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Locating the Nodes From Incomplete Euclidean Distance Matrix Using Bayesian Learning

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ABSTRACT Node localization in wireless sensor networks (WSNs) has received a considerable amount of attention. In this paper, using the natural low-rank properties of the Euclidean distance matrix (EDM), we formulate the node location finding problem from only a small fraction of random entries of the EDM as a low-rank matrix recovery problem. A Bayesian-learning-based method is utilized to recover the original EDM, based on which the relative positions of all the sensor nodes in WSNs are accurately estimated by applying classical multi-dimensional scaling (MDS). In addition, with the location knowledge of anchor nodes, we transform the relative positions into absolute positions. The simulation results illustrate that our proposed approach leads to superior performance over various other methods.

INDEX TERMS Localization, Euclidean distance matrix completion, Bayesian learning, Wireless sensor networks.

I. INTRODUCTION

Wireless sensor networks (WSNs), which are composed of a large number of inexpensive and energy-constrained sensor nodes (SNs), have found important applications in many fields such as environmental monitoring, habitat monitoring, the prediction and detection of natural calamities, medical monitoring and structural health monitoring [1]. To respond to the monitoring data, the location information of sensor nodes should be available at the fusion center (FC), where the actions of WSNs, such as fire alarm, energy transfer, and emergency request, are made. As such, an approach to identify the location information of all nodes at the FC is of crucial importance [2]. Due to the network cost, deployment conditions and node energy constraints, only a few nodes, named anchor nodes, can obtain their global positions by equipping GPS devices. The positions of other unknown nodes are determined using localization algorithms based on the anchor nodes and inter-node range measurements. Most localization algorithms for WSNs can be classified into two categories: range-based and range-free algorithms [3], [4]. Range-based techniques use distance or angle estimates for location estimation, while range-free techniques only use connectivity information between unknown nodes and anchor nodes. In this paper, we focus on range-based localization techniques, as they have high levels of localization accuracy.

Recently, the research in [5] has shown that the underlying geometric structure of the Euclidean distance matrix (EDM), a matrix of the squared distances between points, can be extracted to reconstruct the network topology. The theoretical guarantees have been proven for multi-dimensional scaling (MDS) [6], which can find the best point set representation for a given set of distances. Therefore, in the case where the EDM is complete known, we can simply apply MDS to accurately estimate the relative positions of all the sensor nodes. In WSNs, the EDM can be obtained based on certain measurements, including time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA) and received signal strength (RSS) [7]. However, obtaining the complete range information of the EDM leads to communication-intensive operations by the WSN, as it needs the measurements among all nodes [8]. In addition, due to, e.g., energy constraints and limited communication ranges, not all sensors can communicate with each other. These inevitable factors cause the incompleteness of EDM and thus increase the difficulty of localization implementation.

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In light of this challenge, matrix completion (MC) [9], derived from the famous compressive sensing (CS) theory [10], has attracted significant attention in the signal processing community [11]–[13]. Because the EDM is a low-rank matrix, it can be recovered from an incomplete EDM with a reasonable number of measured entries not far from the information theoretic limit [14]. Recently, various EDM completion approaches have been proposed [15]–[19]. In [15], fixed point continuation with approximation (FPCA) was introduced as an efficient nuclear-norm-based regularized least-squares method. Keshavan *et al.* [16] developed a method called OptSpace based on optimization over the Grasmann manifold, with a theoretical performance guarantee for the noiseless case. In [17], a greedy approach that uses greedy projection to identify a set of rank-one matrices that best represents the original matrix was proposed. The singular value thresholding (SVT) proposed in [18] has been successfully used in many MC problems; the core of this algorithm lies in the fact that the singular values below a threshold are discarded at each iteration, which may cause a low convergence rate and poor completion performance. The accelerated proximal gradient (APG) method in [19] is proposed to address the bottlenecks of SVT; however, the APG is an iterative method that depends heavily on the choice of relevant parameters and achieves suboptimal completion accuracy.

Several sensor network localization techniques have also been developed based on EDM completion. In [20], a connectivity graph together with the shortest path were exploited for the EDM completion. However, the graph should be well connected to obtain a high localization accuracy, that is to say, each sensor node is required to have a distance measuring ability in its vicinity. Thus, a large number of measurements are still necessary, especially for large-scale networks. In [21], localization from an incomplete EDM was formulated as a low-rank matrix recovery problem. Because of the non-convexity of the rank minimization function, the authors applied the nuclear norm as a convex surrogate. This may not be a good approximation because the nuclear norm adds the singular values together and treats them differently, whereas in the rank minimization function, non-zero singular values have equal contributions. In [8] and [22], semi-definite programming (SDP) relaxation and solvers for the same problem have also been proposed, which can achieve high accuracy in estimating the sensor locations. However, as the size of the SDP problem increases, the dimension of the matrix cone increases and the number of variables increases quadratically, the resulting high arithmetic operation complexity indicates that these techniques can only address small-scale node localization problems. Therefore, a special effort has to be made to develop accurate and robust approaches to localize sensor nodes from incomplete EDMs in WSNs.

