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Sparsity-Inducing DOA Estimation of Coherent Signals Under the Coexistence of Mutual Coupling and Nonuniform Noise

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ABSTRACT The sparse reconstruction techniques can improve the accuracy and resolution of the direction of arrival (DOA) estimation using sensor arrays. However, due to reflective objects and nonidealities of the antennas and circuitry, the received signals may be coherent and coupled to each other in nonuniform noise environments, causing severe performance degradation of the signal sparse reconstruction. In this paper, a novel sparsity-inducing DOA estimation method is proposed to adapt to such a challenging scenario. To mitigate the nonuniform noise, its power components are first eliminated by a linear transformation. Then, leveraging the steering vector parametrization based on the banded symmetric Toeplitz structure of the mutual coupling matrix (MCM), a reweighted ℓ_1-norm minimization subject to an error-constrained ℓ_2-norm is designed to determine the DOA estimates, further enhancing the sparsity and providing robustness against the noise. In addition, a new stochastic Cramér–Rao lower bound (CRLB) of the DOA estimation is derived for the considered adverse condition. The simulation results demonstrate the superiority of the proposed method over its state-of-the-art counterparts.

INDEX TERMS Direction of arrival (DOA), mutual coupling, coherent signals, nonuniform noise, sparse reconstruction.

I. INTRODUCTION

Prevailing high-resolution algorithms for direction of arrival (DOA) estimation using antenna arrays, such as MUSIC [1] and ESPRIT [2], work well provided that the array manifold is known *a priori* or well calibrated, and there is no correlation between incident signals. In practice, however, antennas in massive MIMO systems may be closely spaced, and mutual coupling, an intrinsic characteristic between array elements, becomes prominent. Besides, the source signals may undergo multipath propagation that is very common in urban areas, and the resultant multipath signals become coherent to each other accordingly. These nonidealities can significantly deteriorate the estimation performance, and much effort has been devoted to tackle the problem of DOA estimation of coherent signals in the presence of unknown mutual coupling.

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For uniform linear arrays (ULAs) where each sensor is only coupled with a portion of the whole array, "middle subarray" has been recognized to preserve the Vandermonde structure of the array response and be mutual coupling-free [3]–[6], so various rank recovery methods in conjunction with this property are developed to handle the coherency between signals [7]–[9]. However, Dai and Ye [7] utilized forward spatial smoothing only to reconstruct the deficient rank and ignored the conjugate information, resulting in a loss of half of DOFs. Toeplitz matrix reconstruction directly in data domain [8] showed good performance for real-time applications in decorrelating coherent signals without mutual coupling compensation, but at the cost of halving the effective array aperture that is very limiting in practice. To ameliorate the drawbacks mentioned above, a matrix reconstruction method was proposed to improve effective aperture after rank restoration [9], but still constrained by the size of the "middle subarray".

Recently, some novel approaches have been proposed to deal with the issue via exploiting spatial sparsity [10]–[12], that exhibits remarkable superiority in resolution and robustness to coherency. In [10], ℓ_1-SVD is applied to obtain DOA estimates with the help of the ''middle array'' in ULAs. Liu and Zhou first address the DOA estimation problem with three array perturbations under a unified sparse Bayesian learning framework [11]. This work is followed by Liu *et al.*, and a joint estimation of DOAs and array perturbations is solved by sparse matrix completion. More recently, some new sparse DOA estimation algorithms are developed to withstand the mutual coupling effects [12]–[16]. An improved ℓ_1-SVD method is proposed in [12] by using a transformation of the real steering vector with the structured mutual coupling of the whole array, introduced in [4], to form a new dictionary for sparse reconstruction, and performance improvements are achieved since the information of whole array is exploited. Following a quite distinct transformation of the real steering vector derived in [17] and replacing the ℓ_1-SVD with the simultaneous orthogonal matching pursuit (SOMP) [13] and sparse Bayesian learning [14], respectively, with multiple measurement vectors (MMVs), Chen *et al.* provide two off-grid compressed sensing based methods to tackle adverse coupling effects, further improving the estimation accuracy and resolution. Taking advantage of the steering vector transformation in [4], Meng *et al.* [15] present a DOA estimation approach against the unknown coupling based on block sparse recovery of the array covariance vectors by the popular weighted ℓ_1-norm penalty. However, the weighting matrix is constructed by the principle of the Capon estimator, and it may cause a limited resolution and poor estimation performance at low signal-to-noise-ratios (SNRs). By leveraging the uncorrelation between signals and the same transformation of the coupled steering vector, we cast the original problem as a group sparsity reconstruction of an SMV problem, and the developed solution takes advantage of a larger array aperture and mitigates the noise effects, allowing more accurate DOA estimates to be resolved [16].

It should be noted that the aforementioned methods are generally restricted to uniform white Gaussian noise environments. On the other hand, in many applications, the noise power in each channel is no longer identical, i.e., the noise becomes nonuniform, due to the nonhomogeneity of the hardware in receiving channels or prevailing external noise received by the array elements [18]–[20]. However, to the best of our knowledge, the problem of DOA estimation for coherent signals under the coexistence of unknown mutual coupling and nonuniform noise has not been investigated. To bridge this gap, in this letter, we address the issue from a sparse reconstruction perspective. The nonuniform noise is first mitigated by removing its power components. Then, considering the banded symmetric Toeplitz structure of the MCM, a novel dictionary is constructed by parameterizing the virtual steering vector, and a newly designed reweighted ℓ_1-norm minimization subject to an error-constrained ℓ_2 norm is carried out to determine the DOA estimates. In addition, we provide the stochastic CRLB for the coherent signals under the coexistence of mutual coupling and nonuniform noise. The CRLB derivation herein is a natural extension of the well-known results given by Weiss and Friedlander [21] for the case of uniform noise. Simulation results illustrate that the proposed algorithm outperforms its sparsity-inducing counterparts.

The remainder of this paper is organized as follows. In Section II, an array model for coherent signals perturbed by unknown mutual coupling in nonuniform noise is introduced. In Section III, a robust sparsity-aware DOA estimator using weighted ℓ_1-norm minimization of reconstructed covariance vectors is developed. Section IV provides numerical examples for demonstrating the validity and efficiency of our proposed method. Finally, some concluding remarks are given in Section.

