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H_∞ Robust Control of Permanent Magnet Synchronous Motor Based on PCHD

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ABSTRACT The surface permanent magnet synchronous motor (SPMSM) speed regulation system is easily affected by the inner parameter perturbation and the external load disturbance at run time. To solve this problem, the H_∞ robust control strategy is proposed in this paper. First, given the systematic uncertainties, the H_∞ robust current controller based on the Hamilton–Jacobi Inequality is designed to ensure the robustness of current control under the SPMSM nominal mathematical model. This model is expressed as the port-controlled Hamiltonian with the dissipation form; second, the linear matrix inequality-based H_∞ sliding surface and the sliding control law are designed under the extended state space expression of the SPMSM motion equation. Thereby, the robust H_∞ sliding mode speed controller is acquired, thus realizing the robustness of speed control and improving the dynamic characteristics of the system. Finally, the effectiveness and availability of the proposed control strategy are verified by the hardware-in-the-loop simulation experiment.

INDEX TERMS Surface permanent magnet synchronous motor (SPMSM), H_∞ robust control, sliding mode control, linear matrix inequality (LMI), Hamilton-Jacobi Inequality (HJI).

I. INTRODUCTION

The permanent magnet synchronous motor (PMSM) is being widely used in high-tech fields such as automobiles, robotics and aerospace due to its high reliability, high torque-to-inertia ratio, high efficiency, and energy saving. In the occasions where the PMSM is required to both operate stably and track accurately, some uncertainties inside and outside the system may lead to poor control performances of the speed regulation system [1]–[3]. As the conventional PI control scheme is difficult to meet high-performance control requirements of the system, the design of advanced control strategies with high performances has gradually attracted widespread attention of scholars at home and abroad. Compared with the single closed-loop control, the speed and current double closed-loop control structure has advantages of fast dynamic response and strong anti-disturbance ability. Hence, it is generally adopted by engineering designers. Since the current inner loop determines the transient and steady state performances of the system, disturbances in the current loop will affect the current tracking performance and the system robustness. Consequently, how to construct a current control strategy with high stability, good dynamic performances and strong robustness of the system is the key to the servo control.

In recent years, some current control strategies with high performances have been presented successively. In [4] and [5], the current control methods based on the disturbance observer are proposed, which improve the current control performances through the observation and compensation of the system disturbances simultaneously. These methods, although, increase the robustness of the current control significantly, on the contrary, complicate the designs of the controllers. In addition, the stability of the system depends on the disturbance observation accuracy to some extent. In [6]–[11], the model predictive current control algorithms are reported. They solve the problem that the conventional predictive current control algorithm depends on the accuracy of the motor parameters and obtain promising control effect. But the need to solve the optimal equation online brings enormous computation costs. Hamiltonian system theory has offered many feasible control schemes for the academia in nonlinear subjects, and it attracts much attention from the researchers of PMSM nonlinear system [12]–[17]. In [13], based on the principals of the energy shaping and the port controlled Hamiltonian system, the shaping controller is obtained by means of interconnection and damping configuration. Furthermore, in order to restrain the voltage fluctuation in current loop, the H_∞ robust current controller is established, thus enhancing the robustness of the current control [18].

As a conventional control strategy, PI control has several drawbacks, especially the poor dynamic response performance, weak disturbance rejection ability and unsatisfactory robustness [19], [20]. For the sake of handling such problems, the improved control strategies have been put forward [21]. The LMI method, becomes increasingly convenient in calculation, owing to the progress of the theory, the development of the algorithm and a wide range of applications of computers [22]–[25]. So, the LMI theory has been gradually applied to the permanent magnet synchronous motor speed regulation system. In [24], the robust optimal control strategy is proposed, by which the robust optimal controller gain is obtained combined with the LMI theory. Although the proposed control strategy in [24] restrains the external disturbances effectively, it does not consider the inner parameters perturbation. Parameters mismatching between the controller and the motor may affect the controller's performances, perhaps even leading to the system instability. Besides, the sliding mode control solves parameters mismatching between the controller and the motor effectively in PMSM speed regulation system, since it has the robustness to both parameters perturbation and external disturbances [26]–[28]. In [29], the LMI based robust H_∞ sliding mode control strategy is presented which inhibits the mismatched disturbances of the system effectively. Also, it offers a new idea for solving the parameters mismatching between the controller and the motor.

