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Formation Optimization and Control for Maneuvering Target Tracking by Mobile Sensing Agents

HUIZHEN YANG AND YUE WANG
School of Marine Science and Technology, Northwestern Polytechnical University, Xi'an 710072, China
Corresponding author: Huizhen Yang (rainsun-ly@nwpu.edu.cn)

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ABSTRACT This paper presents a geometric formation control strategy of multiple sensing agents for maneuvering target tracking, which ensures agents to track the target with optimal target state estimation. Three sub-problems are solved for achieving the target tracking. First, an IMMCEKF algorithm is proposed to estimate the maneuvering target state by fusing multiple bearings-only measurements. This algorithm combines the interacting multiple-model estimator and the extended Kalman filter-based augmented measurement fusion algorithm, which is a centralized filter. Second, the algorithm of constructing optimal configuration for target tracking is proved and verified by maximizing the determinant of the Fisher information matrix. Third, we transform the problem of target tracking with optimal configuration into the problem of formation control. A geometric formation control approach based on Jacobi vectors is proposed. The formation shape controller and the formation tracking controller are decoupled because we use the formation center to describe the formation motion which is not relative to the formation shape described by the Jacobi vectors. The simulation results show that multiple sensing agents can track the moving target with optimal configuration such that the estimation error is obviously reduced.

INDEX TERMS Target tracking, fisher information matrix, formation control, EKF-based augmented measurement fusion algorithm, Jacobi vectors.

I. INTRODUCTION

Target tracking by multiple mobile agents are desirable for much more applications in the real world, including unmanned pursue systems, warning systems and autonomous cruise systems. Here, a mobile agent can be unmanned ground vehicle (UGV), unmanned aerial vehicle (UAV) and unmanned underwater vehicle (UUV), which is equipped with sensing, computing and communication capabilities [1]–[3]. The sensor here is mostly passive sensor, like passive sonar for UUV. The passive sensor does not emit electromagnetic waves when it works, and has the advantages of long distance of action and high concealment. The bearing of the target is almost the only reliable measure that a passive sensor can get. In the case of bearings-only measurement, a single agent has a strong nonlinearity to the target tracking, and the tracking process is a weak observable process [4]. In order to achieve better tracking effect, the agent needs to move according to a certain trajectory. This puts high demands on the movement of the agent, especially when the target is maneuvered, the trajectory optimization of the agent is basically difficult to achieve. Therefore, it is often necessary to use multiple agents to fuse multiple sensors for target tracking. Recently, researchers have developed algorithms using multiple sensing agents for target localization and tracking [5]–[7].

In this paper, we study the problem of multiple mobile sensing agents tracking a moving target cooperatively. The objective is to control agents to move along with the target in order to ensure the continuous detectability of the target. For implementing cooperative target tracking, there are two key problems: one is estimating the target state by fusing measurements, the other is designing the tracking controller for each agent based on the estimation of the target. However, the agents mounted with sensors and the target are dynamically moving. Then the processes of estimating and tracking are coupled. How to ensure the agents can track the target precisely and continuously is an important research problem.

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Since the dynamical motion of maneuvering target is difficult to be described, the state of maneuvering target is so hard to be estimated accurately. Different approaches have been investigated and evaluated in many literatures [8]–[11]. One common approach is the interacting multiple-model (IMM)-based estimator in which the models are suggested to switch from one to another in a Markov transition probability [12]–[14]. During the last decade, many versions of IMM-based estimator schemes, like IMMKF, IMMEKF, IMMUKF,etc., which use KF (Kalman filter), EKF (Extended Kalman filter), UKF (Unscented Kalman filter) to compute the model-conditioned state estimation, are derived and developed. EKF is widely used as nonlinear filtering algorithms to deal with nonlinear models of state estimation. Therefore, we combined a extended dimensional centralized EKF algorithm [15] and IMM to estimate the maneuvering target with multiple bearings-only measurements.

Accurate state estimation of target is so significant for tracking controller design. In fact, we know that the relative sensor-target geometry can significantly affect the estimation accuracy [16]–[18]. A number of authors have proposed theoretical contributions for the optimal sensor placement problem in which the sensors are supposed to be static. The general idea of optimizing the placement is minimizing the target estimation uncertainty which is usually characterized by the Fisher information matrix (FIM). The FIM is the inverse of the Cramer-Rao lower bound (CRLB), which is the minimum achievable estimation variance [19]. In [16], the optimal sensor-target geometries for bearings-only, range-only and time-of-arrival measurements are analyzed in depth. The main conclusions are: (1) the optimal sensor placement is not unique; (2) for passive sensors when the target and sensors are static and the sensor-target range are equal, the optimal configuration is that sensors are placed around the target. Reference [18] proves necessary and sufficient conditions of optimal placement by using the frame theory.

Once we get the optimal sensor placement, the next problem is how to achieve the optimal configuration. One simple way is allocating each optimal position to each agent, then design a controller to guarantee that the agent goes to the desired position [20]. Reference [21] minimizes the determination of FIM to find optimal orientation angles for UAVs such that multiple UAVs can cooperatively track a moving radio frequency transmitter. Machine learning is used to predict the future position of a moving urban target in [22] and [23], such that the tracking performance of UAVs can be improved according to the simulation results. Reference [24] uses the optimal placement deduced by [16] with the hypothesis that ranges between each agent and the target are the same. In fact, this tracking problem could be transferred into the formation control problem. However, the optimal formation is time-varying due to the maneuverability of the mobile agents and target. In this paper, we propose a geometry formation control strategy to make the agents track the target for the optimal target state estimation. This geometry formation control method is successfully used for formation control of multiple AUVs (Autonomous Underwater Vehicle) in [25] and [26]. We use the Jacobi vectors to describe the formation shape and the position of the entire formation to be the formation center which does not affect the formation shape or orientation. Then the formation shape dynamics and translational dynamics are decoupled which is more helpful for simplifying controller design.

The main contributions are summarized as follows: 1) We proposed a novel framework for maneuvering target tracking by multiple agents which is based on optimal target estimation. 2) We first combined IMM and an extended dimensional centralized EKF algorithm to estimate the state of a maneuvering target. 3) We use a geometric formation control approach based on Jacobi vectors to achieve cooperative target tracking for optimal target estimation in which the formation shape controller and formation tracking controller are decoupled, then the PD control approach is used to achieve the tracking control and formation control, which is easier to be implemented in practice.