Throughout this paper, the following notations will be used: the operators $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^H$, $(\cdot)^+$, $\otimes$, $\odot$, $\mathrm{tr}\{\cdot\}$, $\|\cdot\|_1$, and $\|\cdot\|_2$ denote the operation of transpose, conjugate, conjugate transpose, pseudo-inverse, Kronecker product, Schur-Hadamard product, trace, ℓ_1 norm, and Euclidean (ℓ_2) norm, respectively. The symbol $\mathrm{diag}\{z_1, \cdots, z_N\}$ represents a diagonal matrix with diagonal entries $z_1, \cdots, z_N$ and $\mathrm{blkdiag}\{\mathbf{Z}_1, \mathbf{Z}_2\}$ represents a block diagonal matrix with diagonal entries $\mathbf{Z}_1, \mathbf{Z}_2$. The symbol $\mathbf{I}_K$ stands for an identity matrix of size $K \times K$.

II. PROBLEM FORMULATION

Consider K groups of coherent signals impinging on a ULA with M identical omnidirectional sensors. In the k-th coherent group, the signals coming from direction θ_{kp}, $p = 1, 2, \cdots, P_k$ corresponds to the p-th multipath propagation of the signal source $s_k(t)$, and the complex fading coefficient is α_{kp}. It is apparent that the total number of coherent signals is $N = \sum_{k=1}^{K} P_k$. If there exits mutual coupling between the array elements, the corresponding $M \times 1$ array observation vector is then given by

$$\begin{aligned}\mathbf{x}(t) &= \sum_{k=1}^{K}\sum_{p=1}^{P_k} \mathbf{C}\mathbf{a}(\theta_{kp})\alpha_{kp}s_i(t) + \mathbf{n}(t) \\ &= \mathbf{C}\mathbf{A}\boldsymbol{\Gamma}\mathbf{s}(t) + \mathbf{n}(t)\end{aligned} \tag{1}$$

where $\mathbf{a}(\theta) = \left[1, \beta(\theta), \cdots, \beta^{M-1}(\theta)\right]^T \in \mathbb{C}^M$ is the steering vector, $\beta(\theta) = e^{j\frac{2\pi d}{\lambda}\sin\theta}$ with λ and d being the wavelength of carrier signal and the spacing between adjacent elements, respectively, $\mathbf{C}$ denotes the MCM, $\mathbf{A} = [\mathbf{a}(\theta_1), \cdots, \mathbf{a}(\theta_N)]$ is the array manifold, $\boldsymbol{\Gamma} = \mathrm{blkdiag}\{\boldsymbol{\alpha}_1, \cdots, \boldsymbol{\alpha}_K\}$ with $\boldsymbol{\alpha}_k = \left[\alpha_{k1}, \cdots, \alpha_{kP_k}\right]^T$ containing attenuation information of the k-th coherent group where $\mathrm{blkdiag}\{\cdot\}$ represents a block diagonal matrix, and $\mathbf{s}(t) = [s_1(t), \cdots, s_N(t)]^T$. We assume that $\mathbf{n}(t)$ is a nonuniform zero-mean Gaussian noise vector and uncorrelated to $\mathbf{s}(t)$. From (1), the array covariance matrix is given by

$$\mathbf{R}_x = E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] = \mathbf{C}\mathbf{A}\boldsymbol{\Gamma}\mathbf{R}_s\boldsymbol{\Gamma}^H\mathbf{A}^H\mathbf{C}^H + \mathbf{Q} \tag{2}$$

where $\mathbf{R}_s = E\left[\mathbf{s}(t)\mathbf{s}^H(t)\right] = \text{diag}\left\{\gamma_1^2, \gamma_2^2, \cdots, \gamma_N^2\right\}$ is a diagonal matrix containing the signal powers $\left\{\gamma_i^2\right\}_{i=1}^N$, and $\mathbf{Q} = E\left[\mathbf{n}(t)\mathbf{n}^H(t)\right] = \text{diag}\left\{\sigma_1^2, \sigma_2^2, \cdots, \sigma_M^2\right\}$ is also a diagonal matrix with σ_m^2 being the noise power at the m-th antenna.

As described in [3]–[12], [15], and [16], it is often sufficient to consider the ULA coupling model that has just finite non-zero coefficients, and a banded symmetric Toeplitz matrix can be used to model mutual coupling. By assuming that the largest interelement spacing contributing to mutual coupling to be P, the MCM can be expressed as

$$\mathbf{C} = \text{Toeplitz}\left\{\left[1, c_1, \cdots, c_{P-1}, \mathbf{0}_{1\times(M-P)}\right]\right\} \tag{3}$$

where Toeplitz $\{\mathbf{r}\}$ denotes a symmetric Toeplitz matrix constructed by the vector $\mathbf{r}$, and $0 < |c_1|, |c_2|, \cdots, |c_{P-1}| < c_0 = 1$ are the mutual coupling coefficients.

III. PROPOSED DOA ESTIMATOR USING SPARSE RECONSTRUCTION

A. NONUNIFORM NOISE MITIGATION

Denoting

$$\mathbf{R}_x = \mathbf{CAP} + \mathbf{Q} \tag{4}$$

where $\mathbf{P} = \mathbf{\Gamma R}_s\mathbf{\Gamma}^H\mathbf{A}^H\mathbf{C}^H$, and vectorizing $\mathbf{R}_x$, one has

$$\begin{aligned}\mathbf{y} &= \text{vec}\left(\mathbf{R}_x\right) \\ &= \left(\mathbf{I}_M \otimes (\mathbf{CA})\right)\text{vec}\left(\mathbf{P}\right) + \mathbf{1}_{M^2}\end{aligned} \tag{5}$$

where $\text{vec}\left(\mathbf{P}\right) = \left[\mathbf{p}_1^T, \cdots, \mathbf{p}_M^T\right]^T$ where $\mathbf{p}_i \in \mathbb{C}^N$ is the i-th column of $\mathbf{P}$, and $\mathbf{1}_{M^2} = \left[\sigma_1^2\mathbf{e}_1^T, \cdots, \sigma_M^2\mathbf{e}_M^T\right]^T$ where $\mathbf{e}_i \in \mathbb{R}^M$ is a column vector with 1 at the i-th entry and 0 elsewhere.

