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Linear Quadratic Predictive Fault-Tolerant Control for Multi-Phase Batch Processes

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ABSTRACT This paper proposes a linear quadratic predictive fault-tolerant control (LQPFTC) scheme for multi-phase batch processes with input time-delay and actuator faults. First of all, according to a given model with input time-delay, a new variable is introduced and the given model is transformed into an extended state-space model without time delay. And then, a new 2D switched system model based on an equivalent 2D-Roesser model is constructed by introducing the state error and the output tracking error to solve the actuator fault and realize the optimal control performance. By adjusting the variable in the function, a quadratic performance function based on the model is designed and a linear predictive fault-tolerant controller by combining with the principle of predictive control is proposed. Then, using the Lyapunov function and the average dwell time method, the sufficient conditions for the robust exponential stability of the system along the time and batch direction, and the minimum running time of each phase are derived. Finally, taking the injection molding process as an example, different fault values are selected for simulation. The results show that the 2D controller designed can realize tracking control with different actuator fault values and even a serious one.

INDEX TERMS Model predictive fault-tolerant control, batch process, input time-delay, equivalent 2D-Roesser model, actuator faults.

I. INTRODUCTION

With the small-scale, multi-variety and high value-added production characteristics, the batch process has become the mainstay in modern industrial production [1]. In recent years, research on optimization and advanced control of batch process has also appeared as a new research area [2]–[4].

However, as the complexity of the production process and operating process increases, the probability of fault also does so. In the actual production process, the faults are roughly divided into actuator faults, sensor faults and internal system faults. Since there are friction, saturation, dead zone, etc. everywhere in actual production, the actuator fault is the most common, so its research is more valuable. If an actuator fault cannot be detected and corrected effectively, system control performance will be degraded and even cause serious personnel safety issues. In addition, due to high quality, high precision production requirements, the system control performance requirements will also increase. It is important to find useful approaches to cope with this problem.

In this context, the research on fault-tolerant control (FTC) has attracted people's attention [5]–[11]. The main goal of FTC is that the closed-loop system can still guarantee good control performance under faults. For example, considering the problem of control system/actuator failures in nonlinear processes subject to input constraints, [5] presents two approaches for fault-tolerant control that focus on incorporating performance and robustness considerations, respectively. For FTC of a six-phase permanent-magnet synchronous motor drive system with open phases, an intelligent complementary sliding-mode control was presented in [6]. A novel adaptive fault tolerant controller was studied for nonlinear unknown systems that have multiple actuators [7]. At the same time, some fault diagnosis and control strategies have also been put forward [8]–[11].

As mentioned above, actuator faults especially partial faults are very common in industrial systems. Subject to the complexity of the production process itself, and the immature support technology, the research results related to FTC of batch processes did not occur until 2006 [12]. In [12], to handle the system under actuators, a particular 2D iterative learning reliability controller was proposed. With the wide

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application of iterative learning control [13]–[16] (ILC) and the emergence of fault diagnosis methods for various batch processes, the control problem gradually received attention and successively appeared a series of research results. For the single-phase batch processes with partial actuator faults, unknown disturbances and/or time-delay, the FTC is designed through robust ILC or guaranteed cost performance control using linear matrix inequality (LMI) [17], [18]. The results are also extended into multi-phase batch process [19], [20]. In [19], the FTC problem is transformed into an equivalent switching system based on the 2D equivalent model, and sufficient conditions are given by the average dwell time method, which ensures that the system remains robust along time and batch direction, and gives the minimum run time for each phase.

Most of these results are for the batch process to study the reliable fault-tolerant control (RFTC) in the case of actuator faults. For the model, linear and nonlinear systems have also been studied, and it has been extended to the FTC of the multi-phase batch processes. However, RFTC only allows faults to vary within a certain range. Once the fault is out of range, the controller will not work more effectively and the system control performance will degrade, thus affecting product quality. It is necessary to seek new control methods.

Recently, Model predictive control (MPC) has attracted much attention because it can improve the system performance and be easy to deal with constraints, disturbances [21], [22]. In today's increasingly complex industrial environment, MPC is widely used in industrial control with its unique charm, and there have been many researches on predictive control of batch processes. In order to solve the online computing burden of integrated scheduling and control for batch processes, multi-parameter model predictive control (mp-MPC) can be included in the integration of scheduling and control. Also, the iterative learning model predictive control (ILMPC) has the advantages of ILC and MPC, and has good anti- disturbance ability and constraint processing ability. Nonlinear modeling and identification based Model Predictive Control (NMPC) method can addresses constraints and nonlinearities during the feedback control [23], [24]. In order to solve the multi-phase characteristics of the batch process and reduce the computational burden, the online ILMPC law is proposed and the quadratic programming problem online is solved [25].

The above studies have one thing in common, that is, they are all conducted under normal systems. As the above description shows, actuator fault is inevitable. Predictive control is used for FTC by virtue of its own advantages. Currently, most of predictive fault-tolerant control (PFTC) treat actuator faults as mismatched disturbances and treat batch processing as a one-dimensional (1D) system. For example, for the single-phase batch process with partial actuator faults or unknown disturbances, Zhang *et al.* [26], [27] proposed new linear quadratic control (LQ) and MPC that can be designed based on state space model/minimum optimization respectively. Alternatively, robust model predictive control can be designed for fault handling [28]. Zhang et al. proposed a FTC strategy based on state space model [29] and a predictive function control method [30] and even combine both them [31]. However, 1D predictive control only allows the system to achieve its optimal control performance on the timeline, and performance is not improved in the batch direction. Recently, research on 2D system models-based model predictive control has been emerged [32]–[35]. This method can significantly change the control performance of the batch direction. Particularly, for batch processing with actuator fault and uncertain disturbance, Shi *et al.* [35] proposed robust iterative learning fault-tolerant control (ILFTC) that is studied under the framework of predictive control.

However, the above method research on predictive control does not consider the multi-phase characteristics of the batch process, so it cannot guarantee that the system performance is always optimal under the actuator fault, because the control performance of the previous phase will affect the next phase, thereby affecting the product quality. Compared with single-phase control studies, the results of multi-phase processes are relatively small [36]–[38], especially in the case of system faults [39]–[41]. Further analysis of multi-phase batch processes obviously remains to be explored.

To solve the problems, the paper proposes a linear quadratic predictive fault-tolerant control method for multi-phase batch process based on 2D model theory specific for actuator fault and batch process' repeatability, 2D characteristic and multi-phase characteristic. The method effectively solves the batch process control problem with actuator fault and different phase switching time. The advantages are: (1) the proposed method can update the control law in time, can solve the system deviation problem caused by the disturbance and the serious fault, ensure the optimal control performance of system and achieve high quality production; (2) the controller designed for actuator fault has certain robustness; (3) the method designs the switching signal meeting the system's stability requirement according to the multi-phase characteristic to ensure the control system's robustness and get the minimum running time of system, thus realizes the high-efficiency production. Finally, the paper demonstrates the feasibility and effectiveness of method proposed by modeling and simulating the injection molding process. Different fault values are selected and the simulating results show that the designed control law can still guarantee certain control effects under different actuator fault values.