The organization of this paper is as follows. Problem formulation is given in section II. The proposed IMMCEKF algorithm is described in section III. Then in section IV, optimal formation for target tracking is proved and constructing algorithm is proposed. Geometry formation controllers are designed for agents to achieving target tracking with optimal target estimation in section V. Section VI presents the framework of target tracking by mobile agents with optimal target estimation. Simulation results are presented in section VII and the last section is conclusion.

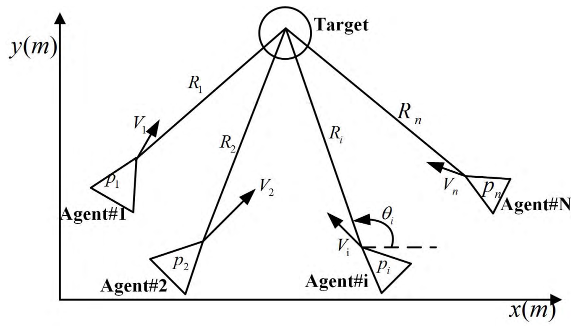

FIGURE 1. Cooperative maneuvering target tracking by multiple mobile agents.

II. PROBLEM FORMULATION

The functionality of cooperative target tracking by mobile agents depends on the following factors: the environment, the target, the agent itself, the sensor onboard the agent, and the agent coordination method. In this paper, we suppose that: 1) The environment is ideal such that the horizontal plane under consideration is infinite, not rasterized, and obstacle-free. 2) The target is maneuvering without prior information. 3) All the agents communicate with each other without time delay. 4) The passive sensor is equipped on the agent such that the bearing-only measurements can be obtained. Fig. 1 shows that a group of agents equipped passive sensors which can measure the bearings-only information are tracking a moving target.

Define $P_T = [x_T \ y_T]^T$ as the Cartesian coordinates of the target and denote the position of the ith agent as $P_i = [x_i \ y_i]^T$, $i = 1, 2, \cdots, N$. Where, x_T, y_T, x_i and y_i are in m i.e. metre. θ_i is the corresponding bearing measurement of the ith agent which is in *rad* i.e. radian. We use $\ddot{P} = U$ to describe the dynamical property of each agent. U is the acceleration control input vector.

To implement cooperative target tracking, we should solve two key problems: target state estimation and tracking controller design. In fact, we know that the sensor placement can significantly affect the estimation accuracy. Here the sensor placement means the relative sensor-target geometry. Since the agents mounted with the sensor and the target are dynamically moving, the estimating process and tracking process are coupled. To ensure the agents can track the target precisely and continuously, we propose a geometry formation control strategy to make the agents track the target for the optimal target state estimation.

The dynamic properties of maneuvering targets cannot be modeled exactly. To simplify this problem, we suppose that the movement of maneuvering target is always described by combination of several typical models such as constant velocity (CV) model, constant acceleration (CA) model, and constant turn (CT) model. The dynamics model commonly assumed for a target in tracking is given by

$$X(k+1) = \Phi X(k) + \gamma(k), \tag{1}$$

where $X = \begin{bmatrix} x_T & y_T & \dot{x}_T & \dot{y}_T \end{bmatrix}^T$ is the state vector of the target, Where, x_T and y_T are in m and $\dot{x}_T$ and $\dot{y}_T$ are in m/s. Φ defines the linear dynamics according to CV, CA, or CT model. $\gamma(k) \sim \mathcal{N}(0, Q)$ is uncorrelated, zero-mean Gaussian noise processes with covariance matrices $Q = \sigma_\gamma^2 I$.

At each time instant k, the bearing measurement of the target by the ith agent is

$$\theta_i^k = arctan\frac{y_T^k - y_i^k}{x_T^k - x_i^k} + \omega_i^k, \quad i = 1, \cdots N, \tag{2}$$

where $\omega_i^k \sim \mathcal{N}(0, \sigma_\omega^2)$ is the measurement noise.

Suppose that each agent can obtain the bearing measurements from the other agents without considering the communication time delay. Let $Y(k) = \begin{bmatrix} \theta_1^k & \theta_2^k & \cdots & \theta_N^k \end{bmatrix}$ be the measurement vector consisting of all the measurements collected from the N agents at time k. Then we have the following measurement equation:

$$Y(k) = h(X_k) + \varepsilon(k), \tag{3}$$

where $h(X_k) = \begin{bmatrix} arctan\frac{y_T^k - y_1^k}{x_T^k - x_1^k} & \cdots & arctan\frac{y_T^k - y_n^k}{x_T^k - x_n^k} \end{bmatrix}$. $\varepsilon(k)$ is the noise vector with the covariance $R = \sigma_\omega^2 I$.

The noise terms $\gamma(k)$ and $\varepsilon(k)$ satisfy:

$$E\left[\begin{pmatrix} \gamma^k \\ \varepsilon^k \end{pmatrix} \begin{pmatrix} \gamma^k & \varepsilon^k \end{pmatrix}\right] = \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix}. \tag{4}$$

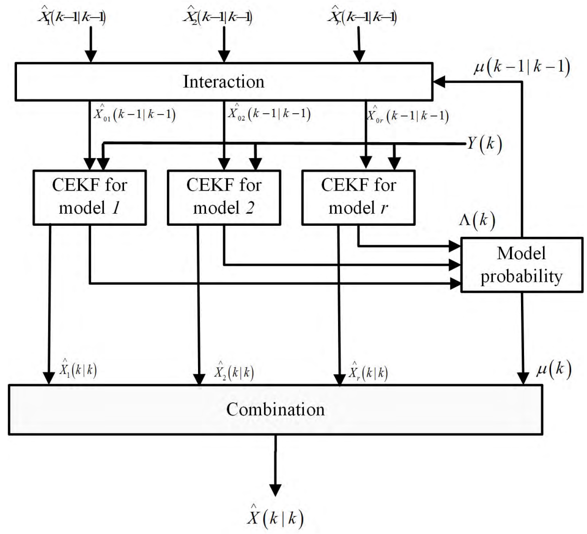

FIGURE 2. Structure of the IMM algorithm.