Motivated by the discussion above, in this paper, we focus on the node localization problem from only a small fraction of random entries of the EDM. Based on the low-rank factorization of the EDM and sparse Bayesian learning (SBL) principles, we propose an EDM completion algorithm. Then, the MDS and Procrustes analysis technique [5] are utilized to localize all the nodes. The main contributions of this paper are outlined as follows:

(1) We propose a novel node localization framework for WSNs. The key idea is to formulate the node localization problem from an incomplete EDM as a low-rank matrix recovery problem subject to a set of linear equality constraints.

(2) Using low-rank matrix factorization, a new Bayesian-learning-based matrix completion (BLMC) method, which favors low-rank solutions, is proposed to recover the original EDM.

(3) Based on the recovered EDM, the relative positions of all nodes in the WSN are accurately estimated by applying classical MDS and then transformed into absolute positions with the location knowledge of the anchor nodes.

The remainder of this paper is organized as follows. The problem formulation is given in Section II. In Section III, our proposed localization approach is described, including the EDM completion algorithm using Bayesian learning and MDS together with the Procrustes analysis technique utilized to localize the nodes. Then, the performance of the proposed algorithm in terms of localization accuracy is demonstrated in Section IV. Finally, Section V concludes the paper.

II. PROBLEM FORMULATION

Consider a typical localization scenario in WSNs, where a large number of sensor nodes are randomly deployed in an area; each sensor node measures the distance information of adjacent nodes and then sends this information to the FC. Node localization proceeds by constructing the topology of the sensor nodes, i.e., estimating the relative positions of the sensors. Among the N sensor nodes in total, M ($M \ll N$) are anchor nodes, whose positions are known, and the other $N - M$ nodes, called source nodes, are position-unknown nodes. We define $\{\boldsymbol{u}_j\}_{j=1}^{M} \in \mathbb{R}^d$ and $\{\boldsymbol{x}_i\}_{i=M+1}^{N} \in \mathbb{R}^d$ as the locations of M anchor nodes and those of the $N - M$ source nodes in a d-dimensional Euclidean space, respectively.

Here, we ignore the distinction between any anchor node and the source node, and we identify $\boldsymbol{u}_j$ with $\boldsymbol{x}_j$. Let

$$\mathbf{P} = [\boldsymbol{p}_1, \cdots, \boldsymbol{p}_N] = [\boldsymbol{u}_1, \cdots, \boldsymbol{u}_M, \boldsymbol{x}_{M+1}, \cdots, \boldsymbol{x}_N] \quad (1)$$

denote the locations of all the sensor nodes. Then, the squared distance between $\boldsymbol{p_i}$ and $\boldsymbol{p_j}$ is given as

$$D_{i,j} = \| \boldsymbol{p}_i - \boldsymbol{p}_j \|^2 \quad (2)$$

where $\| \cdot \|$ denotes the Euclidean norm. By noting that

$$D_{i,j} = (\boldsymbol{p_i} - \boldsymbol{p_j})(\boldsymbol{p_i} - \boldsymbol{p_j})^T = \boldsymbol{p_i}^T \boldsymbol{p_i} - 2\boldsymbol{p_i}^T \boldsymbol{p_j} + \boldsymbol{p_j}^T \boldsymbol{p_j} \quad (3)$$

the EDM $\mathbf{D} = [D_{i,j}]$ can be written as

$$\mathbf{D} \triangleq \mathbf{1}\mathrm{diag}(\mathbf{P}^T\mathbf{P})^T - 2\mathbf{P}^T\mathbf{P} + \mathrm{diag}(\mathbf{P}^T\mathbf{P})\mathbf{1}^T \quad (4)$$

where $\mathbf{1} \in \mathbb{R}^N$ is a column vector of all ones and $\mathrm{diag}(\cdot)$ is a column vector of the diagonal entries.

If the matrix $\mathbf{D}$ is fully known, by applying the MDS method, the relative positions of all nodes in the WSN can be estimated accurately. Then, the relative positions can be transformed into absolute positions with the knowledge of at least $d+1$ anchor nodes [23].

