IEEE Access
Multidisciplinary ⁝ Rapid Review ⁝ Open Access Journal

Received January 30, 2019, accepted February 25, 2019, date of publication March 8, 2019, date of current version March 26, 2019.

Digital Object Identifier 10.1109/ACCESS.2019.2903833

Neural Network-Based Adaptive Finite-Time Consensus Tracking Control for Multiple Autonomous Underwater Vehicles

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This work was supported in part by the National Key Research and Development Plan under Grant 2017YFB1303503, in part by the National Natural Science Foundation of China under Grant 61573204, Grant 61501276, Grant 61603204, and Grant 61673227, in part by the Shandong Province Outstanding Youth Fund under Grant ZR2018JL020, and in part by the Taishan Scholar Special Project Fund under Grant TSQN20161026.

ABSTRACT Considering the problem of consensus tracking control for multiple autonomous underwater vehicle (AUV) system, a neural network-based finite-time nonsingular fast terminal sliding mode control method is proposed. First, in order to elaborate on the communication relationship, the algebraic graph theory is combined with a leader–follower architecture. Next, the modified nonsingular fast terminal sliding mode is adopted to improve the fast response characteristic of the system, and distributed control laws are constructed based on the force analysis of each AUV. Furthermore, neural networks technique is employed to approximate the uncertain dynamics and forces caused by the harsh environment of the ocean. Finally, it is proved that the tracking errors can converge to a small neighborhood of the origin by using the proposed algorithm. The effectiveness and robustness of the proposed algorithm are illustrated by a simulation example.

INDEX TERMS Adaptive neural control, multiple AUV system, nonsingular terminal sliding mode, finite time, graph theory.

I. INTRODUCTION

With the accelerated development of marine science and oceanographic engineering, the autonomous underwater vehicles (AUVs) have increasingly become the focus of attention in recent years. Not only can AUVs overcome the hostile underwater environment, but also they have various huge advantages such as small volume, light weight, and low cost. These reasons make them widely range in marine science, for instance, scientific inspection, rescue operation, shipwreck search, gas exploration, littoral survey, anti-submarine warfare [1]–[20]. The difficulty of these underwater intervention tasks is increased since marine missions require accurate positioning control of AUVs for acquiring good quality of data. Meanwhile, a single AUV is used to implement simple missions because its ability is limited in energy, control or other aspects. However, the multi-AUV system has incomparable advantages over single AUV in terms of spatial distribution, efficiency and flexibility of task execution. Therefore, the importance of the system becomes more and more obvious in the past 20 years.

In recent years, the essential key is how to make them unite and cooperate tasks by coordinating with each other. The multi-AUV system inherits the good points of a single AUV and innovates its own unique characteristics in coordinated formation control, but how to achieve accurate trajectory tracking control of multi-AUV system is a more challenging problem in complex undersea environment. Meanwhile, AUVs may suffer unknown forces such as ocean waves, tides, currents and upward or downward stream. These problems will cause highly coupled nonlinearities, time varying dynamics, parameter uncertainties resulting from the lack of knowledge about hydrodynamic coefficients with different operating conditions.

In the beginning trend of multi-AUV system design, adaptive PD-based control method is used to solve the formation control problems in [4]. Other noteworthy extensions which appeared in the formation control are self tuning

The associate editor coordinating the review of this manuscript and approving it for publication was Jiahu Qin.

control [5], adaptive control [6]–[9], gain scheduling control [10], linear quadratic Gaussian control [11], nonlinear tracking control [12]. In order to solve the consensus tracking control problem, a great deal of algorithms were developed, i.e., linear conventional PD/PID control [13], the method of leader-follower [14], [15], backstepping control [16], integrator backstepping method [17], output-feedback tracking controllers [18], dynamic surface control technique [19], genetic algorithm based control [20], sliding mode control [21], [22], and approach of graph theory analysis [23]. From the existing literature review, it has been investigated that graph theory analysis has great advantages for dealing with distributed control problem because it can simplify communication topology and optimize information model of multi-AUV system. It chooses edges and vertexes to delegate the each independent AUV and their communication topological relations, respectively. Consequently, deployment of graph theory is the appropriate preference for the consensus tracking control of multi-AUV system in complex oceanic environment. Reference [15] analyzed the convergence of multi-AUV systems with leader-follower architecture and graph theory. Reference [17] designed the cooperative and adaptive tracking control for a group of underactuated AUVs by combining the backstepping technique with graph theory. Although various algorithms guarantee the convergence performance, they take a long convergence time.

From the point of view of time optimization, finite-time has great advantage in the convergence of multi-AUV system. Compared with the asymptotic control method, the finite time can provide faster response speed, higher tracking accuracy, and better anti-interference ability [24]. Accordingly, it is particularly important to introduce finite-time techniques into systems which needs to improve the speed of convergence, such as the multi-agent systems [25], [26], the motor system [27], the nonlinear systems [28], [29], the multi robot system [30], the attitude synchronization control of spacecraft [31], [32]. When [23] adopted the finite-time technique into multi-AUV system, however they set external interference and uncertain forces to fixed values, the system can not have strong robustness in the harsh underwater environment. Focusing on this condition, how to combine finite-time technique while dealing with the uncertain forces in the ocean have not yet been reported. In view of all these methods mentioned above, the controller based on nonsingular fast terminal sliding mode and neural networks are designed for multi-AUV system in this paper.

Compared with the communication topology [14], [15], the control methods for dealing with the uncertain forces and external interference in [6]–[9] and [33]–[35], and the consensus methods for multi-AUV system in [36]–[38], the proposed NN-based finite-time consensus tracking approach has the following advantages:

(1) In [6], [14], and [15], the technologies of reinforcement learning and leader-follower are adopted, respectively. The former has a long time to learn and interact with environment, the followers of the latter are independent of each other, there is no direct connection in the system. However, this paper introduce the graph theory to express the communication topology relation in multi-AUV system, it improves the convenience of information interaction and optimizes transfer time of information model in the system.

(2) In [6]–[9], the uncertain forces and external interference were offset by constructing the traditional adaptive laws, and in [33]–[35], they design various sliding modes or algorithm on the basis of full or partial knowledge of hydrodynamic coefficients and unpredictable disturbances. Nevertheless, this paper combines the neural networks approximation technique with the nonsingular fast terminal sliding mode, which holds the advantages of both of the above.

(3) Although in [21], [22], and [36]–[38], various strategies can guarantee the asymptotic convergence of the system, the time of convergence is not ideal. In order to improve the fast response characteristic of the system, the finite-time technology is adopted to modify the nonsingular terminal sliding mode. It can be seen obviously from convergence time that the system has strong robustness compared with classical control strategy.