III. MANEUVERING TARGET STATE ESTIMATION

The IMM algorithm has been shown to be one of the most cost-effective schemes for maneuvering target state estimation. The main idea of IMM is to use dynamic multiple models system with the Markovian switching coefficients to match different motion states of the target. Suppose that the maneuvering target can be described by r models, such as CV model, CA model, and CT model etc., which can be expressed by model $M_1, M_2, \cdots, M_r$. The IMM algorithm consists of the following four steps: interacting (mixing), filtering, mode probability calculation, and combination as illustrated in Fig.2, where $\hat{X}_i$ means the state estimation of model i.

Equation (2) shows that the measurement equation is nonlinear for passive sensors. Since the extended Kalman filter (EKF) requires less computational load a produces more efficient estimates for nonlinear system estimation. We apply the EKF-based augmented measurement fusion algorithm (which is a centralized filter) to estimate the state of the target for each mode based on the Equation (1) and (3). The steps of the fusion algorithm CEKF are listed below [15]:

- Predict the state at time k:

$$\hat{X}^-(k) = \Phi \hat{X}(k-1). \tag{5}$$

- Calculate the covariance matrix of the predication:

$$P^-(k) = \Phi P(k-1)\Phi^T + Q(k-1). \tag{6}$$

- Calculate the Kalman gain matrix:

$$K(k) = P^-(k)H^T(k)[H(k)P^-(k)H^T(k) + R(k)]^{-1}, \tag{7}$$

where $H(k)$ is the Jacobian of the measurement vector with respect to the state of the target, which is also the observability matrix of the system and gives as follows:

$$H(k) = \nabla_s h(X(k)) = \left[\frac{\partial \theta_1^k}{\partial X} |_{X=\hat{X}(k-1)} \quad \cdots \quad \frac{\partial \theta_n^k}{\partial X} |_{X=\hat{X}(k-1)} \right]^T. \tag{8}$$

- Update the covariance matrix:

$$P(k) = P^-(k) - k(k)H(k)P^-(k). \tag{9}$$

- Update the state estimate:

$$\hat{X}(k) = \hat{X}^-(k) + K(k)(Y(k) - h(\hat{X}^-(k))). \tag{10}$$

The IMMCEKF algorithm combines the CEKF algorithm shown above and IMM algorithm to solve the cooperative maneuvering target tracking problem of multiple mobile sensors.

Assuming that the transition between the models describing the motion of the system obeys Markov chain: $P\{M_{k+1} = j|M_k = i\} = \pi_{ij}, i = 1, \cdots r, j = 1, \cdots r$. π_{ij} is the Markov transition probability from model i to model j. Let $\mu_i(k-1)$ be the probability of model i at time $k-1$. The IMMCEKF algorithm updates the state $\hat{X}_i(k)$, covariance $P_i(k)$ and mode probability $\mu_i(k)$ for each CEKF of model i. The detailed steps are shown as follows.

- Interaction:

  Predicted model probability:

$$C_j^- = \sum_{i=1}^{r} \pi_{ij}\mu_i(k-1). \tag{11}$$

  Mixing probability:

$$\mu_{ij} = \pi_{ij}\mu_i(k-1)/C_j^-. \tag{12}$$

  Mixed initial condition for filter j:

$$\hat{X}_{0j}(k-1) = \sum_{i=1}^{r} \hat{X}_i(k-1)\mu_{ij}(k-1)$$

$$P_{0j}(k-1) = \sum_{i=1}^{r} \mu_{ij}(k-1)\{P_i(k-1) + [\hat{X}_i(k-1) - \hat{X}_{0j}(k-1)] \times [\hat{X}_i(k-1) - \hat{X}_{0j}(k-1)]^T\} \tag{13}$$

- Filtering: Each model-matched filter is the CEKF fusion algorithm.

  State estimate and covariance of model j:

$$[\hat{X}_j^-(k), P_j^-(k)] = CEKF(\hat{X}_{0j}(k-1), P_{0j}(k-1), \Phi^j, Q^j),$$

$$[\hat{X}_j(k), P_j(k)] = CEKF(\hat{X}_j^-(k), P_j^-(k), H^j, Y(k), R^j). \tag{14}$$

  Residual:

$$r_j(k) = Y(k) - h(\hat{X}_j^-(k)) \tag{15}$$

  Residual covariance:

$$S_j(k) = H_j(k)P_j^-(k)H_j^T(k) + R_j \tag{16}$$

  Filter gain:

$$K_j(k) = P_j^-(k)H_j^T(k)S_j^{-1}(k) + Q_j \tag{17}$$

- Mode probability update:

  Likelihood function:

$$\Lambda_j(k) \triangleq N(r_j(k); 0, S_j(k)) = \frac{1}{2\pi|S_j(k)|} exp\{-\frac{1}{2}r_j(k)^T S_j^{-1}(k)r_j(k)\} \tag{18}$$

  which is Gaussian probability density function.

  Model probability:

$$\mu_j(k) = \frac{C_j^-}{\sum_{j=1}^{r} \Lambda_j(k)C_j^-} \tag{19}$$

- Combination:

$$\hat{X}(k) = \sum_{j=1}^{r} \hat{X}_j(k)\mu_j(k),$$

$$P(k) = \sum_{j=1}^{r} \mu_j(k)\{P_j(k) + [\hat{X}_j(k) - \hat{X}(k)][\hat{X}_j(k) - \hat{X}(k)]^T\} \tag{20}$$

IV. OPTIMAL FORMATION FOR TARGET TRACKING

Dynamical target tracking by mobile sensing agents is more difficult due to the maneuverability of target and agents. A suitable controller should be designed for agents, as the agents may lose the target when they are moving. Because the relative sensor-target geometry affects the estimation accuracy specially, there exist many contributions about optimal sensor placement of a static localization problem which involves a single static target and multiple static sensors [16]–[18]. For mobile agents, the aim of target tracking is to estimate the target state on one hand and control the agents tracking the target on the other hand. Considering the estimation and tracking are coupled, we propose to control the agents moving with an optimal formation such that the estimation uncertainty is minimized.

The target estimation uncertainty is usually characterized by the Fisher information matrix (FIM), which is the inverse of the Cramer-Rao lower bound (CRLB). The CRLB is the minimum achievable estimation variance: $P = E[(\hat{X} - X)(\hat{X} - X)^T] \geq C = F^{-1}$. Where F is the Fisher information matrix. In [16]–[18], maximizing the determinant of the FIM is used as the criterion for optimal placement in 2D. Frame theory is introduced and applied to optimal placement problem of bearing-only sensors in both 2D and 3D in [18].

