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Robust H_∞ Output Feedback Control of Networked Control Systems With Discrete Distributed Delays Subject to Packet Dropout and Quantization

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ABSTRACT This paper studies the robust H_∞ control problem of networked linear time-delay systems with discrete distributed delays, involving random packet dropout and quantization. Assume that the measured output of the networked time-delay system can be quantized by the logarithmic quantizer before being transmitted through the communication network. In addition, an appropriate compensation strategy is proposed to reduce the effect of the data packet dropout satisfying a Bernoulli distribution. To deal with the quantization issue, the sector bound method can be used to convert the quantized control problem of the networked system into the robust control problem with uncertainty. Then, a novel observer-based H_∞ output feedback controller is designed to ensure that the networked system is exponentially mean-square stable and an expected H_∞ performance constraint is achieved. Finally, a simulation example is given to prove the effectiveness of the proposed design method.

INDEX TERMS Networked control systems, discrete distributed delays, packet dropout, quantization.

I. INTRODUCTION

Network control systems (NCSs) have become a hot research field in recent years [1]–[4], which have found fruitful applications in a series of important areas such as industrial automation, mobile sensor networks, environmental monitoring, underwater robots and unmanned systems, because of the merit of the remote and distributed control. These applications have motivated many scholars to focus on the control and estimation problem for NCSs.

Although NCSs have lots of appealing advantages such as resource sharing, easy maintenance and flexible installation, NCSs have also introduced some challenges due to the limited communication bandwidth [5]. Among all the challenges, time delays, packet dropout and data quantization are always known as primary causes for performance deterioration, which have become some highlight in the literature. This paper will focus on the impact of the packet dropout and data quantization for the controller design of NCSs with distributed delays.

In the literature, packet dropout as one of the main communication constraints is always a hot topic [6]–[12]. Usually, the phenomenon of the data packet dropout is considered as a random process, so some stochastic methods can be used to characterize this process. In these stochastic methods, data packet dropout can be assumed to satisfy the Bernoulli distribution [6]–[8] and Markov chain [9]–[11]. To deal with the quantized control problem of non-linear NCSs with packet dropout represented by T-S model, A fuzzy predictive controller was proposed by [6]. The exponential synchronization problem of complex dynamic networks with packet dropouts and additive time-delay was discussed by [7]. Using the

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Markov chain method, Reference [9] analyzed the stochastic stability of the networked linear system with time-delays and packet-dropout modeled by two Markov chains, and designed a output feedback controller based on Lyapunov method. Furthermore, an optimal energy allocation and an optimal controller were proposed by [10] for linear NCSs with energy constraint and packet loss modeled by Markov chain. However, different from the stochastic characterization of packet dropout, [12] addressed the bounded packet loss issue for robust predictive control.

On the other hand, an important issue taken into account for NCSs is the data quantization in transmission, where the quantization error has an undesirable effect on the performance and stability of NCSs [13]–[18]. Hence, there are lots of good results to eliminate or mitigate the effect. The sector bound approach proposed by [13] played an important role in the quantization issue, and then authors further expand the above results to the robust stabilization for uncertain systems [14]. A robust controller was designed by [15] for singular systems with quantization, based on a event-triggered mechanism. Different from the logarithmic quantizer [16], [17], reference [18] analyzed the dynamic quantization of uncertain systems with time-delays.

In addition, The phenomenon of data quantization and packet dropout coexisting in communication networks is more realistic and reasonable than only considering the packet dropout or data quantization, which can both effect the stability and performance of NCSs [19]–[23]. Reference [20] discussed the trade-off between the coarsest quantization density, packet dropout rate and the stability of the linear plant. Based on the predictive control strategy, [21] proposed a new predictive control algorithm for constrained networked control systems with packet dropout and data quantization which can be described in a unifying framework by using a novel modeling method. Taking into account the limitations on data rate, packet dropout rate and uncertainty, [22] gave some conditions for uncertain systems to be mean-square stable. Reference [23] studied the quadratic stability of uncertain systems under the coarsest logarithmic quantizer.

Motivated by the above works about these issue such as packet dropout and data quantization, the robust H_∞ output feedback control problem for NCSs with discrete distributed delays involving data quantization and packet dropout is investigated in this paper, where discrete distributed delays [24]–[27] as another important type of time delays [28], [29] also attracts many scholars. Similar to [7]–[9], the packet dropout is characterized as a Bernoulli distribution taking as the values of 1 or 0, and the logarithmic quantization of the measured system output is described as an uncertainty by using the sector bound method. We aim at designing an observer-based H_∞ output feedback controller such that the closed-loop networked system is exponentially mean-square stable and satisfies the H_∞ performance constraint. By using the cone complement linearization approach, a subsequent optimization problem can be solved to derive a suboptimal system performance. The main advantages of the paper lie in the following aspects: (1) The system state with the discrete distributed delays can be used for the controller design of the networked system. (2) A novel model of the networked system with discrete distributed delays involving quantization and dropout is proposed, by using the dropout compensation and sector bound method. (3) An observer-based H_∞ output feedback controller is designed to robustly exponentially stabilize the NCSs and satisfy the H_∞ performance constraint.