II. PROBLEM FORMULATION

Considering the multi-phase characteristic, the batch process system model in the i^{th} phase with input delay is described as follows:

$$\begin{aligned} x^i(t+1,k) &= \bar{A}^i x^i(t,k) + \bar{B}^i u^i(t-d,k) \\ y^i(t,k) &= \bar{C}^i x^i(t,k) \end{aligned} \tag{1}$$

where t is the time at the present moment with $0 < t \leq T_i^{'}$, k is the batch, $T_i^{'}$ is the end time point of processing interval in each phase for each batch, $x^i(t,k) \in R^n$, $y^i(t,k) \in R^l$, $u^i(t,k) \in R^m$ are the state, the output and the input variable of batch k in phase i at moment t respectively, d is the time delay of batch process and $\{\bar{A}^i, \bar{B}^i, \bar{C}^i\}$ is the system matrix with appropriate dimensions.

In the industrial production, the system will have faults due to the long-term and repetitive operation of equipment. In the case of system fault, it's hard for the system's input $u^i(t,k)$ to reach the expected value after passing through the actuator, thus affecting the quality of products. The actuator fault is divided into the stuck fault, the partial fault and the complete fault. Therefore, we use α to represent different types of actuator fault. $\alpha > 0$ means the partial fault and $\alpha = 0$ means the complete fault. The paper only studies the partial actuator fault.

The batch process is multi-phase one, so the actuator fault is supposed to be α^i, the input signal through the actuator is $u^{iF}(t,k)$. The system model with fault is expressed in the following form:

$$u^{iF}(t,k) = \alpha^i u^i(t,k) \quad (i = 1, 2, \cdots, n) \tag{2}$$

where $0 < \underline{\alpha} \le \alpha^i \le \bar{\alpha}$, $\underline{\alpha}^i(\underline{\alpha}^i < 1)$ and $\bar{\alpha}^i(\bar{\alpha}^i \ge 1)$ are known variables. Therefore, the system model with actuator faults can be expressed as follows:

$$\begin{aligned} x^i(t+1,k) &= \bar{A}^i x^i(t,k) + \bar{B}^i u^{iF}(t-d,k) \\ y^i(t,k) &= \bar{C}^i x^i(t,k) \end{aligned} \tag{3}$$

Objective: Propose a linear quadratic predictive fault-tolerant control method for multi-phase batch processes with actuator faults to ensure the system's stable running and good control performance in the case of failures.

Conventional Control Method: The conventional method makes the design based on 1D system model (i.e., combining the state error and the output error in the direction of time alone), and then designs the controller using the conventional infinite-time-domain linear quadratic control.

New Control Method: The method proposed makes the design based on 2D system model (i.e., combining the state error and the output error in directions of time and batch), introduces a new state-space variable to transform the model into an equivalent 2D system model without time delay, and then designs a controller using the infinite-time-domain quadratic predictive fault-tolerant control to realize the minimum performance index with the minimum control input.

To realize the objective above, the paper first introduces a new state-space variable to transform model (3) into an equivalent state-space model without time delay first and then an equivalent 2D model. Then, basing on the model, the paper designs a control law and a switching law of the objective and analyzes the system's stability, and finally verifies the feasibility and effectiveness of control method proposed through a simulation. The specific steps are as follows.

Remark 1: The method proposed in this paper is not a reliable control, but a more advanced control method. Reliable control can only solve actuator failures within a certain range, and it adopts the same controller throughout the whole process, cannot solve the system deviation problem. However, the method proposed in this paper can solve the actuator fault in more serious cases and solves the system deviation problem, because the controller can be adjusted in real time.

Remark 2: In the actual industrial production process, the batch process has nonlinear characteristics. In this paper, the linear model is used for preliminary research. The nonlinear problem can be regarded as the problem of disturbance under the linear model. Here the control law can resist disturbance as shown in [25], [26], and [42]. For the nonlinear characteristics, it will be studied in the follow-up work.

A. BUILD A NEW STATE-SPACE MODEL

Two ways are available to solve the time delay problem. The first one is to introduce a new state and transforming the system with time delay into the system without time delay by extending dimensions. This method has the simple model and the easy design of controller. The other is to analyze the system control using the time-delay system control theory, which is dependent of the size of time delay. This method is less conservative but the design of controller is complicated. The paper adopts the first method, i.e. transform the system with input time delay into the state-space model without time delay.

For the new state-space variable $x_m^i(t,k)^T$ without time delay, we have the following form:

$$x_m^i(t,k)^T = \left[\, x^i(t,k)^T \;\; u^i(t-1,k)^T \;\; \cdots \;\; u^i(t-d,k)^T \,\right] \tag{4}$$

Transforming equation (4), we get a new state-space model without time delay in the i^{th} phase, and its form is:

$$\begin{aligned} x_m^i(t+1,k) &= A_f^i x_m^i(t,k) + B_f^i u^i(t,k) \\ y^i(t,k) &= C_f^i x_m^i(t,k) \end{aligned} \tag{5}$$

where

$$A_f^i = \begin{bmatrix} \bar{A}^i & \bar{\mathbf{0}} & \bar{\mathbf{0}} & \cdots & \bar{\mathbf{0}} & \bar{B}^i \alpha^i \\ \underline{\mathbf{0}} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \underline{\mathbf{0}} & \alpha^i & \mathbf{0} & \cdots & \vdots & \mathbf{0} \\ \underline{\mathbf{0}} & \mathbf{0} & \alpha^i & \mathbf{0} & \mathbf{0} & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \underline{\mathbf{0}} & \mathbf{0} & \cdots & \mathbf{0} & \alpha^i & \mathbf{0} \end{bmatrix}$$

$$B_f^i = \left[\, 0 \quad \alpha^i \quad \mathbf{0} \quad \cdots \quad \mathbf{0} \,\right]^{iT},$$

$$C_f^i = \left[\, \bar{C}^i \quad \mathbf{0} \quad \mathbf{0} \quad \cdots \quad \mathbf{0} \,\right],$$

T is the transpose symbol of matrix, and $\bar{\mathbf{0}}$ and $\underline{\mathbf{0}}$ are both the null vectors with appropriate dimensions.

To keep the system stable running in the case of system fault, we design a fault-tolerant controller including the output error $e^i(t,k) = y_r^i(t) - y^i(t,k)$ to make the output track the expected output $y_r^i(t)$ as much as possible and preserve the optimal control even under actuator faults.

