

Received February 26, 2019, accepted March 10, 2019, date of publication March 14, 2019, date of current version April 5, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2904993

# Adaptive Zeroing-Gradient Controller for Ship Course Tracking With Near Singularity Considered and Zero Theoretical Tracking Error

DONGYANG FU<sup>1</sup>, HUAN WANG<sup>2</sup>, XIUCHUN XIAO<sup>1</sup>, SHAN LIAO<sup>1</sup>,  
AND LONG JIN<sup>3</sup>, (Member, IEEE)

<sup>1</sup>School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang 524025, China

<sup>2</sup>School of Oceanography and Meteorology, Guangdong Ocean University, Zhanjiang 524025, China

<sup>3</sup>School of Information Science and Engineering, Lanzhou University, Lanzhou 730000, China

Corresponding authors: Huan Wang (huan1996wang@163.com) and Xiuchun Xiao (xcxiao@hotmail.com)

This work was supported in part by the National Natural Science Foundation of China under Grant 41340049 and Grant 41430968, in part by the National Marine Important Charity Special Foundation of China under Grant 201305019, in part by the Science and Technology Planning Project of Guangdong under Grant 2013B030200002 and Grant 2016A020222016, in part by the Guangdong University Innovation Outstanding Young Talents Program under Grant 2012WYM-0077, in part by the Project of Enhancing School with Innovation of Guangdong Ocean University under Grant GDOU2014050226, in part by the Key Lab of Digital Signal and Image Processing of Guangdong Province under Grant 2016GDDSIPL-02, in part by the Doctoral Initiating Project of Guangdong Ocean University under Grant E13428, in part by the Fundamental Research Funds for the Central Universities under Grant lzujbky-2017-193, and in part by the Project of Enhancing School with Innovation of Guangdong Ocean University under Grant Q15090.

**ABSTRACT** The zeroing-gradient (ZG) method, a combination of two neural dynamics methods, has been leveraged to construct controller for solving ship course tracking problem with near singularity considered. In this paper, the existing ZG controller is presented and revisited with its limitation identified. Specifically, this paper points out that there exist lagging errors for the ZG controller with the theoretical analyses provided. Then, to remedy the weakness, a new controller is proposed with an adaptive coefficient with zero theoretical tracking errors. Moreover, different activation functions are exploited to accelerate the convergence of the proposed controller. Finally, simulations are conducted to verify the superiority of the proposed controller.

**INDEX TERMS** Zeroing dynamics, gradient dynamics, adaptive method, activation function, ship course tracking.

## I. INTRODUCTION

As a fluid, the surface of water is of irregularity, especially in the seas and oceans. When a ship sails on seas and oceans, interactions occur between water and the ship. In this sense, the trajectory of the ship is affected by various factors, wind speed, ocean current, and the mass of the ship. Besides, ship transport plays an important role in world trade [1], of which the safety and economical efficiency should be guaranteed. Therefore, trajectory tracking control of the ship is of particular importance and worth investigating to ensure the sufficient accuracy of the trajectory.

Ship motion control includes ocean autonomous navigation, automatic departure of port area and collision avoidance

The associate editor coordinating the review of this manuscript and approving it for publication was Yan-Jun Liu.

problems [2]. Ocean autonomous navigation mainly includes course control and track control, for which different methods have been proposed, such as neural network control [3]–[5], fuzzy logic control [6], [7], robust control [8], nonlinear control [9], the advanced model reference adaptive control [10], [11]. Due to the unpredictable factors and unknown parameters, much effort has been devoted to proposing methods based on intelligent approaches [12]–[15]. For example, models based on backstepping and Nussbaum gain for solving nonlinear adaptive ship course tracking are investigated in [13]–[15]. A terminal sliding mode fuzzy control based on multiple sliding surfaces is proposed for ship course tracking steering in [16], and a control combined with radial basis function neural network is proposed for course control of ship steering [17]. Note that the ship course control may encounter the near singularity issue during the real-time tracking and

how to remedy the near singularity with predefined accuracy is still a knotty problem.

Recently, zeroing dynamics method or for short ZD method, is put forward in [18], which is based on an error function and recurrently updates the estimation on the solution to the problem with residual error converging to zero. Such a method is widely applied in mathematics and engineering field, which is able to solve online/time-varying problems, such as time-varying Sylvester equation [18], time-varying full-rank matrix inverse [19], time-varying square roots finding [20], robotic applications [21] and so on [22], [23]. In addition, gradient dynamics (GD) is a classical and powerful methods, of which the negative gradient direction is exploited to generate the evolution based on a constructed energy function. The GD method is utilized to solve time-varying problems [24], [25], which often suffers from the large lagging errors. ZD method, originated from the research of zeroing-type recurrent neural networks, does not rely on the norm of the error function, and is able to solve time-varying problems without losing the evolving direction information [26]–[29]. In contrast, the GD method, originated from the gradient descent method, is able to solve a static problem without considering the velocity compensation. ZG (zeroing-gradient) method, as a combination of ZD and GD, is leveraged to construct controller for ship course tracking problem in [30], which is able to handle the near singularity problem. Then, a finite difference formula is designed to construct discrete-time controller for ship course tracking in [31]. However, these controllers can not eliminate the lagging errors and thus may be not enough effective for ship course tracking.

In this paper, we make progress along this direction by proposing an adaptive zeroing-gradient controller for ship course tracking, which is able to handle the near singularity problem with zero theoretical residual error. The remainder of this paper is organized as follows. Section II presents the preliminaries on the ship course tracking system and then revisits the existing solutions. In addition, Section III proposes the adaptive zeroing-gradient controller with near singularity considered and residual error eliminated, which is guaranteed by the corresponding theoretical analyses. To accelerate the convergence speed of the controller, different activation functions are exploited. Computer simulations are conducted in Section IV to verify the effectiveness of the proposed controller. Finally, Section V concludes this paper with final remarks. Before ending this section, the contributions of this work are summarized as follows.