In this study, aiming to improve the dynamic characteristics and robustness of the conventional vector control system, the double closed-loop H_∞ robust control strategy is presented in the presence of the uncertainties. This paper is organized into six sections. Section II describes the PCHD model of SPMSM. Section III develops the HJI based H_∞ robust current controller. Section IV proposes the robust H_∞ sliding mode speed controller based on LMI, including the H_∞ sliding surface and the robust H_∞ sliding mode speed control law. Section V reports the results of the HIL simulation experiment. Section VI presents our conclusions.

II. PCHD MATHEMATICAL MODEL OF SPMSM

The mathematical model of SPMSM in d, q coordinates frame can be represented as follows:

$$\begin{cases} L\dfrac{di_d}{dt} = u_d - R_s i_d + P_n\omega_m L i_q \\ L\dfrac{di_q}{dt} = u_q - R_s i_q - P_n\omega_m\psi_f - P_n\omega_m L i_d \\ J\dfrac{d\omega_m}{dt} = P_n\psi_f i_q - B_0\omega_m - T_{L0} \end{cases} \tag{1}$$

where u_d, u_q and i_d, i_q are the stator d-q axis voltage and current components, respectively, R_s and L are the stator resistance and inductance, respectively, ω_m is the mechanical angular speed, ψ_f is the rotor PM flux linkage, P_n is the number of pole pairs, B_0 is the viscous friction coefficient, J is the moment of inertia and T_{L0} is the initial value of the load torque.

From (1), the nominal PCHD model can be established as follows:

$$\begin{cases} \dot{x} = (J(x) - R(x))\,\partial_x H + I(x)\,u \\ y = I(x)\,\partial_x H \end{cases} \tag{2}$$

where $J(x) = \begin{bmatrix} 0 & 0 & P_n x_2 \\ 0 & 0 & -P_n(x_1+\psi_f) \\ -P_n x_2 & P_n(x_1+\psi_f) & 0 \end{bmatrix}$, $R(x) = \begin{bmatrix} R_s & 0 & 0 \\ 0 & R_s & 0 \\ 0 & 0 & B_0 \end{bmatrix}$, $I(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} Li_d \\ Li_q \\ J\omega_m \end{bmatrix} = D\begin{bmatrix} i_d \\ i_q \\ \omega_m \end{bmatrix}$, $D = \begin{bmatrix} L & 0 & 0 \\ 0 & L & 0 \\ 0 & 0 & J \end{bmatrix}$, $u = \begin{bmatrix} u_d \\ u_q \\ -T_{L0} \end{bmatrix}$.

The Hamiltonian function of the system is given by

$$H(x) = \frac{1}{2}x^T D^{-1} x = \frac{1}{2}\left(\frac{1}{L}x_1^2 + \frac{1}{L}x_2^2 + \frac{1}{J}x_3^2\right) \tag{3}$$

Then

$$\partial_x H = \begin{bmatrix} i_d & i_q & \omega_m \end{bmatrix}^T$$

From the requirement of the maximum torque current control and the reference mechanical angular speed, we can acquire the desired equilibrium point $x^* = \begin{bmatrix} 0 & Li_q^* & J\omega_m^* \end{bmatrix}^T$. Convert (2) into the PCHD nominal model of SPMSM with the desired equilibrium state x^*, and meanwhile take the parameter perturbation and the load disturbance into account, then we obtain the following mathematical model:

$$\begin{cases} \dot{x} = [J_d(x) - R(x)]\,\partial_x H_d + I(x)\,u + Z(x)\,\xi \\ y = I(x)\,\partial_x H_d \end{cases} \tag{4}$$