Unfortunately, because of noisy environments, energy constraints, and limits on the communication range, the measurement of distance data is incomplete, that is to say, only a small fraction of noisy distance information can be transmitted to the FC. Thus, the EDM $\mathbf{D}$ tends to only be partially known; moreover, it is corrupted by noise. In the following, we denote the noisy and incomplete EDM as $\tilde{\mathbf{D}} = \{\tilde{D}_{ij}\}$, which is generated according to

$$\tilde{\mathbf{D}} = \Gamma_{\Omega}(\mathbf{D} + \mathbf{N}) \tag{5}$$

where $\mathbf{N}$ is the dense error matrix with elements N_{ij}; Ω is the set of all known indices of $\tilde{\mathbf{D}}$, where the cardinality of the set Ω is λN^2, with $\lambda \leq 1$, defined as the sparsity level; and $\Gamma_{\Omega}(\cdot)$ is the projection operation, which keeps the known entries unchanged and sets all the unknown entries to zeros, i.e.,

$$\Gamma_{\Omega}(\mathbf{D}) = \begin{cases} D_{i,j} & \text{if } (i,j) \in \Omega \\ 0 & \text{if } (i,j) \notin \Omega \end{cases} \tag{6}$$

Because the rank of $\mathbf{P}$ is at most d (i.e., it has d rows), the rank of $\mathbf{P}^T\mathbf{P}$ is also at most d. The remaining two summands in (4) are of rank one. By rank inequalities, the rank of a sum of matrices cannot exceed the sum of the ranks of the summands. Hence, for the 2-D case, the rank of $\mathbf{D}$ is at most 4 [5], which is substantially smaller than its dimension N. In other words, $\mathbf{D}$ is a low-rank matrix, which can be estimated from $\tilde{\mathbf{D}}$ via the rank minimization problem below

$$\begin{aligned} &\min \ \text{rank}\,(\mathbf{D}) \\ &s.t. \ ||\tilde{\mathbf{D}} - \Gamma_{\Omega}(\mathbf{D})||_F^2 \leq \varepsilon \end{aligned} \tag{7}$$

where rank$(\cdot)$ denotes the rank operator and the estimate of $\mathbf{D}$, $\|\cdot\|_F$ denotes the Frobenius norm, and ε is an estimate of the noise level.

In general, we call (7) the EDM completion problem. Note that it is a non-convex and NP-hard problem; therefore, some researchers have resorted to approximation or relaxation to make the problem feasible. A related problem has been studied in [24], where the authors have shown that the rank minimization problem can be approximately solved using the Nuclear norm:

$$\begin{aligned} &\min \ \|\mathbf{D}\|_* \\ &s.t. \ ||\tilde{\mathbf{D}} - \Gamma_{\Omega}(\mathbf{D})||_F^2 \leq \varepsilon \end{aligned} \tag{8}$$

where $\|\cdot\|_*$ is equal to the sum of the singular values of $\mathbf{D}$.

III. PROPOSED NODE LOCALIZATION APPROACH

A. PROPOSED IDEA

With the above system and data model, in this section, we propose to solve the node localization problem in three steps, as shown in Fig. 1.

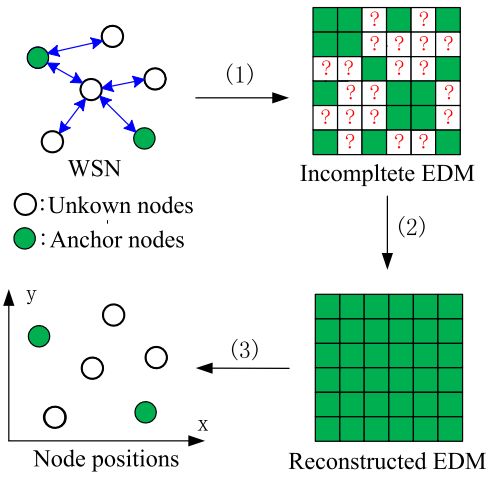

FIGURE 1. The flow chart of the proposed node localization approach.

(1) Acquire an EDM matrix that is in general incomplete and may contain some unknown elements.
(2) Using the low-rank properties of the EDM, a Bayesian-learning-based algorithm is utilized to recover the original EDM from incomplete and noisy distance measurements.
(3) Based on the reconstructed EDM, the MDS and Procrustes analysis technique are applied to determine the absolute positions of all nodes.

B. EDM COMPLETION USING BAYESIAN LEARNING

We can decompose any matrix of rank r by singular value decomposition (SVD) [18]

$$\mathbf{D} = \mathbf{U}\mathbf{S}\mathbf{V}^T = (\mathbf{U}\mathbf{S}^{\frac{1}{2}})(\mathbf{S}^{\frac{1}{2}}\mathbf{V}^T) \tag{9}$$

where $\mathbf{U}$ and $\mathbf{V}$ are $N \times r$ matrices with orthogonal columns and $\mathbf{S}$ is an $r \times r$ diagonal matrix of the non-zero singular values. Thus, $\mathbf{D}$ can be rewritten as

$$\mathbf{D} = \mathbf{A}\mathbf{B}^T \tag{10}$$

where $\mathbf{A}$ and $\mathbf{B}$ are $N \times r$ matrices such that rank$(\mathbf{D}) = r \leq N$. Now, the $(N^2 - \lambda N^2)$ unknown elements in $\mathbf{D}$ need to be filled and reduce to $(2Nr - \lambda N^2)$, where usually $\lambda << 1$; thus, the number of unknown elements to be filled is approximately $2r/N$ of the original's, which will greatly reduce the complexity of the matrix completion. To be equivalent to optimization problem (8), as shown in [25], the matrices $\mathbf{A}$ and $\mathbf{B}$ can then be estimated using

$$\begin{aligned} &\min \ \|\mathbf{A}\|_F^2 + \|\mathbf{B}\|_F^2 \\ &s.t. \ ||\tilde{\mathbf{D}} - \Gamma_{\Omega}(\mathbf{D})||_F^2 \leq \varepsilon \end{aligned} \tag{11}$$