Carefully examining the position of σ_i^2, we find that σ_i^2 is located at the $((i-1)(M+1)+1)$-th entry of $\mathbf{y}$, so the noise components can be eliminated by performing a linear transformation on $\mathbf{y}$ as

$$\bar{\mathbf{y}} = \mathbf{Jy} = \mathbf{J}\left(\mathbf{I}_M \otimes (\mathbf{CA})\right)\text{vec}\left(\mathbf{P}\right) \tag{6}$$

where $\mathbf{J} = \left[\text{blkdiag}\{\mathbf{J}_1, \mathbf{J}_2, \cdots, \mathbf{J}_Q\}, \mathbf{0}_{Q(M-1)\times M(M-Q)}\right] \in \mathbb{R}^{Q(M-1)\times M^2}$ is a selection matrix with $\mathbf{J}_q = [\mathbf{e}_1, \cdots, \mathbf{e}_{i-1}, \mathbf{e}_{i+1}, \cdots, \mathbf{e}_M]^T \in \mathbb{R}^{(M-1)\times M}$ where $\mathbf{e}_i \in \mathbb{R}^M$ is a column vector with 1 at the i-th entry and 0 elsewhere. Here, Q indicates how many columns of $\mathbf{R}_x$ we use and $Q \leq M$.

In order to combat effect of unknown mutual coupling in the ULA, we reparameterize the coupled steering vector as [4]

$$\mathbf{Ca}(\theta) = \mathbf{T}(\theta)\boldsymbol{\alpha}(\theta) \tag{7}$$

where $\mathbf{T}(\theta) \in \mathbb{C}^{M\times(2P-1)}$ and $\boldsymbol{\alpha}(\theta) \in \mathbb{C}^{2P-1}$ are given by

$$\mathbf{T}(\theta) = \text{blkdiag}\left\{\mathbf{T}_1, \mathbf{T}_2, \mathbf{T}_3\right\} \tag{8}$$

$$\boldsymbol{\alpha}(\theta) = \left[\mu_1, \cdots, \mu_{P-1}, \tau(\theta), \alpha_1, \cdots, \alpha_{P-1}\right]^T \tag{9}$$

with

$$\mu_l = 1 + \sum_{k=1}^{P-1} c_k\beta^k(\theta) + \sum_{k=1}^{l-1} c_k\beta^{-k}(\theta) \tag{10}$$

$$\alpha_l = 1 + \sum_{k=1}^{P-1} c_k\beta^{-k}(\theta) + \sum_{k=1}^{P-1-l} c_k\beta^{k}(\theta) \tag{11}$$

$$\tau(\theta) = 1 + \sum_{k=1}^{P-1} c_k\left(\beta^k(\theta) + \beta^{-k}(\theta)\right) \tag{12}$$

$$\mathbf{T}_1 = \text{diag}\left\{1, \beta(\theta), \cdots, \beta^{P-2}(\theta)\right\} \in \mathbb{C}^{(P-1)\times P-1)} \tag{13}$$

$$\mathbf{T}_2 = \left[\beta^{P-1}(\theta), \cdots, \beta^{M-P}(\theta)\right]^T \in \mathbb{C}^{M-2P+2} \tag{14}$$

$$\mathbf{T}_3 = \text{diag}\left\{\beta^{M-P+1}(\theta), \cdots, \beta^{M-1}(\theta)\right\} \in \mathbb{C}^{(P-1)\times P-1)} \tag{15}$$

and generally $\tau(\theta)$ in (9) is assumed to be nonzero as suggested in [4].

Substituting (7) back to (4), one has

$$\bar{\mathbf{y}} = \mathbf{Bg} \tag{16}$$

where $\mathbf{B} = \mathbf{J}\left(\mathbf{I}_M \otimes [\mathbf{T}(\theta_1), \cdots, \mathbf{T}(\theta_N)]\right)$ and $\mathbf{g} = \left(\mathbf{I}_M \otimes \text{blkdiag}\left\{\boldsymbol{\alpha}(\theta_1), \cdots, \boldsymbol{\alpha}(\theta_N)\right\}\right)\text{vec}\left(\mathbf{P}\right)$. In practice, only finite samples are available, and the unknown $\bar{\mathbf{y}}$ is estimated from L snapshots, i.e., $\hat{\bar{\mathbf{y}}} = \mathbf{J}\text{vec}\left(\hat{\mathbf{R}}_x\right)$ where $\hat{\mathbf{R}}_x = \frac{1}{L}\sum_{t=1}^L \mathbf{x}(t)\mathbf{x}^H(t)$.

B. DOA ESTIMATION USING SPARSE RECONSTRUCTION

It is noted that the $\bar{\mathbf{y}}$ can be sparsely represented in the spatial domain over the entire angular grid as

$$\bar{\mathbf{y}} = \mathbf{B}\left(\boldsymbol{\Theta}\right)\bar{\mathbf{p}} \tag{17}$$

where $\mathbf{B}(\boldsymbol{\Theta})$, an over-complete dictionary, is defined as a collection of virtual steering matrices $\mathbf{J}(\mathbf{I}_M \otimes [\mathbf{T}(\tilde{\theta}_1), \cdots, \mathbf{T}(\tilde{\theta}_G)])$ over the entire possible grids with $\boldsymbol{\Theta} = \{\tilde{\theta}_1, \cdots, \tilde{\theta}_G\}$. We assume that the true DOAs are exactly on the sampling grids $\boldsymbol{\Theta}$ and $G \gg M$.