where $J_d(x) = J(x) + J_a(x)$, $J_a(x) = \begin{bmatrix} 0 & 0 & -P_n x_2^* \\ 0 & 0 & P_n x_1^* \\ P_n x_2^* & -P_n x_1^* & 0 \end{bmatrix}$, $H_d(x) = \frac{1}{2}(x - x^*)^T D^{-1}(x - x^*)$, $Z(x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\xi = (1+\Delta)\left\{[J(x) - R(x)]\begin{bmatrix} i_d^* \\ i_q^* \\ \omega_m^* \end{bmatrix} - J_a(x)\,\partial_x H_d\right\} + \begin{bmatrix} f_d \\ f_q \\ f_\omega \end{bmatrix}$,

$$\begin{cases} f_d = -\Delta R_s i_d + P_n\omega_m\Delta L i_q - \Delta L\dfrac{di_d}{dt} \\ f_q = -\Delta R_s i_q - P_n\omega_m\Delta\psi_f - P_n\omega_m\Delta L i_d - \Delta L\dfrac{di_q}{dt} \\ f_\omega = P_n\Delta\psi_f i_q - \Delta B_0\omega_m - \Delta J\dfrac{d\omega_m}{dt} - \Delta T_L \end{cases} \tag{5}$$

III. DESIGN OF H_∞ ROBUST CURRENT CONTROLLER

In order to acquire the H_∞ robust current controller, the lemma 1 below is given:

Lemma 1 [10]: Consider a system described as follows:

$$\begin{cases} \dot{x} = f(x) + Z(x)\xi \\ y = h(x) \end{cases} \tag{6}$$

where $f(x)$ and $h(x)$ are nonlinear. If there exists an $E(x) \geq 0\ (E(x^*) = 0)$ which satisfies the HJI below:

$$(\partial_x E)^T f(x) + \frac{1}{2\gamma^2}(\partial_x E)^T Z(x) Z^T(x)\partial_x E + \frac{1}{2}h^T(x)h(x) \leq 0 \tag{7}$$

then

$$\int_0^\tau \|y(t)\|^2 dt \leq \gamma^2 \int_0^\tau \|\xi\|^2 dt \tag{8}$$

where γ is an H_∞ disturbance attenuation level bound from the disturbance ξ to the output y.

According to Lemma 1 and model (4), define the two functions in (7) as:

$$\begin{cases} f(x) = [J_d(x) - R(x)]\partial_x H_d + I(x)u \\ h(x) = I(x)\partial_x H_d \end{cases} \tag{9}$$

Choose $E(x) = H_d(x) \geq 0$, then $E(x^*) = H_d(x^*) = 0$. Substituting (9) into (7) yields:

$$\begin{aligned}&(\partial_x H_d)^T \{[J_d(x) - R(x)]\partial_x H_d + I(x)u\} \\ &+ \frac{1}{2\gamma^2}(\partial_x H_d)^T Z(x) Z^T(x)\partial_x H_d \\ &+ \frac{1}{2}(\partial_x H_d)^T I^T(x) I(x)\partial_x H_d \leq 0\end{aligned} \tag{10}$$

Then

$$\begin{aligned}&(\partial_x H_d)^T J_d(x)\partial_x H_d - (\partial_x H_d)^T R(x)\partial_x H_d \\ &+ (\partial_x H_d)^T I(x)u + \frac{1}{2\gamma^2}(\partial_x H_d)^T Z(x) Z^T(x)\partial_x H_d \\ &+ \frac{1}{2}(\partial_x H_d)^T I^T(x) I(x)\partial_x H_d \leq 0\end{aligned} \tag{11}$$

Thanks to $J_d(x)$ is a skew-symmetric matrix, we can get

$$\begin{aligned}&-(\partial_x H_d)^T R(x)\partial_x H_d + (\partial_x H_d)^T I(x)u \\ &+ \frac{1}{2\gamma^2}(\partial_x H_d)^T Z(x) Z^T(x)\partial_x H_d \\ &+ \frac{1}{2}(\partial_x H_d)^T I^T(x) I(x)\partial_x H_d \leq 0\end{aligned} \tag{12}$$

Since $R(x) \geq 0$, when

$$(\partial_x H_d)^T I(x)u + \frac{1}{2}\left(1 + \frac{1}{\gamma^2}\right)(\partial_x H_d)^T Z(x) Z^T(x)\partial_x H_d = 0,$$

