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Time-Varying Formation Control for Second-Order Discrete-Time Multi-Agent Systems With Directed Topology and Communication Delay

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ABSTRACT Time-varying formation control protocol design and analysis problems for the second-order discrete-time multi-agent systems with directed interaction topology and communication delay are investigated. A local information-based distributed protocol is designed by utilizing the delayed state information of neighbors. Through system decomposition and stability analysis, an explicit description of the feasible time-varying formation set is given. Necessary and sufficient conditions for the systems with the directed topology and communication delay to achieve time-varying formation are obtained, which are related to the topology of the interaction graph and the feasibility of the predefined formation. Necessary constraints on the gain parameters and the sampling period are proposed, so as to guide the design of parameters in the protocol. The numerical simulation results indicate that the protocol can steer the agents to accomplish the desired time-varying formation and effectively tolerate the relatively large bounded communication delay. Outdoor experiment with quadrotors is presented to demonstrate the effectiveness of the obtained theoretical results with one sampling period delay.

INDEX TERMS Multi-agent systems, consensus control, time-varying formation, directed topology, communication delay.

I. INTRODUCTION

Through efficient coordination, many inexpensive, simple individuals can emerge much better performance than a single monolithic one. Formation control, as one of the most fundamental distributed cooperative control problems for multi-agent systems, is a critical step of cooperation among agents [1], [2]. Therefore, cooperative formation control for multi-agent systems has become a research hotspot and accurate maintenance of a geometric formation between agents has been studied extensively [3]–[5]. In general, the formation control problems for multi-agent systems are to find distributed coordination schemes for networks of agents such that they would reach and maintain some desired, possibly time-varying formation or group configuration. The main challenge in formation control of multi-agent systems is that each agent has to use local information to achieve the desired formation, rather than rely on centralized coordination, especially when there are communication constraints such as time delay and directed topology.

Recently, consensus control for multi-agent systems has attracted great attention from various domains and great advances have been derived already [6]–[8]. Following the boom in the research of consensus control problems, consensus-based formation control approaches are developed. It has been proved in [9] that the traditional leader-follower, behavior and virtual structure based approaches can be regarded as special cases of consensus-based ones, and the weaknesses of the previous approaches can be overcame to some extent. Consensus based time-invariant formation control problems for first-, second- and high-order multi-agent systems have been studied extensively [10]–[15].

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When the time-invariant formation is achieved, velocity components of all agents must be identical. This is not enough because in many practical applications, such as target surveillance and formation reconfiguration, the desired formation requires the velocity components of agents to be different. In these cases, only the time-varying formation can be used. In [16], coherent formation control of multi-agent systems in the presence of time-varying formation was studied. In [17], decentralized time-varying formation control for multi-robot systems with first-order dynamics was presented. Reference [18] dealt with the cooperative control for nonlinear multi-agent systems, whose objective was to stabilize a group of agents to time-varying formation. Formation control problems for second-order swarm systems with general directed topology were addressed in [19]. The time-varying formation problems were studied in [20] under directed topology and an adaptive approach was utilized to develop a fully distributed formation controller for general linear multi-agent systems. Reference [21] studied the time-varying formation tracking problems for multiple manipulator systems under fixed and switching directed graphs with a dynamic leader. Reference [22] considered the time-varying formation control problems for a class of networked systems with unknown parameters and non-identical nonlinear dynamics. Time-varying formation analysis and design problems for general high-order swarm systems with communication constraints were investigated in [23].

Considering the fact that time delay often exists in practical systems due to the limitation of bandwidth, abundant data transmission and asymmetry of communication links, formation control problems with time delay have been investigated in [24]–[26]. Reference [27] researched the distributed formation control problems for multi-agent systems with randomly switching topologies and time delay. Reference [28] considered the problems on formation tracking control of second-order multi-agent systems with communication delay. In [29], the containment control problems were considered for nonlinear multi-agent systems with directed topology and time-delay. Reference [30] investigated the leader-follower formation control problems for a group of networked robots that were subject to bounded time-varying communication delays and an asynchronous clock.

It should be pointed out that in most recent literatures, the systems were described by continuous-time dynamics, such as the works mentioned above. However, in practical formation control applications via interaction networks, continuous states of agents (such as position and velocity) are always represented and updated by their sampled values at a certain interval, which results in discrete-time or sampled-data formulation. The conclusions obtained in continuous-time systems cannot be used to solve such problems directly. Thus, the formation control protocol design and analysis problems for multi-agent systems with discrete-time dynamics are necessary and of practical significance. In [31], the formation control problems without time delay were investigated for discrete-time multi-agent systems with unknown nonlinear dynamics by means of iterative learning approach. Reference [32] presented a distributed control law based on the output regulation control framework to solve the formation control problems of first-order discrete-time nonlinear multi-agent systems without time delay. Reference [33] used a fault tolerant approach to control a group of wheeled mobile robots in a formation without time delay. In [34], the time-invariant formation control for high-order discrete-time multi-agent systems was achieved in the absence of time delay. The time-invariant formation control problems of second-order discrete-time systems with time delay and undirected topology were investigated in [35]. Reference [36] established the necessary and sufficient condition for designing formation of discrete-time second-order multi-agent systems with only one sampling period delay and the desired formation cannot be time-varying. The asynchronous time-invariant formation control problems of second-order discrete-time multi-agent systems with time-varying delays were investigated in [37].

Although some important results and approaches have been established in a few references, research on formation control for discrete-time systems is not as sufficient as for continuous systems, especially the time-varying formation control under conditions with time delay and directed topology. This paper mainly focuses on the time-varying formation control problems for second-order discrete-time multi-agent systems with relatively large bounded communication delay, which is meaningful yet still unresolved. The multi-agent systems in this paper are described by second-order discrete-time dynamics, where agents are governed by both position and velocity states. It is more complicated and conforms to reality as the sampling period and gain parameters can be considered simultaneously. Furthermore, the desired formation can be time-varying and the interaction topology is directed. A distributed protocol is designed by utilizing the delayed states of neighbors. Compared with the previous results, this paper aims to solve the following three problems for second-order discrete-time multi-agent systems with directed topology and communication delay: (i) what are the conditions that guarantee the time-varying formation can be achieved with communication delay; (ii) how to determine whether a desired time-varying formation is feasible; (iii) how to design the parameters in the protocol to achieve the feasible time-varying formation.

The remainder of this paper is organized as follows. In section II, some necessary preliminary results and lemmas are described together with problem description. Section III considers the time-varying formation control analysis and protocol design problems. Numerical examples and outdoor flight experiment are provided in section IV to illustrate the validity of the algorithm and section V summarizes this paper.

II. PRELIMINARY AND PROBLEM DESCRIPTION

A. GRAPH THEORY

Let $G = (W, E, A)$ be a weighted directed graph with vertices set $W = \{1, \cdots, N\}$, edges set $E \subseteq \{(i, j) : i, j \in W\}$

and adjacency matrix $\boldsymbol{A} = [a_{ij}]_{N\times N}$, which describes the interaction topology among vertices, N is the number of vertices. If there exists a directed edge $e_{ij} \in E$ from vertex j to i, then vertex j is called a neighbor of vertex i , i.e., vertex i has information of j . $N_i = \{j|j \in W : e_{ij} \in E\}$ is defined as the neighbor set of vertex i. The adjacency matrix $\boldsymbol{A}$ satisfies $a_{ij} > 0$ if and only if $j \in N_i(j \neq i)$, otherwise $a_{ij} = 0$. A directed graph G is said to have a spanning tree if there is at least one vertex has directed paths to all the other vertices. Laplacian matrix $\boldsymbol{L} = [l_{ij}]_{N\times N}$ plays an important role in the description of neighbor relationship of a graph and it is defined by

$$l_{ij} = \begin{cases} \sum_{k=1,k\neq i}^{N} a_{ik}, & i = j, \\ -a_{ij}, & i \neq j. \end{cases}$$

Before discussing the main problems to be addressed in this paper, we also need some necessary preliminary results and lemmas on graph and matrix theory first.