- An adaptive zeroing-gradient controller is proposed for the ship course control, which is able to handle the near singularity problem with zero theoretical error.
- Different activation functions are exploited to speed up the convergence of the proposed controller for ship course tracking.
- Theoretical analyses are provided to guarantee the availability and validity of the proposed controller.

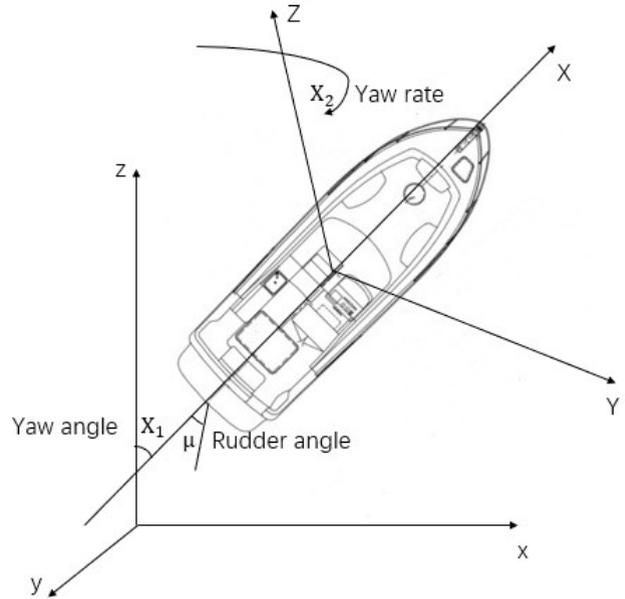


FIGURE 1. Schematic diagram of a simplified ship navigation model.

- Computer simulations are provided to verify the effectiveness and superiority of the proposed controller.

## II. PRELIMINARIES, PROBLEM FORMULATION, AND REVISIT

This section presents the preliminaries on the ship course tracking system and then revisits the existing solutions.

### A. PRELIMINARIES ON SHIP COURSE TRACKING

In this part, the ship course tracking system is provided. Due to the interaction between the ship and water caused by many factors encountered in the ocean, a ship may drift off the desired course, and a rudder is used to manipulate and control the course of the ship to maintain the expected route. The schematic diagram of a simplified ship navigation model is shown as Fig.1. Besides, a function describing the relationship between the rudder angle and the yaw rate for small rudder angles is proposed in [32], which is formulated as

$$\frac{r(s)}{\delta(s)} = \frac{K}{Ts + 1}, \quad (1)$$

where  $r$  denotes the yaw rate;  $\delta$  denotes the rudder angle;  $K$  denotes the gain constant;  $T$  denotes the time constant. The function (1) does not consider many factors influencing the relation between the rudder angle and the yaw rate, e.g., the size of the ship, the depth of water, the salinity of the water and so on. Therefore, a new function for large rudder angle is presented by reverse spiral manoeuvres, in which a parameter  $\alpha$  is termed the Norrbin coefficient [33]:

$$T\ddot{\psi} + \dot{\psi} + \alpha\dot{\psi}^3 = K\delta, \quad (2)$$

where  $\psi$  is the yaw angle and  $\dot{\psi}(t) = r(t)$ . According to (2), we can control the trajectory of the ship by adjusting the rudder angle. In other words, the ship course tracking problem can be converted into the problem of controlling the rudder angle. Then, the ship course system in this paper can be expressed as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = \varphi_0\mu(t) + \varphi_1x_2(t) + \varphi_2x_2^3(t), \\ y(t) = x_1(t). \end{cases} \quad (3)$$

It is worth noting that (3) is an extended expression of (2), where  $x_1(t)$  denotes the yaw angle and  $x_2(t)$  denotes the yaw rate with  $\dot{x}_1(t)$  and  $\dot{x}_2(t)$  being their time derivatives, respectively;  $\mu(t)$  is the control input denoting the rudder angle;  $y(t)$  is the output denoting yaw angle;  $\varphi_0, \varphi_1, \varphi_2$  are parameters.

## B. REVISIT TO EXISTING METHODS

In this part, the ZG controller with near singularity considered for ship course tracking is revisited. Then, the lagging error in the existing ZG controller is derived theoretically.

### 1) ZG CONTROLLER

The construction of ZG controller includes two parts: zeroing dynamics (ZD) method and gradient dynamics (GD) method.

The first step, using the ZD method to construct the error function as  $z_1 = y(t) - y_d(t)$ , where  $y_d(t)$  is the desired output. According to equation (3), error function can be rewritten as

$$z_1 = y(t) - y_d(t) = x_1(t) - y_d(t). \quad (4)$$

Then, aided with the definition of error function  $z_1: \dot{z}_1(t) = -\gamma z_1(t)$  according to ZD method, where  $\gamma > 0 \in \mathbb{R}$ , equation (4) can be rearranged as

$$\dot{x}_1(t) - \dot{y}_d(t) + \gamma(x_1(t) - y_d(t)) = 0. \quad (5)$$

On the basis of (3), (5) can be rewritten as

$$x_2(t) - \dot{y}_d(t) + \gamma(x_1(t) - y_d(t)) = 0. \quad (6)$$

Let  $z_2(t) = x_2(t) - \dot{y}_d(t) + \gamma(x_1(t) - y_d(t))$ . Likewise, by means of the definition of error function, we can obtain:

$$\begin{aligned} \dot{z}_2(t) - \ddot{y}_d(t) + \gamma(\dot{x}_1(t) - \dot{y}_d(t)) \\ = -\gamma(x_2(t) - \dot{y}_d(t) + \gamma(x_1(t) - y_d(t))). \end{aligned} \quad (7)$$

Combining formula (3) with formula (7), we can get the expression of the control input:

$$\begin{aligned} \mu(t) = (1/\varphi_0)(\ddot{y}_d(t) - 2\gamma x_2(t) - \gamma^2 x_1(t) + 2\gamma \dot{y}_d(t) \\ + \gamma^2 y_d(t) - \varphi_1 x_2(t) - \varphi_2 x_2^3(t)). \end{aligned} \quad (8)$$

So far, the ZD method has been used twice to obtain a closed-loop system.