This paper is organized as follows. Section II formulates the problem for networked systems with distributed delays involving quantization and dropout. Section III designs a observer-based H_∞ output feedback controller. Section IV provides an illustrative example. Finally, Section V gives concluding remarks.

II. PROBLEM FORMULATION

The structure of the networked control system in this paper is discussed in Fig. 1.

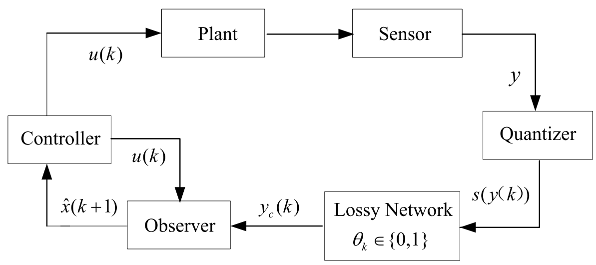

FIGURE 1. The structure of networked control system.

The plant described by a linear discrete-time system with mixed delays is given as follows:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bx(k-d(k)) \\
&\quad + C\sum_{i=1}^{+\infty}\mu_i x(k-i) + Du(k) + Ew(k), \qquad (1)\\
z(k) &= H_1x(k) + E_1w(k), \\
y(k) &= H_2x(k), \\
x(j) &= x_0(j), \quad j < 0,
\end{aligned}
$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^m$ is the control input, $w(k) \in \mathbb{R}^q$ is disturbance input, $z(k) \in \mathbb{R}^p$ is the controlled output, $y(k) \in \mathbb{R}^l$ is the measured system output and A, B, C, D, E, H_1, H_2 are known real matrices with appropriate dimensions. $d(k)$ denotes the time-varying delay with $d_1 \le d(k) \le d_2$, where d_1, d_2 are known positive integers. Furthermore, the constants $\mu_i > 0$ $(i = 1, 2, \ldots)$ satisfy

$$
\sum_{i=1}^{+\infty} i\mu_i < +\infty \quad and\ \overline{\mu} = \sum_{i=1}^{+\infty}\mu_i < +\infty. \qquad (2)
$$

In the Fig. 1, communication in sensor-controller link occurs through unreliable network, which introduces the issues of the quantization and the random packet dropout considered in this paper. For the sake of analysis, assume that the measured output of the system has been quantized by the logarithmic quantizer [13] before the packet dropout caused by lossy network occurs. Furthermore, the Logarithmic quantizer can be defined as follows:

Definition 1: A quantizer is called a logarithmic quantizer if its quantitative set can be described as:

$$h = \{\pm h_i, h_i = \rho^i h_0, i = 0, \pm 1, \ldots\} \cup \{\pm h_0\} \cup \{0\}$$

where $0 < \rho < 1$ is called the quantization density of the logarithmic quantizer, and $h_0 > 0$ is the default output value. Then the associated quantized function $s(\cdot)$ can be defined by

$$s(y(k)) = \begin{cases} h_i, & if\ \frac{1}{1+\delta}h_i < y(k) < \frac{1}{1-\delta}h_i \\ 0, & if\ y(k) = 0 \\ -s(-y(k)), & if\ y(k) < 0 \end{cases} \quad (3)$$

where $\delta = \frac{1-\rho}{1+\rho}$.

Meanwhile, the randomly packet dropout satisfying the Bernoulli distribution [8] is considered in this paper, and the following compensation strategy is proposed for reducing the effect of the data packet dropout. Then, the effective information from the quantized system output can be described as:

$$y_c(k) = (1-\theta_k)s(y(k)) + \theta_k y_c(k-1), \quad (4)$$

where θ_k taking the values of 0 and 1 is an i.i.d. random variable with probability distribution:

$$\begin{aligned} Prob\{\theta_k = 1\} &= E\{\theta_k = 1\} = \bar{\theta}, \\ Prob\{\theta_k = 0\} &= 1 - E\{\theta_k = 1\} = 1 - \bar{\theta}, \\ Var\{\theta_k\} &= E\{(\theta_k - \bar{\theta})^2\} = (1-\bar{\theta})\bar{\theta}. \end{aligned} \quad (5)$$

Here, $\bar{\theta}$ denote the data packet arrival probability at any sampling time, and then the data packet dropout probability is $1-\bar{\theta}$. In this paper, assume that the state of the system (1) can be unmeasured, and then the output feedback controller based on the full-dimensional state observer is proposed as follows:

$$\begin{aligned} \hat{x}(k+1) &= A\hat{x}(k) + Du(k) + L(y_c(k) - H_2\hat{x}(k)), \\ u(k) &= K\hat{x}(k), \end{aligned} \quad (6)$$

where the matrices L and K are the observer and controller gain, respectively.