(12) is established. Then, the H_∞ robust current controller could be derived and presented as follows:

$$u = -\frac{1}{2}\left(1 + \frac{1}{\gamma^2}\right)\partial_x H_d \tag{13}$$

$$u_d = -\frac{1}{2}\left(1 + \frac{1}{\gamma^2}\right)(i_d - i_d^*) \tag{14}$$

$$u_q = -\frac{1}{2}\left(1 + \frac{1}{\gamma^2}\right)(i_q - i_q^*) \tag{15}$$

IV. DESIGN OF ROBUST H_∞ SLIDING MODE SPEED CONTROLLER

A. DESIGN OF H_∞ SLIDING SURFACE

The H_∞ sliding surface denotes that the system satisfies both the robust stability and the H_∞ disturbance attenuation level γ', when the system state is on the sliding surface. The motion equation of the SPMSM on d-q axes can be translated into the dynamic error equation and described as:

$$\dot{e}_{\omega_m} = -\frac{B_0}{J}e_{\omega_m} - \frac{p_n\psi_f}{J}i_q + \frac{1}{J}T_{L0} + \frac{B_0}{J}\omega_m^* \tag{16}$$

where $e_{\omega_m} = \omega_m^* - \omega_m$. Make further efforts, extend (16) and take the parameter perturbation and the external disturbance into consideration, then the state space dynamic expression can be described as follows:

$$\dot{x}' = Ax' + Bu' + M\delta \tag{17}$$

$$y' = Cx' \tag{18}$$

where $A = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{B_0}{J} \end{bmatrix} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ -\frac{P_n\psi_f}{J} \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$, $M = \begin{bmatrix} l \\ \frac{1}{J} \end{bmatrix} = \begin{bmatrix} M_1 \\ M_2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}$, $x' = \begin{bmatrix} x'_1 \\ x'_2 \end{bmatrix} = \begin{bmatrix} \int_0^t e_{\omega_m} d\tau \\ e_{\omega_m} \end{bmatrix}$, $u' = i_q$, $\delta = T_L + B_0\omega_m^* + J\left[-\Delta\left(\frac{B_0}{J}\right)e_{\omega_m} - \Delta\left(\frac{P_n\psi_f}{J}\right)i_q + \Delta\left(\frac{1}{J}\right)T_{L0} + \Delta\left(\frac{B_0}{J}\right)\omega_m^*\right]$ $T_L = T_{L0} + \Delta T_L$, l is the assumed mismatched disturbance coefficient.

Obviously, in (17) and (18):

1) The state x' is controllable and observable.

2) The input matrix B is full column rank, as well as $R(B) = 1 < 2$.

Make the following assumption: The disturbance signal δ is norm bounded, and the bound is δ_0, that is:

$$\|\delta\| \leq \delta_0 \tag{19}$$

Define the H_∞ sliding surface as:

$$s = \Xi x' = \begin{bmatrix} \Xi_1 & \Xi_2 \end{bmatrix} x' \tag{20}$$

During ideal sliding on the surface $s = 0$, we have:

$$\Xi_1 x'_1 + \Xi_2 x'_2 = 0 \tag{21}$$

Hence, if $\Xi_2 \neq 0$, it has:

$$x'_2 = -\frac{\Xi_1}{\Xi_2}x'_1 \tag{22}$$

Substitute (22) into (17) and (18) to yield the following reduced-order state space equation of x'_1(Ξ_2 usually takes 1):

$$\begin{cases} \dot{x}'_1 = (A_1 - A_2\Xi_1)x'_1 + M_1\delta \\ y' = (C_1 - C_2\Xi_1)x'_1 \end{cases} \tag{23}$$

In (23), $-\Xi_1 x'_1$ can be regarded as the state feedback control law. Select the candidate Lyapunov function as:

$$V_{x'_1}(t) = P{x'}_1^2 \tag{24}$$

where P is a positive real constant. The equivalent inequality constraint of the H_∞ state feedback controller gain $-\Xi_1$ is:

$$\begin{cases} \dot{V}_{x'_1}(t) < 0 \\ \int_0^\infty \left({y'}^T y' - {\gamma'}^2 \delta^2 \right) dt < 0 \end{cases} \tag{25}$$