The second step, a ZG controller in the form of  $\dot{\mu}(t)$  is obtained by using GD method based on ZD method.

Equation (8) can be rewritten as

$$\begin{aligned} \varphi_0\mu(t) + \varphi_1x_2(t) + \varphi_2x_2^3(t) - \ddot{y}_d(t) + 2\gamma x_2(t) \\ - 2\gamma \dot{y}_d(t) + \gamma^2 x_1(t) - \gamma^2 y_d(t) = 0. \end{aligned} \quad (9)$$

Then, defined a function  $f(t)$  about time  $t$  as

$$\begin{aligned} f(t) = \varphi_0\mu(t) + \varphi_1x_2(t) + \varphi_2x_2^3(t) - \ddot{y}_d(t) \\ + 2\gamma x_2(t) - 2\gamma \dot{y}_d(t) + \gamma^2 x_1(t) - \gamma^2 y_d(t). \end{aligned} \quad (10)$$

Next, defined an energy function  $\epsilon(t) = f^2(t)/2$  according to the GD method. Hence, a new controller can be carried out with the control input in the form of  $\dot{\mu}(t) = -\nu \partial \epsilon / \partial \mu$ , where the parameter  $\nu > 0 \in \mathbb{R}$  is to control the convergence rate of the GD method [30].

$$\dot{\mu}(t) = -\nu \varphi_0 f(t). \quad (11)$$

The above equation is termed the ZG controller in form of  $\dot{\mu}(t)$  for solving ship course tracking problem. It can be seen from equation (11) that this controller can solve the problem of near singular points since it eliminate the division operation. In other words, no matter how the parameter  $\varphi_0$  change at any time, ZG controller (11) can run smoothly without crashing.

### 2) THEORETICAL ANALYSES ON ZG CONTROLLER

As shown above, ZG controller (11) is able to solve the problem of near singularity occurs in the ship course tracking system effectively when coefficient  $\varphi_0$  is varying with time. However, large lagging residual error may be generated for the ship course tracking aided with the ZG controller (11). The corresponding theoretical analyses are given as follows.

*Theorem 1:* The residual error upper bound of the ZG controller (11) converges globally to an estimated error value instead of zero.

*Proof:* To express more clearly and concisely, equation (10) is reformulated as

$$f(t) = \varphi_0\mu(t) - b(t), \quad (12)$$

where  $-b(t) = \varphi_1x_2(t) + \varphi_2x_2^3(t) - \ddot{y}_d(t) + 2\gamma x_2(t) - 2\gamma \dot{y}_d(t) + \gamma^2 x_1(t) - \gamma^2 y_d(t)$ . Similarly, ZG controller (11) can be reformulated as

$$\dot{\mu}(t) = -\nu \varphi_0(\varphi_0\mu(t) - b(t)). \quad (13)$$

By defining  $f(t) = 0$  in equation (12) theoretically, we have the theoretical value of  $\mu(t)$  termed  $\mu^*(t)$  as

$$\mu^*(t) = \frac{b(t)}{\varphi_0}.$$

Furthermore, a new error function is defined as  $E(t) = \mu(t) - \mu^*(t)$  to indicate the value of residual error between  $\mu(t)$  and  $\mu^*(t)$ . Then, the time derivative of  $E(t)$  can be expressed as  $\dot{E}(t) = \dot{\mu}(t) - \dot{\mu}^*(t)$ .

Define a Lyapunov function  $V(t) = E^2(t)/2$  and its time derivative is computed as

$$\begin{aligned} \dot{V}(t) &= E(t)\dot{E}(t) \\ &= E(t)(\dot{\mu}(t) - \dot{\mu}^*(t)) \\ &= E(t)(-\nu \varphi_0 f(t) - \dot{\mu}^*(t)) \\ &= E(t)(-\nu \varphi_0(\varphi_0\mu(t) - b(t)) - \dot{\mu}^*(t)) \end{aligned}$$

$$\begin{aligned}
 &= E(t)(-v\varphi_0^2(\mu(t) - \frac{b(t)}{\varphi_0}) - \dot{\mu}^*(t)) \\
 &= E(t)(-v\varphi_0^2(\mu(t) - \mu^*(t)) - \dot{\mu}^*(t)) \\
 &= E(t)(-v\varphi_0^2 E(t) - \dot{\mu}^*(t)) \\
 &= -v\varphi_0^2 E^2(t) - E(t)\dot{\mu}^*(t),
 \end{aligned}$$

of which the left part  $-v\varphi_0^2 E^2(t)$  is evidently less than zero. Suppose there is a positive real number  $\Theta$  that, for any  $\dot{\mu}^*(t)$ , there is always  $\Theta > |\dot{\mu}^*(t)|$ . Thus  $\dot{V}(t)$  can be rewritten as

$$\begin{aligned}
 \dot{V}(t) &= -v\varphi_0^2 E^2(t) - E(t)\dot{\mu}^*(t) \\
 &\leq -v\varphi_0^2 E^2(t) + |E(t)|\Theta \\
 &= -|E(t)|(v\varphi_0^2 |E(t)| - \Theta).
 \end{aligned}$$

According to the above equation, the following three situations need to be discussed:

Case 1: suppose  $(v\varphi_0^2 |E(t)| - \Theta) > 0$ , where  $|E(t)| > (\Theta/v\varphi_0^2)$ , and thus  $\dot{V}(t) < 0$ , which indicates that  $V(t)$  is convergent and that  $|E(t)|$  is convergent to  $(\Theta/v\varphi_0^2)$ .

