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# An Optimization Model for Balancing Assembly Lines With Stochastic Task Times and Zoning Constraints

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**ABSTRACT** This paper aims to bridge the gap between theory and practice by addressing a real-world assembly line balancing problem (ALBP), where task times are stochastic and there are zoning constraints in addition to the commonly known ALBP constraints. A mixed integer programming (MIP) model is proposed for each of the straight and U-shaped assembly line configurations. The primary objective in both cases is to minimize the number of stations; minimizing the maximum of stations' meantime; and the stations' time variance is considered as secondary objectives. Four different scenarios are discussed for each model, with differences in the objective function. The models are validated by solving a real case taken from an automobile manufacturing company and some standard test problems available in this paper. The results indicate that both models are able to provide optimum solutions for problems of different sizes. The technique for order preference by similarity to ideal solution (TOPSIS) is used to create reliable comparisons of the different scenarios and valid analysis of the results. Finally, some insights regarding the selection of straight and U-shaped layouts are provided.

**INDEX TERMS** Assembly line balancing, mathematical programming, stochastic, zoning constraints.

## I. INTRODUCTION

Assembly lines have traditionally been used for mass and lean production where one or more product(s) have to be assembled in an arrangement of workplaces called stations, which are usually connected by some kind of material handling device (e.g., conveyors and cranes). The assembly line balancing problem (ALBP) is a well-known decision problem and has significant impact on the performance and productivity of assembly plants. ALBP aims to optimally distribute the assembly tasks among assembly stations while optimizing one or more objectives (e.g., number of workstations or cycle time) without violating certain technological, operational, and spatial constraints [1].

In principle, ALBPs are generally classified into two groups depending on their assumptions, constants, and objectives. The two groups are simple assembly line balancing problems (SALBPs) and generalized assembly line balancing problems (GALBPs) [2], [3].

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SALBPs, which make some simplifying assumptions, are often divided into two types in the literature [4], [5]. In SALBP-1 the cycle time ($CT$) is given and the aim is to minimize the number of stations ($M$). SALBP-2 aims to minimize the $CT$ and $M$ is given. Readers interested in knowing more about SALBP are referred to the comprehensive reviews by Scholl and Becker [6] and Battaïa and Dolgui [7].

GALBPs deal with more practical considerations and additional constraints raised in the real-world such as U-shaped assembly lines, variable task times, and zoning constraints [8], [9]. In other words, all ALBPs that do not belong to SALBP fall within the scope of GALBP. A good classification of GALBPs can be found in Boysen *et al.* [10] and Becker & Scholl [11].

ALBPs can be grouped into single-model and mixed-model product types; in the former a single homogeneous model is assembled, while in the latter more models are produced. In terms of task time, assembly lines can be classified as deterministic or stochastic [12]. According to the literature [13]–[15], the most prominent type of ALBP is SALBP, where task times are deterministic and only basic ALBP constraints are considered. Therefore, it is not surprising that there is still a significant gap between existing research and real-world applications. Both researchers and industrial practitioners are now working toward solving GALBPs as more realistic assembly line settings and features are taken into consideration [7], [16], [17]. The current study is targeted at dealing with GALBP by considering task time variations and zoning constraints as two important characteristics of real-world assembly lines.

In real-life assembly, the existence of different sources of variation threatens assembly targets. One source of variability that has a significant impact on assembly performance is the variation in task time due to human or environmental factors such as workers' tiredness, workers' lack of skills, complex operations, and machine breakdowns. The variation in task time may result in line stoppages, shortages, or overtime if it has not been taken into consideration during the planning phase [18], [19]. Moreover, to cope with real-world ALBPs, some constraints arising from the practical environment also have to be considered in addition to the common ALBP constraints such as the precedence relationship and cycle time. One of these practical constraints is the restrictive relationship between tasks, known as zoning constraint [20]. Two types of zoning constraints exist in real-world assembly, namely, positive zoning, and negative zoning. In cases where some tasks are compatible or linked due to the equipment, line shape or space considerations, a positive zoning constraint should be considered ensuring that they are performed at the same station. On the other hand, there might be some incompatible tasks that cannot be performed at the same station due to different equipment or the nature of the operations (e.g., drilling or measuring), which forces the introduction of a negative zoning constraint [21].

In light of the above discussion and to bridge the existing research gap, this study proposes two mathematical models, one per line configuration (straight and U-shaped) to solve the single-model ALBP while considering both positive and negative zoning constraints as well as stochastic task times in addition to other conventional ALBP constraints. The main optimization criterion for both models is considered to be the number of stations. Due to the stochastic nature of the task times, two additional objectives, namely, the maximum of stations' mean time and the maximum of stations' time variance, are optimized as the secondary objectives.

The remainder of the paper is organized as follows. In Section II, a comprehensive literature review of straight and U-shaped stochastic ALBPs is provided. Section III provides a brief description of the case study. The mathematical models developed are given in Section IV. Computational results and their analysis are presented in Section V. Finally, the concluding remarks are outlined in Section VI.

## II. LITERATURE REVIEW

The literature on ALBP is extensive as different authors have addressed different types of ALBPs over the past decades, proposing different models and algorithms, while considering different assumptions, constraints, and objectives. Moreover, there are a few studies in the literature in which the straight and U-shaped ALBPs have been jointly addressed by proposing meta-heuristic algorithms (e.g., [22], [23]). However, here we will review only studies addressing the single-model stochastic ALBP (St-ALBP) for both straight and U-shaped line configurations, which is the focus of this study. For a comprehensive review of different types of ALBP, the readers are referred to recent review studies [7], [24]–[26].

Liu *et al.* [27] proposed a bidirectional heuristic to cope with a straight assembly line, working with a set number of stations and a pre-defined confidence level to ensure that the workloads at the stations did not exceed the given upper bound for *CT*. The objective of the study was merely to minimize the *CT*. Erel *et al.* [28] proposed a beam search algorithm to minimize the total cost, including the cost of labor and incomplete operations in U-shaped assembly lines. In their study, three different levels were proposed for *CT*, assuming that *CT* can be any positive number due to the stochasticity of the problem. The task incompletion cost was included in the objective function.

Baykasoglu and Özbakir [29] developed a rule-based genetic algorithm (GA) for U-shaped assembly lines to minimize the number of stations for a given *CT*. The algorithm was developed by integrating some priority heuristic rules, GA, and a well-known computer method of sequencing operations for assembly lines (COMSOAL). Ağpak and Gökçen [30] dealt with both straight and U-shaped assembly lines by developing novel chance-constrained binary integer programming. The aim of their study was to minimize the number of stations. The authors also presented a goal programming model to cope with the unreliability of assembly lines caused by the stochastic nature of the task times.

Bagher *et al.* [31] developed a novel algorithm for balancing a U-shaped line with the aim of minimizing the number of stations, the stations' idle time, and the probability of having uncompleted tasks at each station. The algorithm was designed by combining the imperialist competitive algorithm (ICA) with some priority heuristic rules and COMSOAL. A study by Cakir *et al.* [32] targeted balancing a straight assembly line with the possibility of assigning tasks to parallel stations. They developed a hybrid simulated annealing (SA) algorithm with the aim of minimizing the cost of design and the smoothness index (*SI*) as a measure for the variation of workload.

Hazir and Dolgui [19] tackled the problem by proposing two mathematical optimization models and developing a robust decomposition algorithm in order to find the optimum solution. The optimization objective was to minimize *CT* for a given number of workstations in straight assembly lines. A hybrid particle swarm optimization (PSO) algorithm with variable neighborhood search was proposed by Hamta *et al.* [33] to minimize the *CT*, *SI*, and total equipment cost. Task times were assumed to be dependent on worker/machine learning as well as on the task sequences.

**TABLE 1. Summary of the reviewed studies of single-model St-ALBP for straight and U-shaped line configurations.**

| Study | Layout | | Additional constraints | | | Objectives | | | | | | | | Solution approach | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | SL | UL | PZC | NZC | OC / PS | *M* | *CT* | *BD/ SI* | *Cost* | *IP* | *RD* | *SMT* | *STV* | MP | EA | HE | MH |
| Liu et al. [27] | • | | | | | | • | | | | | | | | | • | |
| Erel et al. [28] | | • | | | | | | | • | | | | | | | | • |
| Baykasoğlu & Özbakır [29] | | • | | | | • | | | | | | | | | | | • |
| Ağpak and Gökçen [30] | • | • | | | | • | | | | | | | | • | | | |
| Bagher et al. [31] | | • | | | | • | | • | | • | | | | | | | • |
| Cakir et al. [32] | • | | | | • | | | • | • | | | | | | | | • |
| Hazir and Dolgui [19] | • | | | | | | • | | | | | | | • | • | | |
| Hamta et al. [33] | • | | | | | | • | • | • | | | | | | | | • |
| Zhang et al. [34] | • | | | | | • | | | | | | | | | • | | |
| Zhang et al. [35] | • | | | | | | • | | • | | | | | | | | • |
| Krishnan et al. [36] | • | | | | | | | | | | • | | | | | • | • |
| Aydoğan et al. [12] | | • | | | | • | | | | | | | | | | | • |
| Pınarbaşı et al. [37] | • | | | | | | | • | | | | | | • | • | | |
| Saif et al [38] | • | | | | | | • | | | • | | | | | | | • |
| Zhang et al. [39] | • | | | | | | • | | • | | | | | | | | • |
| Dong et al. [20] | • | | | • | | | • | | • | | | | | • | | | • |
| This study | • | • | • | • | | • | | | | | | • | • | • | | | |

Note: Straight line (SL), U-shaped line (UL), positive zoning constraint (PZC), negative zoning constraint (NZC), other constraints (OC); parallel station (PS); station number (M), cycle time (CT), balance delay (BD) / smoothness index (SI), cost (including labor, incompletion, design, equipment and processing costs), incompletion probability (IP), risk of delay (RD); stations' mean time (SMT), stations' time variance (STV), mathematical programming (MP), exact algorithm (EA), heuristics (HE), meta-heuristics (MH).