It is important to note that the angular positions of the signal arrivals $\theta_i, i = 1, \cdots, N$, are indicated by the non-zero entries in $\bar{\mathbf{p}}$, whose values describe the corresponding coefficients associated with $\mathbf{g}$. It is useful to introduce a vector $\mathbf{p}^{\text{o}} = [p_1^{\text{o}}, p_2^{\text{o}}, \cdots, p_G^{\text{o}}]^T$ with the g-th entry p_g^{o} equal to the ℓ_2-norm of $Q(2P-1)$ entries, from the $(G(q-1)+(g-1))(2P-1)+1$-th to the $(G(q-1)+g)(2P-1)$-th, $q = 1, \cdots, Q$, i.e., $p_g^{\text{o}} = (\sum_{q=1}^{Q}\sum_{q=(G(q-1)+(g-1))(2P-1)+1}^{(G(q-1)+g)(2P-1)} \bar{p}_r^2)^{\frac{1}{2}}$ where $\bar{p}_r$ is the r-th entry in $\bar{\mathbf{p}}$. Generally, the non-zero entries of $\bar{\mathbf{p}}_g = [\bar{p}_{(g-1)(2P-1)+1}, \cdots, \bar{p}_{g(2P-1)}, \cdots, \bar{p}_{(G(Q-1)+(g-1))(2P-1)+1}, \cdots, \bar{p}_{(G(Q-1)+g)(2P-1)}]^T$ take different values to each other but share the same positions because they correspond to the DOAs of the same N signals. Therefore, $\bar{\mathbf{p}}_g$ exhibits a group sparsity that can be coherently described by $\mathbf{p}^{\text{o}}$ with the same sparse structure and hence, seeking a sufficiently sparse $\mathbf{p}^{\text{o}}$ will make $\{\bar{\mathbf{p}}_g\}_{g=1}^G$ consistently fit $\hat{\bar{\mathbf{y}}}$ as sparsely as possible in a manner such that all the entries in $\bar{\mathbf{p}}_g$ tend to be zero or nonzero simultaneously. The DOA estimation of coherent signals with unknown mutual coupling and nonuniform noise can be solved by detecting the locations of nonzero entries of $\mathbf{p}^{\text{o}}$.

Theoretically, one should choose the ℓ_0-norm as an ideal measure of sparsity. However, the ℓ_0-norm minimization problem is nonconvex, NP-hard, and mathematically intractable. Following [22]–[24] and considering the effect of finite snapshots, one can relax this to a simpler reweighted ℓ_1-norm constrained optimization problem, that is,

$$\min_{\mathbf{p}^o} \|\mathbf{W}\mathbf{p}^o\|_1, \qquad \text{s.t.} \left\|\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}\right\|_2^2 \leq \alpha \tag{18}$$

where $\mathbf{W}$ is a diagonal matrix containing weights $\{w_g\}_{g=1}^G$, which can more closely resemble the ℓ_0-norm and further enforce the sparsity, and α is the threshold parameter which determines the upper bound of the fitting error. The selection of $\mathbf{W}$ and α, which guarantees robust sparse recovery, is specified in the following section.

The weights $\{w_g\}_{g=1}^G$, diagonal entries of $\mathbf{W}$, play an essential role in the sparse reconstruction since they tune the entries of $\mathbf{p}^o$. An inappropriate selection of w_g may produce prominent pseudo spikes in the spatial spectrum. A selection guidance we find intuitively plausible is that w_g can adaptively penalize nonzero entries in the sparse vector $\mathbf{p}^o$, that is, large weights punish the entries which are more likely to be zeros, whereas small weights preserve the large entries. Besides, these weights should adapt to the signal environments. By this means, the introduced $\mathbf{W}$ can enhance the sparse solution. To this end, we begin with the selected covariance vector $\mathbf{G}_i\mathbf{y} = \bar{\mathbf{A}}\mathbf{P}\mathbf{e}_i$, where $\mathbf{G}_i = \left[\mathbf{0}_{\bar{M}\times(M(i-1)+P-1)}, \mathbf{I}_{\bar{M}}, \mathbf{0}_{\bar{M}\times(M^2-Mi+P-1)}\right] \in \mathbb{R}^{\bar{M}\times M^2}$ with $\bar{M} = M-2P+2$ and $\bar{\mathbf{A}} = \bar{\mathbf{G}}\mathbf{A}\mathbf{D}$ with $\bar{\mathbf{G}} = \left[\mathbf{I}_{\bar{M}}, \mathbf{0}_{\bar{M}\times(2P-2)}\right]$ and $\mathbf{D} = \text{diag}\left\{\tau(\theta_1)\beta^{P-1}(\theta_1), \cdots, \tau(\theta_N)\beta^{P-1}(\theta_N)\right\}$, since the "middle subarray", of size $M - 2P + 2$, preserves the Vandermonde structure of the array response [3], [4], and $\mathbf{G}_i\mathbf{y}$ is immune to the mutual coupling and nonuniform noise effects when $\{i\}_{i=1}^{P-1} \cup \{i\}_{i=M-P+2}^{M}$. Then an average matrix can be constructed as

$$\begin{aligned}\bar{\mathbf{R}} &= \sum_{i=1}^{P-1} \mathbf{G}_i\mathbf{y}\mathbf{y}^H\mathbf{G}_i^T + \mathbf{G}_{M-P+1+i}\mathbf{y}\mathbf{y}^H\mathbf{G}_{M-P+1+i}^T \\ &= \bar{\mathbf{A}}\left(\sum_{i=1}^{P-1} \mathbf{P}\mathbf{e}_i\mathbf{e}_i^T\mathbf{P}^H + \mathbf{P}\mathbf{e}_{M-P+1+i}\mathbf{e}_{M-P+1+i}^T\mathbf{P}^H\right)\bar{\mathbf{A}}^H \\ &= \bar{\mathbf{A}}\bar{\mathbf{R}}_s\bar{\mathbf{A}}^H \end{aligned} \tag{19}$$

where $\bar{\mathbf{R}}_s = \left(\sum_{i=1}^{P-1} \mathbf{P}\mathbf{e}_i\mathbf{e}_i^T\mathbf{P}^H + \mathbf{P}\mathbf{e}_{M-P+1+i}\mathbf{e}_{M-P+1+i}^T\mathbf{P}^H\right)$. It should be noted that $\bar{\mathbf{R}}$ is rank deficient. To recover the rank deficiency, one can perform the well-known forward-backward spatial smoothing [25] on $\bar{\mathbf{R}}$ as

$$\begin{aligned}\bar{\mathbf{R}}_{fb} &= \frac{1}{2l}\sum_{i=1}^{l} \mathbf{E}_i\left(\bar{\mathbf{R}} + \mathbf{F}\bar{\mathbf{R}}^*\mathbf{F}^T\right)\mathbf{E}_i^T \\ &= \mathbf{E}_1\bar{\mathbf{A}}\left[\sum_{i=1}^{l} \boldsymbol{\Phi}^{i-1}\bar{\mathbf{R}}_s\boldsymbol{\Phi}^{1-i} + \boldsymbol{\Phi}^{2-\bar{m}-i}\bar{\mathbf{R}}_s^*\boldsymbol{\Phi}^{i+\bar{m}-2}\right] \\ &\quad \times \bar{\mathbf{A}}^H\mathbf{E}_1^T \end{aligned} \tag{20}$$