To solve the quantization problem of the networked time-delay systems, the measured output of the system after the logarithmic quantization can be expressed by using the sector bound approach [13]:

$$s(y(k)) = (I + \Delta(k))y(k). \quad (7)$$

Then, the state estimate error is defined by

$$e(k) = x(k) - \hat{x}(k). \quad (8)$$

Taking (4) and (6)-(7) into (1) and (8), the following dynamics of the error system can be obtained:

$$\begin{aligned} x(k+1) &= [A + DK]x(k) - DKe(k) + Bx(k - d(k)) \\ &\quad + C\sum_{i=1}^{+\infty}\mu_i x(k-i) + Ew(k), \\ e(k+1) &= [\theta_k L - L(1-\theta_k)\Delta(k)]H_2 x(k) \\ &\quad + (A - LH_2)e(k) - L\theta_k y_c(k-1) \\ &\quad + Bx(k-d(k)) + C\sum_{i=1}^{+\infty}\mu_i x(k-i) + Ew(k), \\ y_c(k) &= (1-\theta_k)(I + \Delta(k))H_2 x(k) + \theta_k y_c(k-1). \end{aligned} \quad (9)$$

By denoting $\zeta^T(k) = [x^T(k)\ e^T(k)\ y_c^T(k-1)]^T$, the above augmented closed-loop system can be expressed as follows:

$$\begin{aligned} \zeta(k+1) &= \bar{A}\zeta(k) + \bar{B}x(k-d(k)) \\ &\quad + \bar{C}\sum_{i=1}^{+\infty}\mu_i x(k-i) + \bar{E}w(k) \end{aligned} \quad (10)$$

where

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A + DK & -DK & 0 \\ A_1 & A - LH_2 & -\theta_k L \\ A_2 & 0 & \theta_k I \end{bmatrix}, \\ \bar{B} &= \begin{bmatrix} B \\ B \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} C \\ C \\ 0 \end{bmatrix}, \bar{E} = \begin{bmatrix} E \\ E \\ 0 \end{bmatrix}, \\ A_1 &= [\theta_k L - L(1-\theta_k)\Delta(k)]H_2, \\ A_2 &= (1-\theta_k)(I + \Delta(k))H_2. \end{aligned}$$

It is noted that the augmented networked system (10) contains the random parameter θ_k caused by packet loss and the uncertainty $\Delta(k)$ caused by the quantizer. These are different from the traditional deterministic system, so the networked system (10) has greater uncertainty and more complexity.

To deal with the robust control problem of the closed-loop system with stochastic parameter and uncertainty, in the paper we aim to design the robust controller such that the closed-loop system is exponentially mean-square stable and the H_∞ performance constraint is satisfied. That is, the designed robust controller for the closed-loop systems (10) should satisfy the following two conditions:

(a1) The networked closed-loop system (10) with $w(k) = 0$ is exponentially mean-square stable.

(a2) For all non-zero $w(k)$, the controlled output satisfies

$$\sum_{k=0}^{+\infty} E\{\|z(k)\|^2\} < \gamma^2 \sum_{k=0}^{+\infty} E\{\|w(k)\|^2\} \quad (11)$$

under zero initial condition, where $\gamma > 0$ is a given scalar.

To give the main results about the robust control problem of the closed-loop systems (10), the following lemmas [9], [25] are introduced as follows:

Lemma 1: Let $M \in \mathbb{R}^{n\times n}$ be a positive definite matrix, If $x_i \in \mathbb{R}^n$ and $a_i \geq 0$ $(i = 1, 2, \ldots)$ are convergent, the following inequality holds:

$$(\sum_{i=1}^{+\infty} a_i x_i)^T M (\sum_{i=1}^{+\infty} a_i x_i) \leq (\sum_{i=1}^{+\infty} a_i)(\sum_{i=1}^{+\infty} a_i x_i^T M x_i). \quad (12)$$

Lemma 2: Assume that matrices D, E, F are the real matrices with suitable dimensions, and the matrix F satisfies

$F^TF \le I$, then there exists a scalar ε such that the following inequation holds:

$$DFE + E^TF^TD^T \le \varepsilon DD^T + \varepsilon^{-1}E^TE. \tag{13}$$

Lemma 3: Let $V(\eta(k))$ be a Lyapunov functional. If there exist real scalars $\lambda \ge 0$, $\chi > 0$, $\nu > 0$ and $0 < \psi < 1$, such that

$$\chi\|\eta(k)\|^2 \le V(\eta(k)) \le \nu\|\eta(k)\|^2 \tag{14}$$

and

$$E\{V(\eta(k+1)|\eta(k))\} - V(\eta(k)) \le \lambda - \psi V(\eta(k)). \tag{15}$$