Lemma 1 [38]: Let $\mathbf{L}$ *be the Laplacian matrix of a directed graph* G *with eigenvalues denoted as* λ_i, $i = 1, \cdots, N$, *then: (1)* $\mathbf{L}$ *has at least one zero eigenvalue and* $\mathbf{1}_N$ *is the associated eigenvector, that is* $\mathbf{L}\mathbf{1}_N = \mathbf{0}$ *. (2) If* G *has a spanning tree, then 0 is a simple eigenvalue of* $\boldsymbol{L}$, *and all the other eigenvalues have positive real parts, that is* $0 = \lambda_1 < Re(\lambda_i)$, $i = 2, \cdots, N$.

Lemma 2 [38]: If matrix $\boldsymbol{\Lambda}$ *is nonnegative and has the same positive constant row sum* $\mu > 0$, *then* μ *is an eigenvalue of* $\boldsymbol{\Lambda}$ *with an associated eigenvector* $\mathbf{1}$ *and* $\rho(\boldsymbol{\Lambda}) = \mu$, *where* $\rho(\boldsymbol{\Lambda})$ *is the spectral radius. Furthermore, the eigenvalue* μ *of* $\boldsymbol{\Lambda}$ *has algebraic multiplicity equal to one if and only if the graph associated with* $\boldsymbol{\Lambda}$ *has a spanning tree.*

B. PROBLEM FORMULATION

Consider a multi-agent system with N nodes, labeled from 1 to N arbitrarily, define $F = \{2, \cdots, N\}$. The interaction topology among the N agents can be described by graph G, with each agent being a vertex and the interaction from agent j to agent i is represented by the edge e_{ij}, where j is called a neighbor of i. The purpose of this paper is to design a protocol to steer the agents to form a predefined time-varying formation with directed topology and communication delay. In the discrete-time case, using the forward difference approximation as that employed in [39], the dynamics of each agent is described by

$$\begin{cases} \boldsymbol{x}_i(t+\delta) = \boldsymbol{x}_i(t) + \delta \boldsymbol{v}_i(t), \\ \boldsymbol{v}_i(t+\delta) = \boldsymbol{v}_i(t) + \delta \boldsymbol{u}_i(t), \end{cases} \tag{1}$$

where $\boldsymbol{x}_i(t) \in R^n$, $\boldsymbol{v}_i(t) \in R^n$ and $\boldsymbol{u}_i(t) \in R^n$ are the position, velocity and control input vectors of agent i, $i = 1, \cdots, N$. $\delta > 0$ is analogous to the sampling period, in the following we will refer to it as such. The update time instants $t \geq 0$ will be the form $t = t_0 + q\delta$, $t_0 \geq 0$ is the initial moment, $q = 1, 2, \cdots$. $n \geq 1$ is the dimension of the state. In the following, for the sake of convenience in description, let $n = 1$ if not otherwise specified. However, all the conclusions hereafter can be extended to higher dimensional cases directly by using Kronecker product.

The bounded time-varying formation set to be achieved is specified by $\boldsymbol{h}_x(t) = [\boldsymbol{h}_{1x}(t), \boldsymbol{h}_{Fx}^T(t)]^T$, $\boldsymbol{h}_v(t) = [\boldsymbol{h}_{1v}(t), \boldsymbol{h}_{Fv}^T(t)]^T$ and $\boldsymbol{h}_a(t) = [\boldsymbol{h}_{1a}(t), \boldsymbol{h}_{Fa}^T(t)]^T$, where $\boldsymbol{h}_{Fx}(t) = [\boldsymbol{h}_{2x}(t), \cdots, \boldsymbol{h}_{Nx}(t)]^T$, $\boldsymbol{h}_{Fv}(t) = [\boldsymbol{h}_{2v}(t), \cdots, \boldsymbol{h}_{Nv}(t)]^T$ and $\boldsymbol{h}_{Fa}(t) = [\boldsymbol{h}_{2a}(t), \cdots, \boldsymbol{h}_{Na}(t)]^T$. $\boldsymbol{h}_x(t)$, $\boldsymbol{h}_v(t)$ and $\boldsymbol{h}_a(t)$ are so called formation reference vectors with $\boldsymbol{h}_{ix}(t)$, $\boldsymbol{h}_{iv}(t)$ and $\boldsymbol{h}_{ia}(t)$ being the components of position, velocity and acceleration for agent i, respectively. Define $\boldsymbol{h}_i(t) = [\boldsymbol{h}_{ix}(t), \boldsymbol{h}_{iv}(t)]^T$, $\boldsymbol{\xi}_i(t) = [\boldsymbol{x}_i(t), \boldsymbol{v}_i(t)]^T$ and $\boldsymbol{h}(t) = [\boldsymbol{h}_x^T(t), \boldsymbol{h}_v^T(t)]^T$.

Definition 1: Multi-agent system (1) is said to achieve the desired time-varying formation specified by $\boldsymbol{h}(t)$ if and only if for any given bounded initial states and $i, j \in \{1, \cdots, N\}$, $i \neq j$,

$$\lim_{t\to\infty} \left\{ \left(\boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t)\right) - \left(\boldsymbol{\xi}_j(t) - \boldsymbol{h}_j(t)\right) \right\} = \mathbf{0}. \tag{2}$$

Remark 1: It should be noted that $\boldsymbol{h}(t)$ only gives the desired time-varying formation rather than the reference trajectory for each agent to follow, that is $\boldsymbol{h}_i(t)$ only gives the relative offset vector of $\boldsymbol{\xi}_i(t)$. In the case $\boldsymbol{h}(t) = \mathbf{0}$, Definition 1 becomes a consensus seeking problem for discrete-time multi-agent systems.

Due to the limitation of bandwidth, abundant data transmission, motion of agents and congestion of network links, time delay often exists in practical communication networks. Assume that the communication delay only exists in the actually transmitted information and every agent can use its own instantaneous state information. Other constraints such as packet loss and external disturbance are ignored to simplify the problem and for convenience in description and analysis. Each agent's sampled state is encapsulated into a data packet along with its time stamp. The communication delay from agent j to agent i is denoted by τ_{ij} and let $\hbar = \max\{\tau_{ij} : i = 1, \cdots, N, j \in N_i\}$. By introducing an information storer (Fig. 1(a)) to temporarily store the received data packets and each agent runs with a long enough waiting time $\tau(\tau \geq \hbar)$, such that all the data packets can be fully received and updated. In the discrete-time case, it is reasonable to set the value of the waiting time as $\tau = p\delta$, where p is the smallest integer greater than or equal to $\frac{\hbar}{\delta}$. It should be noted that for discrete-time systems, although the communication delay can be reduced by improving the bandwidth or event triggered strategy [40], there exists at least one sampling period delay when the states are exchanged among agents, as shown in Fig. 1(b).