where $\mathbf{E}_i = \left[\mathbf{0}_{\bar{m}\times(i-1)}, \mathbf{I}_{\bar{m}}, \mathbf{0}_{\bar{m}\times(l-i)}\right]$, $\bar{m}$ and l are the number of elements of a subarray and smoothing times, respectively, satisfying $\bar{m} = \bar{M} + 1 - l$, and $\mathbf{F} \in \mathbb{R}^{\bar{M}\times\bar{M}}$ denotes an exchange matrix that has unity entries on the cross diagonal and zeros elsewhere. Referring to [25], one knows that the rank deficiency has been successfully recovered, i.e., rank $\left\{\bar{\mathbf{R}}_{fb}\right\} = N$, provided that $\bar{m} \geq N+1$ and $2l \geq P_{max}$ where $P_{max} = \max\{P_1, P_2, \cdots, P_K\}$. Then we propose to make w_g the following form

$$w_g = \left\|\bar{\mathbf{U}}_n^H\breve{\mathbf{a}}\left(\tilde{\theta}_g\right)\right\|_2, \quad g = 1, 2, \cdots, G \tag{21}$$

where $\bar{\mathbf{U}}_n$ is the eigenvector matrix associated with the smallest $\bar{m} - N$ eigenvalues of $\bar{\mathbf{R}}_{fb}$ and $\breve{\mathbf{a}}(\theta) = [1, \beta(\theta), \cdots, \beta^{\bar{m}-1}(\theta)]^T$. Without loss of generality, the overcomplete dictionary $\mathbf{B}(\boldsymbol{\Theta})$ can be partitioned into two matrices along the columns, one, denoted as $\mathbf{B}_1$, is assumed to contain the N steering matrices of the true DOAs, and the other, denoted as $\mathbf{B}_2$, consists of the remaining steering matrices corresponding to the zero entries in the recovered vector $\bar{\mathbf{p}}$. Denote $\mathbf{w} = [w_1, \cdots, w_G]/\max\{w_1, \cdots, w_G\} = [\mathbf{w}_1, \mathbf{w}_2]/\max\{w_1, \cdots, w_G\}$. As only $\mathbf{B}_1$ is the array manifold of the true DOAs, its corresponding weights in $\mathbf{w}_1/\max\{w_1, \cdots, w_G\}$ should be smaller than those in $\mathbf{w}_2/\max\{w_1, \cdots, w_G\}$. In particular, $\mathbf{w}_1/\max\{w_1, \cdots, w_G\} \rightarrow \mathbf{0}$ when the number of snapshots $L \rightarrow \infty$. The weighting matrix can be constructed as $\mathbf{W} \triangleq \text{diag}\{\mathbf{w}\}$ accordingly, and an adaptive penalization of nonzero entries in $\mathbf{p}^o$ is achieved.

Referring to [26], if $\bar{\mathbf{p}}$ is exactly reconstructed, $\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}$ is amenable to asymptotically normal (AsN) distribution

$$\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}} \sim \text{AsN}\left(\mathbf{0}_{Q(M-1)\times 1}, \bar{\mathbf{R}}\right) \tag{22}$$

where $\bar{\mathbf{R}} = \frac{1}{L}\mathbf{J}\left(\mathbf{R}^T \otimes \mathbf{R}\right)\mathbf{J}^T$, so it can be deduced that

$$\bar{\mathbf{R}}^{-\frac{1}{2}}\left(\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}\right) \sim \text{AsN}\left(\mathbf{0}_{Q(M-1)\times 1}, \mathbf{I}_{Q(M-1)}\right) \tag{23}$$

which directly results in

$$\left\|\bar{\mathbf{R}}^{-\frac{1}{2}}\left(\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}\right)\right\|_2^2 \sim \text{As}\chi^2\left(Q(M-1)\right) \tag{24}$$

where $\text{As}\chi^2\left(Q(M-1)\right)$ represents the asymptotic chi-square distribution with $Q(M-1)$ DOFs. We introduce the parameter β such that the inequality $\left\|\bar{\mathbf{R}}^{-\frac{1}{2}}\left(\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}\right)\right\|_2^2 \leq \beta$ is satisfied with a high probability $\tilde{p}$, that is,

$$\Pr\left\{\chi^2_{Q(M-1)} \leq \beta\right\} = \tilde{p}, \quad \beta = \chi^2_{\tilde{p}}\left(Q(M-1)\right) \tag{25}$$

where $\Pr\{\cdot\}$ denotes the probability of an event. Let $\hat{\bar{\mathbf{R}}} = \frac{1}{L}\mathbf{J}\left(\hat{\mathbf{R}}^T \otimes \hat{\mathbf{R}}\right)\mathbf{J}^T$ be the estimate of $\bar{\mathbf{R}}$. Combining the above derivation and analysis, we arrive at a statistically robust and more tractable formula for DOA estimation as follows

$$\min_{\mathbf{p}^o} \|\mathbf{W}\mathbf{p}^o\|_1, \quad \text{s.t.} \left\|\hat{\bar{\mathbf{R}}}^{-\frac{1}{2}}\left(\hat{\bar{\mathbf{y}}} - \mathbf{B}(\boldsymbol{\Theta})\bar{\mathbf{p}}\right)\right\|_2 \leq \sqrt{\beta}. \tag{26}$$

It is apparent that problem (26) is a second-order cone programming problem and can be efficiently solved by an off-the-shelf optimization solver such as SeDuMi [27].

The major computations involved in the proposed method are to perform the construction of the covariance matrix $\hat{\mathbf{R}}_x$, the eigen-decomposition of $\bar{\mathbf{R}}_{fb}$, a one-dimensional spectral search of $\left\|\bar{\mathbf{U}}_n^H \breve{\mathbf{a}}\left(\tilde{\theta}_g\right)\right\|_2$, and sparse reconstruction in (26), and it thus requires $\mathcal{O}\left(LM^2 + \bar{m}^3 + G\bar{m}^2(\bar{m} - N) + Q^3G^3\right)$ flops in total. Here, a flop stands for a complex-valued floating point multiplication operation, whereas the computational load of other sparse recovery approaches in [10] and [12] are $\mathcal{O}\left(L(M-2P+2)^2 + LN(M-2P+2) + N^3G^3\right)$ and $\mathcal{O}\left(LM^2 + MLN + N^3(2P-1)^3G^3\right)$, respectively.

