsilon sgn(s), \quad (42)$$

where $\hat{c}_2$ and $\hat{F}$ are the estimated value of c_2 and F, respectively. Define the parameters errors $\tilde{c}_2 = c_2 - \hat{c}_2$, $\tilde{F} = F - \hat{F}$. Then they are updated by the following adaptive laws:

$$\dot{\hat{c}}_2 = k_2 s^T sgn(s)\varphi, \quad (43)$$

$$\dot{\hat{F}} = k_3 s(f_1(q_e, p_e) + \alpha p_e + \beta q_e^{\frac{q}{p}})^T, \quad (44)$$

where k_2 is a positive constant, and $k_3 \in \mathbb{R}^{3\times 3}$ satisfies that $k_3 = k_3^T > 0$.

Based on the adaptive FTC scheme designed in (42)-(44), the main result of this subsection can be described as the following theorem.

Theorem 3: Consider the system described by (39) and satisfying the condition (34). Then, the system trajectory will be driven onto the sliding mode surface $s = 0$ in finite time via the adaptive fault tolerant control scheme (42)-(44).

Proof: Choose the Lyapunov function

$$V_3 = \frac{1}{2}s^T Fs + \frac{1}{2}k_2^{-1}\tilde{c}_2^2 + \frac{1}{2}tr(\tilde{F}^T k_3^{-1}\tilde{F}). \quad (45)$$

Taking the time derivative along the system (39), it has

$$\begin{aligned} \dot{V}_3 &= s^T F\dot{s} + k_2^{-1}\tilde{c}_2\dot{\tilde{c}}_2 + tr(\tilde{F}^T k_3^{-1}\dot{\tilde{F}}) \\ &= s^T(F\dot{p}_e + \alpha F\dot{q}_e + \beta Fq_e^{\frac{q}{p}}) + k_2^{-1}\tilde{c}_2\dot{\tilde{c}}_2 + tr(\tilde{F}^T k_3^{-1}\dot{\tilde{F}}) \\ &\le s^T(Ff_1(q_e, p_e) + v_0(t) + c_2 sgn(s)\varphi + \alpha Fp_e \\ &\quad + \beta Fq_e^{\frac{q}{p}}) + k_2^{-1}\tilde{c}_2\dot{\tilde{c}}_2 + tr(\tilde{F}^T k_3^{-1}\dot{\tilde{F}}) \\ &= s^T(\hat{F}f_1(q_e, p_e) + v_0(t) + \hat{c}_2 sgn(s)\varphi + \alpha\hat{F}p_e \\ &\quad + \beta\hat{F}q_e^{\frac{q}{p}}) + s^T(\tilde{F}f_1(q_e, p_e) + \alpha\tilde{F}p_e + \beta\tilde{F}q_e^{\frac{q}{p}}) \\ &\quad + \tilde{c}_2 s^T sgn(s)\varphi + k_2^{-1}\tilde{c}_2\dot{\tilde{c}}_2 + tr(\tilde{F}^T k_3^{-1}\dot{\tilde{F}}). \quad (46) \end{aligned}$$

Then, substituting the fault tolerant controller (42) and the adaptive laws (43)-(44) into (46) yields

$$\begin{aligned} \dot{V}_3 &\le s^T(-ks - \varepsilon sgn(s)) + s^T(\tilde{F}f_1(q_e, p_e) + \alpha\tilde{F}p_e \\ &\quad + \beta\tilde{F}q_e^{\frac{q}{p}}) + \tilde{c}_2 s^T sgn(s)\varphi + k_2^{-1}\tilde{c}_2\dot{\tilde{c}}_2 + tr(\tilde{F}^T k_3^{-1}\dot{\tilde{F}}) \\ &\le -ks^T s - \varepsilon|s| \\ &\le -\frac{2k}{\lambda_{max}(F)}V_3 - \frac{2\varepsilon}{\lambda_{max}(F)}V_3^{\frac{1}{2}}. \quad (47) \end{aligned}$$

According to Lemma 3 again, the trajectory of the system (39) will be driven onto $s = 0$ in finite time

$$T_2 \le \frac{\lambda_{max}(F)}{k} ln \frac{kV_3^{\frac{1}{2}}(0) + \varepsilon}{\varepsilon}, \tag{48}$$

where $V_3(0)$ is the initial value of V_3, $\lambda_{max}(F)$ is the maximum eigenvalue of F.

Then, combining Theorem 2, the quaternion error q_e and the angular velocity error w_e will converge to zero asymptotically. □

Remark 4: In this paper, the $sgn(\cdot)$ is denoted as

$$sgn(\chi) = \begin{cases} 1 & if\, \chi > 0, \\ 0 & if\, \chi = 0, . \\ -1 & if\, \chi < 0. \end{cases} \tag{49}$$

It is obviously that the control law $v_0(t)$ is discontinuous which can cause the chattering, therefore, we can use the following $sat(\cdot)$ for the $sgn(\cdot)$ function:

$$sat(\frac{\chi}{\xi}) = \begin{cases} 1 & if\, \frac{\chi}{\xi} > 1, \\ \chi/\xi & if\, |\frac{\chi}{\xi}| \le 1, \\ -1 & if\, \frac{\chi}{\xi} < -1., \end{cases} \tag{50}$$

where ξ is the boundary layer thickness of the saturation function.