II. FORCE ANALYSIS AND MATHEMATICAL PRELIMINARIES

A. GRAPH THEORY

Consider a distributed multi-AUV system consisting of one leader and n followers, the communication topology can be modeled by a weighted graph $\varsigma = \{M, E, A\}$, where $M = \{m_1, m_2, \cdots, m_n\}$ is the set of vertexes, vertex i $(i \in M)$ denote the ith following AUV. $E \subset M \times M$ represents the set of edges, $A = [a_{ij}] \in R^{n\times n}$ is weighted adjacency matrix of the graph ς. If ith AUV has access to send information directly to jth AUV, the edge between them is denoted as $(m_i, m_j) \in E$. Define $a_{ij} > 0$ (usually $a_{ij} = 1$) if $(m_j, m_i) \in E$, else $a_{ij} = 0$, and suppose $a_{ii} = 0$, $\forall i$. $M_i = \{j|(j, i) \in E\}$ can express the set of adjacent vertexes about vertex m_i and the penetration of vertex m_i is expressed as $d_i = \sum_{j\in N_i}^{n} a_{ij}$. Then the indegree matrix and the Laplacian matrix are $D = \text{diag}\{d_1, d_2, \cdots, d_n\}$, $L = D - A \in R^{n\times n}$, respectively. A path between m_1 to m_k is the sequence (m_1, m_2), (m_2, m_3), (m_{k-1}, m_k), where $(m_{j-1}, m_j) \in E$ for $j = 2, \cdots, k$. If any two vertices (m_i, m_j) are linked with a path, Graph ς is called strongly connected. Define a vertex which can get situations with direct channel from other vertexes as the root vertex, and if weighted graph ς has a root vertex, it contains a directed spanning tree.

In the process of AUV moving, dynamic elements of leader are independent but its situations have a variety of influence for followers. Define m_0 as the leader, and weights between vertex of leader to vertices of followers are defined as $b_{i0} \geq 0$. If leader has a access to ith follower, $b_{i0} = 1$, else $b_{i0} = 0$, and diagonal matrix B and H are defined as $B = \text{diag}\{b_{10}, b_{20}, \cdots, b_{n0}\} \in R^{n\times n}$, $H = L + B$.

The augmented graph $\bar{\varsigma}$ which contains m_0 is defined as $\bar{\varsigma} = \{\bar{M}, \bar{E}, \bar{A}\}$, where $\bar{M} = \{m_0, m_1, m_2, \cdots, m_n\}$, $\bar{E} \subset \bar{M} \times \bar{M}$.

Remark 1: The undirected communication topology of multi-AUV consensus tracking control is constructed in [15] and it is valid to choose $a_{ij} = a_{ji}$. Meanwhile, the stability analysis gets greatly simplify because the terms including a_{ij} in the time derivative of the Lyapunov function are eliminated by mutual cancelation. However, this case is infeasible for directed communication topology. Thus, consensus tracking control of multi-AUV system under directed communication topology is more difficult compared with undirected communication topology.

B. FORCE ANALYSIS

For each AUV, its location and velocity is relatively independent, and in the course of the process, they also have to deal with the location and velocity information from others. Assume all AUVs' attitudes are fixed, so the distributed multi-AUV system eliminates singular points. Without loss of generality and under the Assumption 2, assume that the following AUVs are labeled from 1 to n, and the force analysis is given as follows:

$$\begin{cases} \dot{\eta}_i = R_i(\Theta_i)\mathbf{v}_i \\ M_i\dot{\mathbf{v}}_i = -D_i(\mathbf{v}_i)\mathbf{v}_i + g_i(\Theta_i) + f_i + \xi_i \end{cases} \tag{1}$$

where $\eta_i = [x_i, y_i, z_i]^T$ is the location information in the measurement reference frame. $\mathbf{v}_i = [u_i, v_i, w_i]^T$ and $\Theta_i = [\varphi_i, \theta_i, \psi_i]^T$ represent velocity and attitude (composed by Euler angles, roll, pitch and yaw) vectors in the inertial reference frame. $R_i(\Theta_i)$ is the transition matrix of the two coordinate systems. M_i, $D_i(\mathbf{v}_i)$, ξ_i, $g_i(\Theta_i)$, $f_i = [X_i, Y_i, Z_i]^T \in R^3$ denote the inertia matrix, damping matrix, external disturbance forces, restoring force vector and control force vector, respectively. For an angle $\gamma \in R$, denote $s_\gamma = \sin\gamma$, $c_\gamma = \cos\gamma$ for brevity. In detail,

$$R_i(\Theta_i) = \begin{bmatrix} c_{\psi_i}c_{\theta_i} & -s_{\psi_i}c_{\varphi_i}+s_{\varphi_i}s_{\theta_i}c_{\psi_i} & s_{\psi_i}s_{\varphi_i}+s_{\theta_i}c_{\psi_i}c_{\varphi_i} \\ s_{\psi_i}c_{\theta_i} & c_{\psi_i}c_{\varphi_i}+s_{\varphi_i}s_{\theta_i}s_{\psi_i} & -c_{\psi_i}s_{\varphi_i}+s_{\theta_i}s_{\psi_i}c_{\varphi_i} \\ -s_{\theta_i} & s_{\varphi_i}c_{\theta_i} & c_{\varphi_i}c_{\theta_i} \end{bmatrix}. \tag{2}$$

Define $M_i = \text{diag}[m_{i1}, m_{i2}, m_{i3}]$, $D_i(\mathbf{v}_i) = \text{diag}[d_{L_{i1}} + d_{P_{i1}}|u_i|, d_{L_{i2}} + d_{P_{i2}}|v_i|, d_{L_{i3}} + d_{P_{i3}}|w_i|]$, $m_{ij}, d_{L_{ij}}, d_{p_{ij}} > 0$. $g_i(\Theta_i) = [(W_i - B_i)s_{\theta_i}, -(W_i - B_i)c_{\theta_i}s_{\varphi_i}, -(W_i - B_i)c_{\theta_i}c_{\varphi_i}]^T$, where W_i and B_i represent the gravitational and buoyancy forces, respectively. $i \in M$. Choose $\eta_d \in R$ to denote the trajectory of leader and assume that the first and second time derivatives, $\dot{\eta}_d$ and $\ddot{\eta}_d$, are smooth, bounded and known functions. The second order derivative of η_i can be rewritten as:

$$\begin{cases} \ddot{\eta}_i = -h_i + \bar{f}_i + \bar{g}_i(\Theta_i) \\ \bar{f}_i = R_i(\Theta_i)M_i^{-1}f_i \\ h_i = R_i\Theta_i)M_i^{-1}D_i(\mathbf{v}_i)\mathbf{v}_i - \dot{R}_i\Theta_i)\mathbf{v}_i \\ \bar{g}_i = R_i(\Theta_i)M_i^{-1}\xi_i + R_i(\Theta_i)M_i^{-1}g_i(\Theta_i). \end{cases} \tag{3}$$

The following assumptions are required for consensus tracking control problem of multi-AUV system.

Assumption 1: In multi-AUV system, the augmented graph $\bar{\varsigma}$ includes a spanning tree with the root vertex being the leader vertex.

Assumption 2: The ith follower AUV's external interference ξ_i and restoring force g_i are bounded, that means $\|\xi_i\| \leq \xi_i^*$ and $\|g_i\| \leq g_i^*$, where ξ_i^* and g_i^* are bounded unknown positive constants.

C. SOME LEMMAS

Lemma 1 [23]: H is full rank if and only if graph $\bar{\varsigma}$ is a directed graph contained with a directed spanning tree.

Lemma 2 [39]: Defined $W(t) + hW(t)^\lambda \leq 0$ is a negative semidefinite function defined on $U \in R^n$, where $0 < \lambda < 1$ and $W(t)$ satisfies the condition of smooth and positive definite. Correspondingly, there exists a situation that $W(t)$ starts from $U_0 \in R^n$ will reach $W(t) \equiv 0$ in finite time T_r and $T_r \leq W(t_0)^{1-\lambda}/(h(1-\lambda))$.