Zhang *et al.* [34] proposed an exact enumerative algorithm to balance a straight line and minimize the number of workstations. They simplified the problem by considering the optimal assembly sequence as the precedence graph. Zhang *et al.* [35] proposed a hybrid evolutionary algorithm to balance a straight line with the objective of minimizing the *CT* and the processing cost for a given number of stations. They customized the algorithm to improve the convergence speed by suggesting a unique fitness function strategy and selection mechanism.

Krishnan *et al.* [36] proposed a heuristic approach based on the problem properties to balance a straight line. The optimization objective in this study was to minimize the risk of delays at each station. They also created a simulation model to test the validity of the solution found by the heuristic. Aydoğan *et al.* [12] addressed the problem by proposing a novel PSO algorithm for balancing a U-shaped line, with the aim of minimizing the number of workstations. To overcome the shortcomings of the PSO algorithm, an encoding procedure, adaptive inertia weight, and mutation were incorporated in the algorithm. Pınarbaşı *et al.* [37] proposed a novel approach based on queuing networks and constraint programming (CP) as well as a mathematical model to balance a straight line and equalize station utilization (equivalent to *SI*). The authors examined the effect of service and flow process variations on the line balancing results as well as the service process variation (task time).

A Pareto artificial bee algorithm was proposed by Saif *et al.* [38] to deal with a straight assembly line. The optimization objectives were to minimize *CT* and maximize the sum of the average probability of stations and the probability of the whole assembly line to ensure that the station times would not exceed the *CT*. Zhang *et al.* [39] proposed a hybrid evolutionary algorithm to balance the workload at workstations in a straight assembly line. The optimization criteria in this study were the *CT* and the processing cost over a given and unchangeable number of workstations.

Recently, a hybrid PSO and SA algorithm and a chance-constrained mixed zero-one programming model were proposed by Dong et al. [20] to balance the workload in straight assembly lines. The objectives of both the algorithm and the model were to minimize the *CT* and the equipment cost simultaneously. The authors also considered the negative zoning constraint (i.e., task pairs that must not be performed at the same station) in addition to the conventional ALBP constraints.

A summary of the reviewed papers, including the solution approaches, objectives, line configuration, and any additional constraints other than the conventional ones (i.e., precedence relationship, cycle time, or number of workstations), is given in Table 1.

Reviewing the literature summarized in Table 1 revealed that most of the studies on single-model St-ALBP have been targeted at solving straight line problems using meta-heuristic algorithms. As for constraints and real-world considerations, only two studies went beyond the conventional ALBP constraints (i.e.,[20] and [32]) by taking into account the possibility of having parallel stations and a negative zoning constraint (NZC). The most frequent optimization objectives are minimizing the number of workstations, cycle time, and costs.

The information given in Table 1 also shows that no mathematical optimization model exists in the literature to treat straight and U-shaped single-model St-ALBPs

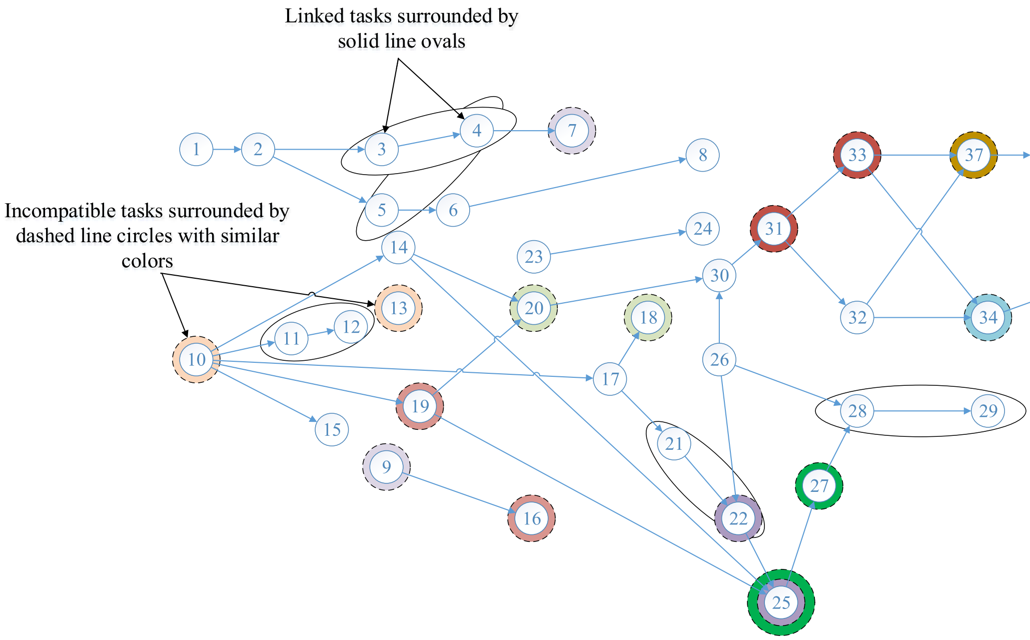


**FIGURE 1.** The precedence network of the real case, including the positive and negative zoning constraints.

while considering both negative and positive zoning constraints (PZC). Moreover, it can be seen that minimizing the maximum of stations' mean time (SMT) and the stations' time variance (STV) has not been addressed in any of the previous studies. This is the case even though these objectives can efficiently evaluate and differentiate the solutions obtained by the optimization models, especially when the resulting solutions are similar in terms of the number of stations (M).

Although most of the previous studies have resorted to meta-heuristic algorithms to cope with St-ALBP complexity and provide approximate solutions to the problem, practical size St-ALBPs can be solved to optimality within reasonable computational time by exact methods with current hardware and software [40]. Thus, to bridge the existing gap in the literature and motivation by a real case in an automotive manufacturing company, this study aims to propose two mixed integer programming models to deal with straight and U-shaped St-ALBP problems considering both NZCs and PZCs.

## III. CASE STUDY EXPLANATION

The case studied was a car engine assembly line of a major Swedish automobile manufacturing company. Although several different models of car engines are manufactured and assembled in this factory, this study is limited to part of the final assembly of a particular engine. The decision makers were seeking the best assembly line configuration (straight or U-shaped) for the chosen part of the car engine's assembly so that the related assembly workloads were efficiently balanced among stations. At the same time, zoning constraints and uncertainty in operation times had to be taken into account as well as the conventional ALBP constraints (e.g., cycle time and precedence relationships).

All the required data, including the task times, precedence relationship between tasks, and the zoning constraints for the product, were collected in collaboration with experts at the company. Figure 1 shows the precedence relations between tasks and the zoning constraints for the chosen part of the engine assembly. In this figure, each node represents an assembly task numbered from 1 to 41, meaning that producing the product involves 41 assembly tasks. The one-directional arrows that connect the nodes show the precedence relationship between tasks, which arise from the technological requirements. Tasks surrounded by ovals with solid lines are linked tasks (e.g., tasks 3 and 4), while tasks surrounded by circles with dashed lines and similar colors are incompatible (e.g., tasks 10 and 13). All the data about the case study can be found in Table 7 in the appendix.

The decision makers were looking for a reliable optimization tool to balance the assembly line for the product by considering two scenarios for cycle time (70 and 65 seconds) as well as two line configurations. The current assembly of the chosen product was being performed in five assembly stations with a cycle time equal to 65 seconds. The decision makers were also interested in knowing whether a higher cycle time (70 seconds) could result in a line with fewer stations or a better workload at the stations.

## IV. PROBLEM DESCRIPTION AND MODEL FORMULATION

In this section, the description of the problem and the formulation of the model are discussed in detail.

### A. PROBLEM DESCRIPTION

The given St-ALBP with zoning constraints, hereafter referred to as St-ALBP-ZC, can be defined as a number of tasks ($j = 1, \ldots, N$), each with a given precedence relationship. It is also assumed that the task times are independent with known means and variances indicated by $t\mu_j$ and $t\sigma_j^2$ respectively. Due to the technological requirements of each product, tasks have to be performed according to a sequence known as the precedence relationship. Furthermore, there are zoning constraints among tasks that have to be satisfied. Thus tasks are assigned to stations ($k = 1, \ldots, K$) according to their precedence relationships and zoning constraints, while ensuring that the probability of the total of task times assigned to stations being greater than $CT$ ($P_k$) should stay below predetermined limits ($\alpha$), as given in Equation (1):

$$P_k \left\{ \sum\nolimits_{i \in W_k} t\mu_i + z_{1-\alpha} \sqrt{\sum\nolimits_{i \in W_k} t\sigma_i^2} \leq CT \right\} \geq 1 - \alpha \quad (1)$$

where $W_k$ is the set of tasks assigned to station $k$ and $Z_{1-\alpha}$ is the $1-\alpha$ quantile of the cumulative standardized normal distribution. It is worth mentioning that equation (1) has been first introduced by Urban and Chiang [41] to model the assembly line balancing problem with stochastic task times. To use this equation as a linear constraint in the model, a few linearization steps are required. The interested readers are referred to Agpak and Gökçen [30] for a detailed explanation about linearization process of equation (1).