IV. SIMULATION

In this section, the simulation results for the proposed adaptive integral-type TSM control schemes are presented. The spacecraft inertia matrix is given as [54]

$$J = \begin{pmatrix} 20 & 1.2 & 0.9 \\ 1.2 & 17 & 1.4 \\ 0.9 & 1.4 & 15 \end{pmatrix} kg \cdot m^2.$$

The case of initial conditions is considered as follows:

$$\begin{aligned} q_{d0} &= [0.3\ 0.2\ 0.5\ 0.7874]^T \\ w_{d0} &= 0.05[sin(0.01\pi t)\ 2sin(0.02\pi t)\ 3sin(0.03\pi t)]^T \\ q_{s0} &= [0.3\ -0.2\ 0.3\ 0.8832]^T. \end{aligned}$$

In addition, the external disturbance $d(t)$ is described as

$$d(t) = \begin{bmatrix} 3cos(0.01t) + 1 \\ 5sin(0.02t) + 3cos(0.02t) + 2 \\ 3sin(0.01t) + 3 \end{bmatrix} \times 10^{-3} N \cdot m.$$

In this paper, two different sets of simulations are presented to demonstrate the effectiveness of the proposed control schemes:

(1) Attitude tracking control using proposed basic fault tolerant control law (21) under fault case 1:

$$E_i = \begin{cases} 1 & t \le 10s \\ 0.75 + 0.1sin(0.5t + i\pi/3) & t > 10s \end{cases}.$$

Here, $i = 1, 2, 3$.

(2) Attitude tracking control using proposed modified fault tolerant control law (42) for the spacecraft with four reaction wheels under fault case 2:

$$\begin{cases} E_1 = 0.4 & 10s < t \le 20s \\ E_4 = 0.2 & 30s < t \le 40s \end{cases}.$$

Furthermore, in order to demonstrate the superior performance of the proposed integral-type TSM control scheme, we compare it with the existing methods, which have been applied to improve the attitude tracking performance of spacecraft with actuator fault such as NTSMC, FTSMC and PID-SMC. The design of the NTSMC, FTSMC and PID-SMC are presented in Appendix. In the following simulation, the selected parameters of these controllers are reported in Table 1, Table 2.

TABLE 1. Selected parameters of the controllers.

Controller	Parameters	Value
NTSMC	$\alpha, p, q, k, \varepsilon, k_1$	1.5, 5, 3, 5, 0.2, 1.2
FTSMC	$\alpha, \beta, p, q, k, \varepsilon, k_1$	1, 4, 3, 5, 5, 0.2, 1.2
PID-SMC	$\alpha, \beta, k, \varepsilon, k_1$	1.8, 4, 5, 0.2, 1.2
Integral-type TSMC	$\alpha, \beta, p, q, k, \varepsilon, k_1$	1, 3.2, 9, 7, 5, 0.2, 1.2

TABLE 2. Selected parameters of the modified controllers.

Controller	Parameters	Value
NTSMC	$\alpha, p, q, k, \varepsilon, k_2, k_3$	1.5, 5, 3, 5, 0.2, 1.5, $100I_3$
FTSMC	$\alpha, \beta, p, q, k, \varepsilon, k_2, k_3$	1, 4, 5, 3, 5, 0.2, 1.5, $100I_3$
PID-SMC	$\alpha, \beta, k, \varepsilon, k_2, k_3$	1, 3.2, 5, 0.2, 1.5, $100I_3$
Integral-type TSMC	$\alpha, \beta, p, q, k, \varepsilon, k_2, k_3$	1.5, 4, 9, 7, 5, 0.2, 1.5, $100I_3$

A. SIMULATIONS FOR BASIC FAULT TOLERANT CONTROL

First, we consider the spacecraft system(12)-(16) with three actuators, the proposed basic control scheme (21) is compared with the NTSMC (54), FTSMC (60), PID-SMC (66) by taking the actuator fault, external disturbance and actuator saturation described previously into account.

Fig.1-Fig.4 show that the time responses of the quaternion error, angular velocity error, and the control torque obtained by four methods, respectively. As what is expected, one can clearly see that each of the controllers can achieve the attitude tracking in this case, but the resulting performance is quite different. For an easier comparison, the convergence time of these control schemes, it can be seen that the NTSMC provides the slowest convergence time. From the Fig.1 (a), (b) and Fig.2 (a), (b), the convergence speed of FTSMC has been obtained faster. However, in Fig.1 (c) and Fig.2 (c), the chattering problem of the control torque has not been effectively solved. Interestingly, the PID-SMC effectively alleviate the chattering problem as shown in Fig.3 (c), but increase the steady state error in Fig.3 (a), (b). Fortunately, as shown in Fig.4, the proposed integral-type TSMC provide faster convergence speed and smaller steady state error compared to the PID-SMC.

From the above simulation results, it can be concluded that the proposed integral-type TSMC control scheme provides