In the following, we solve the problem in (11) using the Bayesian methodology. To obtain low-rank solutions, we employ independent sparsity priors on the individual factors with a common sparsity profile, and we model other elements in the problems using a hierarchical

Bayesian framework. From (10), it is clear that $\mathbf{D}$ is the sum of the outer products of the columns of $\mathbf{A}$ and $\mathbf{B}$, that is,

$$\mathbf{D} = \sum_{i=1}^{k} \boldsymbol{a}_{\cdot i} \boldsymbol{b}_{\cdot i}^T \tag{12}$$

where $k \geq r$ and $\boldsymbol{a}_{\cdot i}$ and $\boldsymbol{a}_{i\cdot}$ are used to denote the ith column and row of $\mathbf{A}$, respectively. Obviously, each outer product increases the rank of $\mathbf{D}$ by at most one. To seek a low-rank estimate of $\mathbf{D}$, a column sparsity in $\mathbf{A}$ and $\mathbf{B}$ should be achieved, which means that most columns in $\mathbf{A}$ and $\mathbf{B}$ are set equal to zero. To enforce this constraint, the columns of $\mathbf{A}$ and $\mathbf{B}$ are associated with Gaussian priors of the precisions (inverse variances) γ_i, that is,

$$p(\mathbf{A}|\boldsymbol{\gamma}) = \prod_{i=1}^{k} \mathbb{N}(a_{\cdot i}|\mathbf{0}, \gamma_i^{-1}\mathbf{I}_N) \tag{13}$$

$$p(\mathbf{B}|\boldsymbol{\gamma}) = \prod_{i=1}^{k} \mathbb{N}(b_{\cdot i}|\mathbf{0}, \gamma_i^{-1}\mathbf{I}_N) \tag{14}$$

where $\mathbf{I}_N$ denotes the $N \times N$ identity matrix. Therefore, the columns of $\mathbf{A}$ and $\mathbf{B}$ have the same sparsity profile enforced by the common precisions γ_i. Because many of the precisions γ_i will assume very large values during inference, the corresponding outer products from $\mathbf{D}$ are effectively removed, which reduces the rank of the estimate.

In addition to (13) and (14), the conjugate Gamma hyperprior is incorporated on the precisions γ_i

$$p(\gamma_i) = \text{Gamma}(a, 1/b) \propto \gamma_i^{a-1} \exp(-b\gamma_i) \tag{15}$$

To obtain broad hyperpriors, the parameters a and b are treated as deterministic constants with small values.

Based on model (5) and the standard assumption, we incorporate white Gaussian noise in the observations $\tilde{\mathbf{D}}$ such that

$$p(\tilde{\mathbf{D}}|\mathbf{A}, \mathbf{B}, \beta) = \prod_{(i,j)\in\Omega} \mathbb{N}(\tilde{D}_{i,j}|D_{i,j}, \beta^{-1}) \tag{16}$$

where $\beta = 1/\varepsilon$ is the noise precision, which is assigned the non-informative Jeffrey's prior [26]

$$p(\beta) = \beta^{-1} \tag{17}$$

Therefore, the joint distribution is

$$p(\tilde{\mathbf{D}}, \mathbf{A}, \mathbf{B}, \boldsymbol{\gamma}, \beta) = p(\tilde{D}|\mathbf{A}, \mathbf{B}, \beta)p(\mathbf{A}|\boldsymbol{\gamma})p(\mathbf{B}|\boldsymbol{\gamma})p(\boldsymbol{\gamma})p(\beta) \tag{18}$$

Because an exact full-Bayesian inference for $\mathbf{A}, \mathbf{B}, \boldsymbol{\gamma}$, and β using joint distributions as (18) is intractable, we resort to the variational Bayesian inference methodology. For each hidden variable, we compute posterior distribution approximations by minimizing the Kullback-Leibler (KL) divergence in an alternating manner [27]. Using the notation $z = (\mathbf{A}, \mathbf{B}, \boldsymbol{\gamma}, \beta)$, which is the vector of all hidden variables, the posterior approximation $q(z_k)$ of each hidden variable $z_k \in z$ is found using

$$\log q(z_k) = \langle \log p(\tilde{\mathbf{D}}, z) \rangle_{z \setminus z_k} + \text{const} \tag{19}$$

where $z \setminus z_k$ denotes the set z with z_k removed, $\langle \cdot \rangle$ denotes the mathematical expectation, and the distribution $p(\tilde{\mathbf{D}}, z)$ is the joint probability distribution given in (18).