We rewrite the measurement Equation (2) is a general form as follows.

$$\hat{M}_i(P_i, P_T) = M_i(P_i, P_T) + \omega_i, \quad i = 1, \cdots n, \tag{21}$$

where $M_i = \frac{P_i - P_T}{\|P_i - P_T\|}$. Then the FIM can be computed by the following equation:

$$F = \sum_{i=1}^{N} (\nabla_{P_T} M_i)^T R^{-1} \nabla_{P_T} M_i \tag{22}$$

where $\nabla_{P_T} M_i$ is the Jacobian of $M_i(P_i, P_T)$ with respect to P_T:

$$\nabla_{P_T} M_i = -\frac{1}{\|r_i\|}(I - M_i M_i^T) \tag{23}$$

with $r_i = P_i - P_T$. Then the FIM can be described by the following equation:

$$F = \sum_{i=1}^{N} c_i^2 (I - M_i M_i^T) \tag{24}$$

with $c_i = \frac{1}{\sigma_\omega \|r_i\|} > 0$ which are scale factors of F.

There are two important properties of the FIM as follows [18].

Proposition 1: F is singular if and only if $\{M_i\}_{i=1}^N$ are collinear.

Proposition 2: F is invariant to the sign change of M_i for all $i \in \{1, \cdots N\}$. The eigenvalues of F are invariant to any orthogonal transformations over $\{M_i\}_{i=1}^N$.

The two properties tell us:

(1) When all sensors and the target are located on the same line, F is singular, which means the estimation uncertainty is infinite. Therefore, this kind of singular placement is the worst and should be avoided.

(2) There exist some equivalent placement because when rotating all sensors the same angle around the target, reflecting all sensors about a line/plane passing through the target, or both combined does not change the eigenvalues of F.

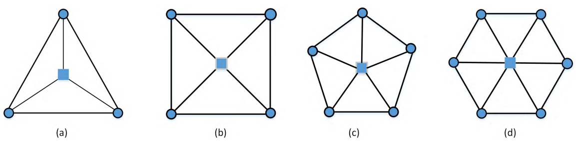

FIGURE 3. Examples of 2D optimal placements: regular polygons. (a) Triangle. (b) Square. (c) Pentagon. (d) Hexagon.

References [16]–[18] deduced the same conclusion: in R^2, an equality weighted placement is optimal in $N(N \geq 3)$ sensors are located at the vertices of an $N-$side regular polygon with the assumption that the sensor ranges are identical as shown in Fig.3. However, this conclusion cannot be applied in the target tracking problem directly, because the agents can't be controlled to move backwards. An algorithm of constructing 2D regular optimal placements is proposed by introducing the frame theory into the problem of that the ranges r_i are arbitrary but $\{c_i\}_{i=1}^N$ is a regular sequence in [18]. Geometrical meaning of a regular sequence is that the distance from each sensor to the target is similar. However, this algorithm can only give a triangle according to the following theorem.

Theorem 1: In $\mathbb{R}^2$, suppose there are N passive sensors which satisfy the measurement equation (21) and $\{c_i\}_{i=1}^N$ is a regular sequence. Let $M_i = [\sin\theta_i \ \cos\theta_i]^T$ be the measurement vector where θ_i is the azimuth angle. Then following statements hold:

(1) A placement $\{M_i\}_{i=1}^N$ is optimal if and only if

$$\sum_{i=1}^{N} c_i^2 M_i M_i^T = \frac{1}{2}\sum_{i=1}^{N} c_i^2 I \tag{25}$$

which is also equivalent to

$$\sum_{i=1}^{N} c_i^2 \bar{M}_i = 0 \tag{26}$$

where $\bar{M}_i = [\sin 2\theta_i \ \cos 2\theta_i]^T$.

(2) There exists the optimal placement $\{\bar{M}_i\}_{i=1}^N$ with $\|\bar{M}_i\| = 1$ solving (26) if and only if

$$c_j^2 \leq \frac{1}{2}\sum_{i=1}^{N} c_i^2 \tag{27}$$

for all $j \in \{1, \cdots, N\}$.

Proof 1: (1) Calculate the Fisher information matrix F. We have

$$F = \sum_{i=1}^{N} c_i^2 \begin{bmatrix} 1-\sin^2\theta_i & -\sin\theta_i\cos\theta_i \\ -\sin\theta_i\cos\theta_i & 1-\cos^2\theta_i \end{bmatrix} \tag{28}$$

Let λ_1, λ_2 be the eigenvalues of F. Then $\det F = \lambda_1\lambda_2 \leq (\frac{\lambda_1+\lambda_2}{2})^2$. Let $\bar{\lambda} = \frac{\lambda_1+\lambda_2}{2}$. The $\det F$ achieves the maximum value when $\lambda_1 = \lambda_2 = \bar{\lambda}$ which means $F - \bar{\lambda}I = 0$.

Base on equation (28),we have $\bar{\lambda} = \frac{\lambda_1+\lambda_2}{2} = \frac{1}{2}trF = \frac{1}{2}\sum_{i=1}^{N} c_i^2$. The optimal placement $\{M_i\}_{i=1}^N$ should satisfy $F = \bar{\lambda}I$ which deduces the following equation based on equation (24):

$$\begin{aligned} F &= \sum_{i=1}^{N} c_i^2 I - \sum_{i=1}^{N} c_i^2 M_i M_i^T = \bar{\lambda} I \\ &\Rightarrow \sum_{i=1}^{N} c_i^2 M_i M_i^T = \sum_{i=1}^{N} c_i^2 I - \bar{\lambda} I = \frac{1}{2}\sum_{i=1}^{N} c_i^2 I \end{aligned} \tag{29}$$

The above equation can be simplified as follows:

$$\begin{aligned} \sum_{i=1}^{N} c_i^2 M_i M_i^T &= \sum_{i=1}^{N} c_i^2 \begin{bmatrix} 1-\sin^2\theta_i & -\sin\theta_i\cos\theta_i \\ -\sin\theta_i\cos\theta_i & 1-\cos^2\theta_i \end{bmatrix} \\ &= \frac{1}{2}\sum_{i=1}^{N} c_i^2 I \\ &\Rightarrow \begin{cases} \sum_{i=1}^{N} 2c_i^2\cos^2\theta_i = \sum_{i=1}^{N} c_i^2 \\ \sum_{i=1}^{N} 2c_i^2 \sin\theta_i\cos\theta_i = 0. \end{cases} \\ &\Rightarrow \begin{cases} \sum_{i=1}^{N} c_i^2(\cos^2\theta_i - \sin^2\theta_i) = 0, \\ \sum_{i=1}^{N} 2c_i^2 \sin\theta_i\cos\theta_i = 0. \end{cases} \\ &\Rightarrow \begin{cases} \sum_{i=1}^{N} c_i^2\cos 2\theta_i = 0, \\ \sum_{i=1}^{N} 2c_i^2 \sin 2\theta_i = 0. \end{cases} \end{aligned} \tag{30}$$

Then Equation (26) is obtained.