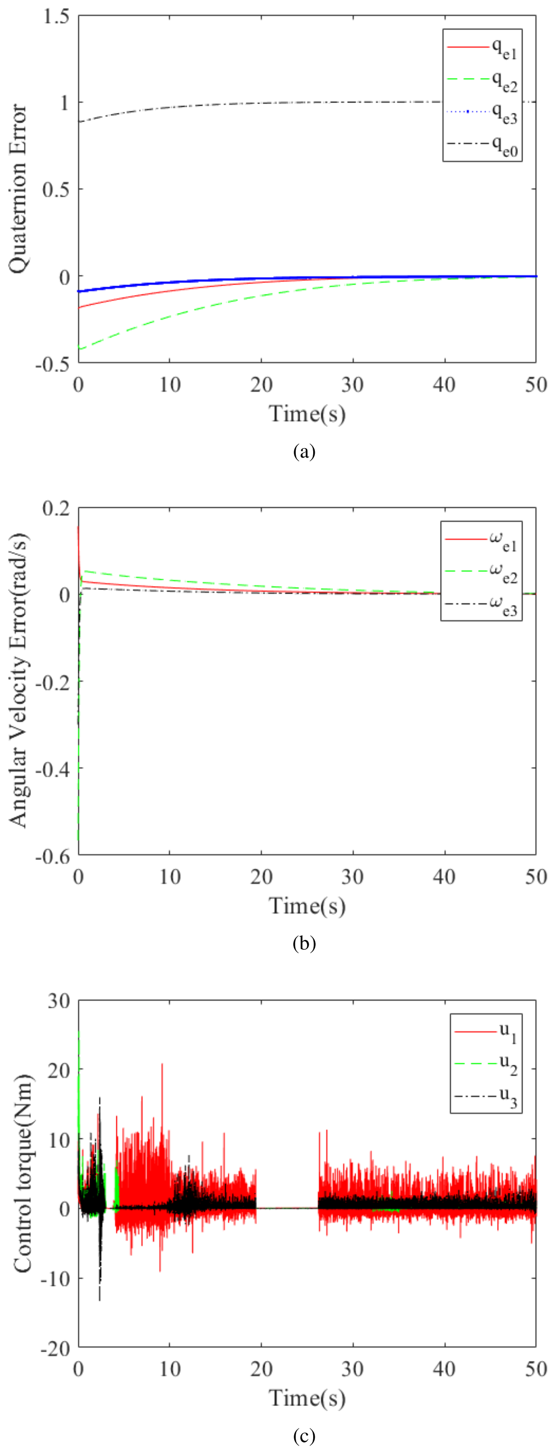

FIGURE 1. Response curves via basic NTSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.

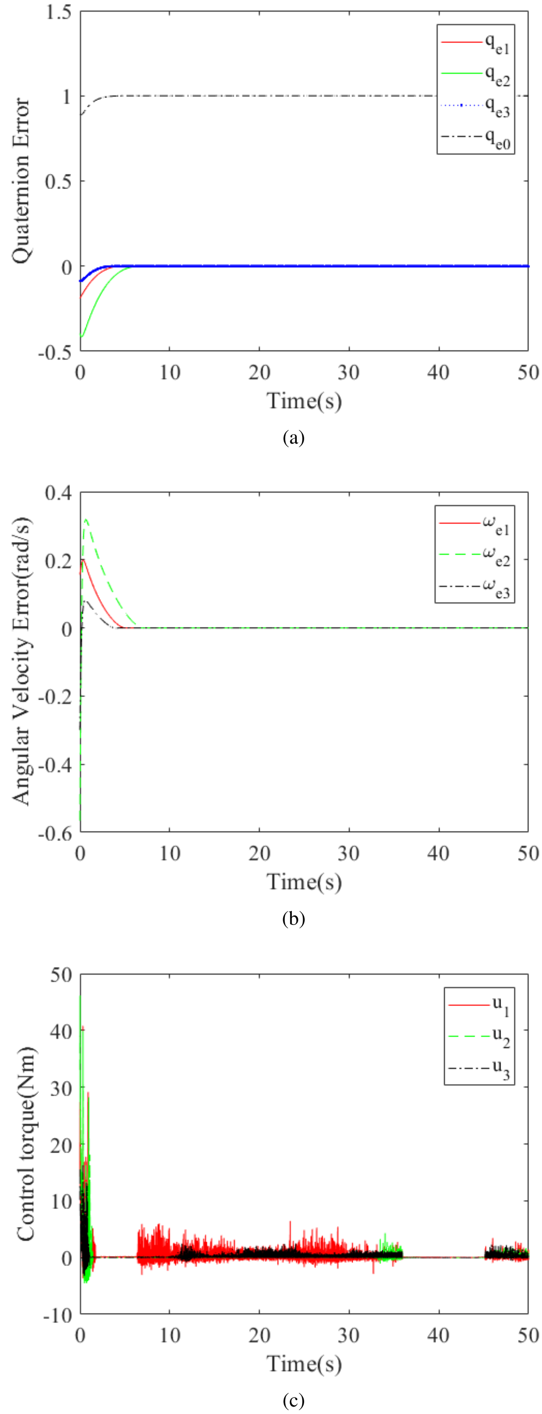

FIGURE 2. Response curves via basic FTSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.

the best control performance compared to the other controllers such as NTSMC, FTSMC, and PID-SMC in terms of small steady state error, fast convergence speed and chattering elimination.