To solve the time-varying formation control problems with directed topology and communication delay, the control protocol using local velocity and position information is designed as follows,

$$\boldsymbol{u}_i(t) = \frac{\boldsymbol{K}}{\sum_{j\in N_i} a_{ij}} \sum_{j=1}^{N} a_{ij} \left\{ \boldsymbol{\xi}_i(t) - \boldsymbol{h}_i(t) - [\boldsymbol{\xi}_j(t-\tau) - \boldsymbol{h}_j(t-\tau)] \right\} + \boldsymbol{h}_{ia}(t), \tag{3}$$

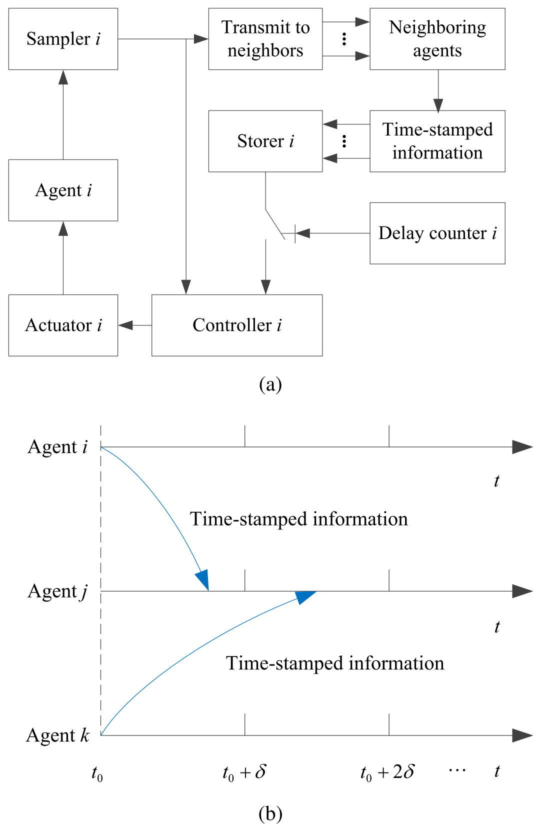


FIGURE 1. A distributed communication delay processing scheme and communication delay in discrete-time systems. (a) A distributed scheme to uniform the communication delay. (b) Communication delay in discrete-time systems.

where $\boldsymbol{K} = [-\alpha, -\beta]$ $(\alpha, \beta > 0)$ is a constant gain matrix, $\tau \geq 0$ is the bounded communication delay.

III. TIME-VARYING FORMATION ANALYSIS AND CONTROL PROTOCOL DESIGN

In this section, necessary and sufficient conditions for system (1) with directed interaction topology and communication delay to achieve the desired time-varying formation $\boldsymbol{h}(t)$ are presented. Then some necessary constraints on the gain parameters and sampling period in the protocol are deduced.

Denote $\boldsymbol{\psi}_{ix}(t) = \boldsymbol{x}_i(t) - \boldsymbol{h}_{ix}(t)$, $\boldsymbol{\psi}_{iv}(t) = \boldsymbol{v}_i(t) - \boldsymbol{h}_{iv}(t)$, $\boldsymbol{\psi}_i(t) = [\boldsymbol{\psi}_{ix}(t), \boldsymbol{\psi}_{iv}(t)]^T$, $\boldsymbol{\varsigma}_{ix}(t) = \boldsymbol{\psi}_{ix}(t) - \boldsymbol{\psi}_{1x}(t)$, $\boldsymbol{\varsigma}_{iv}(t) = \boldsymbol{\psi}_{iv}(t) - \boldsymbol{\psi}_{1v}(t)$, $\boldsymbol{\varsigma}_{Fx}(t) = [\boldsymbol{\varsigma}_{2x}(t), \cdots, \boldsymbol{\varsigma}_{Nx}(t)]^T$, $\boldsymbol{\varsigma}_{Fv}(t) = [\boldsymbol{\varsigma}_{2v}(t), \cdots, \boldsymbol{\varsigma}_{Nv}(t)]^T$, $\boldsymbol{\varsigma}(t) = [\boldsymbol{\varsigma}_{Fx}^T(t), \boldsymbol{\varsigma}_{Fv}^T(t)]^T$. Then protocol (3) has the following form

$$\begin{aligned}\boldsymbol{u}_i(t) &= \frac{\boldsymbol{K}}{\sum\limits_{j\in N_i} a_{ij}} \sum_{j=1}^{N} a_{ij}[\boldsymbol{\psi}_i(t) - \boldsymbol{\psi}_j(t-\tau)] + \boldsymbol{h}_{ia}(t) \\ &= -\frac{1}{\sum\limits_{j\in N_i} a_{ij}} \sum_{j=1}^{N} a_{ij}\left\{\alpha[\boldsymbol{\psi}_{ix}(t) - \boldsymbol{\psi}_{jx}(t-\tau)]\right. \\ &\quad \left. + \ \beta[\boldsymbol{\psi}_{iv}(t) - \boldsymbol{\psi}_{jv}(t-\tau)]\right\} + \boldsymbol{h}_{ia}(t). \end{aligned} \tag{4}$$

Theorem 1: Multi-agent system (1) with directed topology and communication delay achieves time-varying formation $\boldsymbol{h}(t)$ under protocol (3) if and only if for any $i \in \{1, \cdots, N\}$, the formation feasibility condition

$$\begin{cases} \lim\limits_{t\to\infty} [\boldsymbol{h}_{ix}(t) + \delta\boldsymbol{h}_{iv}(t) - \boldsymbol{h}_{ix}(t+\delta)] \\ \qquad - \left[\boldsymbol{h}_{jx}(t) + \delta\boldsymbol{h}_{jv}(t) - \boldsymbol{h}_{jx}(t+\delta)\right] = \boldsymbol{0} \\ \lim\limits_{t\to\infty} [\boldsymbol{h}_{iv}(t) + \delta\boldsymbol{h}_{ia}(t) - \boldsymbol{h}_{iv}(t+\delta)] \\ \qquad - \left[\boldsymbol{h}_{jv}(t) + \delta\boldsymbol{h}_{ja}(t) - \boldsymbol{h}_{jv}(t+\delta)\right] = \boldsymbol{0} \end{cases} \tag{5}$$

is satisfied and the delayed discrete-time system described by

$$\begin{aligned}\boldsymbol{\varsigma}(t+\delta) &= \begin{bmatrix} \boldsymbol{I}_{N-1} & \delta\boldsymbol{I}_{N-1} \\ -\alpha\delta\boldsymbol{I}_{N-1} & (1-\beta\delta)\boldsymbol{I}_{N-1} \end{bmatrix}\boldsymbol{\varsigma}(t) \\ &\quad + \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \alpha\delta\boldsymbol{C} & \beta\delta\boldsymbol{C} \end{bmatrix}\boldsymbol{\varsigma}(t-\tau) \end{aligned} \tag{6}$$

is asymptotically stable, where $\boldsymbol{C} = \boldsymbol{A'}_{11} - \boldsymbol{1}_{N-1}\boldsymbol{A'}_{01}$, $\boldsymbol{A'} = [a'_{ij}]_{N\times N}$, $a'_{ij} = a_{ij}\Big/\sum_{j\in N_i} a_{ij}$, $\boldsymbol{A'}_{11}$ is a $(N-1)\times(N-1)$ matrix formed by the last $N-1$ rows and columns of matrix $\boldsymbol{A'}$, $\boldsymbol{A'}_{01}$ is a row vector formed by the last $N-1$ elements of the first row of matrix $\boldsymbol{A'}$.