The main objective of this St-ALBP-ZC is to minimize the number of assembly stations ($M$). However, it is known that this specific objective reaches a plateau in a few attempts and there are often many solutions with the same $M$, although they may differ in terms of the distribution of tasks among the stations and solution quality [42]. Therefore, to distinguish the best solution among several solutions with the same $M$, some additional objectives are needed. In this study, to guarantee a smooth workload and avoid a high fluctuation in working time at stations, two additional objectives are considered aside from the main objective ($M$). The additional objectives are maximum stations' mean time ($SMT_{max}$) and maximum stations' time variance ($STV_{max}$). The $SMT_{max}$ guides the optimization to find a solution with a smoother station workload in terms of the sum of the mean times of the tasks. The index that matches $SMT_{max}$ identifies the station with the maximum cumulative amount of tasks' mean times. It will force the optimization model to assign the tasks to stations in such a way that all the stations have the same amount of workload in terms of tasks' mean times. On the other hand, it is obvious that smoothing the workload at stations by only considering the tasks' mean times may not provide a valid balance, due to the stochasticity of task-processing times. In other words, it may happen that several tasks with high time variation are assigned to the same stations, resulting in high uncertainty on job completion time, even though the workload is smooth in terms of the cumulative amount of tasks' mean times. In such circumstances the $STV_{max}$ index will come in handy to find the station with the maximum variance in task time. Including this in the optimization model will re-arrange the tasks among the stations so that all the stations have an equal sum of tasks' time variance. The $SMT_{max}$ and $STV_{max}$ are calculated using Equations (2) and (3).

$$SMT_{\max} = \max \left( \sum_{i \in W_k} t\mu_i \right) \quad (2)$$

$$STV_{\max} = \max \left( \sum_{i \in W_k} t\sigma_i^2 \right) \quad (3)$$

The assumptions made to model the problem are as follows:

- The task times are normally and independently distributed with known means and variances.
- The travel times of operators are ignored.
- There are known zoning constraints among the tasks in terms of linked and incompatible tasks sets.
- Parallel tasks and parallel stations are not allowed.
- The cycle time is known and fixed.
- The precedence relationship among tasks is known and unchangeable.
- Tasks cannot be split and each task should be performed at one station from start to finish.
- Only one task can be performed at a station at a given time.
- The tasks' execution times are independent of the assigned station.
- A large quantity of one homogeneous product is produced in a continuous and standardized way (mass production system).

### B. MATHEMATICAL FORMULATION

Two mathematical models are proposed in this section to deal with straight and U-shaped St-ALBP-ZC. To improve the efficiency of the models and decrease the computational time, the models benefit from some realistic upper and lower bounds for the number of stations to reduce the number of decision variables and constraints. Moreover, the earliest and latest stations that each task can be assigned are also considered to further reduce the number of variables and constraints. Four different scenarios are tested in relation to changes in the objectives considered. Due to the importance of the first objective in the real world, the number of stations is set as the primary objective in all the scenarios. The order of objectives in the different scenarios are as follows.

Scenario 1: Minimizing the number of stations

Scenario 2: Minimizing the number of stations and the maximum of stations' mean time

Scenario 3: Minimizing the number of stations and the maximum of stations' time variance

Scenario 4: Minimizing the number of stations and the maximum of stations' mean time and the maximum of stations' time variance

It is worth noting that in scenarios 2 to 4, multiple objectives are dealt with using the lexicographical ordering in which the minimization of the number of stations is performed first, and then the secondary objective(s) are dealt with while ensuring that the primary objective is optimized [43]. Following the lexicographic method, the objectives are arranged based on their importance and optimized in a hierarchical order. Therefore, the problem is first solved by satisfying all the constraints and optimizing the primary objective. Then, the secondary objective is optimized by constraining the problem to the solution space of the primary objective. This process continued until all the objectives are optimized. Interested readers are referred to Coello [44] and Marler and Arora [45] for a good explanation about lexicographic method.

The notations used in modeling the St-ALBP-ZC are presented in Table 2.

**TABLE 2. Notations used in the mathematical model.**

| Notation | Definition |
|---|---|
| Indices | |
| $i,j$ | Task index ($i,j = 1,\ldots,N$) |
| $k,l$ | Station index ($k,l = 1,\ldots,K$) |
| Parameters | |
| $N$ | Number of tasks |
| $CT$ | Given cycle time |
| $K$ | Upper bound on the number of stations; $K \le N$ |
| $Pr_{ij}$ | Precedence matrix: $\begin{cases}1; & \text{if task } i \text{ is the predecessor of task } j\\ 0; & \text{otherwise}\end{cases}$ |
| $ZC^{+}_{ij}$ | PZC matrix: $\begin{cases}1; & \text{if task } i \text{ is linked to task } j\\ 0; & \text{otherwise}\end{cases}$ |
| $ZC^{-}_{ij}$ | NZC matrix: $\begin{cases}1; & \text{if task } i \text{ is incompatible with task } j\\ 0; & \text{otherwise}\end{cases}$ |
| $t_i$ | Processing time of task $i$; $t_i \sim N(t\mu_i, t{\sigma_i}^2)$ |
| $W_k$ | Set of tasks assigned to station $k$ |
| $st_k$ | Time of station $k$; $st_k \sim N(\sum_{i\in W_k} t\mu_i, \sum_{i\in W_k} t\sigma_i^2)$ |
| $m_{min}$ | Theoretical minimum number of workstations |
| $m_{max}$ | Maximum number of workstations |
| $\alpha$ | Upper bound on the probability of total tasks' time assigned to stations exceeds the $CT$ |
| Decision variables | |
| $M$ | Number of stations |
| $SMT_{max}$ | Maximum of the stations' mean time |
| $STV_{max}$ | Maximum of the stations' time variance |
| $x_{ik}$ | $\begin{cases}1; & \text{if task } i \text{ is assigned to station } k \text{ in main precedence network}\\ 0; & \text{otherwise}\end{cases}$ |
| $y_{ik}$ | $\begin{cases}1; & \text{if task } i \text{ is assigned to station } k \text{ in phantom precedence network}\\ 0; & \text{otherwise}\end{cases}$ |
| $S_k$ | $\begin{cases}1; & \text{if station } k \text{ is established}\\ 0; & \text{otherwise}\end{cases}$ |
| $u_{ivk}$ | Auxiliary variable to make sure the feasible assignment of task $i$ and $v$ to station $k$ |

Considering the given St-ALBP-ZC description and assumptions, the following mixed integer programming (MIP) model is proposed for the straight line.

$$Min \begin{cases} (1)\ M = \sum_{k=\lceil m_{\min}\rceil}^{m_{\max}} S_k \\ (2)\ M, SMT_{\max} \\ (3)\ M, STV_{\max} \\ (4)\ M, SMT_{\max} + STV_{\max} \end{cases} \tag{4}$$

$$Subjectto: \sum_{k=E_i}^{L_i} x_{ik} = 1; \quad \forall i = 1,\ldots,N \tag{5}$$

$$\sum_{k=E_i}^{L_i} (m_{\max} - k + 1)x_{ik} - \sum_{k=E_j}^{L_j} (m_{\max} - k + 1)x_{jk} \ge 0; \quad \forall (i,j) \in \mathrm{Pr}_{ij} \tag{6}$$

$$CT^2 - 2CT\sum_{i=1}^{N} t\mu_i x_{ik} + \sum_{i=1}^{N} t\mu_i^2 x_{ik} + 2\sum_{i=1}^{N-1}\sum_{v=i+1}^{N} t\mu_i t\mu_v u_{ivk} - z_{1-\alpha}^2 \sum_{i=1}^{N} t{\sigma_i}^2 x_{ik} \ge 0; \quad \forall k = 1,\ldots,m_{\max} \tag{7}$$

$$CT \times S_k - \sum_{i=1}^{N} t\mu_i x_{ik} \ge 0; \quad \forall k = 1,\ldots,m_{\max} \tag{8}$$

$$x_{ik} + x_{vk} - u_{ivk} \le 1; \quad \forall k = 1,\ldots,m_{\max}; \quad i = E_i,\ldots,L_i; \quad v = E_v,\ldots,L_v;\ i \ne v \tag{9}$$

$$x_{ik} + x_{vk} - 2u_{ivk} \ge 0; \quad \forall k = 1,\ldots,m_{\max}; \quad i = E_i,\ldots,L_i; \quad v = E_v,\ldots,L_v;\ i \ne v \tag{10}$$

$$x_{ik} = x_{jk}; \quad \forall (i,j) \in ZC^{+}_{ij}\ \forall k = 1,\ldots,m_{\max} \tag{11}$$

$$x_{ik} + x_{jk} \le 1; \quad \forall (i,j) \in ZC^{-}_{ij} \forall k = 1,\ldots,m_{\max} \tag{12}$$

$$\sum_{i=1}^{N} t\mu_i x_{ik} \le SMT_{\max}; \quad \forall k = 1,\ldots,m_{\max} \tag{13}$$

$$\sum_{i=1}^{N} t{\sigma_i}^2 x_{ik} \le STV_{\max}; \quad \forall k = 1,\ldots,m_{\max} \tag{14}$$

$$x_{ik}, S_k, u_{ivk} \in \{0,1\}; \quad i,v = 1,\ldots,N\ k = 1,\ldots,m_{\max}; \quad SMT_{\max}, STV_{\max} \in R^+ \tag{15}$$

where $E_i$ and $L_i$ are the earliest and the latest stations for processing task $i$, respectively, calculated by Equations (16) and (17) in which $Pre_i$ and $Suc_i$ indicate the predecessors and the successors of task $i$, respectively. Also, Equations (18) and (19) calculate $m_{min}$ and $m_{max}$, respectively. It is worth mentioning that using the above bounds, searching the problem space for optimality will be performed more efficiently by avoiding redundant task assignments [46].

$$E_i = \left\lceil \frac{t\mu_i + \sum_{j\in \mathrm{Pre}_i} t\mu_j + z_{1-\alpha} \times \sqrt{t\sigma_i^2 + \sum_{j\in \mathrm{Pre}_i} t\sigma_j^2}}{CT} \right\rceil; \quad i = 1,\ldots,N \tag{16}$$

$$L_i = m_{\max} + 1$$

$$- \left\lceil \frac{t\mu_i + \sum_{j \in \mathrm{Suc}_i} t\mu_j + z_{1-\alpha} \times \sqrt{t\sigma_i^2 + \sum_{j \in \mathrm{Suc}_i} t\sigma_j^2}}{CT} \right\rceil; \quad i = 1, \ldots, N \tag{17}$$

$$m_{\min} = \left\lceil \frac{\sum_{i=1}^{N} t\mu_i + z_{1-\alpha} \times \sqrt{\sum_{i=1}^{N} t\sigma_i^2}}{CT} \right\rceil - 1 \leq M \tag{18}$$

$$m_{\max} = \min \left\{ N, \left\lfloor \frac{\sum_{i=1}^{N} t\mu_i}{CT + 1 - \max_{\forall i} t\mu_i} \right\rfloor + 1, \left\lceil 2 \times \frac{\sum_{i=1}^{N} t\mu_i}{CT + 1} \right\rceil + 1 \right\} \tag{19}$$