It is worth noting that the proposed method is developed on the prerequisite $\tau(\theta) \neq 0$. If the condition is not satisfied, i.e., the so-called "blind angle" occurs, the middle subarray cannot receive any signals from those blind angles, resulting in a failure of detection as discussed in [3]. For the issues investigated in this paper, leveraging the whole array of a ULA has a similar problem. As a result, when the incident signals come from any of the blind angles, the proposed algorithm as well as its state-of-the-art counterparts, e.g., [10], [12], [15], [16], fails to identify the DOAs, please refer to [3] and [10] for detailed discussion.

IV. SIMULATION RESULTS AND DISCUSSION

In this section, we present simulation results to show the performance of the proposed sparsity-aware estimator. Simulations are carried out for an eight-element ULA with half-wavelength spacing between adjacent elements. It is assumed that $P = 2$ and the mutual coupling coefficient is $c_1 = 0.1545 + 0.4755j$. The noise is spatially nonuniform and its covariance matrix is $\mathbf{Q} = \mathrm{diag}\{10, 12, 1.5, 2.5, 5, 1, 1.5, 3\}$. The approaches in [10] and [12], referred to as Dai's method and Wang's method, respectively, and the Cramér-Rao lower bound (CRLB) of DOA estimates derived in Appendix are chosen for comparison. To reduce the computational complexity and improve the estimation accuracy, we first use a coarse grid in the range of $[-90°, 90°]$ with a step size of $1°$, and then set a refined grid spacing of $0.02°$ around the estimated peaks. The probability $\tilde{p}$ for β in all the three algorithm is set at 0.999. The accuracy of the DOA estimate is measured from 500 Monte Carlo runs in terms of the root mean-square error (RMSE) which is defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{500N}\sum_{n=1}^{500}\sum_{i=1}^{N}(\hat{\theta}_i^{(n)} - \theta_i)^2} \tag{27}$$

where $\hat{\theta}_i^{(n)}$ is the estimate of θ_i for the n-th trial.

Fig. 1 depicts the spatial spectrum versus DOA for two coherent signals from $[-36°, -18°]$ where the red dash lines mark the true DOAs. The fading coefficients of the coherent signals are $\left[0.6595 + 0.7517j, \text{âĹ'}0.3621 + 0.599j\right]$. The SNR, L, and Q are fixed at 8 dB, 300, and 3, respectively. It can be seen that the proposed method has clear advantages on accuracy and resolution of estimation whereas the other two approaches are also able to detect the two signals but with higher estimation bias.

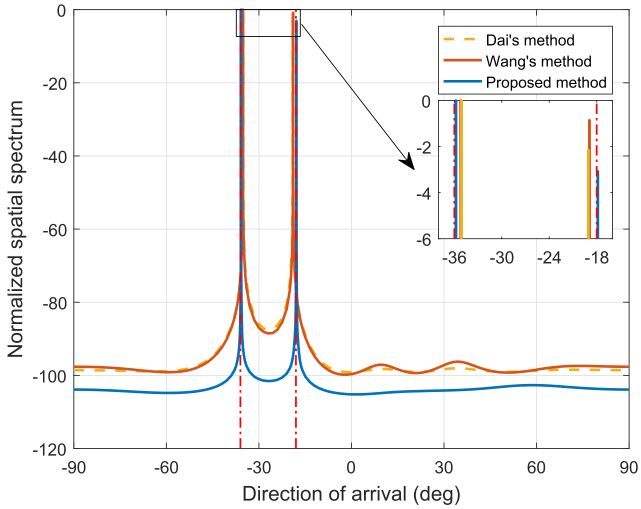

FIGURE 1. Normalized spatial spectra versus DOA. SNR = 8 dB and L = 300. The dashed lines mark the true DOA positions.

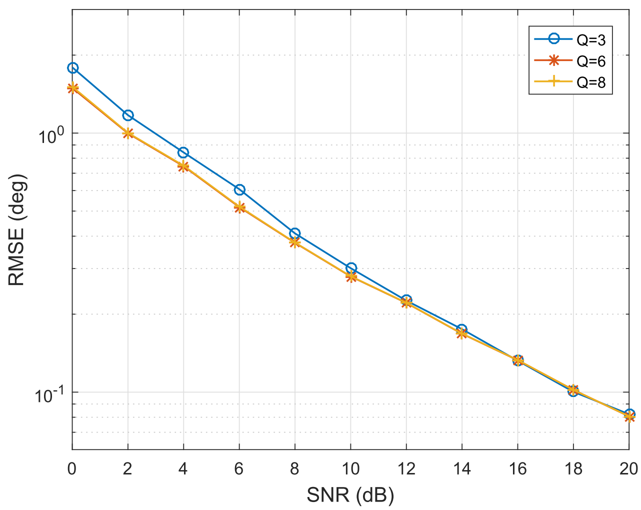

FIGURE 2. RMSE of the proposed method with different Q versus SNR. SNR = 8 dB and L = 300.

The estimation performance of our sparse recovery algorithm is now examined with diverse choices on the value of Q. Other parameters are set the same as the previous scenario. Fig. 2 shows that $Q = 6, 8$ slightly outperform $Q = 3$ while almost the same accuracy achieved by $Q = 6, 8$. This implies that, in general, the larger Q is, the better accuracy one can obtain, but at a cost of greater computations, because more columns offer a better robustness to the errors introduced by the finite snapshots, while the computational complexity of solving (26) involves $\mathcal{O}\left(Q^3G^3\right)$ flops. Therefore, $Q = 3$ is a reasonable tradeoff between the estimation accuracy and computational efficiency.