Case 2: suppose  $(v\varphi_0^2 |E(t)| - \Theta) = 0$ , where  $|E(t)| = (\Theta/v\varphi_0^2)$ , and thus  $\dot{V}(t) \leq 0$  which indicates that  $V(t)$  is convergent similarly.

Case 3: suppose  $(v\varphi_0^2 |E(t)| - \Theta) < 0$ ,  $|E(t)| < (\Theta/v\varphi_0^2)$ , and thus  $\dot{V}(t) > 0$  which indicates that  $V(t)$  is divergent.

From case 3 where  $V(t)$  is divergent, we can get:

$$|E(t)| < \frac{\Theta}{v\varphi_0^2}, \tag{14}$$

it is worth noting that although  $V(t)$  is divergent in the case 3, we can come to a conclusion that the error function  $E(t)$  is bounded by  $\Theta/(v\varphi_0^2)$  according to equation (14). Summarizing the above three cases, we come to the conclusion that there always exists lagging error between the control input  $\mu$  and the theoretical value  $\mu^*(t)$  for the ZG controller (11) in ship course tracking. The proof is complete.  $\square$

### III. ADAPTIVE ZG CONTROLLER AND THEORETICAL ANALYSES

In the above sections, we have discussed ZG controller (11) for solving ship course tracking problems, which indicates that there always exists residual error for the ZG controller (11) in ship course tracking. In this part, we propose an adaptive ZG controller to remedy the weakness of the existing ZG controller (11).

#### A. ADAPTIVE ZG CONTROLLER

In this part, an adaptive coefficient  $\xi(t)$  is exploited to replace the constant coefficient  $v$  in the formula (11), and thus formula (11) is rewritten as

$$\dot{\mu}(t) = -\xi(t)\varphi_0 f(t), \tag{15}$$

where

$$\xi(t) = \frac{c - \dot{b}(t)f(t)}{\varphi_0^2 f^2(t)}, \tag{16}$$

is the adaptive coefficient and  $c > 1$  is a constant. The formula (15) is the adaptive ZG controller, which leverages

a varying parameter to control the convergence speed of the ZG controller, and is able to eliminate the lagging errors. The corresponding theoretical analyses are given as follows.

To accelerate the convergence speed of the proposed adaptive ZG controller (15), different nonlinear activation functions are exploited, which should be monotone increasing and odd. Three activation functions used in this paper are written as follows [34], [35].

1) linear activation function:

$$\sigma_l(x) = x. \tag{17}$$

2) power-sigmoid activation function:

$$\sigma_p(x) = \begin{cases} \frac{1 + \exp(-4)}{1 - \exp(-4)} \frac{1 - \exp(-4x)}{1 + \exp(-4x)}, & |x| < 1 \\ x^3 & |x| \geq 1 \end{cases} \tag{18}$$

3) hyperbolic sine activation function:

$$\sigma_h(x) = \frac{\exp(3x)}{2} - \frac{\exp(-3x)}{2}. \tag{19}$$

Thus, the adaptive ZG controller aided with activation function is presented as

$$\dot{\mu}(t) = -\xi(t)\varphi_0 \sigma(f(t)). \tag{20}$$

#### B. THEORETICAL ANALYSES

In this part, theoretical analyses on the proposed adaptive ZG controller (15) as well as its modification version (20) for ship course tracking problems are provided.

Regarding the performance of the proposed adaptive ZG controller (15) in remedying the lagging error, we offer the following theorem.

*Theorem 2:* The proposed adaptive ZG controller (15) for solving ship course tracking problem (3) is of global convergence and the residual error converges to zero globally.

*Proof:* To prove the convergence of the adaptive ZG controller (15) for solving ship course tracking problem (3), a Lyapunov function is constructed:

$$L(t) = \frac{f^2(t)}{2}, \tag{21}$$

where  $L(t)$  is always positive unless one condition that  $f(t) = 0$ . Then, the time derivative of  $L(t)$  can be derived as

$$\dot{L}(t) = f(t)\dot{f}(t). \tag{22}$$

Finally, formula (22) can be reformulated as

$$\begin{aligned}
 \dot{L}(t) &= f(t)\dot{f}(t) \\
 &= f(t)(\varphi_0(t)\dot{\mu}(t) - \dot{b}(t)) \\
 &= f(t)(\varphi_0(-\xi(t)\varphi_0 f(t)) - \dot{b}(t)) \\
 &= -\xi(t)\varphi_0^2 f^2(t) - \dot{b}(t)f(t) \\
 &= -c|-\dot{b}(t)f(t)| - \dot{b}(t)f(t),
 \end{aligned}$$

where  $c > 1$ , so the above equation is expressed as

$$\dot{L}(t) = -\xi(t)\varphi_0^2 f^2(t) - \dot{b}(t)f(t) < 0. \tag{23}$$

As shown above, due to  $\dot{L}(t) < 0$  as  $c > 1$ ,  $f(t)$  globally converges to zero. In other words, the proposed adaptive ZG controller (15) for solving ship course tracking problem (3) is of global convergence and the residual error converges to zero globally. The proof is complete.  $\square$

Regarding the performance of the activation function aided adaptive ZG controller (20), we offer the following theorems.

**Theorem 3:** The convergence speed of the power-sigmoid activation function aided adaptive ZG controller (20) is faster than that of the adaptive ZG controller (15) for solving ship course tracking problem (3).

*Proof:* According to the definition of the power-sigmoid activation function, i.e., equation (18), the corresponding theoretical analyses can be divided into the following two cases.