In order to obtain the equivalent LMI constraint of (25), we take:

$$\dot{V}_{x'_1}(t) + \lambda + {y'}^T y' - {\gamma'}^2 \delta^2 < 0 \tag{26}$$

where
$\lambda = 2\left({x'}_1 G_1 + \dot{x}'_1 G_2\right)\left[\dot{x}'_1 - (A_1 - A_2\Xi_1){x'}_1 - M_1\delta\right] + \varepsilon_1^2 {x'}_1^2 \geq 0$,
G_1 and G_2 are real variables, $\varepsilon_1 > 0$ is a real constant that needs to be preseted.

Substitute (23) into the left side of (26), it is given by

$$\begin{aligned} &\dot{V}_{x'_1}(t) + \lambda + {y'}^T y' - {\gamma'}^2 \delta^2 \\ &\quad = 2P{x'}_1\dot{x}'_1 + 2\left({x'}_1 G_1 + \dot{x}'_1 G_2\right) \\ &\qquad \cdot \left[\dot{x}'_1 - (A_1 - A_2\Xi_1){x'}_1 - M_1\delta\right] + \varepsilon_1^2 {x'}_1^2 \\ &\qquad + \left[(C_1 - C_2\Xi_1){x'}_1\right]^T (C_1 - C_2\Xi_1){x'}_1 \\ &\qquad - {\gamma'}^2\delta^2 = \theta^T \Sigma \theta \end{aligned} \tag{27}$$

where

$$\theta = \begin{bmatrix} {x'}_1 & \dot{x}'_1 & \delta \end{bmatrix}^T,$$

$$\Sigma = \Gamma - \begin{bmatrix} (C_1 - C_2\Xi_1)^T \\ 0 \\ 0 \end{bmatrix} (-I_2) \begin{bmatrix} (C_1 - C_2\Xi_1)^T \\ 0 \\ 0 \end{bmatrix}^T,$$

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 & -G_1{M'}_1 \\ * & 2G_2 & -G_2{M'}_1 \\ * & * & -{\gamma'}^2 \end{bmatrix},$$

$$\Gamma_1 = -2G_1(A_1 - A_2\Xi_1) + \varepsilon_1^2,$$
$$\Gamma_2 = P + G_1 - G_2(A_1 - A_2\Xi_1).$$

"$*$" denotes the symmetric elements of the matrix. For $\forall\theta \neq 0$, if $\Sigma < 0$, (26)holds.

Integrating both sides of (26) from 0 to ∞, then

$$V_{x'_1}(\infty) - V_{x'_1}(0) + \int_0^\infty \left({y'}^T y' - {\gamma'}^2\delta^2 \right) dt < 0 \tag{28}$$

Obviously in(28), when $V_{x'_1}(0) = 0$, it has:

$$\int_0^\infty \left({y'}^T y' - {\gamma'}^2\delta^2 \right) dt < 0 \tag{29}$$

As a result, (29) can be derived that for δ, the system has the H_∞ disturbance attenuation level γ'. In addition, when $\delta = 0$, (26) indicates that the system has the robust stability.

To sum up, the equivalent constraint of (25) is $\Sigma < 0$ in (27).

Apply the Schur complement to $\Sigma < 0$, then the equivalent condition of $\Sigma < 0$ can be described as:

$$\begin{bmatrix} \Gamma_1 & \Gamma_2 & -G_1M_1 & (C_1 - C_2\Xi_1)^T \\ * & 2G_2 & -G_2M_1 & 0 \\ * & * & -{\gamma'}^2 & 0 \\ * & * & * & -I_2 \end{bmatrix} < 0 \tag{30}$$