Remark 3: In order to improve the shortcomings of reliable control, and based on (5), new MPC is proposed where performance can be improved by adjusting the performance index. Here the stability and shortest running time using 2D theory are derived.

B. EQUIVALENT 2D MODEL

According to actual needs, we extend the dimensions of equation (5) by introducing the system state error and the output tracking error and transform the model into an equivalent 2D model.

First, we introduce the following ILC law:

$$\sum_{ilc} : u^i(t,k) = u^i(t,k-1) + r^i(t,k), \quad u^i(t,0) = 0,$$
$$t = 0, 1, 2, \cdots, T_i^{'}, \quad i = 1 \tag{6}$$

in which $u^i(t,0)$ is the initial value of iteration and its value is generally set as 0, and $r^i(t,k) \in R^m$ is the update law of ILC. The design of ILC law aims to determine $r^i(t,k)$, the update law at moment t of the k^{th} batch with system fault to make system output $y^i(t,k)$ track the expected output $y_r^i(t)$ given.

We define the system state error as

$$f^i(t,k) = f_k(t), \delta(f^i(t,k)) = f^i(t,k) - f^i(t,k-1) \tag{7}$$

From model (5) and ILC law (6), we get

$$\delta(x_m^i(t+1,k)) = A_f^i \delta(x_m^i(t,k)) + B_f^i r^i(t,k) \tag{8}$$

We define the output error $e^i(t,k) = y_r^i(t) - y^i(t,k)$, and then get

$$e^i(t+1,k) = e^i(t+1,k-1) - C_f^i A_f^i \delta(x_m^i(t,k)) - C_f^i B_f^i r^i(t,k) \tag{9}$$

Without considering external disturbance, the equivalent 2D-Roesser model of (8) and (9) can be written in the following form:

$$\begin{bmatrix} \delta(x_m^i(t+1,k)) \\ e^i(t+1,k) \end{bmatrix} = \bar{A}_f^i \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i(t+1,k-1) \end{bmatrix} + \overline{B}_f^i r^i(t,k) \tag{10}$$

where

$$\bar{A}_f^i = \begin{bmatrix} A_f^i & \mathbf{0} \\ -C_f^i A_f^i & I^i \end{bmatrix}, \quad \bar{B}_f^i = \begin{bmatrix} B_f^i \\ -C_f^i B_f^i \end{bmatrix}.$$

And then, the model above can be transformed equivalently into

$$z'^i(t,k) = \overline{A}_f^i z^i(t,k) + \overline{B}_f^i r^i(t,k) \tag{11}$$

in which

$$z'^i(t,k) = \begin{bmatrix} \delta(x_m^i(t+1,k)) \\ e^i(t+1,k) \end{bmatrix} = \begin{pmatrix} z_h^i(t+1,k) \\ z_v^i(t,k+1) \end{pmatrix},$$
$$z^i(t,k) = \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i(t+1,k-1) \end{bmatrix} = \begin{pmatrix} z_h^i(t,k) \\ z_v^i(t,k) \end{pmatrix}.$$

We reproduce the system above into the following switching system model:

$$z'(t,k) = \bar{A}_f^{\sigma(t,k)} z(t,k) + \bar{B}_f^{\sigma(t,k)} r(t,k) \tag{12}$$

in which $\sigma(t,k) : Z^+ \to N := \{1, 2, \cdots, n\}$ represents the switching signal may be correlated to time or system sate, and N is the number of phases of subsystem. $\bar{A}_m^{\vartheta(t,k)}$, $\bar{B}_m^{\vartheta(t,k)}$ are both represented with the switching system model (12) for different phases. The switching sequence is defined as

$$\begin{aligned} \Sigma = \{ & \left[(T_0^{'1}, k_0), \sigma(T_0^{'1}, k_0)\right], \left[(T_0^{'2}, k_0), \sigma(T_0^{'2}, k_0)\right], \cdots \\ & \left[(T_0^{'n}, k_0), \sigma(T_0^{'n}, k_0)\right], \left[(T_1^{'1}, k_1), \sigma(T_1^{'1}, k_1)\right], \\ & \left[(T_1^{'n}, k_1), \sigma(T_1^{'n}, k_1)\right], \cdots, \left[(T_k^{'1}, k_k), \sigma(T_k^{'1}, k_k)\right], \cdots, \\ & \left[(T_k^{'n}, k_k), \sigma(T_k^{'n}, k_k)\right] \cdots \} \end{aligned}$$

in which $\left[(T_i^{'n}, k_i), \sigma(T_i^{'n}, k_i)\right]$ is the connection point between the end of the previous batch and the start of the next batch.

C. DESIGN OF CONTROL LAW

We design the controller for the new 2D model. The design should meet the requirements of stability of closed-loop 2D system and the optimal control performance (13).

The following is the performance index:

$$\begin{aligned} \min_{r^i(t,k)} \Omega_i \hat{=} & [\sum_{j=1}^{P} z'^{iT}(t+j|t,k) Q_i z'^i(t+j|t,k) \\ & + \sum_{j=1}^{M} r^{iT}(t+j-1|t,k) R_i r^i(t+j-1|t,k)] \\ Q_i = & \, diag\left\{q_{j1}^i, q_{j2}^i, \ldots, q_{jn}^i, q_{ju1}^i, q_{ju2}^i, \ldots, q_{jud}^i, q_{je}^i\right\} \\ & 1 \le j \le P \end{aligned} \tag{13}$$

where $Q_i > 0$ is the weighting matrix of process, $R_i \ge 0$ is the input weighting matrix of process state, $q_{j1}^i, q_{j2}^i, \ldots, q_{jn}^i, q_{ju1}^i, q_{ju2}^i, \ldots, q_{jud}^i, q_{je}^i$ is the weighting coefficient of process state and q_{je}^i is the weighting coefficient of output tracking error and $q_{je}^i = 1$. Besides, $P \ge M$.