With this mean field approximation and the posterior factorization $q(z) = \prod q(z_k)$, we estimate the posterior distribution of each hidden variable by holding the other hidden variables fixed using their most recent distributions [28]. The update rules resulting from this inference scheme are presented as follows.

1) ESTIMATION OF FACTORS A AND B

According to (19), with some algebra, the approximation to the posterior distributions of $\mathbf{A}$ and $\mathbf{B}$ decomposes as independent distributions of their rows. Based on the prior in (13) and the observation model in (16), the posterior density of the ith row $\boldsymbol{a}_{i\cdot}$ of $\mathbf{A}$ is found as

$$q(\boldsymbol{a}_{i\cdot}) = \mathbb{N}(\boldsymbol{a}_{i\cdot}|\langle \boldsymbol{a}_{i\cdot} \rangle, \Sigma_i^a) \tag{20}$$

with mean and covariance

$$\langle \boldsymbol{a}_{i\cdot} \rangle^T = \langle \beta \rangle \Sigma_i^a \langle \mathbf{B}_i \rangle^T \tilde{\boldsymbol{d}}_{i\cdot}^T \tag{21}$$

$$\Sigma_i^a = (\langle \beta \rangle \langle \mathbf{B}_i^T \mathbf{B}_i \rangle + \langle \text{ diag}(\gamma_1, \cdots, \gamma_k) \rangle)^{-1} \tag{22}$$

where $\mathbf{B}_i$ contains only the jth rows of $\mathbf{B}$ for which $(i, j) \in \Omega$ such that

$$\langle \mathbf{B}_i^T \mathbf{B}_i \rangle = \sum_{j:(i,j)\in\Omega} \langle \boldsymbol{b}_{j\cdot}^T \boldsymbol{b}_{j\cdot} \rangle = \sum_{j:(i,j)\in\Omega} (\langle \boldsymbol{b}_{j\cdot}^T \rangle \langle \boldsymbol{b}_{j\cdot} \rangle + \Sigma_j^b) \tag{23}$$

where Σ_j^b is the posterior covariance of the jth row of $\mathbf{B}$. In addition, the row vector $\tilde{\boldsymbol{d}}_{i\cdot}$ denotes the ith row of $\tilde{\mathbf{D}}$. Similarly, based on the prior in (14) and the observation model in (16), the posterior density of $\boldsymbol{b}_{j\cdot}$, the jth row of $\mathbf{B}$, is found as a normal distribution

$$q(\boldsymbol{b}_{j\cdot}) = \mathbb{N}(\boldsymbol{b}_{j\cdot}|\langle \boldsymbol{b}_{j\cdot} \rangle, \Sigma_j^b) \tag{24}$$

with parameters

$$\langle \boldsymbol{b}_{j\cdot} \rangle^T = \langle \beta \rangle \Sigma_j^b \langle \mathbf{A}_j \rangle^T \tilde{\boldsymbol{d}}_{\cdot j} \tag{25}$$

$$\Sigma_j^b = (\langle \beta \rangle \langle \mathbf{A}_j^T \mathbf{A}_j \rangle + \langle \text{diag}(\gamma_1, \cdots, \gamma_k) \rangle)^{-1} \tag{26}$$

where $\mathbf{A}_j$ contains the ith rows of $\mathbf{A}$, for which $(i, j) \in \Omega$, and the vector $\tilde{\boldsymbol{d}}_{\cdot j}$ denotes the jth column of $\tilde{\mathbf{D}}$.

2) ESTIMATION OF HYPERPARAMETERS $\boldsymbol{\gamma}$

By combining (13), (14) and (15), the posterior density of γ_i becomes a Gamma distribution

$$q(\gamma_i) = \gamma_i^{(a-1+N)} \exp(-\gamma_i \frac{2b + \langle \boldsymbol{a}_{\cdot i}^T \boldsymbol{a}_{\cdot i} \rangle + \langle \boldsymbol{b}_{\cdot i}^T \boldsymbol{b}_{\cdot i} \rangle}{2}) \tag{27}$$

with mean

$$\langle \gamma_i \rangle = \frac{2(a+N)}{2b + \langle \boldsymbol{a}_{\cdot i}^T \boldsymbol{a}_{\cdot i} \rangle + \langle \boldsymbol{b}_{\cdot i}^T \boldsymbol{b}_{\cdot i} \rangle}) \tag{28}$$