Proof:

Substitute (4) into (1), the closed-loop dynamics of multi-agent system can be written in a compact form as

$$\begin{aligned}\boldsymbol{\psi}_x(t+\delta) &= \boldsymbol{\psi}_x(t) + \delta\boldsymbol{\psi}_v(t) + \boldsymbol{h}_x(t) \\ &\quad - \boldsymbol{h}_x(t+\delta) + \delta\boldsymbol{h}_v(t) \\ \boldsymbol{\psi}_v(t+\delta) &= -\alpha\delta\boldsymbol{\psi}_x(t) + \alpha\delta\boldsymbol{A'}\boldsymbol{\psi}_x(t-\tau) \\ &\quad + (1-\beta\delta)\boldsymbol{\psi}_v(t) + \beta\delta\boldsymbol{A'}\boldsymbol{\psi}_v(t-\tau) \\ &\quad + \boldsymbol{h}_v(t) - \boldsymbol{h}_v(t+\delta) + \delta\boldsymbol{h}_a(t). \end{aligned} \tag{7}$$

Define $\boldsymbol{E} = \begin{bmatrix} 1 & \boldsymbol{0} \\ \boldsymbol{1}_{N-1} & \boldsymbol{I}_{N-1} \end{bmatrix}$, $\bar{\boldsymbol{e}} = \boldsymbol{1}_N$, $\bar{\boldsymbol{E}} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I}_{N-1} \end{bmatrix}$, $\boldsymbol{E}^{-1} = \begin{bmatrix} 1 & \boldsymbol{0} \\ -\boldsymbol{1}_{N-1} & \boldsymbol{I}_{N-1} \end{bmatrix}$, $\tilde{\boldsymbol{e}} = [\,1 \ \ \boldsymbol{0}\,]$, $\tilde{\boldsymbol{E}} = [\,-\boldsymbol{1}_{N-1} \ \ \boldsymbol{I}_{N-1}\,]$. Let $\boldsymbol{\zeta}_x(t) = \boldsymbol{E}^{-1}\boldsymbol{\psi}_x(t)$, $\boldsymbol{\zeta}_v(t) = \boldsymbol{E}^{-1}\boldsymbol{\psi}_v(t)$, that is $\boldsymbol{\psi}_x(t) = \boldsymbol{E}\boldsymbol{\zeta}_x(t)$, $\boldsymbol{\psi}_v(t) = \boldsymbol{E}\boldsymbol{\zeta}_v(t)$. Therefore, from (7), one obtains that

$$\begin{aligned}\boldsymbol{E}\boldsymbol{\zeta}_x(t+\delta) &= \boldsymbol{E}\boldsymbol{\zeta}_x(t) + \delta\boldsymbol{E}\boldsymbol{\zeta}_v(t) + \boldsymbol{h}_x(t) \\ &\quad - \boldsymbol{h}_x(t+\delta) + \delta\boldsymbol{h}_v(t) \\ \boldsymbol{E}\boldsymbol{\zeta}_v(t+\delta) &= -\alpha\delta\boldsymbol{E}\boldsymbol{\zeta}_x(t) + \alpha\delta\boldsymbol{A'}\boldsymbol{E}\boldsymbol{\zeta}_x(t-\tau) \\ &\quad + (1-\beta\delta)\boldsymbol{E}\boldsymbol{\zeta}_v(t) + \beta\delta\boldsymbol{A'}\boldsymbol{E}\boldsymbol{\zeta}_v(t-\tau) \\ &\quad + \boldsymbol{h}_v(h) - \boldsymbol{h}_v(t+\delta) + \delta\boldsymbol{h}_a(t). \end{aligned} \tag{8}$$

Pre-multiplying both sides of (8) by $\boldsymbol{E}^{-1}$ leads to

$$\begin{aligned}\boldsymbol{\zeta}_x(t+\delta) &= \boldsymbol{\zeta}_x(t) + \delta\boldsymbol{\zeta}_v(t) \\ &\quad + \boldsymbol{E}^{-1}[\boldsymbol{h}_x(t) - \boldsymbol{h}_x(t+\delta) + \delta\boldsymbol{h}_v(t)] \\ \boldsymbol{\zeta}_v(t+\delta) &= -\alpha\delta\boldsymbol{\zeta}_x(t) + \alpha\delta\boldsymbol{E}^{-1}\boldsymbol{A'}\boldsymbol{E}\boldsymbol{\zeta}_x(t-\tau) \\ &\quad + (1-\beta\delta)\boldsymbol{\zeta}_v(t) + \beta\delta\boldsymbol{E}^{-1}\boldsymbol{A'}\boldsymbol{E}\boldsymbol{\zeta}_v(t-\tau) \\ &\quad + \boldsymbol{E}^{-1}[\boldsymbol{h}_v(t) - \boldsymbol{h}_v(t+\delta) + \delta\boldsymbol{h}_a(t)]. \end{aligned} \tag{9}$$

Note that

$$\boldsymbol{E}^{-1}\boldsymbol{A'}\boldsymbol{E} = \begin{bmatrix} 1 & \boldsymbol{A'}_{01} \\ \boldsymbol{0} & \boldsymbol{A'}_{11} - \boldsymbol{1}_{N-1}\boldsymbol{A'}_{01} \end{bmatrix} \tag{10}$$

and $\boldsymbol{C} = \boldsymbol{A'}_{11} - \mathbf{1}_{N-1}\boldsymbol{A'}_{01}$, it follows from (9) and (10) that

$$\begin{aligned}\varsigma_{Fx}(t+\delta) &= \varsigma_{Fx}(t) + \delta\varsigma_{Fv}(t)\\ &\quad + \tilde{\boldsymbol{E}}[\boldsymbol{h}_x(t) - \boldsymbol{h}_x(t+\delta) + \delta\boldsymbol{h}_v(t)]\\ \varsigma_{Fv}(t+\delta) &= -\alpha\delta\varsigma_{Fx}(t) + (1-\beta\delta)\varsigma_v(t)\\ &\quad + \alpha\delta C\varsigma_x(t-\tau) + \beta\delta C\varsigma_{Fv}(t-\tau)\\ &\quad + \tilde{\boldsymbol{E}}[\boldsymbol{h}_v(t) - \boldsymbol{h}_v(t+\delta) + \delta\boldsymbol{h}_a(t)], \qquad (11)\end{aligned}$$

which means that

$$\begin{aligned}\varsigma(t+\delta) &= \begin{bmatrix} \boldsymbol{I}_{N-1} & \delta\boldsymbol{I}_{N-1} \\ -\alpha\delta\boldsymbol{I}_{N-1} & (1-\beta\delta)\boldsymbol{I}_{N-1} \end{bmatrix}\varsigma(t)\\ &\quad + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \alpha\delta\boldsymbol{C} & \beta\delta\boldsymbol{C} \end{bmatrix}\varsigma(t-\tau)\\ &\quad + \left(\boldsymbol{I}_2 \otimes \tilde{\boldsymbol{E}}\right)\begin{bmatrix} \boldsymbol{h}_x(t) - \boldsymbol{h}_x(t+\delta) + \delta\boldsymbol{h}_v(t) \\ \boldsymbol{h}_v(t) - \boldsymbol{h}_v(t+\delta) + \delta\boldsymbol{h}_a(t) \end{bmatrix}. \qquad (12)\end{aligned}$$

Based on Definition 1, $\varsigma(t)$ represents the time-varying formation error, that is the formation is achieved if and only if $\varsigma(t) \to 0$ as $t \to +\infty$ for any initial condition $\varsigma(t)$, $t \in [t_0-\tau, t_0]$. From (12), it holds that $\varsigma(t)$ converges to zero if and only if the system described by (6) is asymptotically stable and

$$\begin{aligned}\lim_{t\to\infty}\left(\boldsymbol{I}_2 \otimes \tilde{\boldsymbol{E}}\right)&\left(\begin{bmatrix} \boldsymbol{h}_x(t) + \delta\boldsymbol{h}_v(t) \\ \boldsymbol{h}_v(t) \end{bmatrix}\right.\\ &\left. - \begin{bmatrix} \boldsymbol{h}_x(t+\delta) \\ \boldsymbol{h}_v(t+\delta) \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{N+1} \\ \delta\boldsymbol{h}_a(t) \end{bmatrix}\right) = \mathbf{0} \qquad (13)\end{aligned}$$

is satisfied. By substitute $\tilde{\boldsymbol{E}}$ into (13), it can be obtained that

$$\begin{cases} \lim\limits_{t\to\infty}[\boldsymbol{h}_{ix}(t) + \delta\boldsymbol{h}_{iv}(t) - \boldsymbol{h}_{ix}(t+\delta)] \\ \qquad - [\boldsymbol{h}_{1x}(t) + \delta\boldsymbol{h}_{1v}(t) - \boldsymbol{h}_{1x}(t+\delta)] = \mathbf{0} \\ \lim\limits_{t\to\infty}[\boldsymbol{h}_{iv}(t) + \delta\boldsymbol{h}_{ia}(t) - \boldsymbol{h}_{iv}(t+\delta)] \\ \qquad - [\boldsymbol{h}_{1v}(t) + \delta\boldsymbol{h}_{1a}(t) - \boldsymbol{h}_{1v}(t+\delta)] = \mathbf{0}. \end{cases} \qquad (14)$$