Equation (4) represents the objectives for the four different scenarios. Constraint (5) ensures that each task is assigned to only one station between its earliest and latest possible stations. Constraint (6) guarantees that the precedence relations between tasks are not violated. Constraints (7) and (8) ensure that the probability of exceeding the given $CT$ will always stay under the predetermined limit ($\alpha$). Constraints (9) and (10) are used to make the auxiliary variable ($u_{ivk}$) dependent on $x_{ik}$ and $x_{vk}$ so that using these constraints all the feasible assignments of task $i$ and $v$ to station $k$ will be made possible. Constraint (11) guarantees that the PZCs between the linked tasks are satisfied. Moreover, constraint (12) ensures that the NZCs among tasks are satisfied through maintaining that only one of the unlinked tasks can be assigned to each station. Constraint (13) is used to determine the maximum stations' mean time $SMT_{max}$. In addition, the maximum stations' time variance, $STV_{max}$, is determined by constraint (14). Finally, constraint (15) defines the domains of the decision variables, which are binary and positive real numbers ($R^+$).

Based on the model proposed for the straight line (see Equations 4 to 15), the following MIP model is proposed for the U-shaped line.

Min: objectives given in Equation (4)

$$s.t: p_{ik} = x_{ik} + y_{ik}; \quad \forall i = 1, \ldots, N; \ \forall k = 1, \ldots, m_{\max} \tag{20}$$

$$\sum_{k=E_i}^{L_i} p_{ik} = 1; \quad \forall i = 1, \ldots, N \tag{21}$$

$$\sum_{k=E_i}^{L_i} (m_{\max} - k + 1) p_{ik} - \sum_{k=E_j}^{L_j} (m_{\max} - k + 1) p_{jk} \geq 0; \quad \forall (i,j) \in \mathrm{Pr}_{ij} \tag{22}$$

$$CT^2 - 2CT \sum_{i=1}^{N} t\mu_i p_{ik} + \sum_{i=1}^{N} t\mu_i^2 p_{ik} + 2 \sum_{i=1}^{N-1} \sum_{v=i+1}^{N} t\mu_i t\mu_v u_{ivk} - z_{1-\alpha}^2 \sum_{i=1}^{N} t\sigma_i^2 p_{ik} \geq 0; \quad \forall k = 1, \ldots, m_{\max} \tag{23}$$

$$CT \times S_k - \sum_{i=1}^{N} t\mu_i p_{ik} \geq 0; \quad \forall k = 1, \ldots, m_{\max} \tag{24}$$

$$p_{ik} + p_{vk} - u_{ivk} \leq 1; \quad \forall k = 1, \ldots, m_{\max}; \quad i = E_i, \ldots, L_i; \ v = E_v, \ldots, L_v; \ i \neq v \tag{25}$$

$$p_{ik} + p_{vk} - 2u_{ivk} \geq 0; \quad \forall k = 1, \ldots, m_{\max}; \quad i = E_i, \ldots, L_i; \ v = E_v, \ldots, L_v; \ i \neq v \tag{26}$$

$$p_{ik} = p_{jk}; \quad \forall (i,j) \in ZC_{ij}^{+}; \ \forall k = 1, \ldots, m_{\max} \tag{27}$$

$$p_{ik} + p_{jk} \leq 1; \quad \forall (i,j) \in ZC_{ij}^{-}; \ \forall k = 1, \ldots, m_{\max} \tag{28}$$

$$\sum_{i=1}^{N} t\mu_i p_{ik} \leq SMT_{\max}; \quad \forall k = 1, \ldots, m_{\max} \tag{29}$$

$$\sum_{i=1}^{N} t\sigma_i^2 p_{ik} \leq STV_{\max}; \quad \forall k = 1, \ldots, m_{\max} \tag{30}$$

$$x_{ik}, y_{ik}, p_{ik}, S_k, u_{ivk} \in \{0, 1\}; \quad i, v = 1, \ldots, N \ k = 1, \ldots, m_{\max}; \quad SMT_{\max}, STV_{\max} \in R^{+} \tag{31}$$

The descriptions of Equations (21) to (31) are similar to Equations (5) to (15), except for the decision variable $x_{ik}$ which is replaced by a new zero-one decision variable, $p_{ik}$, calculated by Equation (20).

## V. RESULTS AND ANALYSIS

This section reports on the optimization of the case study using the above MIP models. Some standard test problems will also be analyzed. An efficient multi-attribute decision-making approach called TOPSIS is used to analyze the results and provide managerial insight.

### A. COMPUTATIONAL RESULTS

The proposed models in this study were tested using a set of computational experiments. The case study was addressed first using the MIP models proposed for straight and U-shaped lines. Then a set of standard test problems were solved. The problems can be found at the homepage for assembly line optimization research [47]. The MIP models were coded in GAMS and solved using CPLEX. This is a standard mathematical solver that has proven its performance in solving different optimization problems such as job shop scheduling [48], resource allocation [49] and flight rescheduling [50]. A PC with a Core i7 2.4 GHz processor and 8 GB of RAM was used. Tables 3 and 4 show the results obtained for both straight and U-shaped lines. To investigate the effect of different combinations of objectives on the solutions, all the problems were solved for the four different scenarios defined in Section B. Mathematical formulation. The results