Lemma 3 [40]: For any real numbers $\gamma_1 > 0$, $\gamma_2 > 0$ and $0 < \lambda < 1$, the function $W(t)$ will reach extended Lyapunov condition $\dot{W}(t) + \gamma_1 W(t) + \gamma_2 W(t)^\lambda \leq 0$ in finite time T_r, where $T_r \leq T_0 + \frac{1}{\gamma_1(1-\lambda)} \ln \frac{\gamma_1 W(t_0)^{1-\lambda} + \gamma_2}{\gamma_2}$.

Lemma 4 [32]: For $v_i \in R$, $0 < q \leq 1$. $(\sum_{i=1}^{m} |v_i|)^q \leq \sum_{i=1}^{m} |v_i|^q \leq m^{1-q}(\sum_{i=1}^{m} |v_i|)^q$.

In this brief, the radial basis function neural networks (RBF NNs) technique will be used to approximate the unknown continuous function $f(x)$ [41]–[43]. Let $f(x)$ be the continuous function that delimited on a compact set $\Omega_x \in \mathbf{R}^q$. Then the following equality can be obtained:

$$f(x) = W^{*,T}\varphi(x) + \delta(x). \tag{4}$$

The desired accuracy satisfies that $|\delta(x)| \leq \varepsilon$ and $\varepsilon > 0$, where

$$W^* = arg \min_{W \in R^l} \left\{ \sup_{x \in \Omega_x} \left| f(x) - W^T\varphi(x) \right| \right\}$$

is ideal constant weight matrix and $\delta(x)$ is the approximation error, $W = [W_1, \ldots, W_N]^T$ is weight vector, and$\varphi(x) = [p_1(x), p_2(x), \ldots, p_N(x)]^T / \sum_{i=1}^{N} p_i(x)$ is the basis function vector, with $N > 1$ being the number of RBF NNs rules and p_i are chosen as Gaussian functions, i.e., for $i = 1, 2, \ldots, N$, $p_i(x) = \exp[\frac{-(x-\mu_i)^T(x-\mu_i)}{\gamma_i^2}]$, where$\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ is the center vector, and γ_i is the width of the Gaussian function.

III. THE DESIGN OF THE FINITE TIME NEURAL NETWORKS CONTROLLERS

A. ERROR DEFINITION

Define position and velocity state error measures of multi-AUV system are station-keeping and formation-keeping state errors. The former state errors generate when an individual following AUV compares its state with reference state. It is defined as:

$$c_{1i} = \eta_i - \eta_d, \quad c_{2i} = \dot{\eta}_i - \dot{\eta}_d.$$

The formation-keeping state errors generate when ith following AUV compares its state with jth following AUV. It is described as:

$$c_{1ij} = \eta_i - \eta_j, \quad c_{2ij} = \dot{\eta}_i - \dot{\eta}_j.$$

Thus, the overall neighborhood state errors of ith follower can be expressed as:

$$\begin{aligned}\lambda_{1i} &= \sum_{j=1,j\neq i}^{n} a_{ij}c_{1ij} + b_i c_{1i} = \sum_{j=1,j\neq i}^{n} a_{ij}(\eta_i - \eta_j) + b_i c_{1i}\\ &= \sum_{j=1,j\neq i}^{n} a_{ij}(c_{1i} - c_{1j}) + b_i c_{1i} = \sum_{j=1}^{n} l_{ij}c_{1j} + b_i c_{1i}\\ \lambda_{2i} &= \sum_{j=1,j\neq i}^{n} a_{ij}c_{2ij} + b_i c_{2i} = \sum_{j=1,j\neq i}^{n} a_{ij}(\dot{\eta}_i - \dot{\eta}_j) + b_i c_{2i}\\ &= \sum_{j=1,j\neq i}^{n} a_{ij}(c_{2i} - c_{2j}) + b_i c_{2i} = \sum_{j=1}^{n} l_{ij}c_{2j} + b_i c_{2i}\end{aligned}$$

where b_i is the element of matrix B, $l_{ij} \in L$. The following equalities can be obtained:

$$\begin{aligned}\bar{h}_i &= h_i + \ddot{\eta}_d\\ \dot{c}_{2i} &= -\bar{h}_i + \bar{f}_i + \bar{g}_i, \quad i \in M.\end{aligned} \tag{5}$$

B. NONSINGULAR FAST TERMINAL SLIDING MODE

The nonsingular fast terminal sliding mode vectors are expanded in order to improve the fast response characteristic of the system and guarantee the robustness, they are defined as $\tau = \left[\tau_1^T, \cdots, \tau_n^T\right]^T = 0$, $\tau_i = [\tau_{i1}, \tau_{i2}, \tau_{i3}]^T \in R^3$.

$$\tau_i = \lambda_{1i} + p_{1i}\mathrm{sig}(\lambda_{1i})^{\alpha_1} + p_{2i}\mathrm{sig}(\lambda_{2i})^{\alpha_2} \tag{6}$$

where $1 < \alpha_2 < 2$, $\alpha_2 < \alpha_1$, p_{1i} and p_{2i} are positive constants. $\mathrm{sig}(\lambda_{1i})^{\alpha_1} = [\mathrm{sig}(\lambda_{1i1})^{\alpha_1}, \mathrm{sig}(\lambda_{1i2})^{\alpha_1}, \mathrm{sig}(\lambda_{1i3})^{\alpha_1}]^T$, and $\mathrm{sig}(\lambda_{2i})^{\alpha_2} = [\mathrm{sig}(\lambda_{2i1})^{\alpha_2}, \mathrm{sig}(\lambda_{2i2})^{\alpha_2}, \mathrm{sig}(\lambda_{2i3})^{\alpha_2}]^T$, $i \in M$.

C. CONTROLLERS DESIGN

Considering the buffeting phenomenon and fast response characteristics of the system, construct the controllers as follows: $\bar{f}_i = R_i(\Theta_i)M_i^{-1}f_i$.

$$\begin{aligned}\bar{f}_i = &\Big(\sum_{j=1,j\neq i}^{n} a_{ij} + b_i\Big)^{-1} \times \Big[\sum_{j=1,j\neq i}^{n} a_{ij}\bar{f}_j\\ &- \frac{1}{\alpha_2 p_{2i}}\mathrm{diag}\left\{\mathrm{sig}(\lambda_{2i})^{2-\alpha_2}\right\}\\ &\times (I_1 + \alpha_1 p_{1i}\,|\lambda_{1i}|^{\alpha_1-1}) - p_{2i}\\ &\times |\lambda_{2i}|^{\alpha_2-1}\left(\phi_i\tau_i + \delta_i\mathrm{sig}(\tau_i)^{\alpha_3} + f_{iadp}\right)\Big]\end{aligned} \tag{7}$$

where $0 < \alpha_3 < 1$, $\phi_i > 0$, $\delta_i > 0$, $\mathrm{sig}(\lambda_{2i})^{2-\alpha_2} = [\mathrm{sig}(\lambda_{2i1})^{2-\alpha_2}, \mathrm{sig}(\lambda_{2i2})^{2-\alpha_2}, \mathrm{sig}(\lambda_{2i3})^{2-\alpha_2}]^T$, $\mathrm{sig}(\tau_i)^{\alpha_3} = [\mathrm{sig}(\tau_{i1})^{\alpha_3}, \mathrm{sig}(\tau_{i2})^{\alpha_3}, \mathrm{sig}(\tau_{i3})^{\alpha_3}]^T$.