Setting $G_1 = G_0 < 0$, $G_2 = gG_0 < 0$ and $g > 0$, applying the Schur complement to (30) again, and pre- and post- multiplying it by $diag\left\{ G_0^{-1} \; G_0^{-1} \; 1 \; I_2 \; 1 \right\}$ yield:

$$\begin{bmatrix} \Psi_1 & \Psi_2 & -M_1 & G_0^{-1}(C_1 - C_2\Xi_1)^T & G_0^{-1}\varepsilon_1 \\ * & 2gG_0^{-1} & -gM_1 & 0 & 0 \\ * & * & -{\gamma'}^2 & 0 & 0 \\ * & * & * & -I_2 & 0 \\ * & * & * & * & -1 \end{bmatrix} < 0 \tag{31}$$

where

$$\Psi_1 = -2(A_1 - A_2\Xi_1)G_0^{-1},$$
$$\Psi_2 = G_0^{-2}P + G_0^{-1} - g(A_1 - A_2\Xi_1)G_0^{-1}.$$

Further, letting $G_0^{-1} = Q$, $\Xi_1 Q = Z$, $G_0^{-2}P = N$, ${\gamma'}^2 = \Upsilon$ in (31) yields:

$$\begin{bmatrix} \Phi_1 & \Phi_2 & -M_1 & Q{C_1}^T - Z{C_2}^T & Q\varepsilon_1 \\ * & 2gQ & -gM_1 & 0 & 0 \\ * & * & -\Upsilon & 0 & 0 \\ * & * & * & -I_2 & 0 \\ * & * & * & * & -1 \end{bmatrix} < 0 \tag{32}$$

where

$$\Phi_1 = -2A_1Q + 2A_2Z,$$
$$\Phi_2 = N + Q - gA_1Q + gA_2Z.$$

In conclusion, (32) is the equivalent LMI condition of (25). A_1, A_2, C_1, C_2 and M_1 are known while $Q < 0, Z, N > 0$ and $\Upsilon > 0$ are all one dimensional real variables, and ε_1 is set by 1 (for convenient). Owing to Υ can reflect the robustness of the system, the problem of solving the LMI (32) can be described as the problem of obtaining the minimum (or optimal) value of Υ for a certain g, which can be achieved by mincx of Matlab. Ultimately, all variables can be acquired in the presence of $min\Upsilon$. Further, according to $\Xi_1 = ZQ^{-1}$, we can obtain the H_∞ sliding surface $s = \Xi x' = \begin{bmatrix} \Xi_1 & \Xi_2 \end{bmatrix} x'$.

B. DESIGN OF ROBUST H_∞ SLIDING MODE SPEED CONTROL LAW

Attempt to drive the system state onto the H_∞ sliding surface, the robust H_∞ sliding mode speed control law is designed as:

$$u' = -(\Xi B)^{-1}\left(\|\Xi A\| \left\| x' \right\| + \|\Xi M\| \delta_0 + \beta \right) \cdot sigmoid(s) \tag{33}$$

where $\beta > 0$ and $sigmoid(s) = \frac{2}{1+e^{-as}} - 1$, $a > 0$.

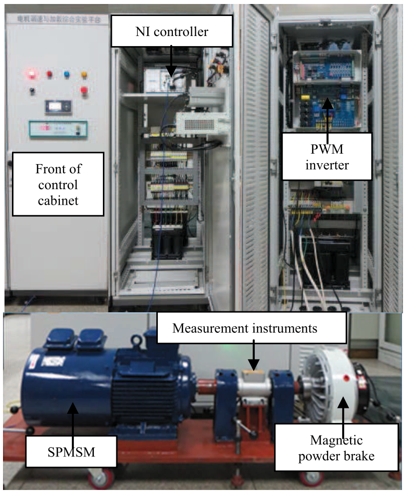

FIGURE 1. Physical maps of the comprehensive experimental platform for motor speed regulation and loading.

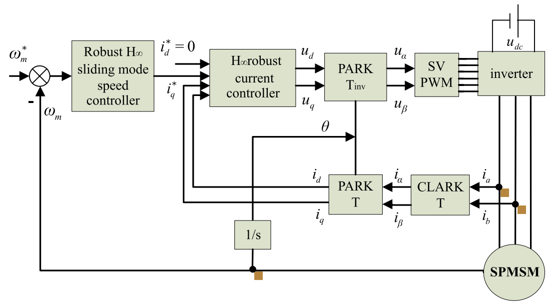

FIGURE 2. The principle diagram of SPMSM speed regulation system.