Then the sequence satisfies

$$E\{\|\eta(k)\|^2\} \le \frac{\nu}{\chi}\|\eta(0)\|^2(1-\psi)^k + \frac{\lambda}{\chi\psi}. \tag{16}$$

III. OBSERVER-BASED H_∞ OUTPUT FEEDBACK CONTROL

Theorem 1: Consider the above closed-loop system (10) with a quantized density $\rho > 0$ and packet-loss rate $\bar{\theta}$. If there exist positive definite matrices $P > 0, Q > 0, Z > 0, R > 0, S > 0, M > 0, N > 0, J > 0$, the control gain K, the observation gain L and a scalar $\varepsilon > 0$ satisfying:

$$\begin{bmatrix} \phi_1 + \delta^2\varepsilon\phi_4^T\phi_4 & \phi_3^T & 0 \\ * & \phi_2 & \phi_6 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \tag{17}$$

$$PM = I, \quad QN = I, \quad JZ = I, \tag{18}$$

where

$$\phi_1 = diag\{-P+(d_2-d_1+1)R+\overline{\mu}S, -Q, -Z, -R, -\frac{S}{\overline{\mu}}\},$$

$$\phi_2 = diag\{-M, -N, -J, -\frac{M}{\bar{\theta}(1-\bar{\theta})}, -\frac{J}{\bar{\theta}(1-\bar{\theta})}\},$$

$$\phi_4 = \begin{bmatrix} H_2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\phi_3 = \begin{bmatrix} A+DK & -DK & 0 & B & C \\ \bar{\theta}LH_2 & A-LH_2 & -\bar{\theta}L & B & C \\ (1-\bar{\theta})H_2 & 0 & \bar{\theta}I & 0 & 0 \\ LH_2 & 0 & -L & 0 & 0 \\ H_2 & 0 & -I & 0 & 0 \end{bmatrix},$$

$$\phi_6^T = \begin{bmatrix} 0 & -(1-\bar{\theta})L^T & (1-\bar{\theta})I & L^T & I \end{bmatrix}.$$

Then, the closed-loop system (10) is exponentially mean-square stable.

Proof: Firstly define the Lyapunov-Krasovskii function as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k) + V_4(k),$$

$$V_1(k) = x^T(k)Px(k) + e^T(k)Qe(k) + y_c^T(k-1)Zy_c(k-1),$$

$$V_2(k) = \sum_{i=k-d(k)}^{k-1} x^T(i)Rx(i),$$

$$V_3(k) = \sum_{j=k-d_2+1}^{k-d_1}\sum_{i=j}^{k-1} x^T(i)Rx(i),$$

$$V_4(k) = \sum_{i=1}^{+\infty}\mu_i\sum_{j=k-i}^{k-1} x^T(j)Sx(j).$$

Thus it has

$$\begin{aligned} E\{\Delta V_1(k)\} &= E\{V_1(k+1)|k\} - V_1(k) \\ &= W_{P_1}^TPW_{P_1} - x^T(k)Px(k) + W_{Q_1}^TQW_{Q_1} \\ &\quad + \bar{\theta}(1-\bar{\theta})W_{Q_2}^TQW_{Q_2} - e^T(k)Qe(k) + W_{Z_1}^TZW_{Z_1} \\ &\quad + \bar{\theta}(1-\bar{\theta})W_{Z_2}^TZW_{Z_2} - y_c^T(k)Zy_c(k), \\ E\{\Delta V_2(k)\} &\le x^T(k)Rx(k) \\ &\quad - x^T(k-d(k))Rx(k-d(k)) \\ &\quad + \sum_{j=k-d_2+1}^{k-d_1} x^T(j)Rx(j), \\ E\{\Delta V_3(k)\} &= (d_2-d_1)x^T(k)Rx(k) \\ &\quad - \sum_{j=k-d_2+1}^{k-d_1} x^T(j)Rx(j), \\ E\{\Delta V_4(k)\} &\le \overline{\mu}x^T(k)Sx(k) \\ &\quad - \frac{1}{\overline{\mu}}(\sum_{i=1}^{+\infty}\mu_ix(k-i))^TS(\sum_{i=1}^{+\infty}\mu_ix(k-i)), \\ E\{\Delta V(k)\} &= E\{\Delta V_1(k)\} + E\{\Delta V_2(k)\} \\ &\quad + E\{\Delta V_3(k)\} + E\{\Delta V_4(k)\} \\ W_{P_1} &= (A+DK)x(k) - DKe(k) + Bx(k-d(k)) \\ &\quad + C\sum_{i=1}^{+\infty}\mu_ix(k-i), \\ W_{Q_1} &= [\bar{\theta}L - L(1-\bar{\theta})\Delta(k)]H_2x(k) + (A-LH_2)e(k) \\ &\quad - L\bar{\theta}y_c(k-1) + Bx(k-d(k)) \\ &\quad + C\sum_{i=1}^{+\infty}\mu_ix(k-i), \\ W_{Q_2} &= [L + L\Delta(k)]H_2x(k) - Ly_c(k-1), \\ W_{Z_1} &= (1-\bar{\theta})[I+\Delta(k)]H_2x(k) + \bar{\theta}y_c(k-1), \\ W_{Z_2} &= [I+\Delta(k)]H_2x(k) - y_c(k-1). \end{aligned} \tag{19}$$