As the agent numbered 1 is arbitrarily chosen, it can be easily verified that (14) is equivalent to (5). Thus the conclusion of Theorem 1 can be obtained. □

Remark 2: Theorem 1 solves the problems (i) and (ii) raised in the Introduction. Equation (5) in Theorem 1 is a description of the feasible time-varying formation set. It indicates that not all desired formation can be achieved. The formation which can be accomplished must meet the constraints that the components of position, velocity and acceleration are compatible without any conflicts. In fact, it is intuitive that a group of agents cannot achieve any formation due to their dynamic limitations. Asymptotically stable of condition (6) ensures the formation error converges to zero and guarantees the time-varying formation can be achieved with communication delay.

Denote $\boldsymbol{H} = \begin{bmatrix} \boldsymbol{I}_{N-1} & \delta\boldsymbol{I}_{N-1} \\ -\alpha\delta\boldsymbol{I}_{N-1} & (1-\beta\delta)\boldsymbol{I}_{N-1} \end{bmatrix}$, $\boldsymbol{P} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \alpha\delta\boldsymbol{C} & \beta\delta\boldsymbol{C} \end{bmatrix}$.

The following lemmas are introduced, which will be useful in the subsequent analysis.

Lemma 3: If $\alpha, \beta, \delta > 0$ satisfy at least one of the following three sets of inequalities,

$$\begin{cases} \beta^2 - 4\alpha \geq 0 \\ 0 < \beta\delta \leq 2 \end{cases} \quad (a),$$
$$\begin{cases} \beta^2 - 4\alpha \geq 0 \\ 2 < \beta\delta < 4 \\ \alpha\delta^2 - 2\beta\delta + 4 > 0 \end{cases} \quad (b),$$
$$\begin{cases} \beta^2 - 4\alpha < 0 \\ \alpha\delta - \beta < 0 \end{cases} \quad (c), \qquad (15)$$

then $\rho(\boldsymbol{H}) < 1$, where $\rho(\boldsymbol{H})$ represents the spectral radius of matrix $\boldsymbol{H}$.

Proof: Let λ be the eigenvalue of $\boldsymbol{H}$, that is,

$$\left|\lambda\boldsymbol{I}_{2(N-1)} - \boldsymbol{H}\right| = \begin{vmatrix} (\lambda-1)\boldsymbol{I}_{N-1} & -\delta\boldsymbol{I}_{N-1} \\ \alpha\delta\boldsymbol{I}_{N-1} & (\lambda-1+\beta\delta)\boldsymbol{I}_{N-1} \end{vmatrix} = 0. \qquad (16)$$

Case 1: If $\lambda = 1$, then it follows that

$$\det\left(\lambda\boldsymbol{I}_{2(N-1)} - \boldsymbol{H}\right) = \alpha\delta^2 = 0, \qquad (17)$$

which is a contradiction.

Case 2: If $\lambda \neq 1$, one can derive from (16) that

$$\left|\lambda\boldsymbol{I}_{2(N-1)} - \boldsymbol{H}\right| = \lambda^2 + (\beta\delta - 2)\lambda + \alpha\delta^2 - \beta\delta + 1 = 0, \qquad (18)$$

then $\lambda_{1,2} = \frac{2-\beta\delta\pm\sqrt{(\beta^2-4\alpha)\delta^2}}{2}$.

(1) If $\beta^2 - 4\alpha \geq 0$, on the basis of the definition of spectral radius of matrix, $\rho(\boldsymbol{H}) < 1$ if and only if

$$\max\left(\left|\frac{2-\beta\delta\pm\sqrt{(\beta^2-4\alpha)\delta^2}}{2}\right|\right) < 1. \qquad (19)$$

It's easy to know that (19) is equivalent to

$$\begin{cases} 2-\beta\delta \geq 0 \\ \dfrac{2-\beta\delta+\sqrt{(\beta^2-4\alpha)}\delta}{2} < 1 \end{cases}$$

or

$$\begin{cases} 2-\beta\delta < 0 \\ \dfrac{2-\beta\delta-\sqrt{(\beta^2-4\alpha)}\delta}{2} > -1. \end{cases}$$

After some calculation, it can be obtained that $\rho(\boldsymbol{H}) < 1$ if and only if $\alpha, \beta, \delta > 0$ satisfy (a) or (b) in (15).

(2) If $\beta^2 - 4\alpha < 0$, it's easy to know that $\rho^2(\boldsymbol{H}) = \alpha\delta^2 - \beta\delta + 1$ and one can obtain that$\rho(\boldsymbol{H}) < 1$ if and only if $\alpha, \beta, \delta > 0$ satisfy (c) in (15). The proof of Lemma 3 is completed □

Lemma 4: If $\rho(\boldsymbol{H}) < 1$, then there exist positive constants $M \geq 1$ and $\gamma \in (0, 1)$ such that $||\boldsymbol{H}||^{t-t_0} \leq M\gamma^{t-t_0}$, $t \geq t_0$.

Lemma 5: Inequality $\theta^{\tau+\delta} - \gamma\theta^{\tau} - l > 0$ exists at least one solution $\theta \in (\gamma, 1)$ if $1 - \gamma - l > 0$.

Lemma 6: Equation (6) has a unique equilibrium 0 as $t \to +\infty$ if the interaction graph of agents G has a spanning tree.

Proof: It is equivalent to verify

$$\lim_{t\to\infty} \left\{ (\boldsymbol{I}_{2(N-1)} - \boldsymbol{H} - \boldsymbol{P})\varsigma(t) \right\} = \boldsymbol{0}, \tag{20}$$

that is

$$\lim_{t\to\infty} \begin{bmatrix} \boldsymbol{0} & -\delta \boldsymbol{I}_{N-1} \\ \alpha\delta(\boldsymbol{I}_{N-1} - \boldsymbol{C}) & \beta\delta \boldsymbol{I}_{N-1} - \boldsymbol{C} \end{bmatrix} \begin{pmatrix} \varsigma_{Fx}(t) \\ \varsigma_{Fv}(t) \end{pmatrix} = \boldsymbol{0}. \tag{21}$$

has a unique solution $\boldsymbol{0}$ if the interconnection graph has a spanning tree.

It is easy to obtain that $\lim_{t\to\infty} \varsigma_{Fv}(t) = \boldsymbol{0}$ from (21), then we only need to prove that $\lim_{t\to\infty} \left\{ (\boldsymbol{I}_{N-1} - \boldsymbol{C})\varsigma_{Fx}(t) \right\} = \boldsymbol{0}$ if and only if $\lim_{t\to\infty} \varsigma_{Fx}(t) = \boldsymbol{0}$, that is $\boldsymbol{C}$ has no eigenvalue 1. Since the interaction graph G associated with $\boldsymbol{A}$ has a spanning tree, it indicates that matrix $\boldsymbol{A'}$ has the eigenvalue 1 with algebraic multiplicity 1 based on the definition of $\boldsymbol{A'}$ and Lemma 2. Thus 1 is not the eigenvalue of $\boldsymbol{C}$ by its definition and some simple matrix operation. The proof of Lemma 6 is completed. □

Theorem 2: Under control protocol (3), for any bounded time delay, there exist α, β and δ such that the time-varying formation of system (1) is reached asymptotically if and only if the interconnection graph G has a directed spanning tree.