Case 1: when  $|x| \geq 1$ , the power-sigmoid activation function degrades to  $\sigma_p(x) = x^3$ . Thus, for the case of  $|f(t)| \geq 1$ , by combining formula (12) and the time derivative of formula (21), we have

$$\begin{aligned} \dot{L}_2(t) &= f(t)\dot{f}(t) \\ &= f(t)(\varphi_0\dot{\mu}(t) - \dot{b}(t)) \\ &= f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_p(f(t))) - \dot{b}(t)) \\ &= -\xi(t)\varphi_0^2f^4(t) - f(t)\dot{b}(t) \\ &< -\xi(t)\varphi_0^2f^2(t) - f(t)\dot{b}(t) \\ &= f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_l(f(t))) - \dot{b}(t)). \end{aligned} \quad (24)$$

Therefore, for the case of  $|f(t)| \geq 1$ , the convergence speed of the power-sigmoid activation function aided adaptive ZG controller (20) is faster than that of the adaptive ZG controller (15) for solving ship course tracking problem (3).

Case 2: when  $|x| < 1$ , the power-sigmoid activation function degrades to  $\sigma_p(x) = (1 + \exp(-4))(1 - \exp(-4x))/((1 - \exp(-4))(1 + \exp(-4x)))$ . Thus, when  $|f(t)| < 1$ , by combining formula (12) and the time derivative of formula (21), we have

$$\begin{aligned} \dot{L}_2(t) &= f(t)\dot{f}(t) \\ &= f(t)(\varphi_0\dot{\mu}(t) - \dot{b}(t)) \\ &= f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_p(f(t))) - \dot{b}(t)) \\ &= -\xi(t)\varphi_0^2f(t)\frac{1 + \exp(-4)}{1 - \exp(-4)} \\ &\quad \times \frac{1 - \exp(-4f(x))}{1 + \exp(-4f(x))} - f(t)\dot{b}(t) \\ &< -\xi(t)\varphi_0^2f^2(t) - f(t)\dot{b}(t) \\ &= f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_l(f(t))) - \dot{b}(t)). \end{aligned}$$

Therefore, for the case of  $|f(t)| < 1$ , the convergence speed of the power-sigmoid activation function aided adaptive ZG controller (20) is faster than that of the adaptive ZG controller (15) for solving ship course tracking problem (3). The proof is complete.  $\square$

**Theorem 4:** The convergence speed of the hyperbolic sine activation function aided adaptive ZG controller (20) is faster than that of the adaptive ZG controller (15) for solving ship course tracking problem (3).

*Proof:* Using Taylor Expansion to the hyperbolic sine activation function generates

$$\begin{aligned} \sigma(x) &= \frac{\exp(3x)}{2} - \frac{\exp(-3x)}{2} \\ &= \frac{1}{2}\left(\sum_{n=0}^{+\infty} \frac{(3x)^n}{n!} - \sum_{n=0}^{+\infty} \frac{(-3x)^n}{n!}\right) \\ &= \sum_{n=1}^{+\infty} \frac{(3x)^{2n-1}}{(2n-1)!} \\ &> 3x. \end{aligned} \quad (25)$$

Combining formula (12) and the time derivative of formula (21) leads to

$$\begin{aligned} \dot{L}_2(t) &= f(t)\dot{f}(t) \\ &= f(t)(\varphi_0\dot{\mu}(t) - \dot{b}(t)) \\ &= f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_h(f(t))) - \dot{b}(t)) \\ &< -3\xi(t)\varphi_0^2f^2(t) - f(t)\dot{b}(t) \\ &< f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_p(f(t))) - \dot{b}(t)) \\ &< f(t)(\varphi_0(-\xi(t)\varphi_0\sigma_l(f(t))) - \dot{b}(t)). \end{aligned} \quad (26)$$

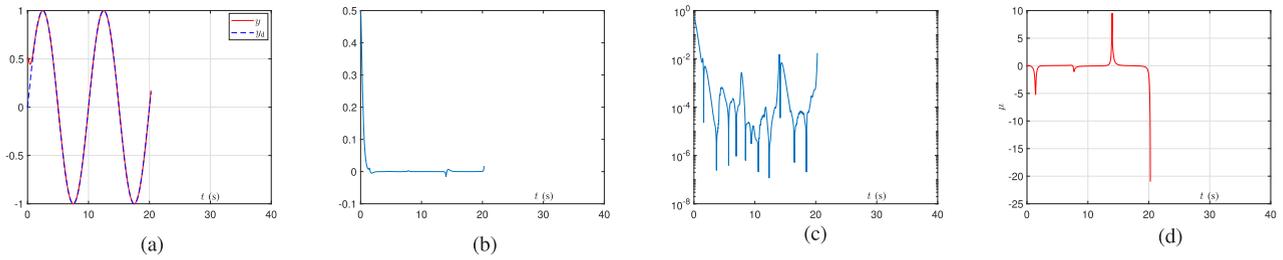
Therefore, the convergence speed of the hyperbolic sine activation function aided adaptive ZG controller (20) is faster than that of the adaptive ZG controller (15) for solving ship course tracking problem (3). The proof is complete.  $\square$