B. MODIFIED FAULT TOLERANT CONTROL

In this subsection, the spacecraft system with four reaction wheels is adopted to illustrate the effectiveness of the proposed modified adaptive integral-type TSM control

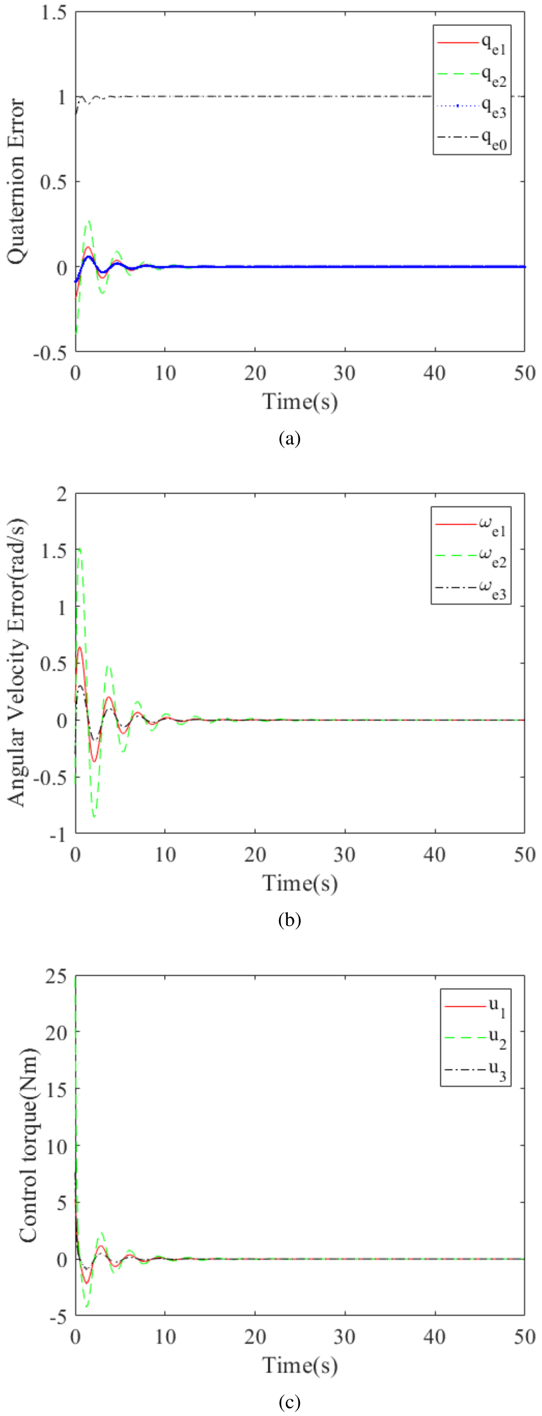

FIGURE 3. **Response curves via basic PID-SMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

scheme (42)-(44). Consider the distribution matrix as

$$D = \begin{bmatrix} 1 & 0 & 0 & 1/\sqrt{3} \\ 0 & 1 & 0 & 1/\sqrt{3} \\ 0 & 0 & 1 & 1/\sqrt{3} \end{bmatrix}.$$

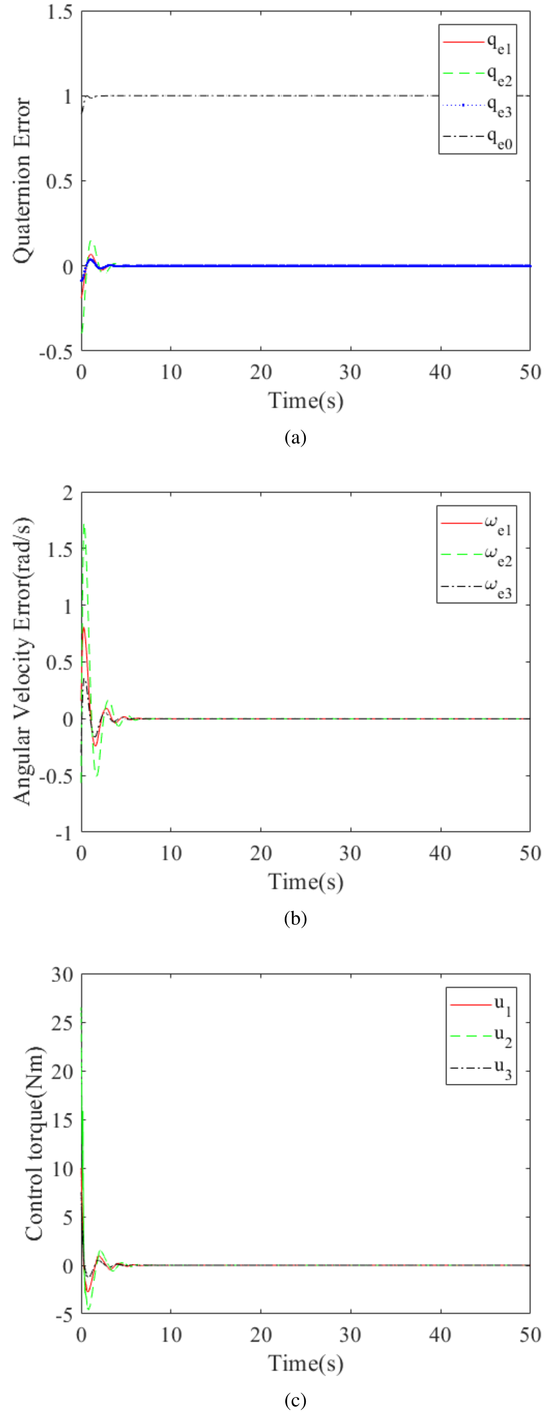

FIGURE 4. **Response curves via basic integral-type TSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