The unit matrix is $I_1 = [1, 1, 1]^T$, the parts of above controllers used to remove the impact of uncertainty and estimate the upper bound parameter of external disturbances are $f_{iadp} = [f_{iadp1}, f_{iadp2}, f_{iadp3}]^T$.

$$f_{iadp} = \sum_{i=1}^{n}\sum_{\chi=1}^{3}\frac{1}{2l_i^2}\hat{w}_i\tau_{i\chi}. \tag{8}$$

Applying the knowledge of Kronecker product to the matrix transformations of multi-AUV system [44], then the distributed controllers can be represented as follows by combining the graph theory:

$$\begin{aligned}\bar{F} = &\left[(D+B)^{-1} \otimes I\right] \times \Big[(A \otimes I)\bar{F}\\ &- \frac{1}{\alpha_2}\mathrm{diag}\left\{\frac{1}{P_2}\mathrm{sig}\,(\lambda_2)^{2-\alpha_2}\right\}\\ &\times \left(\bar{I}_1 + \alpha_1 P_1\,|\lambda_1|^{\alpha_1-1}\right) - P_2\\ &\times |\lambda_2|^{\alpha_2-1}\left(\Phi\tau + \delta\mathrm{sig}\,(\tau)^{\alpha_3} + F_{adp}\right)\Big].\end{aligned} \tag{9}$$

The simplifications of the finite time neural networks controllers can be obtained:

$$\begin{aligned}\bar{F} = &[(L+B) \otimes I]^{-1}\\ &\times\Big[-\frac{1}{\alpha_2}\mathrm{diag}\left\{\frac{1}{P_2}\mathrm{sig}\,(\lambda_2)^{2-\alpha_2}\right\}\\ &\times \left(\bar{I}_1 + \alpha_1 P_1\,|\lambda_1|^{\alpha_1-1}\right) - P_2\\ &\times |\lambda_2|^{\alpha_2-1}\left(\Phi\tau + \delta\mathrm{sig}\,(\tau)^{\alpha_3} + F_{adp}\right)\Big]\end{aligned} \tag{10}$$

where

$$\begin{aligned}\delta &= \mathrm{diag}\,\{\delta_1 \otimes I, \cdots, \delta_n \otimes I\},\\ \Phi &= \mathrm{diag}\,\{\phi_1 \otimes I, \cdots, \phi_n \otimes I\},\\ P_1 &= \mathrm{diag}\,\{p_{11} \otimes I, \cdots, p_{1n} \otimes I\},\\ \frac{1}{P_2} &= \mathrm{diag}\left\{\frac{1}{p_{21}} \otimes I, \cdots, \frac{1}{p_{2n}} \otimes I\right\},\\ |\lambda_1|^{\alpha_1-1} &= \left[|\lambda_{11}|^{\alpha_1-1,T}, \cdots, |\lambda_{1n}|^{\alpha_1-1,T}\right]^T,\\ |\lambda_2|^{\alpha_2-1} &= \left[|\lambda_{21}|^{\alpha_2-1,T}, \cdots, |\lambda_{2n}|^{\alpha_2-1,T}\right]^T,\\ \mathrm{sig}(\tau)^{\alpha_3} &= [\mathrm{sig}(\tau_1)^{\alpha_3,T}, \cdots, \mathrm{sig}(\tau_n)^{\alpha_3,T}]^T,\\ \mathrm{sig}(\lambda_{2i})^{2-\alpha_2} &= [\mathrm{sig}(\lambda_{21})^{2-\alpha_2,T}, \cdots, \mathrm{sig}(\lambda_{2n})^{2-\alpha_2,T}]^T.\end{aligned}$$

Then, the Theorem 1 is given as follows:

Theorem 1: Considering the sliding mode designed as (6) and the finite time neural networks controllers designed as (7), if the Assumption 1 and 2 set up, then each sliding mode τ_i can be reached to the desired regions in finite time. Furthermore, the tracking errors of ith follower can converge to regions near the sliding surface in setting time. Then, they can converge to the origin point along the sliding surface in finite time.

Proof : The Lyapunov function is introduced as:

$$\begin{aligned}V &= V_1 + V_2\\ V_1 &= \frac{1}{2}\tau^T\tau\\ V_2 &= \frac{1}{2}\alpha_2\tilde{w}^2.\end{aligned} \tag{11}$$

Step 1: Take derivative of V_1 [31]:

$$\begin{aligned}\dot{V}_1 = \tau^T & \left\{\lambda_2 + \alpha_1 \text{diag}\left\{P_1 |\lambda_1|^{\alpha_1 - 1}\right\}\lambda_2\right. \\ & + \alpha_2 \text{diag}\left\{P_2 |\lambda_2|^{\alpha_2 - 1}\right\} \\ & \left.\times\ [(L+B)\otimes I]\left(\bar{F} + \bar{G} - \bar{H}\right)\right\} \end{aligned} \tag{12}$$

where $\bar{G} = \left[\bar{g}_1^T, \cdots, \bar{g}_n^T\right]^T$ and $\bar{H} = \left[\bar{h}_1^T, \cdots, \bar{h}_n^T\right]^T$. Replacing the equality (10) into (12), the following equalities can be obtained:

$$\begin{aligned}\dot{V}_1 = & -\alpha_2 \tau^T \text{diag}\left\{P_2^2 |\lambda_2|^{2\alpha_2 - 2}\right\}\Phi\tau \\ & - \alpha_2 \tau^T \text{diag}\left\{P_2^2 |\lambda_2|^{2\alpha_2 - 2}\right\}\delta \text{sig}(\tau)^{\alpha_3} \\ & - \alpha_2 \tau^T \text{diag}\left\{P_2^2 |\lambda_2|^{2\alpha_2 - 2}\right\} F_{adp} \\ & + \alpha_2 \tau^T \text{diag}\left\{P_2 |\lambda_2|^{\alpha_2 - 1}\right\} \\ & \times [(L+B)\otimes I](\bar{G} - \bar{H}) \\ \leq & -\alpha_2 \sum_{i=1}^{n} p_{2i}^2 \phi_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \\ & - \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \delta_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \left|\tau_{i\chi}\right|^{\alpha_3 + 1} \\ & - \alpha_2 \sum_{i=1}^{n}\sum_{\chi=1}^{3} p_{2i}^2 \tau_{i\chi}^T \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} f_{iadp} \\ & + \alpha_2 p_{2i} \sum_{i=1}^{n}\sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{\alpha_2 - 1} \left|\tau_{i\chi}\right| \\ & \times \left\{\left\|\sum_{i=1}^{n}\left[\sum_{j=1}^{n} l_{ij}\left(\bar{g}_j - \bar{h}_j\right) + b_i\left(\bar{g}_i - \bar{h}_i\right)\right]\right\|\right\}. \end{aligned} \tag{13}$$