Proof: (Sufficiency) By (6), one can derive that

$$\varsigma(t) = \boldsymbol{H}^q \varsigma(t_0) + \sum_{s=0}^{q-1} \boldsymbol{H}^{q-1-s} \boldsymbol{P}\varsigma(t_0 + s\delta - \tau), \tag{22}$$

where $t = t_0 + q\delta$, $q = 1, 2, \cdots$.

By Lemma 3, if $\alpha, \beta, \delta > 0$ satisfy (15), then $\rho(\boldsymbol{H}) < 1$. Noticing from Lemma 4 that there exist constants $0 < \gamma < 1$ and $M \geq 1$ such that $\|\boldsymbol{H}\|^{t-t_0} \leq M\gamma^{t-t_0}$, $t \geq t_0$. Therefore by (22), we have

$$\begin{aligned} \|\varsigma(t)\| \leq{} & M\gamma^q \|\varsigma(t_0)\| \\ & + \sum_{s=0}^{q-1} M\gamma^{q-1-s} \|\boldsymbol{P}\| \, \|\varsigma(t_0 + s\delta - \tau)\|. \end{aligned} \tag{23}$$

For $1 - \gamma - M\|\boldsymbol{P}\| > 0$, by Lemma 5, there exists a positive constant $\theta \in (\gamma, 1)$ such that

$$\theta^{\tau+\delta} - \gamma\theta^{\tau} - M\|\boldsymbol{P}\| > 0.$$

In the following, we will show that

$$\|\varsigma(t)\| \leq M\|\boldsymbol{f}\|\theta^{t-t_0}, \quad t \geq t_0, \tag{24}$$

where $\|\boldsymbol{f}\| = \sup_{t\in[t_0-\tau, t_0]} \|\varsigma(t)\|$.

It is obvious that $\|\varsigma(t)\| \leq \|\boldsymbol{f}\| \leq M\|\boldsymbol{f}\|\theta^{t-t_0}$ for $t \in [t_0 - \tau, t_0]$.

Next, we need to show for any $\eta > 1$,

$$\|\varsigma(t)\| \leq \eta M\|\boldsymbol{f}\|\theta^{t-t_0} \triangleq \varphi(t), \quad t \geq t_0. \tag{25}$$

If (25) is not true, then there must exist $t^* = t_0 + q^*\delta$, $q^* > 0$, such that $\|\varsigma(t)\| < \varphi(t)$ for $t \in [0, t^*)$ and $\|\varsigma(t^*)\| = \varphi(t^*)$.

Then by (23), one can obtain that

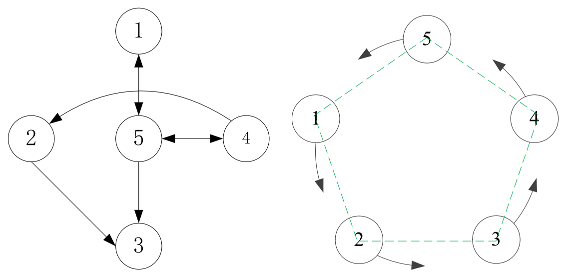
$$\begin{aligned} \varphi(t^*) ={} & \|\varsigma(t^*)\| \\ \leq{} & M\gamma^{q^*} \|\varsigma(t_0)\| \\ & + \sum_{s=0}^{q^*-1} M\gamma^{q^*-s-1} \|\boldsymbol{P}\| \cdot \|\varsigma(t_0 + s\delta - \tau)\| \\ \leq{} & M\gamma^{q^*} \|\boldsymbol{f}\| \\ & + \sum_{s=0}^{q^*-1} M\gamma^{q^*-s-1} \|\boldsymbol{P}\| \cdot (\eta M\|\boldsymbol{f}\|\theta^{s\delta-\tau}) \\ <{} & \eta M\gamma^{q^*} \|\boldsymbol{f}\| \\ & + \sum_{s=0}^{q^*-1} M\gamma^{q^*-s-1} \|\boldsymbol{P}\| \cdot (\eta M\|\boldsymbol{f}\|\theta^{s\delta-\tau}) \\ ={} & \eta M\gamma^{q^*} \|\boldsymbol{f}\| \\ & + \frac{\eta M^2 \|\boldsymbol{P}\| \|\boldsymbol{f}\| \gamma^{q^*}}{\gamma\theta^{\tau}} \sum_{s=0}^{q^*-1} \left(\frac{\theta^{\delta}}{\gamma}\right)^s \\ ={} & \eta M\|\boldsymbol{f}\| \left(\gamma^{q^*} + \frac{M\|\boldsymbol{P}\|}{\theta^{\tau}} \cdot \frac{\gamma^{q^*} - (\theta^{\delta})^{q^*}}{\gamma - \theta^{\delta}}\right). \end{aligned} \tag{26}$$

Based on Lemma 5, for $1 - \gamma - M\|\boldsymbol{P}\| > 0$, there exists a positive constant $\theta \in (\gamma, 1)$ such that $\theta^{\tau+\delta} - \gamma\theta^{\tau} - M\|\boldsymbol{P}\| > 0$, substitute into (26), it follows that

$$\begin{aligned} \varphi(t^*) <{} & \eta M\|\boldsymbol{f}\| \left(\gamma^{q^*} + \frac{M\|\boldsymbol{P}\|}{\theta^{\tau}} \cdot \frac{\gamma^{q^*} - (\theta^{\delta})^{q^*}}{\gamma - \theta^{\delta}}\right) \\ <{} & \eta M\|\boldsymbol{f}\| \left(\gamma^{q^*} + \frac{\theta^{\tau+\delta} - \gamma\theta^{\tau}}{\theta^{\tau}} \cdot \frac{\gamma^{q^*} - (\theta^{\delta})^{q^*}}{\gamma - \theta^{\delta}}\right) \\ ={} & \eta M\|\boldsymbol{f}\|\theta^{q^*\delta} \\ ={} & \varphi(t^*), \end{aligned} \tag{27}$$

which is a contradiction, therefore the assumption is invalid. That is if $\alpha, \beta, \delta > 0$ satisfy (15) and $1 - \gamma - M\|\boldsymbol{P}\| > 0$, (25) is true. Thus, for any $\eta > 1$, (25) holds. Let $\eta \to 1$, (24) holds. That is $\varsigma(t) \to 0$ as $t \to +\infty$ for any initial condition $\varsigma(t)$, $t \in [t_0 - \tau, t_0]$. Since there has a globally reachable node in graph G, then based on Lemma 6, the $\boldsymbol{0}$ equilibrium is unique. That is the error system converges to zero and the time-varying formation is achieved.

(Necessity) By way of contradiction, suppose that the graph G has no spanning tree, and then there are at least two disconnected subgraphs. Without loss of generality, we consider the special case, that is $\tau = 0$ and there are exactly two disconnected agents in graph, denoted as w_1 and w_2. Given the initial condition satisfies $x_1(t_0) = v_1(t_0) = e_1$, $x_2(t_0) = v_2(t_0) = e_2$, the desired formation is set as $\boldsymbol{h}_1(t) = \boldsymbol{h}_2(t) = \boldsymbol{0}$. Simple calculation indicates that $v_1(t) = e_1$, $x_1(t) = (1 + p\delta)e_1$, $v_2(t) = e_2$, $x_2(t) = (1 + p\delta)e_2$. If $e_1 \neq e_2$, then the time-varying formation cannot be achieved, it is in contradiction with the assumption that the formation can be achieved. That is the graph G has a spanning tree. The proof of Theorem 2 is completed. □



(a) (b)

FIGURE 2. Directed interaction topology and desired formation. (a) Interaction topology among agents. (b) Desired time-varying formation.