#### IV. SIMULATION VERIFICATIONS

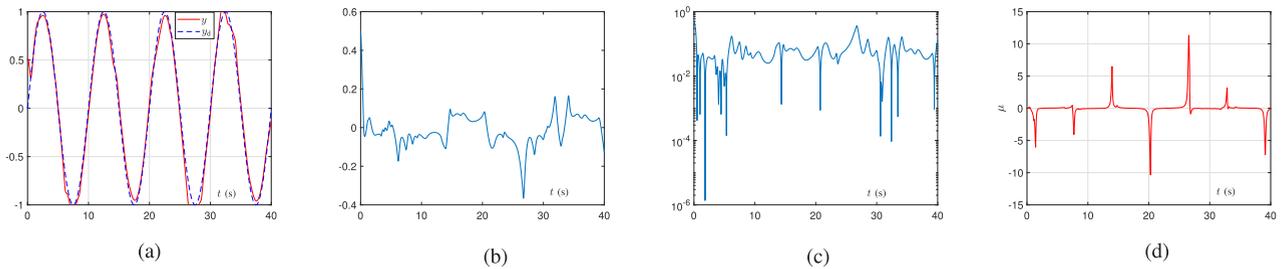
In above sections, theoretical analyses are conducted to illustrate that the proposed adaptive ZG controller (15) as well as its modification (20) can remedy the large lagging error for solving ship course tracking problem (3). In this section, computer simulations are carried out to further verify the effectiveness and superiority of adaptive gradient-based controller (15) and gradient-based controller with activation function (20). In general, initially  $x_1(0) = x_2(0) = 0$ ,  $\mu(0) = 0$ ,  $\varphi_1 = \varphi_2 = 100$ , time  $t = 40$  s, and without loss of generality,  $\varphi_0 = 1000 * (\sin(t - 3) + 1.01)$  for ship course system (3); the desired path is set as  $y_d(t) = \sin(0.2\pi t)$ ;  $\gamma = 5$ ;  $\nu = 2$ ;  $c = 2$ .

The existing ZG controller (11) can solve the near singularity problem when the parameter  $\varphi_0$  changes with time. As shown in Fig. 2, the ship course system (3) encounters a crash with time  $t = 20$  s approximately. Specifically, as visualized in Fig. 2(a), the output path  $y(t)$  converges to the desired path  $y_d(t)$  quickly and then stops due to the crash of the whole system. It can be observed from Fig. 2(b) and (c) that the residual error converges to zero rapidly before 20 s. Fig. 2(d) illustrates the control-input trajectory, which encounters crash at around 20 s.

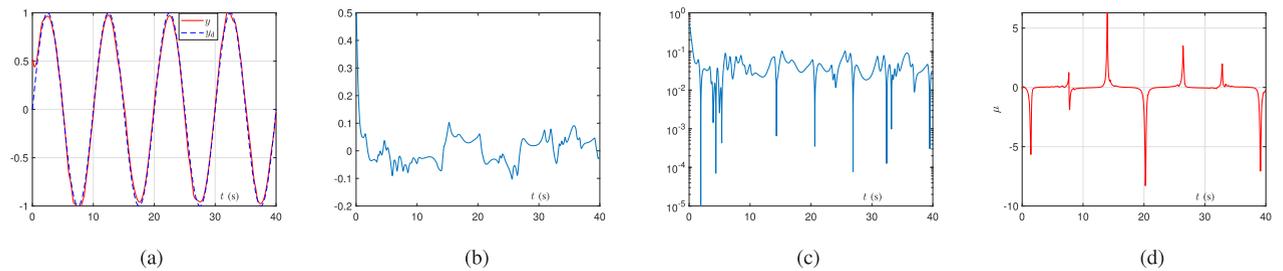
With the same initial conditions, simulation results synthesized by the proposed adaptive ZG controller (15) for solving ship course tracking problem are shown as Fig. 3. Specifically, as visualized in Fig. 3(a), the output path  $y(t)$  converges



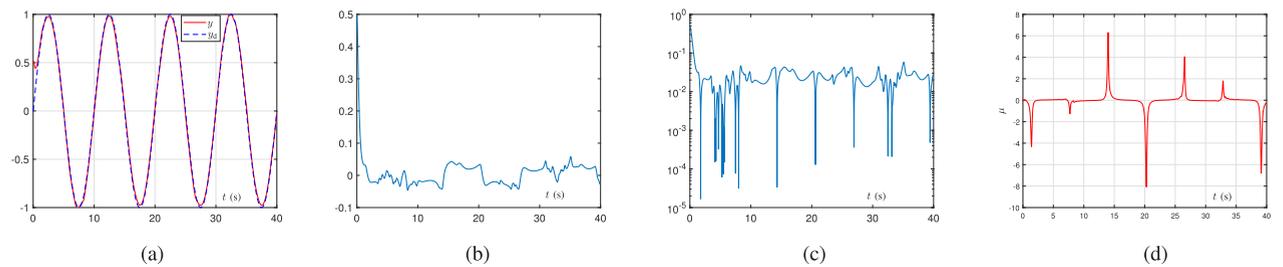
**FIGURE 2.** Output-tracking performance of ZG controller (11) for ship course system (3) with desired path  $y_d(t) = \sin(0.2\pi t)$ . (a) Output trajectory and desired path. (b) Output-tracking error. (c) Output-tracking error in log scale. (d) Control-input trajectory.



**FIGURE 3.** Output-tracking performance of adaptive ZG controller (15) for ship course system (3) with desired path  $y_d(t) = \sin(0.2\pi t)$ . (a) Output trajectory and desired path. (b) Output-tracking error. (c) Output-tracking error in log scale. (d) Control-input trajectory.



**FIGURE 4.** Output-tracking performance of adaptive ZG controller aided with power-sigmoid activation function (20) for ship course system (3) with desired path  $y_d(t) = \sin(0.2\pi t)$ . (a) Output trajectory and desired path. (b) Output-tracking error. (c) Output-tracking error in log scale. (d) Control-input trajectory.