Theorem 1: To meet the requirements of optimal control performance with system fault, the linear quadratic fault-tolerance control law is designed as

$$r^i = -(\Psi_i^T \overline{Q}_i \Psi_i + \overline{R}_i)^{-1} \Psi_i^T \overline{Q}_i E^i z^i \left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix} |t,k \right) \tag{14}$$

$$\begin{aligned} K_i &= (\Psi_i^T \overline{Q}_i \Psi_i + \overline{R}_i)^{-1} \Psi_i^T \overline{Q}_i E^i \\ u^i(t,k) &= u^i(t,k-1) + r^i(t,k) \end{aligned} \tag{15}$$

Proof: According to the equivalent 2D model, the output model within predictive range is set to

$$\begin{bmatrix} \delta\left(x_m^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix} |t,k\right)\right) \\ e^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix} |t,k\right) \end{bmatrix} = E^i \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix} |t,k-1\right) \end{bmatrix} + \Psi_i r^i \left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix} |t,k \right) \tag{16}$$

in which

$$\Psi_i \triangleq \begin{bmatrix} (\bar{B}_f^i)^0 & \mathbf{0} & \cdots & \mathbf{0} \\ (\bar{B}_f^i)^1 & (\bar{B}_f^i)^0 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ (\bar{B}_f^i)^{M-1} & (\bar{B}_f^i)^{M-2} & \cdots & (\bar{B}_f^i)^0 \\ \vdots & \vdots & \ddots & \vdots \\ (\bar{B}_f^i)^{P-1} & (\bar{B}_f^i)^{P-2} & \cdots & \sum_{j=0}^{P-M} (\bar{B}_f^i)^j \end{bmatrix}_{(P\times M)},$$

$$E^i \triangleq \begin{bmatrix} \overline{A}_f^i \\ (\overline{A}_f^i)^2 \\ \vdots \\ (\overline{A}_f^i)^P \end{bmatrix}_{(P\times 1)}, \quad (\overline{A}_f^i)^s = \begin{bmatrix} (A_f^i)^s & \mathbf{0} \\ -C_f^i(A_f^i)^s & I^i \end{bmatrix},$$

$$\left(\bar{B}_f^i\right)^s = \begin{bmatrix} (A_f^i)^s B_f^i \\ -C_f^i(A_f^i)^s B_f^i \end{bmatrix},$$

$$\sum_{s=0}^{P-M} (\bar{B}_f^i)^s = \begin{bmatrix} \sum_{s=0}^{P-M} (A_f^i)^s B_f^i \\ -C^i \sum_{s=0}^{P-M} (A_f^i)^s B_f^i \end{bmatrix},$$

$$\delta\left(x_m^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \triangleq \begin{bmatrix} \delta\left(x_m^i(t+1|t,k)\right) \\ \vdots \\ \delta\left(x_m^i(t+P|t,k)\right) \end{bmatrix},$$

$$e^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) = \begin{bmatrix} e^i(t+1|t,k) \\ \vdots \\ e^i(t+P|t,k) \end{bmatrix},$$

$$r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right) \triangleq \begin{bmatrix} r^i(t|t,k) \\ \vdots \\ r^i(t+M-1|t,k) \end{bmatrix}_{(M\times 1)},$$

$$z'^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) = \begin{bmatrix} \delta\left(x_m^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \\ e^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \delta\left(x_m^i(t+1|t,k)\right) \\ e^i(t+1|t,k) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \delta\left(x_m^i(t+P|t,k)\right) \\ e^i(t+P|t,k) \end{bmatrix} \end{bmatrix}$$

and

$$z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) = \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k-1\right) \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i(t+1|t,k-1) \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \delta(x_m^i(t,k)) \\ e^i(t+P|t,k-1) \end{bmatrix} \end{bmatrix}$$

Then, the above output model is equivalent to

$$z'^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) = E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) + \Psi_i r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)$$

From the performance index equation (13) and (16), it has

$$\begin{aligned} \Omega_i = &\left(E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) + \Psi_i r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)\right)^T \overline{Q}_i \\ &\times \left(\begin{matrix} E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) + \\ \Psi_i r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right) \end{matrix}\right) \\ &+ r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)^T \overline{R}_i r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right) \\ = & r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)^T \left[\Psi_i^T \overline{Q}_i \Psi_i + \overline{R}_i\right] \\ &\times, r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right) \\ &+ 2\left(E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right)^T \overline{Q}_i \Psi_i r^i \\ &\times \left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right) \\ &+ \left(E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right)^T \overline{Q}_i \left(E^i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \end{aligned} \tag{17}$$

where

$$\overline{Q}_i \triangleq \begin{bmatrix} Q_i & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & Q_i & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & Q_i \end{bmatrix}_{(P\times P)},$$

$$\overline{R}_i \triangleq \begin{bmatrix} R_i & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & R_i & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & R_i \end{bmatrix}_{(M\times M)}.$$

From the equation above, let $\frac{\partial \Omega_i}{\partial r^i} = 0$ and then we get

$$r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)^* = -[\Psi_i^T\overline{Q}_i\Psi_i + \overline{R}_i]^{-1}\Psi_i^T\overline{Q}_iE^i \times z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \hat{=} -K_iz^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \quad (18)$$

Therefore, Theorem 1 is proved.

To get the new input variable $u^i(t,k)$, we first make input variable $u^i(t,k-1)$ act as the controlled object and then evaluate $u^i(t,k)$ with the cycle solving method according to the update law $r^i(t,k)$ obtained.

D. DESIGN OF SWITCHING LAW

For different phases, the switching signal can be designed as $\sigma(t,k)$ and, for switched system (12), we suppose

$$r^i\left(\begin{bmatrix} t \\ t+M-1 \end{bmatrix}|t,k\right)^* \hat{=} -K_iz^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \quad (19)$$

where $K_i = (\Psi_i^T\overline{Q}_i\Psi_i + \overline{R}_i)^{-1}\Psi_i^T\overline{Q}_iE^i$.

Then, we get the switched system in each phase of i:

$$z'^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) = (E^i - \Psi_iK_i)z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \quad (20)$$

To determine the stability of switching system, for the i^{th} subsystem, we use the following Lyapunov function:

$$V^i\left(z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) = z^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \times P^iz^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \quad (21)$$

in which $P^i(t,k)$, $i \in \underline{N}, \underline{N} := \{1,2,\cdots,N\}$ is the matrix dependent of residence time τ^i. Then, we have the following theorem:

Theorem 2: For the given $0 < \beta_1^i < 1, 0 < \beta_2^i < 1$ and convergence index θ^i (the horizontal convergence index is no bigger than β_1^i and the vertical convergence index is no bigger than β_2^i), if there is a diagonal matrix $W^i = diag\{W_h^i, W_v^i\} > 0$ in which $W_h^i \in R^{n_1\times n_1}, W_v^i \in R^{n_2\times n_2}$, satisfying the following matrix inequalities

$$\begin{bmatrix} -W^i(\beta_1^i,\beta_2^i) & W^i(E^i-\Psi_iK_i) \\ * & -W^i \end{bmatrix} < 0$$
$$V_{p^i}(z(t,k)) \le \mu_iV_{p^j}(z(t,k)) \quad (22)$$

where $W^i(\beta_1^i,\beta_2^i) = diag\{\beta_1^iW_h^i, \beta_2^iW_v^i\}$, $\mu^i > 1$, $\theta^i = \max\{\beta_1^i,\beta_2^i\}$ and the average residence time τ_i satisfies the following inequality:

$$\tau_i^a \ge (\tau_i^a)^* = -\frac{\ln\mu^i}{\ln\theta^i}. \quad (23)$$

then the closed-loop system is not only fault-tolerant controlled but also exponentially stable of 2D.