The statistical performance of the three algorithms as well as CRLB as a function of SNR and L in the first scenario

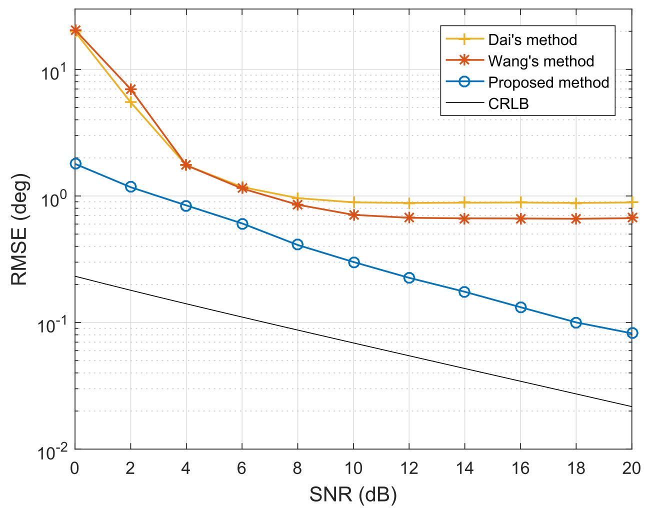

FIGURE 3. RMSE of the DOA estimates versus SNR when $L = 300$ and $Q = 3$.

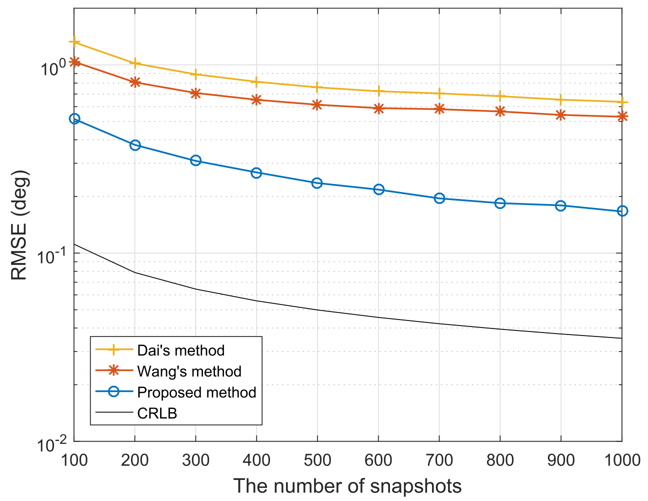

FIGURE 4. RMSE of the DOA estimates versus the number of snapshots when SNR = 10 dB and $Q = 3$.

is illustrated in Figs. 3 and 4. The resultant RMSE versus SNR for 300 snapshots is compared in Fig. 3. It is noticed that the proposed method outperforms its counterparts for all SNRs as the former exploits reweighted ℓ_1-norm to enhance the reconstruction sparsity and the linear transformation to eliminate the main contribution of the nonuniform noise, but there is still a clear margin between the DOA estimates of sparsity-aware technique and the CRLB, while the latter underestimates the noise variance and has relatively large errors, especially at low SNRs. Additionally, Dai's technique performs better than Wang's algorithm up to 4 dB, but slightly worse above 6 dB, and the RMSEs of Dai's and Wang's approaches at high SNRs tend to saturate to 0.9° and 0.65°, respectively, as SNR increases.

The results in Fig. 4 demonstrate that the RMSE of DOA estimation of the three techniques, fixing SNR = 10 dB, asymptotically approach the CRLB as L increases, and the proposed method is still superior to the other two approaches for all snapshot sizes. We also note that Wang's algorithm has lower RMSEs than Dai's approach as the former takes advantage of the whole array aperture, which is consistent with the tests in [12].

V. CONCLUSION

This correspondence has exhibited a novel sparsity-aware DOA estimator for coherent signals under the coexistence of mutual coupling and nonuniform noise. Our study indicated that the issue can resolved by a group sparsity reconstruction for multiple measurement vector that results from the noise-free covariance vectors, by decoupling the virtual steering vector from the MCM with a banded symmetric Toeplitz structure. Since the proposed technique leverages the whole array aperture and mitigates the nonuniform noise, better estimation performance is provided without mutual coupling compensation. Simulation results demonstrate that the proposed method bears a distinct advantage over state-of-the-art solutions.

APPENDIX
DERIVATION OF STOCHASTIC CRLB FOR THE COHERENT SIGNALS UNDER THE COEXISTENCE OF MUTUAL COUPLING AND NONUNIFORM NOISE

In the problem of DOA estimation of coherent signals under the coexistence of mutual coupling and nonuniform noise, the vector including the unknown parameters of interest is

$$\boldsymbol{\eta} = \left[\boldsymbol{\theta}^T, \mathbf{c}^T, \boldsymbol{\sigma}^T\right]^T \tag{28}$$

where $\boldsymbol{\theta} = [\theta_1, \cdots, \theta_N]^T$, $\mathbf{c} = [c_1, \cdots, c_{P-1}]^T$, and $\boldsymbol{\sigma} = \left[\sigma_1^2, \cdots, \sigma_M^2\right]^T$ are the vectors of unknown DOAs, mutual coupling coefficients, and noise powers, respectively.

If the observations follow random zero-mean Gaussian processes, then the (m, n)-th entry of the Fisher information matrix (FIM) is given by [28]

$$\mathbf{F}_{mn} = \sum_{i=1}^{L} \mathrm{tr}\left\{\mathbf{R}_x^{-1}\frac{\partial \mathbf{R}_x}{\partial \eta_m}\mathbf{R}_x^{-1}\frac{\partial \mathbf{R}_x}{\partial \eta_n}\right\} \tag{29}$$

For convenience of formulation, we define the following notations:

$$\dot{\mathbf{A}} = \left[\left.\frac{d\mathbf{a}(\theta)}{d\theta}\right|_{\theta=\theta_1}, \left.\frac{d\mathbf{a}(\theta)}{d\theta}\right|_{\theta=\theta_2}, \cdots, \left.\frac{d\mathbf{a}(\theta)}{d\theta}\right|_{\theta=\theta_N}\right] \tag{30}$$

$$\mathbf{R}_c = \boldsymbol{\Gamma}\mathbf{R}_s\boldsymbol{\Gamma}^H. \tag{31}$$

Additionally, define $\boldsymbol{\kappa} = [\kappa_1, \cdots, \kappa_{P-1}]^T \triangleq \mathrm{Re}\{\mathbf{c}\}$, $\boldsymbol{\varsigma} = [\varsigma_1, \cdots, \varsigma_{P-1}]^T \triangleq \mathrm{Im}\{\mathbf{c}\}$, and

$$\dot{\mathbf{C}}_{\kappa_i} = \mathrm{Toeplitz}\left\{\left[0, \bar{\mathbf{e}}_i^T, \mathbf{0}_{1\times(M-P)}\right]\right\} \tag{32}$$

$$\dot{\mathbf{C}}_{\varsigma_i} = j\mathrm{Toeplitz}\left\{\left[0, \bar{\mathbf{e}}_i^T, \mathbf{0}_{1\times(M-P)}\right]\right\} \tag{33}$$

where $\bar{\mathbf{e}}_i \in \mathbb{R}^{P-1}$ is a column vector with 1 at the i-th entry and 0 elsewhere.