The required expectations are given by

$$\langle \boldsymbol{a}_{\cdot i}^T \boldsymbol{a}_{\cdot i} \rangle = \langle \boldsymbol{a}_{\cdot i} \rangle^T \langle \boldsymbol{a}_{\cdot i} \rangle + \sum_j (\Sigma_j^a)_{ii} \tag{29}$$

$$\langle \boldsymbol{b}_{\cdot i}^T \boldsymbol{b}_{\cdot i} \rangle = \langle \boldsymbol{b}_{\cdot i} \rangle^T \langle \boldsymbol{b}_{\cdot i} \rangle + \sum_j (\Sigma_j^b)_{ii} \tag{30}$$

3) ESTIMATION OF NOISE PRECISION β

We can also use the Bayesian methodology to estimate the noise precision. From (19), the posterior approximation is assumed to be a Gamma distribution with mean

$$\langle \beta \rangle = \frac{\lambda N^2}{\langle \left\| \tilde{\mathbf{D}} - \Gamma_\Omega(\mathbf{A}\mathbf{B}^T) \right\|_F^2 \rangle} \tag{31}$$

In summary, the EDM completion algorithm consists of the successive update the estimation of the rows of $\mathbf{A}$ and $\mathbf{B}$ using (21) and (25), respectively, followed by the estimation of the precisions γ_i using (28) and the noise precision β using (31).

C. LOCATING THE NODES BY MDS

With the reconstructed EDM above, all the pairwise distances in the network are available. According to the classical MDS method, the reconstructed $\hat{\mathbf{D}}$ is first double centered as

$$\mathbf{W} = -\frac{1}{2}\mathbf{J}\hat{\mathbf{D}}\mathbf{J} \tag{32}$$

where $\mathbf{J} = \mathbf{I} - \frac{1}{N}\mathbf{1} \times \mathbf{1}^T$, $\mathbf{I}$ is the $N \times N$ identity matrix, and $\mathbf{1}$ is a column vector of all ones. Here, $\mathbf{W}$ is symmetric and positive semidefinite; given the SVD of $\mathbf{W}$ as $\mathbf{W} = \mathbf{L}\Lambda\mathbf{L}^T$, we obtain the relative positions of each node as follows:

$$\mathbf{R} \triangleq \mathbf{L}_d \Lambda_d^{1/2} \tag{33}$$

where $\mathbf{R} = [\boldsymbol{r}_1, \boldsymbol{r}_2, \cdots, \boldsymbol{r}_N]$, $\mathbf{L}_d$ denotes the $N \times d$ left singular matrix corresponding to the d largest singular values, and Λ_d denotes the $d \times d$ diagonal matrix, with the d largest singular values in the diagonal.

Next, position alignment is conducted based on the anchor nodes. Clearly, $\boldsymbol{r}_i$ can be mapped to $\boldsymbol{p}_i$ by scaling, rotating and shifting the relative positions, also called Procrustes analysis techniques [29]. Because nodes $\{\boldsymbol{u}_j\}_{j=1}^M$ $(M \geq d+1)$ are anchor nodes, we have

$$\mathbf{Q} = \frac{[\boldsymbol{u}_2 - \boldsymbol{u}_1, \boldsymbol{u}_3 - \boldsymbol{u}_1, \cdots, \boldsymbol{u}_M - \boldsymbol{u}_1]}{[\boldsymbol{r}_2 - \boldsymbol{r}_1, \boldsymbol{r}_3 - \boldsymbol{r}_1, \cdots, \boldsymbol{r}_M - \boldsymbol{r}_1]} \tag{34}$$

where $\mathbf{Q}$ is the coordinate-transform matrix. Applying the transformation to all the unknown nodes, we can easily obtain their absolute coordinates as

$$\boldsymbol{x}_i = \mathbf{Q}(\boldsymbol{r}_i - \boldsymbol{r}_1) + \boldsymbol{u}_1, \quad i = M+1, M+2, \cdots, N \tag{35}$$

IV. PERFORMANCE EVOLUTION

In this section, we demonstrate the effectiveness and properties of our proposed localization approach by simulation using Matlab2015b. In our experiment, we generate the WSN shown as Fig. 2, with 100 sensor nodes, whose positions are randomly drawn from a uniform distribution on [0, 1], among which 4 anchor nodes are placed at the four corners for position alignment. We assume that the communication range between the nodes is limited; thus, the EDM obtained in FC might be incomplete. In addition, the EDM's entries might be corrupted by additive zero-mean Gaussian noise with variance σ^2. For simplicity, the incomplete EDM $\tilde{\mathbf{D}}$ is constructed by randomly choosing the entries from $\mathbf{D}$ with

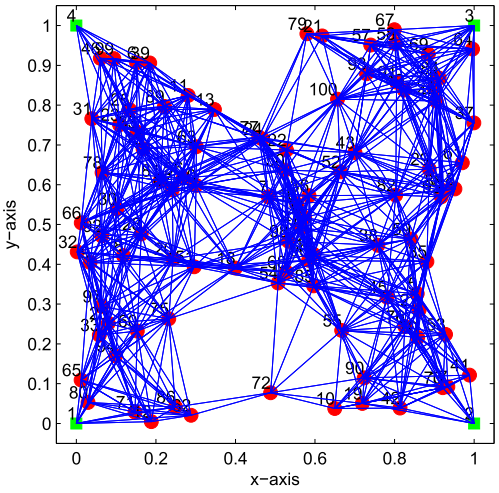

FIGURE 2. Geometry of a wireless sensor network for numerical evaluation.