By adding up the above (19) together, it gives

$$E\{\Delta V(k)\} = E\{V(k+1)|k\} - V(k) = \eta^T(k)\varphi_1\eta(k), \tag{20}$$

where

$$\eta(k) = \begin{bmatrix} \zeta^T(k) & x^T(k-d(k)) & \sum_{i=1}^{+\infty}\mu_ix^T(k-i) \end{bmatrix}^T,$$

$$\varphi_1 = \phi_1 - (\phi_3+\phi_5)^T\phi_2^{-1}(\phi_3+\phi_5),$$

$$\phi_5 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ -(1-\bar{\theta})L\Delta(k)H_2 & 0 & 0 & 0 & 0 \\ (1-\bar{\theta})\Delta(k)H_2 & 0 & 0 & 0 & 0 \\ L\Delta(k)H_2 & 0 & 0 & 0 & 0 \\ \Delta(k)H_2 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

If $E\{\Delta V(k)\} < 0$ is satisfied, it means $\varphi_1 < 0$, which is equivalent to the following one

$$\begin{bmatrix} \phi_1 & (\phi_3+\phi_5)^T \\ * & \phi_2 \end{bmatrix} < 0. \tag{21}$$

Moreover, (21) can be decomposed into:

$$\begin{bmatrix} \phi_1 & \phi_3^T \\ * & \phi_2 \end{bmatrix} + \begin{bmatrix} \phi_4^T \\ 0 \end{bmatrix} \Delta(k) \begin{bmatrix} 0 & \phi_6^T \end{bmatrix} + \begin{bmatrix} 0 \\ \phi_6 \end{bmatrix} \Delta(k) \begin{bmatrix} \phi_4 & 0 \end{bmatrix} < 0. \quad (22)$$

Then, according to lemma 2, if inequality (22) holds, the following inequality is satisfied:

$$\begin{bmatrix} \phi_1 & \phi_3^T \\ * & \phi_2 \end{bmatrix} + \delta^2 \varepsilon \begin{bmatrix} \phi_4^T \\ 0 \end{bmatrix} \begin{bmatrix} \phi_4 & 0 \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} 0 \\ \phi_6 \end{bmatrix} \begin{bmatrix} 0 & \phi_6^T \end{bmatrix} < 0, \quad (23)$$

which can be equivalent to (17). It can be seen from (17) that $\varphi_1 < 0$ and hence

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(k+1)|k\} - V(k) = \eta^T(k)\varphi_1\eta(k) \\ &\leq -\lambda_{min}(-\varphi_1)\eta(k)^T\eta(k) < -\alpha\eta(k)^T\eta(k), \end{aligned} \quad (24)$$

where

$$\begin{aligned} &0 < \alpha < \min\{\lambda_{min}(-\varphi_1), \sigma\}, \\ &\sigma = \max\{\lambda_{max}(P), \lambda_{max}(Q), \lambda_{max}(Z), \lambda_{max}(R), \lambda_{max}(S)\}. \end{aligned} \quad (25)$$

On the basis of (24), it gives

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(k+1)|k\} - V(k) \\ &< -\alpha\eta(k)^T\eta(k) < -\frac{\alpha}{\sigma}V(k) = -\psi V(k). \end{aligned} \quad (26)$$

Therefore, it is very evident from lemma 3 that the above system (10) is exponentially mean-square stable, and then the proof is completed. ∎

Remark 1: From the inequality (17), it can see that $\bar{\theta}(1-\bar{\theta})$ cannot be equal to zero. That is, if $\bar{\theta}(1-\bar{\theta}) = 0$ is satisfied, then the inequality (17) is not any meaning at all. However, in this paper the issues of the data packet dropout and the data quantization is all considered, so $\bar{\theta}$ can be not taken as the values of 0 or 1. $\bar{\theta} = 0$ showing that the data packet dropout never happens and $\bar{\theta} = 1$ from (9) showing that the data quantization never happens, indicates that the issues of the data packet dropout and the data quantization never happens at the same time, which violates the research meaning of this paper. Therefore, $\bar{\theta}(1-\bar{\theta}) \neq 0$ is considered in this paper.