Typically, suppose that the unknown internal and external interference factors of the system are bounded, and the bounds are certain to the conventional nonlinear terminal sliding mode control laws. However, the upper bounds of uncertain forces and external interferences can not be calculated accurately in practical engineering. Even the controllers implement based on certain boundaries, the results will bring chattering problem into multi-AUV system. Thus, we choose the following nonlinear function to integrate the unknown internal and external interference factors:

$$f_i(x_i) = \left\|\sum_{i=1}^{n}\left[\sum_{j=1}^{n} l_{ij}\left(\bar{g}_j - \bar{h}_j\right) + b_i\left(\bar{g}_i - \bar{h}_i\right)\right]\right\| \tag{14}$$

where $x_i = \left(\eta_i, \eta_j, \eta_d, \dot{\eta}_i, \dot{\eta}_j, \dot{\eta}_d, \ddot{\eta}_d\right)$, $i \in M$. In practical, the nonlinear function $f_i(x_i)$ is always uncertain and unknown due to the inertial uncertainties. To increase the precision of controllers, NNs approximation technology is adopted to deal with the unknown forces and nonlinear functions in the system. Then, the nonlinear functions can be approximated by: $f_i(x_i) = W_i^T \varphi_i(x_i) + \delta(x_i)$, W_i is the optimal approximation weight and $\delta(x_i) > 0$ are approximation errors and satisfy that $|\delta(x_i)| < \varepsilon_i$. By Young's inequality, the following inequalities can be obtained:

$$\begin{aligned}\dot{V}_1 \leq & -\alpha_2 \sum_{i=1}^{n} p_{2i}^2 \phi_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \\ & - \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \delta_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \left|\tau_{i\chi}\right|^{\alpha_3 + 1} \\ & - \alpha_2 \sum_{i=1}^{n}\sum_{\chi=1}^{3} p_{2i}^2 \tau_{i\chi}^T \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} f_{iadp} \\ & + \alpha_2 \sum_{i=1}^{n}\sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{\alpha_2 - 1} \left|\tau_{i\chi}\right| p_{2i}(W_i^T \varphi_i(x_i) + \varepsilon_i) \\ \leq & -\alpha_2 \sum_{i=1}^{n} p_{2i}^2 \phi_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \\ & - \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \delta_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \left|\tau_{i\chi}\right|^{\alpha_3 + 1} \\ & - \alpha_2 \sum_{i=1}^{n}\sum_{\chi=1}^{3} p_{2i}^2 \tau_i^T \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} f_{iadp} \\ & + \alpha_2 \sum_{i=1}^{n}[(p_{2i}^2 \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \frac{\|W_i\|^2}{2l_i^2} \varphi_i^T \varphi_i) \\ & + \frac{l_i^2}{2} + \frac{\varepsilon_i^2}{2} + \frac{1}{2} p_{2i}^2 \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2]. \end{aligned} \tag{15}$$

Moreover, put the NNs adaptive laws into the distributed adaptive controllers:

$$\begin{aligned}\dot{V}_1 \leq & -\alpha_2 \sum_{i=1}^{n} p_{2i}^2 \phi_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \\ & - \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \delta_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \left|\tau_{i\chi}\right|^{\alpha_3 + 1} \\ & + \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \times \left(\|W_i\|^2 - \hat{w}_i\right) \frac{\varphi_i^T \varphi_i}{2l_i^2} \\ & + \alpha_2 \sum_{i=1}^{n} \left(\frac{l_i^2}{2} + \frac{\varepsilon_i^2}{2} + \frac{1}{2} p_{2i}^2 \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2\right). \end{aligned} \tag{16}$$

Design $w_i = \max\left\{\|W_1\|^2, \|W_2\|^2, \ldots, \|W_n\|^2\right\}$, $\hat{w}_i$ is the estimation of w_i and $\tilde{w}_i = w_i - \hat{w}_i$ denote the weight matrix estimation errors. Thus, the following inequalities can be further obtained:

$$\begin{aligned}\dot{V}_1 \leq & -\alpha_2 \sum_{i=1}^{n} p_{2i}^2 \phi_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \tau_{i\chi}^2 \\ & - \alpha_2 \sum_{i=1}^{n} p_{2i}^2 \delta_i \sum_{\chi=1}^{3} \left|\lambda_{2i\chi}\right|^{2\alpha_2 - 2} \left|\tau_{i\chi}\right|^{\alpha_3 + 1} \end{aligned}$$

$$+\alpha_2\sum_{i=1}^{n}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\tilde{w}_i\frac{\varphi_i^T\varphi_i}{2l_i^2}$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right). \quad (17)$$

Taking the time derivative of V_2 yields:

$$\dot{V}_2=-\alpha_2\tilde{w}_i\dot{\hat{w}}_i. \quad (18)$$

Then the time derivative of Lyapunov candidate function can be rewritten as:

$$\dot{V}=\dot{V}_1+\dot{V}_2\leq-\alpha_2\sum_{i=1}^{n}p_{2i}^2\phi_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2$$
$$-\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\left|\tau_{i\chi}\right|^{\alpha_3+1}$$
$$+\alpha_2\tilde{w}_i\left(\sum_{i=1}^{n}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\frac{\varphi_i^T\varphi_i}{2l_i^2}-\dot{\hat{w}}_i\right)$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right). \quad (19)$$

Choosing the adaptive law as:

$$\dot{\hat{w}}_i=\sum_{i=1}^{n}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\frac{\varphi_i^T\varphi_i}{2l_i^2}-m_1\hat{w}_i. \quad (20)$$

Then the equality (19) becomes:

$$\dot{V}\leq-\alpha_2\sum_{i=1}^{n}p_{2i}^2\phi_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2$$
$$-\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\left|\tau_{i\chi}\right|^{\alpha_3+1}$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right)$$
$$+\alpha_2m_1\tilde{w}_i\hat{w}_i. \quad (21)$$

The following inequalities will be employed to derive the proof of stability.

$$\begin{aligned}\alpha_2m_1\tilde{w}_i\hat{w}_i&=\alpha_2m_1\tilde{w}_i\left(-\tilde{w}_i+w_i\right)\\&=\alpha_2m_1\left(-\tilde{w}_i^2+\tilde{w}_iw_i\right)\\&\leq\alpha_2m_1\left(-\tilde{w}_i^2+\frac{1}{4}\tilde{w}_i^2+w_i^2\right)\\&\leq-\frac{3\alpha_2m_1}{4}\tilde{w}_i^2+\alpha_2m_1w_i^2. \quad (22)\end{aligned}$$

The above inequality can be expressed as:

$$\dot{V}\leq-\alpha_2\sum_{i=1}^{n}p_{2i}^2\phi_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2$$
$$-\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\left|\tau_{i\chi}\right|^{\alpha_3+1}$$
$$+\alpha_2m_1\tilde{w}_i\hat{w}_i-\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}$$
$$+\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right)$$
$$\leq-\alpha_2\sum_{i=1}^{n}p_{2i}^2\phi_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2$$
$$-\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\left|\tau_{i\chi}\right|^{\alpha_3+1}$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right)$$
$$-\frac{\alpha_2m_1}{4}\tilde{w}_i^2+\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}$$
$$-\frac{\alpha_2m_1}{2}\tilde{w}_i^2+\alpha_2m_1w_i^2$$
$$-\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}. \quad (23)$$

If $\frac{\alpha_2m_1}{2}\tilde{w}_i^2>1$, the following inequalities can be obtained:

$$\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}-\frac{\alpha_2m_1}{2}\tilde{w}_i^2+\alpha_2m_1w_i^2$$
$$<\frac{\alpha_2m_1}{2}\tilde{w}_i^2-\frac{\alpha_2m_1}{2}\tilde{w}_i^2+\alpha_2m_1w_i^2\leq\alpha_2m_1w_i^2.$$

If $\frac{\alpha_2m_1}{2}\tilde{w}_i^2\leq1$, the following inequalities can be further obtained:

$$\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}-\frac{\alpha_2m_1}{2}\tilde{w}_i^2<1-\frac{\alpha_2m_1}{2}\tilde{w}_i^2<1.$$