The disturbance is the same as mentioned above. The nominal inertia matrix and parameter uncertainties are selected as [52]

$$J_0 = \begin{pmatrix} 20 & 0 & 0.9 \\ 0 & 17 & 0 \\ 0 & 0.9 & 15 \end{pmatrix} kg \cdot m^2,$$

$$\Delta J = diag[3sin(0.3t), 2sin(0.2t), sin(0.1t)]kg \cdot m^2.$$

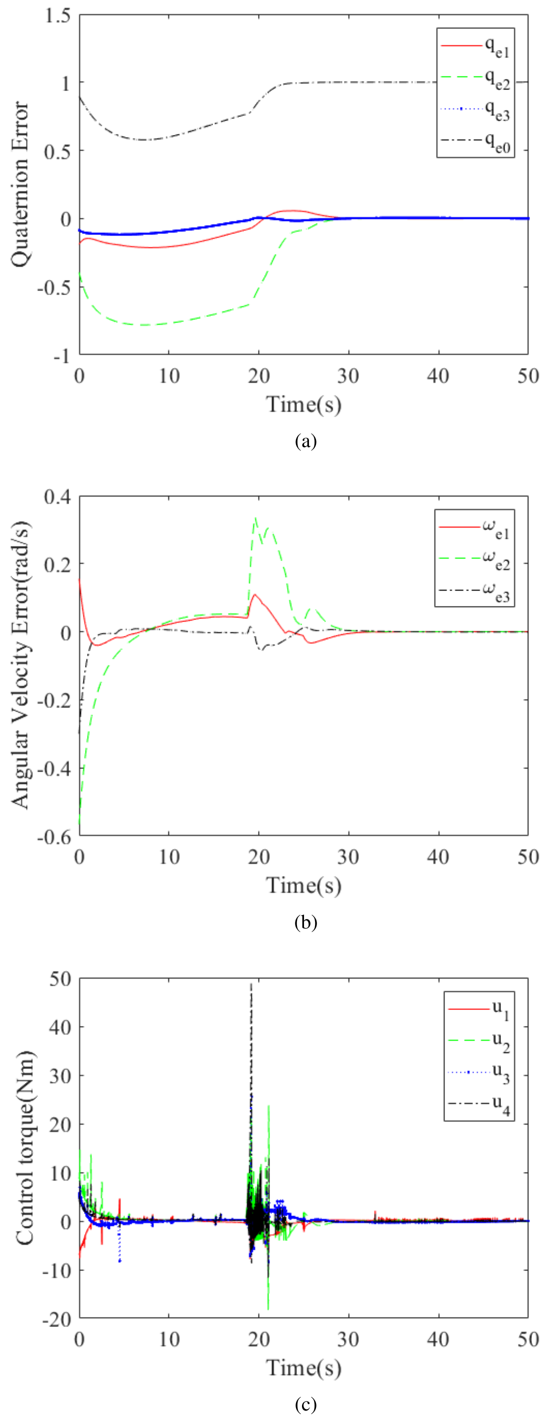

FIGURE 5. **Response curves via modified NTSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

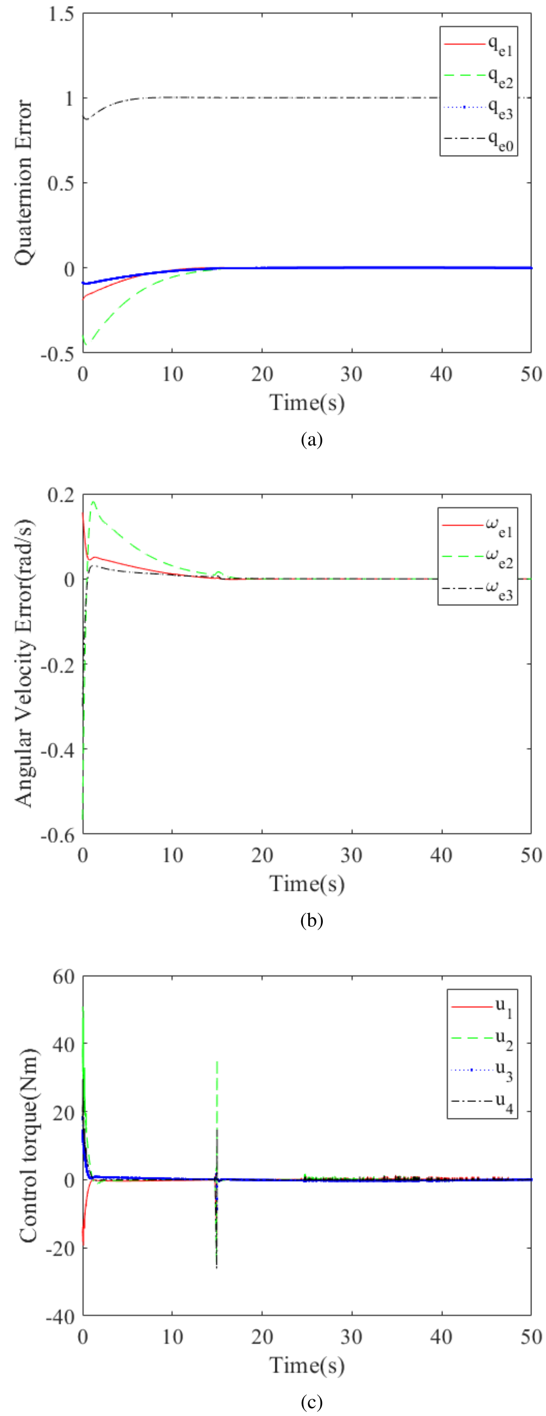

FIGURE 6. **Response curves via modified FTSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

For the simulation, the initial conditions are

$$\hat{F}(0) = \begin{bmatrix} 1530 & -107 & -213 \\ -107 & 1614 & -386 \\ -213 & -386 & 1516 \end{bmatrix}.$$