**TABLE 3.** Results of the proposed MIP model for four scenarios of the straight assembly line.

| Problem | $CT$ | $Z_{1-\alpha}$ | $m_{min}$ | $m_{max}$ | # of Var. | # of Con. | (1) $M$ | | | | (2) $M$,$SMT_{max}$ | | | | (3) $M$,$STV_{max}$ | | | | (4) $M$,$SMT_{max}$+$STV_{max}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time |
| Case study | 70 | 1.64 | 4 | 6 | 4388 | 474 | 5 | 69.6 | 8.00 | 0.16 | 5 | 63.4 | 9.17 | 14.64 | 5 | 69.8 | 7.42 | 1 | 5 | 64.3 | 7.55 | 1.48 |
| | | 1.96 | 4 | 6 | 4388 | 474 | 5 | 68.4 | 9.27 | 0.09 | 5 | 63.4 | 10.54 | 20.66 | 5 | 69.1 | 7.42 | 1.44 | 5 | 64.3 | 7.55 | 1.73 |
| | 65 | 1.64 | 4 | 7 | 4418 | 594 | 5 | 64.7 | 10.44 | 0.28 | 5 | 63.4 | 9.06 | 6.2 | 5 | 65.0 | 7.55 | 2.48 | 5 | 64.3 | 7.55 | 3.33 |
| | | 1.96 | 4 | 7 | 4418 | 594 | 5 | 64.4 | 8.94 | 0.22 | 5 | 63.4 | 10.72 | 6.83 | 5 | 65.0 | 7.55 | 2.05 | 5 | 64.3 | 7.55 | 2.22 |
| Jackson | 21 | 1.64 | 2 | 4 | 213 | 85 | 3 | 18.0 | 4.11 | 0.06 | 3 | 16.0 | 4.08 | 0.09 | 3 | 18.0 | 3.96 | 0.09 | 3 | 16.0 | 4.08 | 0.14 |
| | | 1.96 | 2 | 4 | 213 | 85 | 3 | 18.0 | 4.11 | 0.08 | 3 | 16.0 | 4.08 | 0.11 | 3 | 18.0 | 3.96 | 0.09 | 3 | 16.0 | 4.08 | 0.06 |
| | 14 | 1.64 | 3 | 6 | 311 | 217 | 4 | 14.0 | 3.96 | 0.05 | 4 | 13.0 | 3.96 | 0.16 | 4 | 14.0 | 3.96 | 0.09 | 4 | 13.0 | 3.96 | 0.38 |
| | | 1.96 | 3 | 6 | 311 | 217 | 4 | 14.0 | 3.96 | 0.06 | 4 | 13.0 | 3.96 | 0.25 | 4 | 14.0 | 3.96 | 0.06 | 4 | 13.0 | 3.96 | 0.12 |
| Mitchell | 39 | 1.64 | 2 | 4 | 702 | 94 | 3 | 38.0 | 3.55 | 0.09 | 3 | 35.0 | 3.80 | 0.06 | 3 | 38.0 | 3.33 | 0.25 | 3 | 36.0 | 3.33 | 0.28 |
| | | 1.96 | 2 | 4 | 702 | 94 | 3 | 38.0 | 3.55 | 0.03 | 3 | 35.0 | 3.80 | 0.08 | 3 | 38.0 | 3.33 | 0.09 | 3 | 36.0 | 3.33 | 0.13 |
| | 26 | 1.64 | 4 | 8 | 1238 | 404 | 5 | 26.0 | 3.43 | 0.03 | 5 | 21.0 | 2.76 | 0.06 | 5 | 25.0 | 2.51 | 0.11 | 5 | 21.0 | 2.62 | 0.33 |
| | | 1.96 | 4 | 8 | 1238 | 404 | 5 | 26.0 | 3.43 | 0.09 | 5 | 21.0 | 2.76 | 0.55 | 5 | 25.0 | 2.51 | 0.84 | 5 | 21.0 | 2.62 | 0.97 |
| Buxey | 47 | 1.64 | 6 | 15 | 3601 | 2397 | 7 | 47.0 | 9.68 | 0.77 | 7 | 47.0 | 10.17 | 3.83 | 7 | 47.0 | 8.08 | 12.78 | 7 | 47.0 | 8.08 | 11.81 |
| | | 1.96 | 6 | 15 | 3601 | 2397 | 7 | 47.0 | 9.15 | 0.36 | 7 | 47.0 | 8.93 | 0.69 | 7 | 47.0 | 8.08 | 0.8 | 7 | 47.0 | 8.08 | 6.3 |
| | 41 | 1.64 | 7 | 17 | 4331 | 3594 | 8 | 41.0 | 9.27 | 0.88 | 8 | 41.0 | 9.08 | 1.14 | 8 | 41.0 | 8.06 | 8.27 | 8 | 41.0 | 8.06 | 18.03 |
| | | 1.96 | 7 | 17 | 4331 | 3594 | 8 | 41.0 | 9.11 | 0.66 | 8 | 41.0 | 8.95 | 0.48 | 8 | 41.0 | 8.06 | 1.75 | 8 | 41.0 | 8.06 | 0.92 |
| Gunther | 49 | 1.64 | 10 | 21 | 8440 | 6415 | 11 | 48.0 | 14.91 | 3.67 | 11 | 48.0 | 14.92 | 25.2 | 11 | 48.0 | 14.91 | 3.56 | 11 | 48.0 | 14.91 | 31.06 |
| | | 1.96 | 10 | 21 | 8440 | 6415 | 11 | 49.0 | 14.94 | 1.91 | 11 | 48.0 | 14.93 | 14.56 | 11 | 49.0 | 14.91 | 3.44 | 11 | 48.0 | 14.91 | 27.88 |
| | 44 | 1.64 | 11 | 23 | 9411 | 7807 | 12 | 44.0 | 14.93 | 6.48 | 12 | 44.0 | 14.93 | 25.61 | 12 | 44.0 | 14.91 | 5.53 | 12 | 44.0 | 14.91 | 20.69 |
| | | 1.96 | 11 | 23 | 9411 | 7807 | 12 | 44.0 | 14.91 | 3.33 | 12 | 44.0 | 14.91 | 3.95 | 12 | 44.0 | 14.91 | 4.14 | 12 | 44.0 | 14.91 | 7.28 |
| Killbrid. | 56 | 1.64 | 9 | 21 | 11893 | 6796 | 10 | 56.0 | 13.04 | 4.5 | 10 | 56.0 | 13.38 | 9.97 | 10 | 56.0 | 12.96 | 10.68 | 10 | 56.0 | 12.96 | 6.22 |
| | | 1.96 | 9 | 21 | 11893 | 6796 | 10 | 56.0 | 13.38 | 2.01 | 10 | 56.0 | 12.98 | 2.05 | 10 | 56.0 | 12.96 | 9.13 | 10 | 56.0 | 12.96 | 4.64 |
| | 62 | 1.64 | 8 | 19 | 11563 | 5478 | 10 | 62.0 | 13.44 | 2.64 | 10 | 56.0 | 13.00 | 3.31 | 10 | 61.0 | 12.95 | 4.36 | 10 | 56.0 | 12.95 | 153.83 |
| | | 1.96 | 8 | 19 | 11563 | 5478 | 10 | 62.0 | 13.20 | 2.77 | 10 | 56.0 | 13.89 | 7.66 | 10 | 62.0 | 12.95 | 5.28 | 10 | 56.0 | 12.95 | 34.3 |
| Tonge | 527 | 1.64 | 6 | 10 | 17510 | 1438 | 7 | 524.0 | 66.06 | 0.33 | 7 | 503.0 | 73.18 | 6.24 | 7 | 526.0 | 59.91 | 8.89 | 7 | 521.0 | 59.91 | 12.79 |
| | | 1.96 | 6 | 10 | 17510 | 1438 | 7 | 521.0 | 77.63 | 0.34 | 7 | 502.0 | 64.59 | 2.86 | 7 | 527.0 | 59.28 | 4.01 | 7 | 521.0 | 59.91 | 6.37 |
| | 364 | 1.64 | 9 | 17 | 25413 | 3587 | 10 | 364.0 | 65.32 | 1.15 | 10 | 352.0 | 58.56 | 43.88 | 10 | 364.0 | 58.18 | 52.63 | 10 | 363.0 | 58.18 | 69.01 |
| | | 1.96 | 9 | 17 | 25413 | 3587 | 10 | 364.0 | 61.07 | 1.22 | 10 | 352.0 | 60.73 | 44.05 | 10 | 364.0 | 58.18 | 52.96 | 10 | 363.0 | 58.18 | 70.89 |

**TABLE 4.** Results of the proposed MIP model for four scenarios of the U-shaped assembly line.

| Problem | $CT$ | $Z_{1-\alpha}$ | $m_{min}$ | $m_{max}$ | # of Var. | # of Con. | (1) $M$ | | | | (2) $M$,$SMT_{max}$ | | | | (3) $M$,$STV_{max}$ | | | | (4) $M$,$SMT_{max}$+$STV_{max}$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time | $M$ | $SMT_{max}$ | $STV_{max}$ | CPU time |
| Case study | 70 | 1.64 | 4 | 6 | 4798 | 721 | 5 | 69.0 | 9.33 | 0.33 | 5 | 63.4 | 8.66 | 9.16 | 5 | 69.6 | 7.35 | 6.05 | 5 | 63.8 | 7.55 | 6.58 |
| | | 1.96 | 4 | 6 | 4798 | 721 | 5 | 69.4 | 8.12 | 0.28 | 5 | 63.4 | 8.49 | 17.69 | 5 | 70.0 | 7.35 | 3.39 | 5 | 63.8 | 7.55 | 11.26 |
| | 65 | 1.64 | 4 | 7 | 4828 | 841 | 5 | 64.7 | 7.94 | 0.36 | 5 | 63.4 | 8.66 | 26.44 | 5 | 65.0 | 7.55 | 11.4 | 5 | 63.8 | 7.55 | 18.44 |
| | | 1.96 | 4 | 7 | 4828 | 841 | 5 | 64.9 | 8.72 | 0.37 | 5 | 63.4 | 8.31 | 55.85 | 5 | 64.6 | 7.55 | 9.94 | 5 | 63.8 | 7.55 | 21.29 |
| Jackson | 21 | 1.64 | 2 | 4 | 279 | 131 | 3 | 21.0 | 4.09 | 0.16 | 3 | 16.0 | 4.08 | 0.11 | 3 | 20.0 | 3.96 | 0.09 | 3 | 16.0 | 4.08 | 0.11 |
| | | 1.96 | 2 | 4 | 279 | 131 | 3 | 21.0 | 4.16 | 0.22 | 3 | 16.0 | 4.08 | 0.09 | 3 | 20.0 | 3.96 | 0.11 | 3 | 16.0 | 4.08 | 0.2 |
| | 14 | 1.64 | 3 | 6 | 399 | 274 | 4 | 14.0 | 3.98 | 0.08 | 4 | 13.0 | 3.96 | 0.13 | 4 | 13.0 | 3.96 | 0.11 | 4 | 13.0 | 3.96 | 0.16 |
| | | 1.96 | 3 | 6 | 399 | 274 | 4 | 13.0 | 3.96 | 0.06 | 4 | 13.0 | 3.96 | 0.15 | 4 | 14.0 | 3.96 | 0.26 | 4 | 13.0 | 3.96 | 0.19 |
| Mitchell | 39 | 1.64 | 2 | 4 | 828 | 184 | 3 | 39.0 | 3.81 | 0.06 | 3 | 35.0 | 3.79 | 0.19 | 3 | 38.0 | 3.33 | 0.2 | 3 | 36.0 | 3.33 | 0.17 |
| | | 1.96 | 2 | 4 | 828 | 184 | 3 | 39.0 | 3.81 | 0.09 | 3 | 35.0 | 3.79 | 0.17 | 3 | 38.0 | 3.33 | 0.23 | 3 | 36.0 | 3.33 | 0.23 |
| | 26 | 1.64 | 4 | 8 | 1448 | 536 | 5 | 26.0 | 3.20 | 0.14 | 5 | 21.0 | 2.96 | 3.37 | 5 | 25.0 | 2.51 | 3.56 | 5 | 21.0 | 2.62 | 5.18 |
| | | 1.96 | 4 | 8 | 1448 | 536 | 5 | 26.0 | 3.20 | 0.19 | 5 | 21.0 | 2.96 | 4.19 | 5 | 25.0 | 2.51 | 4.17 | 5 | 21.0 | 2.62 | 6.06 |
| Buxey | 47 | 1.64 | 6 | 15 | 4007 | 2636 | 7 | 47.0 | 9.18 | 1.87 | 7 | 47.0 | 10.73 | 10.64 | 7 | 47.0 | 8.17 | 52.49 | 7 | 47.0 | 8.17 | 115.99 |
| | | 1.96 | 6 | 15 | 4007 | 2636 | 7 | 47.0 | 11.18 | 1.92 | 7 | 47.0 | 8.76 | 12.36 | 7 | 47.0 | 8.17 | 56.61 | 7 | 47.0 | 8.17 | 245.98 |
| | 41 | 1.64 | 7 | 17 | 4795 | 3862 | 8 | 41.0 | 9.74 | 2.29 | 8 | 41.0 | 9.74 | 27.97 | 8 | 41.0 | 8.11 | 43.52 | 8 | 41.0 | 8.11 | 175.7 |
| | | 1.96 | 7 | 17 | 4795 | 3862 | 8 | 41.0 | 9.98 | 3.31 | 8 | 41.0 | 9.27 | 29.31 | 8 | 41.0 | 8.11 | 45.66 | 8 | 41.0 | 8.11 | 456.97 |
| Gunther | 49 | 1.64 | 10 | 21 | 9210 | 6843 | 10 | 49.0 | 14.96 | 5.76 | 10 | 49.0 | 14.93 | 11.68 | 10 | 49.0 | 14.91 | 364.59 | 10 | 49.0 | 14.91 | 456.63 |
| | | 1.96 | 10 | 21 | 9210 | 6843 | 10 | 49.0 | 14.95 | 12.63 | 10 | 49.0 | 14.94 | 20.47 | 10 | 49.0 | 14.91 | 372.31 | 10 | 49.0 | 14.91 | 501.79 |
| | 44 | 1.64 | 11 | 23 | 10251 | 8270 | 12 | 44.0 | 14.91 | 126.34 | 12 | 41.0 | 14.91 | 315 | 12 | 41.0 | 14.91 | 573.69 | 12 | 41.0 | 14.91 | 567.96 |
| | | 1.96 | 11 | 23 | 10251 | 8270 | 12 | 44.0 | 14.91 | 326 | 12 | 41.0 | 14.91 | 515 | 12 | 41.0 | 14.91 | 639 | 12 | 41.0 | 14.91 | 794 |
| Killbrid. | 56 | 1.64 | 9 | 21 | 12793 | 7306 | 10 | 56.0 | 13.45 | 8.51 | 10 | 56.0 | 13.32 | 21.3 | 10 | 56.0 | 12.95 | 0.06 | 10 | 56.0 | 12.95 | 2.13 |
| | | 1.96 | 9 | 21 | 12793 | 7306 | 10 | 56.0 | 13.71 | 12.76 | 10 | 56.0 | 13.48 | 8.52 | 10 | 56.0 | 12.95 | 328.16 | 10 | 56.0 | 12.95 | 627 |
| | 62 | 1.64 | 8 | 19 | 12463 | 5990 | 9 | 62.0 | 13.37 | 3.6 | 9 | 62.0 | 13.72 | 95.46 | 9 | 62.0 | 12.96 | 977.17 | 9 | 62.0 | 12.96 | 473.88 |
| | | 1.96 | 8 | 19 | 12463 | 5990 | 9 | 62.0 | 15.02 | 3.53 | 9 | 62.0 | 13.15 | 28.49 | 9 | 62.0 | 12.96 | 197.67 | 9 | 62.0 | 12.96 | 353.08 |
| Tonge | 527 | 1.64 | 6 | 10 | 18490 | 2014 | 7 | 527.0 | 72.20 | 1.27 | 7 | 503.0 | 73.26 | 39.51 | 7 | 526.0 | 59.91 | 35.62 | 7 | 521.0 | 59.91 | 36.4 |
| | | 1.96 | 6 | 10 | 18490 | 2014 | 7 | 527.0 | 64.84 | 1.03 | 7 | 502.0 | 62.64 | 32.84 | 7 | 526.0 | 59.91 | 71.95 | 7 | 521.0 | 59.91 | 45.74 |
| | 364 | 1.64 | 9 | 17 | 26813 | 4373 | 10 | 363.0 | 60.68 | 10.06 | 10 | 352.0 | 59.38 | 120.32 | 10 | 364.0 | 58.18 | 279.29 | 10 | 363.0 | 58.18 | 329.33 |
| | | 1.96 | 9 | 17 | 26813 | 4373 | 10 | 364.0 | 59.72 | 2.15 | 10 | 352.0 | 60.73 | 165.32 | 10 | 364.0 | 58.18 | 325.73 | 10 | 363.0 | 58.18 | 595.92 |