Proof: We use the functional equation:

$$\begin{aligned} &V^i\left(z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \\ &= z^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P^iz^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \\ &= V_h^i\left(z_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) + V_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right). \end{aligned} \quad (24)$$

where $P^i = diag\{P_h^i, P_v^i\}$,

$$V_h^i\left(z_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) = z_h^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P_h^iz_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)$$

represents the variable in direction T and

$$V_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) = z_v^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \times P_v^iz_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)$$

represents the variable in direction K.

$$\begin{aligned} &\Delta V^i\left(z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k+1\right)\right) \\ &= V_h^i\left(z_h^i\left(\begin{bmatrix} t+2 \\ t+P+1 \end{bmatrix}|t,k\right)\right) + V_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k+1\right)\right) \\ &\quad - V_h^i\left(z_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) - V_v^i\left(z_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \end{aligned} \quad (25)$$

From the switching system in phase i (20) and Lyapunov function (21), we get

$$\begin{aligned} &\Delta V^i \\ &\le V_h^i\left(z_h^i\left(\begin{bmatrix} t+2 \\ t+P+1 \end{bmatrix}|t,k\right)\right) + V_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k+1\right)\right) \\ &\quad - \beta_1^iV_h^i\left(z_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) - \beta_2^iV_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \\ &\le z_h'^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P_h^iz_h'^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \\ &\quad + z_v'^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P_v^iz_v'^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \\ &\quad - \beta_1^iz_h^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P_h^iz_h^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \\ &\quad - \beta_2^i\left(z_v^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right)P_v^iz_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \\ &\le z^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)(E^i-\Psi_iK_i)^TP^i(E^i-\Psi_iK_i)z^i \\ &\quad \times \left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) - z^{iT} \\ &\quad \times \left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)P^i(\beta_1^i,\beta_2^i)z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right) \end{aligned}$$

$$\leq z^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)[(E^i-\Psi_iK_i)^TP^i \times (E^i-\Psi_iK_i)-P^i(\beta_1^i,\beta_2^i)]\times z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)$$
$$\leq z^{iT}\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\Theta_i z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)$$

in which $P^i(\beta_1^i,\beta_2^i) = diag\{\beta_1^iP_h^i,\beta_2^iP_v^i\}$. If the switching system is table, there must be $\Delta V^i < 0$ which is equivalent to

$$\Theta_i=(E^i-\Psi_iK_i)^TP^i(E^i-\Psi_iK_i)-P^i(\beta_1^i,\beta_2^i)<0 \quad (26)$$

Here we add the following lemma:

Lemma 1 ((Schur Complement Lemma)): Suppose W, L and V are given matrices with appropriate dimensions in which W and L are positive definite symmetric matrices, then the necessary and sufficient conditions for the establishment of

$$L^TVL-W<0$$

is

$$\begin{bmatrix} -W & L^T \\ L & -V^{-1} \end{bmatrix}<0 \text{ or } \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix}<0.$$

Then, we multiply the left and right sides of matrix (21) by $diag\{P^i,I^i\}$ and $diag\{P^i,I^i\}$ respectively, and according to Lemma 1, $(E^i-\Psi_iK_i)^TP^i(E^i-\Psi_iK_i)-P^i(\beta_1^i,\beta_2^i)<0$ is satisfied and then we get $\Delta V^i<0$, so the switched system is stabilized.

Let $\theta^i=\max\{\beta_1^i,\beta_2^i\}$, we can get the following inequality:

$$V_h^i\left(z_h^i\left(\begin{bmatrix} t+2 \\ t+P+1 \end{bmatrix}|t,k\right)\right)+V_v^i\left(z_v^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k+1\right)\right) < \theta^iV^i\left(z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \quad (27)$$

For arbitrary integers $M_0>0, N_0>0, y>0$, according to functional equations (5-24), the following inequality is obtained

$$\begin{aligned}
&V_h^i\left(z_h^i\left(\begin{bmatrix} M_0+2 \\ M_0+P+1 \end{bmatrix}|M_0,N_0+y\right)\right)+V_v^i\left(z_v^i\left(\begin{bmatrix} M_0+1 \\ M_0+P \end{bmatrix}|M_0,N_0+y+1\right)\right)\\
&< \theta^iV^i\left(z^i\left(\begin{bmatrix} M_0+1 \\ M_0+P \end{bmatrix}|M_0,N_0+y\right)\right)\\
&\vdots\\
&V_h^i\left(z_h^i\left(\begin{bmatrix} M_0+y+2 \\ M_0+P+y+1 \end{bmatrix}|M_0,N_0\right)\right)+V_v^i\left(z_v^i\left(\begin{bmatrix} M_0+y+1 \\ M_0+P+y \end{bmatrix}|M_0,N_0+1\right)\right)\\
&< \theta^iV^i\left(z^i\left(\begin{bmatrix} M_0+y+1 \\ M_0+P+y \end{bmatrix}|M_0,N_0\right)\right) \quad (28)
\end{aligned}$$

The above inequalities are summed to obtain the following inequalities:

$$\begin{aligned}
&\sum_{\substack{t+k=M_0+N_0+y+2\\ M_0\leq t\leq M_0+y\\ N_0\leq k\leq N_0+y}} V^i\left(z^i\left(\begin{bmatrix} t+2 \\ t+P+1 \end{bmatrix}|t,k\right)\right)\\
&\leq \sum_{\substack{t+k=M_0+N_0+y+2\\ M_0\leq t\leq M_0+y\\ N_0\leq k\leq N_0+y}} V^i\left(z^i\left(\begin{bmatrix} t+2 \\ t+P+1 \end{bmatrix}|t,k\right)\right)\\
&\quad+V_v^i\left(z_v^i\left(\begin{bmatrix} M_0+1 \\ M_0+P \end{bmatrix}|M_0,N_0+y+1\right)\right)\\
&\quad+V_h^i\left(z_h^i\left(\begin{bmatrix} M_0+y+2 \\ M_0+P+y+1 \end{bmatrix}|M_0,N_0\right)\right)\\
&<\theta^i\sum_{\substack{t+k=M_0+N_0+y+1\\ M_0\leq t\leq M_0+y\\ N_0\leq k\leq N_0+y}} V^i\left(z^i\left(\begin{bmatrix} t+1 \\ t+P \end{bmatrix}|t,k\right)\right) \quad (29)
\end{aligned}$$