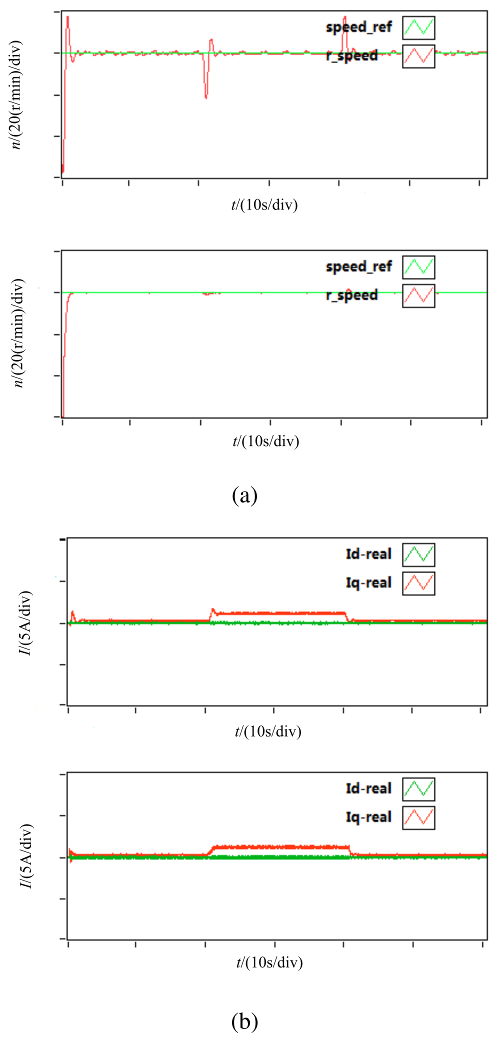

FIGURE 3. Curves of motor with loading and unloading. (a) speed curves. (b) i_d, i_q curves.

Consider the Lyapunov function candidate as:

$$V_s(t) = \frac{1}{2}s^2 \tag{34}$$

The derivative of $V_s(t)$ is given by

$$\dot{V}_s(t) = s\dot{s} = s\Xi\left(Ax' + M\delta\right) + s\Xi Bu' \tag{35}$$

Substituting (33) into (35) yields:

$$\begin{aligned}\dot{V}_s(t) &= s\dot{s} = s\Xi\left(Ax' + M\delta\right) \\ &\quad - \left(\|\Xi A\|\left\|x'\right\| + \|\Xi M\|\,\delta_0 + \beta\right) ssigmoid\,(s) \\ &\approx s\Xi\left(Ax' + M\delta\right) - \left(\|\Xi A\|\left\|x'\right\| + \|\Xi M\|\,\delta_0 + \beta\right)\|s\| \\ &\leq -\beta\,\|s\| \leq 0 \end{aligned} \tag{36}$$

The equal sign holds only in $s = 0$.

V. THE HIL SIMULATION EXPERIMENT RESEARCH

The comprehensive experimental platform of SPMSM speed regulation and loading is displayed in Fig.1. Mainly includes: NI controller, PWM inverter, speed control and loading mechanical platform of SPMSM (including SPMSM, magnetic powder brake, and torque and speed measurement instruments), monitoring host PC and distribution lines. With this platform, the speed regulation and loading of SPMSM HIL simulation experiment can be carried out.

Before starting up the platform, VeriStand should be firstly installed in the PXI controller, and run the corresponding programs. Then the motor control algorithm program can be loaded into the NI controller through the NI MAX software.

The principle diagram of SPMSM speed regulation system is shown in Fig.2.

In order to further verify the superiority of the proposed control strategy and the rationality of the theoretical analysis, the comparative research of two cases in the HIL simulation experiment platform was carried out.

Case 1: Conventional vector control.

Case 2: H_∞ robust control.

(In each comparison group, the above and below inserted figures are curves of case 1 and 2, respectively.)