The compared simulation results under case 2 are depicted in Fig.5-Fig.8. The control performance of the proposed control scheme is illustrated for the spacecraft attitude control system in the presence of the inertia uncertainty and actuator redundancy, even actuator fault and external disturbance. The proposed control scheme is also compared with NTSMC (56), FTSMC (62), and PID-SMC (68). With the gains selected as Table 2, all four controllers can achieve the attitude tracking performance. However, compared with

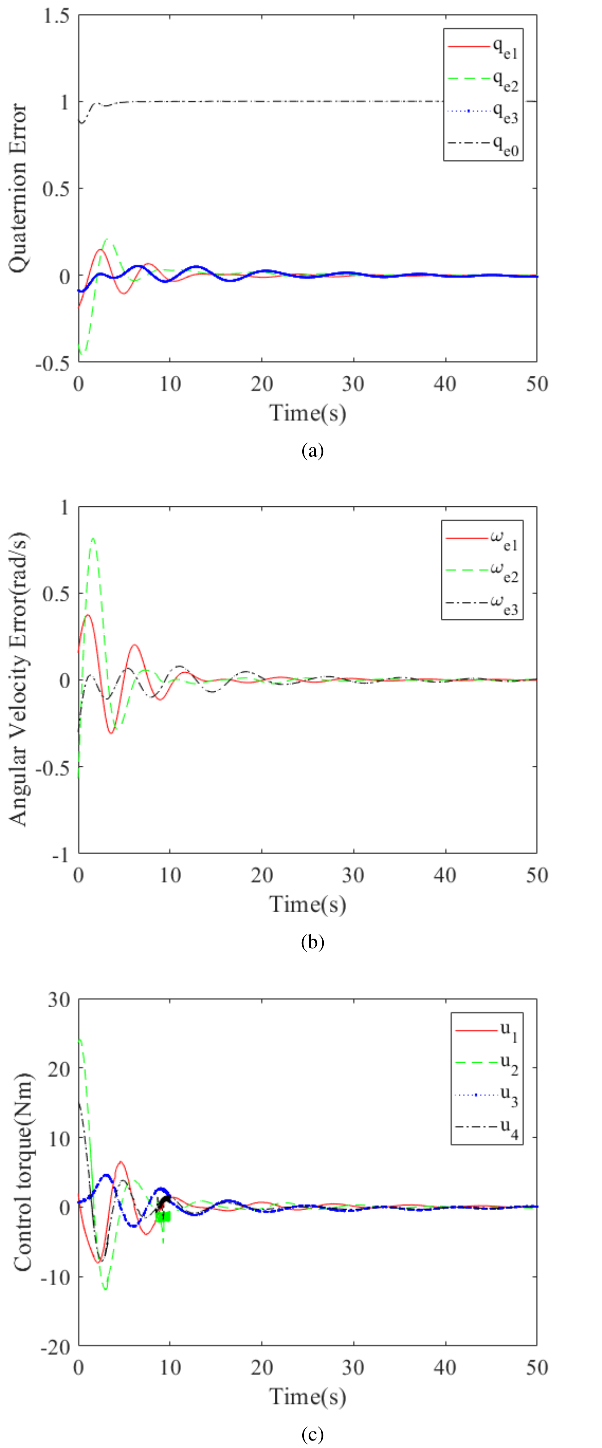

FIGURE 7. **Response curves via modified PID-SMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

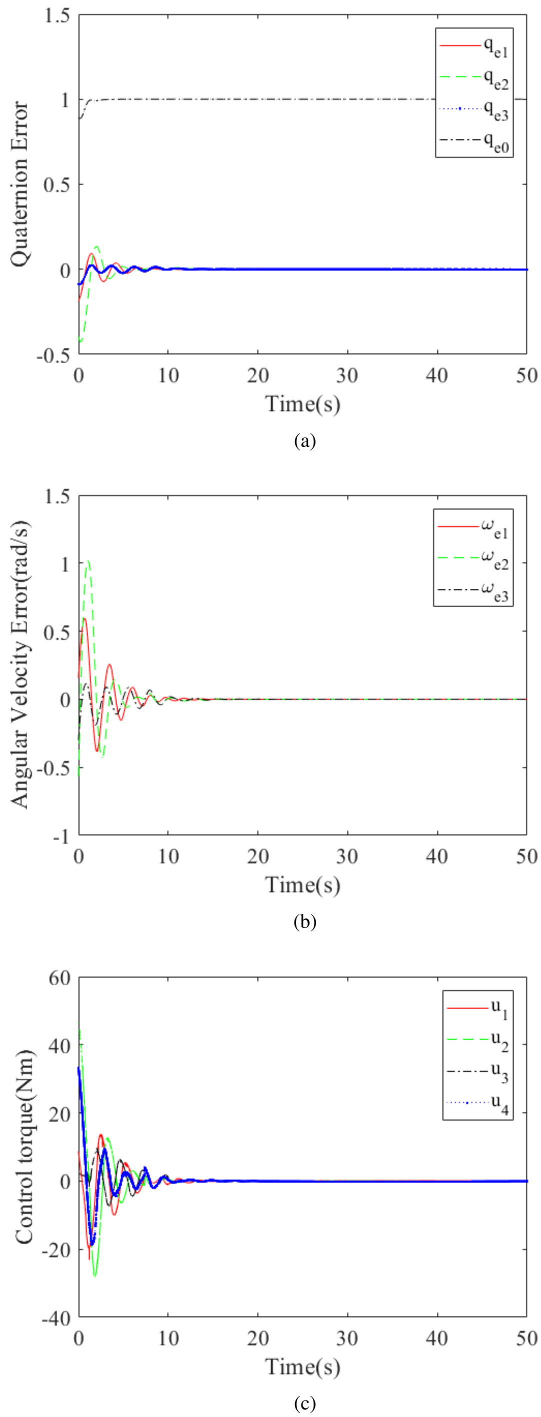

FIGURE 8. **Response curves via modified integral-type TSMC. (a) Quaternion error. (b) Angular Velocity error. (c) Control Torque.**

the above basic FTC scheme, it can be seen that the convergence time of the system trajectories takes a much longer time since the adaptive algorithm is used to estimate the actuator fault.