To get the form of switching point, we use k_{l-f+1} and k_l represent the initial batch and the last batch, respectively, and then $N_\eta(w,F)$ represent the number of switching at the switching signal with the time interval of $[w,F]$, which is obtained as follows:

$$\left[T_{k_{l-f+1}}^s,k_{l-f+1}\right],\left[T_{k_{l-f+1}}^{s+1},k_{k-f+1}\right],\ldots,\left[T_{k_{l-f+1}}^{p-1},k_{k-f+1}\right],\left[T_{k_{l-f+1}}^{p},k_{k-f+1}\right],\left[T_{k_{l-f+2}}^{s},k_{l-f+2}\right],..,\left[T_{k_l}^{p},k_l\right],..$$

in which $\left[T_{k_{l-f+1}}^p,k_{l-f+1}\right]$ and $\left[T_{k_{l-f+1}},k_{l-f+1}\right]$ have the same significance, both representing the end moment of the previous phase and the initial moment of the next phase. Besides, $m_{k_q}^{i-1}=T_{k_q}^{i-1}+k_q$ and $m_{k_q}=T_{k_q}+k_q$ where $q=l-f+1,l-f+2,\cdots,l-1,l$, so the time interval of batch k_q in phase i is $\left[T_{k_q}^{i-1},T_{k_q}\right]$. Let $z(t,k)=z_{t,k}$ for convenience. In the form above, $F\in\left[m_{k_q}^{i-1},m_{k_q}\right]$, $V^{\sigma(T_k^i,k_l)}(z_{t,k})$ and $V^{\sigma\left((T_k^i)^-,k_l\right)}(z_{t,k})$ represent the i^{th} phase and the $(i-1)^{\text{th}}$ phase of batch k_l, respectively. $\sigma\left(\left(T_{k_l}^i\right)^-,k_l\right)$ represents the Lyapunov function of batch k_{l-1} in the i^{th} phase. From Document [34], we get:

$$\sum_{t+k=F} V^{\sigma(T_{kl}^i,k_l)}(z_{t,k})<\left(\theta^i\right)^{F-m_{k_l}^{i-1}}\sum_{t+k=m_{k_l}^{i-1}} V^{\sigma(T_{k_l}^i,k_l)}(z_{t,k}). \quad (30)$$

According to Document [42], at switching moment $m_{k_q}^{i-1}=T_{k_q}^{i-1}+k_q$, we can get

$$\sum_{t+k=m_{k_l}^{i-1}} V^{\sigma(T_{kl}^i,k_l)}(z_{t,k})\leq\mu_i\sum_{t+k=\left(m_{k_l}^{i-1}\right)^-} V^{\sigma\left(\left(T_{k_l}^i\right)^-,k_l\right)}(z_{t,k}). \quad (31)$$

We first add the following definition:

Definition 1: Define $\chi_l = \sup\left\{\left\|X_{t,k}\right\| : \bar{t}+k = w, \bar{t}, k \geq 1\right\}$, and for the given $w \geq 0$ and arbitrary $F \geq w$, if there is a positive definite matrix φ satisfying $\lim\limits_{l\to F} \chi_F \leq \kappa\varphi^{(F-w)}\chi_w$, then the closed-loop 2D system is exponentially stable at switching signal $\sigma(\cdot,\cdot)$ and $\|\bullet\|$ represents the Euclidean norm here.

Thus, from Definition 1, we have

$$
\begin{aligned}
&\sum_{t+k=F} V^{\sigma\left(T_{k_l}^{i},k_l\right)}(z_{t,k}) \\
&< \left(\theta^{i}\right)^{F-m_{k_l}^{i-1}} \sum_{t+k=m_{k_l}^{i-1}} V^{\sigma(T_{kl}^{i},k_l)}(z_{t,k}) \\
&\leq \mu_i \left(\theta^{i}\right)^{F-m_{k_l}^{i-1}} \sum_{t+k=m_{k_l}^{i-1}} V^{\sigma\left(\left(T_{k_l}^{i}\right)^{-},k_l\right)}(z_{t,k}) \\
&\leq \mu_i \left(\theta^{i}\right)^{F-m_{k_l}^{i-1}} \left(\theta^{i-1}\right)^{m_{k_l}^{i-1}-m_{k_l}^{i-2}} \\
&\quad \times \sum_{t+k=m_{k_l}^{i-2}} V^{\sigma\left(\left(T_{k_l}^{i-1}\right),k_l\right)}(z_{t,k}) \\
&\quad \vdots \\
&\leq \prod_{p=1}^{i} \left(\mu_p\right)^{\frac{T_p(F,w)}{\tau_p}} \times \prod_{p=1}^{i} \left(\theta^{p}\right)^{T_p(F,w)} \\
&\quad \times \sum_{t+k=w} V^{\sigma\left(\left(T_{l-f+1}^{1}\right)^{-},k_{l-f+1}\right)}(z_{t,k}) \\
&= \prod_{p=1}^{i} \left(\left(\mu_p\right)^{\frac{1}{\tau_p}} \left(\theta^{p}\right)\right)^{T_p(F,w)} \sum_{t+k=w} V^{\sigma\left(T_{l-f+1}^{1},k_{l-f+1}\right)}(z_{t,k})
\end{aligned}
\tag{32}
$$

Also, we know $\left(\mu_p\right)^{\frac{1}{\tau_p}} \left(\theta^{p}\right) < 1$ and let $\varphi^{p} = \max_{p\in N}\left(\left(\mu_p\right)^{\frac{1}{\tau_p}} \left(\theta^{p}\right)\right)$, and then we have

$$
\begin{aligned}
&\sum_{t+k=F} V^{\sigma\left(T_{k_l}^{i},k_l\right)}(z_{t,k}) \\
&\qquad \leq \kappa\left(\varphi^{i}\right)^{F-w} \sum_{t+k=w} V^{\sigma\left(T_{l-f+1}^{1},k_{l-f+1}\right)}(z_{t,k}).
\end{aligned}
\tag{33}
$$

We can see that if the switching signal satisfies equation (31) of theorem condition, $V^{\sigma\left(T_{k_l}^{i},k_l\right)}$ is convergent. Therefore, the closed-loop system designed is asymptotically stable of 2D.

E. CONCULSION

Designing an easy, real-time and flexibly adjustable controller according to different phases to improve control quality can solve existing methods' defect of nonadjustable controller gain throughout the whole process and offer a residence time method based on Lyapunov function. With the method, we can get values directly without getting the settings of other parameters, thus not only ensure the optimal control performance of system but also reduce system running time and realize efficient production.