Therefore, combining the above two inequalities yields [45]:

$$\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}-\frac{\alpha_2m_1}{2}\tilde{w}_i^2+\alpha_2m_1w_i^2\leq\alpha_2m_1w_i^2+1.$$

Thus, from these inequalities, the time derivative of V can be rewritten as:

$$\dot{V}\leq-\alpha_2\sum_{i=1}^{n}p_{2i}^2\phi_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2$$
$$-\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\gamma_2-2}\left|\tau_{i\chi}\right|^{\alpha_3+1}$$
$$+\alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}+\frac{1}{2}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}\tau_{i\chi}^2\right)$$
$$-\frac{\alpha_2m_1}{4}\tilde{w}_i^2+\alpha_2m_1w_i^2-\left(\frac{\alpha_2m_1}{2}\tilde{w}_i^2\right)^{\alpha_3+1/2}+1$$
$$\leq-aV-bV^{\frac{\alpha_3+1}{2}}+c, \quad (24)$$

where

$$a=\min\left\{\left(2\phi_i-1\right)\alpha_2\sum_{i=1}^{n}p_{2i}^2\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2},\frac{1}{2}m_1\right\},$$

$$b = \min\left\{2^{\frac{\alpha_3+1}{2}}\alpha_2\sum_{i=1}^{n}p_{2i}^2\delta_i\sum_{\chi=1}^{3}\left|\lambda_{2i\chi}\right|^{2\alpha_2-2}, m_1\right\},$$
$$c = \alpha_2\sum_{i=1}^{n}\left(\frac{l_i^2}{2}+\frac{\varepsilon_i^2}{2}\right)+\alpha_2 m_1 w_i^2+1, \quad i \in M.$$

Then, the next thing to prove is that $\tau_{i\chi}$ will not leave the region as soon as $\tau_{i\chi}$ enter the neighborhood of $\left|\tau_{i\chi}\right| \leq \Delta_1$. The proof is divided into two conditions:

Condition 1: When $\tau_{i\chi} \notin \left(\left|\tau_{i\chi}\right| \leq \Delta_1\right)$ and$\lambda_{2i\chi} \neq 0$, where $\Delta_1 = \min\left(\sqrt{\frac{2c}{a}}, \sqrt{2}\left(\frac{c}{b}\right)^{\frac{1}{\alpha_3+1}}\right)$. It is obvious that$\dot{V} \leq -aV - bV^{\frac{\alpha_3+1}{2}}$, which means that $\tau_{i\chi}$ will converge to regions containing origin in setting time by combing Lemma 3.

Condition 2: When $\lambda_{2i\chi} = 0$, it needs to prove that $\lambda_{2i\chi} = 0$, $\tau_{i\chi} \neq 0$ is not an attractor and the reachability of (6) can be ensured.

$$\begin{aligned}\dot{\lambda}_2 &= [(L+B)\otimes I]\left(-\bar{H}+\bar{G}+\bar{F}\right)\\ &= [(L+B)\otimes I]\left(\bar{G}-\bar{H}\right)\\ &= \sum_{i=1}^{n}\sum_{j=1}^{n}a_{ij}\left(\bar{g}_j-\bar{h}_j\right)+b_i\left(\bar{g}_i-\bar{h}_i\right). \end{aligned} \tag{25}$$

Thus, $\dot{\lambda}_{2i\chi} \neq 0$ is hold for any $\lambda_{2i\chi} = 0$.

Under the Condition 1 and Condition 2, if $\tau_{i\chi} \notin \left(\left|\tau_{i\chi}\right| \leq \Delta_1\right)$, the time derivative of the Lyapunov function V is satisfied that $\dot{V} \leq -aV - bV^{\frac{\alpha_3+1}{2}}$. According to Lemma 1 and Lemma 2, the finite time T_{r1}^* exists, and for any $t > T_{r1}^*$, $\left|\tau_{i\chi}\right| \leq \Delta_1$ is hold, that is, the states of all followers will converge to the neighborhood of sliding surface.

Step 2: It needs to prove that the formation-keeping state errors $\lambda_{1i\chi}$ and $\lambda_{2i\chi}$ of following AUVs can converge to local regions of sliding surface in finite time.

Condition l: When $\left|\tau_{i\chi}\right| \leq \Delta_1$, $\lambda_{2i\chi} \neq 0$, the following equality can be obtained:

$$\lambda_{1i}+p_{1i}\mathrm{sig}(\lambda_{1i})^{\alpha 1}+p_{2i}\mathrm{sig}(\lambda_{2i})^{\alpha 2} = \mu_i \tag{26}$$

where $|\mu_i| \leq \Delta_1$, $i = 1, \cdots, n$, $\chi = 1, 2, 3$. This above equation can be transformed into:

$$\begin{aligned}&\lambda_{1i\chi}+\left(p_{1i}-\frac{1}{2}\mu_i\mathrm{sig}\left(\lambda_{1i\chi}\right)^{-\alpha_1}\right)\mathrm{sig}\left(\lambda_{1i\chi}\right)^{\alpha_1}\\ &\quad+\left(p_{2i}-\frac{1}{2}\mu_i\mathrm{sig}\left(\lambda_{2i\chi}\right)^{-\alpha_2}\right)\mathrm{sig}\left(\lambda_{2i\chi}\right)^{\alpha_2} = 0. \end{aligned} \tag{27}$$

If $p_{1i}-\frac{1}{2}\mu_i\mathrm{sig}\left(\lambda_{1i\chi}\right)^{-\alpha_1} > 0$ and $p_{2i}-\frac{1}{2}\mu_i\mathrm{sig}\left(\lambda_{2i\chi}\right)^{-\alpha_2} > 0$, it turns to the form of nonsingular fast terminal sliding mode. Then the location and velocity of multi-AUV system $\lambda_{1i\chi}$ and $\lambda_{2i\chi}$ can converge to local regions in finite time: $\left|\lambda_{1i\chi}\right| \leq \left(\frac{\mu_i}{2p_{1i}}\right)^{\frac{1}{\alpha_1}}$, $\left|\lambda_{2i\chi}\right| \leq \left(\frac{\mu_i}{2p_{2i}}\right)^{\frac{1}{\alpha_2}}$.

Condition 2: When $\lambda_{2i\chi} = 0$, combining the condition of $\left|\tau_{i\chi}\right| \leq \Delta_1$, $\lambda_{1i\chi}$ will converge to this regions in finite time: $\left|\lambda_{1i\chi}\right| \leq \min\left(\Delta_1, \left(\frac{\Delta_1}{p_{1i}}\right)^{\frac{1}{\alpha_1}}\right)$.

By integrating the Condition 1 and Condition 2, $\lambda_{1i\chi}$ and $\lambda_{2i\chi}$ will converge to local regions of sliding surface in finite time T_r^*, where $T_r^* = T_{r1}^* + T_{r2}^*$. For any $t > T_r^*$, these conditions will be established: $\left|\lambda_{1i\chi}\right| \leq \Psi_{1i} \triangleq \Delta_2 \cup \Delta_3$, $\left|\lambda_{2i\chi}\right| \leq \Psi_{2i} \triangleq \Delta_4$, where $\Delta_2 = \left(\frac{\mu_i}{2p_{1i}}\right)^{\frac{1}{\alpha 1}}$, $\Delta_3 = \min\left(\Delta_1, \left(\frac{\Delta_1}{p_{1i}}\right)^{\frac{1}{\alpha 1}}\right)$, $\Delta_4 = \left(\frac{\mu_i}{2p_{2i}}\right)^{\frac{1}{\alpha 2}}$.