Remark 3: Theorem 2 solves the problem (iii) raised in the Introduction. Some necessary constraints on the gain parameters and sampling period are obtained, which give a practical way to design the parameters in the proposed protocol to achieve the feasible time-varying formation.

IV. SIMULATION STUDY AND OUTDOOR EXPERIMENT

In this section, numerical simulations and outdoor experiment with quadrotors are given to illustrate the effectiveness and validity of the theoretical results.

A. SIMULATION STUDY

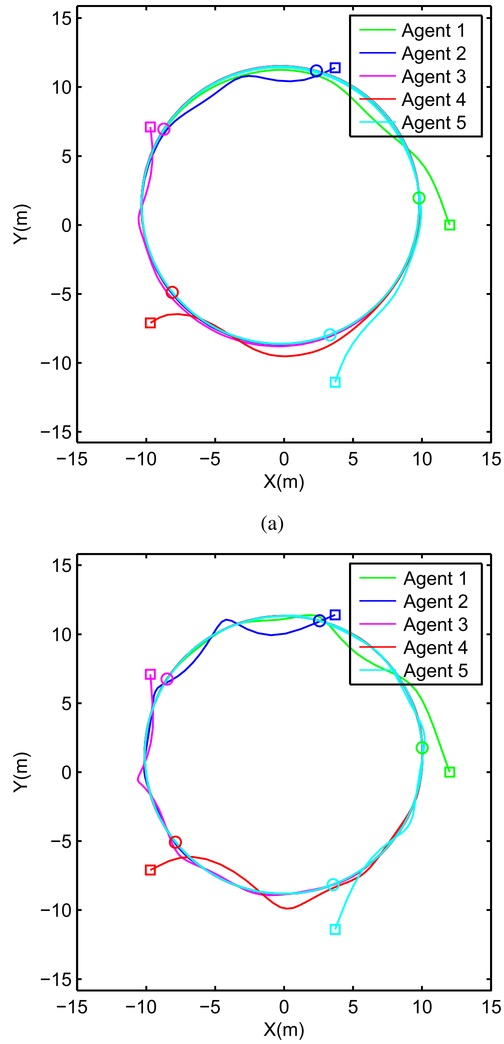
Consider a system consists of 5 agents and the interaction topology G with 0-1 weights is shown in Fig. 2(a). Clearly, G is directed and has a spanning tree. The initial positions and velocities of all the agents are set as $\boldsymbol{\xi}_i(t) = [12\cos(\frac{2\pi(i-1)}{5}), 12\sin(\frac{2\pi(i-1)}{5}), 0, 0]^T, i = 1, \cdots, 5, t \in [t_0 - \tau, t_0]$.

These five agents are supposed to approach a pentagonal formation and at the same time keep rotation around the formation reference point, as shown in Fig. 2(b). The time-varying formation is described by $\boldsymbol{h}_{ix}(0) = [10\cos(\frac{2\pi(i-1)}{5}), 10\sin(\frac{2\pi(i-1)}{5})]^T$, $\boldsymbol{h}_{iv}(t) = [-\frac{\pi}{3}\sin(\frac{2\pi t}{60} + \frac{2\pi(i-1)}{5}), \frac{\pi}{3}\cos(\frac{2\pi t}{60} + \frac{2\pi(i-1)}{5})]^T, i = 1, \cdots, 5$. On the basis of Theorem 1, let

$$\begin{cases} \boldsymbol{h}_{ix}(t+\delta) - \boldsymbol{h}_{ix}(t) - \delta\boldsymbol{h}_{iv}(t) = 0 \\ \boldsymbol{h}_{iv}(t+\delta) - \boldsymbol{h}_{iv}(t) - \delta\boldsymbol{h}_{ia}(t) = 0, \quad i = 1, \cdots, 5. \end{cases}$$

which satisfies (5), thus $\boldsymbol{h}_{ix}(t)$ and $\boldsymbol{h}_{ia}(t)$ for all $t > 0$ can be calculated.

Let $\alpha = 0.2$, $\beta = 0.5$ and $\delta = 0.5s$, after some calculation, we know that these parameters satisfy the constraints given in Lemma 3. In order to compare the performance of the algorithm under different time delays, simulations are carried out with $\tau = 0.5s$ and $\tau = 1.0s$, respectively. The trajectories of positions and velocities of all the agents are shown in Fig. 3 and Fig. 4. The squares and circles in Fig. 3 represent the initial and final positions of agents, respectively. It can be seen from Fig. 3 and Fig. 4 that the designed control protocol (3) steers all the agents to approach the desired time-varying formation successfully. Through the comparison and analysis of Fig. 3 and Fig. 4, we can find that the formation is accomplished within about $15s$ when $\tau = 0.5s$ and $30s$ when $\tau = 1.0s$, respectively. Increasing the communication delay to $1.5s$ and $2.0s$, by simulation experiments, we also find that the formation is finally accomplished within about $80s$ and $200s$, respectively. This shows that our designed protocol is valid and can effectively tolerate relatively large bounded communication delay. But with the increasing of communication delay, the convergence rate decreases and the convergence time gets appreciably longer.



(a)

(b)

FIGURE 3. Positions of agents in simulation. (a) $\tau = 0.5s$. (b) $\tau = 1.0s$.

Remark 4: Compared with existing technology, the proposed method can adapt to longer sampling period (More energy efficient and lower requirements for communication system) and larger communication delay (larger than one sampling period) while achieving the desired time-varying formation. For example, the method proposed in works by Xu *et al.* [36] only solves the problem with one sampling period delay and the desired formation is time-invariant.

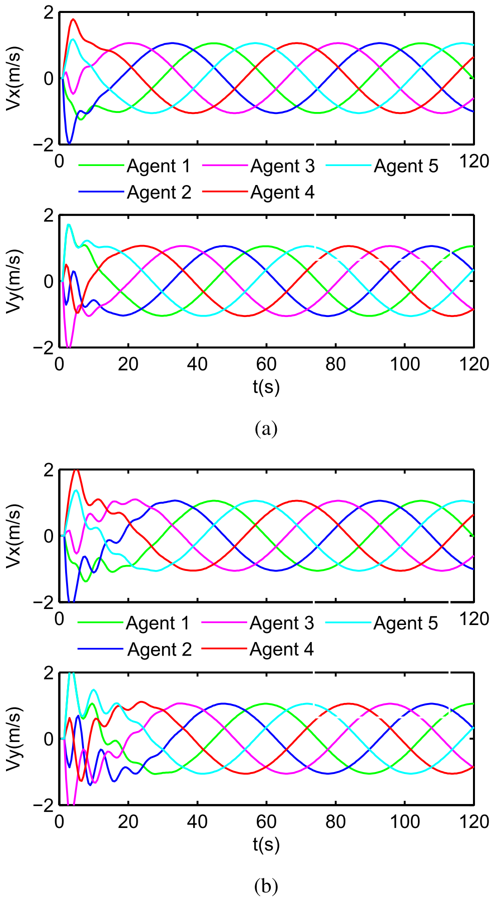


FIGURE 4. Velocities of agents in simulation. (a) $\tau = 0.5s$. (b) $\tau = 1.0s$.