III. CASE STUDIES

A. CASE ANALYSIS

Using the example of injection molding, the paper analyzes the fault-tolerant control for multi-phase batch process. A batch of injection molding process mainly includes the following three phases: the injection phase, the pressure holding phase and the cooling phase. The control of injection and pressure holding phases affects the final quality of products directly, and the cooling phase only cools the high-temperature finished products without control measures. Therefore, the paper only focuses on the injection phase and pressure holding phase of injection molding, studies the switch from the injection phase to the pressure holding phase of system with actuator fault, and builds the corresponding mixed state-space model by combining with the 2D model theory. We select different fault values and compare their experimental images. The comparison results with different actuator faults show that the 2D method proposed not only ensures the stable running of system but also has advantages of fast convergence, short running time and quick tracking, and thus realizes the high-efficiency production. The mathematical models of injection phase and pressure holding phase of injection molding process are described as follows:

The model of injection speed IV and valve opening VO of injection phase is

$$
\frac{IV}{VO} = \frac{8.687z^{-1} - 5.617z^{-2}}{1 - 0.9291z^{-1} - 0.03191z^{-2}}
$$

$$
IV(t+1,k) - 0.9291IV(t,k) - 0.0319IV(t-1,k)
$$

i.e.

$$
= 8.687VO(t,k) - 5.617VO(t-1,k).
$$

The model of cavity pressure NP and injection speed IV of injection phase is

$$
\frac{NP}{IV} = \frac{0.1054z^{-1}}{1 - z^{-1}}
$$

i.e. $NP(t+1,k) - NP(t,k) = 0.1054IV(t,k)$, where the set point for injection speed IV of injection phase is 40mm/s and the set point for cavity pressure NP of holding phase is 300bar.

We suppose

$$
\begin{cases}
x_1^1(t,k) = IV(t,k) \\
x_2^1(t,k) = IV(t-1,k) \\
x_3^1(t,k) = NP(t,k) \\
u(t,k) = VO(t,k),
\end{cases}
\quad
x^1(t+1,k) = \begin{bmatrix} x_1^1(t+1,k) \\ x_2^1(t+1,k) \\ x_3^1(t+1,k) \\ u(t,k) \end{bmatrix}
$$

and then the state-space model of injection phase is as follows

$$\begin{cases} x^1(t+1,k) = \begin{bmatrix} 0.9291 & 0.0319 & 0 & -5.617\alpha \\ 1 & 0 & 0 & 0 \\ 0.1054 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \qquad \times x^1(t,k) + \begin{bmatrix} 8.687 \\ 0 \\ 0 \\ 1 \end{bmatrix} u^{1F}(t,k) \\ y^1(t,k) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x^1(t,k) \end{cases}$$

The model of cavity pressure NP and valve opening VO of pressure holding phase is

$$\frac{NP}{VO} = \frac{171.8z^{-1} - 156.8z^{-2}}{1 - 1.317z^{-1} + 0.3259z^{-2}}$$

i.e.

$$\begin{aligned} NP(t+1,k) &- 1.317NP(t,k) + 0.3259NP(t-1,k) \\ &= 171.8VO(t,k) - 156.8VO(t-1,k). \end{aligned}$$

We suppose

$$\begin{cases} x_1^2(t,k) = NP(t,k) \\ x_2^2(t,k) = NP(t-1,k) \\ u(t,k) = VO(t,k) \end{cases}$$

and then have the following state-space model of pressure holding phase:

$$\begin{cases} x^2(t+1,k) = \begin{bmatrix} 1.317 & -0.3259 & -156.8\alpha \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ \qquad \times x^2(t,k) + \begin{bmatrix} 171.8\alpha \\ 0 \\ 1 \end{bmatrix} u^{2F}(t,k) \\ y^2(t,k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x^2(t,k) \end{cases}$$

Therefore, we adopt the switching condition of $\begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix} x^i(t,k) \geq 350$, i.e. once the cavity pressure exceeds 350, the system will switch from the injection phase to the pressure holding phase. Because the switch between the state-space matrices of injection phase and pressure holding phase is the transition of different dimensions, we can realize it with the state-transition matrix [42]. The paper proposes a linear quadratic tracking predictive fault-tolerant control method for multi-phase batch process based on 2D model theory. Using the strategy combining the 2D-model-based ILC with the linear quadratic predictive fault-tolerant tracking control, the paper studies the batch process with time delay and actuator fault. The paper simulates the injection phase and pressure holding phase of injection molding process and uses the simulation example with different fault values to prove the effectiveness of the method proposed.

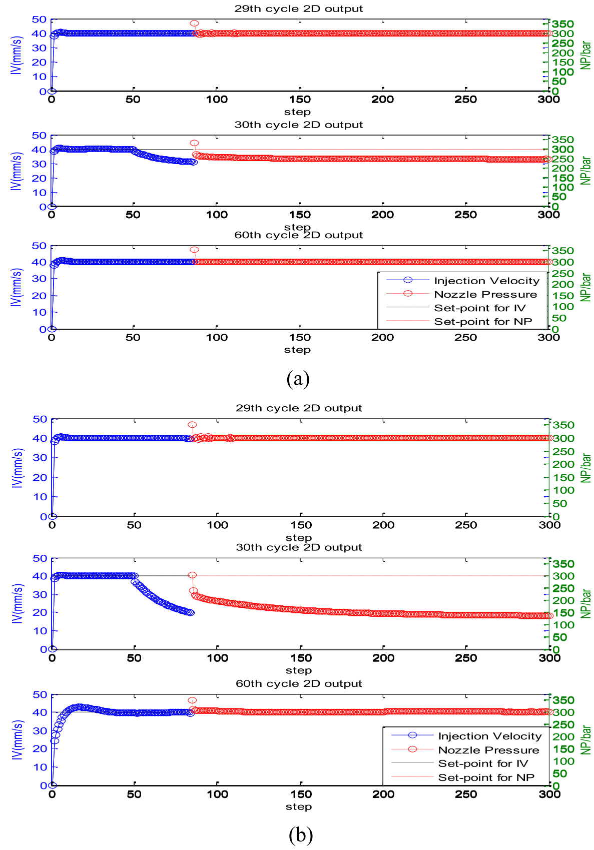

FIGURE 1. (a) System output for the 29th, 30th and 60th batches ($\alpha = 0.6$). (b) System output for the 29th, 30th and 60th batches ($\alpha = 0.2$).

B. SIMULATION RESULTS

To show the control effect of 2D method proposed, we select different fault values for the simulation, which are selected are 0.2 and 0.6. The batch with fault is the 30^{th} batch. We select the 29^{th}, 30^{th} and 60^{th} batches as the previous batch of fault batch, the fault batch and the batch after fault. Next, we compare the system output, the tracking error and the system input with the two fault values through the simulation. Simulation figures are as follows.

Figure 1a shows the system output before and after fault. In the previous batch of fault, the system output is almost a smooth straight line approaching the set value with small fluctuations in the switching point at the 86^{th} step. In the fault batch, the system output value fluctuates greatly and is much lower than the set point 40mm/s after the 50^{th} step. Besides, the cavity pressure is also much less than the set point 300bar after switch. After a period of running, in the 60^{th} batch after fault, the system output is a smooth straight line similar to the control effect of the 29^{th} batch before fault. We can see that the method proposed is effective.

Figure 1b shows that in the 30^{th} batch with fault, the system output presents greater fluctuations than the case with actuator fault of 0.6. The control effect is poor but the system output can still track the given value.