For the simulation results, we can see that the NTSMC provides the worst tracking performance in the condition of the unknown actuator fault. As shown in Fig.5, it takes more than 30s for the method of NTSMC to stabilize the quaternion error and angular velocity error to zero, and the control torque has a severe chattering problem. After that, although the FTSMC can provide a faster convergence time than NTSMC in Fig.6, the chattering problem has not been solved yet. Further, as shown in Fig.7, it can be observed that the smoother response is obtained by PID-SMC controller.

However, the convergence time is more than 35s. Compared to these above methods, as shown in Fig.8, the proposed integral-type TSM controller provides a better performance and has significant flexibility in compensating and better robustness against actuator faults.

Apparently, all the preceding comparisons show that the proposed control scheme still has the best performance in spite of the actuator fault, external disturbance, and the inertia uncertainty.

Remark 5: Faster convergence of the attitude tracking error along the proposed control scheme can be achieved by increasing α and β or decreasing $\frac{q}{p}$. Once reach the sliding surface, that is to say, $s = 0$, note that the larger α and β can lead to a faster convergence speed. Specifically, if $\beta = 0$, the sliding surface becomes the conventional linear sliding surface, $s = p_e + \alpha q_e$. In addition, the value of $\frac{q}{p}$ plays an important role on reaching a faster convergence speed. Because of $0 < \frac{q}{p} < 1$, the integral term $\beta \int_0^t q_e^{\frac{q}{p}} d\tau$ plays a more important role when $0 < |q_e| < 1$. Hence, decreasing $\frac{q}{p}$ can lead to a faster convergence speed and smaller steady state error. Specifically, if $p = q$ ($\frac{q}{p} = 1$), the sliding mode surface has the form of the PID-SMC surface, $s = p_e + \alpha q_e + \beta \int_0^t q_e \, d\tau$. It is obvious that the proposed integral-type TSMC can provide a faster convergence speed than PID-SMC.

Remark 6: The proposed control method assumes the availability of full-state measurements. However, if the angular velocity sensor fails, it may lead to wrong angular velocity measurements. Therefore, the velocity-free feedback attitude control is necessary to relax this requirement and this will be investigated using output feedback sliding mode approach as [55] in our future works.

V. CONCLUSIONS

In this paper, two FTC schemes are proposed based on novel integral-type TSM control technique for the spacecraft attitude tracking. When the actuator fault, the external disturbance, and actuator saturation are considered, the basic fault tolerant control scheme has been developed to compensate the effect of the synthetic uncertainty. Then, a modified fault tolerant controller with the adaptive laws has been designed to achieve attitude tracking, in the event that the actuator redundancy and the inertia uncertainty are considered. The actuator fault is estimated by adaptive law directly without the exact fault diagnosis procedure. The main contribution of the proposed control scheme is that it can integrate the benefits of the NTSMC, FTSMC, PID-SMC, so that the proposed control schemes can provide many great features such as nonsingularity, fast convergence speed and small steady state error. From the simulation comparison with other existing control methods, it is verified that the proposed integral-type TSM FTC scheme has a superior control performance. For future work, the effects of the sensor faults and actuator mixed faults to the control system is planned to be studied.

APPENDIX

Some lemmas applied in the progress of the control scheme are introduced in this part.

Lemma 1 [56]: If $x_i \in \mathbb{R}, i = 1, \cdots, n, 0 < p \leq 1$ is positive number, the following inequality holds $(|x_1| + \cdots + |x_n|)^p \leq |x_1|^p + \cdots + |x_n|^p \leq n^{1-p}(|x_1| + \cdots + |x_n|)^p$.

Lemma 2 [57]: If $\kappa \in (0, 1)$, then the inequality as following form holds $\sum_{i=1}^3 |x_i|^{1+\kappa} \geq (\sum_{i=1}^3 |x_i|^2)^{(1+\kappa)/2}$.

Lemma 3 [58]: For any real numbers $\mu_1 > 0$, $\mu_2 > 0$ and $0 < \iota < 1$, an extend Lyapunov function of finite-time stability can be given in the form of fast terminal sliding mode (FTSM) as

$$\dot{V}(x) + \mu_1 V(x) + \mu_2 V^{\iota}(x) \leq 0, \tag{51}$$

where the setting time can be given by

$$T_\gamma \leq \frac{1}{\mu_1(1-\iota)} ln \frac{\mu_1 V^{1-\iota}(x_0) + \mu_2}{\mu_2}. \tag{52}$$

Lemma 4 (Barbalat Lemma) [59]:

(1) If a scalar function $f(t)$ is uniformly continuous, such that $\lim_{t\to\infty} \int_0^t f(\tau)d\tau$ exists and is finite, then one has that $\lim_{t\to\infty} f(t) = 0$.