obtained for each scenario appear in Table 3 and 4 in the columns (1) $M$; (2) $M$, $SMT_{max}$; (3) $M$, $STV_{max}$; and (4) $M$, ($SMT_{max} + STV_{max}$), respectively.

As for the test problems, the means of the task times ($t\mu_i$) are considered to be equal to the deterministic task times given in the literature by assuming a normal distribution for task times. The variances of the task times ($t\sigma_i^2$) are generated using the uniform distribution $U(0, (t\mu_i/2)^2)$.

In Tables 3 and 4, the problem name and given $CT$s (two $CT$s for each problem) are reported in the first two columns.

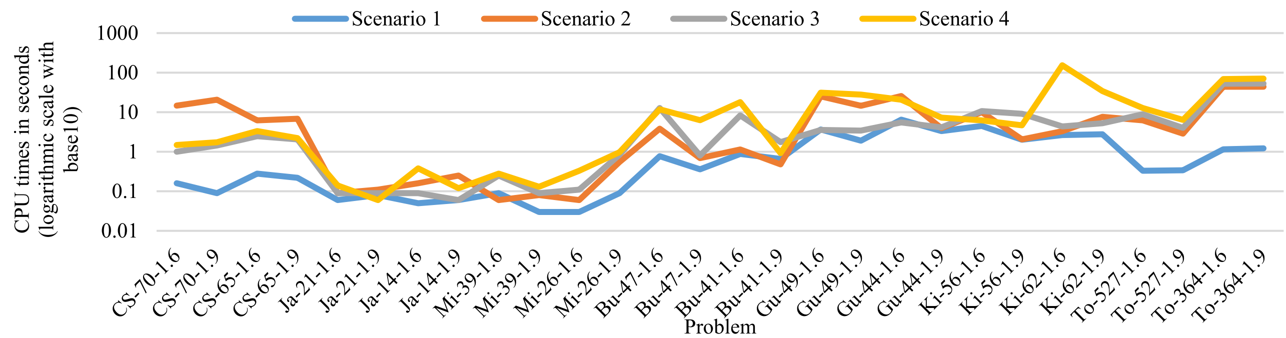


**FIGURE 2.** CPU times obtained by straight line model for each scenario and each problem.

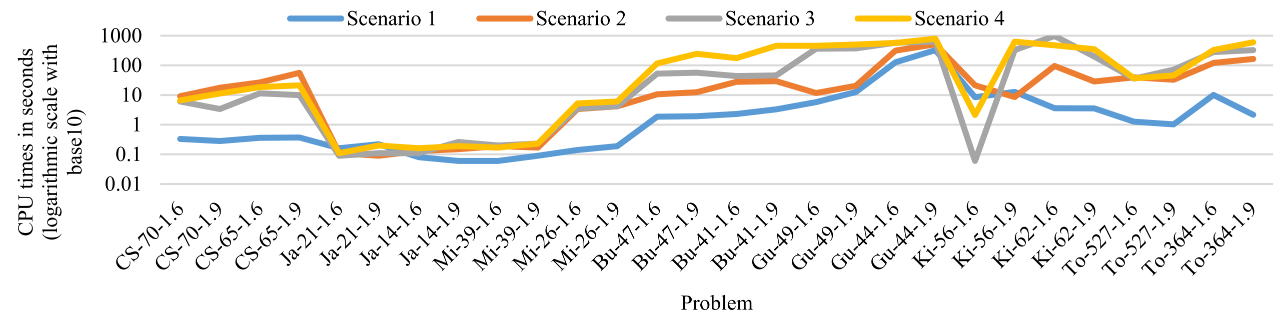


**FIGURE 3.** CPU times obtained by U-shaped line model for each scenario and each problem.

It is worth mentioning that considering the stochastic nature of tasks' times, a safety level is used in this study to assure that the processing times at the stations will not exceed $CT$. For instance, a safety level of 0.95 ($\alpha = 0.05$) means that the stations' times will not exceed the $CT$ 95% of the time. Thus, each problem was solved for two different levels of safety, (95% and 97.5%) which are associated with safety factors of $Z_{1-\alpha} = 1.64$ and $Z_{1-\alpha} = 1.96$, respectively, and shown in column $Z_{1-\alpha}$.

Columns $m_{min}$ and $m_{max}$ specify the minimum and the maximum number of stations, respectively, for the corresponding $CT$ and safety levels, calculated using Equations (18) and (19). Moreover, the number of variables and constraints for each problem are reported under the columns # of Var. and # of Con., respectively.

According to Tables 3 and 4, for all the scenarios the resulting number of stations ($M$) is the same for each value of $CT$. This result was predictable given the lexicographical ordering of objectives and the assignment of higher optimization priority to the number of stations. Different values of the other two objectives, $SMT_{max}$ and $STV_{max}$, were obtained for each scenario. According to the dominance concept, when comparing different solutions in terms of a few objectives with a minimization purpose, solution $g$ is said to be dominated by solution $g'$, if and only if, for all the objectives $Obj^l_{(g')} \leq Obj^l_{(g)}$ ($l = 1, \ldots,$ number of objectives). For example, for the case study in Table 3, comparing the results obtained for scenarios (2) to (4) with the results obtained for scenario (1), one can observe that scenario (4) is the only scenario that could dominate scenario (1) in terms of all objectives (i.e., $M_{(4)} \leq M_{(1)}$; $SMTmax_{(4)} \leq SMTmax_{(1)}$; $STVmax_{(4)} \leq STVmax_{(1)}$). However, scenarios (2) and (3) could not dominate scenario (1) in terms of $STV_{max}$ because $STVmax_{(2)} > STVmax_{(1)}$. The same is true for $SMT_{max}$ because $SMTmax_{(3)} > SMTmax_{(1)}$. Comparing scenario (1) with scenarios (2) to (4) for all the problems solved and for both line configurations (straight and U-shaped), it can be observed that scenario (4) dominates scenario (1) in all problems. Apart from this, no further judgment can be made regarding the dominance of scenario (4) over scenarios (2) and (3). Therefore, to make a reliable and comprehensive comparison of all the scenarios, a multi-attribute decision-making approach is used to analyze the results in the next section.

Tables 3 and 4 show a considerable difference between the minimum and maximum computational times to reach optimality for different problem sizes. The smallest CPU time is less than one second and the largest is 977 seconds. The minimum and maximum obtained CPU times were 0.09 and 55.85 seconds for the case study, respectively. These obtained times were respectively related to scenario (1) for the straight line and scenario (2) for the U-shaped line.

Figure 2 and 3 show the CPU times in logarithmic scale (base 10) of scenarios (1) to (4) for all the problems solved by the straight and U-shaped line models, respectively. The horizontal axis indicates the problems. The labels were created by abbreviating the problem's name and adding the $CT$ value and the safety factor, separated by hyphens.