**FIGURE 5.** Output-tracking performance of adaptive ZG controller with hyperbolic sine activation function (20) for ship course system (3) with desired path  $y_d(t) = \sin(0.2\pi t)$ . (a) Output trajectory and desired path. (b) Output-tracking error. (c) Output-tracking error in log scale. (d) Control-input trajectory.

to the desired path  $y_d(t)$  quickly, which does not encounter crash. In addition, it can be found in Fig. 3(b) and (c) that, the residual error converges to zero in a very short time, whose magnitude is acceptable. Fig. 3(d) plots the control input trajectory synthesized by the proposed controller. These results illustrate the superiority of the adaptive ZG controller (15) compared with the existing ZG controller (11).

Fig. 4 and Fig. 5 show the computer simulation results synthesized by the nonlinear activation function aided adaptive ZG controller (20) for solving ship course tracking problem, with the initial conditions same as those in Fig. 3. It can be observed from these two figures that the convergence speed is accelerated via the nonlinear activation functions, as compared to that in Fig. 3, which further verify the correctness of Theorems 3 and 4.

## V. CONCLUSIONS

In this paper, the existing ZG controller (11) have been revisited and investigated for solving ship course tracking problem (3). Then, theoretical analyses have been provided to illustrate that the ZG controller (11) can not eliminate the residual error. In order to handle the lagging errors problem, a new adaptive ZG controller (15) has been proposed with its convergence performance verified theoretically. Moreover, different activation functions have been leveraged to accelerate the convergence speed of the proposed adaptive ZG controller (15) for solving ship course tracking problem. Finally, experimental results have been conducted to show the superiority of the proposed adaptive ZG controller (15) and its modification (20).

## ACKNOWLEDGMENT

The authors would like to thank all the reviewers for their comments.

## REFERENCES

- [1] J. Q. Huang, *Adaptive Control Theories and its Applications in Ship System*. Beijing, China: National Defense Industry Press, 1992, pp. 5–9.
- [2] X. L. Jia and Y. S. Yang, *Ship Motion Mathematical Model*. Dalian, China: Dalian Maritime Univ. Press, 1998, ch. 6.
- [3] Y. Zhang, P. Sen, and G. E. Hearn, “An on-line trained adaptive neural controller,” *IEEE Control Syst.*, vol. 15, no. 5, pp. 67–75, Oct. 1995.
- [4] Y. Zhang, G. E. Hearn, and P. Sen, “A neural network approach to ship track-keeping control,” *IEEE J. Ocean. Eng.*, vol. 21, no. 4, pp. 513–527, Oct. 1996.
- [5] M. A. Unar and D. J. Murray-Smith, “Automatic steering of ships using neural networks,” *Int. J. Adapt. Control Signal Process.*, vol. 13, no. 4, pp. 203–218, 1999.
- [6] J. R. Layne and K. M. Passino, “Fuzzy model reference learning control for cargo ship steering,” *IEEE Control Syst. Mag.*, vol. 13, no. 6, pp. 23–34, Dec. 1993.
- [7] J. Velagic, Z. Vukic, and E. Omerdic, “Adaptive fuzzy ship autopilot for track-keeping,” *Control Eng. Pract.*, vol. 11, no. 4, pp. 433–443, 2003.
- [8] Y. S. Yang, X. Jia, and X. Yu, “Robust adaptive control algorithm applied to ship steering autopilot with uncertain nonlinear system,” *Shipbuilding China*, vol. 41, no. 1, pp. 21–25, 2000.
- [9] T. I. Fossen and A. Grovlen, “Nonlinear output feedback control of dynamically positioned ships using vectorial observer backstepping,” *IEEE Trans. Control Syst. Technol.*, vol. 6, no. 1, pp. 121–128, Jan. 1998.
- [10] J. van Amerongen and A. J. U. ten Cate, “Model reference adaptive autopilots for ships,” *Automatica*, vol. 11, no. 5, pp. 441–449, 1975.
- [11] J. van Amerongen, “Adaptive steering of ships—A model reference approach,” *Automatica*, vol. 20, no. 1, pp. 3–14, 1984.
- [12] Y. Xudong and J. Jingping, “Adaptive nonlinear design without a priori knowledge of control directions,” *IEEE Trans. Autom. Control*, vol. 43, no. 11, pp. 1617–1621, Nov. 1998.
- [13] J. Du and C. Guo, “Nonlinear adaptive ship course tracking control based on backstepping and Nussbaum gain,” in *Proc. Amer. Control Conf.*, Boston, MA, USA, vol. 4, Jun. 2004, pp. 3845–3850.
- [14] J. Li et al., “Robust adaptive backstepping design for course-keeping control of ship with parameter uncertainty and input saturation,” in *Proc. Int. Conf. Soft Comput. Pattern Recognit. (SoCPar)*, Dalian, China, Oct. 2011, pp. 63–67.
- [15] J. Du, C. Guo, and C. Yang, “Adaptive robust backstepping nonlinear algorithm applied to ship steering,” *IFAC Proc. Volumes*, vol. 38, no. 1, pp. 61–66, 2005.
- [16] L. Yuan and H.-S. Wu, “Terminal sliding mode fuzzy control based on multiple sliding surfaces for nonlinear ship autopilot systems,” *J. Mar. Sci. Appl.*, vol. 9, no. 4, pp. 425–430, 2010.
- [17] Z. Li, J. Hu, and X. Huo, “PID control based on RBF neural network for ship steering,” in *Proc. World Congr. Inf. Commun. Technol.*, Dalian, China, Oct. 2012, pp. 1076–1080.
- [18] Y. Zhang, D. Jiang, and J. Wang, “A recurrent neural network for solving Sylvester equation with time-varying coefficients,” *IEEE Trans. Neural Netw.*, vol. 13, no. 5, pp. 1053–1063, Sep. 2002.
- [19] Y. Zhang, W. Ma, and B. Cai, “From Zhang neural network to Newton iteration for matrix inversion,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 56, no. 7, pp. 1405–1415, Jul. 2009.
- [20] Y. Zhang et al., “Time-varying square roots finding via Zhang dynamics versus gradient dynamics and the former’s link and new explanation to Newton–Raphson iteration,” *Inf. Process. Lett.*, vol. 110, no. 24, pp. 1103–1109, 2010.
- [21] Z. Xie, L. Jin, X. Du, C. Xiao, H. Li, and S. Li, “On generalized RMP scheme for redundant robot manipulators aided with dynamic neural networks and nonconvex bound constraints,” *IEEE Trans. Ind. Inform.*, to be published.
- [22] L. Jin, S. Li, H. M. La, X. Zhang, and B. Hu, “Dynamic task allocation in multi-robot coordination for moving target tracking: A distributed approach,” *Automatica*, vol. 100, pp. 75–81, Feb. 2019.
- [23] L. Jin, S. Li, B. Hu, M. Liu, and J. Yu, “A noise-suppressing neural algorithm for solving the time-varying system of linear equations: A control-based approach,” *IEEE Trans. Ind. Informat.*, vol. 15, no. 1, pp. 236–246, Jan. 2019.
- [24] L. Jin and S. Li, “Distributed task allocation of multiple robots: A control perspective,” *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 5, pp. 693–701, May 2018.
- [25] L. Jin, S. Li, H. M. La, and X. Luo, “Manipulability optimization of redundant manipulators using dynamic neural networks,” *IEEE Trans. Ind. Electron.*, vol. 64, no. 6, pp. 4710–4720, Jun. 2017.
- [26] L. Jin, Y. Zhang, and S. Li, “Integration-enhanced Zhang neural network for real-time-varying matrix inversion in the presence of various kinds of noises,” *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 27, no. 12, pp. 2615–2627, Dec. 2016.
- [27] L. Jin, Y. Zhang, S. Li, and Y. Zhang, “Noise-tolerant ZNN models for solving time-varying zero-finding problems: A control-theoretic approach,” *IEEE Trans. Autom. Control*, vol. 62, no. 2, pp. 992–997, Feb. 2017.
- [28] L. Jin, S. Li, B. Liao, and Z. Zhang, “Zeroing neural networks: A survey,” *Neurocomputing*, vol. 267, pp. 579–604, Dec. 2017.
- [29] L. Jin, S. Li, L. Xiao, R. Lu, and B. Liao, “Cooperative motion generation in a distributed network of redundant robot manipulators with noises,” *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 48, no. 10, pp. 1715–1724, Oct. 2018.
- [30] Y. Yin, Q. Xie, Y. Wang, D. Chen, and Y. Zhang, “ZG control for ship course tracking with singularity considered and solved,” in *Proc. IEEE 11th Int. Conf. Dependable, Auton. Secure Comput.*, Chengdu, China, Nov. 2013, pp. 352–357.
- [31] Y. Zhang, H. Xiao, J. Wang, J. Li, and P. Chen, “Discrete-time control and simulation of ship course tracking using ZD method and ZFD formula 4NgSFD,” in *Proc. IEEE 3rd Inf. Technol. Mechatronics Eng. Conf. (ITOEC)*, Chongqing, China, Oct. 2017, pp. 6–10.
- [32] K. Nomoto, K. Taguchi, K. Honda, and S. Hirano, “On the steering qualities of ships,” *Int. Shipbuilding Prog.*, vol. 4, no. 35, pp. 354–370, 1957.
- [33] T. I. Fossen, *Guidance and Control of Ocean Vehicles*, vol. 199, no. 4. New York, NY, USA: Wiley, 1994.
- [34] L. Jin and Y. Zhang, “Continuous and discrete Zhang dynamics for real-time varying nonlinear optimization,” *Numer. Algorithms*, vol. 73, pp. 115–140, Sep. 2016.
- [35] L. Jin, S. Li, and B. Hu, “RNN models for dynamic matrix inversion: A control-theoretical perspective,” *IEEE Trans. Ind. Informat.*, vol. 14, no. 1, pp. 189–199, Jan. 2018.