(2) If $f(t), \dot{f}(t) \in \mathcal{L}_\infty$, i.e., $f(t), \dot{f}(t)$ are bounded, and there is $f(t) \in \mathcal{L}_p$ for $p \in [1, \infty]$, then $\lim_{t\to\infty} f(t) = 0$.

A. APPENDIX A

The following is the design procedure of the NTSMC.

1) BASIC FAULT TOLERANT CONTROL

First, the NTSM surface can be selected as

$$s = q_e + \tfrac{1}{\alpha} p_e^{\frac{q}{p}}, \tag{53}$$

where α is a positive scalar, p, q are both the odd integers, and $p > q$.

Then, the NTSM control scheme can be designed as

$$\begin{aligned} v(t) &= M^{-1}(-ks - \varepsilon sgn(s) - \alpha \frac{q}{p} p_e^{2-q/p} - \hat{c}_1 sgn(s) \\ &\quad - f(q_e, p_e)), \quad (54)\\ \dot{\hat{c}}_1 &= k_1 s^T sgn(s), \quad (55) \end{aligned}$$

where $\hat{c}_1$ is the estimated value of c_1, and k_1 is a positive scalar.

2) MODIFIED FAULT TOLERANT CONTROL

The NTSM control scheme can be given as

$$\begin{aligned} v_0(t) = -\hat{F} f_1(q_e, p_e) - \hat{c}_2 sgn(s)\varphi - \alpha \frac{q}{p} \hat{F} p_e^{2-q/p} - ks \\ - \varepsilon sgn(s). \quad (56) \end{aligned}$$

The adaptive laws can be designed as

$$\begin{aligned} \dot{\hat{c}}_2 &= k_2 s^T sgn(s)\varphi, \quad (57)\\ \dot{\hat{F}} &= k_3 s (f_1(q_e, p_e) + \alpha \frac{q}{p} p_e^{2-q/p})^T, \quad (58) \end{aligned}$$

where k_2 is a positive constant, and $k_3 \in \mathbb{R}^{3\times 3}$ satisfies that $k_3 = k_3^T > 0$.

B. APPENDIX B

The following is the design procedure of the FTSMC.

1) BASIC FAULT TOLERANT CONTROL

First, the FTSM surface can be selected as

$$s = p_e + \alpha q_e + \beta q_e^{\frac{q}{p}}, \tag{59}$$

where α and β are both the positive scalars, p, q are both the odd integers, and $p > q$.

Then, the FTSM control scheme can be designed as

$$v(t) = M^{-1}(-ks - \varepsilon sgn(s) - \alpha p_e - \beta \frac{q}{p} q_e^{q/p-1} p_e - \hat{c}_1 sgn(s) - f(q_e, p_e)), \tag{60}$$

$$\dot{\hat{c}}_1 = k_1 s^T sgn(s), \tag{61}$$

where $\hat{c}_1$ is the estimated value of c_1, and k_1 is a positive scalar.

2) MODIFIED FAULT TOLERANT CONTROL

The FTSM control scheme can be given as

$$v_0(t) = -\hat{F} f_1(q_e, p_e) - \hat{c}_2 sgn(s)\varphi - \alpha \hat{F} p_e - \beta \frac{q}{p} \hat{F} q_e^{q/p-1} p_e - ks - \varepsilon sgn(s). \tag{62}$$

The adaptive laws can be designed as

$$\dot{\hat{c}}_2 = k_2 s^T sgn(s)\varphi, \tag{63}$$

$$\dot{\hat{F}} = k_3 s (f_1(q_e, p_e) + \alpha p_e + \beta \frac{q}{p} q_e^{q/p-1} p_e)^T, \tag{64}$$

where k_2 is a positive constant, and $k_3 \in \mathbb{R}^{3\times3}$ satisfies that $k_3 = k_3^T > 0$.

C. APPENDIX C

The following is the design procedure of the PID-SMC.

1) BASIC FAULT TOLERANT CONTROL

First, the PID-SMC surface can be selected as

$$s = p_e + \alpha q_e + \beta \int_0^t q_e \, d\tau, \tag{65}$$

where α and β are both the positive scalars.

Then, the PID-SMC control scheme can be designed as

$$v(t) = M^{-1}(-ks - \varepsilon sgn(s) - \alpha p_e - \beta q_e - \hat{c}_1 sgn(s) - f(q_e, p_e)), \tag{66}$$

$$\dot{\hat{c}}_1 = k_1 s^T sgn(s), \tag{67}$$

where $\hat{c}_1$ is the estimated value of c_1, and k_1 is a positive scalar.

2) MODIFIED FAULT TOLERANT CONTROL

The PID-SMC control scheme can be given as

$$v_0(t) = -\hat{F} f_1(q_e, p_e) - \hat{c}_2 sgn(s)\varphi - \alpha \hat{F} p q_e - \beta \hat{F} q_e - ks - \varepsilon sgn(s). \tag{68}$$

The adaptive laws can be designed as

$$\dot{\hat{c}}_2 = k_2 s^T sgn(s)\varphi, \tag{69}$$

$$\dot{\hat{F}} = k_3 s (f_1(q_e, p_e) + \alpha p_e + \beta q_e)^T, \tag{70}$$

where k_2 is a positive constant, and $k_3 \in \mathbb{R}^{3\times3}$ satisfies that $k_3 = k_3^T > 0$.

Remark 7: The stability proofs of the controllers in the Appendix are omitted since it is similar to this paper.

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