Remark 2: Due to the equality (18) in theorem 1, the constraint conditions of the theorem 1 are not strict LMI forms. Then, the cone complement linearization approach can transform the above of the non-convex optimization (17) and (18) for the control deign problem of the systems (10) into a nonlinear minimization problem based on the strict LMI forms.

After the cone complement linearization, the optimization problem of theorem 1 becomes the following one:

$$\min tr(PM + QN + JZ) \quad (27)$$

satisfying the inequality (17) and the following constraints:

$$\begin{bmatrix} P & I \\ I & M \end{bmatrix} \geq 0, \quad \begin{bmatrix} Q & I \\ I & N \end{bmatrix} \geq 0, \quad \begin{bmatrix} J & I \\ I & Z \end{bmatrix} \geq 0. \quad (28)$$

The above optimization problem (30) and (31) can be solved by the iterative approach.

Next, some sufficient conditions for the H_∞ performance constraint of the closed-loop system will be analyzed:

Theorem 2: Consider the above closed-loop system (10) with a quantized density $\rho > 0$ and packet-loss rate $\bar{\theta}$. If there exists positive definite matrices $P > 0, Q > 0, Z > 0, R > 0, S > 0, M > 0, N > 0, J > 0$, the control gain K, the observation gain L and scalars $\varepsilon > 0$, $\gamma > 0$ satisfying:

$$\begin{bmatrix} \lambda_3 + \delta^2\varepsilon\lambda_7^T\lambda_7 & \lambda_4^T & 0 \\ * & \lambda_6 & \lambda_8 \\ * & * & -\varepsilon I \end{bmatrix} < 0, \quad (29)$$

$$PM = I, \quad QN = I, \quad JZ = I, \quad (30)$$

where

$$\begin{aligned} \lambda_1 &= \begin{bmatrix} E_1^T H_1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \lambda_2 &= \begin{bmatrix} H_1 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \lambda_3 &= \begin{bmatrix} \phi_1 & \lambda_1^T \\ * & -\gamma^2 I \end{bmatrix}, \lambda_4^T = \begin{bmatrix} \phi_3^T & 0 & \lambda_2^T & 0 \\ 0 & 0 & 0 & E_1^T \end{bmatrix}, \\ \lambda_5^T &= \begin{bmatrix} \phi_5^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \lambda_6 = diag\{\phi_2, -I, -I, -I\}, \\ \lambda_7 &= \begin{bmatrix} H_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \lambda_8^T &= \begin{bmatrix} 0 & -(1-\bar{\theta})L^T & (1-\bar{\theta})I & L^T & I & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Then the above system (10) is exponentially mean-square stable, and the H_∞ performance constraint can be achieved for all nonzero $w(k)$.

Proof: By theorem 1, inequality (29) contains inequality (17). That is, the closed-loop system (10) is gradually stable in the mean square if the inequality (28) holds.

When $w(k) \neq 0$, let's take the following Lyapunov-Krasovskii function as $V(k) = V_1(k)+V_2(k)+V_3(k)+V_4(k)$, we have

$$\begin{aligned} E\{\Delta V(k)\} &= E\{V(k+1)|k\} - E\{V(k)\} + E\{z^T z(k)\} \\ &\quad -\gamma^2 E\{w^T(k)w(k)\} = \pi^T(k)\varphi_2\pi(k), \end{aligned} \quad (31)$$

where

$$\pi(k) = \begin{bmatrix} \zeta^T(k) & w^T(k) & x^T(k-d(k)) & \sum_{i=1}^{+\infty}\mu_i x_{k-i}^T \end{bmatrix}^T,$$

$$\varphi_2 = \lambda_3 - (\lambda_4 + \lambda_5)^T\lambda_6^{-1}(\lambda_4 + \lambda_5).$$

By Schur complement, $\varphi_2 < 0$ is equivalent to

$$\begin{bmatrix} \lambda_3 & (\lambda_4 + \lambda_5)^T \\ * & \lambda_6 \end{bmatrix} < 0, \quad (32)$$

which can be decomposed into:

$$\begin{bmatrix} \lambda_3 & \lambda_4^T \\ * & \lambda_6 \end{bmatrix} + \begin{bmatrix} \lambda_7^T \\ 0 \end{bmatrix} \Delta(k) \begin{bmatrix} 0 & \lambda_8^T \end{bmatrix} + \begin{bmatrix} 0 \\ \lambda_8 \end{bmatrix} \Delta(k) \begin{bmatrix} \lambda_7 & 0 \end{bmatrix} < 0. \quad (33)$$