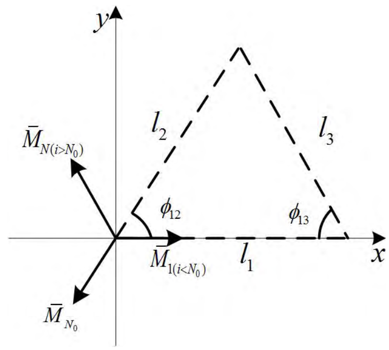

FIGURE 4. Geometric meaning of the optimal triangle placement.

(2) First prove the necessity. If $\sum_{i=1}^{N} c_i^2 \bar{M}_i = 0$, then $c_j^2 \bar{M}_j = \sum_{i=1,i\neq j}^{N} c_i^2 \bar{M}_i$ for all $j \in \{1, \cdots, N\}$. Such that the following inequality exists.

$$c_j^2 = \|c_j^2 \bar{M}_j\| = \|\sum_{i=1,i\neq j}^{N} c_i^2 \bar{M}_i\| \leq \sum_{i=1,i\neq j}^{N} \|c_i^2 \bar{M}_i\| = \sum_{i=1,i\neq j}^{N} c_i^2 \tag{31}$$

The sufficiency is more difficult to be proved. However, [18] proved the sufficiency by dividing the N sensors into three groups.

Since $c_j^2 + \sum_{i=1,i\neq j}^{N} c_i^2 = \sum_{i=1}^{N} c_i^2$, we have $c_j^2 \leq \frac{1}{2}\sum_{i=1}^{N} c_i^2$ by considering equation (31). Then there always exits an index N_0, $2 \leq N_0 \leq N$ such that

$$c_1^2 + \cdots + c_{N_0-1}^2 \leq \frac{1}{2}\sum_{i=1}^{N} c_i^2 \tag{32}$$

$$c_1^2 + \cdots + c_{N_0}^2 \geq \frac{1}{2}\sum_{i=1}^{N} c_i^2 \tag{33}$$

The N sensors separated into three groups: $1, \cdots, N_0$, N_0, and $N_0+1, \cdots, N$. Let

$$\begin{aligned} l_1 &= c_1^2 + \cdots + c_{N_0-1}^2 \\ l_2 &= c_{N_0}^2 \\ l_3 &= c_{N_0+1}^2 + \cdots + c_N^2 \end{aligned} \tag{34}$$

The three line segments with lengths respectively as l_1, l_2, l_3 can form a triangle.

Let $\bar{M}_i = \bar{M}_1$ for $i = 1, \cdots, N_0 - 1$ (the first group) and $\bar{M}_i = \bar{M}_N$ for $i = N_0 + 1, \cdots, N$ (the third group). Thus equation (26) becomes

$$l_1 \bar{M}_1 + l_2 \bar{M}_{N_0} + l_3 \bar{M}_N = 0 \tag{35}$$

If we choose $\bar{M}_1, \bar{M}_{N_0}, \bar{M}_N$ that align with the three sides of the triangle with side length as l_1, l_2, l_3 (see Fig.4), $l_1\bar{M}_1, l_2\bar{M}_{N_0}, l_3\bar{M}_N$ form a triangle such that the above equation (35) is solved. □

Based on the theorem 1, an optimal placement can be constructed by the following algorithm. However, the optimal formation is always a triangle.

Algorithm 1:

Step 1: Choose N_0 satisfying (32) and (33);

Step 2: Compute l_1, l_2, l_3 according to equation (34).

Step 3: Compute the internal angles ϕ_{12} and ϕ_{13} of the triangle with side lengths as l_1, l_2, l_3 as shown in Fig.

$$\begin{cases} \phi_{12} = \arccos(\dfrac{l_1^2 + l_2^2 - l_3^2}{2l_1 l_2}), \\ \phi_{13} = \arccos(\dfrac{l_1^2 + l_3^2 - l_2^2}{2l_1 l_3}), \end{cases} \tag{36}$$

Step 4: Choose and compute

$$M_i = \begin{cases} [1, 0]^T, \quad i = 1, \cdots, N_0 - 1 \\ \begin{bmatrix} \cos\dfrac{\pi + \phi_{12}}{2} & \sin\dfrac{\pi + \phi_{12}}{2} \end{bmatrix}^T, \quad i = N_0 \\ \begin{bmatrix} \cos\dfrac{\pi - \phi_{13}}{2} & \sin\dfrac{\pi - \phi_{13}}{2} \end{bmatrix}^T, \\ \quad i = N_0 + 1, \cdots, N \end{cases} \tag{37}$$

We verify the algorithm by the following example.

Example 1: Suppose there are three sensors which are located at $p_1 = (-10, 0)$, $p_2 = (0, 0)$, $p_3 = (0, 10)$. A target is at $p_T = (5, 5)$.The measurement noise variance of each sensor is $\sigma_i = 0.1$. Then $c_1^2 = 0.4$, $c_2^2 = 2$, $c_3^2 = 2$. It's easy to check that $c_j^2 \leq \frac{1}{2}\sum_{i=1}^{N} c_i^2 = 0.22$. Choose $N_0 = 2$, we get $l_1 = 0.04$, $l_2 = 0.2$, $l_3 = 0.2$. Then calculate $\phi_{12} = \phi_{13} = 1.5708rad$ by using the cosine theorem. Next calculate the optimal azimuth angle $M_1 = (1, 0)^T$, $M_2 = (-0.7071, 0.7071)^T$, $M_3 = (0.7071, 0.7071)^T$. Finally, get the optimal position of sensors $p_1^{optimal} = (-10.8114, 5)^T$, $p_2^{optimal} = (10, 0)^T$, $p_3^{optimal} = (0, 0)^T$.