the sparsity level λ. To investigate the completion accuracy of the reconstructed EDM, the average completion error (ACE) is defined as

$$\text{ACE} = E\{||\mathbf{D} - \hat{\mathbf{D}}||_F / ||\mathbf{D}||_F\} \tag{36}$$

where $E\{\cdot\}$ is the expectation operator and $\hat{\mathbf{D}}$ represents the reconstructed EDM. To compare the localization performance of the proposed algorithm, we use the average localization error (ALE) to calculate the localization accuracy, which is evaluated by

$$\text{ALE} = E\{\frac{1}{N}||\mathbf{P} - \hat{\mathbf{P}}||_F\} \tag{37}$$

where $\mathbf{P}$ is the actual position of the sensor nodes and $\hat{\mathbf{P}}$ is their estimated global position. All the simulations are repeated 100 times to obtain faithful results.

Fig. 3 shows the ACE of the reconstructed EDM based on the proposed Bayesian learning methodology with respect to

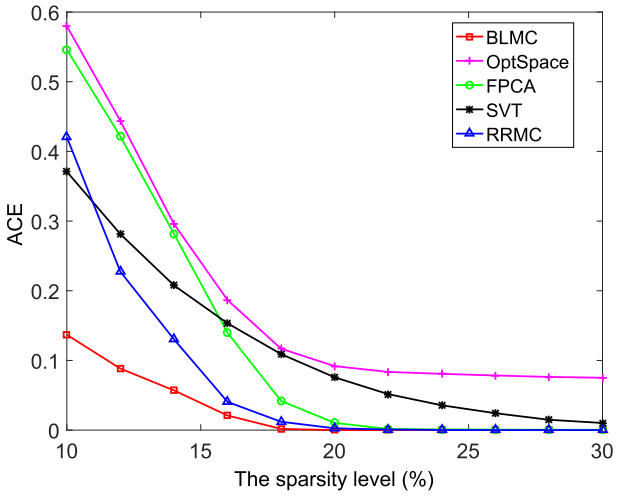

FIGURE 3. ACE versus the sparsity level.

the sparsity level λ (the proportion of entries that are known in $\mathbf{D}$) without noise. We also compare the proposed scheme with traditional FPCA [15], OptSpace [16], SVT [18], and robust rank-k matrix completion (RRMC) [30]. It is demonstrated that all the ACE decline with increasing λ. For FPCA, RRMC and our proposed scheme, only 20% of entries of the EDM are necessary to achieve an accurate reconstruction. Under the same number of measurements (fewer than 20%), the ACE of the proposed scheme is the smallest, therein outperforming the other completion methods.

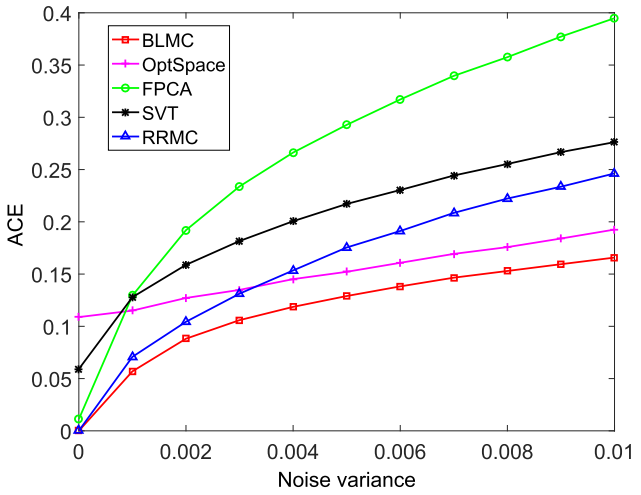

FIGURE 4. ACE versus noise variance.

When the known entries are corrupted by noise, we plot the ACE curves of different algorithms versus the noise variance, where $\lambda = 0.2$. As shown in Fig. 4, all the ACE increase with increasing noise variance, whereas the FPCA achieves the worst completion performance with $\sigma^2 > 0.001$. Compared with OptSpace, the completion accuracy has been improved by at least 5% using our proposed scheme. In conclusion, our proposed Bayesian learning scheme is an accurate and efficient completion method, thereby supporting the arguments in Section III.

Fig. 5 shows the ACE curves of the proposed completion scheme under different network sizes, where $\sigma^2 = 0.002$. It is demonstrated that the proposed Bayesian learning scheme achieves a superior performance in terms of both ACE and the reduced number of entries, especially when the scale of the network is large. For example, an ACE of 10% is obtained for small-scale networks with $N = 100$ when the proportion of entries reaches 20%, while only 8% is needed for a slightly larger network with $N = 250$. This means that fewer communication operations are necessary between sensors, and the energy consumption will be reduced for the whole network.