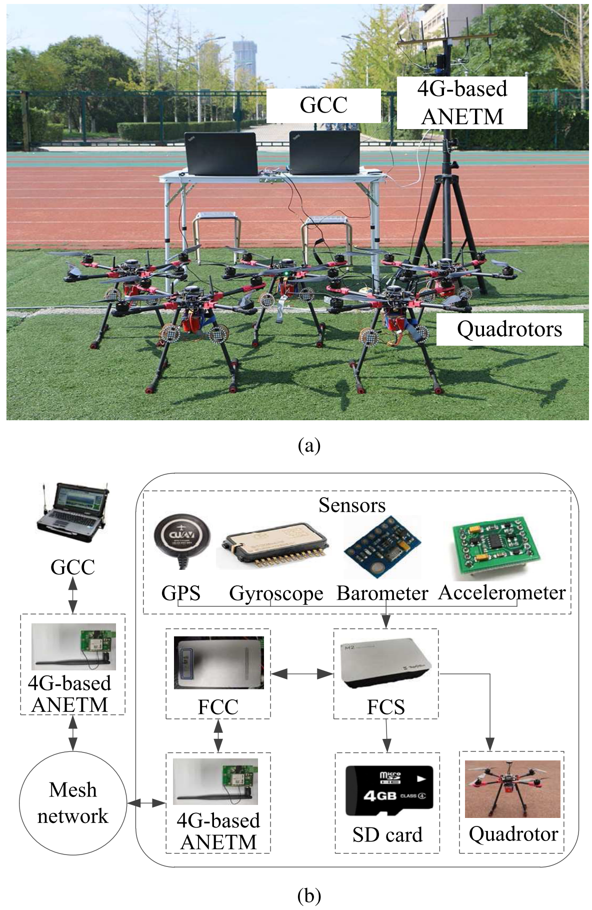


FIGURE 5. The formation flying experimental system. (a) Outdoor experimental platform. (b) Hardware structure of the system.

B. OUTDOOR EXPERIMENT WITH QUADROTORS

Outdoor experiment with quadrotors is presented to demonstrate the effectiveness of the obtained theoretical results with each UAV being regarded as an agent. Based on the hierarchical control architecture, a demonstration and verification system for formation control is constructed. Fig. 5(a) shows the experimental platform which includes one ground command center (GCC) and five quadrotors. Each of the quadrotors has been equipped with a flight control system (FCS), a formation control coordinator (FCC) and a 4G-based ad hoc networking module (ANETM), as shown in Fig. 5(b). The GCC can fly and land the quadrotors one after another or at the same time as required, hence no remote controller is needed. The FCS implements the attitude and altitude controllers and has an embedded SD card to record the key flight parameters at a rate of 10 Hz. The 4G-based ANETM is connected to FCC and used for wireless communication among quadrotors and the GCC. Through the wireless mesh network, commands and mission information can be sent to a specified quadrotor or to all quadrotors as needed, and the states of each quadrotor can be monitored by GCC in real time. Based on the commands sent by GCC and the state information received from neighbors, the control input is calculated by FCC and sent to FCS through serial port. The tip-to-tip wingspan of each quadrotor is 1200 mm. The quadrotors are powered by lithium battery and the duration is about 15 minutes. The maximum take-off weight and payload is about 3500 g and 500 g, respectively. The position and velocity in the horizontal plane (XY plane) of each quadrotor are measured by a GPS module at a rate of 10 Hz with accuracy of 1.2 m CEP.

The experimental procedure for the formation flying is as follows. First, the GCC gives TAKEOFF command to the quadrotors. The vehicles take off automatically and hover at the initial positions. Then, the GCC gives START command to the vehicles and the quadrotors start to perform the formation approaching process. The supervisor can monitor the parameters in real time through the GCC in case of emergency. When the flight test is accomplished or any unexpected situation happens, GCC gives the RETURN command to the vehicles and the quadrotors return to their respective launch positions automatically.

Due to the limitation of flying space, consider a system with 5 quadrotors. For the purpose of comparison and verification, all the initial conditions and parameters are kept the same as section 4.1. According to network conditions, the average delay of the ANETM is about 50ms-180ms, that is $\hbar = 180ms$. Thus, it can be obtained that $\tau = \delta = 0.5s$, which means only one sampling period communication delay needs to be considered. Fig. 6(a) and Fig. 6(b) depict the

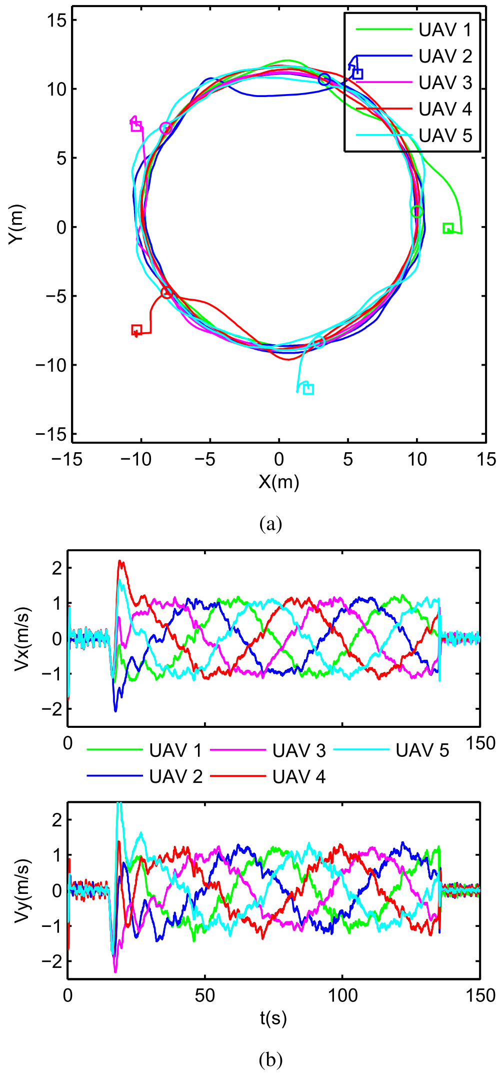


FIGURE 6. Positions and velocities of UAVs in experiment($\tau = 0.5s$). (a) Evolution of positions. (b) Evolution of velocities.

FIGURE 7. Captured image of the quadrotors in experiment.

trajectories and velocities of the quadrotors in experiment, respectively. The initial and final positions of the five quadrotors are marked by squares and circles. It can be observed that the time-varying formation is asymptotically approached. Fig. 7 shows a captured image of the formation in the outdoor flying experiment. Due to the sensor errors and external disturbances, etc., there are some errors between the experimental results and the simulation results, which are acceptable for practical applications.

V. CONCLUSIONS

Time-varying formation control problems for second-order discrete-time multi-agent systems with communication delay are investigated, where the interaction topology is directed and the desired formation is time-varying. By utilizing the delayed state of neighbors, a local state information based distributed formation control protocol is proposed. An explicit description of the feasible time-varying formation set is given. Necessary and sufficient conditions for the systems with directed topology and uniformed communication delay to achieve time-varying formation are obtained, which are related to the topology of the interaction graph and the feasibility of the predefined formation. Necessary constraints on the gain parameters and sampling period are proposed as guidance to the design of parameters in the protocol. Numerical simulations show that the proposed protocol can steer the agents to accomplish the desired time-varying formation and effectively tolerate relatively large bounded communication delay. But with the increasing of communication delay, the convergence rate decreases and the convergence time gets appreciably longer. Finally, outdoor experiment with quadrotors is presented to verify the effectiveness of the obtained theoretical results with one sampling period delay.

There are still a number of issues need to be further investigated and extensions to systems with switching topologies and time-varying delays are currently under investigation. Another thing needs to be discussed in the future is that other constraints such as measurement error, external disturbance and input saturation should be taken into account.

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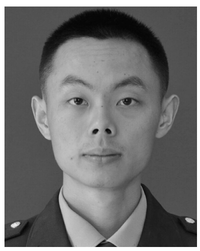

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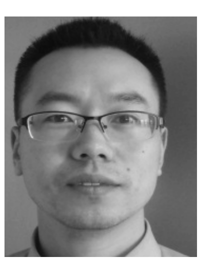

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