 

According to Figure 2, scenario (4) resulted in a higher CPU times for most of the test problems except the case study and a very few other exceptions. This higher computational time can be justified because all the three optimization objectives are included in scenario (4).

Figure 3 for the U-shaped line also shows that a higher CPU time was required by scenario (4) compared to other scenarios for the majority of the problems solved.

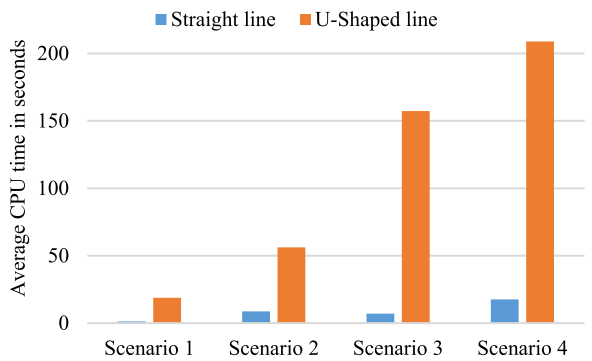


**FIGURE 4.** **Comparison of average CPU times for different scenarios for the straight and U-shaped lines.**

To investigate the effect of the line configuration on the computational time, the average CPU times of each scenario for the straight and the U-shaped line configurations are presented in Figure 4. According to Figure 4, the U-shaped line model took more time to find the optimum solution in all four scenarios compared to the straight line model. This higher computation time was expected as there are more possibilities for assigning tasks to stations in the U-shaped line model, which has a larger solution space.

### *B. ANALYSIS OF THE RESULTS*

A multi-attribute decision-making (MADM) approach is needed to compare the results and rank the different scenarios in terms of all the considered objectives. Although a variety of MADM approaches can be found in the literature (see Zahedi Khameneh and Kılıçman [51] for the most recent review of MADM approaches), the technique for order of preference by similarity to ideal solution (TOPSIS) was chosen in this study. This choice was made mainly due to the proven performance of TOPSIS in dealing with different types of ALBPs [52]–[54]. It also has outstanding ability to distinguish between different scenarios and rank them. Moreover, TOPSIS has been shown to be a successful approach in cases where the objectives have different dimensions or one objective is much larger than the others (e.g., $STV_{max} \ll SMT_{max}$ in most of the problems in this study) [54].

TOPSIS relies on the concept that the best scenario should have a minimum distance from the positive ideal solution and a maximum distance from the negative ideal solution. Interested readers are referred to Hwang and Yoon [55] for a good explanation and detailed information about TOPSIS. In this study and following standard TOPSIS implementation procedure, there are seven steps in ranking the scenarios.

*Step 1:* For each test problem, the results obtained by scenarios (1) to (4) are used to form matrix $D$ for TOPSIS calculations as shown in Equation (32).

$$D = \begin{array}{c c} & \begin{array}{ccc} M & SMT_{\max} & STV_{\max} \end{array} \\ \begin{array}{c} s_1 \\ s_2 \\ s_3 \\ s_4 \end{array} & \begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \end{bmatrix} \end{array} \tag{32}$$

where $x_{sl}$ is the $l$th objective of the scenario $s$ for each test problem.

*Step 2:* The normalized decision matrix $R$ with element $r_{sl}$ is calculated using Equation (33):

$$r_{sl} = \frac{x_{sl}}{\sqrt{\sum_{l=1}^{3} (x_{sl})^2}}; \quad s = 1, 2, 3, 4; \; l = 1, 2, 3 \tag{33}$$

*Step 3:* The weighted normalized decision matrix $V$ with element $v_{sl}$ is calculated using Equation (34):

$$v_{sl} = w_l . r_{sl}; \quad s = 1, 2, 3, 4; \; l = 1, 2, 3 \tag{34}$$

where $w_l$ indicates the weight of the $l$th objective. The sum of the weights of the objectives has to be equal to one, that is, $\sum_{\forall l} w_l = 1$. Since minimizing $M$ is prioritized over minimizing $SMT_{max}$ and $STV_{max}$, after some pilot studies the weights for objectives $M$, $SMT_{max}$ and $STV_{max}$, were set to 0.5, 0.25 and 0.25, respectively.

*Step 4:* The positive and the negative ideal solutions are calculated using Equations (35) and (36).

$$V^+ = \{v_1^+, v_2^+, v_3^+\} = \{(\max v_{sl} | l \in L), (\min v_{sl} | l \in L')\}; \quad s = 1, 2, 3, 4; \quad l = 1, 2, 3 \tag{35}$$

$$V^- = \{v_1^-, v_2^-, v_3^-\} = \{(\min v_{sl} | l \in L), (\max v_{sl} | l \in L')\}; \quad s = 1, 2, 3, 4; \quad l = 1, 2, 3 \tag{36}$$

where $L$ and $L'$ are associated with positive and negative measures, respectively. Since all the objectives in this study are negative measures that have to be minimized (i.e., $M$, $SMT_{max}$ and $STV_{max}$), the positive and the negative ideal solutions are the solutions with the minimum and the maximum values, respectively, for all objectives.

*Step 5:* Two measures called distances from the positive and the negative ideal solutions for each scenario are calculated using Equations (37) and (38).

$$D_s^+ = \sqrt{\sum_{l=1}^{3} (v_{sl} - v_l^+)^2}; \quad s = 1, 2, 3 \tag{37}$$

$$D_s^- = \sqrt{\sum_{l=1}^{3} (v_{sl} - v_l^-)^2}; \quad s = 1, 2, 3 \tag{38}$$

*Step 6:* Finally, the relative closeness ($RC$) index of scenario $s$ from the negative ideal solution is calculated using Equation (39).

$$RC_s = D_s^- / D_s^- + D_s^+; \quad s = 1, 2, 3 \tag{39}$$

The $RC$ lies between zero and one. The larger the value of $RC$, the better the performance of the corresponding scenario $s$.

*Step 7:* Finally, the scenarios are ranked in descending order based on their $RC$ values in which ranks 1 and 4 are assigned to the best and the worst scenarios, respectively.

The basic version of TOPSIS assumes that no interaction exists among criteria which is rarely true for the real-world problems. This is due to the Euclidean distance applied in calculation of distance from the positive and negative ideal solutions [56]. To overcome this disadvantage, a new measure called Mahalanobis distance is applied in this study which are calculated by Equations (40) and (41) as a substitute for Equations (37) and (38), respectively.

$$D_s^+ = \sqrt{(r_{sl} - v_l^+)^T \Omega^T \sum^{-1} \Omega (r_{sl} - v_l^+)}; \quad s = 1, 2, 3 \tag{40}$$

$$D_s^- = \sqrt{(r_{sl} - v_l^-)^T \Omega^T \sum^{-1} \Omega (r_{sl} - v_l^-)}; \quad s = 1, 2, 3 \tag{41}$$

where $T$ indicates the transpose and $\Omega$ is the diagonal matrix of weight with elements $\Omega = \text{diag}(\sqrt{w_1}, \sqrt{w_2}, \sqrt{w_3})$. The $\sum^{-1}$ is the inverse of the covariance matrix which includes the correlations between criteria.

**TABLE 5.** Ranking of scenarios (1) to (4) for each test problem by TOPSIS for the straight and U-shape lines.

| Problem | $CT$ | $Z_{1-\alpha}$ | Straight line (1) | (2) | (3) | (4) | U-shaped line (1) | (2) | (3) | (4) |
|---|---|---|---|---|---|---|---|---|---|---|
| Case study | 70 | 1.64 | 2 | 4 | 3 | 1 | 4 | 3 | 2 | 1 |
| | | 1.96 | 3 | 4 | 2 | 1 | 2 | 4 | 3 | 1 |
| | 65 | 1.64 | 4 | 3 | 2 | 1 | 3 | 4 | 2 | 1 |
| | | 1.96 | 3 | 4 | 2 | 1 | 4 | 3 | 2 | 1 |
| Jackson | 21 | 1.64 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| | | 1.96 | 3 | 1 | 2 | 1 | 3 | 1 | 2 | 1 |
| | 14 | 1.64 | 2 | 1 | 3 | 1 | 2 | 1 | 1 | 1 |
| | | 1.96 | 2 | 1 | 3 | 1 | 1 | 1 | 2 | 1 |
| Mitchell | 39 | 1.64 | 2 | 4 | 3 | 1 | 4 | 3 | 2 | 1 |
| | | 1.96 | 2 | 4 | 3 | 1 | 4 | 3 | 2 | 1 |
| | 26 | 1.64 | 4 | 3 | 2 | 1 | 4 | 3 | 1 | 2 |
| | | 1.96 | 4 | 3 | 2 | 1 | 4 | 3 | 1 | 2 |
| Buxey | 47 | 1.64 | 2 | 3 | 1 | 1 | 2 | 3 | 1 | 1 |
| | | 1.96 | 3 | 2 | 1 | 1 | 3 | 2 | 1 | 1 |
| | 41 | 1.64 | 3 | 2 | 1 | 1 | 2 | 2 | 1 | 1 |
| | | 1.96 | 3 | 2 | 1 | 1 | 3 | 2 | 1 | 1 |
| Gunther | 49 | 1.64 | 2 | 3 | 1 | 1 | 3 | 2 | 1 | 1 |
| | | 1.96 | 4 | 2 | 3 | 1 | 3 | 2 | 1 | 1 |
| | 44 | 1.64 | 2 | 3 | 1 | 1 | 3 | 2 | 1 | 1 |
| | | 1.96 | 3 | 2 | 1 | 1 | 2 | 1 | 1 | 1 |
| Killbridge | 56 | 1.64 | 2 | 3 | 1 | 1 | 3 | 2 | 1 | 1 |
| | | 1.96 | 3 | 2 | 1 | 1 | 3 | 2 | 1 | 1 |
| | 62 | 1.64 | 4 | 2 | 3 | 1 | 3 | 4 | 2 | 1 |
| | | 1.96 | 4 | 2 | 3 | 1 | 3 | 2 | 1 | 1 |
| Tonge | 527 | 1.64 | 3 | 4 | 2 | 1 | 4 | 3 | 2 | 1 |
| | | 1.96 | 4 | 3 | 2 | 1 | 4 | 3 | 2 | 1 |
| | 364 | 1.64 | 4 | 1 | 3 | 2 | 4 | 3 | 2 | 1 |
| | | 1.96 | 4 | 3 | 2 | 1 | 2 | 4 | 3 | 1 |
| | Average rank | | 3.0 | 2.6 | 2.0 | 1.0 | 3.0 | 2.5 | 1.6 | 1.1 |

Table 5 shows the ranking of the scenarios by applying TOPSIS for each test problem for both straight and U-shaped lines. For example, in the case study with $CT$ of 70 and safety factor of 1.64 for the straight line, the ranking of scenarios (1) to (4) by TOPSIS is 2, 4, 3 and 1, respectively. In the same way rankings 4, 3, 2 and 1 are found for scenarios (1) to (4) for the U-shaped line.