On the basis of lemma 2, inequality (33) can be satisfied, if the following inequality holds:

$$\begin{bmatrix} \lambda_3 & \lambda_4^T \\ * & \lambda_6 \end{bmatrix} + \delta^2 \varepsilon \begin{bmatrix} \lambda_7^T \\ 0 \end{bmatrix} \begin{bmatrix} \lambda_7 & 0 \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} 0 \\ \lambda_8 \end{bmatrix} \begin{bmatrix} 0 & \lambda_8^T \end{bmatrix} < 0, \quad (34)$$

which is equivalent to the inequality (29). Therefore, we have from (31) that

$$E\{V(k+1)|k\} - E\{V(k)\} + E\{z(k)^T z(k)\} - \gamma^2 E\{w^T(k)w(k)\} < 0. \quad (35)$$

Summing up (35) from 0 to ∞ leads to

$$\sum_{k=0}^{+\infty} E\{z^T(k)z(k)\} < \gamma^2 \sum_{k=0}^{+\infty} E\{w^T(k)w(k)\} + E\{V(0)\} - E\{V(\infty)\}. \quad (36)$$

Because the closed-loop system (10) is exponentially mean-square stable, it can be easily seen from (36) that

$$\sum_{k=0}^{+\infty} E\{z^T(k)z(k)\} \le \gamma^2 \sum_{k=0}^{+\infty} E\{w^T(k)w(k)\}, \quad (37)$$

which implies the specified H_∞ performance constraint can be achieved, and the proof is then completed. ■

Remark 3: The stability of the networked time-delay system (10) with distributed delays is analyzed in the theorem 2 based on the Lyapunov theory and the Schur complements. Furthermore, the design method of robust H_∞ controller (6) for networked time-delay system (10) with random packet loss and quantization is presented. In a word, the virtue of the presented results depends on the discrete distributed delays, and much work involving the construction of new Lyapunov functions has been used to handle the discrete distributed delays, which is very meaningful for practical NCSs.

Although the above method gives the sufficient conditions for robust H_∞ controller design, the designed parameters such as the observer gain L, the controller gain K and the minimum H_∞ performance bound $\gamma > 0$ are also not be obtained by the LMIs tool, which should be solved by using a cone complementary linearization approach. By the above transformation approach, the minimization optimization problem based on LMI conditions is given:

$$\min tr(PM + QN + JZ) \quad (38)$$

satisfying the inequality (32) and the following constraints:

$$\begin{bmatrix} P & I \\ I & M \end{bmatrix} \ge 0, \quad \begin{bmatrix} Q & I \\ I & N \end{bmatrix} \ge 0, \quad \begin{bmatrix} J & I \\ I & Z \end{bmatrix} \ge 0. \quad (39)$$

IV. NUMERICAL EXAMPLE

The networked system proposed by (1) is as follows, in which the relevant parameters are given as:

$$A = \begin{bmatrix} 0.6 & -0.1 & 0 \\ 0 & -0.8 & 0.5 \\ 0.2 & 0 & -0.7 \end{bmatrix}, \quad B = \begin{bmatrix} 0.2 & -0.1 & 0 \\ 0.1 & -0.1 & 0 \\ 0 & -0.2 & 0.1 \end{bmatrix},$$

$$C = \begin{bmatrix} -0.2 & 0 & 0.1 \\ -0.2 & -0.1 & 0.1 \\ 0 & 0.2 & -0.1 \end{bmatrix},$$

$$D^T = \begin{bmatrix} 1.9 & 0.2 & -3.5 \end{bmatrix},$$

$$E^T = \begin{bmatrix} -0.0034 & -0.000756 & 0.05 \end{bmatrix}, \quad E_1 = 1,$$

$$H_1 = \begin{bmatrix} -2.23 & 5.6 & -0.3 \end{bmatrix}, \quad d(k) = 2 + \frac{1 + (-1)^k}{2},$$

$$H_2 = \begin{bmatrix} -0.23 & -2.7 & 3.9 \end{bmatrix}, \quad \mu_i = 2^{-(i+1)}.$$

Obviously, it obtains $d_1 = 2, d_2 = 3, \overline{\mu} = 0.5$. In this paper, the quantization density is given as $\rho = 0.6$ and the initial state is $x(0) = [0.22 \;\; 0.1 \;\; -0.25]^T$, $\hat{x}(0) = [0 \;\; 0 \;\; 0]^T$ and the disturbance input is expressed by $w(k) = \frac{1}{k}$.

TABLE 1. Different values of designed parameters.