A. MOTOR WITH LOADING AND UNLOADING HIL SIMULATION EXPERIMENT

The desired speed was given as 60 r/min, the load torque of $5N \cdot m$ was injected into the system at 0.4s, and was

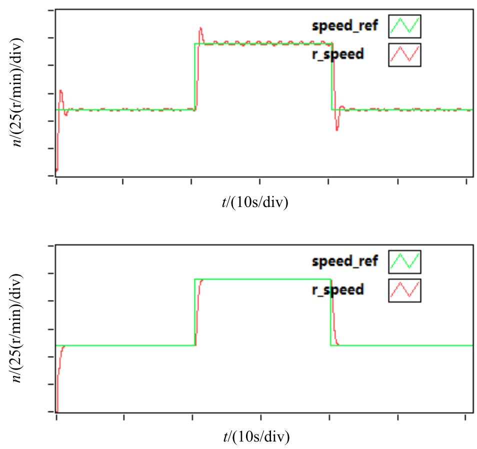

FIGURE 4. **Speed curves of motor up and down.**

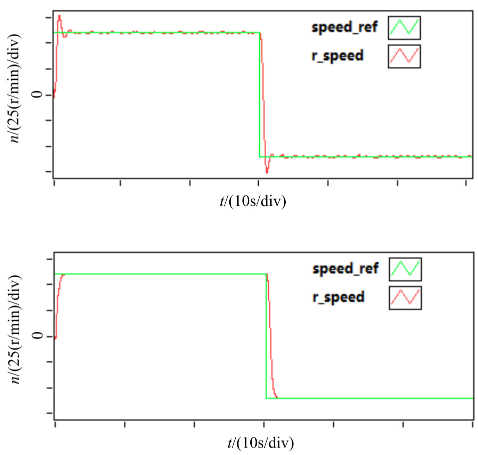

FIGURE 5. **Speed curves of motor positive and negative.**

removed suddenly at 0.7s. Curves of speed, i_d, i_q were shown in Fig.3.

It can be seen from Fig.3 that there was about 28% of overshoot in the speed curve of case 1, while case 2 almost had no overshoot during the starting process in Fig.3 (a). At the moment of load torque variation, 33% of the speed drop occurred in case 1, while the small fluctuation was generated in case 2 of Fig.3 (a). In Fig.3 (b), the current curves of both case 1 and 2 showed the smooth transition processes when the load torque changed, which signified the strong robustness of both cases.

B. MOTOR UP AND DOWN, AND POSITIVE AND NEGATIVE HIL SIMULATION EXPERIMENTS

The desired speed was given as 60 r/min, changed the desired speed value as 120 r/min at 0.3s, and backed to 60 r/min again at 0.7s. Speed curves of motor up and down were shown in Fig.4; The desired speed was given as 60 r/min and changed it as -60 r/min at 0.5s. Speed curves of motor positive and negative were shown in Fig.5.

It can be observed from Fig.4 and Fig.5 that the speed curves of case 1 generated about 28% of overshoot, while the speed curves of case 2 had promising and smooth transition processes when the desired speeds changed, which indicated the good dynamic performances of the proposed control strategy.

VI. CONCLUSION

In this paper, we put forward the H_∞ robust control strategy on account of the vector control system of SPMSM. In the design of speed controller, the robust H_∞ sliding mode speed controller is acquired to ensure the robustness of the speed control, as the LMI based H_∞ sliding surface and control law are constructed. In the design of current controller, the H_∞ robust current controller based on HJI is represented considering the systematic uncertainties under the PCHD nominal model, which guarantees the robustness of the current control. In the end, the HIL simulation experiment is carried out to verify the effectiveness of the proposed control method.

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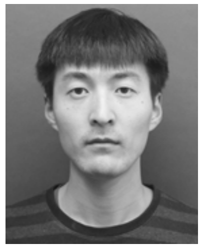

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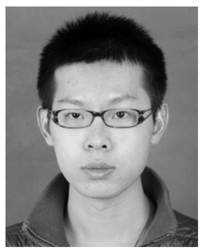

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