In this appendix, we derive all blocks composing the FIM with the unknown parameters to calculate the CRLB. The idea of deriving the blocks follows the norm in [28], so the detailed deducing is omitted here for succinctness.

$$\begin{aligned}\mathbf{F}_{\boldsymbol{\theta\theta}} &= 2L\mathrm{Re}\Big\{\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)\odot\left(\mathbf{R}_c\mathbf{A}^H\right.\\ &\quad\left.\times\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)^T+\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\right)\\ &\quad\odot\left(\dot{\mathbf{A}}^H\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)^T\Big\}\end{aligned}\tag{34}$$

$$\begin{aligned}\mathbf{F}_{\kappa_i\kappa_j} &= 2L\mathrm{Re}\Big\{\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_j}\right.\\ &\quad\left.\times\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right)+\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right.\\ &\quad\left.\times\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\kappa_j}^H\right)\Big\}\end{aligned}\tag{35}$$

$$\begin{aligned}\mathbf{F}_{\varsigma_i\varsigma_j} &= 2L\mathrm{Re}\Big\{\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_j}\right.\\ &\quad\left.\times\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right)+\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right.\\ &\quad\left.\times\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\varsigma_j}^H\right)\Big\}\end{aligned}\tag{36}$$

$$\mathbf{F}_{\boldsymbol{\sigma\sigma}} = L\left(\mathbf{R}_x^{-1}\odot\left(\mathbf{R}_x^{-1}\right)^T\right)\tag{37}$$

$$\begin{aligned}\mathbf{F}_{\boldsymbol{\theta}\kappa_j} &= 2L\mathrm{Re}\Big\{\mathrm{diag}\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_j}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\right.\\ &\quad\left.\times\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)+\mathrm{diag}\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\right.\\ &\quad\left.\times\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\kappa_j}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)\Big\}\end{aligned}\tag{38}$$

$$\begin{aligned}\mathbf{F}_{\boldsymbol{\theta}\varsigma_j} &= 2L\mathrm{Re}\Big\{\mathrm{diag}\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_j}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\right.\\ &\quad\left.\times\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)+\mathrm{diag}\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\mathbf{C}\right.\\ &\quad\left.\times\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\varsigma_j}^H\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)\Big\}\end{aligned}\tag{39}$$

$$\begin{aligned}\mathbf{F}_{\boldsymbol{\theta\sigma}} &= L\Big\{\left(\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{R}_x^{-1}\mathbf{C}\dot{\mathbf{A}}\right)^T\\ &\quad+\left(\dot{\mathbf{A}}^H\mathbf{C}^H\mathbf{R}_x^{-1}\right)\odot\left(\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\right)^T\Big\}\end{aligned}\tag{40}$$

$$\begin{aligned}\mathbf{F}_{\kappa_i\varsigma_j} &= 2L\mathrm{Re}\Big\{\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_j}\right.\\ &\quad\left.\times\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right)+\mathrm{tr}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\right.\\ &\quad\left.\times\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\varsigma_j}^H\right)\Big\}\end{aligned}\tag{41}$$

$$\begin{aligned}\mathbf{F}_{\kappa_i\boldsymbol{\sigma}} &= L\Big\{\mathrm{diag}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\kappa_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\right)^T\\ &\quad+\mathrm{diag}\left(\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\kappa_i}^H\mathbf{R}_x^{-1}\right)^T\Big\}\end{aligned}\tag{42}$$

$$\begin{aligned}\mathbf{F}_{\varsigma_i\boldsymbol{\sigma}} &= L\Big\{\mathrm{diag}\left(\mathbf{R}_x^{-1}\dot{\mathbf{C}}_{\varsigma_i}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\mathbf{C}^H\mathbf{R}_x^{-1}\right)^T\\ &\quad+\mathrm{diag}\left(\mathbf{R}_x^{-1}\mathbf{C}\mathbf{A}\mathbf{R}_c\mathbf{A}^H\dot{\mathbf{C}}_{\varsigma_i}^H\mathbf{R}_x^{-1}\right)^T\Big\}.\end{aligned}\tag{43}$$

As a result, the FIM can be expressed as

$$\mathbf{F} = \begin{bmatrix}\mathbf{F}_{\boldsymbol{\theta\theta}} & \mathbf{F}_{\boldsymbol{\theta\kappa}} & \mathbf{F}_{\boldsymbol{\theta\varsigma}} & \mathbf{F}_{\boldsymbol{\theta\sigma}}\\ \mathbf{F}_{\boldsymbol{\theta\kappa}}^T & \mathbf{F}_{\boldsymbol{\kappa\kappa}} & \mathbf{F}_{\boldsymbol{\kappa\varsigma}} & \mathbf{F}_{\boldsymbol{\kappa\sigma}}\\ \mathbf{F}_{\boldsymbol{\theta\varsigma}}^T & \mathbf{F}_{\boldsymbol{\kappa\varsigma}}^T & \mathbf{F}_{\boldsymbol{\varsigma\varsigma}} & \mathbf{F}_{\boldsymbol{\varsigma\sigma}}\\ \mathbf{F}_{\boldsymbol{\theta\sigma}}^T & \mathbf{F}_{\boldsymbol{\kappa\sigma}}^T & \mathbf{F}_{\boldsymbol{\varsigma\sigma}}^T & \mathbf{F}_{\boldsymbol{\sigma\sigma}}\end{bmatrix}.\tag{44}$$

Consequently, the CRLB can be obtained by taking the inverse of the FIM as

$$\mathrm{CRLB}_\theta = \sqrt{\tfrac{1}{N}\textstyle\sum_{i=1}^{N}\left[\mathbf{F}^{-1}\right]_{ii}}.\tag{45}$$

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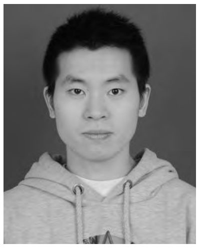

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