According to Table 5, scenario (4) was ranked first in all the test problems for straight line except for the "Tonge" problem with a $CT$ of 364 and a safety factor of 1.64. Similarly, scenario (4) has been ranked first in almost all problems for U-shaped line except for Mitchell with a $CT$ of 26 and safety factors of 1.64 and 1.96. It is worth mentioning that when two or more scenarios have the same rank, the other scenarios are ranked after them. For instance, in the "Jackson" problem with a $CT$ of 21 and a safety factor of 1.64, scenarios (2) and (4) have been equally ranked (i.e., first in this example) and the ranks of the remaining scenarios have been decreased by one (i.e., becoming 2 and 3 instead of 3 and 4).

In the last row of Table 5, the average rank of each scenario for both line configurations is presented. According to the calculated average ranks, for both straight and U-shaped lines, scenario (4) was ranked first followed by scenarios (3), (2) and (1), which are placed in the second to fourth rank, respectively.

To provide managerial insight into the effective selection of line configuration as well as to help the decision makers in making a wise decision, the results obtained by scenario (4) for the U-shaped line are compared against the results achieved for the straight line. Table 6 shows the results of the comparison. The findings are reported separately for the case study and the standard test problems. The problems were compared for all three objectives and the results are presented as the percentage of worse, equal, and better solutions.

**TABLE 6.** Comparison of U-shaped versus straight line results obtained by scenario (4) in terms of different objectives.

| | Case study $M$ | $SMT_{max}$ | $STV_{max}$ | Test problems $M$ | $SMT_{max}$ | $STV_{max}$ |
|---|---|---|---|---|---|---|
| Worse | 0% | 0% | 0% | 0% | 17% | 33% |
| Equal | 100% | 0% | 100% | 83% | 75% | 58% |
| Better | 0% | 100% | 0% | 17% | 8% | 8% |

In the case study, when the results for the U-shaped line were compared with the results of the straight line in terms of $M$, the results were equal (i.e., 5 stations). Thus, no worse and better results are reported (i.e., 0%). In terms of $SMT_{max}$, the U-shaped line shows its superiority over the straight line by providing better results for all the solved case study instances (4 instances). However, there is no difference between the results obtained by straight and U-shaped lines in terms of $STV_{max}$. Increasing the $CT$ from 65 to 70 seconds does not improve the quality of the solutions and provides no advantage either in relation to $M$ or $SMT_{max}$ and $STV_{max}$, as the value of all the objectives in scenario (4) remained unchanged for both line configurations and both safety factors. In summary, these results imply that the company which requested the case study is currently working with the correct values of $M$, $CT$, and line configuration. However, the results

**TABLE 7.** Case study information.*

| Task # | Predecessor | $t\mu$ (Sec.) | $t\sigma$ (Sec.) | Linked task | Incomp. task | Task # | Predecessor | $t\mu$ (Sec.) | $t\sigma$ (Sec.) | Linked task | Incomp. task |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | - | 5.3 | 2.0 | | | 22 | 21,26 | 11 | 1.0 | 21 | |
| 2 | 1 | 3.1 | 1.0 | | | 23 | - | 4.9 | 1.0 | | |
| 3 | 2 | 6 | 2.0 | | | 24 | 23 | 4.5 | 1.0 | | |
| 4 | 3 | 7.1 | 1.0 | 3 | | 25 | 19,14,22 | 7.2 | 2.0 | | 22 |
| 5 | 2 | 6.5 | 1.0 | 4 | | 26 | - | 10.3 | 1.0 | | |
| 6 | 5 | 7.1 | 1.0 | | | 27 | 25 | 2.4 | 1.0 | | 25 |
| 7 | 4 | 11.1 | 2.0 | | | 28 | 26,27 | 5.1 | 1.0 | | |
| 8 | 6 | 11.1 | 1.0 | | | 29 | 28 | 8.8 | 1.0 | 28 | |
| 9 | - | 11.2 | 4.0 | | 7 | 30 | 20,26 | 15.7 | 7.0 | | |
| 10 | - | 3.3 | 1.0 | | | 31 | 30 | 11.9 | 2.0 | | |
| 11 | 10 | 12.6 | 1.0 | | | 32 | 31 | 5.3 | 2.0 | | |
| 12 | 11 | 15.5 | 5.0 | 11 | | 33 | 31 | 4.9 | 1.0 | | 31 |
| 13 | - | 3.6 | 1.0 | | 10 | 34 | 32,33 | 18.1 | 3.0 | | |
| 14 | 10 | 2.7 | 1.0 | | | 35 | 34 | 13.5 | 1.0 | | |
| 15 | 10 | 16.5 | 7.0 | | | 36 | 35 | 3.2 | 1.0 | | 34 |
| 16 | 9 | 9.9 | 3.0 | | | 37 | 32,33 | 2.7 | 1.0 | | |
| 17 | 10 | 8.5 | 1.0 | | | 38 | 37 | 4.5 | 2.0 | | |
| 18 | 17 | 4.4 | 1.0 | | | 39 | - | 3.3 | 1.0 | | 37 |
| 19 | 10 | 2.7 | 1.0 | | 16 | 40 | 38 | 4.7 | 2.0 | | |
| 20 | 14,19 | 9.4 | 2.0 | | 18 | 41 | 40 | 8.4 | 4.0 | | |
| 21 | 17 | 8.9 | 2.0 | | | | | | | | |

*The zoning constraints as well as the means and the variances of the task times were randomly generated due to the confidentiality of the company data.

of this study suggest that the assignment of tasks to stations can be reconsidered to minimize the $SMT_{max}$ and consequently smooth the workload at stations.

For test problems, Table 6 suggests that the U-shaped line resulted in equal or fewer stations for all the solved test problems. Thus, it is safe to recommend the use of the U-shaped line if minimizing the number of stations is the prime or only objective; but it is hard to draw a general conclusion about the other objectives. However, the straight line provided better solutions for a slightly larger portion of the problems than the U-shaped line. In regard to $SMT_{max}$, both U-shaped and straight lines resulted in the same value for the majority of the test problems solved (75%). However, the straight line showed better performance in 17% of the problems solved, while this number was only 8% for the U-shaped line. In terms of $STV_{max}$, the same results were found by straight and U-shaped lines for slightly more than half of the test problems solved (58%), while the U-shaped line resulted in 8% better and 33% worse results compared to the straight line.

Overall, the analysis of results by TOPSIS showed that scenario number (4) is almost always ranked first. It is therefore the preferred scenario. The comparative analysis of the U-shaped and straight lines for scenario (4) reevaluated the advantage of using the U-shaped line in the case study. Additionally, the U-shaped line appeared to be a good choice for reducing the number of stations for assembly lines with different sizes and characteristics. However, a coherent and general conclusion could not be reached with regard to $STV_{max}$ and $SMT_{max}$. Therefore the decision should be made case by case.

## VI. CONCLUSIONS

This study, which was inspired by a real problem encountered in an automotive manufacturing company, contributes to the body of ALBP knowledge. It bridges the gap in the literature by proposing two mathematical programming models for balancing straight and U-shaped assembly lines where the stochastic task times and the zoning constraint (ZC) between tasks are considered simultaneously. The main objective of the proposed models is to minimize the number of stations. In addition, other objectives including the maximum of stations' mean time and the maximum of stations' time variance were also optimized. To show the effect of the objectives on the St-ALBP-ZC solutions, four different scenarios were designed by considering various combinations of objectives, and then solved for both straight and U-shaped lines. The computational results on the real case and some standard test problems taken from the literature showed the effectiveness of the proposed models in solving St-ALBP-ZC within a reasonable computational time.

A multi-attribute decision-making approach (TOPSIS) was used to obtain a reliable analysis of the results for each scenario for both straight and U-shaped lines. The TOPSIS analysis showed that scenario (4) (i.e., using all the objectives) resulted in better solutions for both straight and U-shaped lines. In addition, by comparing the results obtained for scenario (4) for both the straight and U-shaped lines in the case study, the U-shaped line was found to be the preferred configuration as it resulted in better or equal solutions for all the objectives. It was also found that the U-shaped line always provided equal or better solution in terms of the number of stations compared to the straight line for all the solved problems. Therefore, the U-shaped line is the recommended line configuration where the prime optimization objective is the number of stations. However, the straight line showed equal or better performance in the majority of the test problems solved compared to the U-shaped line, in terms of both the maximum of stations' time variance and the maximum of stations' mean time. But, considering that the U-shaped line also resulted in better solutions in a few cases including the case study, no general recommendation can be provided regarding the line preference to optimize the two secondary objectives considered. Therefore, a case-based study is recommended where objectives other than the number of stations should be optimized.

Future studies can consider extending the proposed models to include other real-world restrictions such as equipment, or ergonomic considerations. In addition, efficient meta-heuristics could be developed to address the St-ALBP-ZC and compare their performance with the optimization models proposed in this study for both straight and U-shaped lines.

## APPENDIX

See Table 7.

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