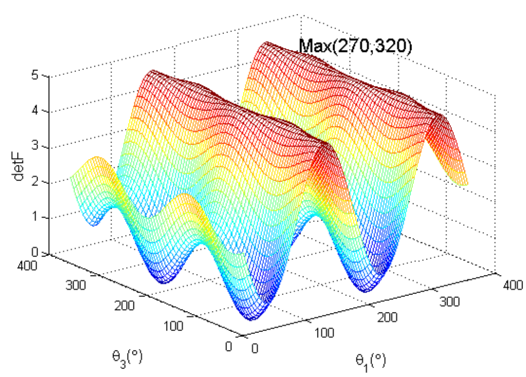

FIGURE 5. The value of the FIM with different θ_1 and θ_3.

We enumerate azimuthal angles θ_1, θ_3 with fixed r_1 and r_3 and calculate $\det F$ to find the maximum of $\det F$. Let $p_2 = (0, 0)$ and $p_T = (5, 5)$. The results is shown by Fig.5. When $\theta_1 = 273°$, $\theta_3 = 321°$, the value of $\det F$ is maximum. Such that the position of the two sensors are

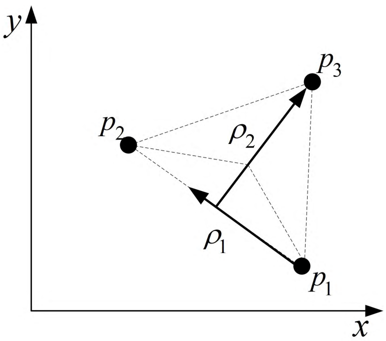

FIGURE 6. The formation described by the Jacobi vectors.

$(-10.7898, 5.8256)^T$ and $(9.5474, -0.4149)^T$. The results obtained by enumeration method is close to the constructing algorithm. The validity of the algorithm is verified.

This algorithm constructs the optimal placement by dividing sensors into three groups. The sensors in the same group will be distributed on the same line. Investigate Fig.3 and consider the *Proposition 2*. When $N = 3, 4, 6$, we can reflect some sensors about the target such that a triangle formation becomes. This triangle form is useful for the target tracking.

V. COOPERATIVE TARGET TRACKING OF MULTIPLE AGENTS FOR OPTIMAL TARGET ESTIMATION

In the previous section, the optimal formation algorithm for multi-agents collaborative target observation is given. Based on this, we design a controller to make the agents track the target with optimal placement formation.

A geometric formation control which is based on the Jacobi vectors has been applied in *multi-AUVs* formation control in our previous works [25], [26]. The formation shape is described by Jacobi vectors that satisfy

$$\begin{bmatrix} \rho_1 & \cdots & \rho_{N-1} & q_c \end{bmatrix}^T = \Phi \begin{bmatrix} p_1 & \cdots & p_{N-1} & p_N \end{bmatrix}^T, \tag{38}$$

where $q_c = \frac{1}{N}\sum_1^N p_i$ is the formation center, Φ is the Jacobi transform matrix. ρ_i are the Jacobi vectors which describe the formation geometry. For example, if we deploy three agents, the Jacobi vectors can be defined as follows and the formation is shown in Fig6.:

$$\begin{aligned} \rho_1 &= \frac{1}{\sqrt{2}}(p_2 - p_3) \\ \rho_2 &= \frac{1}{\sqrt{6}}(2p_1 - p_2 - p_3) \end{aligned} \tag{39}$$

The formation state equation can be deduced by (38) as follows:

$$\begin{bmatrix} \ddot{\rho}_1 & \cdots & \ddot{\rho}_{N-1} & \ddot{q}_c \end{bmatrix}^T = \begin{bmatrix} \bar{u}_1 & \cdots & \bar{u}_{N-1} & \bar{u}_c \end{bmatrix}^T \tag{40}$$

Then the PD controller can achieve the goal $\rho_i \to \rho_{id}$ and $q_c \to q_{cd}$ where ρ_{id} and q_{cd} are the desired Jacobi vectors and formation center position respectively.

$$\begin{aligned} \bar{u}_i &= -k_1^{\rho}(\rho_i - \rho_{id}) - k_2^{\rho}\dot{\rho} \\ \bar{u}_c &= -k_1^{q_c}(q_c - q_{cd}) - k_2^{q_c}\dot{q}_{cd} \end{aligned} \tag{41}$$

The acceleration inputs of the agents are obtained by

$$[u_1, \cdots, u_N]^T = \Phi[\bar{u}_1, \cdots, \bar{u}_c]. \tag{42}$$

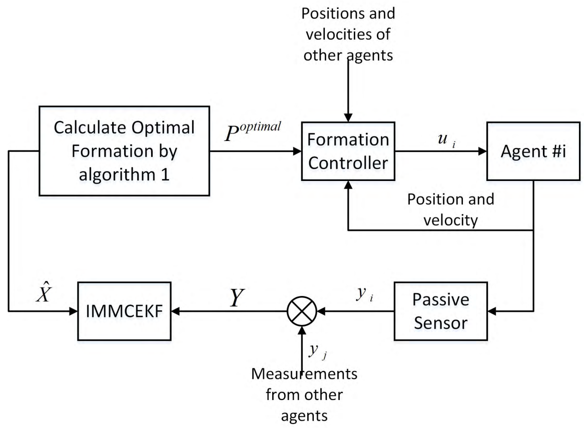

FIGURE 7. The whole process of target tracking by optimal formation.

VI. FRAMEWORK OF TARGET TRACKING BY MOBILE AGENTS WITH OPTIMAL TARGET ESTIMATION

Suppose the agent is mounted with a passive sensor and it also has the localization sensor. The whole process is explained by the following Fig.7. First, measurement vector Y combines the measurement of its passive sensor and other measurements communicated from other agents. Then the estimated target state $\hat{X}$ is obtained by IMMCEKF. The positions of optimal configuration $P^{optimal}$ is calculated by algorithm 1. Finally, the control of each agent u_i is calculated by the formation controller with positions and velocities of itself and other agents. The IMMCEKF filter, algorithm 1 and formation controller can be implemented in each agent, then the proposed framework is decentralized without considering communication delay among agents.