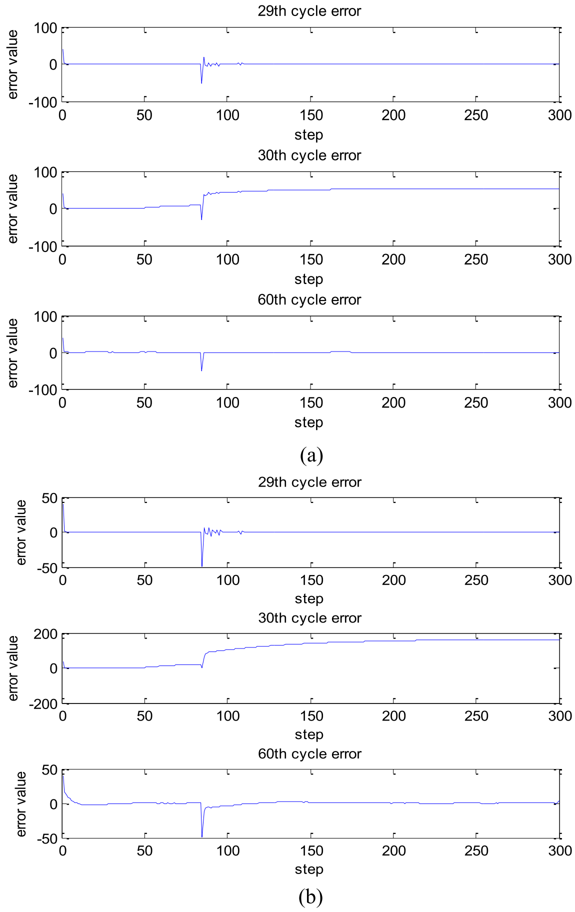

FIGURE 2. (a) Tracking error for the 29th, 30th and 60th batches ($\alpha = 0.6$). (b) Tracking error for the 29th, 30th and 60th batches ($\alpha = 0.2$).

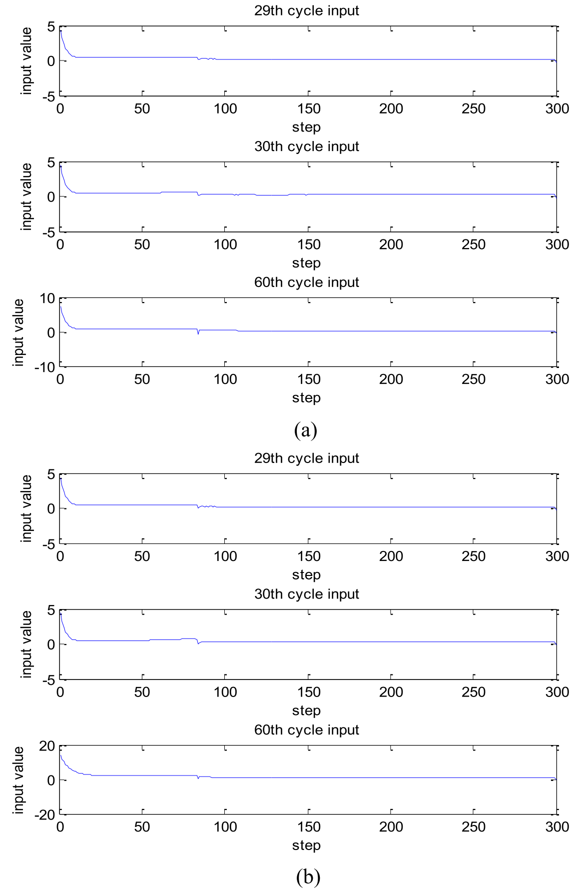

FIGURE 3. (a) System input for the 29th, 30th and 60th batches ($\alpha = 0.6$). (b) System input for the 29th, 30th and 60th batches ($\alpha = 0.2$).

Figure 2a shows the tacking error for the batches before and after fault batch. In the previous batch of fault, the tracking error stabilizes on a fixed value quickly in the initial phase, and then has fluctuations after switch, but becomes a straight line approaching 0.2 infinitely after running a dozen steps. In the fault batch, the error has obviously greater fluctuations and the control becomes poor. In the 60$^{\text{th}}$ batch after fault, the error has small fluctuations initially, but the fluctuations are obviously smaller than those in the 29$^{\text{th}}$ batch, the previous batch of fault batch, and the error presents a stable straight line approaching 0.1 after running a few steps. Figure 2b has a more serious fault than figure 2a, so its fluctuations at the point of switch in the previous batch of are greater than those in figure 2a, but the value stabilizes on the fixed value 0.3 after a few steps. In the fault batch, the tracking error is obviously bigger than that in figure 2a. In the 60$^{\text{th}}$ batch after fault, the error tracking is slow and then stabilizes on a straight line approaching the fixed value of 0.1 in the 194$^{\text{th}}$ step. Although the tracking velocity is slower than that in figure 2a, the control effect is still good.

Figure 3a shows the system input for the batches before and after fault. From the figure we can see that in the previous batch of fault, the input value is a smooth straight line approaching 0.17. In the fault batch, the input value increases and approaches a fixed value 0.23 with many fluctuations. In the 60$^{\text{th}}$ batch, the batch after fault, the input value shows as a smooth straight line approaching 0.28. Figure 3b shows that in the batch with fault, the system input has obvious fluctuations and presents a straight line approaching 0.35. In the 60$^{\text{th}}$ batch after fault, the input value has smaller fluctuations and presents a smooth straight line approaching 0.85. Because the fault hasn't been eliminated and needs adjustments continuously, the input value increases continuously. Although the input value increases, the stability is still good. In conclusion, the method designed has a good control effect on different actuator faults, thus proving the effectiveness of the method proposed.

IV. CONCLUSION

The actuator fault is very common in the industrial production, so we need to reduce the probability of fault to reduce the risks caused by fault. The paper proposes a linear quadratic predictive fault-tolerant control method for multi-phase batch process based on 2D model theory specific for the problems of actuator fault and input time delay in bath process production. According to the multi-phase batch process model with time delay and fault, the paper introduces a new variable to build a new state-space model with no time delay, and considers the closed-loop system as a 2D-Roesser model containing the state error and the output tracking error based on 2D theory and then designs an easy, real-time and flexibly adjustable controller according to different phases to improve control quality, and thus solves existing methods' defect of nonadjustable controller gain throughout the process. Using a simulation example, the paper proves that the method proposed can ensure the system's good tracking performance and optimal control performance with different actuator faults to meet the requirements of industrial production and enhance productivity, and thus can improve modern industrial production greatly.

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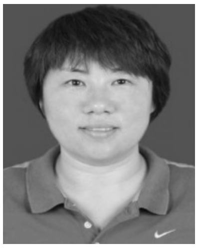

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