Step 3: It needs to prove that the station-keeping state errors of each following AUV, i.e., $c_{1i\chi}$ and $c_{2i\chi}$, can converge to local regions of sliding surface in the condition of $\forall t > T_r^*$. Because $\bar{\varsigma}$ satisfies the Assumption 1 and Lemma 1, the conclusions that H is full rank and $[(L+B)\otimes I]^T\,[(L+B)\otimes I]$ is positive can be obtained. Choose θ to express the eigenvalues of H, so the following inequalities can be obtained:

$$\theta_{\min}\left([(L+B)\otimes I]^T\,[(L+B)\otimes I]\right)c_1^Tc_1 \leq 3\sum_{i=1}^{n}\Psi_{1i}^2$$
$$\theta_{\min}\left([(L+B)\otimes I]^T\,[(L+B)\otimes I]\right)c_2^Tc_2 \leq 3\sum_{i=1}^{n}\Psi_{2i}^2.$$

The above equations mean that:

$$\begin{aligned}\left|c_{1i\chi}\right| &\leq \sqrt{\frac{3\sum_{i=1}^{n}\Psi_{1i}^2}{\theta_{\min}\left([(L+B)\otimes I]^T\,[(L+B)\otimes I]\right)}}\\ \left|c_{2i\chi}\right| &\leq \sqrt{\frac{3\sum_{i=1}^{n}\Psi_{2i}^2}{\theta_{\min}\left([(L+B)\otimes I]^T\,[(L+B)\otimes I]\right)}}\end{aligned} \tag{28}$$

Then the Theorem 1 has been proved and the tracking errors of multi-AUV system converge to a small enough neighborhood in finite time.

Remark 2: The radiuses of regions are depended on following control parameters: the region of Δ_1 can get shorten if ϕ_i, δ_i, p_{2i} turn larger or α_3 turns smaller. The range of the station-keeping state errors will be reduced if α_1, α_2 become smaller and p_{1i}, p_{2i} become larger. Although the region can be rendered as small as desired variables, two inevitable issues still need concern. Firstly, chattering problem will be more serious when ϕ_i, δ_i get bigger. Secondly, if α_2 approaches to 1 and α_1 approaches to α_2, the nonsingular fast terminal sliding mode will turn to conventional linear sliding mode.

Remark 3: We choose position and velocity, which is the derivative of position, as the system degrees of freedom in this paper. Compared with underactuated case, the freedom is equal to the independent control variable f_i of each AUV. All results are established based on the assumption of fixed attitudes and we will consider the underactuated case by assuming that the attitudes are not fixed in future work.

IV. SIMULATION

To validate the simulation model conducted for the finite time neural network controllers. Considering the multi-AUV system made up of 3 followers and 1 leader, and communication relationship of system is shown clearly in Fig. 1. Choosing interference forces of each follower as $\xi_i = [0.01\sin(it)$, $0.02\cos(it)$, $0.03\sin(2it)]$, and the transition matrix are

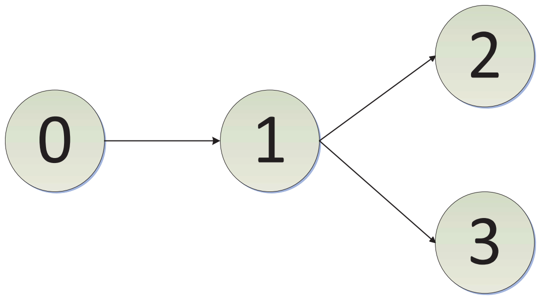

FIGURE 1. Communication relationship.

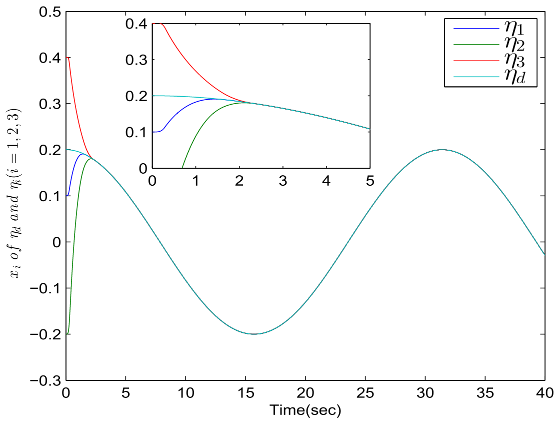

FIGURE 2. x_i of η_d and η_i. i = 1, 2, 3.

chosen as $\varphi_i = \pi/5$, $\theta_i = -\pi/10$, $\psi_i = \pi/12$. Command other matrices as $M_i = \text{diag}\{1.76\,, 1.47, 1.38\}$, and $D_i = \text{diag}\{1.19 + 0.8\,|u_i|\,, 0.89 + 0.92\,|v_i|\,, 1.47 + 1.18\,|w_i|\}$. The initial reference trajectories are selected as $\eta_d = \frac{1}{5}[\cos(\frac{1}{5}t),\ \sin(\frac{1}{5}t)\ ,\ \frac{1}{2}\sin(\frac{1}{2}t)]^T$, $W_i = 1.27$, and $B_i = 1.18$. Combining the communication topology, Laplacian matrix L and B are given as:

$$L = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\eta_1(0) = [\frac{1}{10}, \frac{3}{5}, \frac{7}{10}]^T$, $\eta_2(0) = [-\frac{1}{5}, \frac{2}{5}, \frac{1}{2}]^T$, and$\eta_3(0) = [\frac{2}{5}, \frac{7}{10}, -\frac{1}{10}]^T$ are starting locations of ith followers. Then, suppose that the starting acceleration does not exist, that is $\ddot{\eta}_i = [0, 0, 0]$. The coefficients of multi-AUV system are chosen as: $\alpha_1 = 2$, $\alpha_2 = 1.67$, $\alpha_3 = \frac{3}{5}$, $p_{1i} = 2$, $p_{2i} = 2$, $\phi_i = 5$, $\delta_i = 5$, $\varepsilon_i = \frac{1}{2}$, $l_i = \frac{1}{2}$, $m_1 = 0.0028$, $i = 1, \cdots, 3$.

To clearly observe the performance comparison between above control laws and nonlinear terminal sliding mode, define the latter as: $\tau_i = \lambda_{1i} + p_{2i}\,\text{sig}(\lambda_{2i})^{\alpha 2}$. Then the control laws are constructed as:

$$\begin{aligned} \bar{f}_i = (\sum_{j=1, j\neq i}^{n} a_{ij} + b_i)^{-1} \times [\sum_{j=1, j\neq i}^{n} a_{ij}\bar{f}_j \\ - \frac{1}{\alpha_2 p_{2i}} \text{diag}\left\{\text{sig}(\lambda_{2i})^{2-\alpha_2}\right\} \\ - p_{2i}\,|\lambda_{2i}|^{\alpha_2 - 1}\left(\phi_i \tau_i + \delta_i \text{sig}(\tau_i)^{\alpha_3} + f_{iadp}\right)]. \end{aligned} \tag{29}$$

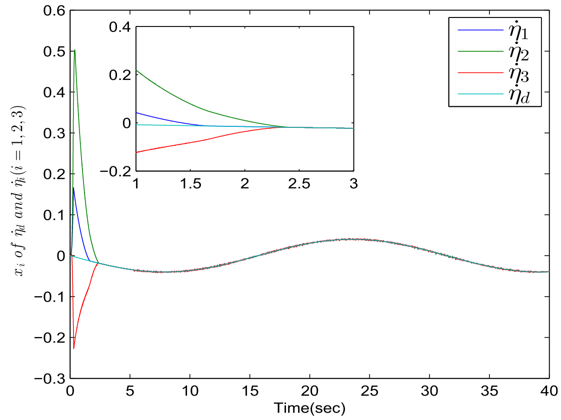

FIGURE 3. x_i of $\dot{\eta}_d$ and $\dot{\eta}_i$. i=1, 2, 3.