VII. SIMULATIONS

We present simulation results to verify the proposed tracking control strategy. We consider that there are three mobile sensing agents tracking a moving target which has the following four cases: (1) the target moves with constant velocity; (2) the target moves with constant acceleration; (3) the target moves with constant angular velocity; (4) the target moves with several maneuver modes.

Suppose the initial position of the agents and the target are at $p_1 = [0, 5]^T$, $p_2 = [5, -2]^T$, $p_3 = [0, 0]^T$, $p_T = [5, 5]^T$ respectively, which are in m. The initial velocity of agents equals to the target, which is set to be $100m/s$. The initial orientation angle of the target is $\frac{\pi}{4} rad$, so the target's speed is $[100\cos(\frac{\pi}{4}), 100\sin(\frac{\pi}{4})]^T$ which is in m/s. The Gaussian

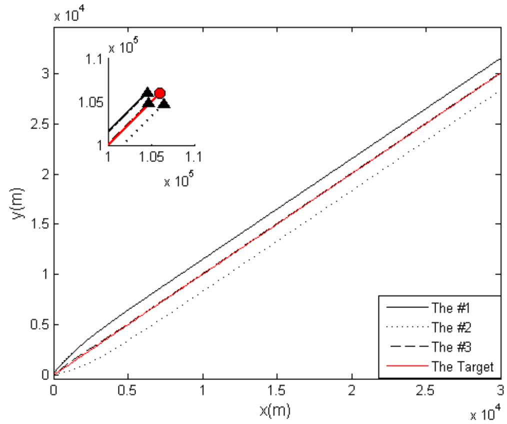

FIGURE 8. The trajectories of agents and target with CV motion.

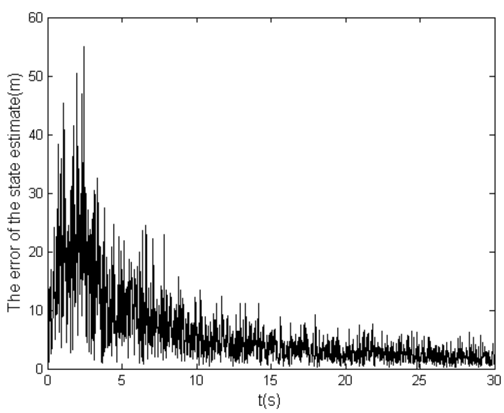

FIGURE 9. The state estimation error of the target with CV motion.

white noise of the state and measurement equations satisfies $E = \begin{bmatrix} 10I & 0 \\ 0 & \sqrt{200}I \end{bmatrix}$.

In case 1, the constant speed of target is given as $[100\cos(\frac{\pi}{4}), 100\sin(\frac{\pi}{4})]^T$, which are in m/s. Fig.8 shows the trajectories of the target and agents, and Fig.9 shows the position error of estimation by IMMCEKF. The position error is calculated by $e = \sqrt{(x_T - \hat{x}_T)^2 + (y_T - \hat{y}_T)^2}$ which represents the distance between the real position of target and the estimated position by IMMCEKF filter.

In case 2, the orientation angle of the target is also set to be $\frac{\pi}{4}rad$, the constant acceleration is $[5, 4]^T$ which is in m/s^2. Fig.10 and 11 show the results.

In case 3, the angular velocity of the target is set to be $\omega = 0.25rad/s$. Fig.12 and 13 show the results.

In Fig.8, 10 and 12, the enlarged view shows the final positions of the agents and the target, in which the triangle represents the agent and the circle represents the target. According to the Fig.9,11,13, we can see that multi-agent have been adjusting the formation at the beginning. When the agents form the optimal formation, the target state estimation error reduces significantly and the cooperative target tracking for optimal target estimation by multiple agents is achieved. However, we also can find that the estimation error became

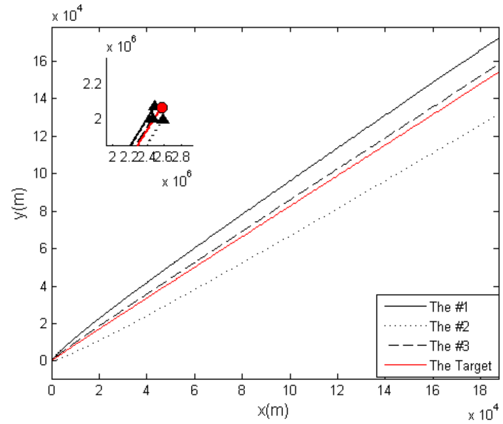

FIGURE 10. The trajectories of agents and target with CA motion.

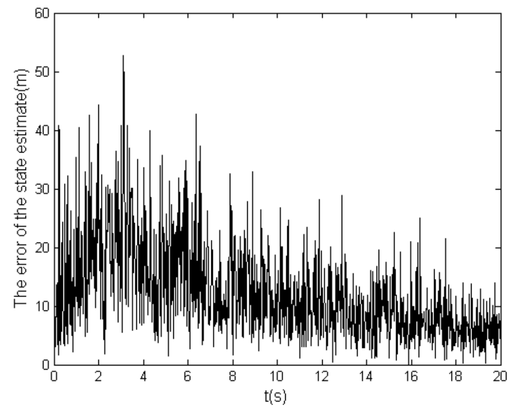

FIGURE 11. The state estimation error of the target with CA motion.

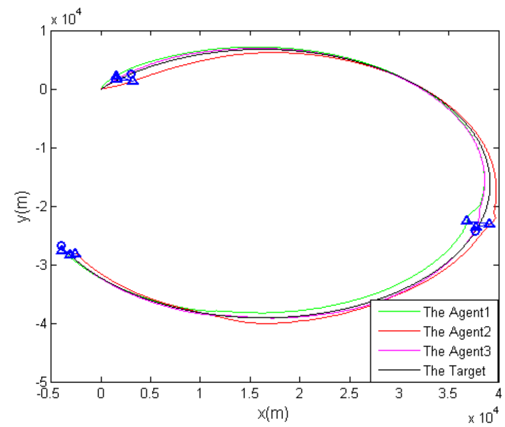

FIGURE 12. The trajectories of agents and target with constant angular velocity motion.

more smaller in case 1 than case 2 and 3, because the final error is maintained at around $5m$ in the case 1 while the error in Case 2 and Case 3 are around $10m$. In fact, the target is moving without maneuvers in case 1, but in case 2, the target's velocity is changing and in case 3, the target's orientation is changing. Comparing these results, we say that estimation error of a maneuvering target is larger than a non-maneuvering target.