$\bar{\theta}$	$E\{(\theta_k - \bar{\theta})^2\}$	L^T	γ_{min}
0.9	0.009	$[0.0042 \;\; 0.1227 \;\; -0.0835]^T$	0.8572
0.8	0.22	$[0.0051 \;\; 0.2428 \;\; -0.1643]^T$	3.4670
0.6	0.24	$[0.0095 \;\; 0.3243 \;\; -0.2188]^T$	11.2762

Based on the theorem 2, three observer-based H_∞ output feedback controllers (6) for networked system can be designed with the same quantization density $\rho = 0.6$ and three different data packet arrival probabilities $\bar{\theta}$ respectively taken as 0.9, 0.8 and 0.6. For the above given parameters, by solving the optimization problem in the theorem 2, the optimization results for robust H_∞ controllers are shown in Table 1 and the corresponding controller gains are derived as follows

$$K_1 = \begin{bmatrix} -0.0138 & 0.0093 & -0.1112 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} -0.0206 & -0.0158 & -0.0816 \end{bmatrix},$$
$$K_3 = \begin{bmatrix} -0.0210 & -0.0251 & -0.0566 \end{bmatrix}.$$

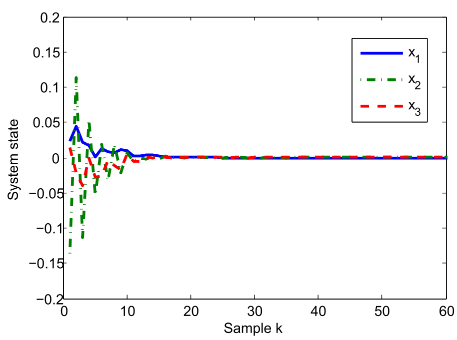

FIGURE 2. States of graph $\gamma_{min} = 0.8572$.

From the above, three sets of different values of the controller gain K, the observer gain L and the minimum H_∞ performance index γ_{min} are obtained respectively. It can be easily seen that the minimum H_∞ performance index γ_{min} increases when data packet arrival probability $\bar{\theta}$ decreases, because of the serious data packet dropout. Figures 1–3 respectively show the dynamical behaviors of the system state for three cases under the same quantization density $\rho = 0.6$. From the

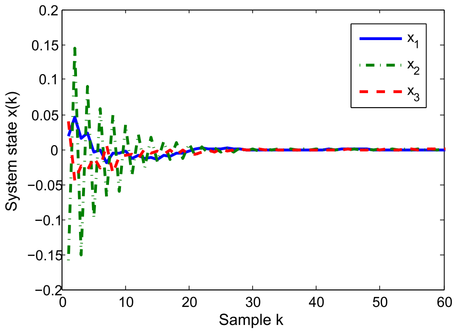

FIGURE 3. State trajectories with $\gamma_{min} = 3.4670$.

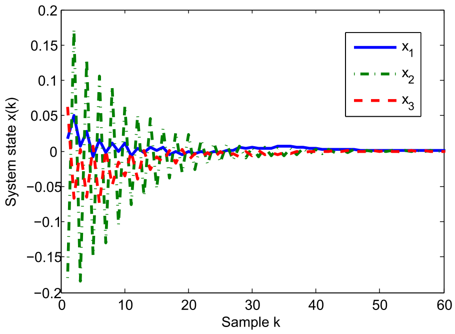

FIGURE 4. State trajectories with $\gamma_{min} = 11.2762$.

Figures 1–3, it can be seen that the state trajectories asymptotically tend to zero, which implies that the networked system can be exponentially mean-square stable by using the proposed robust controller (6). However, when the packet dropout becomes more serious, the state trajectories of the closed-loop networked system can also be close to zero, while it will take a longer time for the system state to tend to zero and the jitters of state variables become more and more serious in Figures 1–3. It indicates that the data packet dropout has a great influence on the robustness of networked system when the system performance inevitably deteriorates, although the proposed controller ensures that the networked system can be exponentially mean-square stable and the prescribed H_∞ performance constraint is achieved.

V. CONCLUSION

This paper has dealt with the robust H_∞ control problem of the networked time-delay systems with quantization and random packet dropout. Relying on the sector bound method to represent the quantization effect and a Bernoulli distribution to characterize the packet dropout phenomenon, the quantization control problem of the networked system with random packet dropout has been transformed into the robust control problem of the networked uncertain time-delay system with a random parameter. To deal with this feedback control problem for the networked system, an observer-based H_∞ output feedback controller has been designed to ensure the exponential stability of the closed-loop system in mean-square sense and achieve the expected H_∞ performance index. Finally, a numerical simulation has verified the effectiveness of the proposed method.

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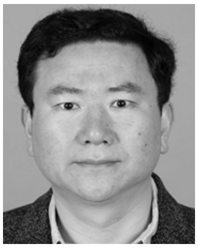

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