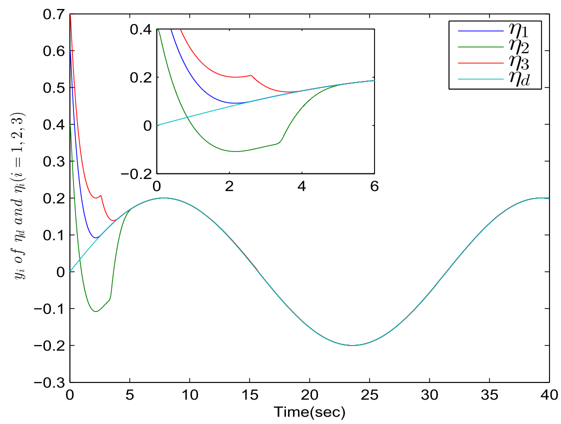

FIGURE 4. y_i of η_d and η_i. i = 1, 2, 3.

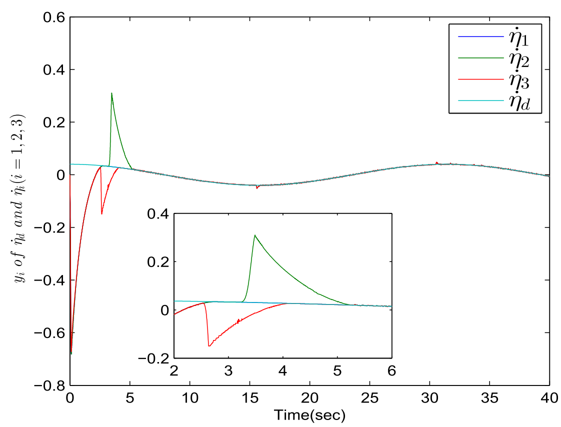

FIGURE 5. y_i of $\dot{\eta}_d$ and $\dot{\eta}_i$. i = 1, 2, 3.

The parameters of the equality (29) are chosen the same as those given in equality (7). Considering the station-keeping errors metric as the reference objects, it is defined as: $SKEM = \sqrt{\sum_{i=1}^{n} \|c_{1i}\|^2}$, the differences can be obviously seen from the performance comparison.

Fig. 1 is the communication topology of the multi-AUV system. By using the proposed control method, the results of simulation are clearly presented in Figs. 2-7, where

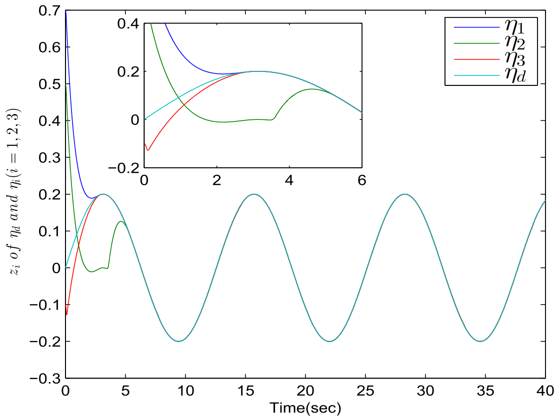

FIGURE 6. z_i of η_d and η_i. i = 1, 2, 3.

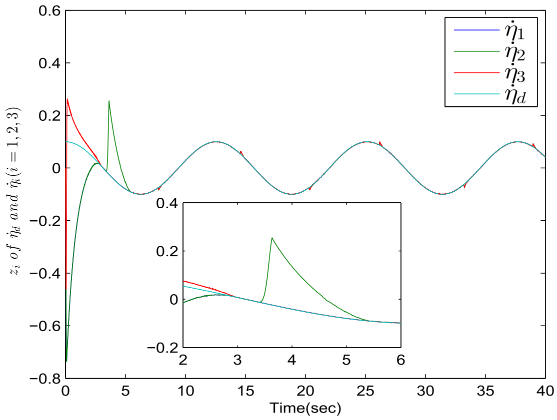

FIGURE 7. z_i of $\dot{\eta}_d$ and $\dot{\eta}_i$. i = 1, 2, 3.

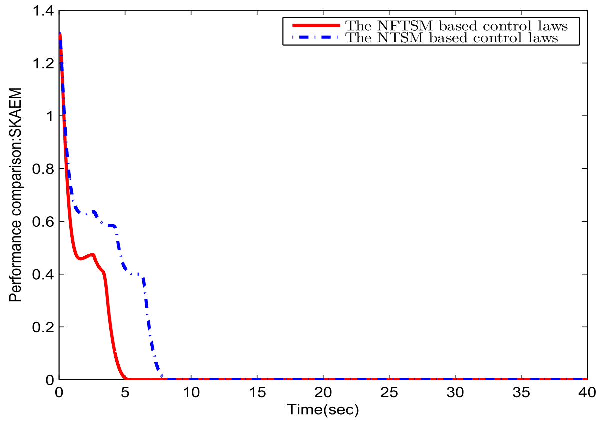

FIGURE 8. Performance comparison.

Figs. 2-4 display the trajectories of the leader η_d and the followers η_i, and Figs. 5-7 reflect the velocity signals of the leader $\dot{\eta}_d$ and the followers $\dot{\eta}_i$. These Figs. show that the desired reference leader signals can be tracked well by the followers. Furthermore, it can clearly see that the advantages of nonsingular fast terminal sliding mode(NFTSM) in convergence time from Fig. 8, it is much better than nonsingular terminal sliding mode (NTSM).

Remark 4: The experimental results are obtained by trial and error for multiple simulation and the problem of collision is excluded. The simulation results mentioned above indicate clearly that the controllers have better robustness to resist unknown internal and external interference factors compared with the traditional control method.

V. CONCLUSION

In this paper, a distributed control method based on RBF NNs technique and nonsingular fast terminal sliding mode has been applied for multi-AUV system. In order to eliminate the impacts of unknown internal and external interference factors and lower accuracy caused by complex and changeable marine environment, the algebraic graph theory and the NNs technique are integrated into the distributed controllers. By combining the nonsingular terminal sliding mode with the finite-time technique, the region of original point where the location and velocity tracking signals converge to in finite time are obtained. In addition, the convergence time has been greatly shortened comparing with which constructed by classical nonlinear terminal sliding mode. The simulation model clearly demonstrates the availability of the proposed method and robustness of the multi-AUV system. Besides, in practice, collisions between AUVs may occur in the process of completing the tasks together, which are very undesired. The future research problems we considered include the algorithm simplification and practical application.

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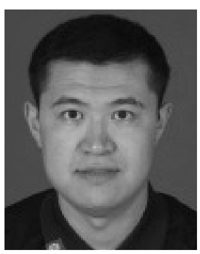

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