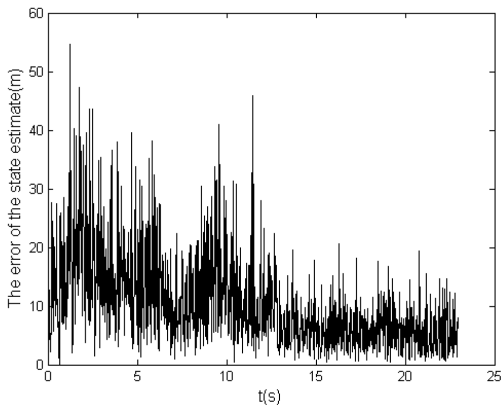

FIGURE 13. The state estimation error of the target with constant angular velocity motion.

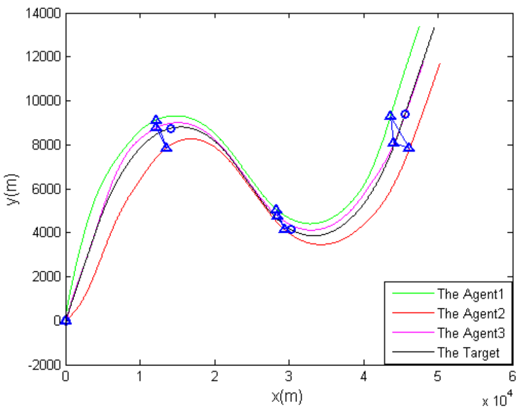

FIGURE 14. The trajectories of agents and target with multiple motions.

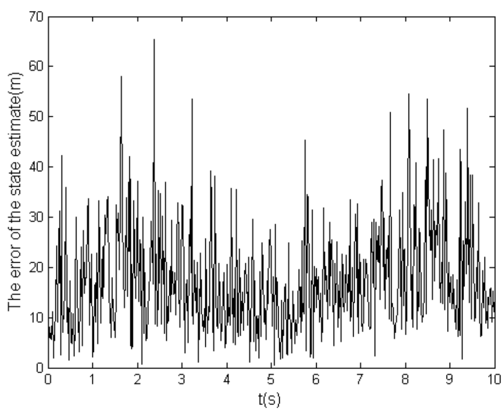

FIGURE 15. The state estimation error of the target with multiple motions.

In case 4, the trajectory of target includes five segments: (1) CV model with the given initial velocity $[100\cos(\frac{\pi}{4})m/s, 100\sin(\frac{\pi}{4})m/s]^T$ for $1s$; (2) CT model with the angular velocity $\omega = 0.35rad/s$ for $3.5s$; (3) CV model for $1s$; (4) CT model with the angular velocity $\omega = -0.35rad/s$ for $3.5s$; (5) CV model for $1s$. Fig.14 shows the trajectories of the agents and the target. Fig.15 shows the position error of estimation, which doesn't reduce obviously like the previous three cases. However, investigating the position error carefully, we find that the magnitude of the error is related to whether the target motion model is switched. For example, when the target motion model is switched at $1s$, the IMM algorithm cannot match the correct motion model immediately, which leads to the increase of estimation error. After a period of time, when the correct model is matched, the estimation error will gradually decrease. This is the characteristic of the IMM algorithm. Another reason is that the speed of convergence of the optimal formation does not keep up with the speed of the target turning.

VIII. CONCLUSION

Using multiple mobile sensing agents to track a maneuvering target is a challenge task in many practical applications. The sensing and tracking processes are coupled due to the mobility of the agents and the target. The optimal placement of static sensing agents for target tracking is investigated in many literatures. The optimal placement means those relative sensor-target geometries which result in a measure of the uncertainty ellipse being minimized. To the dynamical agents, we proposed a geometry formation control strategy for maneuvering target tracking, which ensures agents to track the target with optimal target state estimation. The main contribution is that we transfer the problem into a geometry formation control problem. By using the Jacobi vectors to describe the formation shape of the agents and the formation center to describe the formation motion which isn't relative to the formation shape described by the Jacobi vectors, we decouple the formation shape controller and the formation tracking controller designing. We combine the IMM estimator and the centralized EKF-based fusion algorithm to estimate the maneuvering target state. Based on the target state estimation, an optimal formation is deduced by maximizing the determinant of FIM and the constructing algorithm is verified by using the frame theory.

Simulation results show that the classical IMM filtering method cannot be matched to the correct model immediately when the target model is switched. This is not only related to the accuracy of the model, but also directly related to the estimation of the input interaction, and the Markov transition probability has a great impact on the input interaction. The Markov probability matrix in this paper is known a priori, and the filtering parameters determined by the priori information are a tradeoff between the target model switching and the non-switching. Because the current measurement information of the system contains the current mode information, in order to obtain more accurate conversion between models, the current measurement information of the system can be used to estimate the Markov transition probability matrix in real time. Therefore, IMM algorithm can be improved according to the above analysis in the future work.

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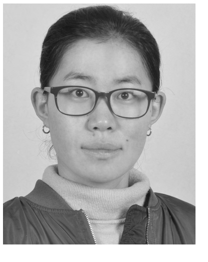
HUIZHEN YANG received the B.S., M.S., and Ph.D. degrees from Northwestern Polytechnical University (NWPU), Xi'an, China, in 1995, 1998, and 2005, respectively, where she is currently an Associate Professor with the School of Marine Science and Technology.

She was a Visiting Scholar with the School of Electrical and Computer Engineering, Georgia Institute Technology, from 2008 to 2009. She has authored three teaching books, more than 50 articles, and two patents. Her research interests include cooperative target tracking by multiple AUVs, cooperative control of multi-AUVs systems, and cyber-physical systems.

Dr. Yang received the Most Satisfied Faculty Award of NWPU, in 2014. She is the member of the Council of China Simulation Federation.



YUE WANG was born in Datong, Shanxi, China, in 1993. She received the B.S. degree in automation from Chang'an University, Xi'an, China, in 2017. She is currently pursuing the M.S. degree in automation engineering with Northwestern Polytechnic University. Her research interests include cooperative target tracking by multiple AUVs and cooperative control of multi-AUVs' systems.