Based on the Bayesian learning EDM completion approach, we plot the overall performance of our proposed localization approach in terms of the ALE with respect to the communication range r under different noise variances in Fig. 6, where $N = 100$. It is shown that the ALE of

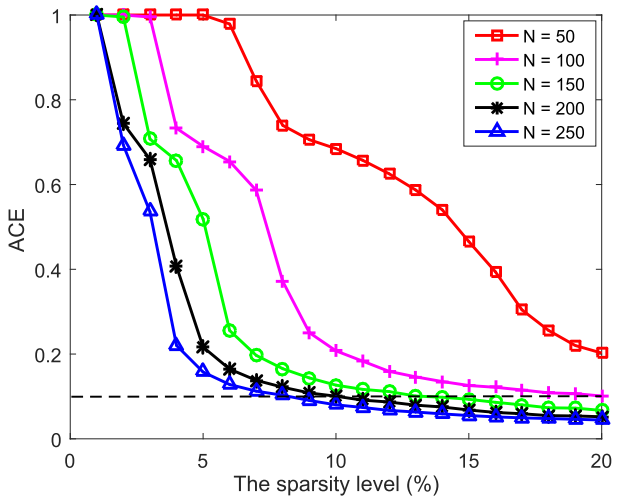

FIGURE 5. ACE versus the sparsity level for different network sizes.

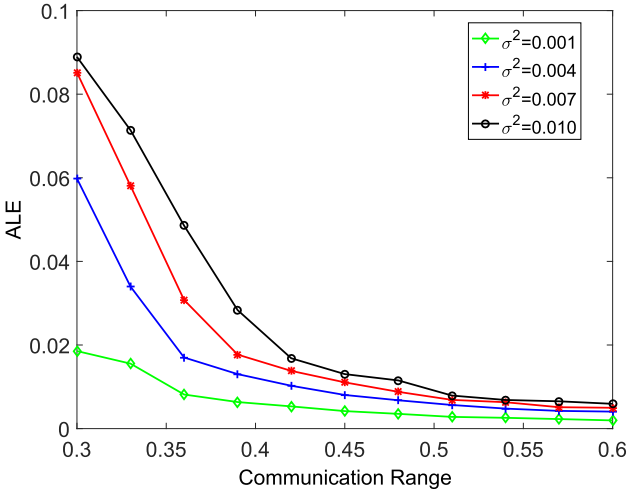

FIGURE 6. ALE versus communication range for different noise variances.

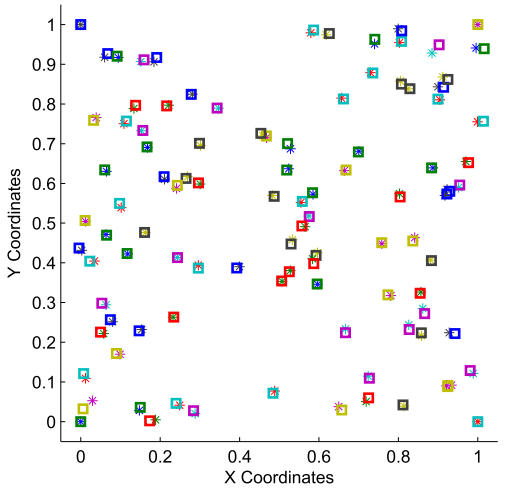

FIGURE 7. An intuitive node localization result.

our proposed localization approach decreases sharply with increasing communication range. This is because for large r, more entries in $\mathbf{D}$ are known, and the EDM completion

becomes more accurate. The ALE also declines with decreasing noise variance; an ALE of 0.002 is obtained for $\sigma^2 = 0.001$. In addition, the ALE presents no obvious change when $r > 0.5$.

Fig. 7 shows an intuitive node localization result of the WSN shown as Fig. 2 provided by the proposed localization approach. The communication range is set to 0.5, and $\sigma^2 = 0.001$. The asterisks are the actual positions of these 100 nodes, while squares represent the estimated locations. As illustrated in the figure, most nodes are accurately localized, therein demonstrating the effectiveness of our proposed location estimation scheme.

V. CONCLUSION

This paper proposed a novel node localization algorithm for WSNs based on EDM completion. Using the unique low-rank feature of the EDM, a new Bayesian-learning-based method is first used to recover the original EDM from incomplete and noisy range measurements. Then, the relative positions of the nodes are obtained by applying the classical MDS method. Next, based on the anchor node location knowledge, the translation, rotation and reflection of the relative positions are used to determine the absolute positions of the unknown nodes. The simulation results show that, compared with other algorithms, our proposed algorithm improves the localization accuracy under the same number of range measurements, and the network energy consumption can be reduced, especially for large-scale WSNs.

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