**DONGYANG FU** received the Ph.D. degree from the South China Sea Institute of Oceanology, Chinese Academy of Sciences, and the Ph.D. degree from the State Key Laboratory of Satellite Ocean Environment Dynamics, Second Institute of Oceanography, State Oceanic Administration, Guangzhou, China. He is currently a Professor with the School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China. His current research interests include ocean color remote sensing and its application, remote sensing in offshore water quality, response of upper ocean to typhoon, and neural networks.



**HUAN WANG** received the B.E. degree from Shandong Jiaotong University, Jinan, China, in 2018. She is currently pursuing the M.E. degree in physical oceanography with the School of Oceanography and Meteorology, Guangdong Ocean University, Zhanjiang, China. Her current research interests include the steady-state control of the ship, neural networks, and ocean color remote sensing.



**XIUCHUN XIAO** received the Ph.D. degree in communication and information system from Sun Yat-sen University, Guangzhou, China, in 2013. He is currently an Associate Professor with the School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China. His current research interests include image processing, artificial neural networks, and computer vision.



**SHAN LIAO** received the master's degree in software engineering from the Guangdong University of Technology, Guangzhou, China, in 2012. She is currently with the School of Electronics and Information Engineering, Guangdong Ocean University, Zhanjiang, China, where she is an Experimentalist. Her current research interests include remote sensing signal processing and algorithm, artificial neural networks, and communication technology.



**LONG JIN** (M'17) received the B.E. degree in automation and the Ph.D. degree in information and communication engineering from Sun Yat-sen University, Guangzhou, China, in 2011 and 2016, respectively. He was a Postdoctoral Fellow with the Department of Computing, The Hong Kong Polytechnic University, Hong Kong. He is currently a Full Professor with the School of Information Science and Engineering, Lanzhou University, Lanzhou, China. His main research interests include neural networks, robotics, and intelligent information processing.

...